Dear Steffen Rebennack,
Editor of EJOR

Thanks for the great opportunity of having our work reviewed by EJOR's editorial board, which provided great comments and suggestions that surely enhanced the quality of our work.

Besides the modifications in the text to address the issues raised by the reviewers, we have also made some small corrections and changes of notation on our own in the hopes of improving the manuscript. In the text, we annotated these changes with the color blue.

The other modifications were made to address the issues raised by the reviewers, and we highlighted them as follows:

- The red highlights are corrections concerning the points made by reviwer #1.

- The green highlights are corrections concerning the points made by reviwer #2.

- The orange highlights are corrections concerning points made by both reviewers.

Please see in the next pages the answers to both reviewers.

On behalf of the authors and with our best regards,
Danilo Tedeschi.

Answer to reviewer # 1

First, thanks for dedicating your time to revise our work, it is a great honor for us to continue to improve our work with your help. We have split your review into two sections, which we called "comments", one regarding the lack of a real-world application connection, and the other regarding potential extensions to the problems studied by us. Please find below our responses to them.

Comment #1: "The authors indicate that 'it is fair to say that PMCLP with elliptical coverage has not been vastly studied as only two articles have been found on it.' I would like the authors to expand on this. In particular, addressing under which real-world conditions the elliptical shape is the ideal shape to model coverage. The paper is technically correct, and it was interesting to follow the development of the model; however, the authors did not really intent to connect to a real-world application where elliptical coverage is needed or desired. Maybe there's little research because there's no interest on the particular problem? In the absence of a practical application motivating the extension to ellipses, this paper has limited interest."

Thanks a lot for this great detailed feedback. Upon receiving your input, we did question ourselves about the interest there is on our work, and we ended up coming to an agreement that a real-world connection is fundamental to making a case this problem is worth being published. We addressed this issue by adding a new paragraph to our Introduction citing the work "*Mustafa S. Canbolat and Michael von Massow. Planar maximal covering with ellipses.*", which covers with great detail the practical applications of the problems studied in our work:

"In the real world, this problem comes up when a telecommunications company has to buy and place antennas over a region to distribute a signal to a population, not necessarily everyone, aiming profit maximization. More details about the applications of PMCLP can be found in [5], which also ponders about the practical importance of considering the PMCLP with elliptical coverage."

This change can be found in page 3 of the manuscript. Also, to better fit this addition, we had to modify the paragraph after the added one; we do not display it here because it is not relevant to this comment.

Comment #2: "I also would like the authors to discuss potential extensions, specially if there's evidence that the elliptical shape is indeed useful to model facility coverage. Given fixed coverage areas, is it possible to allow minor and major axes to be decision variables to determine with shapes cover a particular space better? The aspect ratio and how elongated or circular the ellipses are at the final solution given different space densities might be an interesting issue to explore, rather than fixing the shape of the ellipses from the onset."

Thanks for this idea of an extension to our problem, we think it is very interesting, and we did devote some time to think about it. We believe that there is not a trivial adaptation to any of our methods that would make it possible to work for this variation of the problem; at least for us, it seems like it would require extensive work. We hypothesized that with this addition of a

new variable (the ellipse's semi-major or semi-minor), we could have a similar result for MCER: there is a constant number of let's call locations (center, angle of rotation, and the new parameter), which makes four points be on the ellipse with a fixed area. Although, We could not prove it nor disprove it. We also thought about the possibility of having an optimization method on top of our current algorithm to decide the new parameter, but we could not think of a way we could make a model like that work. Even though we could not make too much out of the suggestion, we still believe that it could be a good idea for future works on the subject. Therefore, we added a paragraph to the conclusion to pass this idea on:

"We believe that fixing only the area of each ellipse, instead of its shape, can be an interesting extension of MCE and MCER; having only one additional decision variable –the major or minor axis– might still allow a similar model to be employed, even though extensive work would be necessary to adapt, somehow, the theory developed for this version, especially of MCER."

Answer to reviewer # 2

First, thanks for dedicating your time to revise our work. We found every one of the points made of great importance for us to achieve a top quality paper.

