Eigenimages

- Unitary transforms
- Karhunen-Loève transform and eigenimages
- Sirovich and Kirby method
- Eigenfaces for gender recognition
- Fisher linear discriminant analysis
- Fisherimages and varying illumination
- Fisherfaces vs. eigenfaces

Image recognition using linear projection

- To recognize complex patterns (e.g., faces), large portions of an image (say N pixels) have to be considered
- High dimensionality of "image space" results in high computational burden for many recognition techniques

  Example: nea rest-neighbor search requires pairwise comparison with every image in a database

- Transform $\vec{c} = Wf$ is a projection on a J-dimensional linear subspace that greatly reduces the dimensionality of the image space $J << N$
- Idea: tailor the projection to a set of representative training images and preserve the salient features by using Principal Component Analysis (PCA)

$$W_{opt} = \underset{W}{\operatorname{argmax}} \left(\det\left(W R_{ff} W^H \right) \right)$$

Mean squared value of projection

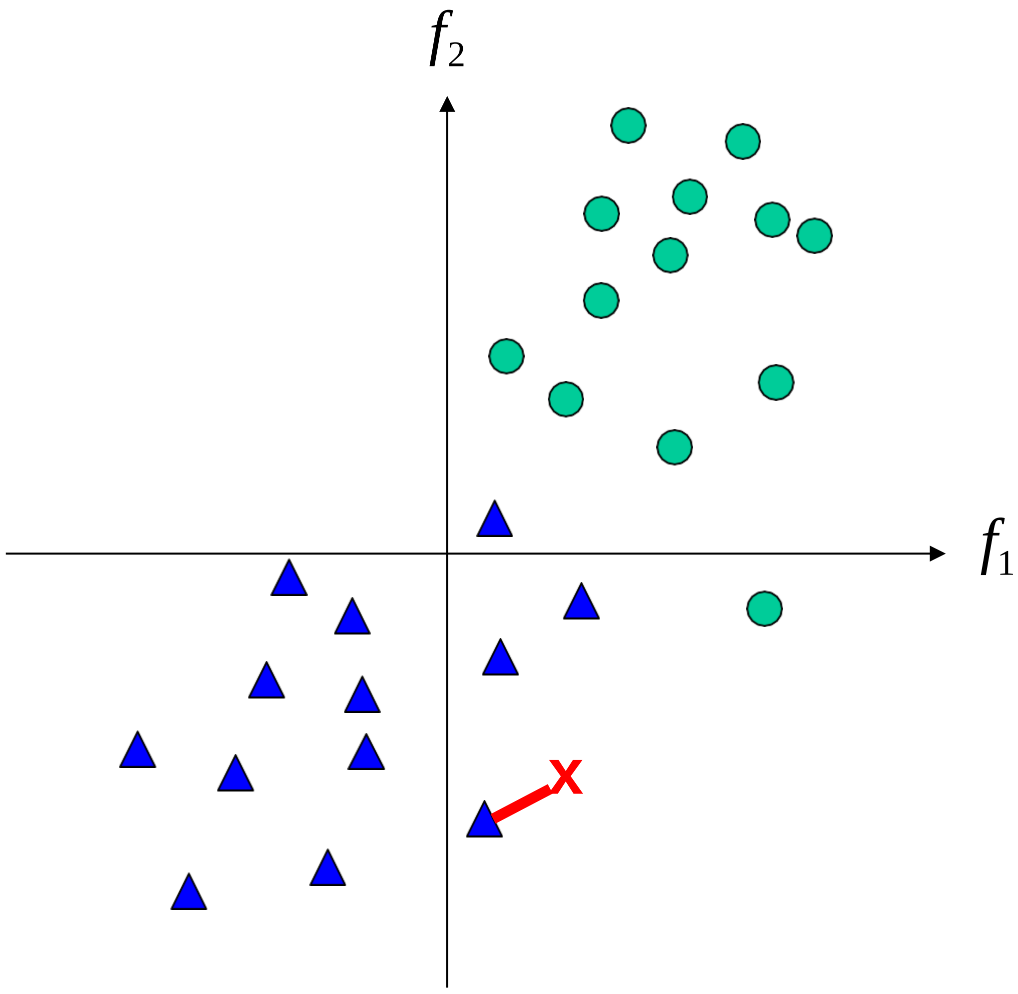
JxN projection matrix with orthonormal rows.

Autocorrelation matrix of image

Image recognition using linear projection

<u>2-d example:</u>

Goal: project samples on a 1-d subspace, then perform classification.



