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SYMPOSIUM ON LOCATION PROBLEMS: IN MEMORY OF LEON COOPER

THE PLANAR MAXIMAL COVERING LOCATION PROBLEM

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1. INTRODUCTION

Public facility location modeling has received a great deal of interest in the last decade. The main difference among many of the different approaches to public facilities location modeling is in the measures of effectiveness used. These measures are used to spell out how effective a particular location configuration is with respect to the overall purpose of the service and to the area the facilities are intended to serve. In contrast, private facility location models are usually cast in terms of minimizing cost, maximizing return, or maximizing revenue [ReVelle, Marks, and Liebman (1970)]. Numerous measures have been developed, the use of a particular one being dependent on the type of service provided. One example of an effectiveness measure that has been widely used is the total weighted distance or time for travel to the facilities [Hakimi (1965)]. The smaller the total weighted distance or time, the more accessible the facilities are in general. Another example of an effectiveness measure is the distance or time that the user most distant from a facility would have to travel to reach that facility, that is, the maximal service distance [ReVelle et al. (1977)]. For a given location configuration, the maximum distance which any user would have to travel to reach a facility would reflect the worst possible performance of the system. Many of the location problems incorporating the maximal service distance concept can be loosely defined as belonging to the class of covering problems.

One covering problem which has received attention is the location set covering problem developed by Toregas and ReVelle [Toregas (1971), Toregas and ReVelle (1972, 1973)]. This problem identifies the minimal number and the location of facilities, which insures that no demand point will be farther than the maximal service distance from a facility. White and Case (1974) have designated this as the total covering problem. When facility costs are constant, then the location set covering problem objective is equivalent to minimizing the costs of facility placement subject to serving all demand within the maximum service distance. This problem embodies one of the basic criteria established by the insurance

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industry in rating the coverage of fire services. In fact, the location set covering model has been requested by over 100 U.S. cities for the location planning of fire services [ReVelle et al. (1977)]. Kolesar and Walker (1974) designed a real-time operations management model for fire equipment relocation in New York city to back up stations that have dispatched all equipment to a call for assistance. Moving equipment to cover empty stations is performed to provide adequate coverage in the event of another fire in the same area and decrease response in the event of an additional alarm at any existing fires. This now famous model utilizes the concepts of the location set covering problem. Although the location set covering problem has been defined within a spatial context, it has also been used as the basis of selecting establishments to be inspected by inspection teams in the FDA [Cohen (1978)].

Recognizing that in many circumstances, it is not possible to provide the number of facilities to cover totally all demand within a desired maximal service distance, Church and ReVelle (1974) defined the maximal covering location problem. This problem may be stated as: *maximize covering (population covered) within a desired maximal service distance* S *by locating a fixed number of facilities.* When facility costs are constant, the constraint on locating a fixed number of facilities is equivalent to a budget constraint [Church and ReVelle (1974)]. Under constant facility costs the maximal covering problem can then be viewed as maximizing a service objective by facility location subject to a budget constraint. The concept of coverage embraces criteria proposed in legislation and planning agencies for emergency medical service delivery [ReVelle et al. (1977)]. Maximal covering has been applied in ambulance location [Eaton and Daskin (1980); Hamon, Eaton, and Church (1979)], fire station location [Schilling (1976)], rain gage network design for the Tennessee Valley Authority [Courtney (1978)], health clinic location [Eaton et al. (1981)], solid waste planning area determination [Church (1980)], list selection in advertising [Dwyer and Evans (1981)], air pollution monitor network design [Hoagland and Stephens (1976)], and buying center location in the coffee marketing system in Uganda [Migereko (1983)].

Church and ReVelle (1974) presented an integer-linear formulation for the maximal covering problem which can be applied to either a network or euclidean plane problem. For either case, potential facility sites are predefined and finite in number and demand points desirous of coverage are fixed and finite in number. Each demand point has a number or weight (e.g., population at that point) assigned to it which is the value associated with covering that point with a facility. A demand point is covered when the closest facility to that point is at a distance less than or equal to the desired maximal service distance. A demand node is uncovered when the closest facility to that node is at a greater distance than the desired maximal service distance. Their objective is to maximize the number served or covered within the desired maximal service distance by locating a fixed number of facilities. Church and ReVelle demonstrated that an analysis of marginal coverage values may lead a decisionmaker to consider locating fewer facilities than originally planned because of the decreasing marginal coverage as the number of facilities increase.

White and Case (1974) reported on a problem related to the maximal covering problem which they have designated as the partial covering problem. Whereas the total covering problem involves the determination of the minimum number and location of facilities such that all demand points are covered, the partial covering problem seeks to determine the location of a given number of facilities such that a maximum number of demand points is covered. The partial covering problem is actually a special case of the maximal covering problem where the covering weights assigned to each demand point are equal to one. White and Case (1974) have reported on the use of the Ignizio Heuristic in solving the partial cover problem.

Most covering models have been cast under the assumption of uncapacitated facilities or equipment. In circumstances where this is not true, the concept of coverage needs to be cast in a probabilistic framework. Church and Bianchi (1982) have given a complete survey of the covering models dealing with emergency services and expected coverage. Many models have been developed and include the expected covering model of Daskin (1981), the stochastic location set covering model of Chapman and White (1974) and the expected-coverage equipment and station location model of Church and Bianchi (1982). Church and Roberts (1982) presented a discussion of the definitions of coverage including expected coverage and multiobjective coverage. They presented a general model where coverage values are a function of distance.

