

Planar Maximal Covering with Ellipses

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- Covering problems
 - Set Cover Problem
 - Maximal Covering Problem
- Maximal Covering Location Problem (MCLP)
- Planar Maximal Covering Location Problem (PMCLP)
 - One disk: $\mathcal{O}(n^2)$ and $\mathcal{O}(n^2 \log n)$ algorithms
 - m disks: $\mathcal{O}(n^{2m-1} \log n)$ algorithm
- Goals
 - Develop a $\mathcal{O}(n^2 \log n)$ algorithm for the one disk case
 - Adapt it for the m ellipses case

Norms

Let $u \in \mathbb{R}^2$ and Q a 2×2 positive definite matrix

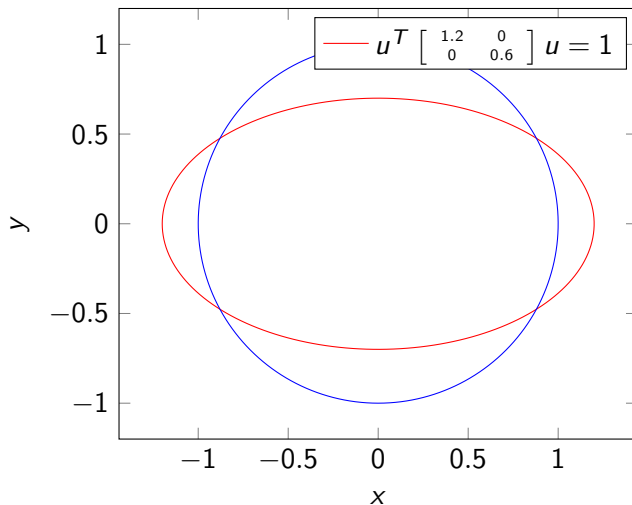
- Euclidean

$$\|u\|_2 = \sqrt{u^T u}$$

- Elliptical

$$\|u\|_Q = \sqrt{u^T Q u}$$

Figure: The elliptical and euclidean norms.



Ellipse

Given a center $c \in \mathbb{R}^2$ and a 2×2 p.d. matrix Q , an ellipse is the set of points that satisfy

$$\|u - c\|_Q = 1,$$

with \leq representing the set of covered points

Axis-parallel ellipse

Any 2 by 2 diagonal d.p. matrix determines an axis-parallel ellipse, which can also be described by

$$\frac{(x - c_x)^2}{a^2} + \frac{(y - c_y)^2}{b^2} = 1,$$

where (a, b) are the shape parameters and $c = (c_x, c_y)$ is the center.

Maximal Covering by Disks

One disk

$MCD(\mathcal{P}, 1)$ is the problem of placing one disk on the plane to cover a subset of a demand set \mathcal{P} maximizing the weights of the covered points.

$$\max_q w(\mathcal{P} \cap D(q)),$$

- $\mathcal{P} = \{p_1, \dots, p_n\}$ is the demand set with weights $w(p_i) > 0$
- $w(A)$, $A \subset \mathcal{P}$, is the sum of weights of the points in A
- $D(q)$ is a unit disk with center at point q
- We will introduce an equivalent problem...

Maximum Weight Clique Problem

Let $\mathcal{D} = \{D_1, \dots, D_n\}$ be a set of n unit disks with weights $w_i > 0$. The maximum weight clique is defined as

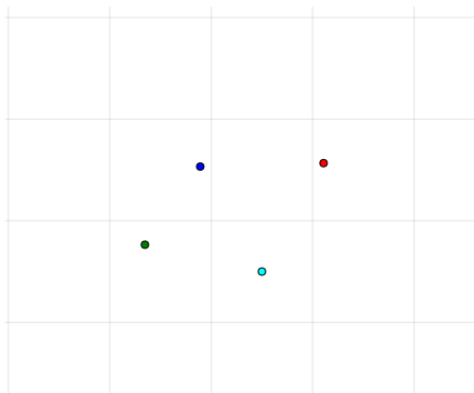
$$\max_q \sum_{D_k \cap q \neq \emptyset} w_k,$$

- The disks are fixed with centers at $\mathcal{P} = \{p_1, \dots, p_n\}$ with $w_k = w(p_k)$
- A clique is a non-empty intersection area of a subset of disks. We search for only a point in an optimal clique.
- An optimal solution for the maximum weight clique is an optimal solution for $MCD(\mathcal{P}, 1)$.

