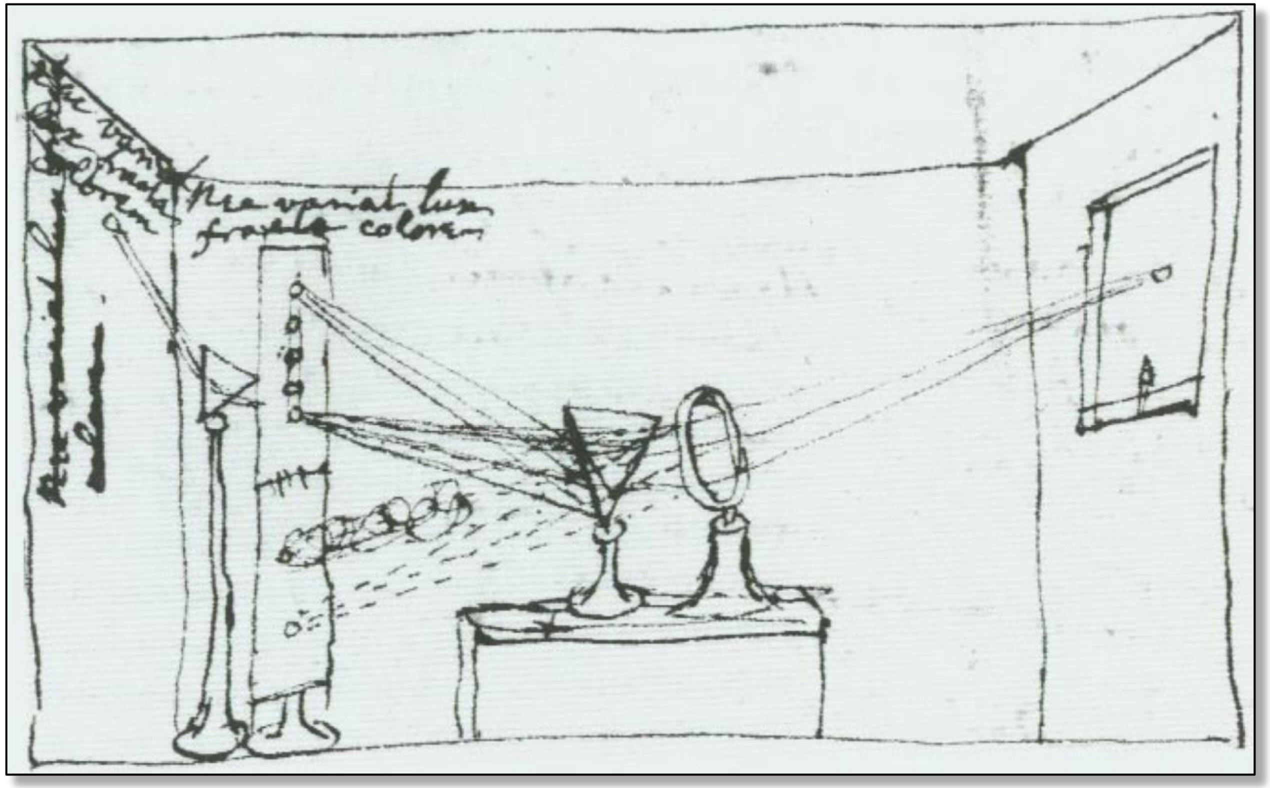
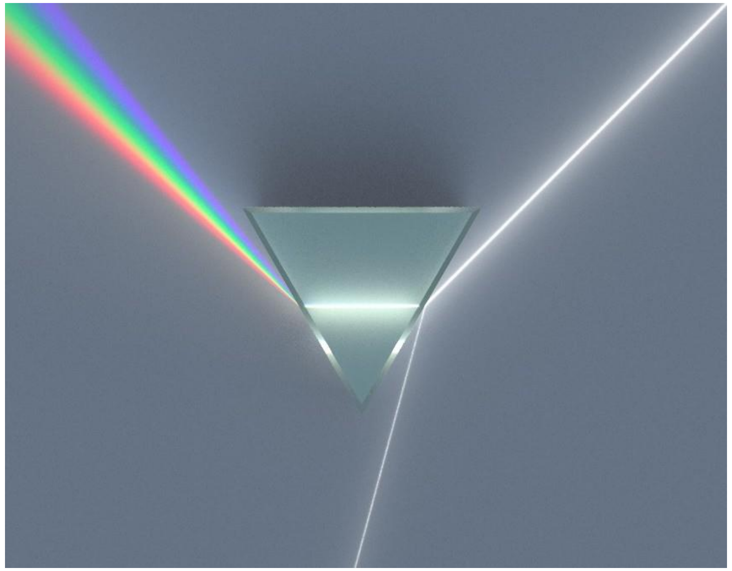
Introduction to color science

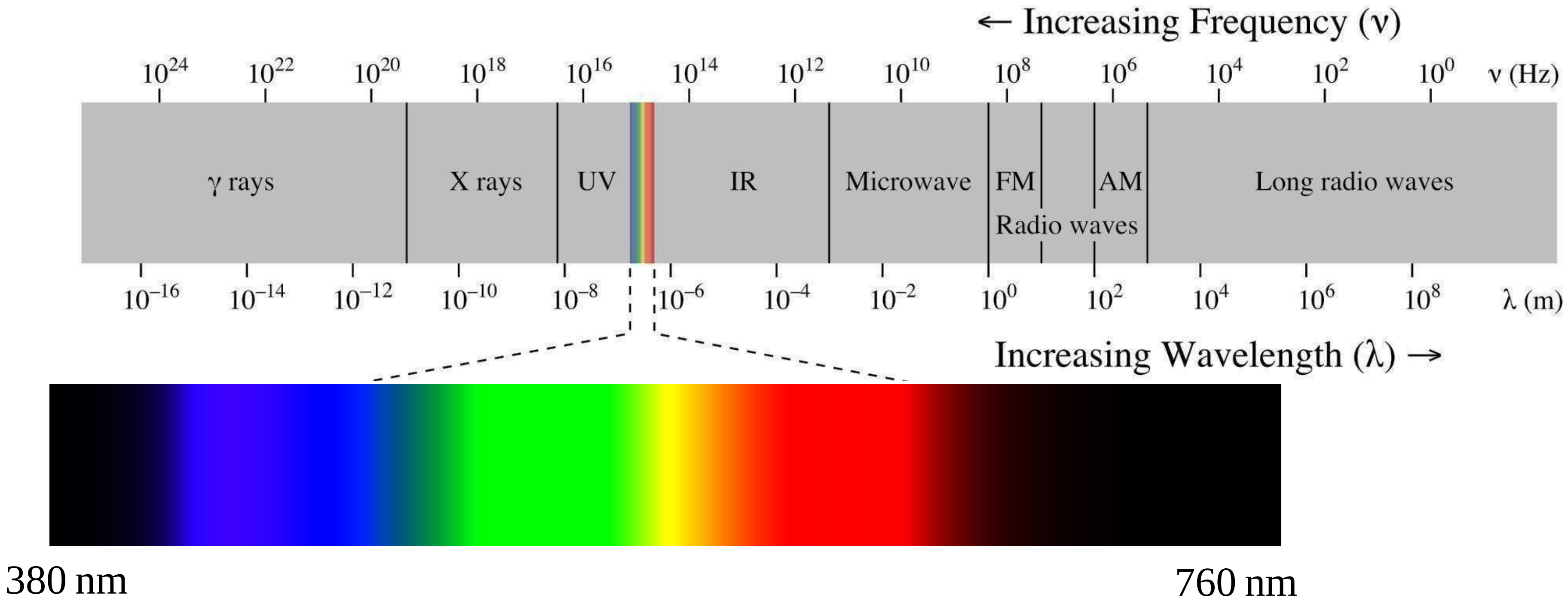
- Trichromacy
- Spectral matching functions
- CIE XYZ color system
- xy-chromaticity diagram
- Color gamut
- Color temperature
- Color balancing algorithms

