1 Intro

$$A(x_1^2 - x_2^2) + B(x_1y_1 - x_2y_2) + C(y_1^2 - y_2^2) + D(x_1 - x_2) + E(y_1 - y_2) = 0$$
$$A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3) + E(y_1 - y_3) = 0$$
$$A(x_2^2 - x_3^2) + B(x_2y_2 - x_3y_3) + C(y_2^2 - y_3^2) + D(x_2 - x_3) + E(y_2 - y_3) = 0$$

From them we get:

$$D = -\frac{A(x_1^2 - x_2^2) + B(x_1y_1 - x_2y_2) + C(y_1^2 - y_2^2) + E(y_1 - y_2)}{x_1 - x_2}$$
$$E = -\frac{A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3)}{y_1 - y_3}$$

Inserting E into D:

$$\begin{aligned} D(x_1 - x_2) = -A(x_1^2 - x_2^2) - B(x_1y_1 - x_2y_2) - C(y_1^2 - y_2^2) + \\ \frac{A(x_1^2 - x_3^2) + B(x_1y_1 - x_3y_3) + C(y_1^2 - y_3^2) + D(x_1 - x_3)}{y_1 - y_3}(y_1 - y_2) \\ D(x_1 - x_2 - \frac{(y_1 - y_2)(x_1 - x_3)}{y_1 - y_3}) = \\ A(\frac{x_1^2 - x_3^2}{y_1 - y_3} - x_1^2 + x_2^2) + \\ B(\frac{x_1y_1 - x_3y_3}{y_1 - y_3} - x_1y_1 + x_2y_2) + \\ C(\frac{y_1^2 - y_3^2}{y_1 - y_3} - y_1^2 + y_2^2) \end{aligned}$$

Renaming things:

$$\begin{aligned} \alpha_d = x_1 - x_2 - \frac{(x_1 - x_3)(y_1 - y_2)}{y_1 - y_3} \\ \alpha_a = \frac{x_1^2 - x_3^2}{y_1 - y_3} - x_1^2 + x_2^2 \\ \alpha_b = \frac{x_1y_1 - x_3y_3}{y_1 - y_3} - x_1y_1 + x_2y_2 \\ \alpha_c = \frac{y_1^2 - y_3^2}{y_1 - y_3} - y_1^2 + y_2^2 \\ D\alpha_d = A\alpha_a + B\alpha_b + C\alpha_c \\ D = \frac{A\alpha_a + B\alpha_b + C\alpha_c}{\alpha_d} \end{aligned}$$

Going back to (E):

$$E(y_1-y_3) = -A(x_1^2-x_3^2-(x_1-x_3)\frac{\alpha_a}{\alpha_d}) - B(x_1y_1-x_3y_3-(x_1-x_3)\frac{\alpha_b}{\alpha_d}) - C(y_1^2-y_3^2-(x_1-x_3)\frac{\alpha_c}{\alpha_d})$$

Renaming things:

$$\begin{aligned}
\beta_e &= y_1-y_3 \\
\beta_a &= -x_1^2+x_3^2+(x_1-x_3)\frac{\alpha_a}{\alpha_d} \\
\beta_b &= -x_1y_1+x_3y_3+(x_1-x_3)\frac{\alpha_b}{\alpha_d} \\
\beta_c &= -y_1^2+y_3^2+(x_1-x_3)\frac{\alpha_c}{\alpha_d} \\
E\beta_e &= A\beta_a+B\beta_b+C\beta_c \\
E &= \frac{A\beta_a+B\beta_b+C\beta_c}{\beta_e}
\end{aligned}$$

Using the third equation:

$$\begin{aligned}
A(x_2^2-x_3^2)+B(x_2y_2-x_3y_3)+C(y_2^2-y_3^2)+ \\
\frac{A\alpha_a+B\alpha_b+C\alpha_c}{\alpha_d}(x_2-x_3)+ \\
\frac{A\beta_a+B\beta_b+C\beta_c}{\beta_e}(y_2-y_3) = 0
\end{aligned}$$

Now, isolating B:

$$\begin{aligned}
A(x_2^2-x_3^2+(x_2-x_3)\frac{\alpha_a}{\alpha_d}+(y_2-y_3)\frac{\beta_a}{\beta_e})+ \\
B(x_2y_2-x_3y_3+(x_2-x_3)\frac{\alpha_b}{\alpha_d}+(y_2-y_3)\frac{\beta_b}{\beta_e})+ \\
C(y_2^2-y_3^2+(x_2-x_3)\frac{\alpha_c}{\alpha_d}+(y_2-y_3)\frac{\beta_c}{\beta_e}) = 0
\end{aligned}$$

$$\begin{aligned}
\gamma_a &= x_2^2-x_3^2+(x_2-x_3)\frac{\alpha_a}{\alpha_d}+(y_2-y_3)\frac{\beta_a}{\beta_e} \\
\gamma_b &= (x_2y_2-x_3y_3+(x_2-x_3)\frac{\alpha_b}{\alpha_d}+(y_2-y_3)\frac{\beta_b}{\beta_e} \\
\gamma_c &= y_2^2-y_3^2+(x_2-x_3)\frac{\alpha_c}{\alpha_d}+(y_2-y_3)\frac{\beta_c}{\beta_e} \\
A\gamma_a+B\gamma_b+C\gamma_c &= 0 \\
B &= -\frac{A\gamma_a+C\gamma_c}{\gamma_b}
\end{aligned}$$