Following we provide answers to each one of the comments you have given us:

Comment #1: " The highlights (contributions of the paper) and organization of the remaining sections should be presented at the end of Section 1."

We address this issued by adding a paragraph to the end of our Introduction:

"This paper is organized as follows. Section 2 presents the definitions of both problems and 3introduces some notations used throughout the rest of the work. An algorithm for MCE is developed in Section 3, and an algorithm for MCER in Section 5 with the observation that its development depends on a subproblem whose algorithm is developed in Section 4. Improvements suggestions to the implementations are given in Section 6 while the implementation details are given in Section 7. Numerical results are presented and analyzed in Section 8. Finally, a conclusion is given in Section 9."

This paragraph can be found in the manuscript in pages 3 and 4.

Comment #2: "In my opinion 'fixed shape' ellipses should be replaced by 'fixed shape parameters' ellipses, or ellipses with the given semi-major and semi-minor axes. Alternately the term 'fixed shape' ellipses should be explained."

We agree that using "fixed shape" without clearly defining what it is can be confusing to the reader. To address this issue we rewrote the pieces of text where this was being used:

- In the first paragraph of the abstract (page 2), we used the term "given their major and minor axis" to better convey the same idea.

- In the Introduction section (page 3), we changed the description of our problem to: "(...) each ellipse is defined to have fixed major and minor axis and an undefined location (...)."

- Section 4 had in its title (page 7) "(...) of An Ellipse Given Its Shape and (...)", we replaced it by "(...) of An Ellipse Given Its Shape Parameters and (...)."

Comment #3: "Please, extend the description of practical applications."

We address this issued by adding a new paragraph to our Introduction citing the work "*Mustafa S. Canbolat and Michael von Massow. Planar maximal covering with ellipses.*", which covers with great detail the practical applications of the problems studied in our work:

"In the real world, this problem comes up when a telecommunications company has to buy and place antennas over a region to distribute a signal to a population, not necessarily everyone, aiming profit maximization. More details about the applications of PMCLP can be found in [5], which also ponders about the practical importance of considering the PMCLP with elliptical coverage."

This change can be found in page 3 of the manuscript. Also, to better fit this addition, we had to modify the paragraph after the added one; we do not display it here because it is not relevant to this comment.

Comment #4: "The literature review in Introduction is a bit limited and should to be extended analyzing more approaches and references on the subject. It can be expected that beginning from the first publication [8] in 1984 a reasonable number of references on the subject have been published. See, e.g. papers:

'A Combinatorial Maximum Cover Approach to 2D Translational Geometric Covering' by K. Daniels A. Mathur R. Grinde

'Covering a polygonal region by rectangles' by G. Scheithauer, Y. Stoyan, T. Romanova, A. Krivulya

'Translational polygon containment and minimal enclosure' using mathematical programming by V.J. Milenkovic , K. Daniels

and references in 2007 PhD thesis of Daoqin Tong from the Ohio State University."

This issued is addressed in the Introduction section on page 2. We agree with you that the literature review needed improvement, and we believe that we were able to improve on that regard with the following changes:

- We added a citation to "H Younies and G O Wesolowsky. Planar maximal covering location problem under block norm distance measure". This shows another example of versions of PMCLP other than the classical one using Euclidean norm.

- We added a citation to the survey paper: "Alan T. Murray. Maximal coverage location problem: Impacts, significance, and evolution.". Instead of giving an extensive review, we use a common practice of citing a survey paper.

Now, after these additions, the piece of our Introduction doing literature review of PMCLP is as follows:

"A continuous extension of MCLP, where the facility and demand sets are located on the plane and coverage is determined by a norm function, was introduced in [8] and named Planar Maximal Covering Location Problem (PMCLP) following the advances on covering location problems on the plane that were being made at the time.

PMCLP can be seen as a class of problems, as, in general, facilities can have their coverage defined by any norm function, and the demand set, although initially defined as points, could have a more generic definition; in [4], for example, the demand set is defined as a set of rectangles; and in [20], a version of PMCLP where coverage is determined using block norms is studied. For a more comprehensive review on PMCLP and MCLP, please see [15].

