Planar Maximal Covering with Ellipses

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April 25, 2019

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- Goals
  - Develop a $\mathscr{O}(n^2 \log n)$ algorithm for the one disk case
  - Adapt it for the m ellipses case

Norms

Let $u \in \mathbb{R}^2$ and Q a 2 by 2 positive definite matrix

- Euclidean

$$||u||_2 = \sqrt{u^T u}$$

- Elliptical

$$||u||_Q = \sqrt{u^T Q u}$$

Preliminaries

Ellipse

Given a center $c \in \mathbb{R}^2$ and a p.d. matrix Q, an ellipse is the set of points that satisfy

$$||u - c||_Q = 1$$

with $\leq$ representing the set of covered points

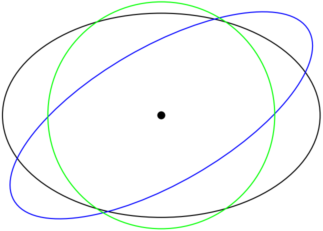
Axis-parallel ellipse

Any 2 by 2 diagonal d.p. matrix determines an axis-parallel ellipse, which can also be described by

$$\frac{(x - c_x)^2}{a^2} + \frac{(y - c_y)^2}{b^2} = 1$$

Preliminaries

Figure: Three curves representing the points that have distance equal to one.



Maximal Covering by Disks

One disk

$MCD(\mathscr{P}, 1)$ is the problem of placing one disk on the plane to cover a subset of a demand set $\mathscr{P}$ maximizing the weights of the covered points.

$$\max_q w(\mathscr{P} \cap D(q)),$$

- $\mathscr{P} = \{p_1, \ldots, p_n\}$ is the demand set with weights $w(p_i) > 0$
- $w(A)$, with $A \subset \mathscr{P}$ is the sum of weights of the points in A
- $D(q)$ is a disk with radius one with center at point q
- We will introduce an equivalent problem...

Maximum Weight Clique Problem

Let $\mathscr{D} = \{D_1, \ldots, D_n\}$ be a set of n unit disks with weights $w_i > 0$. The maximum weight clique is defined as

$$\max_{q} \sum_{D_k \cap q \neq \emptyset} w_k,$$

- The disks are fixed at $\mathscr{P} = \{p_1, \ldots, p_n\}$ with $w_k = w(p_k)$
- A clique is a non-empty intersection area of a subset of disks
- An optimal solution for the maximum weight clique is an optimal solution for $MCD(\mathscr{P}, 1)$.

Cobertura Maximal por Ellipses

content... trabalhos passados

content... algoritmo

Trabalhos Futuros