

# On model reduction and equivalence in ABMs

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## 1 Poisson models and Markov processes

Let a Poisson model be a dynamic system on some discrete state space such that the probability that the process will transition from state  $x$  to state  $x'$  in time  $dt$  is given by  $\rho(x'|x)dt$ . If we let  $\rho_{ij}$  be a transition matrix such that

$$\rho_{ij} = \begin{cases} \rho(i|j) & \text{if } i \neq j \\ \sum_{k \neq j} \rho(k|j) & \text{if } i = j \end{cases}$$

and let  $X$  be a vector representing a probability distribution over the state space, then we can define the Poisson model as a continuous time dynamic system

$$\frac{dX}{dt} = \rho X$$

so that

$$X(t) = e^{\rho t} X(0) = e^{(\mu - I)rt} X(0) = \sum_{k=0}^{\infty} \frac{(rt)^k e^{-rt}}{k!} \mu^k X(0)$$

where  $r = \max_j \sum_{k \neq j} \rho(k|j)$  is a scalar and  $\mu = \frac{\rho}{r} + I$ . It can be seen that each entry of  $\mu$  is non-negative and the sum of entries in each column is 1, so  $\mu^k X(0)$  is the state of a discrete time Markov process at time  $k$ .

Since  $\mu r t$  commutes with  $I r t$  we can separate and expand to give

$$X(t) = e^{-rt} e^{\mu r t} X(0) = \sum_{k=0}^{\infty} \frac{(rt)^k e^{-rt}}{k!} \mu^k X(0)$$

which we recognize as a sum of powers of  $\mu$  weighted with a Poisson distribution. So, each continuous-time Poisson model  $\rho$  has an associated discrete-time Markov process  $\mu = \frac{\rho}{r} + I$ . The two are related by the fact that, for a given start state distribution, the state of the Poisson model at time  $t$  is the weighted sum of states in a trajectory of the Markov process, where the weights are given by a Poisson distribution with rate  $rt$ .

This means, among other things, that as  $t \rightarrow \infty$  the state distribution of a Poisson model tends to a uniform distribution over the attractor of its associated Markov model. So every Poisson model tends to a steady state distribution (i.e. a single point in distribution space).

## 2 Social norms and social agents

Let a *social role* be a tuple  $\langle S, \sigma_0, A, Q \rangle$  where  $S$  is a domain of social states of the actor of which  $\sigma_0$  is the default state given at the beginning of an episode,  $A$  is a domain of social actions that can be performed by the actor and  $Q$  is a social-quality function in  $S \times A \rightarrow \mathcal{R}$ .

Let a *social norm* be defined as a tuple  $\langle r_1, r_2, \tau \rangle$  where  $r_1$  and  $r_2$  are social roles and  $\tau$  is a state transition function in  $S_1 \times A \times S_2 \rightarrow S_1 \times S_2$  saying that if two agents playing roles  $r_1$  and  $r_2$  are in states  $s_1$  and  $s_2$  respectively and agent 1 performs action  $a$  on agent 2 then, after the action the agents have state  $s'_1$  and  $s'_2$  respectively.

Let a *social connection* be a tuple  $\langle N, s_1, s_2 \rangle$  where  $N$  is a social norm,  $s_1$  is the social state of the first agent in  $N$  and  $s_2$  is the social state of the second.

Let a *social network* be a directed graph, where each edge is associated with a social connection. So, each agent in a social network can be thought of as being in a number of social states (one for each edge) and each social state comes with a constraint on behaviour in the form of the social Q-functions.

Let a *social agent* be a node in a social network. The agent's internal state is the set of social states on its edges. The agent's behaviour towards each agent it has a social connection with is constrained by the Q-function associated with that connection.

Agents in a social network need not, and often do not, physically exist. God, the government and Microsoft do not exist but many people model them as agents to which they have social connections. There raises no difficulty as long as the physical meaning of social actions involving the non-existent agent don't require its physical presence, and there's no reason it should need to. These actions can be instrumental in the coordination of behaviour between physically existent agents. I can be employed by, or sue Microsoft without ever expecting to meet face to face.

### 2.1 Social network as closure

If an agent models its world and assumes every other agent models the world in the same way, then every agent in the model contains an instance of the model and we have an infinite regression, so we need some kind of closure.

If we assume that the state of the social network is public knowledge, through observation, gossip etc. (where  $X$  is public knowledge if everyone knows  $X$  and it's public knowledge that everyone knows  $X$ ) then, under discounting, agents can model their world as consisting of agents that act according to the public knowledge. For example, agents can use the social norm as the off-tree approximator and run Monte-Carlo tree search to account for potential breaking of the social norm. In this way, agents are able to come to the same equilibrium.

## 2.2 Social network as abstraction

In many situations, we're only interested in the social actions that each agent performs, so we can model a set of agents as interacting via social actions. In reality, these social actions are grounded in physical actions and if we're interested in the details of this, then each social action must come with an extension which is a set of sequences of physical actions that qualify as this social action. The agent must then choose a physical action in consideration of the social interpretations of that action. In general, the physical substrate of the agent's social world adds additional constraints to the sequence of social actions it may perform.

Modelling at the physical level also allows agents to perform actions that have multiple social meanings, so a single physical action or sequence of actions can perform many social actions. It is also possible for a physical action to be unclassifiable as a social action yet state changing in the social norm (e.g. I sit down at a restaurant, the waiter brings me the menu, I set fire to it). In this case the norm is considered null and new norms need to be negotiated.

## 3 How do agents learn social norms?

[Language, telling stories. By accidentally breaking social norms and being told off.]

A learner of social norms that only has access to observed physical behaviours must somehow learn the abstraction function, the social quality function and the transition function. If thrown into a world where a social norm is already prevalent and stable, and we assume people are guided by social norms, then all this can theoretically be done by minimising the error in predicting other's behaviours.

Alternatively, can can abstract over the physical and assume agents make observations at the social level of abstraction. This glosses over the learning of the abstraction function, which we learn from gossip etc. and focusses on the social dynamics. If agents are able to make social observations of their surroundings and their interactions then an agent can construct a probabilistic policy for each social norm (or if the norm type is itself negotiated, and we have a universal state and action space, then this can be a single policy...although it's hard to see the advantage of this compared to stratification by norm).

## 4 how do agents negotiate roles to form new social connections?

Context. Verbally.

Again, we can abstract over the physical details of this and assume there is a public vocabulary of norms, and connections have an associated social norm type-id.

## 5 How do social norms evolve?

If agents choose to create social connections at a rate proportional to their expected reward then some social norms will be popular and others not. Ultimately, some may die out.

New norms can be created by more or less intelligent mutation of existing norms (perhaps even unintentional misunderstandings or chance events), or specialisation of general-purpose norms: [this assumes there's a hierarchy of abstraction among norms, which we haven't talked about yet...] for example, hunting large game and sharing the meat may emerge as a specialisation of you-help-me-and-i'll-help-you norm...?

### 5.1 Punishment and praise in social norms

In a purely Q-learning environment, punishment and praise after the fact is ineffective in one-shot games. However, if a social norm consists of a whole policy, and is agreed upon at the beginning of an interaction, then a credible threat of contingent punishment and/or praise can be instrumental in making the policy a Nash equilibrium.