HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY
FALTY OF COMPUTER SCIENCE AND ENGINEERING

CAPSTONE PROJECT
COMPUTER ENGINEERING

Motion Planning around Obstacles

Semester 251

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Abstract

Trajectory optimization offers mature tools for motion planning in high-dimensional spaces under dynamic constraints. However, in obstacle-cluttered environments, the non-convexity of the free space typically forces roboticists to rely on sampling-based planners, which struggle with high dimensions and differential constraints. Here, we demonstrate that convex optimization can reliably solve these problems through the convex decomposition of the free space. Specifically, we decompose the collision-free configuration space into finite convex regions, organizing them into a Graph of Convex Sets (GCS). By combining Bézier curves with this graph structure, we formulate the motion planning problem as a compact mixed-integer convex program that naturally handles collision avoidance, velocity, and duration constraints. We validate GCS across a spectrum of environments with increasing complexity, ranging from simple 2D obstacles and cluttered mazes to dynamical quadrotors and high-dimensional 7-DoF manipulators. Numerical experiments demonstrate that GCS consistently outperforms widely-used sampling-based planners (e.g., PRM). Specifically, in high-dimensional tasks, GCS reduces online query time by up to an order of magnitude compared to standard PRM, while finding globally optimal trajectories that are significantly shorter (up to 40% reduction in length) than those from post-processed sampling-based planners [1]. Ultimately, this work brings the reliability and speed of convex optimization to the traditionally challenging domain of nonconvex motion planning.

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Chapter 1

Introduction

In chapter 1, the overview, objectives and goals of the research's project are illustrated. The outline of the report is also presented.

1.1 Motivation and Technical Challenges

Making optimal decisions in real-time is a cornerstone of modern engineering. The necessity of tightly coupling **discrete decisions** with **continuous control** is vividly illustrated in advanced robotics:

- **Humanoid Robots (e.g., Boston Dynamics' Atlas):** Navigating complex terrain, such as a parkour course, requires the robot to simultaneously select footstep locations (discrete) and compute the trajectories for its center of mass and limbs (continuous). Currently, solutions often rely on "hand-coding" the discrete components, which severely limits the robot's autonomy in novel environments [2].

- **Industrial Manipulators (e.g., Amazon's Robin):** Sorting efficiency depends on jointly optimizing the sequence of bins (discrete) and the arm's trajectory (continuous) [3]. A mere **1% increase** in operating speed through superior optimization translates to millions of additional packages processed annually.

However, solving these coupled problems simultaneously remains an open challenge, typically necessitating manual engineering interventions or settling for suboptimal solutions.

Algorithmically, selecting a motion planning method currently forces researchers to compromise between conflicting features: dimensionality, dynamic constraints, and completeness.

- **Trajectory Optimization:** These methods excel at handling robot dynamics and high-dimensional spaces. However, when facing obstacle-cluttered environments—which render the problem inherently non-convex—they often get trapped in **local minima**, failing to identify a collision-free trajectory.[4]
- **Sampling-based Planners:** To overcome these limitations, the robotics community frequently resorts to sampling-based methods (e.g., RRT, PRM) due to their "probabilistic completeness". However, this comes at a cost: the resulting trajectories are often significantly **suboptimal**, and imposing **continuous differential constraints** on discrete samples remains notoriously difficult.[5], [6]

Mixed-Integer Convex Programming (MICP) has emerged as a promising solution that bridges this gap. It combines the strengths of both worlds: the completeness of sampling-based algorithms and the dynamic handling capabilities of trajectory optimization, while guaranteeing **global optimality** within a unified framework. Nevertheless, the adoption of MICP is severely limited by its prohibitive computational costs, often requiring minutes to solve even small-scale problems.

The objective of this research is to break this computational barrier. We focus on a crucial class of motion planning problems involving differential constraints and propose an MICP-based planner capable of reliably solving high-dimensional problems in a matter of seconds through a **Single Convex Program**.

1.2 Goals

1.3 Scope

1.4 Thesis structure

There are five chapters in this capstone project:

- Chapter 1 briefly describes the project about problem's motivation, as well as the project's objectives and scope
- Chapter 2 is dedicated to presenting the foundational knowledge about convex optimization, mixed-integer optimization, and graph theory that are essential for understanding the proposed method.
- Chapter 3 arranges the discussion of related works on the same task to deepen the understanding of existing methods and their constraints.
- Chapter 4 describes the GCS framework and the formulation of our MICP. From that, we present the experimental results and analysis of our method in various environments with different complexity levels.
- Chapter 5 summarizes whole project and the plan for future development.

Chapter 2

Theoretical Background

In Chapter 2, it presents about preliminary knowledge in this project.

2.1 Convex analysis and optimization

2.2 Mixed-integer optimization

2.3 Graphs

Chapter 3

Related works

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3.1 Sampling-Based Methods

3.2 Discussion

In this chapter, we have seen three distinct approaches to enhance...

Chapter 4

Proposed Solution

As mentioned in Chapter 3, the enhancement of knowledge of . Chapter 4 presents the proposed solution for enhancing .

4.1 Baseline methods

4.2 Proposed solution: ...

4.3 Experiment setup

4.3.1 Implementation details

4.4 Results and analysis

Chapter 5

Conclusion

5.1 Summary

5.2 Future Work

Future research will focus on...

- ...
- ...

References

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Appendix A

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A.1 ...

A.2 ...