They are due to the coupling between inelasticity and gradients that exists in the stationary state, in such a way that strong inelasticity implies large gradients of the hydrodynamic fields. This coupling is a peculiarity of the steady states of inelastic fluids, following from the balance between the energy dissipated because of inelastic cooling and the hydrodynamic energy fluxes.

A previous analysis, carried out in [19], focussed on the macroscopic description of the granular gas in terms of the density and granular temperature fields, the velocity field being zero. At this level of description, the role of the movable piston on top of the gas is to partially determine the boundary conditions needed to solve the hydrodynamic equations for the steady state under consideration. Here, the interest will be on the fluctuations of the movable piston, namely on its position fluctuations. Some results for the velocity fluctuations have been reported elsewhere [14]. There, it was shown that the steady state velocity fluctuations of the piston are gaussian with zero mean for $\alpha_P \geq 0.6$ and $\alpha \geq 0.8$. Nevertheless, no simple relationship between the second moments of the velocity distributions of the piston and the gas next to it was found. It is worth to remark that there is no reason to expect such a relation to exist at a macroscopic level of description, i.e. involving only the hydrodynamic fields and the parameters of the system. Actually, the simulation results reported in [14] indicate that the details of the velocity distribution of the gas, beyond its first few moments, are relevant to determine the second moment of the velocity distribution of the piston.

It is clear that the position fluctuations of the movable piston are related with the volume fluctuations of the inelastic gas. Actually, this relationship can be made direct and exact by properly choosing the nature of the vibrating wall located at the bottom of the system. The mission of the latter is to energize the system, keeping the particles fluidized. The expectation is that the behavior in the bulk of the system is independent of the details of the way in which this wall is being vibrated. Consequently, the simplest possible choice has been used in all the results to be reported in the following. The bottom wall is vibrated with a sawtooth velocity profile, having a velocity v_W . This means that all the particles colliding with the wall find it moving upwards with that velocity [20, 21]. Besides, the amplitude of the wall motion is considered much smaller than the mean free path of the particles in its vicinity, so that the position of the wall can be taken as fixed at z = 0. Therefore, the dynamics of the vibrating wall at the bottom does not induce directly any change in the volume (or area) occupied by the granular gas. Also, and again for the sake of simplicity,