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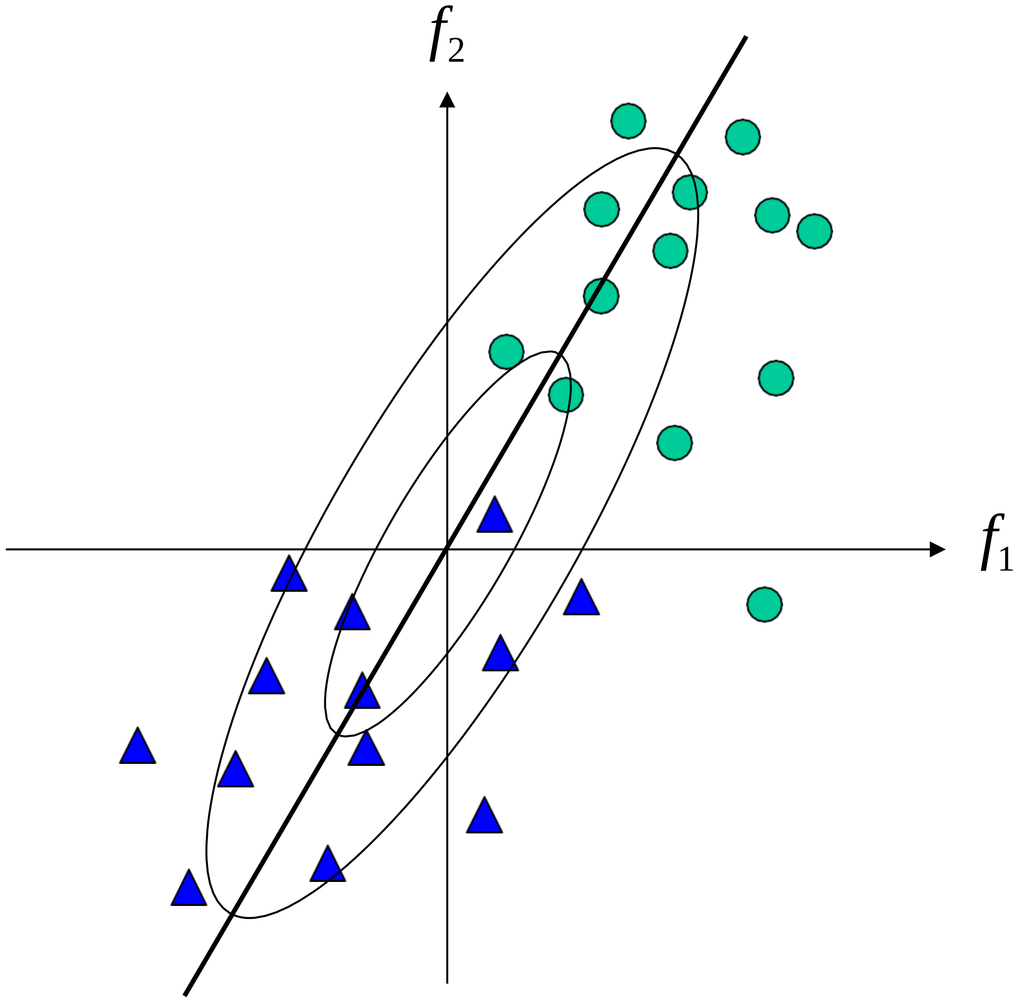

Image recognition using linear projection

2-d example:

Goal: project samples on
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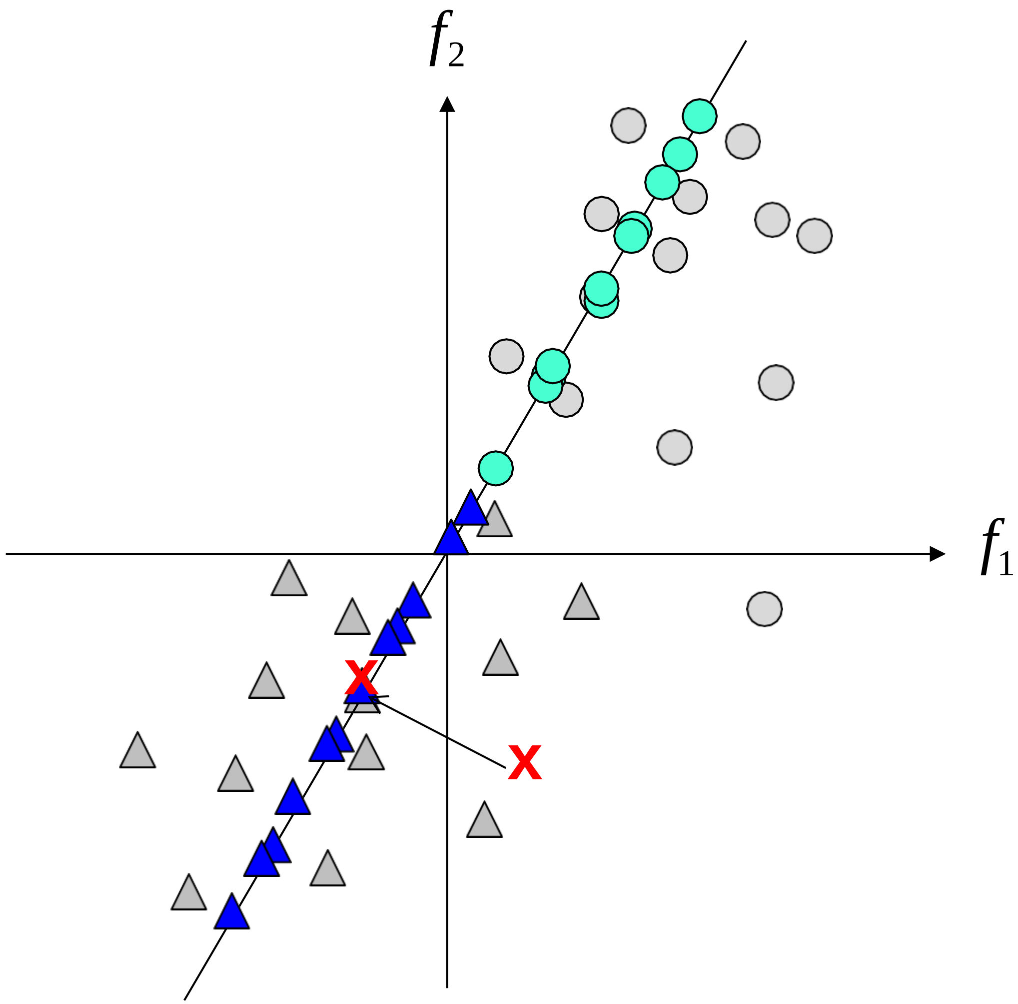
classification.



Unitary transforms

- Sort pixels $f[x,y]$ of an image into column vector f of length N
- Calculate N transform coefficients

$$\vec{c} = Af$$

where $\boldsymbol{A}$ is a matrix of size NxN

- The transform A is unitary, iff

$$A^{-1} = \underbrace{A^{*T} \equiv A^{H}}_{\text{Hermitian conjugate}}$$

- If A is real-valued, i.e., $A=A^*$, transform is „orthonormal“

Energy conservation with unitary transforms

- For any unitary transform $\vec{c} = A\vec{f}$ we obtain

$$\|\vec{c}\|^2 = \vec{c}^H \vec{c} = \vec{f}^H A^H A \vec{f} = \|\vec{f}\|^2$$

- Interpretation: every unitary transform is simply a rotation of the coordinate system (and, possibly, sign flips)
- Vector length is conserved.
- Energy (mean squared vector length) is conserved.

