

# Planar Maximal Covering with Ellipses

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April 26, 2019

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- Covering problems
  - Set Cover Problem
  - Maximal Covering Problem
- Maximal Covering Location Problem (MCLP)
- Planar Maximal Covering Location Problem (PMCLP)
  - One disk:  $\mathcal{O}(n^2)$  and  $\mathcal{O}(n^2 \log n)$  algorithms
  - $m$  disks:  $\mathcal{O}(n^{2m-1} \log n)$  algorithm
- Goals
  - Develop a  $\mathcal{O}(n^2 \log n)$  algorithm for the one disk case
  - Adapt it for the  $m$  ellipses case

## Norms

Let  $u \in \mathbb{R}^2$  and  $Q$  a  $2 \times 2$  positive definite matrix

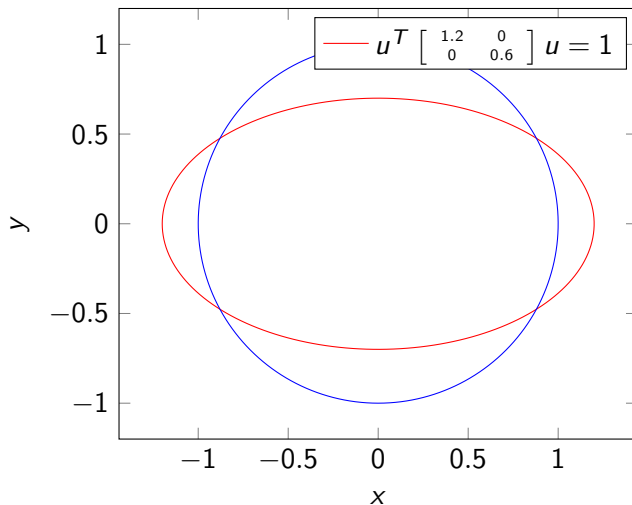
- Euclidean

$$\|u\|_2 = \sqrt{u^T u}$$

- Elliptical

$$\|u\|_Q = \sqrt{u^T Q u}$$

Figure: The elliptical and euclidean norms.



## Ellipse

Given a center  $c \in \mathbb{R}^2$  and a  $2 \times 2$  p.d. matrix  $Q$ , an ellipse is the set of points that satisfy

$$\|u - c\|_Q = 1,$$

with  $\leq$  representing the set of covered points

## Axis-parallel ellipse

Any  $2$  by  $2$  diagonal d.p. matrix determines an axis-parallel ellipse, which can also be described by

$$\frac{(x - c_x)^2}{a^2} + \frac{(y - c_y)^2}{b^2} = 1,$$

where  $(a, b)$  are the shape parameters and  $c = (c_x, c_y)$  is the center.

# Maximal Covering by Disks

## One disk

$MCD(\mathcal{P}, 1)$  is the problem of placing one disk on the plane to cover a subset of a demand set  $\mathcal{P}$  maximizing the weights of the covered points.

$$\max_q w(\mathcal{P} \cap D(q)),$$

- $\mathcal{P} = \{p_1, \dots, p_n\}$  is the demand set with weights  $w(p_i) > 0$
- $w(A)$ ,  $A \subset \mathcal{P}$ , is the sum of weights of the points in  $A$
- $D(q)$  is a unit disk with center at point  $q$
- We will introduce an equivalent problem...

# Maximum Weight Clique Problem

Let  $\mathcal{D} = \{D_1, \dots, D_n\}$  be a set of  $n$  unit disks with weights  $w_i > 0$ . The maximum weight clique is defined as

$$\max_q \sum_{D_k \cap q \neq \emptyset} w_k,$$

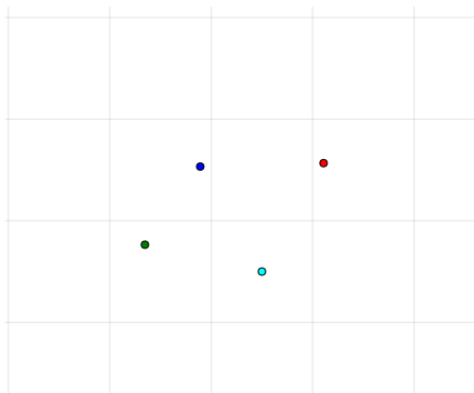
- The disks are fixed with centers at  $\mathcal{P} = \{p_1, \dots, p_n\}$  with  $w_k = w(p_k)$
- A clique is a non-empty intersection area of a subset of disks. We search for only a point in an optimal clique.
- An optimal solution for the maximum weight clique is an optimal solution for  $MCD(\mathcal{P}, 1)$ .