Another problem that can be regarded as a total covering problem is the well-known minimax facility location problem. In the single facility problem, the facility is placed in order to minimize the largest distance to any user. The multifacility problem deals with locating facilities to minimize the maximum distance that any user is to the closest facility. Elzinga and Hearn (1972), Brady and Rosenthal (1980), Nair and Chandrasekaran (1971), Francis (1972), Chakraborty and Chaudhuri (1981), Drezner and Wesolowsky (1980b), Jacobsen (1982), and Wesolowsky (1973) are examples dealing with the single facility minimax case under different assumptions of metric (e.g., rectilinear, euclidean, and l_p). Dearing and Francis (1974), Wesolowsky (1972), Drezner and Wesolowsky (1978), Charalambous (1981), Love, Wesolowsky, and Kramer (1973), Elzinga, Hearn, and Randolph (1976), and Elzinga and Hearn (1972) have dealt with the multifacility case. A related problem is the minisum problem with maximum distance constraints where the total weighted distance is minimized subject to constraints on the maximum distance any point is to the closest facility. Schaefer and Hurter (1974) and Hurter, Schaefer, and Wendell (1975) give an approach for this type of problem defined on the plane.

Although the minimax problem on the plane has been the subject of considerable research, the maximal covering, partial covering, and expected coverage problems defined on the plane have not. In the case of the maximal covering approach given by Church and ReVelle and the partial covering approach developed by White and Case, the existing solution techniques are designed for only the case where the potential facility sites are restricted to a finite number of prespecified points. This paper deals with the development of an optimal solution technique to solve the maximal covering and partial covering problems defined on a plane using euclidean distance measure or rectilinear distance measure where all points are potential facility sites rather than restricted to a finite set of prespecified points. These problems will be called the planar maximal covering location problem under euclidean distance measure (PMCE) and planar maximal covering location problem under rectilinear distance measure (PMCR), respectively. First, the solution technique developed by Church and ReVelle for the MCLP where potential sites are finite and fixed in advance is reviewed. Then, extensions of this technique will be made for solving the unrestricted site problem. Third, computational experience is reported for this technique and several sample solutions are given. Fourth, several extensions of this approach are noted. Finally, interesting comparisons are made between several solutions of the maximal covering problem where facilities can be placed anywhere.

2. SOLVING THE MAXIMAL COVERING LOCATION PROBLEM WITH PRESPECIFIED FACILITY SITES

The maximal covering location problem was formulated by Church and ReVelle (1974) in the following manner:

$$(MC) \qquad \text{Minimize } Z = \sum_{i \in I} a_i y_i$$

subject to

$$(1) \qquad \sum_{J \in N_i} x_j + y_i \geq 1 \qquad (\text{for all } i \in I)$$

$$(2) \qquad \sum_{j \in J} x_j = p$$

$$(3) \qquad x_j = (0,1) \qquad (\text{for all } j \in J)$$

$$(4) \qquad y_i = (0,1) \qquad (\text{for all } i \in I)$$

where

I = the set of demand points,
J = the set of facility sites,
S_i = distance beyond which a demand point i is considered uncovered,
d_{ij} = shortest distance from point i to site j,
$x_j = \begin{cases} 1 \text{ if a facility is allocated to site } j, \\ 0 \text{ otherwise,} \end{cases}$
$N_i = \{j \in J \mid d_{ij} \leq S_i\}$, the set of facility sites eligible to provide coverage to point i,
$y_i = \begin{cases} 1 \text{ if demand point } i \text{ is not covered,} \\ 0 \text{ if demand point } i \text{ is covered within the maximal service distance } S_i, \end{cases}$
a_i = population to be served at demand point i, and
p = the number of facilities to be located.

The objective is structured to minimize the number of people not covered within the desired distance. Church and ReVelle (1974) have shown that minimizing the number not covered is equivalent to maximizing the number that are covered within the maximal service distance. The first type of constraint requires

y_i to equal one unless one or more facilities are established at sites in the set N_i. The number of facilities allocated is restricted to equal p in constraint (2).

This formulation has been utilized in optimally solving the maximal covering location problem using linear programming and a branch-and-bound algorithm. Computational experience and further refinements of the approach are given in Church and ReVelle (1974) and Church (1974). The above formulation can also be used to solve optimally the partial covering problem defined by White and Case (1974) by assigning $a_i = 1$ for all $i \in I$.

Church and ReVelle (1974) have shown how the maximal covering location problem can be defined as a p-median problem. This means that all solution procedures for the p-median problem can be used as well for the maximal covering problem—these procedures include lagrangian relaxation with subgradient optimization and the generalized assignment problem approach, among others. Klastorin (1979) has shown how the MCLP can be structured as a generalized assignment problem.