Energy distribution for unitary transforms

- Energy is conserved, but, in general, unevenly distributed among coefficients.
- Autocorrelation matrix

$$R_{cc} = E\left[\vec{c}\vec{c}^H\right] = E\left[A\vec{f} \cdot \vec{f}^H A^H\right] = AR_{ff}A^H$$

- Diagonal of R_{cc} comprises mean squared values („energies") of the coefficients c_i

$$E\left[c_i^2\right] = \left[R_{cc}\right]_{i,i} = \left[AR_{ff}A^H\right]_{i,i}$$

Eigenmatrix of the autocorrelation matrix

<u>Definition:</u> eigenmatrix $\mathbf{\Phi}$ of autocorrelation matrix R_{ff}

- $\mathbf{\Phi}$ is unitary
- The columns of $\mathbf{\Phi}$ form a set of eigenvectors of R_{ff}, i.e.,

$$\boxed{R_{ff}\theta = \theta\Lambda}$$

Λ is a diagonal matrix of eigenvalues λ_i

$$\Lambda = \begin{pmatrix} \lambda_0 & & & 0 \\ & \lambda_1 & & \\ & & \ddots & \\ 0 & & & \lambda_{N-1} \end{pmatrix}$$

- R_{ff} is normal matrix, i.e., $R_{ff}^H R_{ff} = R_{ff} R_{ff}^H$,
  hence unitary eigenmatrix exists („spectral theorem“)
- R_{ff} is symmetric nonnegative definite, hence $\lambda_i \geq 0 \quad \text{for all } i$

Karhunen-Loève transform

- Unitary transform with matrix

$$A = \theta^{H}$$

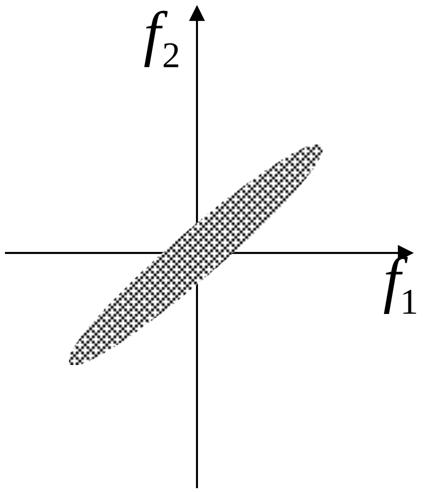
- Transform coefficients are pairwise uncorrelated

$$R_{cc} = A R_{ff} A^{H} = \theta^{H} R_{ff} \theta = \theta^{H} \theta \Lambda = \Lambda$$

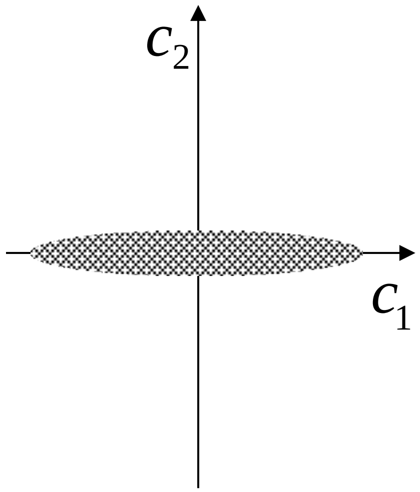
- Columns of $\mathbf{\Phi}$ are ordered according to decreasing eigenvalues.
- Energy concentration property:
  - No other unitary transform packs as much energy into the first J coefficients.
  - Mean squared approximation error by keeping only first J coefficients is minimized.
  - Holds for any J.

Illustration of energy concentration

Strongly correlated samples,
equal energies



$$\mathbf{A} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$



<u>After KLT:</u>
uncorrelated samples,
most of the energy in
first coefficient

Basis images and eigenimages

- For any transform, the inverse transform

$$\vec{f} = A^{-1}\vec{c}$$

can be interpreted in terms of the superposition of columns of A^{-1} („basis images“)

- For the KL transform, the basis images are the eigenvectors of the autocorrelation matrix R_{ff} and are called „eigenimages.“
- If energy concentration works well, only a limited number of eigenimages is needed to approximate a set of images with small error. These eigenimages span an optimal linear subspace of dimensionality J.
- Eigenimages can be used directly as rows of the projection matrix

