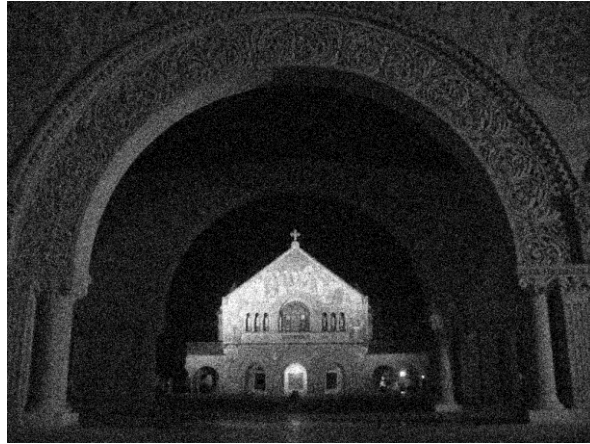


# Point operations for combining images

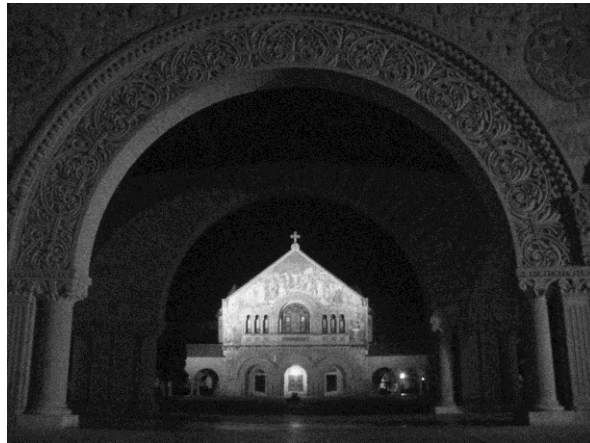
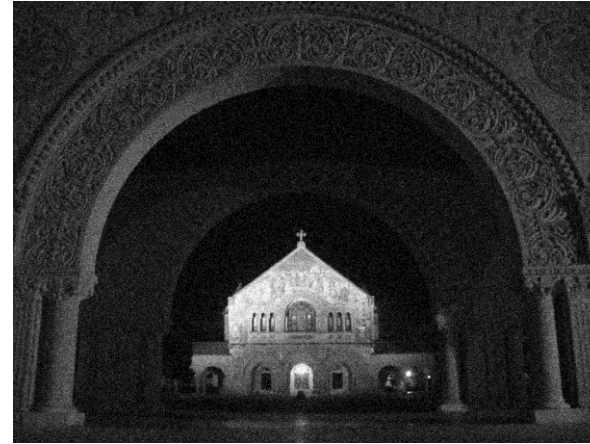
- Image averaging for noise reduction
- Combination of different exposures for high-dynamic range imaging
- Image subtraction for change detection
- Need for accurate alignment
- Displacement estimation