3. A GENERAL APPROACH TO SOLVING THE PLANAR MAXIMAL COVERING LOCATION PROBLEM UNDER EUCLIDEAN DISTANCE MEASURE

Now let us consider a planar problem where we have a given number of demand points and where we will allow any point on the plane to be a potential site. Let P_i denote the ith demand point ($i = 1, 2, \ldots, n$), D_i the closed circular disk of radius S_i centered at P_i, and K_i the circle which is the circumference of disk D_i. The circle intersect point set (CIPS) is defined to consist of all demand points plus all points in which some pair K_i, K_j ($i \neq j$) of circles meet. Thus CIPS is a finite set; indeed, it has at most n^2 members. One key observation is the following:

Theorem 1: There is at least one optimal solution to the PMCE problem in which all facility locations lie in the CIPS.

Proof: Consider any optimal solution. We show there is another solution, at least as good (hence, also optimal), with all locations in the CIPS. If some point P of the solution lies in no disk D_i, then it covers no demand point; replacing P by any P_i yields a solution at least as good. In what follows, assume all such replacements have already been made.

If some point P of the solution lies in exactly one disk D_i, then it covers P_i but no other P_j; replacing P with P_i yields a solution just as good. Assume all such replacements have already been made.

If some point P of the solution lies in two or more of the D_i's but not on any K_i, move it in any direction until it first meets the boundary of some D_i containing its original position. Replacing P by its new position yields a solution at least as good. Assume all such replacements have already been made.

If some point P of the solution lies on exactly one K_i and in the interior of one or more D_j's, move it along the K_i until it meets the boundary of one of the D_j's or until a complete rotation is made about P_i without encountering a boundary of one of the D_j's. If a move along K_i meets the boundary of another D_j, replacing P with this intersection point yields a solution at least as good. If a complete rotation about P_i on D_i is made without encountering another boundary D_j, then disk D_i lies wholly within the other disks D_j, and replacing P by P_i yields a solution at least as good. Assume that all such replacements have been made. Then the current solution has all facility locations in the CIPS, and the theorem is proved.

The implications of Theorem 1 are valuable. Theorem 1 implies that one way of obtaining an optimal solution to the PMCE problem is to obtain the best solution comprised entirely of points in the intersect point set (i.e., obtain the best intersect point set solution). Therefore, the following solution technique can be outlined for solving the p-facility PMCE problem:

1. Develop the circle intersect point set for the desired problem. This set has at most n^2 points where n equals the number of demand points.
2. Obtain the best p-facility solution from the above set using formulation MC and linear programming.

The above approach is similar to what Hakimi (1965) developed for the network p-median problem. Essentially he showed that there is at least one optimal solution to a p-median problem that is composed entirely of nodes of the network. Thus, he showed that one way of determining an optimal solution to the p-median problem is to find the best all node solution.

This result is also similar to that of Wendell and Hurter (1973) proving that for a rectilinear multifacility Weber problem only certain intersection points need to be considered. These points were defined by the set of points (x,y) such that $x \in \{x_i\}$ and $y \in \{y_i\}$ where (x_i,y_i) are the coordinates of the demand points. Hansen, Perreur, and Thisse (1980) have described further properties associated with this type of problem.

The use of intersecting circles is not new in solving constrained planar problems, except that they have been limited to analyzing one-facility problems. Schaefer and Hurter (1974) use the concept of intersecting circles in dealing with a one-facility Weber problem with maximum distance constraints. It is interesting to note that the enclosed region used by Drezner and Wesolowsky (1980a) in solving a one-facility maximin problem would be the same region analyzed in a PMCE problem if all demand could be covered by a single facility. In fact, one of the CIP on the boundary or interior to this region would be identified as an optimal covering point. This region was also analyzed by computer graphics in Brady and Rosenthal (1980) in a one-facility case where a constrained minimax problem was solved.

In solving the PMCE problem with the above approach, a difficulty arises with the size of the CIPS. For $n = 50$, there could be 2500 points, which means that formulation MC could have 2500 x_j variables. There then seems to be an obvious limit to the approach if all points in the CIPS need to be included in Step 2.

4. A REVISED APPROACH TO SOLVING THE PLANAR MAXIMAL COVERING LOCATION PROBLEM UNDER EUCLIDEAN DISTANCE MEASURE

Suppose that there are two circle intersect points P_u and P_v such that P_u covers exactly the same demand points as P_v and possibly more, i.e., $C_i(P_u) \geq C_i(P_v)$ for all i where $C_i(P_k) = 1$ if point P_k can cover demand point i and 0 otherwise. In this case,

P_u is said to dominate P_v since P_u covers the same demand points as P_v plus possibly more. If $C_i(P_u) = C_i(P_v)$ for all i, then P_u can cover exactly the same demand as P_v. For this instance, either one can dominate the other. Either can be selected as the dominating one.

A new set can be developed from the CIPS by dropping all dominated points. This set, called the reduced circle intersect point set (RCIPS), has the same important property that the CIPS has. Namely, an optimal solution exists to the euclidean MCLP which consists of points belonging entirely to the RCIPS. The proof of this fact follows the same type of argument given for Theorem 1. Essentially, an optimal solution to the PMCE can be generated where all sites are members of the RCIPS, by replacing each dominated site with a dominating one.

Using the above information, a revised approach to solving the PMCE can be outlined as follows:

a. Generate the CIPS for the desired problem.
b. Develop a RCIPS from the CIPS.
c. Use formulation MC and linear programming to identify an optimal PMCE solution using the points in RCIPS.