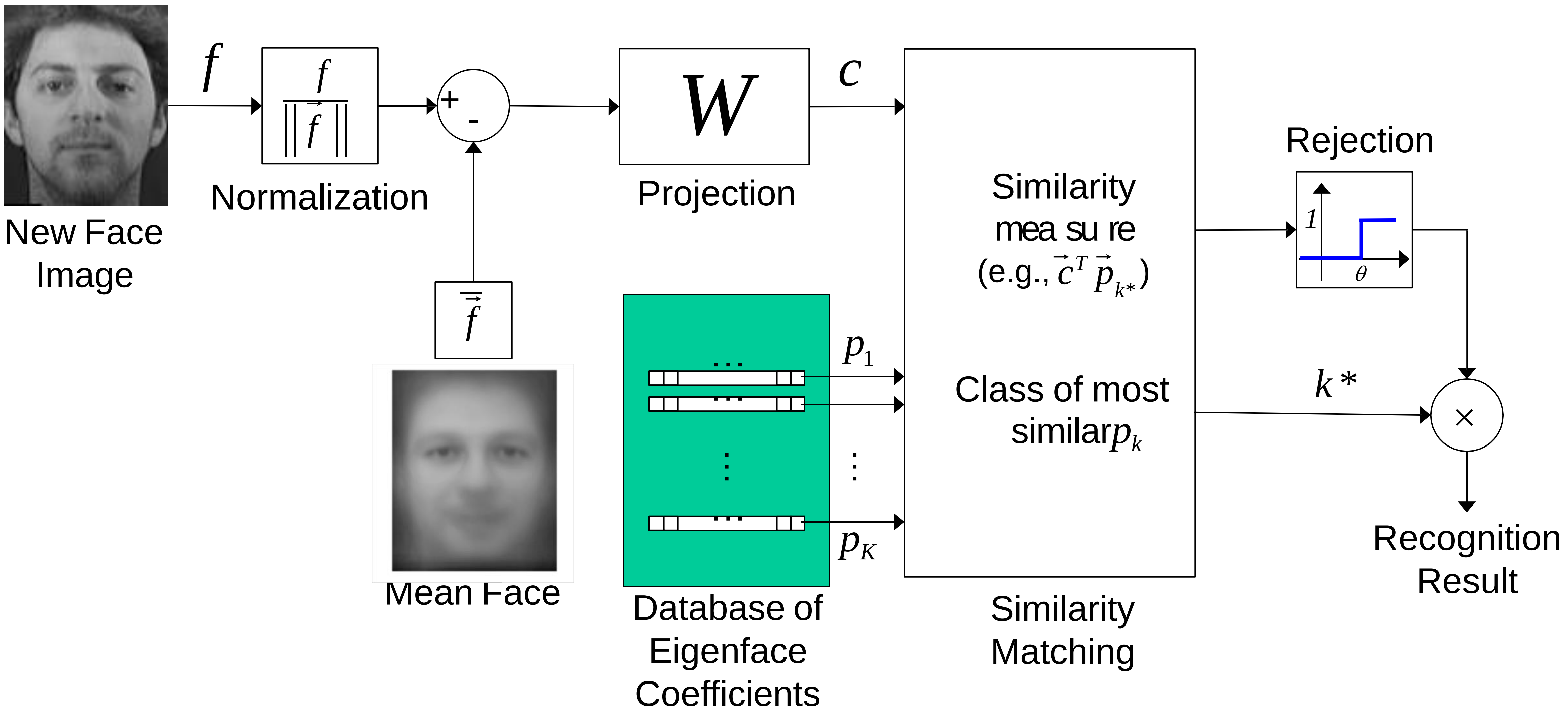
$$W_{opt} = \underset{W}{\operatorname{argmax}} \left(\det\left(W R_{ff} W^H \right) \right)$$

Mean squared value of projection

JxN projection matrix with orthonormal rows

Autocorrelation matrix of image

Eigenimages for face recognition



Computing eigenimages from a training set

- How to obtain NxN covariance matrix?
  - Use training set $\Gamma_1, \Gamma_2, \ldots, \Gamma_{L+1}$ (each column vector represents one image)
  - Let μ be the mean image of all $L+1$ training images
  - Define training set matrix $S = \left(\vec{\Gamma}_1 - \vec{\mu}, \vec{\Gamma}_2 - \vec{\mu}, \vec{\Gamma}_3 - \vec{\mu}, \ldots, \vec{\Gamma}_L - \vec{\mu}\right)$

    and calculate scatter matrix $R = \sum_{l=1}^{L} \left(\vec{\Gamma}_l - \vec{\mu}\right)\left(\vec{\Gamma}_l - \vec{\mu}\right)^H = SS^H$

  Problem 1: Training set size should be $L+1 >> N$

  If $L < N$, scatter matrix R is rank-deficient

  Problem 2: Finding eigenvectors of an NxN matrix.

- Can we find a small set of the most important eigenimages from a small training set $L << N$?

Sirovich and Kirby algorithm

- Instead of eigenvectors of SS^H, consider the eigenvectors of S^HS, i.e.,

$$S^H S\vec{v}_i = \lambda_i \vec{v}_i$$

- Premultiply both sides by S

$$SS^H S\vec{v}_i = \lambda_i S\vec{v}_i$$

- By inspection, we find that $S\vec{v}_i$ are eigenvectors of SS^H

Sirovich and Kirby Algorithm (for $L << N$)

- Compute the LxL matrix S^HS
- Compute L eigenvectors v_i of S^HS
- Compute eigenimages corresponding to the $L_0 \leq L$ largest eigenvalues as a linear combination of training images Sv_i

L. Sirovich and M. Kirby, "Low-dimensional procedure for the characterization of human faces," Journal of the Optical Society of America A, 4(3), pp. 519-524, 1987.

Example: eigenfaces

- The first 8 eigenfaces obtained from a training set of 100 male and 100 female training images