# Image averaging for noise reduction

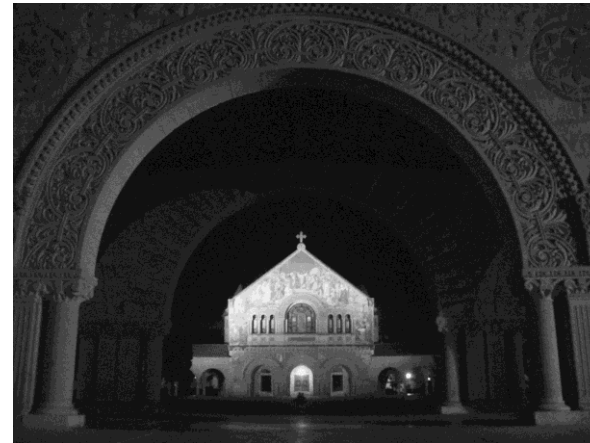
1 image



2 images



8 images



32 images



# High-dynamic range imaging



-8 f-stops



-2 f-stops



+2 f-stops



+4 f-stops



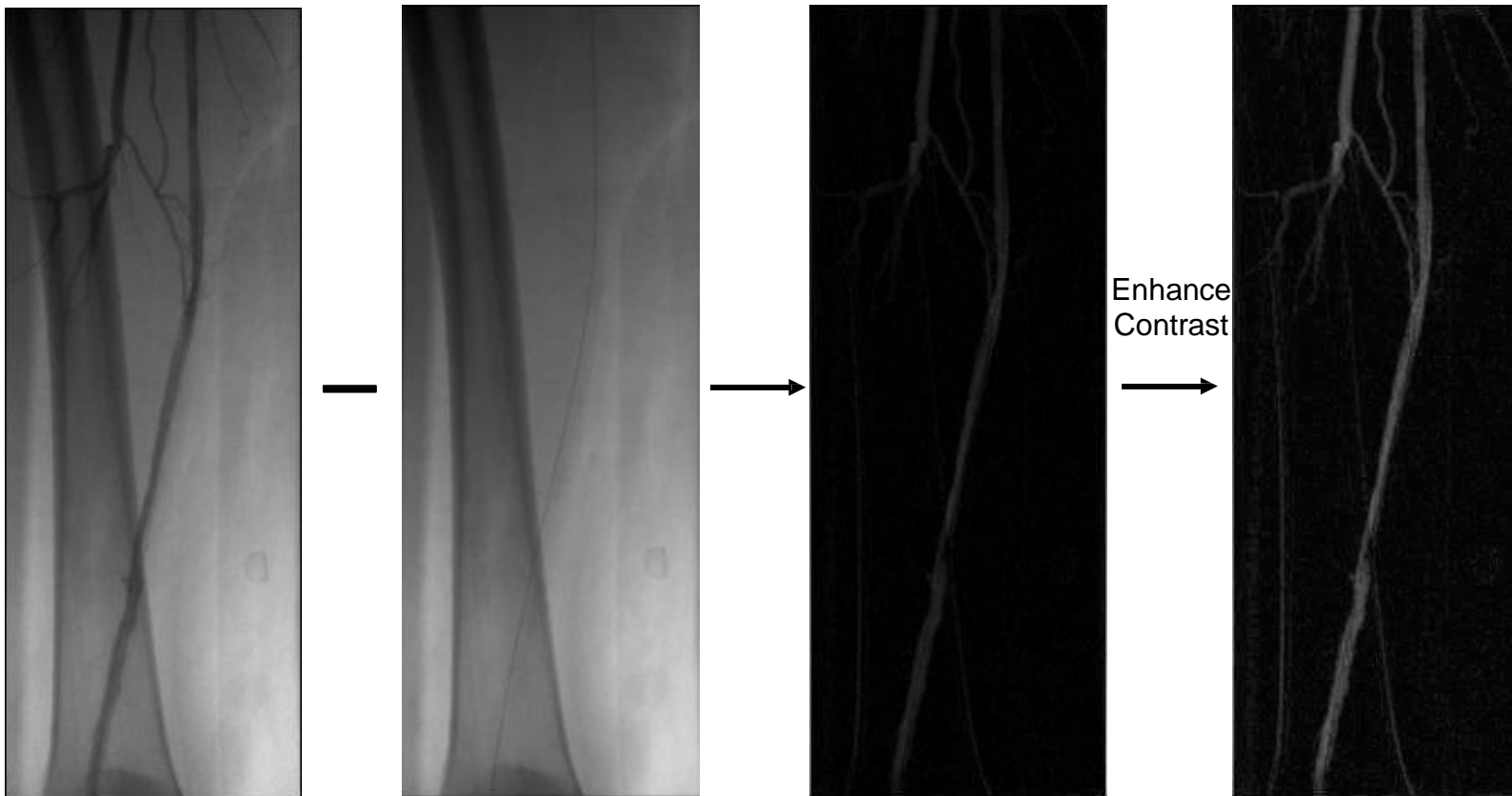
Blended image from  
Exposure Fusion

*[Tom Mertens et al. 2007]*

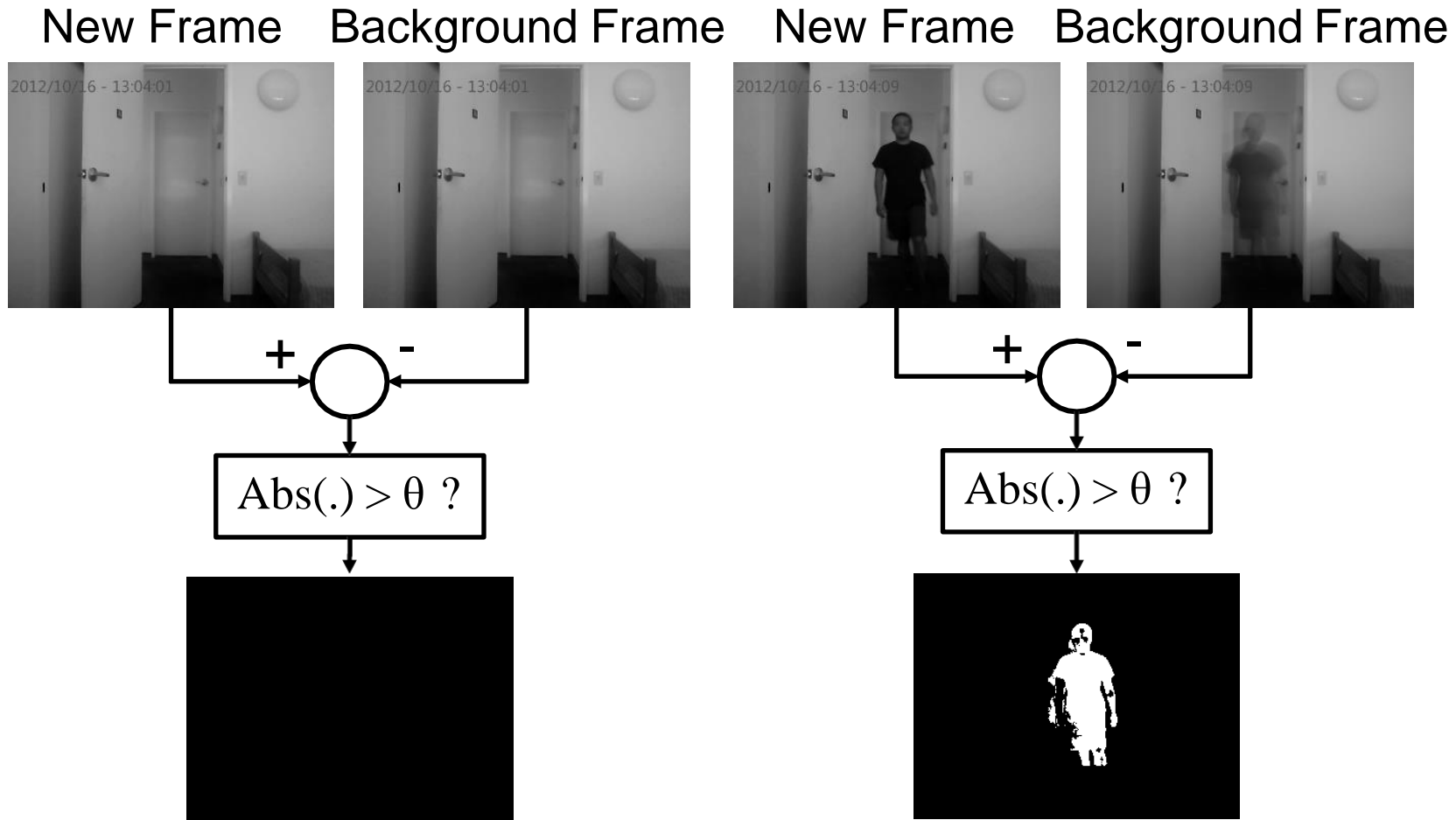


# Image subtraction

- Find differences/changes between 2 mostly identical images
- Example: digital subtraction angiography