It should be noted here that similar types of reduction techniques have been used in solving the location set covering problem. Toregas (1971) and Toregas and ReVelle (1973) and Roth (1969) have given detailed analyses of reduction techniques. The domination principle used above is the same as the COLDOM procedure used by Toregas (1971).

5. COMPUTATIONAL EXPERIENCE

The above optimal three-step procedure has been applied to many euclidean maximal covering location problems based on the 55-node problem used by Swain (1971). For the first step, a FORTRAN IV program called EUGENE was developed to generate the CIPS for any given problem. The second step was performed by a PL1 program called REDUCE. Then using a linear programming generating program (LPGEN) a problem file of formulation MC was developed in such a manner that the IBM Mathematical Programming System (MPS360) could access the problem file and perform the necessary linear programming algorithm. The specific details of using MPS360 and formulation MC are given in Church (1974) and Church and ReVelle (1974). Each process step was performed on the IBM 360 model 65.

When the solution for more than one number of facilities was desired for a given problem and stated set of service distances, postoptimal routines of the MPS360 system were used. For example, suppose that the desired range in the number of facilities was $p = 1$ to 10. The problem file was set up for the linear program with $p = 1$. After the MPS360 system obtained an optimal solution for this value of p, the option for parameterization of the right-hand side was used to obtain solutions for $p = 2$ to 10. The cost of computing each additional solution ($p = 2$ to $p = 10$) is only a small increment over that for the first solution ($p = 1$). In obtaining all integer optimal solutions, fractional linear programming solutions were resolved by a branch-and-bound algorithm.

TABLE 1: Computational Results of Solving Several Planar Maximal Covering Location Problems Based on the 55-Node Problem

	Step 1		Step 2		Step 3			
Value of S*	Number of Intersect Points	EUGENE Execution Time (secs.) IBM 360/65**	Number of Points in RCIPS	REDUCE Execution Time (secs.) IBM 360/65**	Range of P and Number of Solutions	Number of Fractional Solutions	Branch and Bound Execution Time (secs.) IBM 360/65**	Total MPS360 Time (secs.) IBM 360/65**
10.00	1628	6.24	101	51.01	1 to 7	1	14	40
12.00	2047	6.67	91	63.33	1 to 5	—	—	34
15.00	2526	8.23	73	59.33	1 to 4	—	—	27.77
19.00	2863	8.19	41	50.66	1 to 3	—	—	13.22
20.00	2908	8.65	30	43.00	1 to 3	—	—	9.70
				Total Number of Solutions	22	1		

*The maximal service distance for each demand was fixed at the same value (S).
**All execution times include input and output functions.

Table 1 gives the performance of the linear programming step in solving maximal covering problems based on the 55-node problem defined by Swain (1971). More than 90 percent of the time the linear programs terminated all integer optimal. In Table 1, computation times are given for the MPS360 step as well as set up times and statistics involved for Steps 1 and 2. It is important to note that the amount of reductions are numerous and the size of the RCIPS is usually quite small. The amount of time used in producing a CIPS by EUGENE is generally small, but the reduction step requires considerably more execution time. However, the REDUCE program is admittedly inefficient and it may be possible to lower the amount of needed CPU time by a considerable amount. Nevertheless, the average cost in obtaining one of the solutions given in Table 1 is relatively small. It should be noted that the RCIPS is independent of the a_i values and need not be regenerated to handle several sets of population estimates.

6. EXAMPLE SOLUTIONS

For any desired set of maximal service distances S_i and a given number of facilities to be located, a planar maximal covering location problem using euclidean distance measure can be defined. By holding the set of maximal service distances fixed and solving over a range of p-values, one can create a cost-effectiveness curve which outlines the change in effectiveness (in terms of coverage) as the system cost is altered by increasing or decreasing the number of available facilities. The use of this type of curve in decision making is described by Church and ReVelle (1974).

Figure 1 shows two cost-effectiveness curves for the 55-node problem where the maximal service distance for each demand is fixed at 10. The dashed line curve is associated with the MCLP where potential facility points are restricted to be at demand points. The solid line curve is associated with the PMCE problem where facilities can be placed anywhere. For both curves, the increase in the number of people covered by the addition of one facility decreases as the total number of facilities increases. For the dashed line curve, the solution for $p = 9$ represents the

