Describing phenology with beta, beta-binomial, and beta-hypergeometric distributions

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Characterizing phenology

CONCEPTS & SYNTHESIS

EMPHASIZING NEW IDEAS TO STIMULATE RESEARCH IN ECOLOGY

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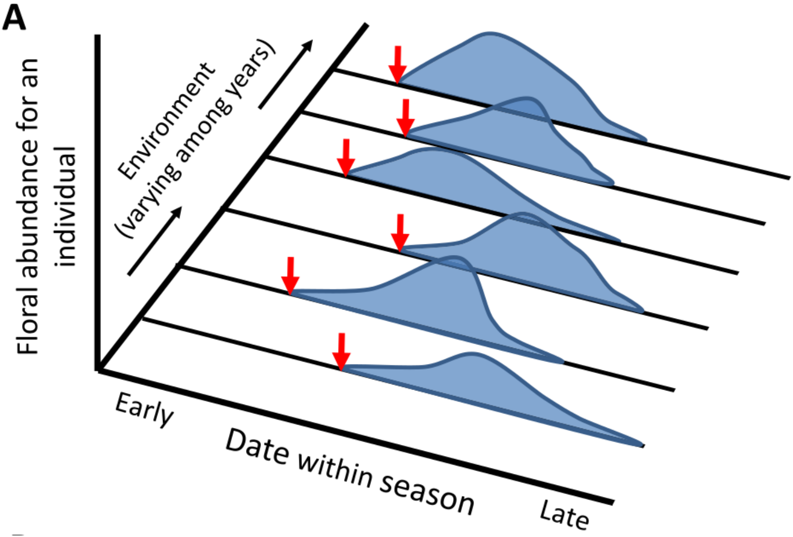
Phenology as a process rather than an event: from individual reaction norms to community metrics

Brian D. Inouye,[1,2,3,5] Johan Ehrlén,[2,4] and Nora Underwood[1,2,3]

- Phenology is usually characterized with discrete measures such as *first flowering*
- This approach can obscure many important features
- More useful to describing phenology as a distribution

Phenology as a distribution



Source: *Inouye et al. 2019*

Advantages of phenology as a distribution

- More comprehensive (accurate?) description
- Use more of the available data/information
- Fit the distribution to data
- Parameters of the distribution will (hopefully) be interpretable
- Given a distribution, we can estimate measures like mean, variance, correlation
- May allow comparisons over time/environments

Objective

Find an appropriate, general, and above all interpretable distribution for describing phenology

Where to start

- With the available data, of course

  e.g., percent budburst over time

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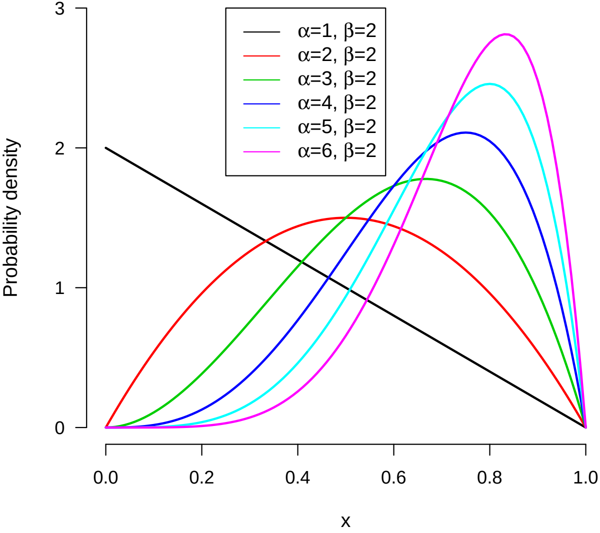
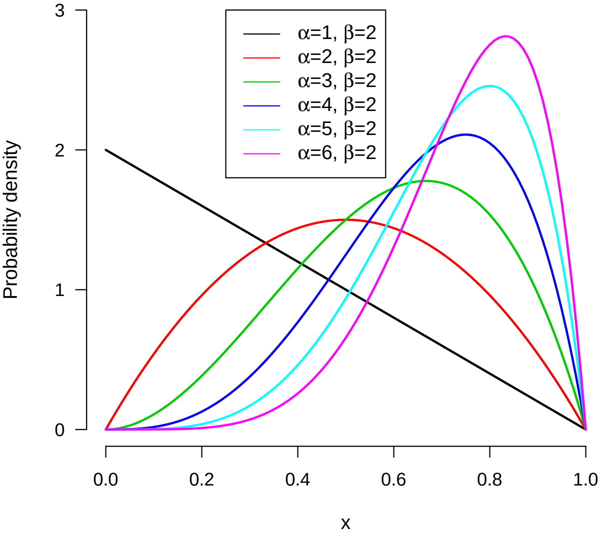
- *Beta distribution*