# Video background subtraction



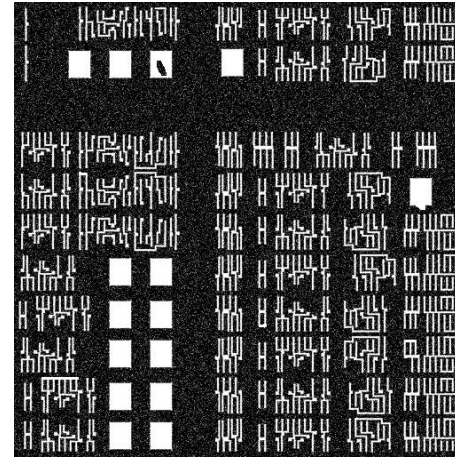
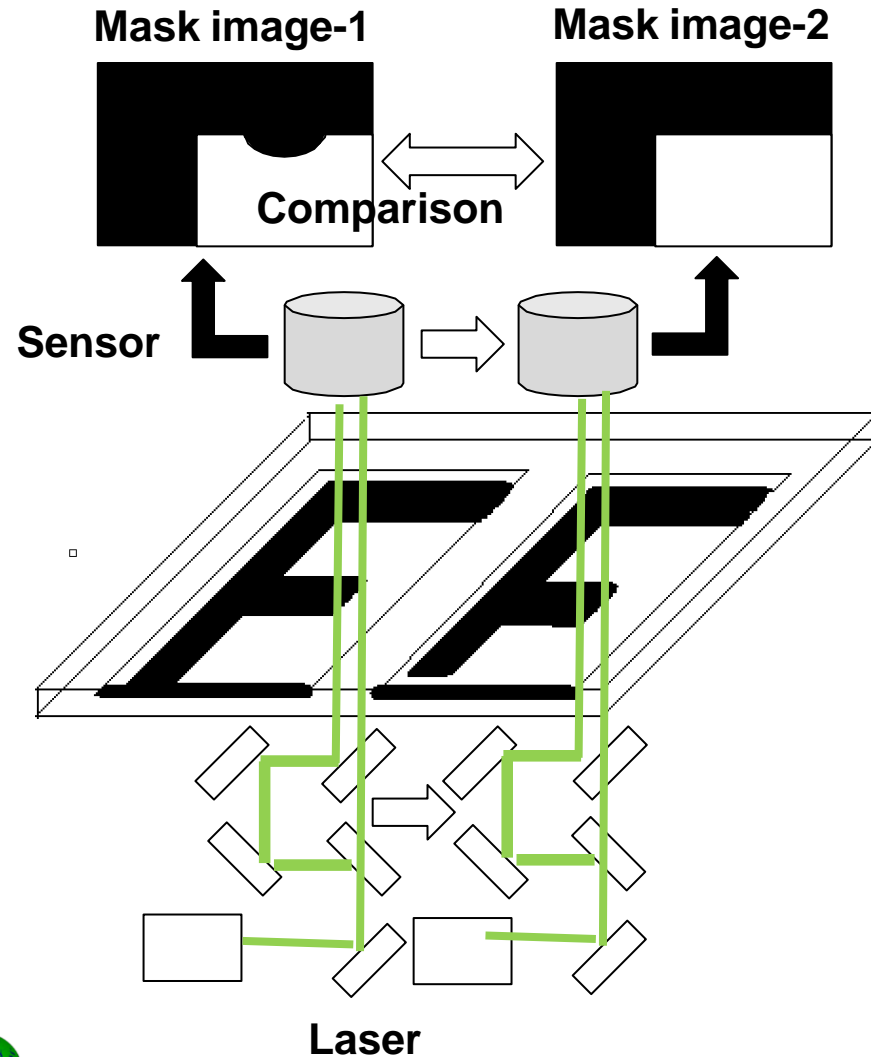
**Update:**

$$\text{Background}[t] := \alpha \text{ Background}[t-1] + (1 - \alpha) \text{ New}[t]$$

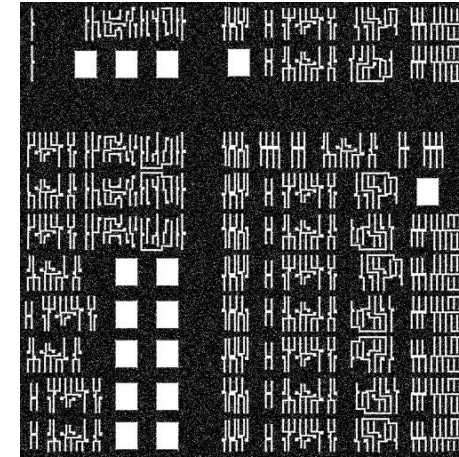




# Image subtraction in IC manufacturing: inspection of photomasks



Mask image-1



Mask image-2



Difference image



# Where is the defect?

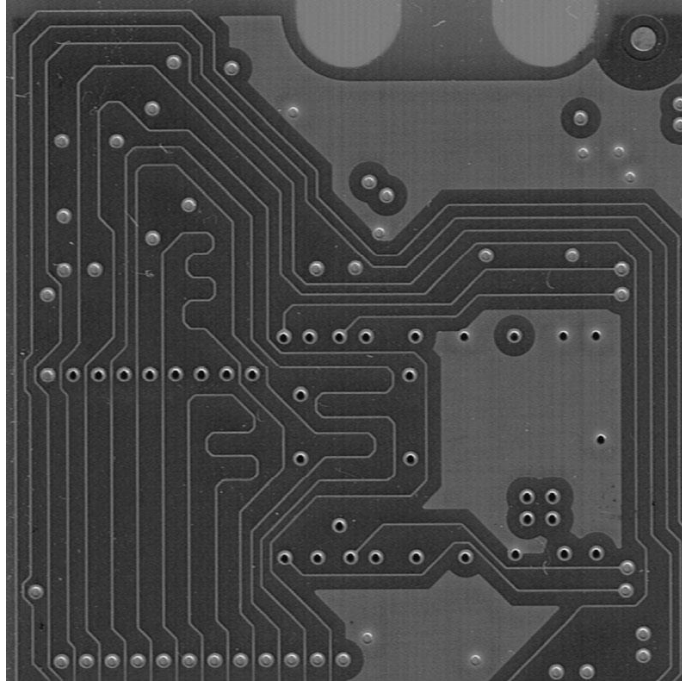


Image  $g[x,y]$  (no defect)

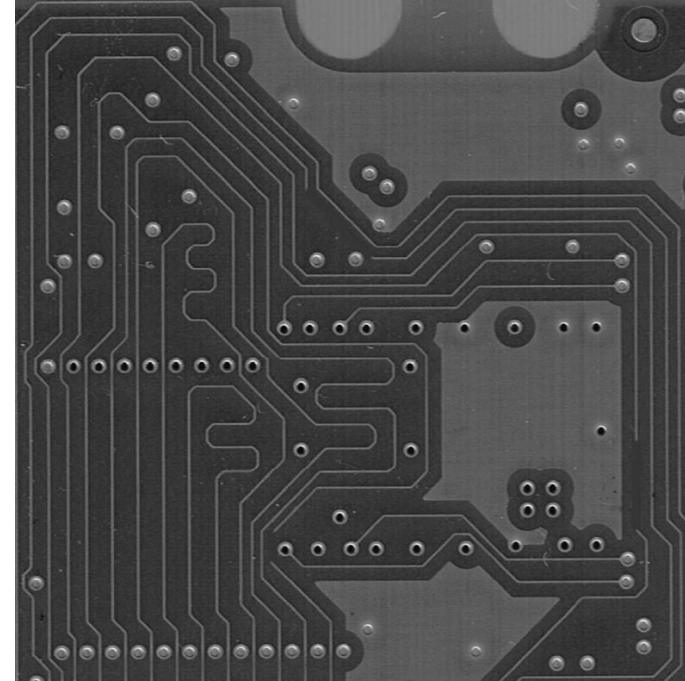
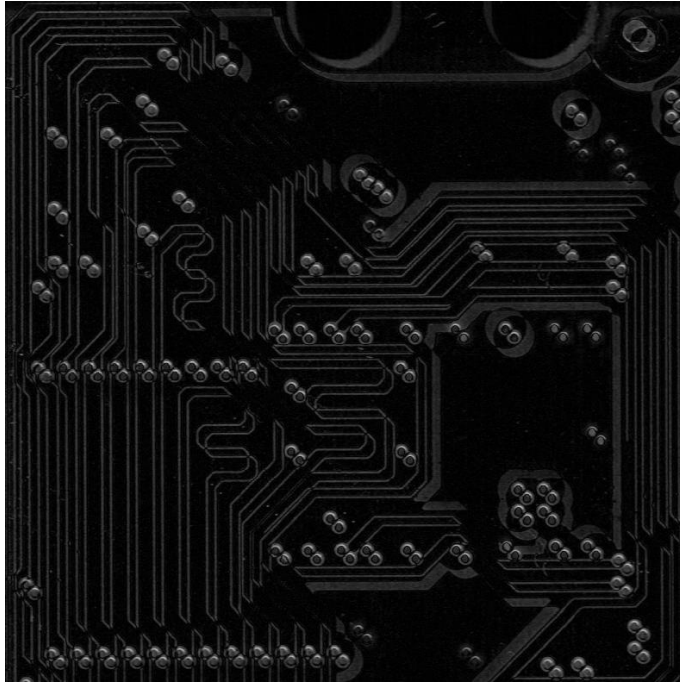