Newton’s Prism Experiment - 1666





Color: visible range of the electromagnetic spectrum



Radiometric quantities

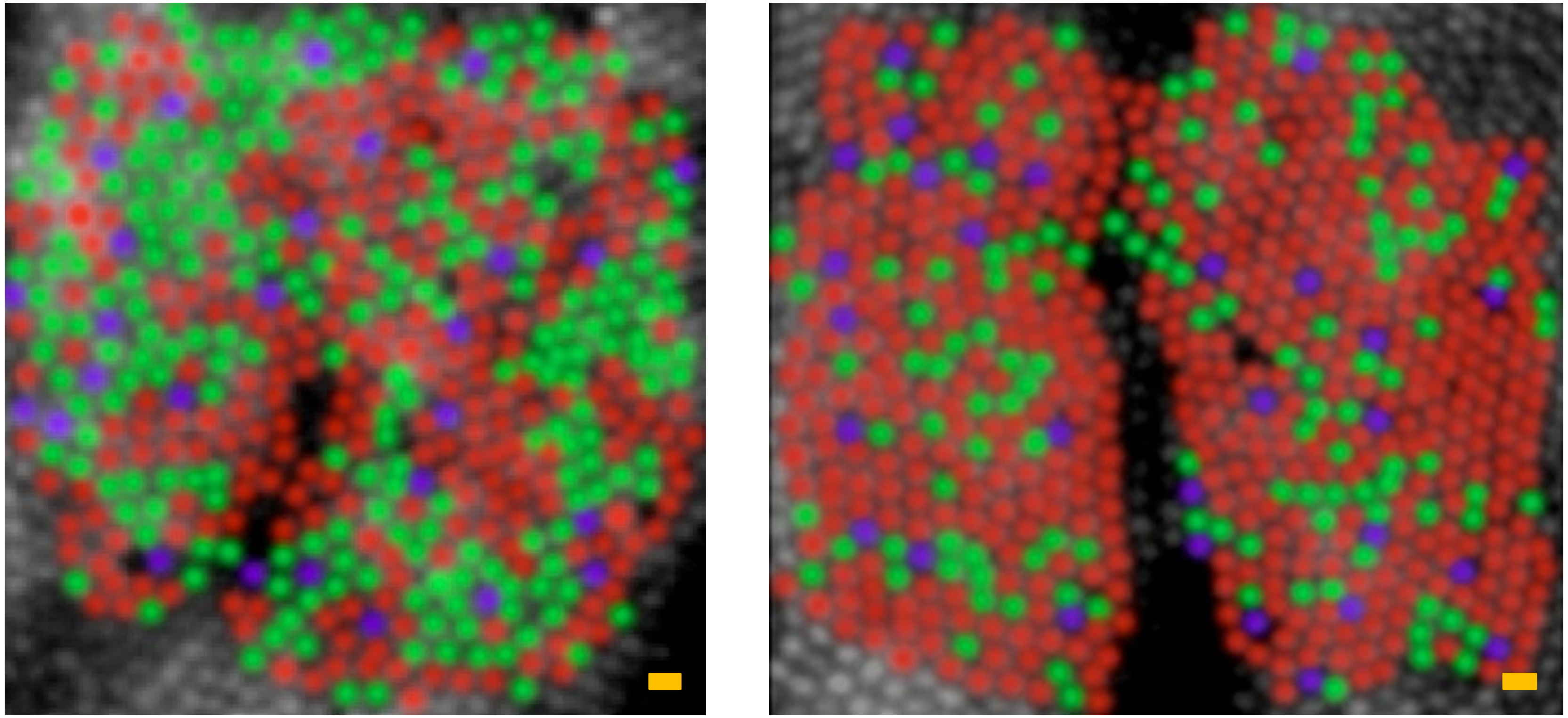
Name	Symbol	Unit name	Unit symbol	Dimension symbol	Notes
Radiance	$L_{e,\Omega}$[nb 5]	watt per steradian per square metre	$W \cdot sr^{-1} \cdot m^{-2}$	$\mathbf{M} \cdot \mathbf{T}^{-3}$	Radiant flux emitted, reflected, transmitted or received by a *surface*, per unit solid angle per unit projected area. This is a *directional* quantity. This is sometimes also confusingly called "intensity".
Spectral radiance	$L_{e,\Omega,\nu}$[nb 3] or $L_{e,\Omega,\lambda}$[nb 4]	watt per steradian per square metre per hertz *or* watt per steradian per square metre, per metre	$W \cdot sr^{-1} \cdot m^{-2} \cdot Hz^{-1}$ *or* $W \cdot sr^{-1} \cdot m^{-3}$	$\mathbf{M} \cdot \mathbf{T}^{-2}$ *or* $\mathbf{M} \cdot \mathbf{L}^{-1} \cdot \mathbf{T}^{-3}$	Radiance of a *surface* per unit frequency or wavelength. The latter is commonly measured in $W \cdot sr^{-1} \cdot m^{-2} \cdot nm^{-1}$. This is a *directional* quantity. This is sometimes also confusingly called "spectral intensity".
Irradiance	E_e[nb 2]	watt per square metre	W/m^2	$\mathbf{M} \cdot \mathbf{T}^{-3}$	Radiant flux *received* by a *surface* per unit area. This is sometimes also confusingly called "intensity".

Quantity		Unit		Dimension	Notes
Name	Symbol[nb 1]	Name	Symbol	Symbol	
Radiant energy					
Radiant energy dens					
Radiant flux					
Spectral flux					
Radiant intensity					
Spectral intensity					
Radiance					
Spectral radiance					
Irradiance					
Spectral irradiance					
Radiosity					
Spectral radiosity					
Radiant exitance					
Spectral exitance	or $M_{e,\lambda}$[nb 4]	or watt per square metre, per metre	or W/m^3	or $\mathbf{M} \cdot \mathbf{L}^{-1} \cdot \mathbf{T}^{-3}$	"Spectral emittance" is an old term for this quantity. This is sometimes also confusingly called "spectral intensity".
Radiant exposure	H_e	joule per square metre	J/m^2	$\mathbf{M} \cdot \mathbf{T}^{-2}$	Radiant energy received by a *surface* per unit area, or equivalently irradiance of a *surface* integrated over time of irradiation. This is sometimes also called "radiant fluence".
Spectral exposure	$H_{e,\nu}$[nb 3] or $H_{e,\lambda}$[nb 4]	joule per square metre per hertz *or* joule per square metre, per metre	$J \cdot m^{-2} \cdot Hz^{-1}$ *or* J/m^3	$\mathbf{M} \cdot \mathbf{T}^{-1}$ *or* $\mathbf{M} \cdot \mathbf{L}^{-1} \cdot \mathbf{T}^{-2}$	Radiant exposure of a *surface* per unit frequency or wavelength. The latter is commonly measured in $J \cdot m^{-2} \cdot nm^{-1}$. This is sometimes also called "spectral fluence".

Photometric quantities

Quantity		Unit		Dimension	Notes
Name	Symbol[nb 1]	Name	Symbol	Symbol	
Luminous energy	Q_v [nb 2]	lumen second	lm·s	$\mathbf{T}\cdot\mathbf{J}$ [nb 3]	Units are sometimes called *talbots*.
Luminous flux / Luminous power	Φ_v [nb 2]	lumen (= cd·sr)	lm	$\mathbf{J}$ [nb 3]	Luminous energy per unit time.
Luminous intensity	I_v	candela (= lm/sr)	cd	$\mathbf{J}$ [nb 3]	Luminous power per unit solid angle.
Luminance	L_v	candela per square metre	cd/m^2	$\mathbf{L}^{-2}\cdot\mathbf{J}$	Luminous power per unit solid angle per unit *projected* source area. Units are sometimes called *nits*.
Illuminance	E_v	lux (= lm/m^2)	lx	$\mathbf{L}^{-2}\cdot\mathbf{J}$	Luminous power *incident* on a surface.
Luminous exitance / Luminous emittance	M_v	lux	lx	$\mathbf{L}^{-2}\cdot\mathbf{J}$	Luminous power *emitted* from a surface.
Luminous exposure	H_v	lux second	lx·s	$\mathbf{L}^{-2}\cdot\mathbf{T}\cdot\mathbf{J}$	
Luminous energy density	ω_v	lumen second per cubic metre	lm·s·m^{-3}	$\mathbf{L}^{-3}\cdot\mathbf{T}\cdot\mathbf{J}$	
Luminous efficacy	η [nb 2]	lumen per watt	lm/W	$\mathbf{M}^{-1}\cdot\mathbf{L}^{-2}\cdot\mathbf{T}^{3}\cdot\mathbf{J}$	Ratio of luminous flux to radiant flux.
Luminous efficiency / Luminous coefficient	V			1	

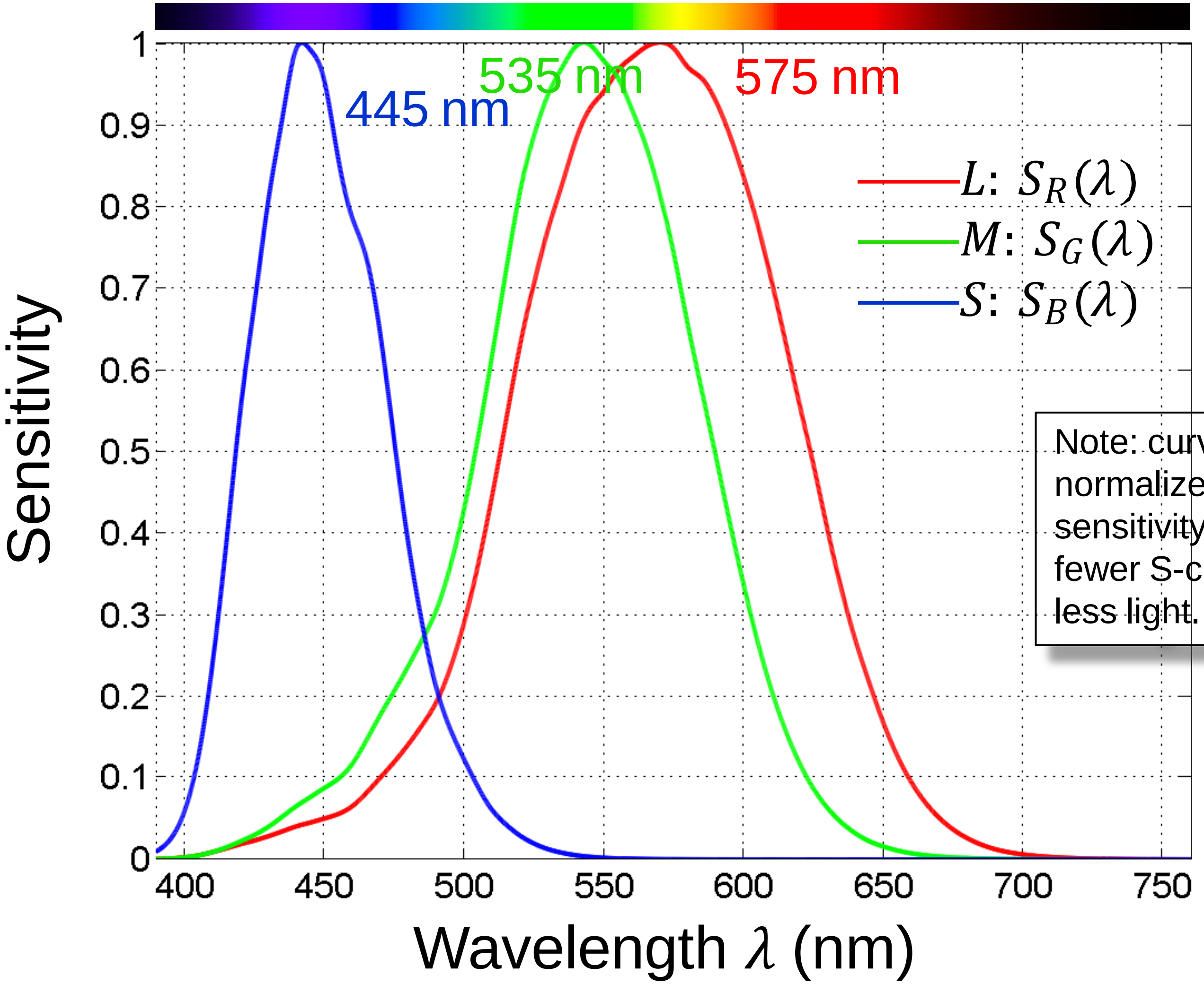
Human retina



[Roorda, Williams, 1999]

Pseudo-color image of nasal retina,
1 degree eccentricity, in two male subjects, scale bar 5 micron

Absorption of light in the cones of the human retina



Note: curves are normalized. **Much** lower sensitivity to Blue, since fewer S-cones absorb less light.

