MAT1856/APM466 Assignment 1

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Fundamental Questions - 25 points

1.

   (a) Printing more money promotes faster inflation whereas issuing bonds does not, with the latter being better for society.

   (b) If the government plans to implement policies to slow down inflation, the long term part of a yield curve will flatten out.

   (c) The US Fed used quantitative easing, a bond buying program that helps stimulate the economy by increasing cash flow, since cash flow during COVID drastically decreased. [1]

2. The bonds we will use for constructing the curves will be

   - CAN 0.50 Mar 1, 2022 CAN 0.25 Nov 11, 2022 CAN 0.25 Apr 30, 2023
   - CAN 0.50 Oct 31, 2023 CAN 0.25 Apr 1, 2024 CAN 0.75 Sep 30, 2024
   - CAN 1.25 Feb 28, 2025 CAN 0.50 Aug 31, 2025 CAN 0.25 Feb 28, 2026
   - CAN 1.00 Sep 1, 2026 CAN 1.25 Mar 1, 2027

   We selected these bonds since to get a yield curve from 0-5 years we want bonds with maturity dates spaced out through this time frame. We also want the coupon values to be close. Finally, we want the issue dates to be close as well for the most accuracy. All of these bonds were issued within the last 3 years (except the first issues in 2016) making these bonds a good choice for constructing the curves.

3. The eigenvectors represent directions that the stochastic curve take. The eigenvalues represent how important that direction is to the movement of the stochastic curve. A larger eigenvalue means that the eigenvector associated to it is a better predictor of overall movement in the stochastic curve. Typically, the two largest eigenvalues and the associated eigenvectors are most looked at as they are able to describe most of the movement.

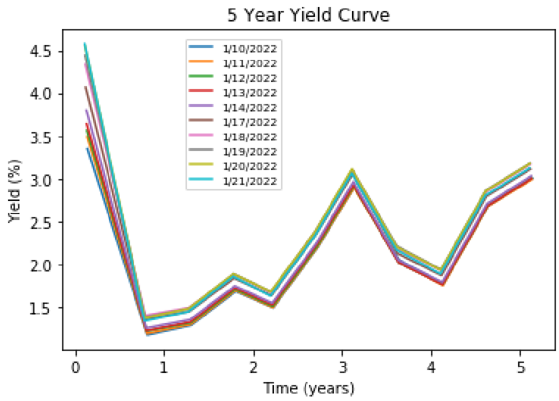
Empirical Questions - 75 points

4.

   (a) Let r be the ytm. We can write the following equation involving ytm:

$$P = \sum_i p_i e^{-rt_i}$$

   From the lecture slides. The p_i will be the coupon payments except the last one which also include the full value of 100 and the t_i are the time till payments. The P is the price obtained from our bond data. We can solve this numerically using binary search since the expression on the right side decreases as r increases so there will be a unique solution. Next, since we picked maturity dates equally spaced out throughout the five years, between points the curve will be approximately linear. Thus, it makes sense to use linear interpolation. If (d_1, r_1) and (d_2, r_2) are consecutive points on the curve with d_i being maturity dates and r_i being ytm, then we just draw the line between them. After doing everything above, we obtain the plot below.



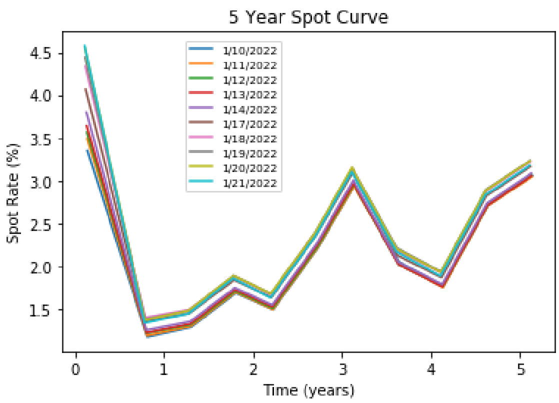
(b) We can derive the spot rate using a bootstrapping method. [2] The spot rate s_1 for zero-coupon bonds is simple to calculate. We can calculate further spot rates using bootstrapping. For example, the spot rate for bonds with one coupon, call it s_2, will satisfy:

$$\text{Price} = \frac{\text{Coupon}}{e^{s_1 t_1}} + \frac{\text{Coupon} + 100}{e^{s_2 t_2}}$$

Where the coupon comes from the bond data and the t_i are time till payments. Since we are bootstrapping, s_1 is known so this just needs to be solved for s_2. This can be done with a binary search to find s_2 since this equation will have a unique solution. Further formulas will look like:

$$\text{Price} = \sum_i \frac{\text{Coupon}}{e^{s_i t_i}} + \frac{100}{e^{s_n t_n}}$$

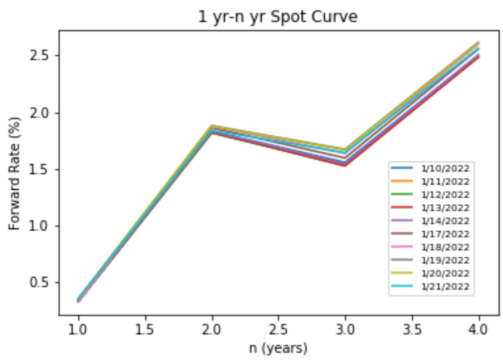
Where n is the number of coupon payments. We use continuous compounding since it makes the equation easier to write (and works as an approximation). Again, this can be solved using binary search since only s_n is unknown and the solution will be unique. We do this until we reach 5 years, so we will need to write an equation for each bond. Then we just use linear interpolation to fill in since that is what we used for the yield curve. Doing all of this gives the following plot.