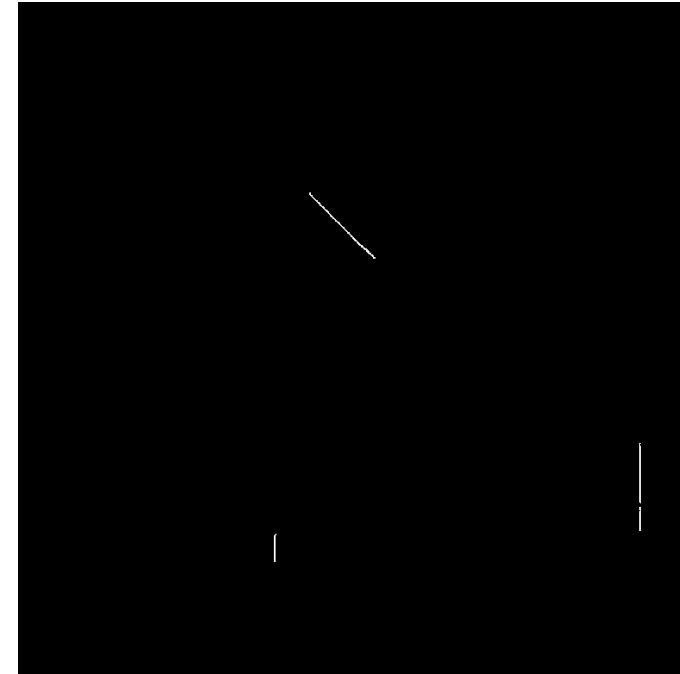
Image  $f[x,y]$  (w/ defect)



# Absolute difference between two images



$|f-g|$  w/o alignment



$|f-g|$  w/ alignment





# Displacement estimation by block matching

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

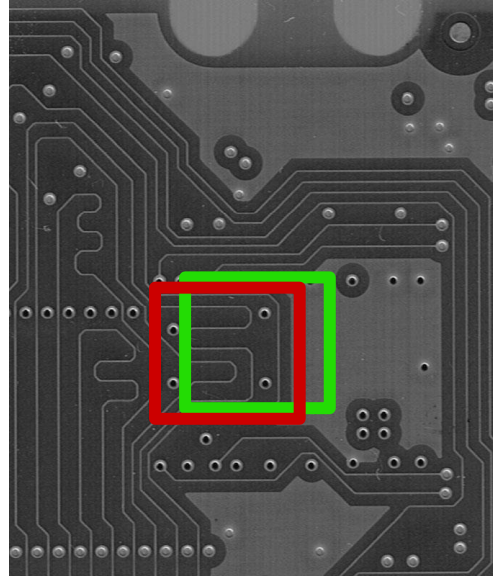


Image  $g[x, y]$

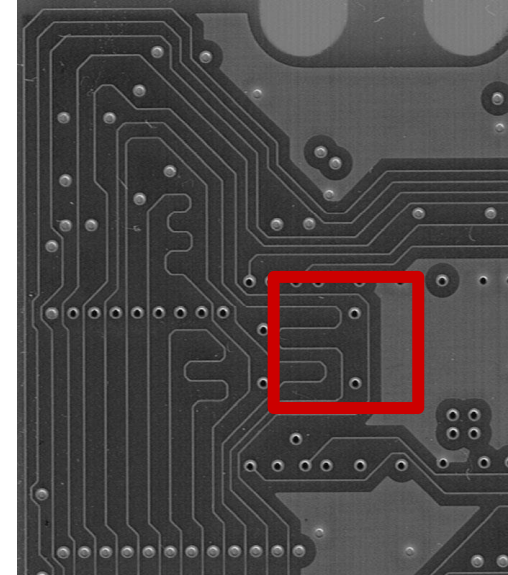


Image  $f[x, y]$

Rectangular array of pixels is selected as a measurement window

# Displacement estimation by block matching

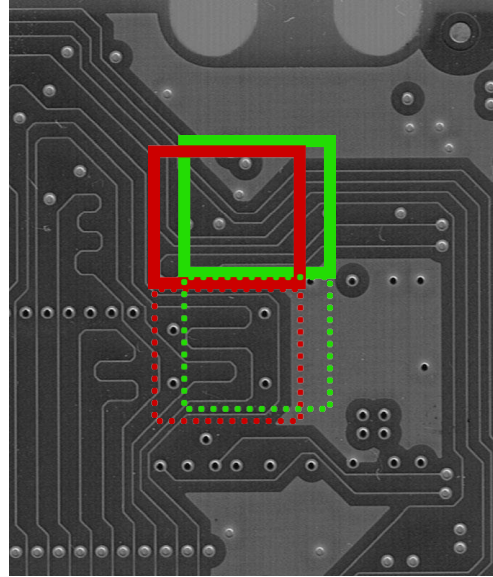


Image  $g[x,y]$

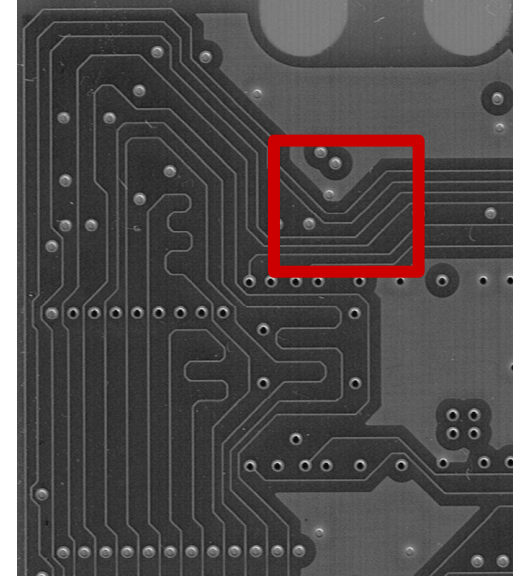


Image  $f[x,y]$

. . . process repeated for another measurement window position.