Three-receptor model of color perception

$$C(\lambda)$$

Spectral energy distribution of incident light

$$\int_\lambda S_R(\lambda)C(\lambda)d\lambda \rightarrow \alpha_R$$

$$\int_\lambda S_G(\lambda)C(\lambda)d\lambda \rightarrow \alpha_G$$

$$\int_\lambda S_B(\lambda)C(\lambda)d\lambda \rightarrow \alpha_B$$

Effective cone stimulation (“tristimulus values”)

[T. Young, 1802] [J.C. Maxwell, 1890]

- Different spectra can map into the same tristimulus values and hence look identical (“metamers”)
- Three numbers suffice to represent any color

Color matching

- Suppose 3 primary light sources with spectra $P_k(\lambda)$, $k = 1,2,3$
- Intensity of each light source can be adjusted by factor β_k
- How to choose β_k, $k = 1,2,3$, such that desired tristimulus values (α_R, α_G, α_B) result ?

$$C(\lambda) = \beta_1 P_1(\lambda) + \beta_2 P_2(\lambda) + \beta_3 P_3(\lambda)$$

$$\boxed{\int_\lambda S_R(\lambda) C(\lambda) d\lambda} \rightarrow \alpha_R$$

$$\boxed{\int_\lambda S_G(\lambda) C(\lambda) d\lambda} \rightarrow \alpha_G$$

$$\boxed{\int_\lambda S_B(\lambda) C(\lambda) d\lambda} \rightarrow \alpha_B$$

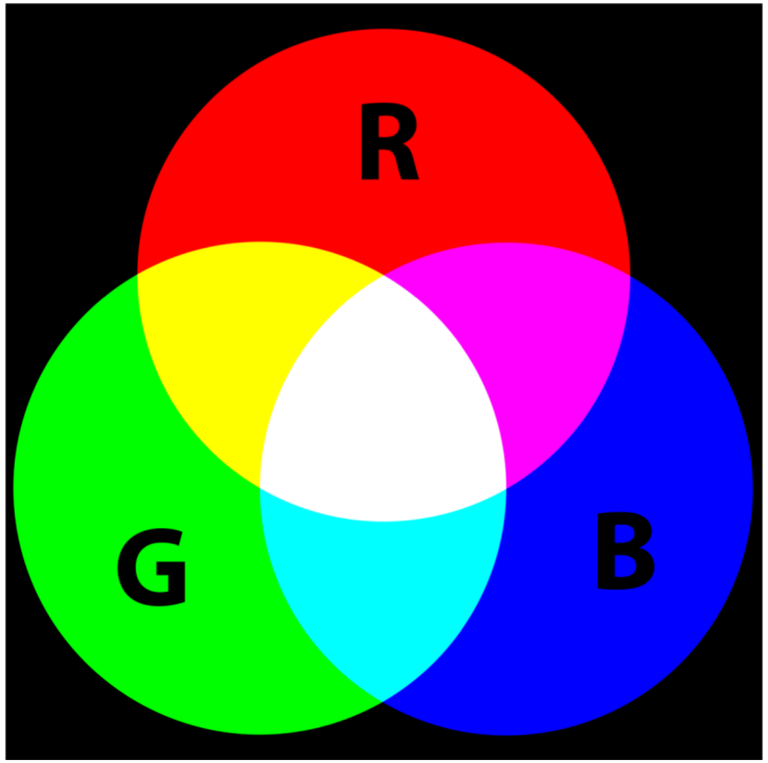
$$\alpha_i = \int_\lambda S_i(\lambda) \left[\beta_1 P_1(\lambda) + \beta_2 P_2(\lambda) + \beta_3 P_3(\lambda) \right] d\lambda$$

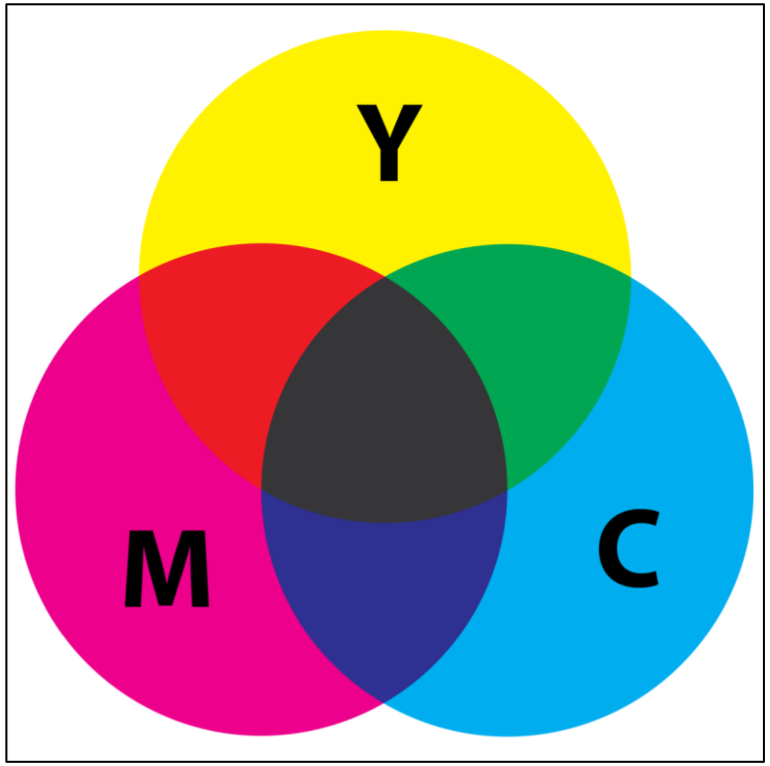
$$= \beta_1 \cdot K_{i,1} + \beta_2 \cdot K_{i,2} + \beta_3 \cdot K_{i,3}$$

$$\text{with } K_{i,j} = \int_\lambda S_i(\lambda) P_j(\lambda) d\lambda$$

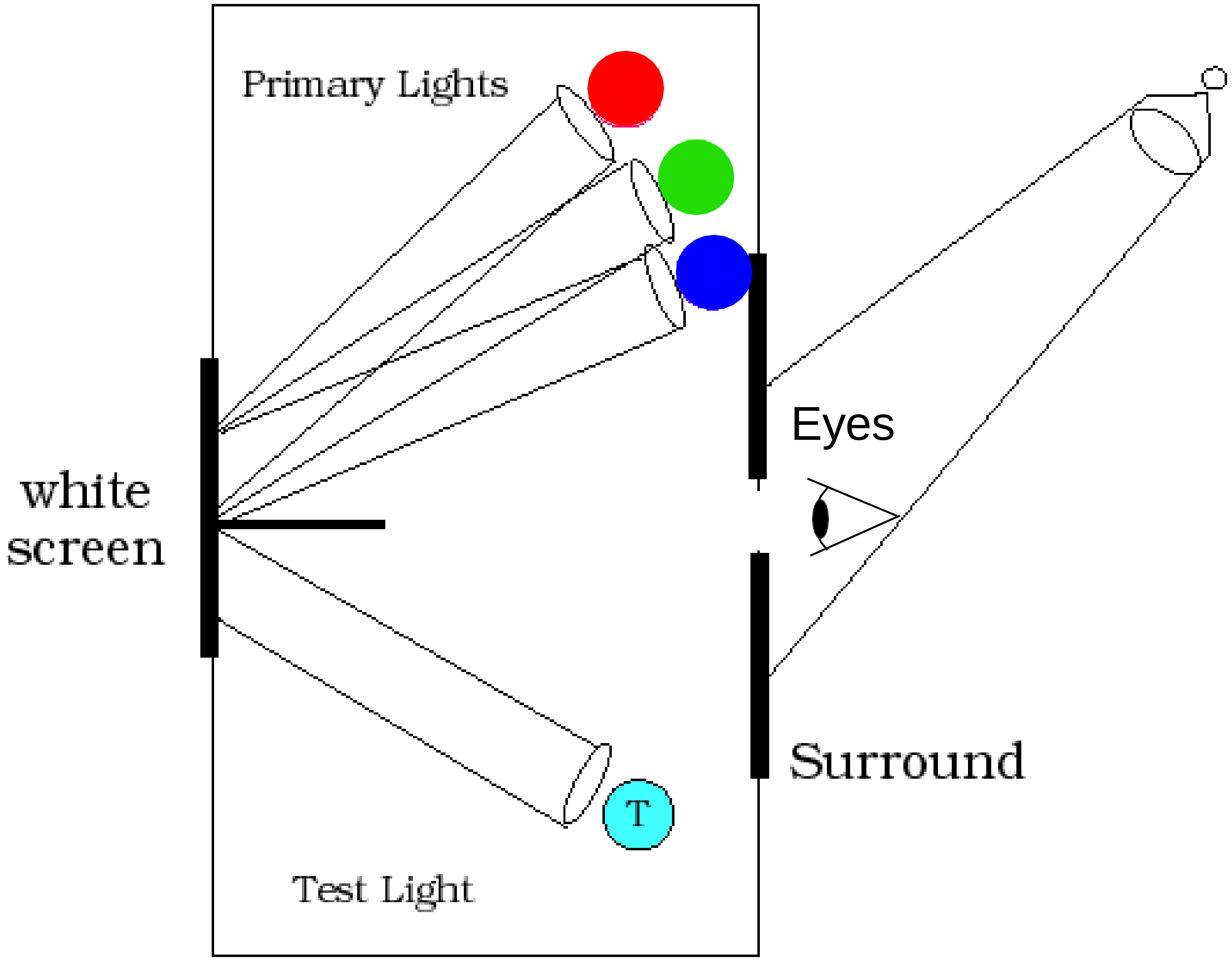
Color matching is linear!

