1 Intro

$$A(x_1^2 - x_2^2) + B(x_1y_1 - x_2y_2) + C(y_1^2 - y_2^2) + D(x_1 - x_2) + E(y_1 - y_2) = 0$$
$$A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3) + E(y_1 - y_3) = 0$$
$$A(x_2^2 - x_3^2) + B(x_2y_2 - x_3y_3) + C(y_2^2 - y_3^2) + D(x_2 - x_3) + E(y_2 - y_3) = 0$$

From them we get:

$$D = -\frac{A(x_1^2 - x_2^2) + B(x_1y_1 - x_2y_2) + C(y_1^2 - y_2^2) + E(y_1 - y_2)}{x_1 - x_2}$$
$$E = -\frac{A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3)}{y_1 - y_3}$$

Inserting E into D:

$$D(x_1 - x_2) = -A(x_1^2 - x_2^2) - B(x_1y_1 - x_2y_2) - C(y_1^2 - y_2^2) +$$
$$\frac{A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3)}{y_1 - y_3}(y_1 - y_2)$$
$$D(x_1 - x_2 - \frac{(y_1 - y_2)(x_1 - x_3)}{y_1 - y_3}) =$$
$$A(\frac{x_1^2 - x_3^2}{y_1 - y_3} - x_1^2 + x_2^2) +$$
$$B(\frac{x_1y_1 - x_3y_3}{y_1 - y_3} - x_1y_1 + x_2y_2) +$$
$$C(\frac{y_1^2 - y_3^2}{y_1 - y_3} - y_1^2 + y_2^2)$$

Renaming things:

$$\alpha_d = x_1 - x_2 - \frac{(x_1 - x_3)(y_1 - y_2)}{y_1 - y_3}$$
$$\alpha_a = \frac{x_1^2 - x_3^2}{y_1 - y_3} - x_1^2 + x_2^2$$
$$\alpha_b = \frac{x_1y_1 - x_3y_3}{y_1 - y_3} - x_1y_1 + x_2y_2$$
$$\alpha_c = \frac{y_1^2 - y_3^2}{y_1 - y_3} - y_1^2 + y_2^2$$
$$D\alpha_d = A\alpha_a + B\alpha_b + C\alpha_c$$
$$D = \frac{A\alpha_a + B\alpha_b + C\alpha_c}{\alpha_d}$$

Going back to (E):

$$E(y_1 - y_3) = -A(x_1^2 - x_3^2 - (x_1 - x_3)\frac{\alpha_a}{\alpha_d}) - B(x_1y_1 - x_3y_3 - (x_1 - x_3)\frac{\alpha_b}{\alpha_d}) - C(y_1^2 - y_3^2 - (x_1 - x_3)\frac{\alpha_c}{\alpha_d})$$

Renaming things:

$$\begin{aligned}
\beta_e &= y_1 - y_3 \\
\beta_a &= -x_1^2 + x_3^2 + (x_1 - x_3)\frac{\alpha_a}{\alpha_d} \\
\beta_b &= -x_1y_1 + x_3y_3 + (x_1 - x_3)\frac{\alpha_b}{\alpha_d} \\
\beta_c &= -y_1^2 + y_3^2 + (x_1 - x_3)\frac{\alpha_c}{\alpha_d} \\
E\beta_e &= A\beta_a + B\beta_b + C\beta_c \\
E &= \frac{A\beta_a + B\beta_b + C\beta_c}{\beta_e}
\end{aligned}$$

Using the third equation:

$$\begin{aligned}
A(x_2^2 - x_3^2) + B(x_2y_2 - x_3y_3) + C(y_2^2 - y_3^2) + \\
\frac{A\alpha_a + B\alpha_b + C\alpha_c}{\alpha_d}(x_2 - x_3) + \\
\frac{A\beta_a + B\beta_b + C\beta_c}{\beta_e}(y_2 - y_3) = 0
\end{aligned}$$

Now, isolating B:

$$\begin{aligned}
A(x_2^2 - x_3^2 + (x_2 - x_3)\frac{\alpha_a}{\alpha_d} + (y_2 - y_3)\frac{\beta_a}{\beta_e}) + \\
B(x_2y_2 - x_3y_3 + (x_2 - x_3)\frac{\alpha_b}{\alpha_d} + (y_2 - y_3)\frac{\beta_b}{\beta_e}) + \\
C(y_2^2 - y_3^2 + (x_2 - x_3)\frac{\alpha_c}{\alpha_d} + (y_2 - y_3)\frac{\beta_c}{\beta_e}) = 0
\end{aligned}$$

$$\begin{aligned}
\gamma_a &= x_2^2 - x_3^2 + (x_2 - x_3)\frac{\alpha_a}{\alpha_d} + (y_2 - y_3)\frac{\beta_a}{\beta_e} \\
\gamma_b &= (x_2y_2 - x_3y_3 + (x_2 - x_3)\frac{\alpha_b}{\alpha_d} + (y_2 - y_3)\frac{\beta_b}{\beta_e} \\
\gamma_c &= y_2^2 - y_3^2 + (x_2 - x_3)\frac{\alpha_c}{\alpha_d} + (y_2 - y_3)\frac{\beta_c}{\beta_e} \\
A\gamma_a + B\gamma_b + C\gamma_c &= 0 \\
B &= -\frac{A\gamma_a + C\gamma_c}{\gamma_b}
\end{aligned}$$