# Maximum Weight Clique Problem

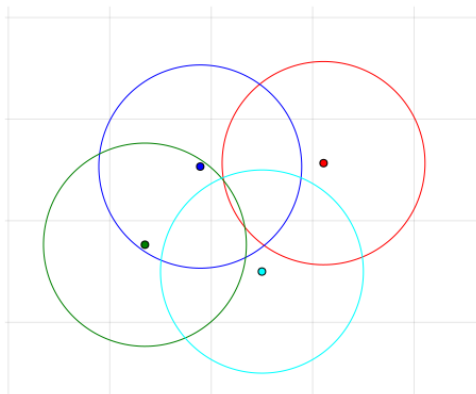
## Equivalence

Figure: An instance of  $MCD(\mathcal{P}, 1)$ .



# Maximum Weight Clique Problem

Figure: An instance of  $MCD(\mathcal{P}, 1)$ .

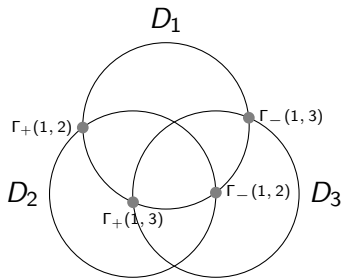


# Maximum Weight Clique Problem

## Algorithm

Let  $\Gamma_+(i, j)$  and  $\Gamma_-(i, j)$  be the opening and closing angles of intersections of disks  $i$  and  $j$ . Also,  $\Gamma_+(i, j), \Gamma_-(i, j) \in [0, 2\pi]$ .

Figure: Three disks and their intersection points.



# Maximum Weight Clique Problem

## Algorithm

For a disk  $D_i$ , a counter-clockwise traversal visits every  $\Gamma_+(i, j)$  and  $\Gamma_-(i, j)$  in counter-clockwise order.

- An intersection region of disks is bounded by arcs.
- The arc  $\Gamma_+(i, j), \Gamma_-(i, j)$  (counter-clockwise) determines a region where  $i$  and  $j$  intersect.
- In a counter-clockwise traversal, the arcs where  $\Gamma_+(i, j) > \Gamma_-(i, j)$  can be a problem for the implementation. Work-around: repeat it.

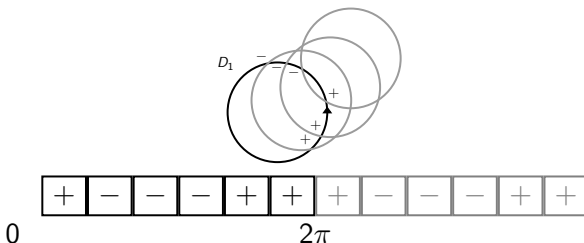
# Maximum Weight Clique Problem

## Algorithm

The algorithm is described simply as:

For every disk, traverse the sorted list of intersection angles twice, keeping a set of active disks, and the current best solution.

**Figure:** The intersections list of a disk with three other disks.



# Maximum Weight Clique Problem

## Algorithm

The run-time complexity of the algorithm is  $\mathcal{O}(n^2 \log n)$ .

- There are  $\mathcal{O}(n^2)$  intersections among  $n$  disks
- Sorting takes  $\mathcal{O}(n^2 \log n)$
- The traversal takes  $\mathcal{O}(n)$  for every disk.
- It can be implemented in  $K \log n$  where  $K$  is the number of intersections.

### Multiple disks

For every disk, try every possible assignment that the traversal goes through.

# Maximal Covering by Ellipses

Let  $MCE(\mathcal{P}, a, b)$  be an instance of the maximal covering by one ellipse, with  $E$  being an ellipse with shape parameters  $(a, b) \in \mathbb{R}_{>0}^2$ , an optimal solution of  $MCE(\mathcal{P}, a, b)$  is given by

$$\max_q |\mathcal{P} \cap (q)|,$$

- $E(q)$  is an axis-parallel ellipse with shape parameters  $(a, b) \in \mathbb{R}_{>0}^2$ .
- Assuming unit weights for now
- Same algorithm for one disk

# Cobertura Maximal por Ellipses

$m$  ellipses

content... algoritmo



# Trabalhos Futuros