Applied Mechanics and Materials Vols 110-116 (2012) pp 4861-4866 Online: 2011-10-24
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doi:10.4028/www.scientific.net/AMM.110-116.4861

Planar maximal covering location problem with inclined ellipses

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Keywords:omponent; Maximal covering location problem; Elliptical coverage; inclined ellipses; mixed integer nonlinear programming; Heuristic algorithm

Abstract:aximum coverage location problem is considered in this study. Extension of this problem is investigated for situations that coverage areas are elliptical; these ellipses can locate anywhere on the plane with any angle. Mixed integer nonlinear programming (MINLP) is applied for formulation. This problem can be used in many practical situations such as locating wireless transmitter towers. A heuristic algorithm named MCLPEA for solving this problem was designed. This algorithm can produce very good results in efficient running time. Finally, the comparison of the results for this study was carried done.

Introduction

Maximal covering location problem (MCLP) has been used when there are insufficient resources for servicing to all demand points; so the objective is finding the best places for resources to maximize servicing. Moreover, the possibility of covering maximum demand points will be investigated by this method. Thus, any facility within the coverage areas will serve but outside of it won't serve. Sometimes did the objective function mix with minimizing costs so resource costs will take in objective function. This is due to some of resources are usually wanted to select from all available resources so by considering of their coverage areas and cost, best resources will choose.

This problem presented in three different versions of network, planar and spheres. There are many researches regarding to investigate of network version but the other variables haven't been investigated a lot because they are more complicated than network version. In the spheres and planar versions, assumption is that the resources can be located in anywhere on the plane and the only different between these two versions is the type of distance function. Indeed, spheres version is more realistic in the very large zones because it considers spherical of the earth in calculations. On the other hand, in the very large zones the planar version may cover a demand point theoretically but it's not covered in practice. That is why the spheres error of the earth must be considered in distance function. Anyway, it can be mentioned that the planar version is a very good estimation for not very large zones.

First network version of this problem introduced by Church and ReVelle [4] and then planar version presented by Mehrez and Stulman [7]. A few years later extended model was proposed by Church [3] the problem was formulated by them in the mixed integer programming (MIP).After that many researches were done by considering different coverage areas. Some of them are about the practical usages of this problem such as locating emergency warning sirens [5] or locating signal detectors [6].

One of the most important usages of maximal covering Problem is in the locating of power or wireless transmitter. This method was presented by Aguado-Agelet, Varela, Alvarez-Vazquez, Hernando and Formella [1] and was extended by Mustafa S.Canbolat, Michael von Massow [2].

In 2009, a new model was introduced by Mustafa S.Canbolat, and Michael von Massow[2] for locating Wireless transmitter and it was formulated by mixed integer nonlinear programming (MINLP).the objective in this problem was maximizing covered points with minimum costs. For the first time, elliptical shape for coverage area was considered by them, and it is clearly observed that this model is very realistic. Furthermore, they introduced spheres version and related distance function for the first time. In the considered study they didn't take into account this fact that elliptical coverage areas can locate in any angle on the plane so formulation of the problem just let ellipses locate horizontally or vertically.

In this paper, Maximal covering location problem was extended with ellipses (MCLPE) and supposed the ellipses can locate with any angle on the plane. In 2^{nd} step of this paper, the problem was formulated. In fact the most important point of this study was completion previous study[2] and It was preferred to use their definitions and notation for formulating the problem .In section.3 a very efficient heuristic algorithm named MCLPEA was introduced. It's important to notice MCLPEA is a heuristic algorithm and may not get global optimum. In section.4 it was concluded and the results of this study was compared by previous research [2].

problem description

This problem includes n demand points with nonnegative weights w_j that spread on the plane. In fact each point shows a center of population and its weight is computable from its potential customers. The objective is choosing K ellipses between m available ellipses and finding the best place for them ,in order to maximizing covering and minimizing the costs.

Notation. Tabel.1 shows the notation was utilized for formulation of the proposed model.

Formulation. The proposed formulation for MCLP by inclined ellipses is as follows

$$\text{Maximize } \sum\sum w_j z_{ij} - \sum c_i y_i \quad (0)$$

$$l_2(f_{1i},p_j)+l_2(f_{2i},p_j)-2a_i \le (1-z_{ij})M;\ \forall i,j \quad (1)$$

$$\sum y_i = k \quad (2)$$

$$Z_{ij}-y_i \le 0;\ \forall i,j \quad (3)$$

$$f_2x_i = f_1x_i + 2a_i\cos\alpha_i\ (1-(b_i^2/a_i^2))\ ^\wedge.5;\ \forall i \quad (4)$$

$$f_2y_i = f_1y_i - 2a_i\sin\alpha_i\ (1-(b_i^2/a_i^2))\ ^\wedge.5;\ \forall i \quad (5)$$

$$\sum z_{ij} \le 1;\ \forall j \quad (6)$$

$$Y_i, z_{ij} \in \{0,1\};\ \forall i,j \quad (7)$$

$$0 \le \alpha_i \le 180 \quad (8)$$

The objective value ensures the maximum covering of weighted demand points by minimum total ellipses selection cost will happen. Constraint (1) is the most important constraint and ensures when the sum of Euclidian distances of point P_j from two foci of ellipse i is not greater than $2a_i$,

