

1 Inter

We have ellipses E_0, E_1, E_2 , where E_0 is centered at $(0, 0)$. Let the intersection points between E_0 and E_i be r_i , we want $r_1 = r_2$.

Let h_i, k_i be the center of E_i .

we have the r_i

$$y = \frac{h_1 h_2^2 - h_2 h_1^2 + q(h_1 k_2^2 - h_2 k_1^2)}{2q(h_1 k_2 - h_2 k_1)} \quad (1)$$

$$x_i = y_i \frac{-2qk_i}{2h_i} + \frac{h_i^2 + qk_i^2}{2h_i} \quad (2)$$

Also, let's already use the rotation:

$$h_i = \cos \theta x_i - \sin \theta y_i, k_i = \sin \theta x_i + \cos \theta y_i \quad (3)$$

Using the fact that $h_i^2 + k_i^2 = a^2 + b^2$ and simplifying (with wolfram) the denominator we have, we have:

$$y = \frac{(q-1)(h_2^2 h_1 - h_1^2 h_2) + q(a^2 + b^2)(h_1 - h_2)}{2q(x_1 y_2 - x_2 y_1)} =$$

$$y = \frac{(h_2 - h_1)((q-1)h_2 h_1 - q(a^2 + b^2))}{2q(x_1 y_2 - x_2 y_1)}$$