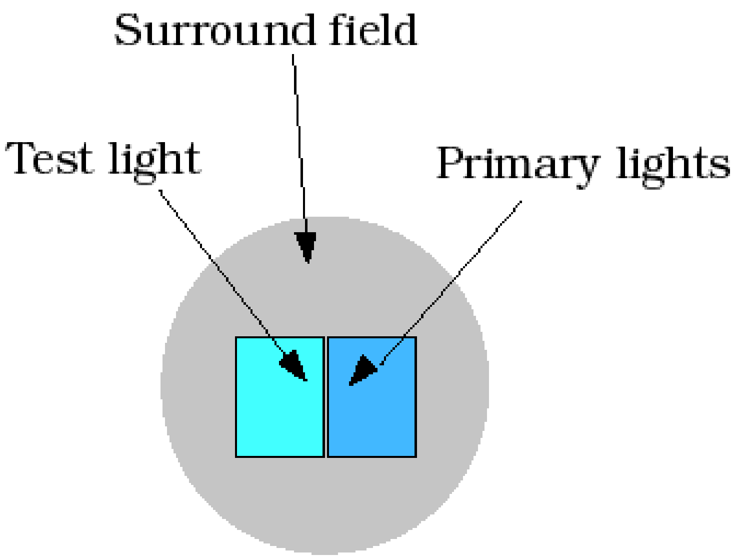
Additive vs. subtractive color mixing





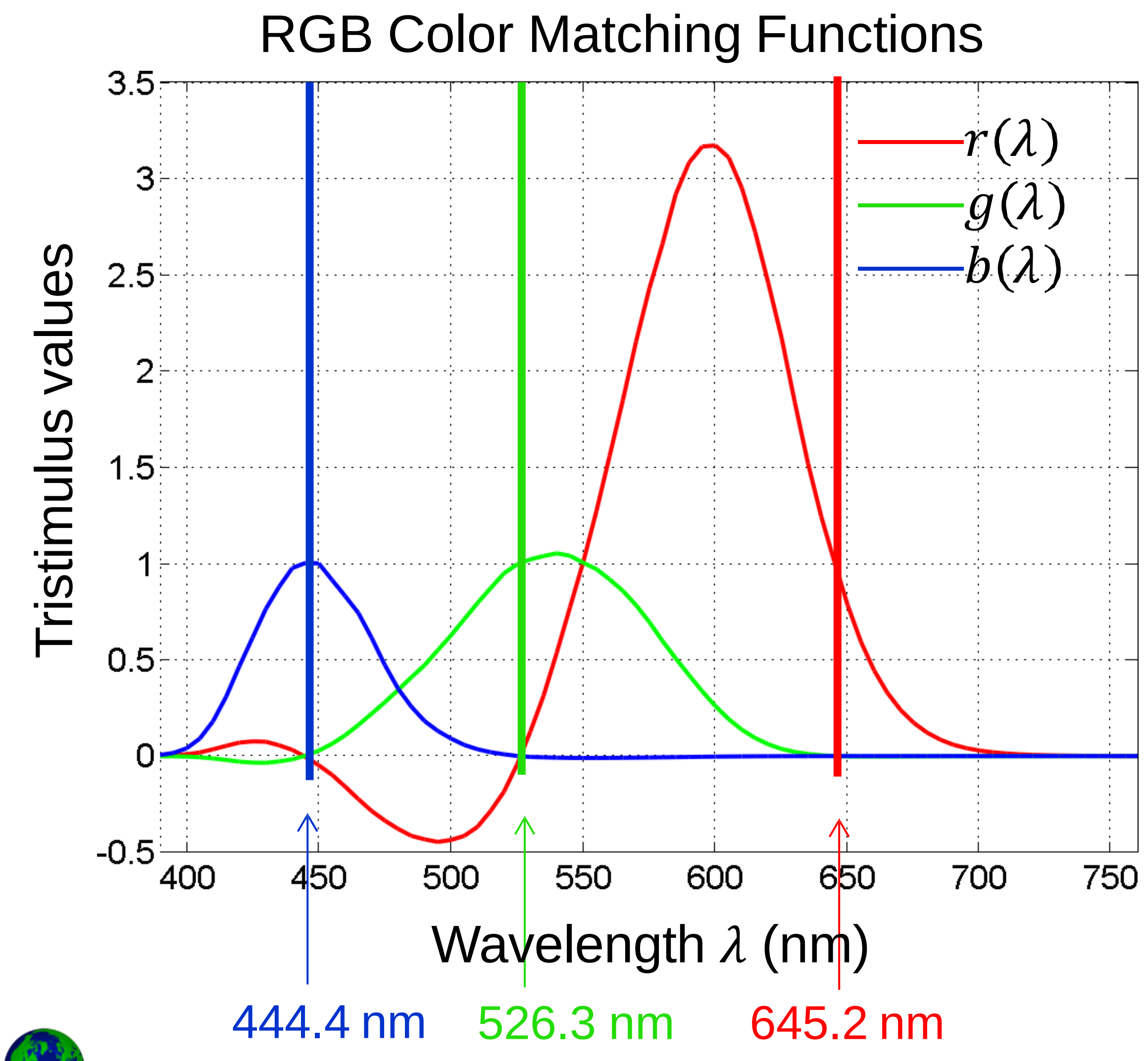
Color matching experiment





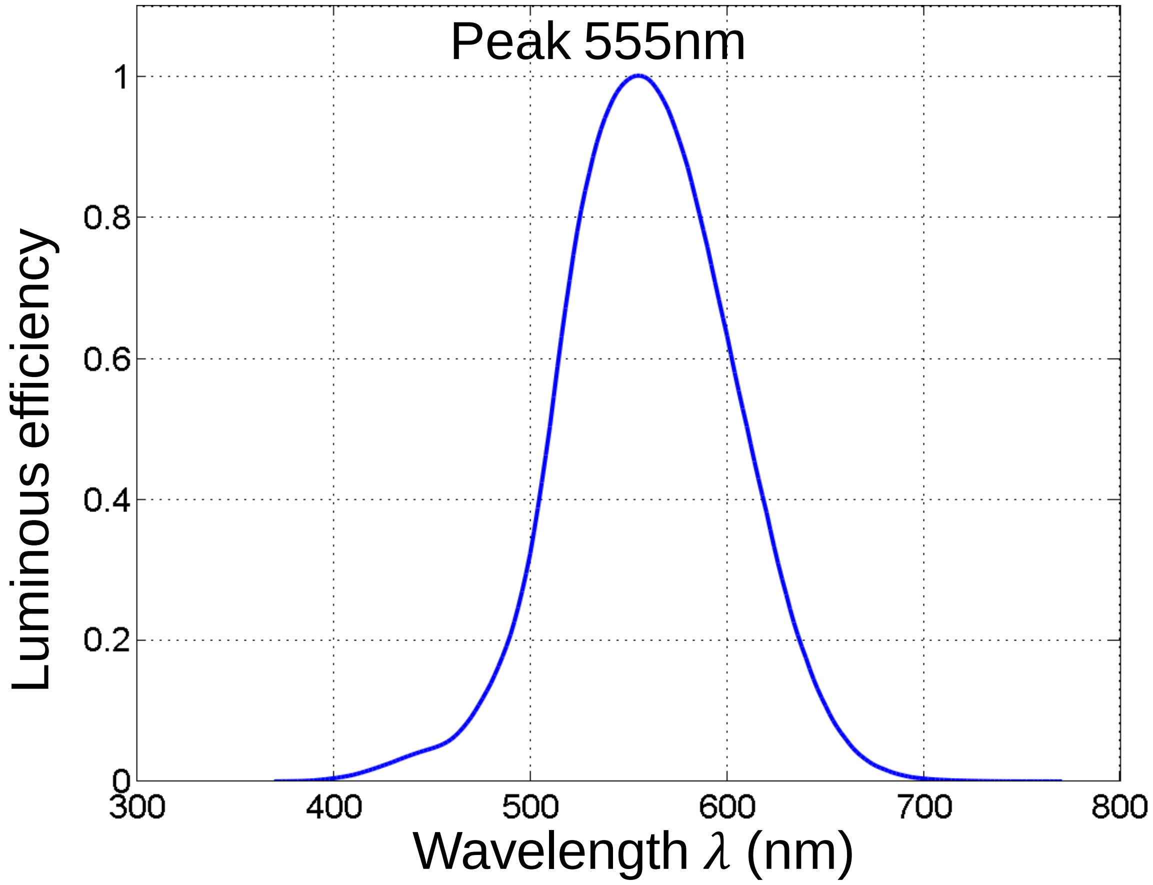
Courtesy B. Wandell, from [Foundations of Vision, 1996]

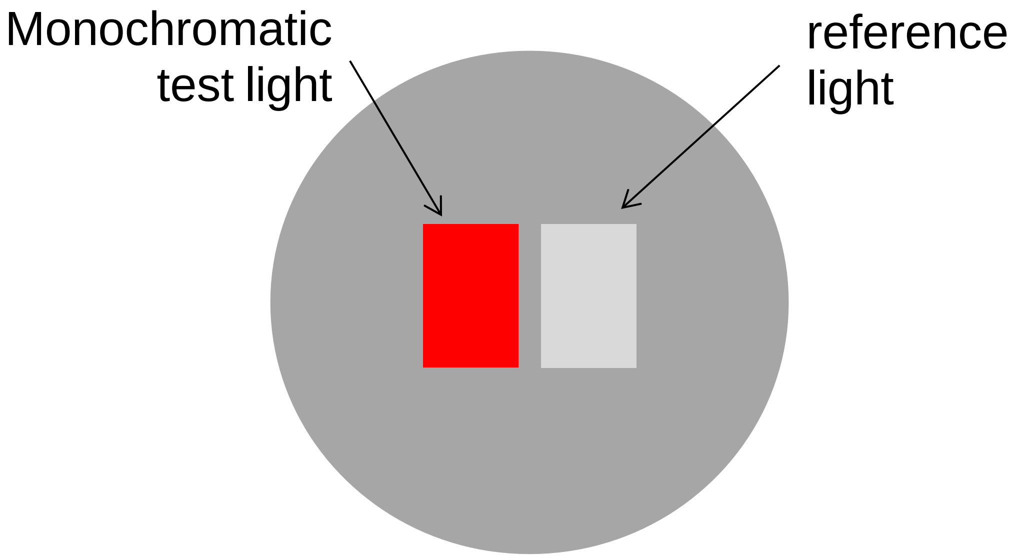
Spectral matching functions



- Color matching experiment: Monochromatic test light vs. mixture of 3 monochromatic primaries
- "Negative intensity": color is added to test color
- CIE (Commision Internationale de L'Eclairage), 1931: Spectral RGB primaries (scaled, such that $R_\lambda = G_\lambda = B_\lambda$ matches spectrally flat white).