TABLE I.Notation [2]

Inputs	Explanation
m	The number of available ellipses
n	The number of demand points
w_j ; j=1,…,n	Weighted for demand point j
C_i ; i=1,…,m	Cost of choosing ellipse i
a_i ; i=1,…, m	Semimajor axis of ellipse i
b_i ; i=1,…,m	Semiminor axis of ellipse i
M	A big number
K ; K≤m	The number of ellipses which are to be located
Decision variables	**Explanation**
Z_{ij}	Binary decision variable, equal to 1if demand point i is covered by ellipses j, and 0 otherwise
y_i	Binary decision variable, equal to 1 if ellipse I is selected, and 0 otherwise
f_1x_i	x-coordinate for focus 1 of ellipse i
f_1y_i	y-coordinate for focus 1 of ellipse i
	Angle of ellipse i (fig.2)
Other variables	**Explanation**
f_2x_i	X-coordinate for focus 2 of ellipse i which can be calculated from x-coordinate of focus 1 of ellipse i
f_2y_i	y-coordinate for focus 2 of ellipse i which can be calculated from y-coordinate of focus 1 of ellipse i

then Z_{ij} is equal to 1 ,and it means demand j is covered by ellipse i [2].Constraint (2) determines the number of ellipses that should be chosen. Constraint (3) ensures no point is assigned to an unselected ellipse. Constraint (4) and (5) are for calculating two variable of f_2x_i, f_2y_i by considering of slope of related ellipse. The constraint (6) ensures the demand point j is covered by at most one ellipse. Constraint (7) shows that y_i and Z_{ij} are binary variables. Finally, constraint (8) shows that angle for each ellipse can be in the span of [0,180] and there is no need for other angles, because these angles can cover other angles. Other variables are unrestricted sign.

MCLPEA ALGORITHM

It is clear MCLP with inclined ellipses is more complicated than its basic form that cannot be inclined. As shown in [2] it is not efficient to use exact method for solving the complicated problems because they need a lot of time; so for solving this kind of problems, MCLPEA algorithm was designed by authors of this paper that can solve all problems in a very efficient time, and it is possible to find reasonable answers for these problems by this method. Heuristic algorithm consists of two stages in order to find the good answers. In the first stage, the algorithm clusters the points and in the second stage, it searches each cluster for finding the best answer. The clustering is because of finding the angle in each group and all of searches in that group will be done by that angle. On the other hand, if the center of ellipse i is located in the related zone of cluster j, the algorithm uses the angle of cluster j for producing inclined ellipses. After many tests it was found that one of the best angles in each cluster is the angle of the regression line slope so the algorithm finds this angle by considering the weight of each point in each cluster.

Clustering stage. There are many methods to cluster the points and naturally they have a lot of cons and pros in compare to each other. After many tests the following method is chosen for clustering:

Step.0: Make the list of “existence points” and put all points in it.

Step.1: Find the maximum semimajor axis between available ellipses and consider twice of this number as a “neighborhood criterion”; so two points are neighbor if the distance between them isn’t greater than this criterion.

Example.1: If c be the largest semimajor axis between available ellipses then point b is neighbor of point a if Euclidian distance of b from a isn’t greater than 2c.

Step.2: Choose one point from the list of existence points randomly and assign a numbers as group number to it; this number must be different from all previous groups’ numbers.

Step.3: Choose one point from all neighbor points of selected point in step.2 randomly and assign it group number of step.2. It is important to notice the neighbor points must be in the list of existence points and if there is no neighbor points go to step 6.

Step.4: In this step the definition of neighborhood change and we will accept a point as a neighbor of group k if it will be neighbor of all previous selected points in group k. Choose one point from all neighbor points randomly and assign it group number of step.2, the neighbor points must be in the list of existence points.

Step.5: Repeat step.4 until there is no neighbor point.

Step.6: Consider selected points as a one group and omit them from the list of existence points.

Step.7: Repeat step.2 to step.6 until there is at least one point in the list of existence points.

Example.2: function of clustering stage is as follows:

Suppose two ellipses and some points are available as shown in table.2.It is clear the neighborhood criterion is equal to 1.5 and one of the clustering can be as table.3. The number of points are selected and shown in table respectively in basis of clustering stage. It is clear from table.3, distances between every two points in a one group isn’t greater than 1.5.