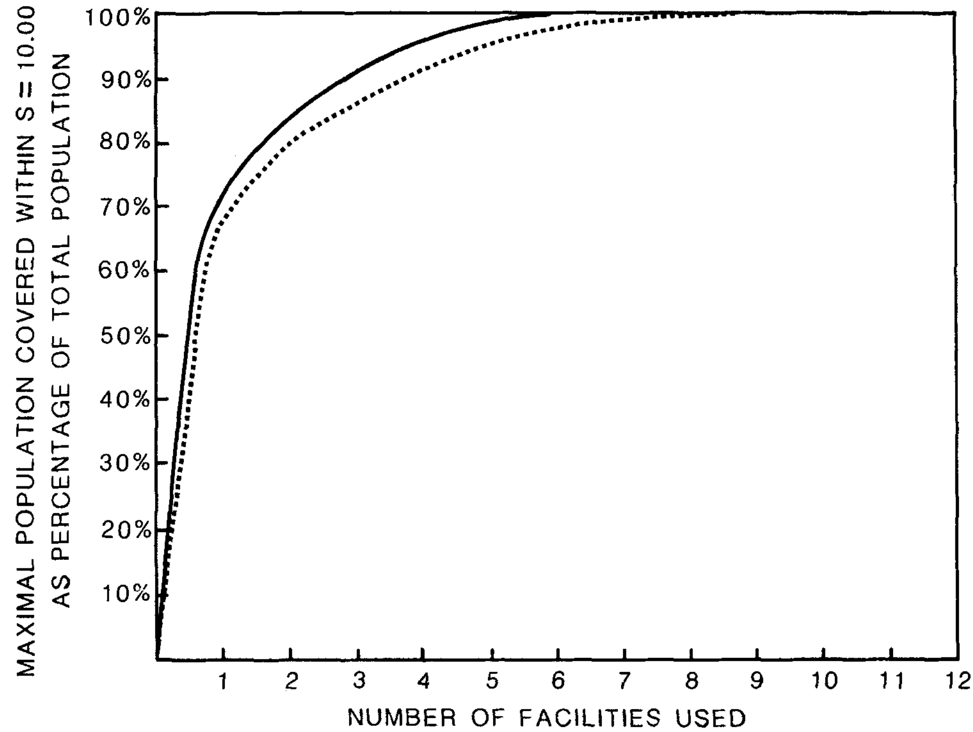


FIGURE 1: Two Cost-Effectiveness Curves Derived for a Maximal Service of 10 for Each Demand Point.

Note: Data for 55-node problem from Swain (1971).

lowest p-value for which 100 percent coverage is obtainable when facilities are restricted to demand points. This solution, therefore, represents an optimal solution to the location set covering problem where facilities are restricted to demand points. Similarly, the solution for $p = 7$ on the solid line curve represents a solution for the location set covering problem where facilities may be placed anywhere. Most important though, is the fact that only seven facilities are required for the PMCE in comparison to the nine facilities required for the MCLP. This is a 22 percent difference in the number of facilities necessary for total coverage.

Since the PMCE problem is a less restricted version of the MCLP, one already knows that coverage obtainable under the PMCE would be greater than or equal to the coverage obtainable under the MCLP for the same given number of facilities. However, for most of our test problems coverage under the PMCE has been strictly greater than the coverage obtainable under the more restrictive MCLP. Table 2 gives maximal coverage values for the 55-node problem for the PMCE and MCLP over a range of p-values (number of facilities). Notice that for each value of p the relative difference in coverage is small.

Although the relative difference in the maximum coverage is small between the two optimal solutions for any value of p, there is still a large difference between

TABLE 2: A Comparison of Coverage Values for a Range of Values of P for the PMCE and MCLP Problems Where the Maximal Service Distance was Fixed at 10 for Each Demand

P-Value	Potential Facilities Placed Only at Nodes Cover Value	Potential Facilities Placed Anywhere Cover Value
1	425	437
2	502	523
3	548	568
4	581	608
5	609	626
6	625	636
7	633	640
8	638	640
9	640	640

the two types of solutions in the amount of facilities necessary to cover a given number of population. For example, assume that coverage of 625 is desired. For the MCLP, six facilities are necessary. However, for the PMCE only five facilities are required. Note that beyond $p = 4$ it requires fewer facilities under the PMCE problem to obtain the same coverage possible by the p-facility MCLP. This is an important result: not only does the PMCE solution cover more for each value of p, but beyond $p = 4$, the MCLP in many cases requires $p + 1$ facilities to cover what the PMCE can do with p facilities. The basic reason for the improvement in coverage of the PMCE over that of the MCLP is that the planar problem is less constrained where facilities can be placed anywhere (as opposed to being located at the demand points). This flexibility allows placement that yields an increase in coverage. The increase in coverage associated with this flexibility in the PMCE in some cases is greater than the marginal coverage provided by an additional facility in the MCLP. Hence, the p-facility PMCE solution is sometimes superior to the $p + 1$ facility MCLP solution.

Figure 2 gives a comparison of two solutions where the dashed line partitions represent an optimal solution to the five-facility MCLP and the solid line partitions represent an optimal solution to the five-facility PMCE problem. For both problems the maximal service distance for each demand was fixed at 10. The partitions denote which demand points are served by a facility within the distance of 10. The heavily-circled points in Figure 2 represent the demand points chosen to house facilities in the optimal MCLP solution. The five sites indicated by squares represent the optimal PMCE solution. For example, point 55 is in a solid line partition. This means that the closest facility to point 55 is the square in that same partition and that point 55 is within a distance of 10 of that facility. However, point 55 is not in a dashed line partition. This means that point 55 is not covered by the optimal MCLP solution. The optimal PMCE solution covers five more demand points than the optimal MCLP solution where facilities are restricted to nodes. In addition, the maximum coverage possible increases from 609 to 626 by allowing facilities to be placed anywhere rather than only at demand points.