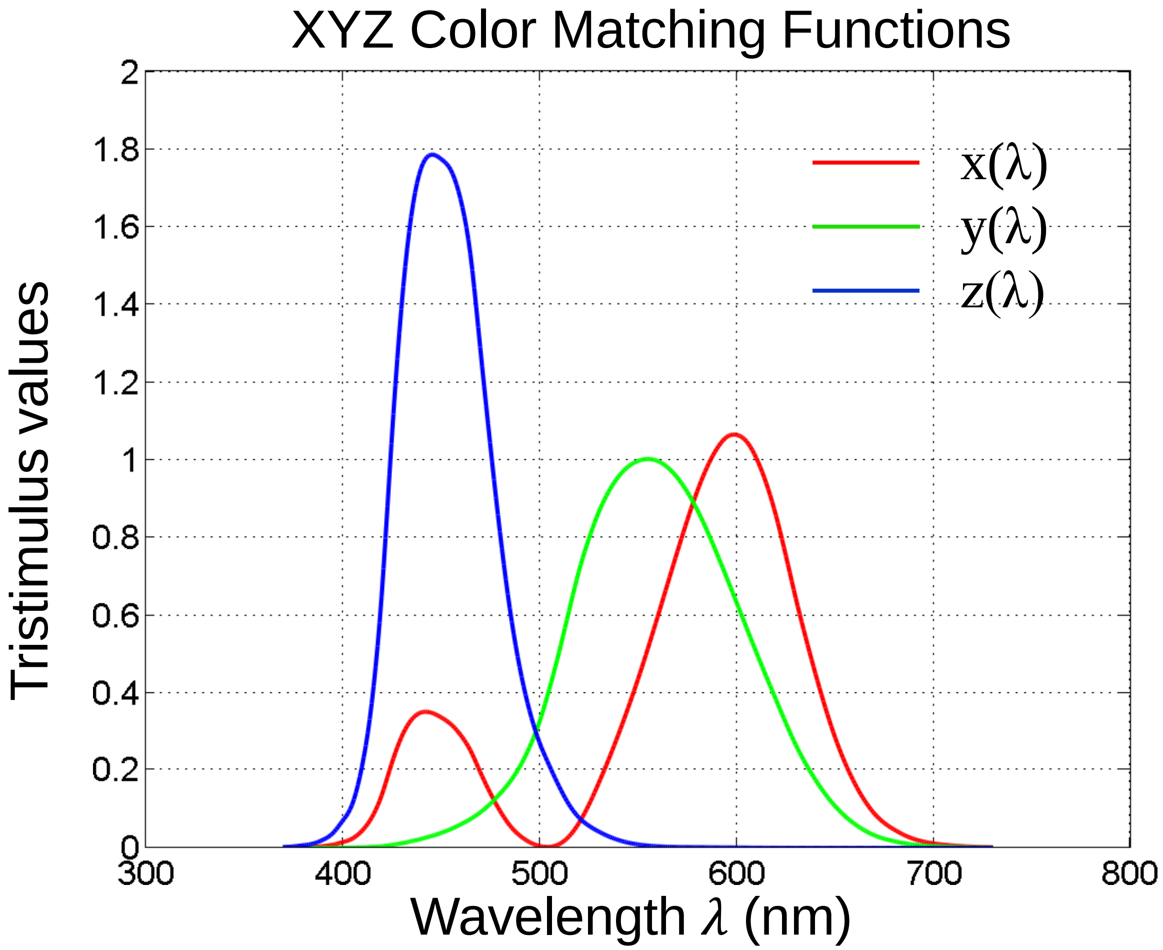
Luminosity function





- <u>Experiment:</u>
  Match the brightness of a white reference light and a monochromatic test light of wavelength λ
- Links photometric to radiometric quantities

CIE 1931 XYZ color system



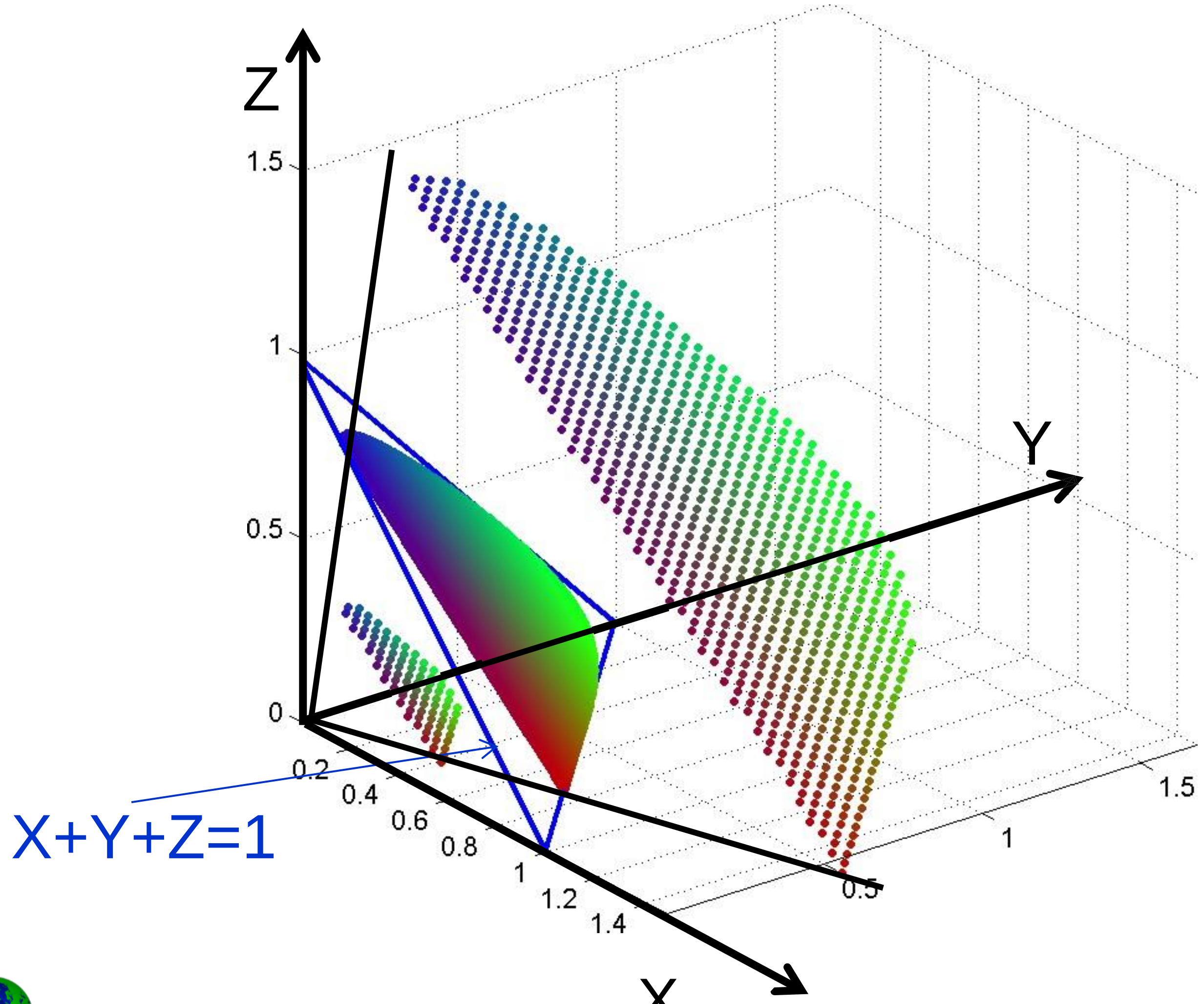
Properties:

- All positive spectral matching functions

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} .490 & .310 & .200 \\ .177 & .813 & .011 \\ .000 & .010 & .990 \end{pmatrix} \begin{pmatrix} R_\lambda \\ G_\lambda \\ B_\lambda \end{pmatrix}$$