Mean Face

Eigenface 1

Eigenface 2

Eigenface 3

Eigenface 4

Eigenface 5

Eigenface 6

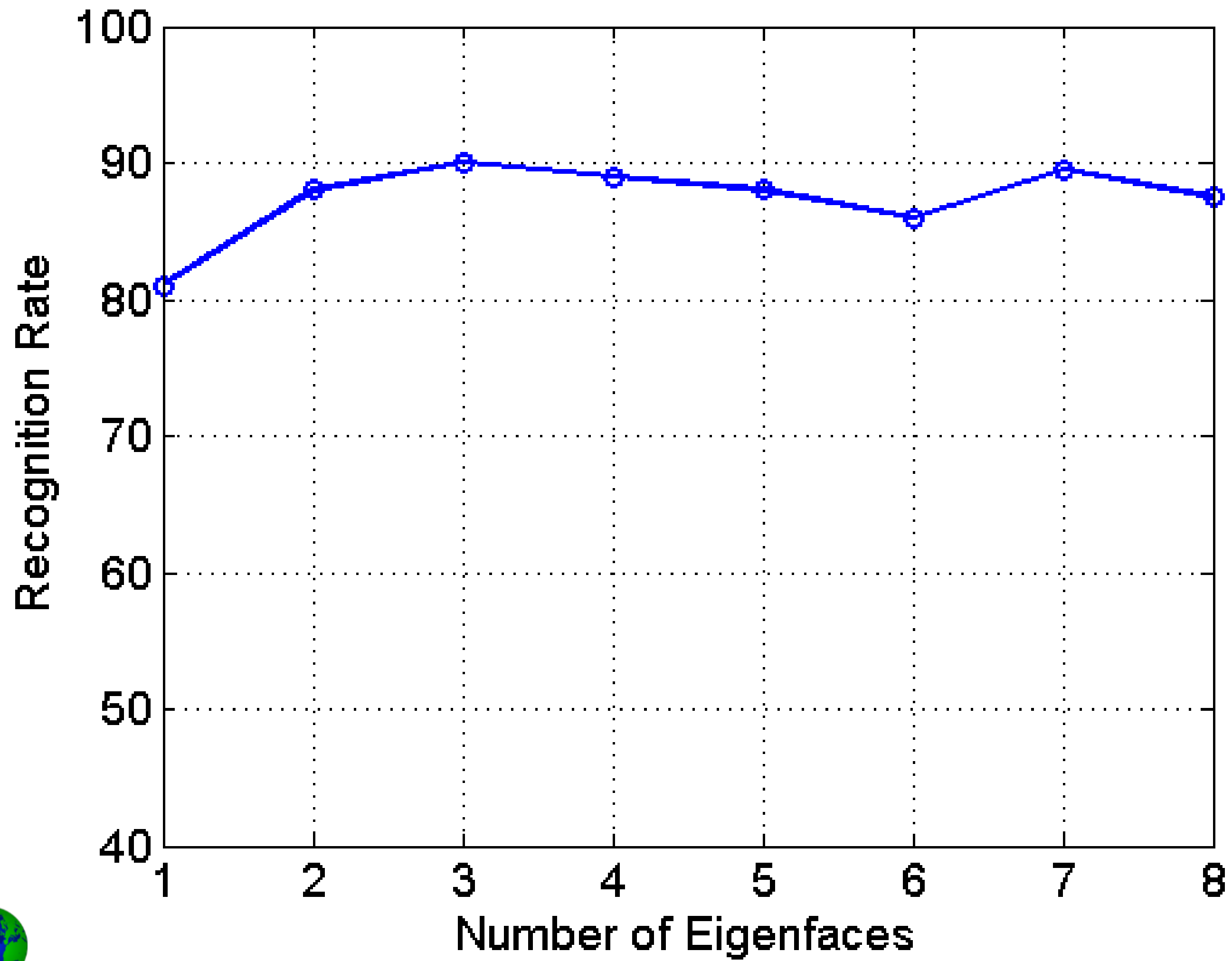
Eigenface 7

Eigenface 8

- Can be used to generate faces by adjusting 8 coefficients.
- Can be used for face recognition by nearest-neighbor search in 8-d „face space.“

Gender recognition using eigenfaces

Nearest neighbor search in “face space”



Female face samples

Male face samples

Fisher linear discriminant analysis

- Eigenimage method maximizes "scatter" within the linear subspace over the entire image set – regardless of classification task

$$W_{opt} = \arg\max_{W} \left(\det \left(W R W^{H} \right) \right)$$

- Fisher linear discriminant analysis (1936): maximize between-class scatter, while minimizing within-class scatter

$$W_{opt} = \arg\max_{W} \left(\frac{\det \left(W R_B W^{H} \right)}{\det \left(W R_W W^{H} \right)} \right)$$

$$R_B = \sum_{i=1}^{c} N_i \left(\vec{\mu}_i - \vec{\mu} \right) \left(\vec{\mu}_i - \vec{\mu} \right)^{H}$$

Samples in class i

Mean in class i

$$R_W = \sum_{i=1}^{c} \sum_{\vec{\Gamma}_l \in Class(i)} \left(\vec{\Gamma}_l - \vec{\mu}_i \right) \left(\vec{\Gamma}_l - \vec{\mu}_i \right)^{H}$$

Fisher linear discriminant analysis (cont.)

- Solution: Generalized eigenvectors w_i corresponding to the J largest eigenvalues $\{\lambda_i \mid i = 1, 2, ..., J\}$, i.e.

$$\boxed{R_B \overrightarrow{w_i} = \lambda_i R_W \overrightarrow{w_i} \ , \quad i = 1, 2, ..., J}$$

- Problem: within-class scatter matrix R_w at most of rank L-c, hence usually singular.
- Apply KLT first to reduce dimensionality of feature space to L-c (or less), proceed with Fisher LDA in lower-dimensional space