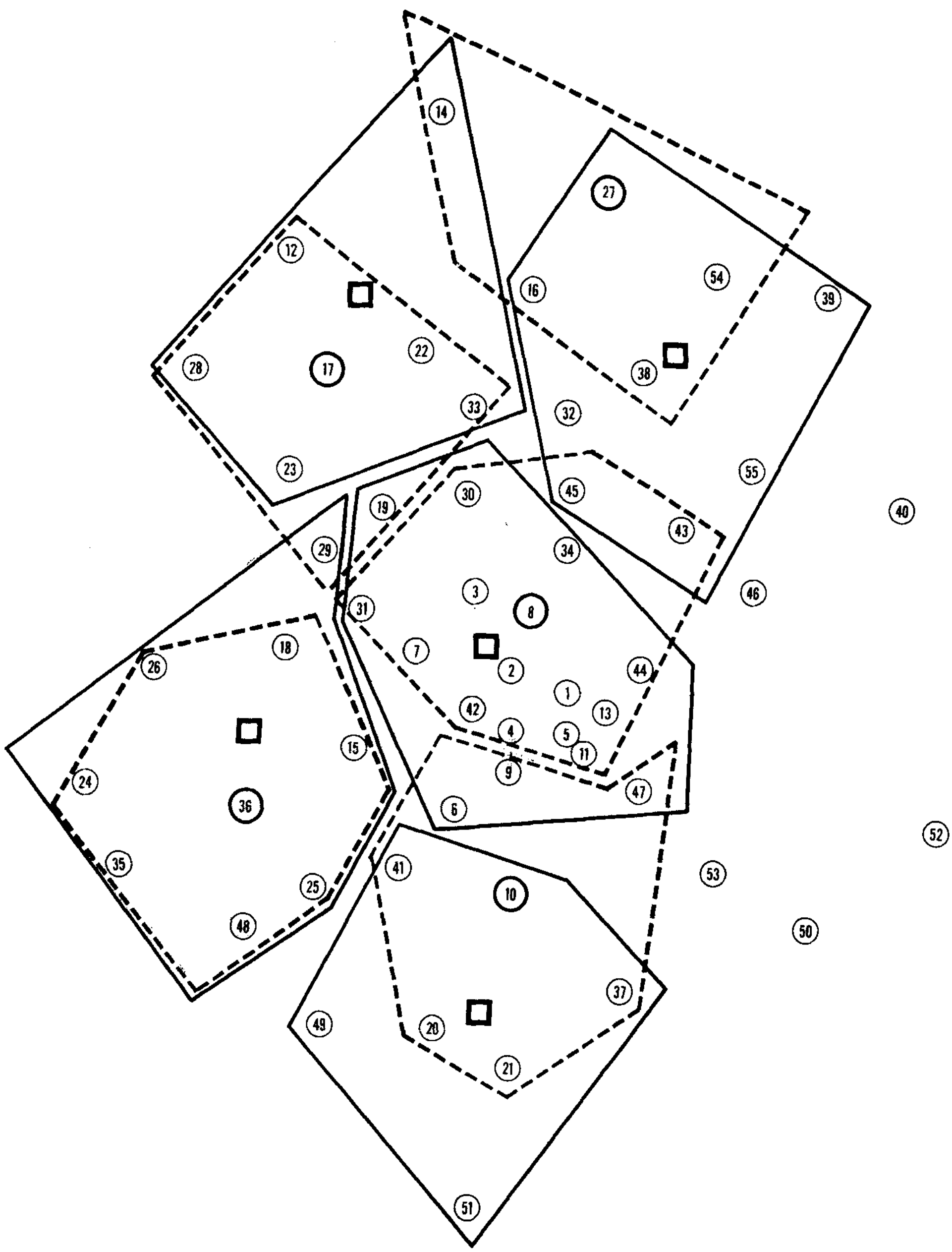


FIGURE 2: A Comparison of an Optimal 5-Facility MCLP Solution and an Optimal 5-Facility PMCE Solution Where the Maximal Service Distance Was Fixed at 10 for Each Demand.

As mentioned previously, it takes six facilities for the problem where facilities are restricted to demand points to cover as much as five facilities can cover when site placement is unrestricted. A comparison of these two solutions is given in Figure 3. Notice that the five-facility PMCE and six-facility MCLP cover exactly the same demand points except points 32, 40, 46, and 49. The amount of coverage obtained by both solutions is essentially the same. The main difference is that the MCLP needs one more facility than the PMCE to provide that same coverage.

8. EXTENSIONS

It was pointed out in the presentation of the cost-effectiveness curve given in Figure 1 that the seven-facility optimal solution for the PMCE represents an optimal location set covering solution where facility placement is not restricted to demand points. This is true because this is the smallest number of facilities such that 100 percent coverage is achieved for the desired set of maximal service distances. This implies that optimal solutions to the planar location set covering problems where facility placement is unrestricted can be identified by using the above PMCE solution approach. Furthermore, it can be seen that there exists an optimal solution to the planar location set covering problem which is comprised entirely of points in the RCIPS. This means that the reduction algorithm of Toregas and ReVelle (1973) can be applied to either the CIPS or the RCIPS to identify optimal location set covering solutions for the unrestricted site problem.

In preceding sections a technique was developed to solve the PMCE where facility placement can be anywhere on the plane. Now consider the use of the metropolitan metric (rectilinear) distance measure for the planar maximal covering location problem instead of the euclidean distance measure. This problem is designated the planar maximal covering location problem with rectilinear distance measure (PMCR). The metropolitan or rectilinear distance (sometimes called grid distance or Manhattan metric) between two points is defined to be the sum of the absolute values of the differences of the vertical and horizontal coordinates of the two points. The metropolitan metric measure has been the subject of previous research [Beaumont (1981)]. Figure 4a gives a pictorial view of rectilinear measure between points P_k and P_l. The distance between P_k and P_l, designated $D(P_k, P_l)$, is $a + b$.