(c) Let r be the forward rate. We use the following formula to calculate forward rates:

$$e^{r_2 t_2} = e^{r_1 t_1} e^{r(t_2 - t_1)} \implies r = \frac{r_2 t_2 - r_1 t_1}{t_2 - t_1}$$

What this equation means is that we take the spot rate r_1 for the first time t_1 and the spot rate r_2 for the later time t_2. Then we solve for r the forward rate. We obtain this formula by using continuous compounding, adapted from investopedia. [3] In our case, $t_1 = 1$ since we are looking at 1 year to n years. Then we just plug in the correct values to calculate each of the forward rates. For 1yr-1yr, we would take $t_2 = 2$ and the corresponding spot rate. For 1yr-2yr, it would be $t_2 = 3$ and so on. We also do this for each day and we obtain the following plot.



5. We obtain the following covariance matrices for daily log returns of yield and forward rates. The 5x5 matrix is for yield while the 4x4 is forward rates. The number in front of each matrix is a scalar.

$$10^{-4}\begin{pmatrix} 8.43 & 6.30 & 3.91 & 4.58 & 3.49 \\ 6.30 & 5.02 & 3.05 & 3.80 & 2.76 \\ 3.91 & 3.05 & 2.02 & 2.37 & 1.81 \\ 4.58 & 3.80 & 2.37 & 3.40 & 2.36 \\ 3.49 & 2.76 & 1.81 & 2.36 & 1.76 \end{pmatrix} \qquad 10^{-4}\begin{pmatrix} 8.85 & 0.22 & 1.47 & 0.07 \\ 0.22 & 0.94 & 1.31 & 1.01 \\ 1.47 & 1.31 & 2.96 & 1.81 \\ 0.07 & 1.01 & 1.81 & 1.31 \end{pmatrix}$$

6. We have the following eigenvalues and eigenvectors for the matrices above in order from largest to smallest eigenvalue.

Yield Eigenvalue	Yield Eigenvector	Forward Eigenvalue	Forward Eigenvector
$1.95 \cdot 10^{-3}$	[6.45, 5.02, 3.13, 3.88, 2.88]	$9.27 \cdot 10^{-4}$	[9.59, 0.75, 2.61, 0.77]
$7.85 \cdot 10^{-5}$	[5.89, 0.29, -0.43, -7.35, -3.31]	$4.48 \cdot 10^{-4}$	[2.65, -3.94, -7.07, -5.24]
$1.67 \cdot 10^{-5}$	[0.94, -6.02, 5.62, -2.14, 5.16]	$2.79 \cdot 10^{-5}$	[-0.72, -8.12, 5.53, -1.71]
$1.10 \cdot 10^{-5}$	[-4.64, 5.37, 6.08, -3.45, -0.92]	$2.80 \cdot 10^{-6}$	[0.63, -4.23, -3.56, 8.31]
$1.63 \cdot 10^{-6}$	[-1.13, 3.10, -4.64, -3.79, 7.30]		

The largest eigenvector corresponds to the main movement of the variables. From the table above, the largest eigenvectors all have positive entries which means that the variables move up and down together. This makes sense in the bond market since if one bond is going up, others will tend to go up as well. The largest eigenvalue represents how strong this movement is. Both $1.95 \cdot 10^{-3}$ and $9.27 \cdot 10^{-4}$ are not big numbers, indicating that the movement from day to day is not that much. This also makes sense since from the bond values often only changed by a few cents daily, so yield and forward rates did not differ much day to day.

GitHub Link to Code

`https://github.com/danezhou/MAT1856HW1`

References

[1] "U.S. Fed Plans to Slowly Start Making Borrowing More Expensive - but Rate Hikes Still a Ways Away — CBC News." CBCnews, CBC/Radio Canada, 3 Nov. 2021, `https://www.cbc.ca/news/business/federal-reserve-qe-bonds-1.6235777`.

[2] Hayes, Adam. "Understanding the Spot Rate Treasury Curve." Investopedia, 19 May 2021, `https://www.investopedia.com/terms/s/spot_rate_yield_curve.asp`.

[3] Ross, Sean. "The Formula for Converting Spot Rate to Forward Rate." Investopedia. `https://www.investopedia.com/ask/answers/043015/how-do-i-convert-spot-rate-forward-rate.asp`.

[4] "Markets Insider: Stock Market News, Realtime Quotes and Charts." Business Insider, `https://markets.businessinsider.com`.