# Integer pixel shifts

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

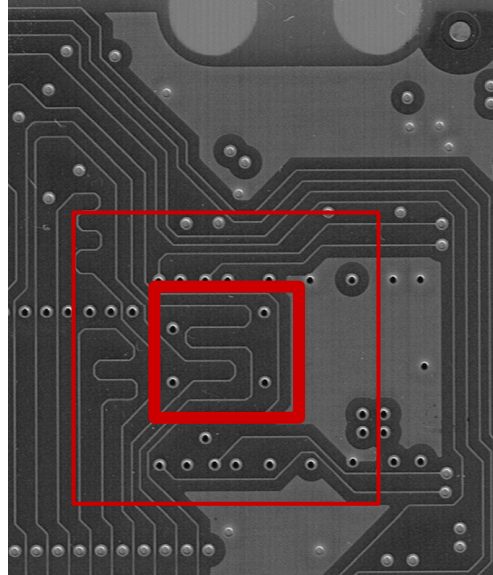


Image  $g[x,y]$

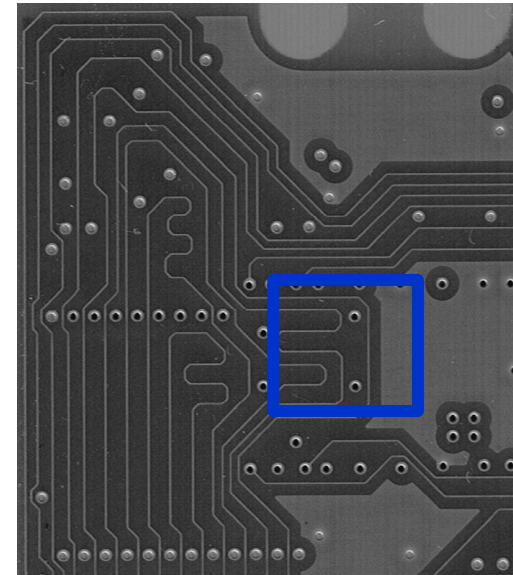


Image  $f[x,y]$

Rectangular array of pixels is selected as a measurement window

# Integer pixel shifts

28	42	42	43	44	40	32	20	29	32	22
30	44	45	45	45	42	30	21	26	27	18
35	54	54	54	52	50	50	50	48	21	25
40	63	62	62	62	60	60	60	60	31	25
74	121	120	120	120	120	120	120	120	80	17
79	127	130	130	130	130	130	130	130	80	23
80	129	131	131	131	131	131	131	131	98	49
50	78	77	77	77	75	75	75	75	65	29
22	37	37	37	39	40	40	41	41	38	25

54	53	52	49	31	21
62	63	59	60	44	33
120	114	112	111	80	32
130	128	124	125	88	24
131	124	127	127	96	42
77	71	73	75	63	52

Rectangular array of pixels is selected as a measurement window

Measurement window is compared with a shifted array of pixels in the other image, to determine the best match

# Error metric

- *Sum of Squared Differences*

Sum all values in measurement window

$$SSD[\Delta_x, \Delta_y] = \sum_{[x,y] \in \text{msmnt window}} \left( f[x, y] - g[x + \Delta_x, y + \Delta_y] \right)^2$$