A diamond-shaped boundary can be developed for demand point i using a fixed distance S_i (i.e., $S_i = a + b$) such that any point on the boundary is exactly distance S_i from that demand point. An example of such a boundary is given in Figure 4a for point P_l. Any point lying solely within the diamond-shaped boundary centered around point P_l is less than distance S_i from point P_l. Therefore, any facility located on or within the boundary can cover point P_l. This is essentially the same property outlined for the circle of radius S_i about demand point i in the PMCE problem.

Points P_3 and P_4 in Figure 4 are called diamond intersect points (DIP). These points are the points of intersections of the diamond-shaped boundaries defined for fixed distances S_1 and S_2 centered around the demand points, P_1 and P_2, respectively. Figure 4b gives an infinite point set intersection of two diamond

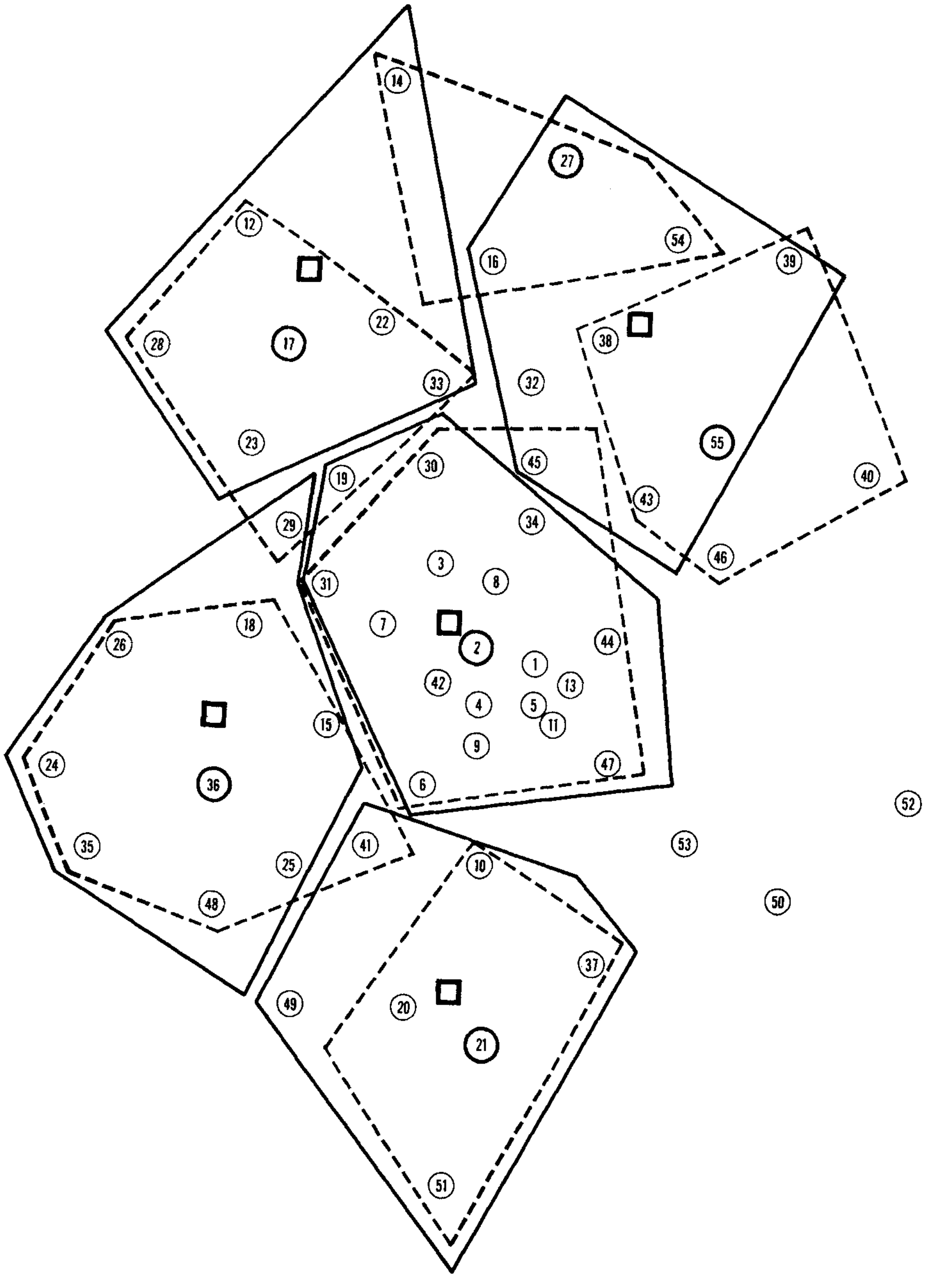
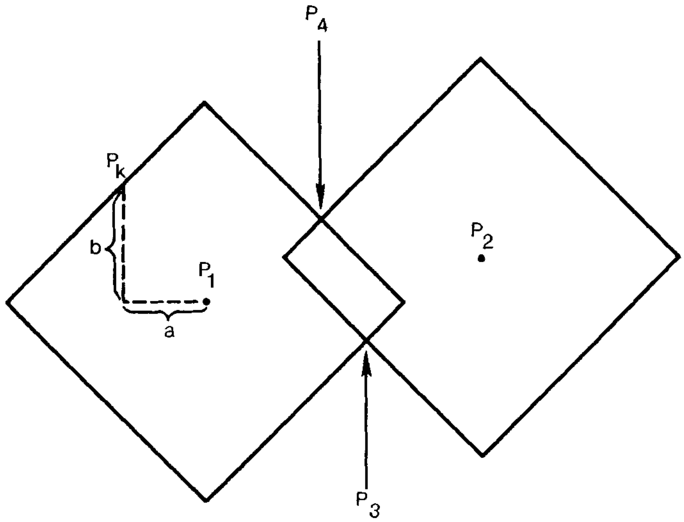
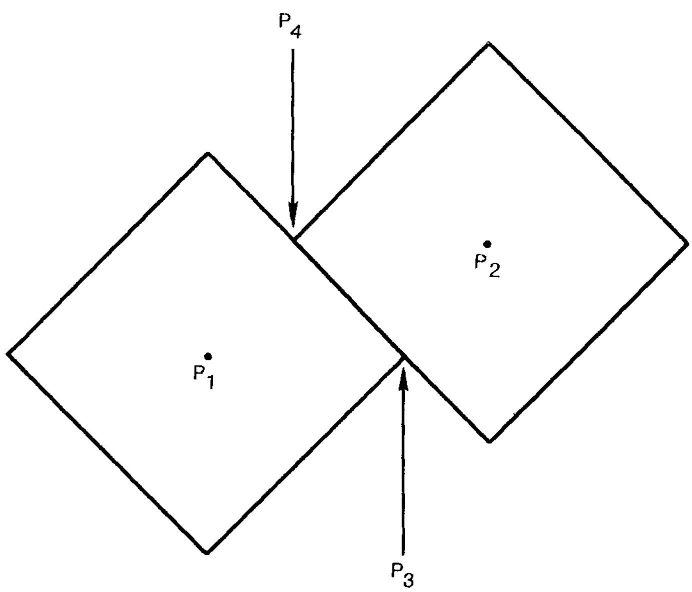


