

1 Intro

$$\begin{aligned}
A(x_1^2 - x_2^2) + B(x_1y_1 - x_2y_2) + C(y_1^2 - y_2^2) + D(x_1 - x_2) + E(y_1 - y_2) &= 0 \\
A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3) + E(y_1 - y_3) &= 0 \\
A(x_2^2 - x_3^2) + B(x_2y_2 - x_3y_3) + C(y_2^2 - y_3^2) + D(x_2 - x_3) + E(y_2 - y_3) &= 0
\end{aligned}$$

From them we get:

$$\begin{aligned}
D &= -\frac{A(x_1^2 - x_2^2) + B(x_1y_1 - x_2y_2) + C(y_1^2 - y_2^2) + E(y_1 - y_2)}{x_1 - x_2} \\
E &= -\frac{A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3)}{y_1 - y_3}
\end{aligned}$$

Inserting E into D :

$$\begin{aligned}
D(x_1 - x_2) &= -A(x_1^2 - x_2^2) - B(x_1y_1 - x_2y_2) - C(y_1^2 - y_2^2) + \\
&\quad \frac{A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3)}{y_1 - y_3}(y_1 - y_2) \\
D(x_1 - x_2 - \frac{(y_1 - y_2)(x_1 - x_3)}{y_1 - y_3}) &= \\
&\quad A(\frac{x_1^2 - x_3^2}{y_1 - y_3} - x_1^2 + x_2^2) + \\
&\quad B(\frac{x_1y_1 - x_3y_3}{y_1 - y_3} - x_1y_1 + x_2y_2) + \\
&\quad C(\frac{y_1^2 - y_3^2}{y_1 - y_3} - y_1^2 + y_2^2)
\end{aligned}$$

Renaming things:

$$\begin{aligned}
\alpha_d &= x_1 - x_2 - \frac{(x_1 - x_3)(y_1 - y_2)}{y_1 - y_3} \\
\alpha_a &= \frac{x_1^2 - x_3^2}{y_1 - y_3} - x_1^2 + x_2^2 \\
\alpha_b &= \frac{x_1y_1 - x_3y_3}{y_1 - y_3} - x_1y_1 + x_2y_2 \\
\alpha_c &= \frac{y_1^2 - y_3^2}{y_1 - y_3} - y_1^2 + y_2^2 \\
D\alpha_d &= A\alpha_a + B\alpha_b + C\alpha_c \\
D &= \frac{A\alpha_a + B\alpha_b + C\alpha_c}{\alpha_d}
\end{aligned}$$

Going back to (E) :

$$E(y_1 - y_3) = -A(x_1^2 - x_3^2 - (x_1 - x_3)\frac{\alpha_a}{\alpha_d}) - B(x_1y_1 - x_3y_3 - (x_1 - x_3)\frac{\alpha_b}{\alpha_d}) - C(y_1^2 - y_3^2 - (x_1 - x_3)\frac{\alpha_c}{\alpha_d})$$

Renaming things:

$$\begin{aligned}\beta_e &= y_1 - y_3 \\ \beta_a &= -x_1^2 + x_3^2 + (x_1 - x_3)\frac{\alpha_a}{\alpha_d} \\ \beta_b &= -x_1y_1 + x_3y_3 + (x_1 - x_3)\frac{\alpha_b}{\alpha_d} \\ \beta_c &= -y_1^2 + y_3^2 + (x_1 - x_3)\frac{\alpha_c}{\alpha_d} \\ E\beta_e &= A\beta_a + B\beta_b + C\beta_c \\ E &= \frac{A\beta_a + B\beta_b + C\beta_c}{\beta_e}\end{aligned}$$

Using the third equation:

$$\begin{aligned}A(x_2^2 - x_3^2) + B(x_2y_2 - x_3y_3) + C(y_2^2 - y_3^2) + \\ \frac{A\alpha_a + B\alpha_b + C\alpha_c}{\alpha_d}(x_2 - x_3) + \\ \frac{A\beta_a + B\beta_b + C\beta_c}{\beta_e}(y_2 - y_3) = 0\end{aligned}$$

Now, isolating B :

$$\begin{aligned}A(x_2^2 - x_3^2 + (x_2 - x_3)\frac{\alpha_a}{\alpha_d} + (y_2 - y_3)\frac{\beta_a}{\beta_e}) + \\ B(x_2y_2 - x_3y_3 + (x_2 - x_3)\frac{\alpha_b}{\alpha_d} + (y_2 - y_3)\frac{\beta_b}{\beta_e}) + \\ C(y_2^2 - y_3^2 + (x_2 - x_3)\frac{\alpha_c}{\alpha_d} + (y_2 - y_3)\frac{\beta_c}{\beta_e}) = 0 \\ \gamma_a = x_2^2 - x_3^2 + (x_2 - x_3)\frac{\alpha_a}{\alpha_d} + (y_2 - y_3)\frac{\beta_a}{\beta_e} \\ \gamma_b = (x_2y_2 - x_3y_3 + (x_2 - x_3)\frac{\alpha_b}{\alpha_d} + (y_2 - y_3)\frac{\beta_b}{\beta_e}) \\ \gamma_c = y_2^2 - y_3^2 + (x_2 - x_3)\frac{\alpha_c}{\alpha_d} + (y_2 - y_3)\frac{\beta_c}{\beta_e} \\ A\gamma_a + B\gamma_b + C\gamma_c = 0 \\ B = -\frac{A\gamma_a + C\gamma_c}{\gamma_b}\end{aligned}$$