- Y corresponds to luminance
- Equal energy white: X=Y=Z
- Virtual primaries

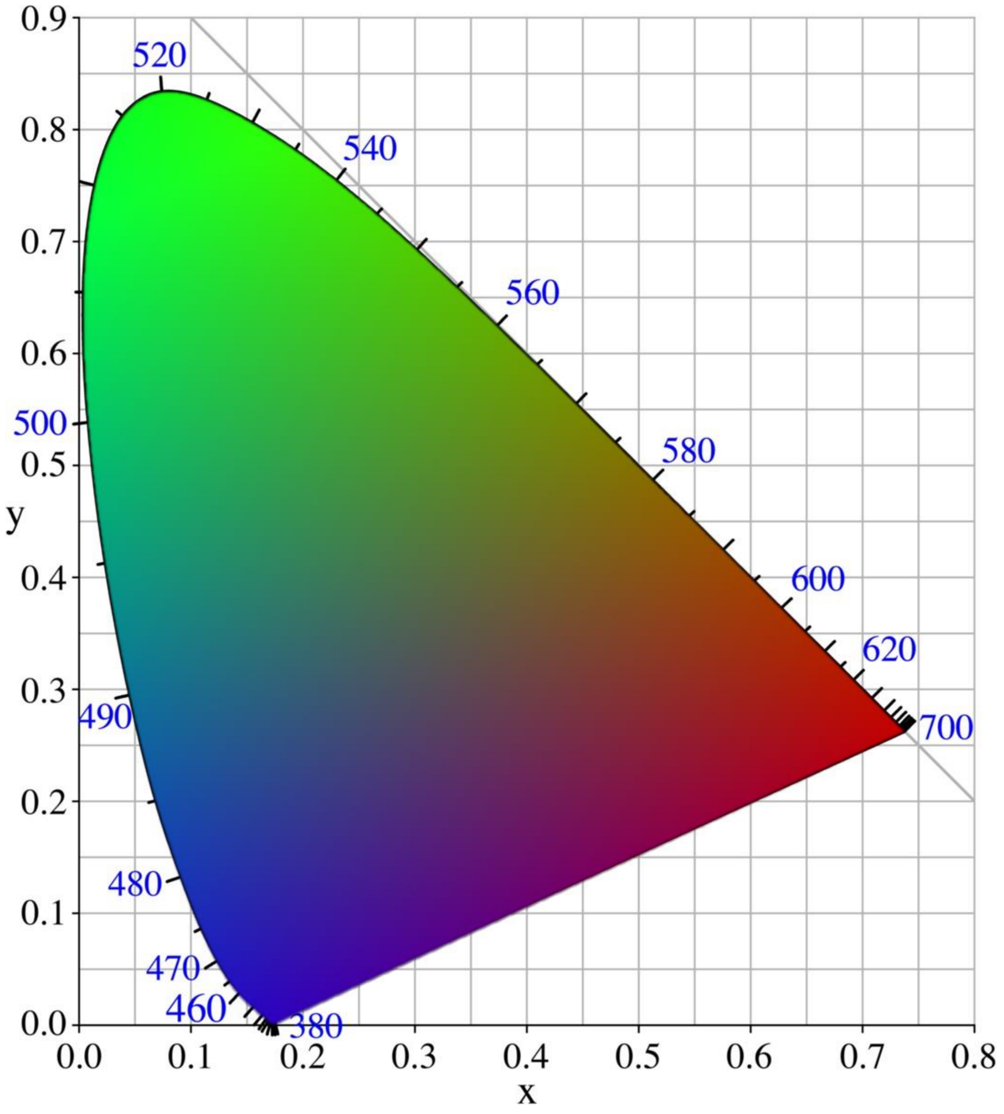
Color gamut and chromaticity



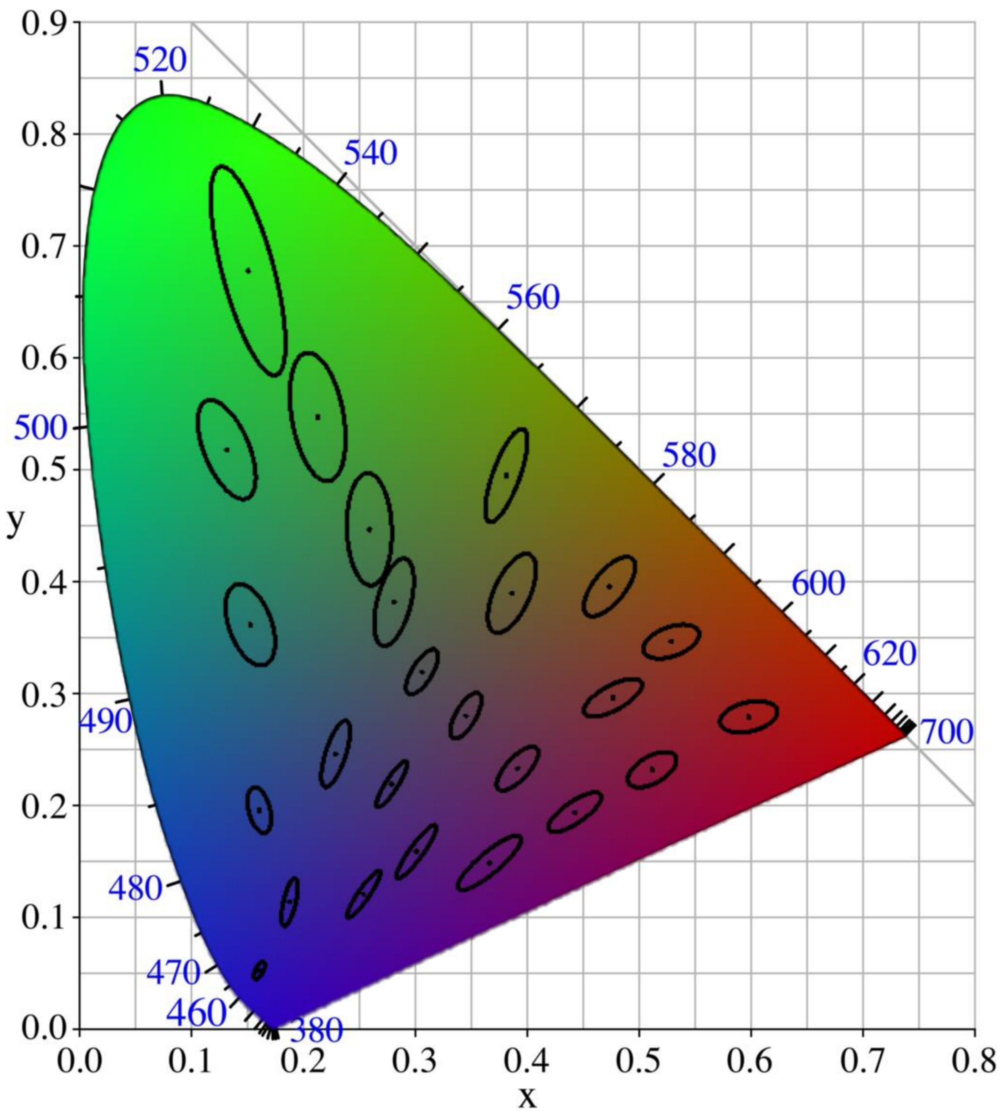
$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

CIE chromaticity diagram



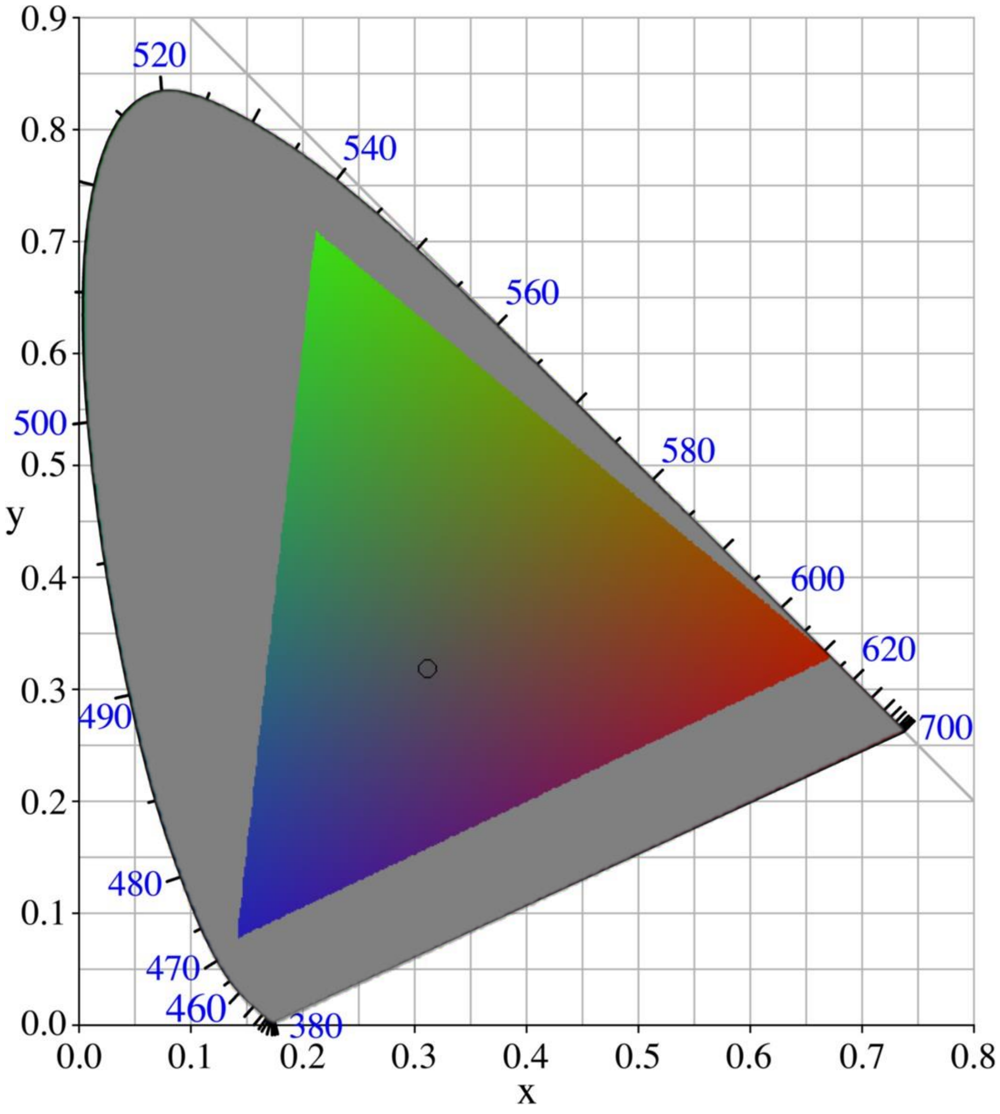
Perceptual non-uniformity of xy chromaticity



Just noticeable chromaticity differences (10X enlarged)

[MacAdam, 1942]