FIGURE 3. A Comparison of an Optimal 6-Facility MCLP Solution and an Optimal 5-Facility PMCE Solution Where the Maximal Service Distance Was Fixed at 10 for Each Demand.



$$D(P_k, P_1) = |X_k - X_1| + |y_K - y_1| = a + b$$

a. Definition of Diamond Intersect Points.



b. An Example of an Intersection Set Being a Line Segment.

FIGURE 4: An Example of Diamond-Shaped Boundaries Defined by Rectilinear Distance Measure.

boundaries. For this case the two endpoints of the intersection line segment are the diamond intersect points. Even though the intersection of the diamond boundaries in Figure 4b is a line segment, the diamond intersect points that represent this intersection will be the endpoints of the line segment. This restriction is made so that there will be at most two diamond intersect points defined by any intersection of two boundaries. The set of all diamond intersect points plus the demand points comprises the diamond intersect point set (DIPS). Thus, the DIPS has at most n^2 points where n is the number of demand points. It can be easily shown that there exists an optimal solution to the PMCR problem which consists of points belonging entirely to the DIPS. The proof of this fact is analogous to that presented for Theorem 1. In addition, the reduction procedure used in the solution method for the PMCE problem can be utilized in developing a reduced diamond intersect point set such that an optimal solution to the PMCR problem exists which is comprised entirely of points in this set. The validity of this fact can be established in the same manner as the reasoning stated for the use of the RCIPS in the PMCE problem. Thus, an optimal approach to the PMCR can be stated as follows:

1. Generate the diamond intersect points set associated with the maximal service distances S_i on the desired problem.
2. Use the reduction algorithm to develop a reduced diamond intersect point set.
3. Use formulation MC and linear programming to identify an optimal p-facility PMCR solution.

In addition to being able to solve optimally the PMCR problem, the planar location set covering problem under rectilinear distance measure where facility placement is unrestricted can be solved by using the Toregas and ReVelle reduction approach applied to the DIPS or RDIPS.

9. CONCLUSIONS

In summary, it has been shown that existing techniques for solving the maximal covering problem can be utilized in solving the planar maximal covering location problem defined under euclidean distance measure (PMCE). This approach rests on the basis that an optimal solution exists which is comprised entirely of points in the circle intersect point sets. A proof of this fact has been presented. Furthermore, optimal solutions to the PMCE problem have been determined which show the importance of the PMCE problem relative to the MCLP. Computational experience is cited which indicates that solutions to the PMCE problem can be generated for a relatively small cost. In addition, it has been indicated that the above approach is also applicable for the planar maximal covering problem utilizing a rectilinear distance measure (PMCR). Furthermore, it has been stated that the reduced circle intersect point set (RCIPS) and the reduced diamond intersect point set (RDIPS) can be used with the reduction technique of Toregas and ReVelle to solve planar location set covering problems where facility placement is unrestricted and where distance is measured either by the euclidean measure or by the rectilinear measure.

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