

Title: Transit Timing and Duration Variations for the Discovery and Characterization of Exoplanets

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Abstract Transiting exoplanets in multi-planet systems have non-Keplerian orbits which can cause the times and durations of transits to vary. We review the theory and observations of transit timing variations (TTV) and transit duration variations (TDV).

Introduction

Here we discuss some aspects of planetary orbital physics, to set the stage for TTV and TDV. Consider the vector stretching from the star of mass M_* to the planet of mass M_p to be $\mathbf{r} = (x, y, z)$, with a distance r and direction $\hat{\mathbf{r}}$. The Keplerian potential, $\phi = -GM/r$ (where $M \equiv M_*M_p/(M_* + M_p)$ is called the reduced mass), is one of only two radial, power-law potentials that gives rise to closed orbits¹. This means that, in the absense of perturbations, there is a strict periodicity $\mathbf{r}(t + P) = \mathbf{r}(t)$. Moreover, Kepler showed that Tycho Brahe's excellent data for planetary positions were consistent with Copernicus' idea of a heliocentric system only if the planets (including the Earth) followed elliptical paths of semi-major axis a , and one focus on the Sun. Newton was successful at finding the principle underlying such orbits, a force law $\ddot{\mathbf{r}} = -GMr^{-2}\hat{\mathbf{r}}$, which results in a period $P = 2\pi a^{3/2}G^{-1/2}(M_* + M_p)^{-1/2}$ (i.e. with the a -scaling Kepler found the planets actually obeyed).

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¹ the other one, the harmonic potential $-kr$ would only have relevance for collisionless orbits within a homogeneous massive body

This research program was thrown into some doubt by the “Great Inequality,” the fact that the orbits of Jupiter and Saturn did not fit the fixed Keplerian ellipse model. This was overcome by the perturbation theory of Lagrange, which resulted in the first characterization of the masses of those planets (check). We can recreate the main effect of this insight by writing an additional force to that of gravity of the Sun:

$$\mathbf{F}_1 = -GMr_1^{-2}\hat{\mathbf{r}}_1 - \mathbf{F}_{21}, \quad (1)$$

where we now specify forces and distances explicitly to planet 1, and add a force of planet 2 on planet 1. This latter force consists of two terms: **I’m not sure the following equation is correct as stands - if $r_1 < r_2$, then first term would be positive**

$$\mathbf{F}_{21} = -GM(r_1 - r_2)^{-3}(\mathbf{r}_2 - \mathbf{r}_1) + GM_2r_2^{-2}\hat{\mathbf{r}}_2. \quad (2)$$

The first term on the right-hand-side is the direct gravitational acceleration of planet 1 due to planet 2. The second is a frame-acceleration effect, due to the acceleration the Sun feels due to the second planet. Since the Sun is fixed at the zero of the frame, this acceleration is manifested by acceleration in the opposite direction of planet 1. Can we average the force over the 5:2 resonant conjunction timescale, and see what it amounts to for each of the orbital elements?

Likewise, Leverrier and Adams used the same technique, dynamical perturbations, to discover the first planet by gravitational means. In this case, they did not know the zeroth order solution (i.e. the Keplerian ellipse) for the yet-to-be-discovered Neptune. In its place, they assumed the Titius-Bode rule held, and sought only the phase of the orbit. This worked because they only wanted to see how the acceleration, then deceleration, as Uranus passed Neptune, would betray its position on the sky to optical observers. [Say later: the task that researchers set for themselves to discover planets by TTV is a bit more demanding. We do not have any hints as to what the planet’s orbit might be (neither circular nor roughly obeying some spacing law). Additionally, the data per orbit is considerably noisy; in only a few cases are the orbit-by-orbit “chopping” signal statistically significant after just three transits. Finally, the orbit is only sampled at the transit phase, so opportunities for aliasing of the signal are abundant.]

The discovery of transits marks the first time that data on exoplanets could be precise enough to notice gravitational interactions.²

- Definition of TTVs/TDVs [DF] (Figure? O-C method) (??) - History: theory, observation (Schneider TTV, Miralda-Escude’ TTV) [EA] (??)

² Only around the same time (2000) were perturbations noticed in the resonant interaction of the planets of GJ 876.

