Basic Properties and Facts

Arithmetic Operations

$$ab + ac = a(b + c)$$
 $a\left(\frac{b}{c}\right) = \frac{ab}{c}$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$
 $\frac{a}{\left(\frac{b}{c}\right)} = \frac{a}{b}$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd} \qquad \qquad \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

$$\frac{a-b}{c-d} = \frac{b-a}{d-c} \qquad \qquad \frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{ab+ac}{a} = b+c, \ a \neq 0 \quad \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}$$

Exponent Properties

$$a^n a^m = a^{n+m} (ab)^n = a^n b^n$$

$$(a^n)^m = a^{nm}$$
 $a^0 = 1, a \neq 0$

$$\frac{a^n}{a^m} = a^{n-m} = \frac{1}{a^{m-n}} \qquad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$a^{\frac{n}{m}} = \left(a^{\frac{1}{m}}\right)^n = (a^n)^{\frac{1}{m}} \quad \frac{1}{a^{-n}} = a^n$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n} \quad a^{-n} = \frac{1}{a^n}$$

Properties of Radicals

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad \qquad \sqrt[n]{ab} = \sqrt[n]{a} \sqrt[n]{b}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^n} = a$$
 if n is odd

$$\sqrt[n]{a^n} = |a| \text{ if } n \text{ is even}$$

Properties of Inequalities

If
$$a < b$$
 then $a + c < b + c$ and $a - c < b - c$

If
$$a < b$$
 and $c > 0$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

If
$$a < b$$
 and $c < 0$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

Properties of Absolute Value

$$|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a & \text{if } a < 0 \end{cases}$$

$$|a| \ge 0 \qquad \qquad |-a| = |a|$$

$$|ab| = |a| |b|$$
 $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$

$$|a+b| \le |a| + |b|$$
 Triangle Inequality

Distance Formula

If $P_1=(x_1,y_1)$ and $P_2=(x_2,y_2)$ are two points the distance between them is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Complex Numbers

$$i=\sqrt[3]{-1}$$
 $i^2=-1$ $\sqrt{-a}=i\sqrt{a}$, $a\geq 0$

$$(a + bi) + (c + di) = a + c + (b + d)i$$

$$(a + bi) - (c + di) = a - c + (b - d)i$$

$$(a+bi)(c+di) = ac - bd + (ad+bc)i$$

$$(a + bi) (a - bi) = a^2 + b^2$$

$$|a+bi|=\sqrt{a^2+b^2}$$
 Complex Modulus

$$\overline{(a+bi)}=a-bi$$
 Complex Conjugate

$$\overline{(a+bi)}(a+bi) = |a+bi|^2$$

Logarithms and Log Properties

Definition

$$y = \log_b(x)$$
 is equivalent to $x = b^y$

Example

$$log_5(125) = 3$$
 because $5^3 = 125$

Special Logarithms

$$ln(x) = log_e(x)$$
 natural log

$$\log(x) = \log_{10}(x)$$
 common log

where
$$e = 2.718281828...$$

Logarithm Properties

$$\log_b(b) = 1 \qquad \qquad \log_b(1) = 0$$

$$\log_b(b^x) = x \qquad \qquad b^{\log_b(x)} = x$$

$$\log_b(x^r) = r \log_b(x)$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$$

The domain of $\log_b(x)$ is x > 0

Factoring and Solving

Factoring Formulas

$$x^{2} - a^{2} = (x + a)(x - a)$$

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^{2} + (a + b) x + ab = (x + a) (x + b)$$

$$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$$

$$x^{3} - 3ax^{2} + 3a^{2}x - a^{3} = (x - a)^{3}$$

$$x^{3} + a^{3} = (x+a)(x^{2} - ax + a^{2})$$

$$x^{3} - a^{3} = (x - a)(x^{2} + ax + a^{2})$$

$$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$$

If n is odd then.

$$x^{n} - a^{n} = (x - a) (x^{n-1} + ax^{n-2} + \dots + a^{n-1})$$

$$(x+a)(x^{n-1}-ax^{n-2}+a^2x^{n-3}-\cdots+a^{n-1})$$

Quadratic Formula

Solve
$$ax^2 + bx + c = 0$$
, $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 - 4ac > 0$ – Two real unequal solns.

If
$$b^2 - 4ac = 0$$
 – Repeated real solution.

If
$$b^2 - 4ac < 0$$
 – Two complex solutions.

Square Root Property

If
$$x^2 = p$$
 then $x = \pm \sqrt{p}$

Absolute Value Equations/Inequalities

If *b* is a positive number

$$|p| = b \Rightarrow p = -b \text{ or } p = b$$

$$|p| < b \implies -b < p < b$$

$$|p|>b \quad \Rightarrow \quad p<-b \quad \text{or} \quad p>b$$

Completing the Square

Solve
$$2x^2 - 6x - 10 = 0$$

(1) Divide by the coefficient of the x^2

$$x^2 - 3x - 5 = 0$$

(2) Move the constant to the other side.

$$x^2 - 3x = 5$$

(3) Take half the coefficient of \boldsymbol{x} , square it and add it to both sides

$$x^{2} - 3x + \left(-\frac{3}{2}\right)^{2} = 5 + \left(-\frac{3}{2}\right)^{2} = 5 + \frac{9}{4} = \frac{29}{4}$$

(4) Factor the left side

$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

(5) Use Square Root Property

$$x - \frac{3}{2} = \pm \sqrt{\frac{29}{4}} = \pm \frac{\sqrt{29}}{2}$$

(6) Solve for x

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

Functions and Graphs

Constant Function

$$y = a$$
 or $f(x) = a$

Graph is a horizontal line passing through the The graph is a parabola that opens right if point (0, a).