In our work, because of the similarities with the problems studied by us, we are particularly interested in the Euclidean PMCLP defined with coverage being determined by the Euclidean norm. In [8], a method for this version of PMCLP was developed following the demonstration of a theorem, which showed

that a finite set of locations could be constructed to contain at least one optimal solution transforming PMCLP into a combinatorial optimization problem."

We did consider the list of suggested works we could look at to improve our literature review you have given us, however, even though we think they are great suggestions, we decided to cite two works we were more familiar with, also with the intention of not extending this part of the Introduction too much. It is worth mentioning though, that some works of Daoqin Tong are cited in the survey paper we are using.

Comment #5: "Please, clarify what is the critical parameter for your algorithm: the number of ellipses or the number of demand points?"

We addressed this issue in section 8.2 "New instances" (page 19):

"We were able to run experiments with up to 700 demand points; however, it is clear that the number of ellipses is the critical parameter of both algorithms, as even for small demand sets, we could not go over 7 ellipses."

Comment #6: "In the paper overlapping of covering ellipses is allowed, i.e. a demand point can be covered by more than one ellipse. However, it seems natural that the weight of the demand point covered by various ellipses can differ from the weight of the same point covered by only one ellipse."

Thanks for this comment, we, too, think that depending on the viewpoint, considering the weight of each point to be a function which depends on the set of ellipses covering is more natural. We believe that our methods could be trivially adapted to work with this different definition, however, we also think that we have to ponder about doing this change because this is not the traditional definition of any other PMCLP we have based our developments on; meaning that we would have to have a strong motive for this not to look like an arbitrary change.

Comment #7: "What happens if no overlapping between covering ellipses is allowed. Please, comment on this."

Thanks for the great question, we thought a lot about this, and we came up with some arguments to show you what happens if we add this constraint to our problem.

First, for an example of covering without overlapping, please see the paper "Covering point sets with two disjoint disks or squares" by Cabello, S., Díaz Báñez, J.M., Seara, C., Sellarès, J.A., Urrutia, J., Ventura, I.

This work deals with the problem of covering points using two Euclidean disks with the constraint of no overlapping being allowed. This article is cited by a work we are citing in our Introduction: "Covering many or few points with unit disks." by Mark de Berg, Sergio Cabello, and Sariel Har-Peled. This work develops an algorithm for the overlap-allowed covering of points by multiple Euclidean disks. In its Introduction, the authors cite the previous work that does not allow overlap saying: "This condition changes the problem significantly, because now issues related to packing problems arise". Based on this, we are safe to say that we should expect that adding a no-overlap constraint would change our problem significantly too.

The way our methods work is to look for solutions with points on the ellipses, this type of solution is not favorable when we have this constraint as, for

example, see the counter-example in this figure:
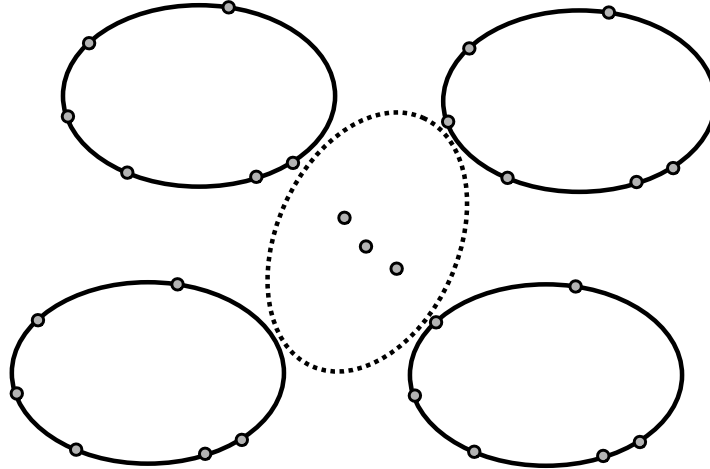
It is possible to see that there are non-overlapping solutions, however, our algorithm would fail to find any. Notice that moving the ellipse with no points on its coverage region's border in order to put at least two points on it would cause overlap.

We believe methods for the problems with this additional constraint would have to come from a different base idea than ours do.