Preliminaries

Since the gravitational interactions between planets occurs on the orbital timescale, the amplitude of transit timing variations is proportional to the orbital period of each planet, as well as a function of other dimensionless quantities. Thanks to Newton's second law and Newton's law of gravity, the acceleration of a body does not depend on its own mass. Thus, the transit timing variations of each planet scale with the masses of the *other* bodies in the system. In a two-planet system, then, to lowest order in mass ratio,

$$\begin{aligned}\delta t_1 &\propto P_1 \frac{m_2}{m_0} f_1(), \\ \delta t_2 &\propto P_2 \frac{m_1}{m_0} f_2(),\end{aligned}\tag{3}$$

where the masses of the star and planets are m_0, m_1 , and m_2 , and f_1 and f_2 are a function of the period ratio and the angular orbital elements of the planets.

- Energy/angular momentum conservation [DF]
- Linear TTV (independently adds from different planets, off resonance) - - Applications: [EA] - Detection - Confirmation - Characterization - Sensitive to density since time-dependent: transits are sensitive to density of star; TTV are sensitive to mass ratio; transit depth radius ratio - so we get density of planet from transits + TTV. Dimensions of G are density and time. - TTV + RV gives Mass + radius ; CBPs as example

Theory

- TTVs: - Inner Keplerian variation; CBPs as example (Kepler-16) [DF] - Near-resonant TTVs - Lithwick et al. [DF] (Figure - mechanism + data) - Degeneracy - multiple resonances can give same solution (Kepler-19); Breaking degeneracy with TDV as well [DF] - Chopping/other harmonics - KOI 1353 / KOI-872 [EA] (Figure) - Resonance - Kepler-30? Ne'svorny (1603.07306); Boue'+2012 - Kepler-223 (resonant chain - to fit data & stability); room for more work on this. [DF] - Exomoons [EA] - Light time? Borkovits deconvolution [DF] - Borkovits(?) - KOI 1474 cleaner example? Or leave out? Future - circumstellar planets in binaries; Schwartz et al. w/ Haghighipour. [DF] - TDVs - Precession - Kepler-108 1606.04485 / KOI-142 Nesvorny / KOI-13 (Mazeh) - and CBPs turning on or off. [DF] Ragozzine/Wolf/Pal/Koscis/Jordan - GR precession - Heyl & Gladman; J2 (Figure - CBP? - Kepler-47? Kostov? Kepler-35? Try them out.) - Exomoons [EA]

Observations/Practical considerations

Confirmation of multi-planet systems in Kepler anti-correlated sinusoids, Ford GPs [DF] [Some firsts to history section; some best-cases as examples in theory section] - Timing precision: [EA] - Comes from steepest part of lightcurve ingress/egress - Signal-to-noise of TTV/TDV measurements (Carter/Winn; Rogers/Page) - Finite-exposure time effects - Effects of stellar variability: flux variability, star spots.

Science Results

- Best characterization, specifically mass: Kepler-36 - conjunctions/impulse/Hill approximation (N-body) [EA] - Other favorite systems? Kepler-11 puffy/packed planets [DF] - Best eccentricity constraint for a super-Earth? Kepler-36? Include? - Ensemble TTV analysis: Xie - differing architecture for the single-transiters due to less frequent TTV, Hadden-Lithwick - eccentricity distribution; Hot Jupiters lonely (Steffen); Latham - gas giants less frequent in multi-transiting (no TTVs) [DF] - Measuring masses - Steffen bias? [DF] - N-body modeling of Kepler-systems: Jontof-Hutter [DF] (Mass-radius Figure? - ask Daniel Jontof-Hutter) Transparency to avoid big error bars visually dominating. EA will make the figure. Referenced Wayne Hu figure on cosmo constraints. - CBPs [DF]

Future

- More thorough TTV analysis: GPs - for measuring transit times - Follow-up of Kepler targets - Comparison of TTV masses with RV masses: better constraints and confidence in both methods? - MCMC with high-multiplicity systems - TESS, JWST, CHEOPS, PLATO, ? - TTV/TDV of exomoons - HZ exoplanets - Smaller CBPs - Stellar/planet characterization: TTV + RV

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Neptune: Bouvard/Adams/Le Verrier/Galle

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