Searching stage. Step.0: make two lists include: list of “existence ellipses “that consists all available ellipses and the list of “existence points” that consist all points.

Step.1: Determine searching zone for each groups. These zones are similar to rectangle and by considering the minimum and maximum of x-coordinate and y-coordinate of each group it will construct, by investigation of the example.2 this step will be resulted as table.4.

TABLE II.Data of available points and ellipses

ellipses			points		
number	*Semimajor axis*	*Semiminor axis*	*number*	*x-coordinate*	*y-coordinate*
1	0.75	0.3	1	0.5	0.5
2	0.7	0.4	2	0.75	0.5
			3	1	1
			4	2	1
			5	2.5	0.75
			6	3	1

TABLE III.The results of clustering stage

Number of the points	Number of the groups
5	1
4	1
6	1
2	2
1	2
3	2

Step.2: Calculate the regression line slope and related angle for each group zone by taking into account of the included weighted points of each group zones.

$$Slope_j=(\sum w_jx_jy_j-((\sum w_jx_j\sum w_jy_j)/\sum w_j))/(\sum w_jx_j^2-((\sum w_jx_j)^2/\sum w_j)) \quad (9)$$

$$A_j=\tan^{-1}(slope_j) \quad (10)$$

Step.3: Choose several points for each ellipses in the each group zone and keep in mind those points as a center of assigned ellipses. Moreover, the ellipses must be in the list of existence ellipses.

Step.4: calculate f_1x_i , f_1y_i, f_2x_i and f_2y_i for each ellipses by making inquiries about its center and group zone angle. The ellipses must be in the list of existence ellipses

$$f1xi=cx_i-(a_i\cos\alpha_j)(1-(b_i^2/a_i^2))^{.5} \quad (11)$$

$$f2xi= cxi+(a_i\cos\alpha_j)(1-(b_i^2/a_i^2))^{.5} \quad (12)$$

$$f1yi=cyi-(a_i\sin\alpha_j)(1-(b_i^2/a_i^2))^{.5} \quad (13)$$

$$f2yi= cyi+(a_i\sin\alpha_j)(1-(b_i^2/a_i^2))^{.5} \quad (14)$$

Step.5: compute the sum of the weights of the covered points by each ellipse then subtract the cost of related ellipses from this value.

TABLEIV. Results of step.1 at searching stage

Group number	Group zone	
	x	y
1	[2‹3]	[.75‹1]
2	[.5‹1]	[.5‹1]

Note: as mentioned previously, if the sum of Euclidian distances of point P_j from two foci of ellipse i is not greater than $2a_i$ then demand j is covered by ellipse i .

Step.6: compare the values of previous step and select the maximum value. So, the best ellipses from existence ellipses and best location for it will find.

Step.7: Omit selected ellipse and all covered points of this ellipse from the lists of existence points and existence ellipses.

Step.8: Repeat step.3 to step.7 until satisfying the second constraint then calculate the objective value in view of all selected ellipses and covered points.

Step.9: repeat clustering stage and searching stage for several times. That is due to sensitivity of this algorithm to type of clustering and we think 30 times repeat leads to very good results.

Conclusion and comparing the results

We presented maximal covering location problem with inclined ellipses and designed a heuristic algorithm for solving this problem. MCLPEA results (table.5) were compared with S.Canbolat, Michael von Massow [2] results (table.6), and it is shown MCLPEA algorithm gets better results rather than the basic form of MCLPE that ellipses can locate horizontally or vertically. For this comparison, their data [2] was used.

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TABLE V.MCLPEA result

k	n	# of points covered			Average running times (s)	Best solution base on objective function		
		Worst	*Average*	*Best*		*Selected facility(ies)*	*#of covered points*	*Objective function*
1	25	2	3.53	6	0.3798271	1	4	2
1	50	7	8.53	10	0.76485728	3	10	5.2
1	100	12	14.6	17	2.4324	3	17	12.2
2	25	3	6.43	10	0.57296625	1,2	10	4.8
2	50	12	14.6	19	1.847339	1,3	19	12.2
2	100	23	27.1	31	7.12436	1,3	31	24.2

TABLE VI. S.Canbolat, Michael von Massow[2]

k	n	# of points covered			Selected facility(ies)	Average runnig times(s)	Optimality
		Worst	*Average*	*Best*			
1	25	2	2.5	3	1	1.6	Optimal
1	50	6	7.9	9	3	1.92	optimal
1	100	12	15.8	17	3	2.76	optimal
2	25	6	7.3	9	1,2	2.41	optimal
2	50	12	14.7	15	1,3	4.12	Best integer solution
2	100	23	26.2	28	2,3	7.28	Best integer solution

Mechanical and Aerospace Engineering, ICMAE2011
10.4028/www.scientific.net/AMM.110-116

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10.4028/www.scientific.net/AMM.110-116.4861

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