Eigenimages vs. Fisherimages

<u>2-d example:</u>

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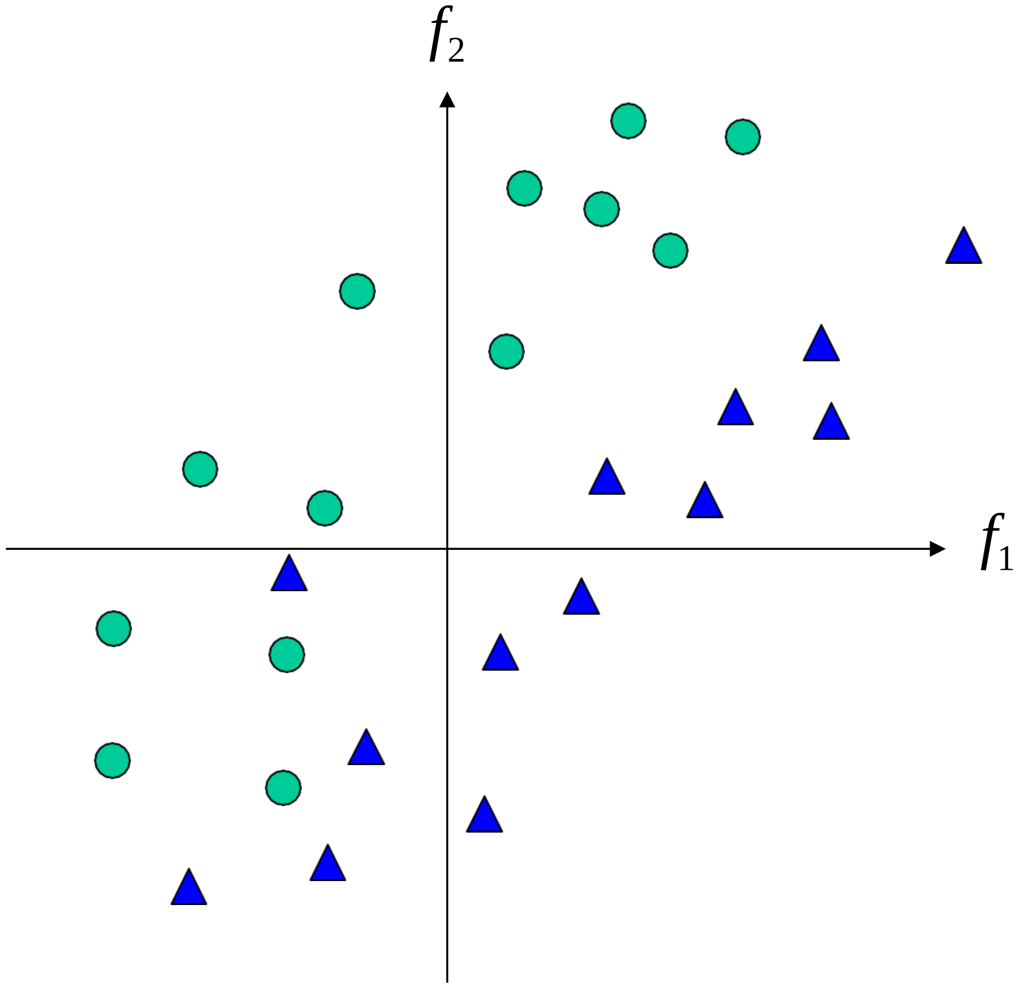
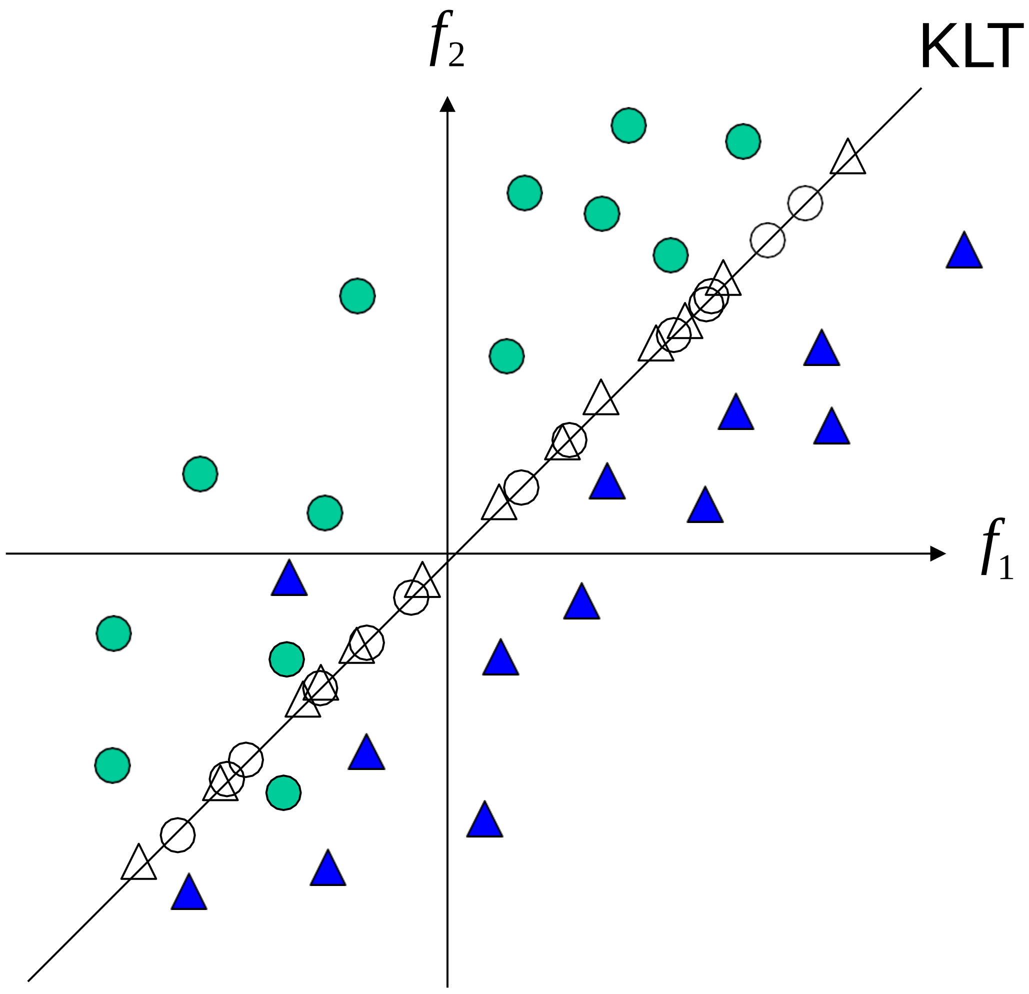


Eigenimages vs. Fisherimages

<u>2-d example:</u>

Goal: project samples on a 1-d subspace, then perform classification.

The KLT preserves maximum energy, but the 2 classes are no longer distinguishable.