Horizontal displacement

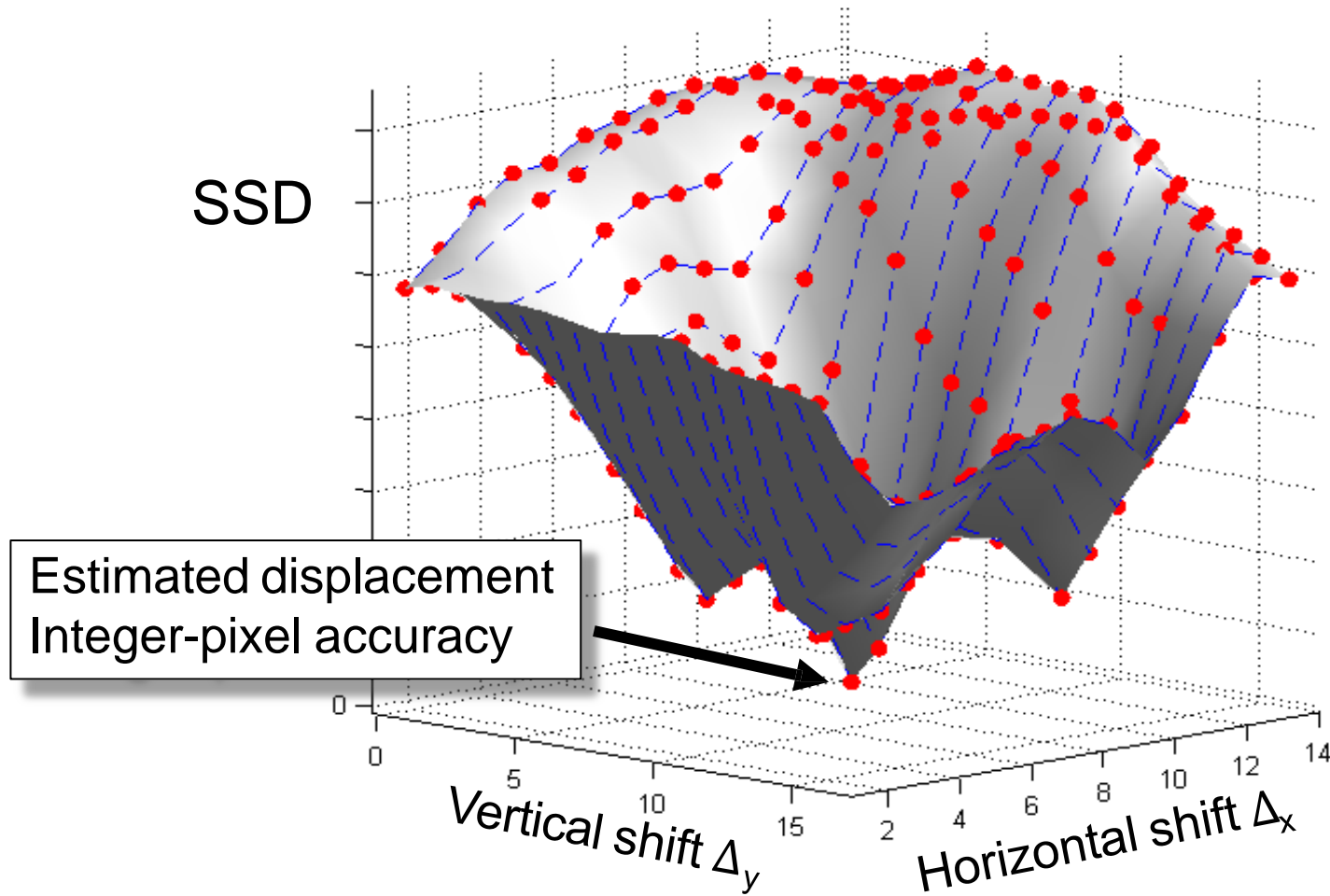
Vertical displacement

The diagram shows the SSD formula with three callout boxes. The first box, 'Sum all values in measurement window', points to the summation symbol  $\sum$ . The second box, 'Horizontal displacement', points to  $\Delta_x$ . The third box, 'Vertical displacement', points to  $\Delta_y$ .

- Alternatives: SAD (*Sum of Absolute Differences*), cross correlation, mutual information . . .
- Robustness against outliers: sum of saturated squared differences, median of squared differences . . .

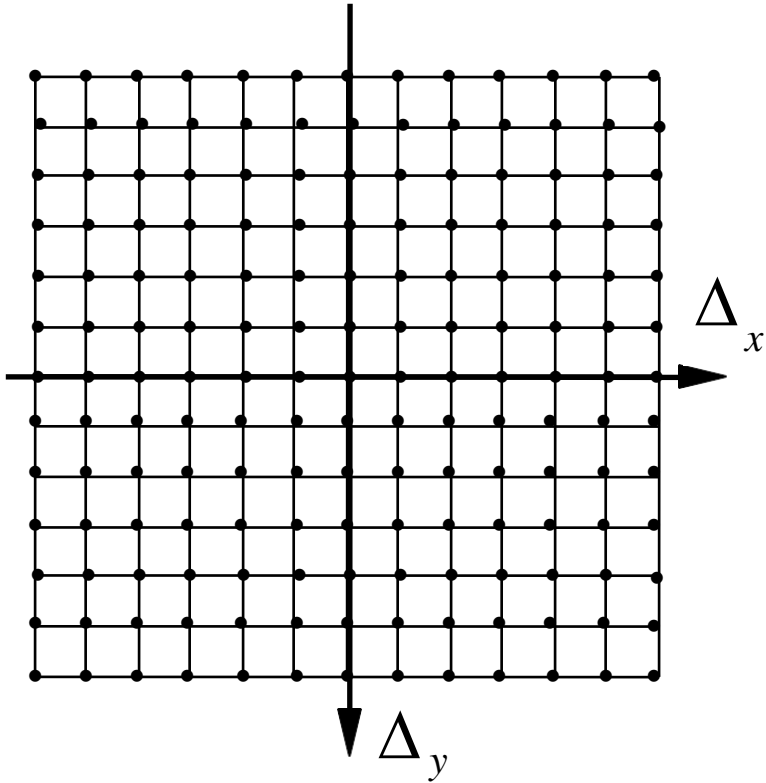


# SSD values resulting from block matching



# Block matching: search strategies

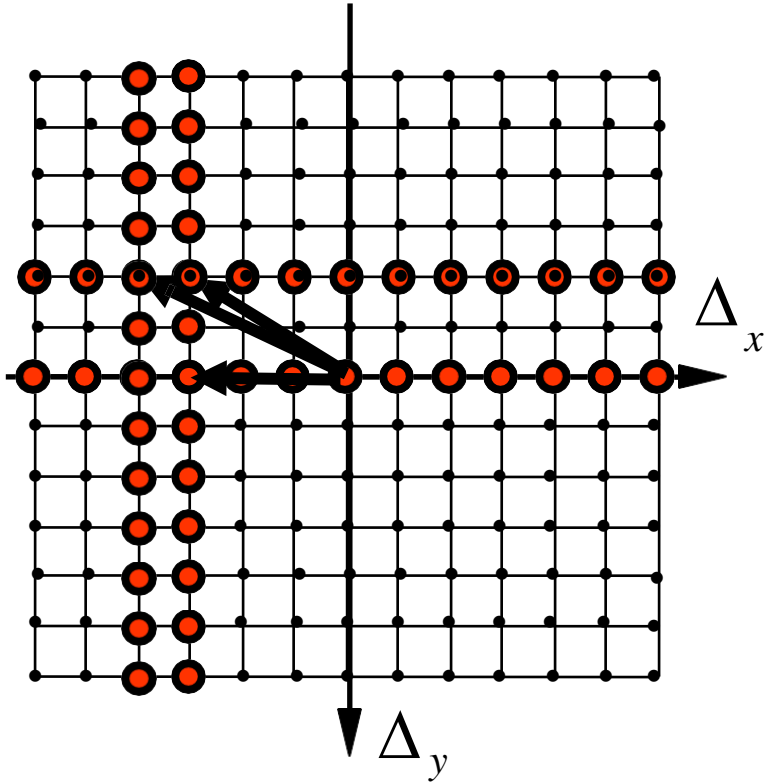
## Full search



- All possible displacements within the search range are compared.
- Computationally expensive
- Highly regular, parallelizable

