parametric rate $n^{-1/2}$, n being the sample size. We also characterize classes of alternative hypotheses against which our tests have no power, and argue that such classes are rather exceptional. Finally, we show that critical values can be computed with the assistance of a multiplier-type bootstrap that is easy to implement.

3.1 Asymptotic null distribution

The asymptotic null distributions of our tests are the limiting distributions of continuous functionals of $\hat{R}_n^p(u)$ under H_0. To derive the asymptotic results, we adopt the following notation. For a generic set $\mathcal{G}$, let $l^\infty(\mathcal{G})$ be the Banach space of all uniformly bounded real functions on $\mathcal{G}$, equipped with the uniform metric $\|f\|_{\mathcal{G}} \equiv \sup_{z\in\mathcal{G}} |f(z)|$. We study the weak convergence of $\hat{R}_n^p(u)$ and its related processes as elements of $l^\infty(\Pi)$, where $\Pi \equiv [0,1]$. Let "$\Rightarrow$" denote weak convergence on $(l^\infty(\Pi), \mathcal{B}_\infty)$ in the sense of J. Hoffmann-Jϕrgensen, where $\mathcal{B}_\infty$ denotes the corresponding Borel σ-algebra - see e.g. Definition 1.3.3 in van der Vaart and Wellner (1996).

We assume the following regularity conditions. Let Θ_0 be an arbitrarily small neighborhood around θ_0 such that $\Theta_0 \subset \Theta$. For any $d_1 \times d_2$ matrix $A = (a_{ij})$, let $||A||$ denote its Euclidean norm, i.e. $||A|| = [\mathrm{tr}(AA')]^{1/2}$.

Assumption 3.1 *(i) The parameter space Θ is a compact subset of $\mathbb{R}^k$; (ii) the true parameter θ_0 belongs to the interior of Θ; and (iii) $\left\|\hat{\theta}_n - \theta_0\right\| = O_p(n^{-1/2})$.*

Assumption 3.2 *The parametric propensity score function $q(x,\theta)$ is twice continuously differentiable in Θ_0 for each $x \in \mathcal{X}$, with its first derivative $g(x,\theta) = \partial q(x,\theta)/\partial\theta = (g_1(x,\theta),\ldots,g_k(x,\theta))'$ satisfying $\mathbb{E}[\sup_{\theta\in\Theta_0} ||g(X,\theta)||] < \infty$ and its second derivative satisfying $\mathbb{E}[\sup_{\theta\in\Theta_0} ||\partial g(X,\theta)/\partial\theta||] < \infty$. Furthermore, the matrix $\Delta(\theta) \equiv \mathbb{E}[g(X,\theta)g'(X,\theta)]$ is nonsingular in Θ_0.*

Assumption 3.3 *The function $F_\theta(u) = \mathbb{P}(q(X,\theta) \le u)$ satisfies $\sup_{u\in\Pi} |F_{\theta_1}(u) - F_{\theta_2}(u)| \le C||\theta_1 - \theta_2||$, where C is a bounded positive number, not depending on θ_1 and θ_2.*

Assumptions 3.1-3.3 are weaker than related conditions in the literature. For instance, Assumption 3.1 only requires $\sqrt{n}\left(\hat{\theta}_n - \theta_0\right) = O_p(1)$, but does not require $\sqrt{n}\left(\hat{\theta}_n - \theta_0\right)$ to admit an asymptotically linear representation. Assumption 3.2 is a condition concerning the degree of smoothness of the propensity score $q(x,\theta)$, and is satisfied for standard parametric models such as the Probit and the Logit specifications. It also only requires finite first moment of $g(X,\theta)$, instead of more than four moments as in Shaikh et al. (2009). Assumption 3.3 simply imposes a Lipschitz type continuity condition on the CDF of the parametric propensity score.