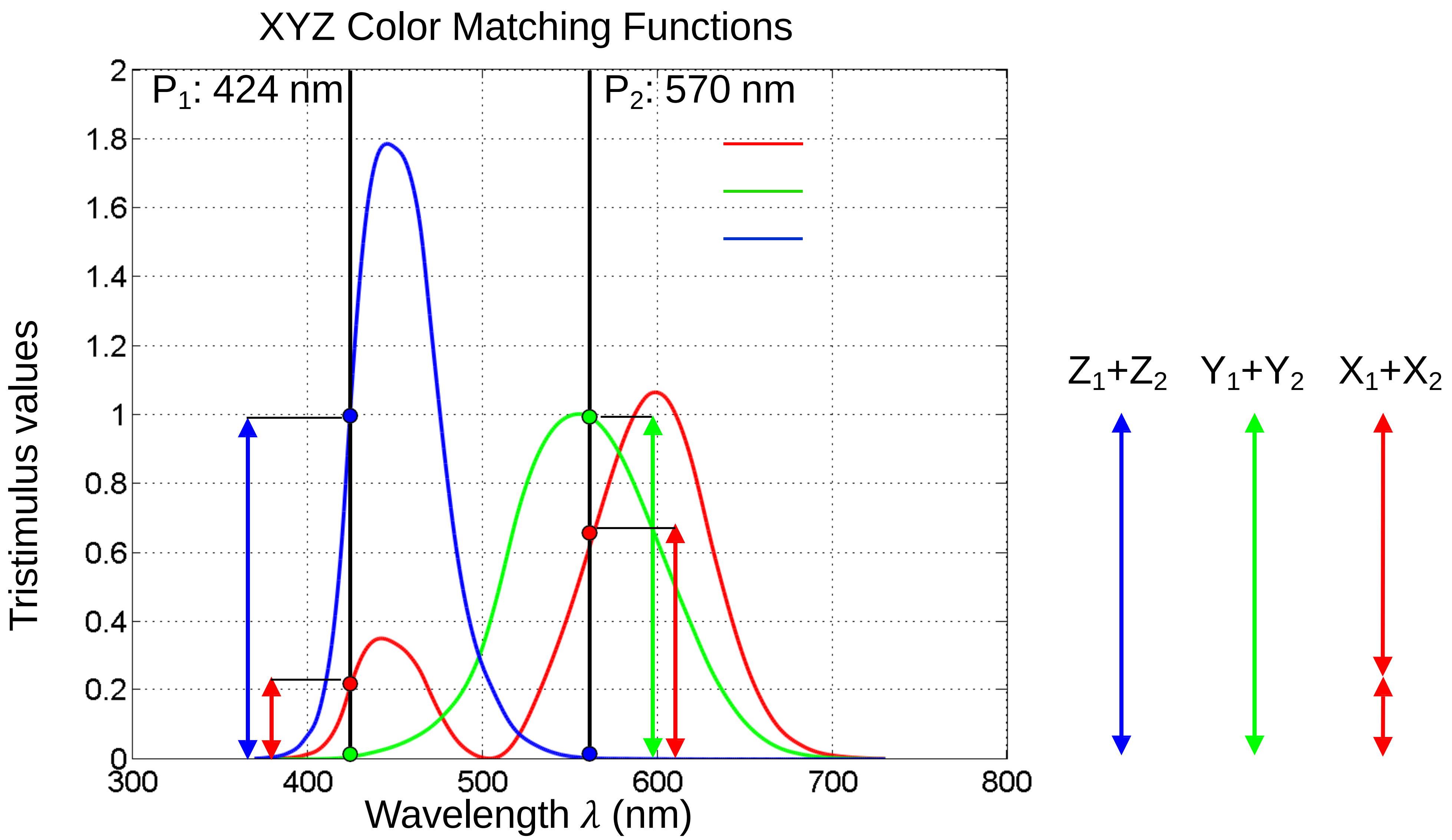
Color gamut



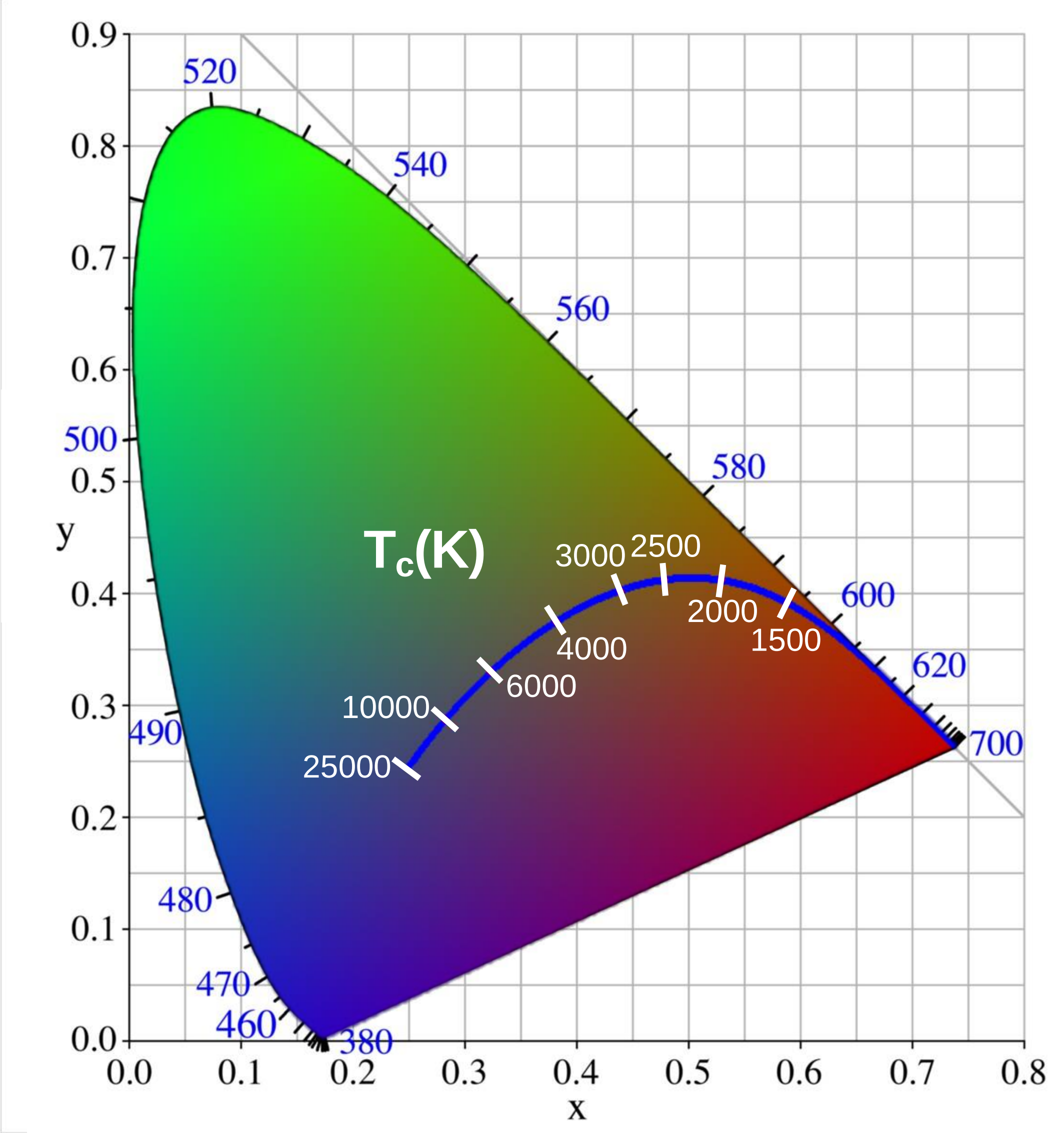
<u>NTSC phosphors</u>

R: x=0.67, y=0.33
G: x=0.21, y=0.71
B: x=0.14, y=0.08

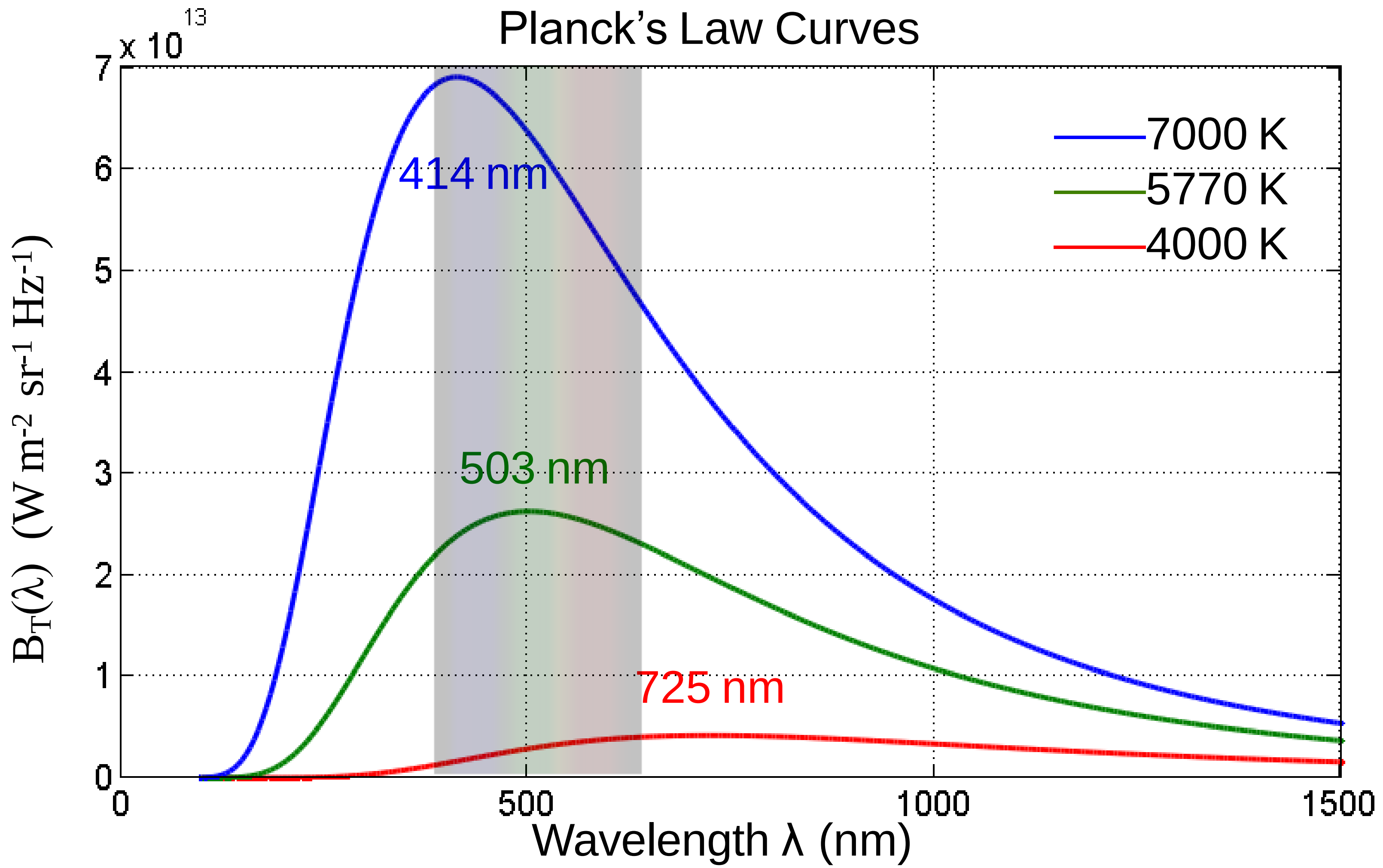
Reference white:
x=0.31, y=0.32
Illuminant C


XYZ Color Matching Functions
P1: 424 nm
P2: 570 nm
Tristimulus values
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
300
400
500
600
700
800
Wavelength λ (nm)
Z1+Z2
Y1+Y2
X1+X2