# Block matching: search strategies

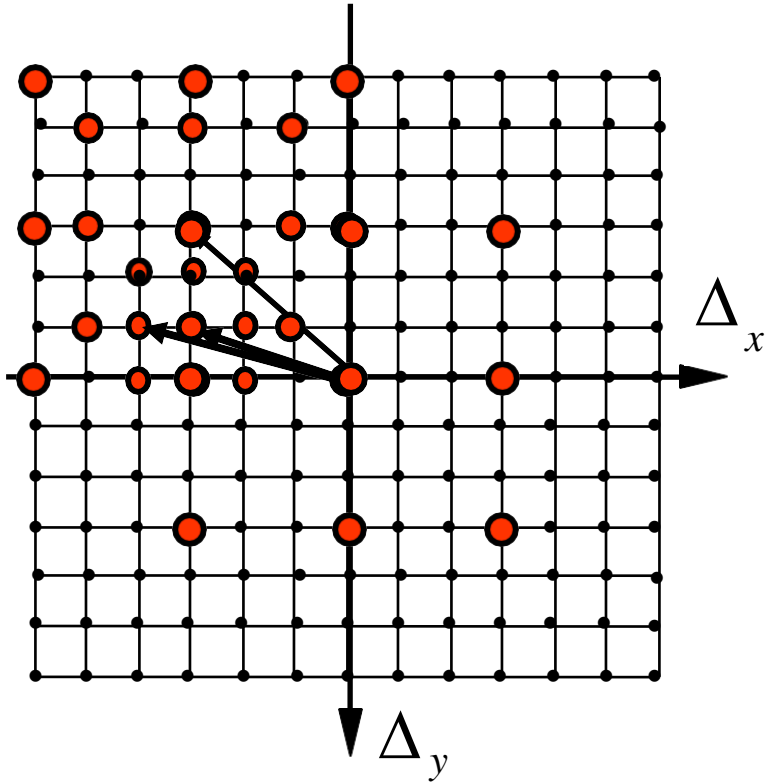
## Conjugate direction search



- Alternate search in x and y directions
- Stop when there is no further improvement

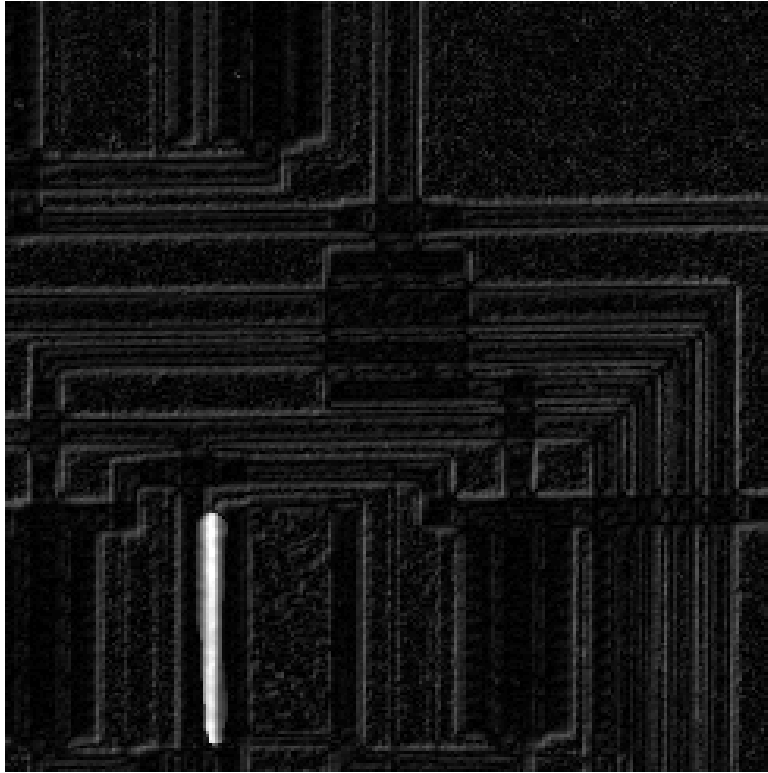
# Block matching: search strategies

## Coarse-to-fine

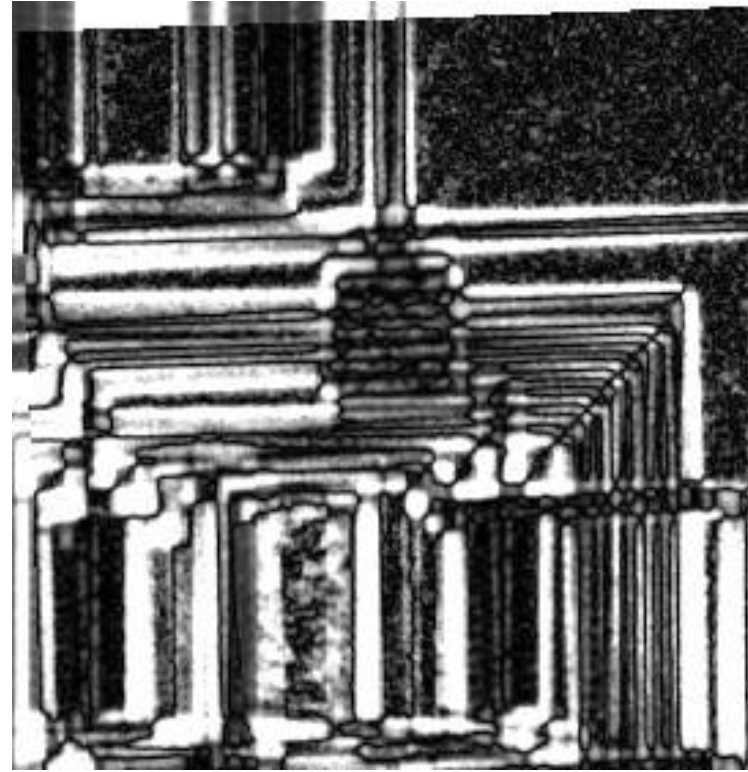


- Start with coarsely spaced candidate displacements
- Smaller pattern when best match is in the middle
- Stop when desired displacement accuracy is reached