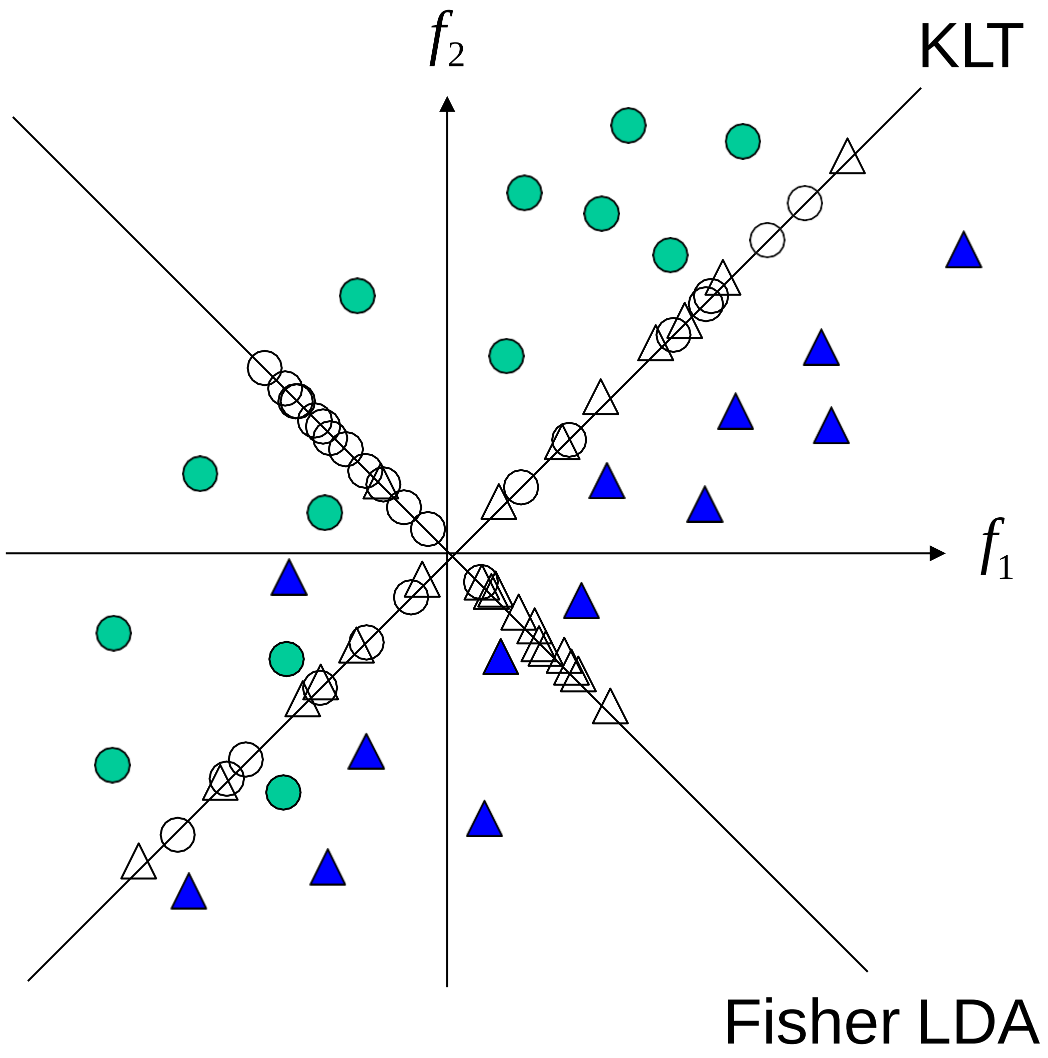
Eigenimages vs. Fisherimages

<u>2-d example:</u>

Goal: project samples on a 1-d subspace, then perform classification.

The KLT preserves maximum energy, but the 2 classes are no longer distinguishable.

Fisher LDA separates the classes by choosing a better 1-d subspace.

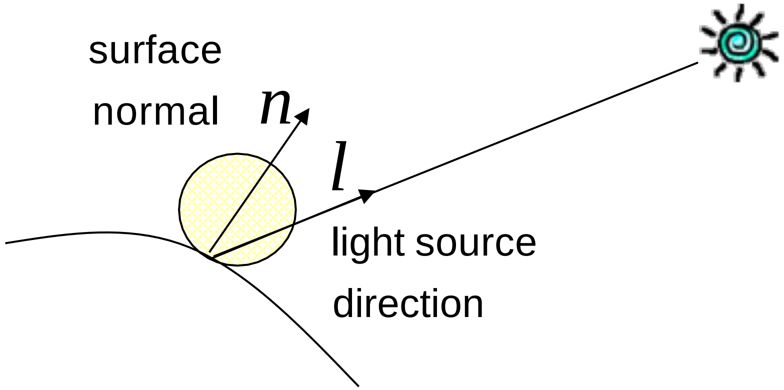


Fisherimages and varying illlumination

Differences due to varying illumination can be much larger than differences among faces!

Fisherimages and varying illlumination

- All images of same Lambertian surface with different illumination (without shadows) lie in a 3d linear subspace
- Single point source at infinity



Light source intensity

$$f(x,y) = a(x,y)\left(\vec{l}^{\,T}\vec{n}(x,y)\right)L$$

Surface albedo

- Superposition of arbitrary number of point sources at infinity still in same 3d linear subspace, due to linear superposition of each contribution to image
- Fisherimages can eliminate within-class scatter