Beta distribution



Beta distribution



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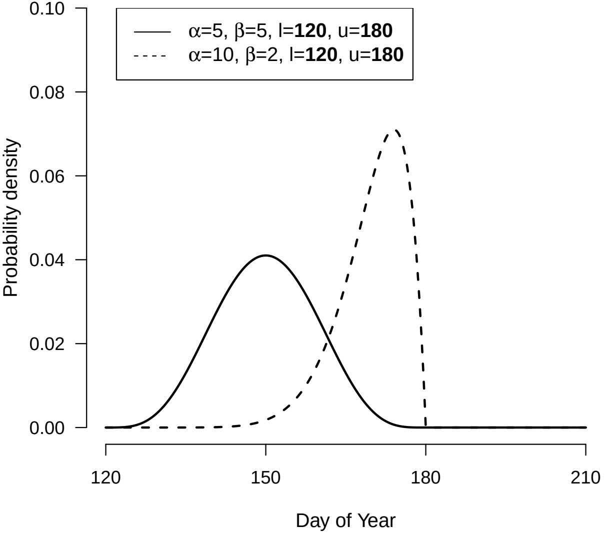
- **Problem 2: Area under the curve is 1**

Beta distribution (non-standardized)



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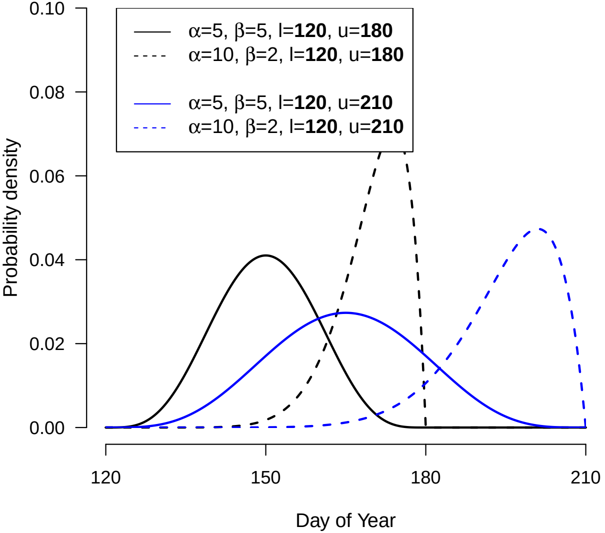

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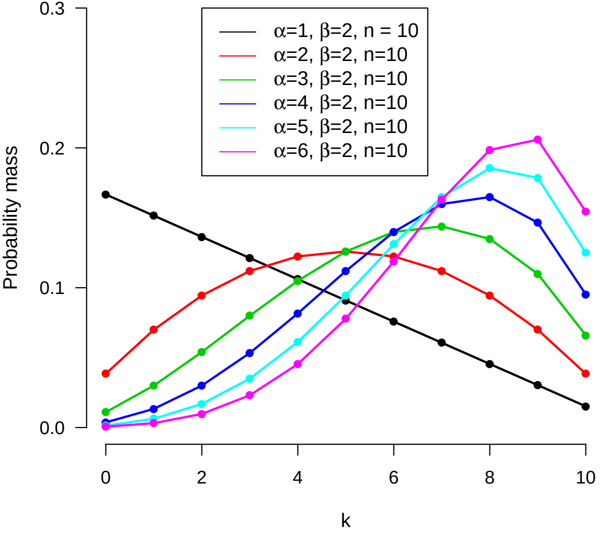
- **Problem 1: *x* ranges between 0 and 1**

  Solved!
- **Problem 2: Area under the curve is 1**
- **Problem 3: Assumes data are continuous**

  Hmmm...

Beta-binomial distribution



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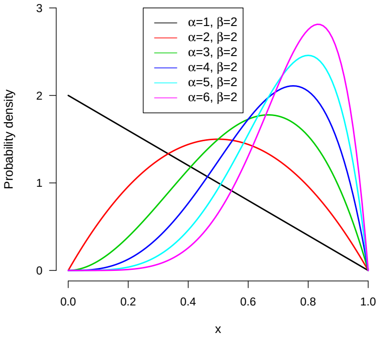

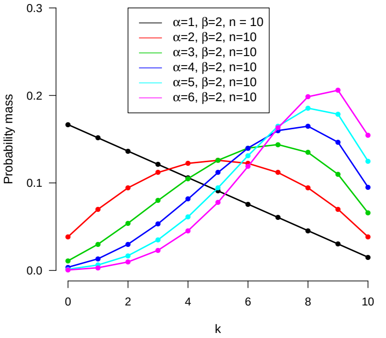

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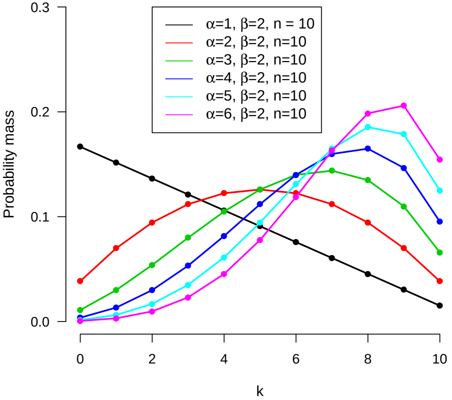

- Compound distribution
- Binomial random variable, $B(n, p)$, p is beta-distributed

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- *Problem: This uses the standardized beta distribution (range 0 to 1)*

- **We probably want the beta-hypergeometric (which doesn't exist yet, but soon will!)**