Line/Linear Function

$$y = mx + b$$

$$y = mx + b$$
 or $f(x) = mx + b$

Graph is a line with point (0, b) and slope m.

Slope

Slope of the line containing the two points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\mathsf{rise}}{\mathsf{run}}$$

Slope – intercept form

The equation of the line with slope m and y-intercept (0, b) is

$$y = mx + b$$

Point - Slope form

The equation of the line with slope m and passing through the point (x_1, y_1) is

$$y = y_1 + m\left(x - x_1\right)$$

Parabola/Quadratic Function

$$y = a \left(x - h \right)^2 + k$$

$$f(x) = a(x-h)^2 + k$$

The graph is a parabola that opens up if a > 0or down if a < 0 and has a vertex at (h, k).

Parabola/Quadratic Function

$$y = ax^2 + bx + c$$

$$f\left(x\right) = ax^2 + bx + c$$

The graph is a parabola that opens up if a > 0or down if a < 0 and has a vertex at

$$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$$
.

Parabola/Quadratic Function

$$x = ay^2 + by + c$$

$$g(y) = ay^2 + by + c$$

a>0 or left if a<0 and has a vertex at

$$\left(g\left(-\frac{b}{2a}\right), -\frac{b}{2a}\right).$$

$$(x - h)^2 + (y - k)^2 = r^2$$

Graph is a circle with radius r and center (h, k).

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

Graph is an ellipse with center (h, k) with vertices a units right/left from the center and vertices b units up/down from the center.

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Graph is a hyperbola that opens left and right, has a center at (h, k), vertices a units left/right of center and asymptotes that pass through center with slope $\pm \frac{b}{-}$.

Hyperbola

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Graph is a hyperbola that opens up and down, has a center at (h, k), vertices b units up/down from the center and asymptotes that pass through center with slope $\pm \frac{b}{2}$.

Common Algebraic Errors

| 2 2 | Reason/Correct/Justification/Example |
|---|--|
| $\frac{2}{0} eq 0$ and $\frac{2}{0} eq 2$ | Division by zero is undefined! |
| $-3^2 \neq 9$ | $-3^2 = -9$, $(-3)^2 = 9$ Watch parenthesis! |
| $(x^2)^3 \neq x^5$ | $(x^2)^3 = x^2 x^2 x^2 = x^6$ |
| $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$ | $\frac{1}{2} = \frac{1}{1+1} \neq \frac{1}{1} + \frac{1}{1} = 2$ |
| $\frac{1}{x^2 + x^3} \neq x^{-2} + x^{-3}$ | A more complex version of the previous error. |
| $\frac{\cancel{d} + bx}{\cancel{d}} \neq 1 + bx$ | $\frac{a+bx}{a} = \frac{a}{a} + \frac{bx}{a} = 1 + \frac{bx}{a}$ Beware of incorrect canceling! |
| $-a(x-1) \neq -ax - a$ | $-a\left(x-1\right) =-ax+a$ Make sure you distribute the "-"! |
| $(x+a)^2 \neq x^2 + a^2$ | $(x+a)^2 = (x+a)(x+a) = x^2 + 2ax + a^2$ |
| $\sqrt{x^2 + a^2} \neq x + a$ | $5 = \sqrt{25} = \sqrt{3^2 + 4^2} \neq \sqrt{3^2} + \sqrt{4^2} = 3 + 4 = 7$ |
| $\sqrt{x+a} \neq \sqrt{x} + \sqrt{a}$ | See previous error. |
| $(x+a)^n \neq x^n + a^n$ and $\sqrt[n]{x+a} \neq \sqrt[n]{x} + \sqrt[n]{a}$ | More general versions of previous three errors. |
| $2(x+1)^2 \neq (2x+2)^2$ | $2(x+1)^2 = 2(x^2+2x+1) = 2x^2+4x+2$ $(2x+2)^2 = 4x^2+8x+4$ Square first then distribute! |
| $(2x+2)^2 \neq 2(x+1)^2$ | See the previous example. You can not factor out a constant if there is a power on the parenthesis! |
| $\sqrt{-x^2 + a^2} \neq -\sqrt{x^2 + a^2}$ | $\sqrt{-x^2+a^2}=\left(-x^2+a^2\right)^{\frac{1}{2}}$ Now see the previous error. |
| $\frac{a}{\left(\frac{b}{c}\right)} \neq \frac{ab}{c}$ | $\frac{a}{\binom{b}{c}} = \frac{\binom{a}{1}}{\binom{b}{c}} = \binom{a}{1} \binom{c}{b} = \frac{ac}{b}$ |
| $\frac{\left(\frac{a}{b}\right)}{c} \neq \frac{ac}{b}$ | $\frac{\left(\frac{a}{b}\right)}{c} = \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{1}\right)} = \left(\frac{a}{b}\right)\left(\frac{1}{c}\right) = \frac{a}{bc}$ |