Fisherface trained to recognize gender

Female face samples

Male face samples

Mean image

μ

Female mean

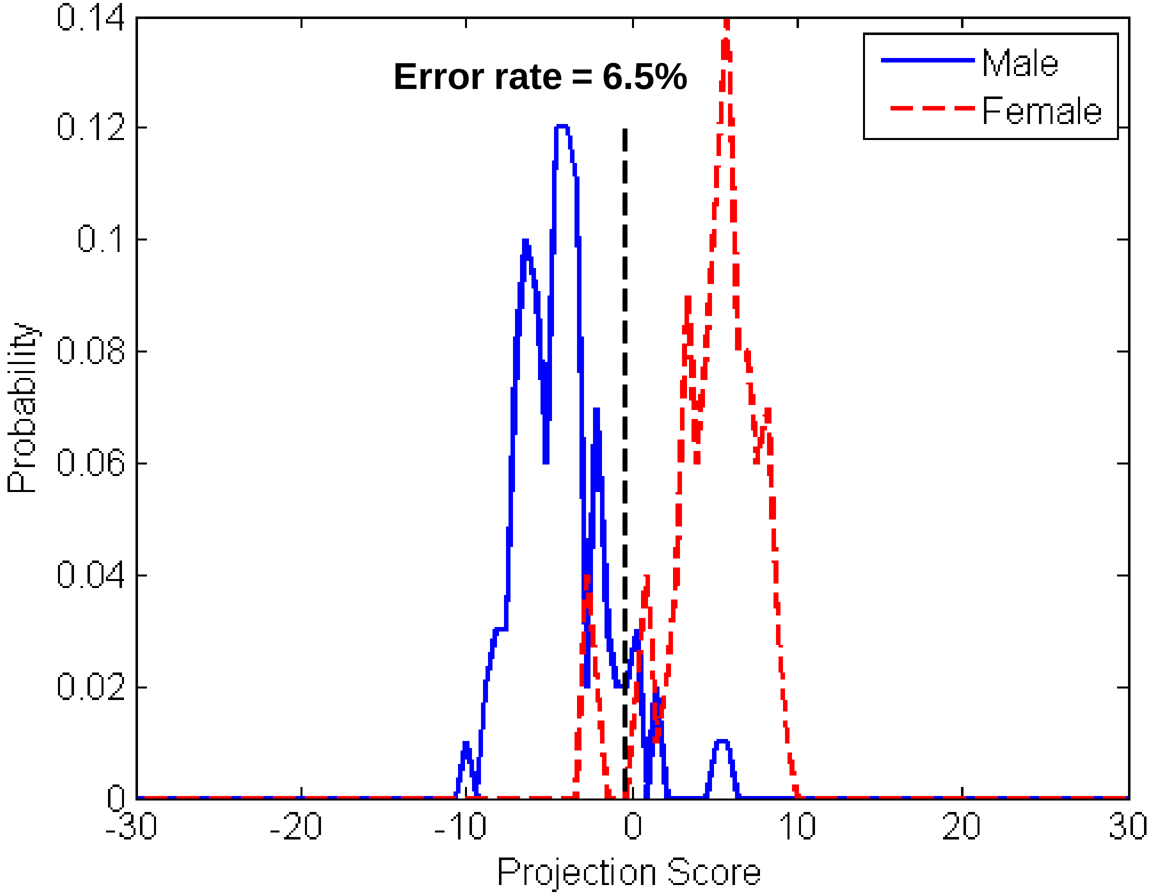
$\vec{\mu}_1$

Male mean

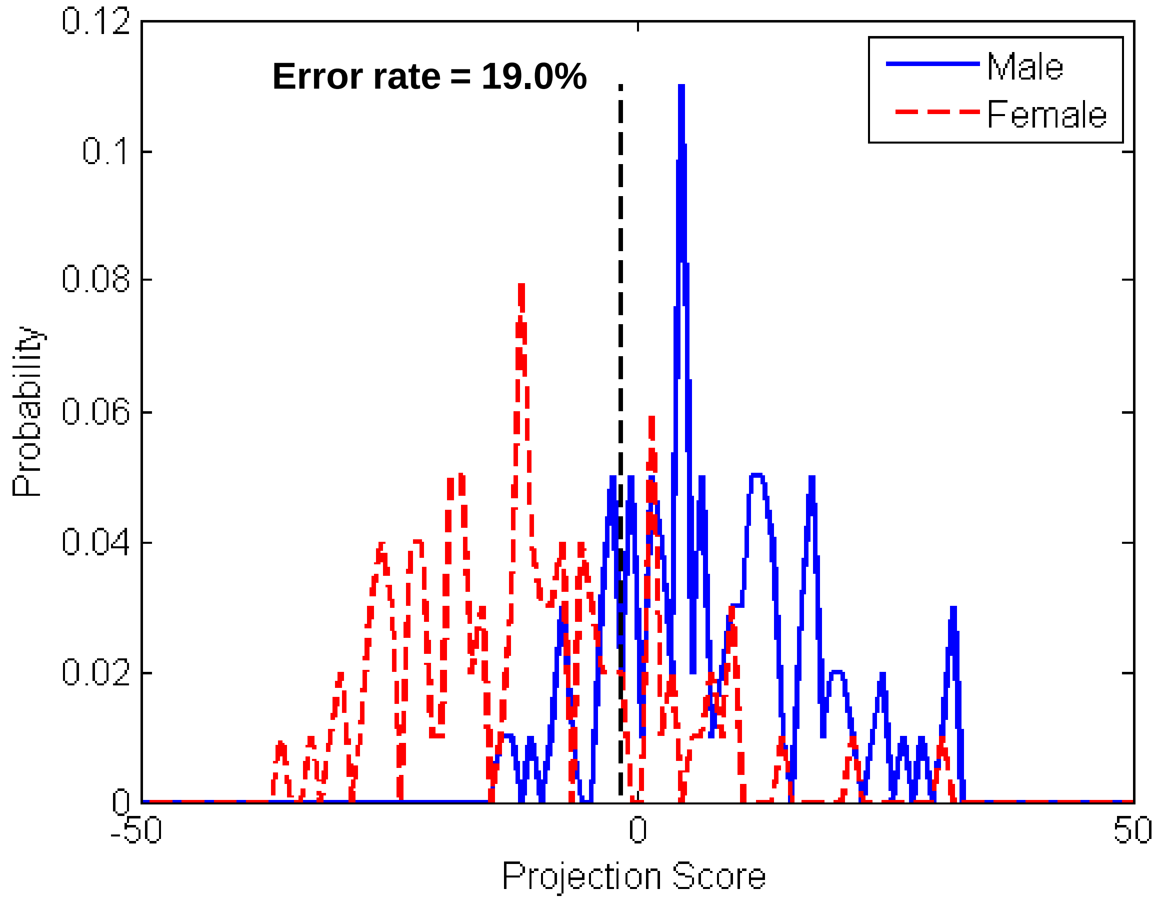
$\vec{\mu}_2$

Fisherface

Gender recognition using 1st Fisherface



Gender recognition using 1st eigenface



Person identification with Fisherfaces and eigenfaces