# Absolute difference between images



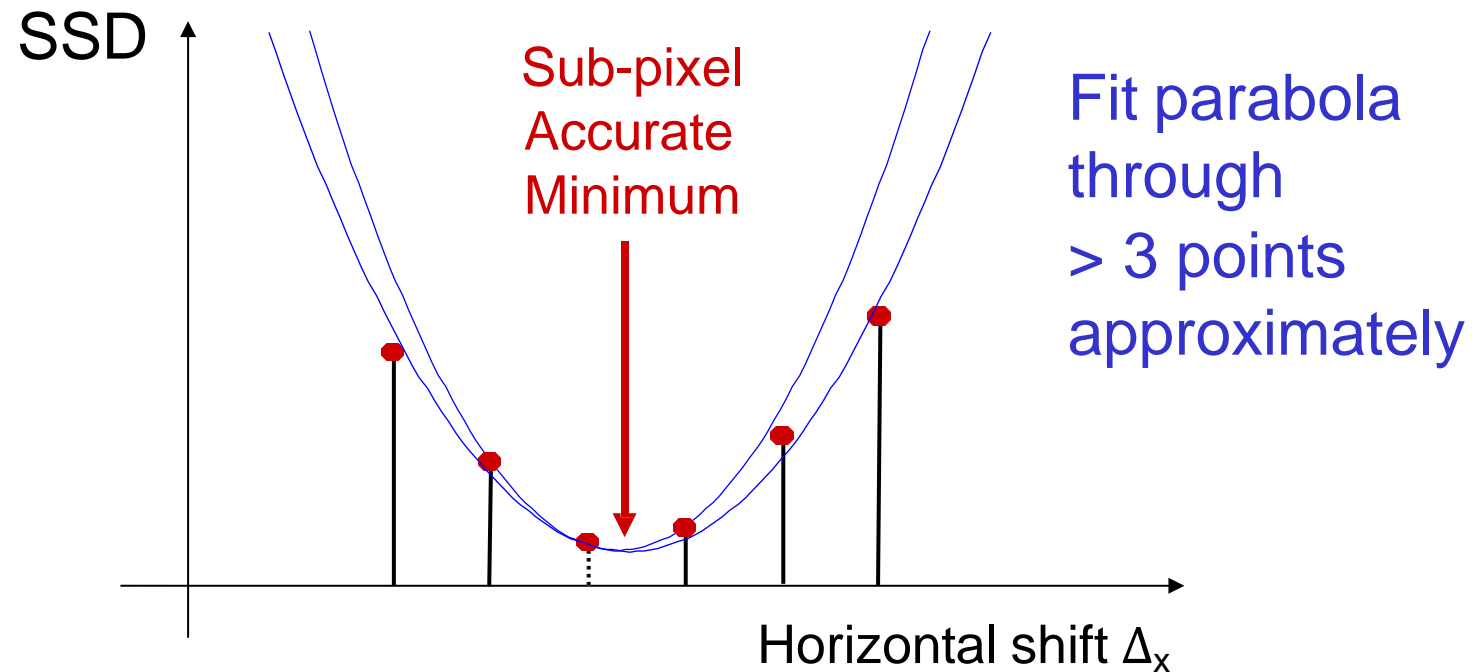
w/ integer-pixel alignment



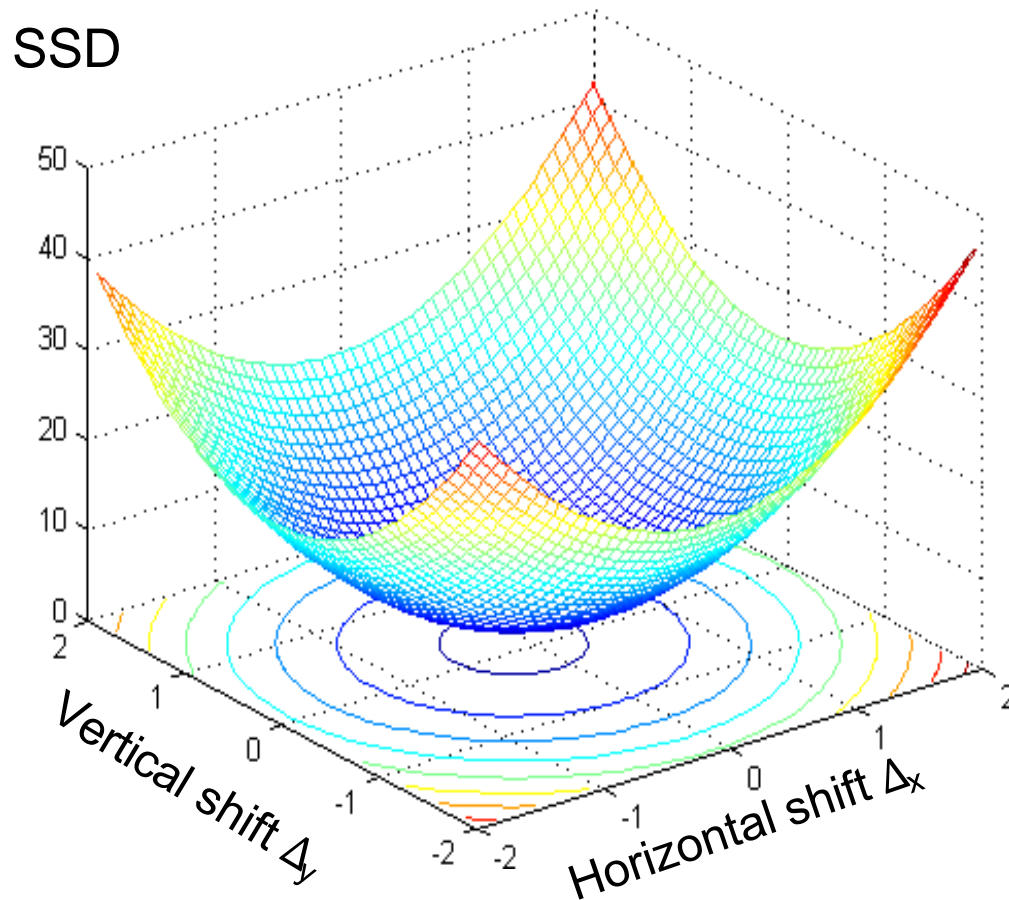
w/o alignment



# Interpolation of the SSD Minimum



# 2-d Interpolation of SSD Minimum

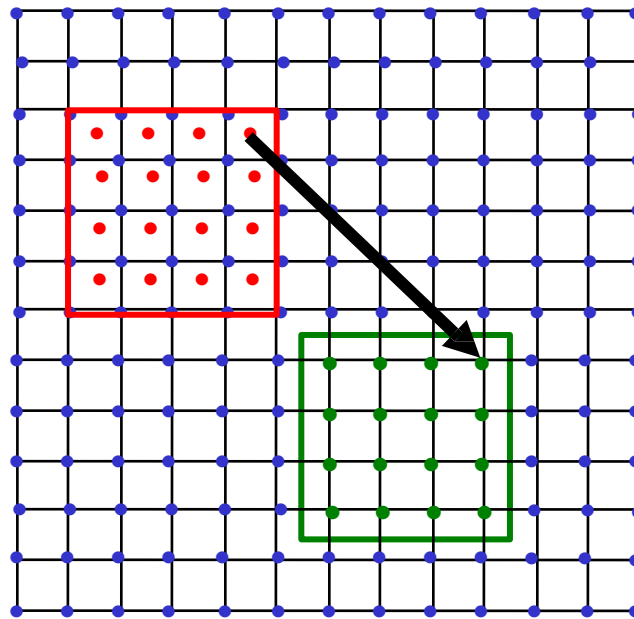


## Paraboloid

- Perfect fit through 6 points
- Approximate fit through  $> 6$  points

# Sub-pixel accuracy

- Interpolate pixel raster of the reference image to desired sub-pixel accuracy (e.g., by bi-linear or bi-cubic interpolation)
- Straightforward extension of displacement vector search to fractional accuracy
- Example: half-pixel accurate displacements



$$\begin{pmatrix} \Delta_x \\ \Delta_y \end{pmatrix} = \begin{pmatrix} 4.5 \\ 4.5 \end{pmatrix}$$