White at different color temperatures



Blackbody radiation



Planck's Law, 1900

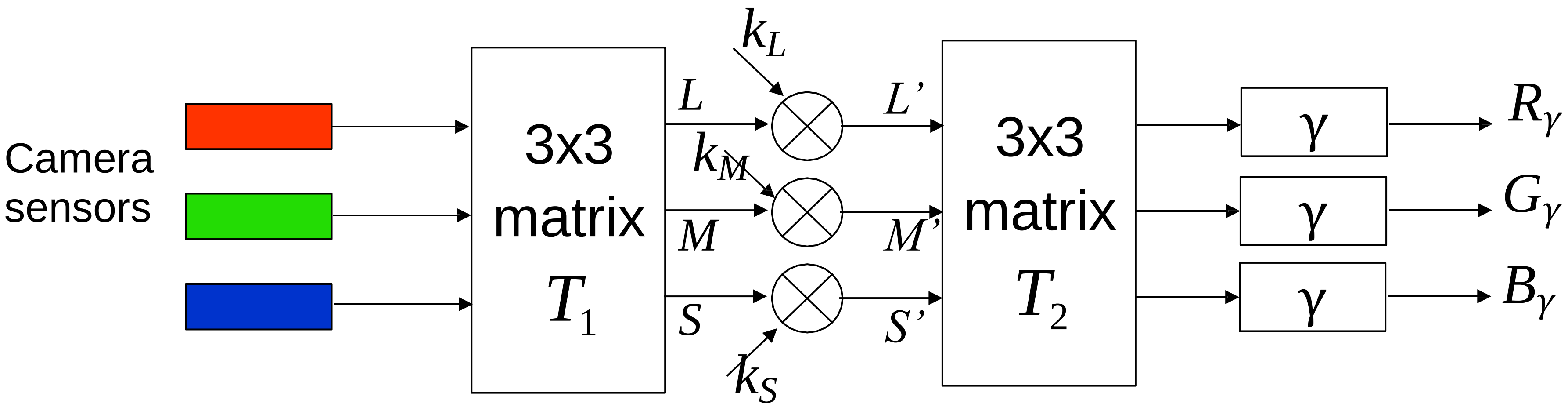
$$B_T(\lambda) = \frac{2hc^2 / \lambda^5}{e^{hc/\lambda kT} - 1}$$

Wien's Law

$$\lambda_{peak}[nm] = \frac{2,900,000}{T[K]}$$

Color balancing

- Effect of different illuminants can be cancelled only in the spectral domain (impractical)
- Color balancing in 3-d color space is practical approximation
- Color constancy in human visual system: gain control in cone space LMS *[von Kries,1902]*
- Von Kries hypothesis applied to image acquisition devices (cameras, scanners)



- How to determine k_L, k_M, k_S automatically?

Color balancing (cont.)

- Von Kries hypothesis

$$\begin{pmatrix} L' \\ M' \\ S' \end{pmatrix} = \begin{pmatrix} k_L & 0 & 0 \\ 0 & k_M & 0 \\ 0 & 0 & k_S \end{pmatrix} \begin{pmatrix} L \\ M \\ S \end{pmatrix}$$

- If illumination (or a patch of white in the scene) is known, calculate

$$k_L = \frac{L_{desired}}{L_{actual}}; \quad k_M = \frac{M_{desired}}{M_{actual}}; \quad k_S = \frac{S_{desired}}{S_{actual}}$$

Color balancing with unknown illumination

- Gray-world

$$k_L \sum_{x,y} L\left[x,y\right] = k_M \sum_{x,y} M\left[x,y\right] = k_S \sum_{x,y} S\left[x,y\right]$$

- Scale-by-max

$$k_L \max_{x,y} L\lfloor x,y \rfloor = k_M \max_{x,y} M\lfloor x,y \rfloor = k_S \max_{x,y} S\lfloor x,y \rfloor$$

- Shades-of-gray
  [Finlayson, Trezzi, 2004]

$$k_L \left(\sum_{x,y} L^p \lfloor x,y \rfloor \right)^{\frac{1}{p}} = k_M \left(\sum_{x,y} M^p \left[x,y\right] \right)^{\frac{1}{p}} = k_S \left(\sum_{x,y} S^p \left[x,y\right] \right)^{\frac{1}{p}}$$

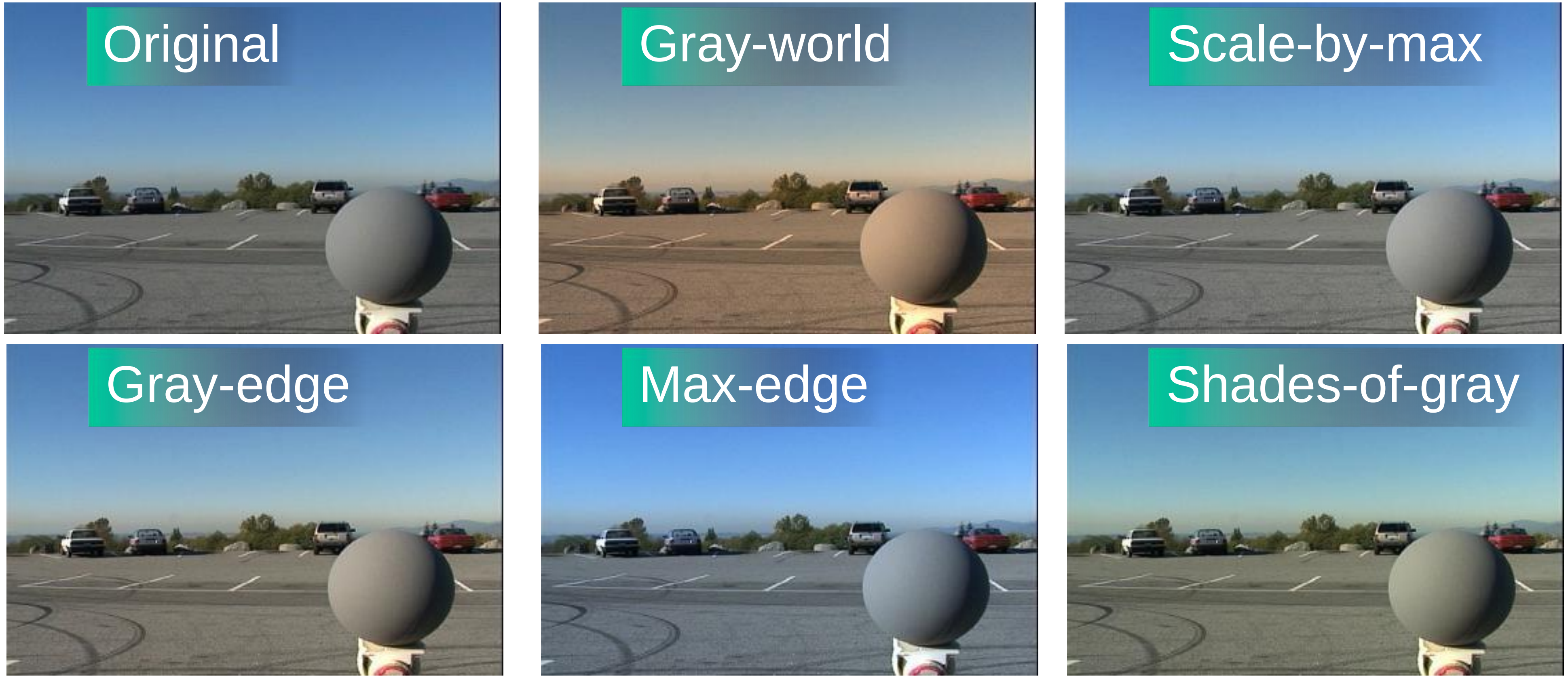
  » Special cases: gray-world ($p = 1$), scale-by-max ($p = \infty$)
  » Best performance for $p \approx 6$

- Refinements:
  smooth image, exclude saturated color/dark pixels,
  use spatial derivatives instead (“gray-edge,” “max-edge”)
  [van de Weijer, 2007])

Color balancing example

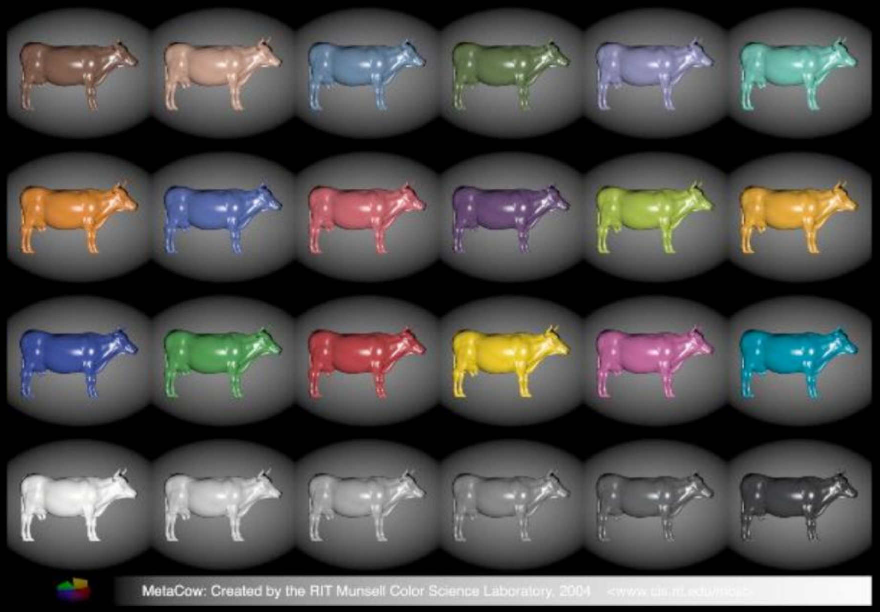
Original Gray-world Scale-by-max Gray-edge Max-edge Shades-of-gray

Color balancing example



Original image courtesy Ciurea and Funt

Daylight D65
CIE observer



Daylight D65
cheap camera

Illuminant A
CIE observer

Color conversion cheat sheet (*e.g.*, for HW2)

- Great website for insights, every possible color conversion scheme, and much more: www.brucelindbloom.com

- Spectrum to CIE XYZ: (no illuminant)

$$X = \int_\lambda \bar{x}(\lambda) P(\lambda) d\lambda$$

$$Y = \int_\lambda \bar{y}(\lambda) P(\lambda) d\lambda$$

$$Z = \int_\lambda \bar{z}(\lambda) P(\lambda) d\lambda$$

CIE XYZ to CIE xyY:

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$Y = Y$$

- CIE XYZ to CIE RGB:

$$\begin{bmatrix} R_{linear} \\ G_{linear} \\ B_{linear} \end{bmatrix} = M^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

approximation of CIE gamma:

$$\{R, G, B\} = \{R, G, B\}_{linear}^{1/\gamma}$$

- CIE RGB to CIE XYZ:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = M \begin{bmatrix} R_{linear} \\ G_{linear} \\ B_{linear} \end{bmatrix}$$

$$M = \begin{bmatrix} .490 & .310 & .200 \\ .177 & .813 & .011 \\ .000 & .010 & .990 \end{bmatrix}$$