Maximum Weight Clique Problem

Equivalence

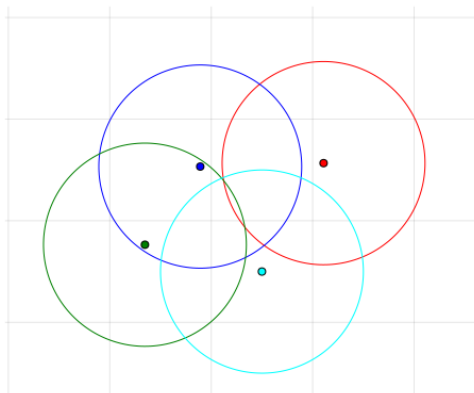
Figure: An instance of $MCD(\mathcal{P}, 1)$.



Maximum Weight Clique Problem

Equivalence

Figure: An instance of $MCD(\mathcal{P}, 1)$.

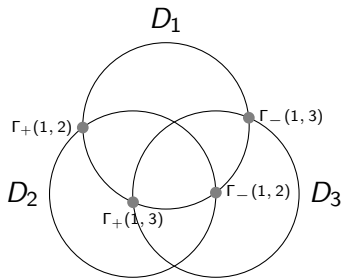


Maximum Weight Clique Problem

Algorithm

Let $\Gamma_+(i, j)$ and $\Gamma_-(i, j)$ be the opening and closing angles of intersections of disks i and j . Also, $\Gamma_+(i, j), \Gamma_-(i, j) \in [0, 2\pi]$.

Figure: Three disks and their intersection points.



Maximum Weight Clique Problem

Algorithm

For a disk D_i , a counter-clockwise traversal visits every $\Gamma_+(i, j)$ and $\Gamma_-(i, j)$ in counter-clockwise order.

- An intersection region of disks is bounded by arcs.
- The arc $\Gamma_+(i, j), \Gamma_-(i, j)$ (counter-clockwise) determines a region where i and j intersect.
- In a counter-clockwise traversal, the arcs where $\Gamma_+(i, j) > \Gamma_-(i, j)$ can be a problem for the implementation. Work-around: repeat it.

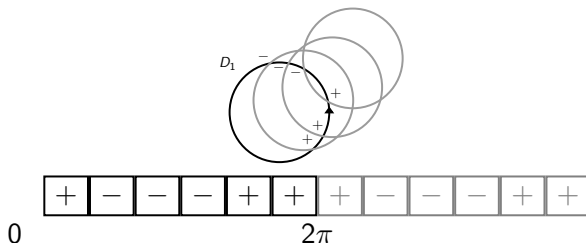
Maximum Weight Clique Problem

Algorithm

The algorithm is described simply as:

For every disk, traverse the sorted list of intersection angles twice, keeping a set of active disks, and the current best solution.

Figure: The intersections list of a disk with three other disks.



Maximum Weight Clique Problem

Algorithm

The run-time complexity of the algorithm is $\mathcal{O}(n^2 \log n)$.

- There are $\mathcal{O}(n^2)$ intersections among n disks
- Sorting takes $\mathcal{O}(n^2 \log n)$
- The traversal takes $\mathcal{O}(n)$ for every disk.
- It can be implemented in $K \log n$ where K is the number of intersections.

Multiple disks

For every disk, try every possible assignment that the traversal goes through.

Maximal Covering by Ellipses

Let $MCE(\mathcal{P}, a, b)$ be an instance of the maximal covering by one ellipse, with E being an ellipse with shape parameters $(a, b) \in \mathbb{R}_{>0}^2$, an optimal solution of $MCE(\mathcal{P}, a, b)$ is given by

$$\max_q |\mathcal{P} \cap (q)|,$$

- $E(q)$ is an axis-parallel ellipse with center point q
- Assuming unit weights for now
- Same algorithm for one disk

Cobertura Maximal por Ellipses

m ellipses

content... algoritmo

