Here we quoted the string breaking distance $R_{\rm br}$ which was determined by the lattice simulations [16], Eq. (6). The condition (12) is satisfied at

$$eB_u \simeq 6m_\pi^2\,, \qquad eB_d = 12m_\pi^2\,, \tag{13}$$

for u and d quarks, respectively.

Notice that the magnitude of the magnetic field expected to be created at noncentral heavy-ion collisions at ALICE/LHC (1) is even greater than the thresholds (13), so that effects of the transverse quark localization may in principle be accessible in the LHC experiment. The localization effects should be much less visible at the RHIC experiment due to the much weaker magnetic fields (1).

What is the effect of the strong magnetic field on the process of the QCD string breaking? The breaking occurs due to the light $q\bar q$ pair creation. In the absence of the magnetic field the created light quark (antiquark) gets attracted to the heavy antiquark (quark) and the long string disappears completely, Fig. 1. However, in a sufficiently strong magnetic field B the distance between the light quark and antiquark in the $\mathbf{B}$-transverse plane is limited by the B–dependent magnetic length l_q, Eq. (11). Therefore if the string is located in the $\mathbf{B}$-transverse plane, then the created $q\bar q$ pair cannot destroy the whole string, because the light quark and antiquark are bounded together by the magnetic field at the mutual separation l_q, Fig. 2.

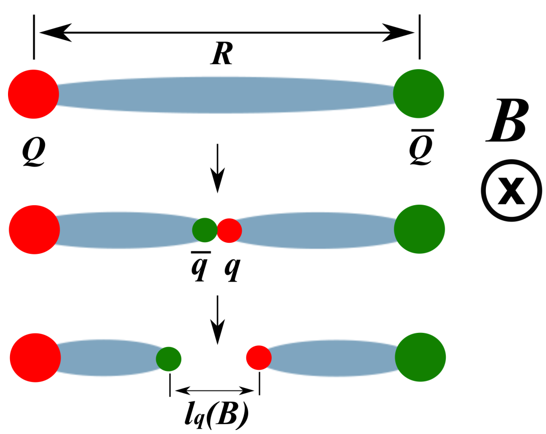

FIG. 2. The (partial) breaking of the QCD string in the background magnetic field. The distance between created quark q and antiquark $\bar q$ is restricted by the magnetic length (11). The axis of the magnetic field is perpendicular to the plane.

In principle, the distance between the light quark and the light antiquark can be increased in the transverse directions by jumping to higher Landau levels. This is, however, an energetically costly process because of the presence of the magnetic gap (9). Therefore the creation of the $q\bar q$ pair from the vacuum may only remove a piece of a sufficiently long QCD string, Fig. (2). Moreover, due to the $(1+1)$ dimensional character of the motion of the light quarks in the external magnetic field, the $q\bar q$ pair creation leads to appearance of two nonlocal (elongated) heavy-light mesons, Fig. 2. On the contrary, in the absence of the background magnetic field the string breaking leads to formation of the tightly-bound heavy-light mesons, Fig. 1.

An increase in the magnetic field causes a decrease in the length (11) of the string piece that can be "removed" by the pair creation. Thus, as the magnetic field increases the system gets less gain in energy due to the string breaking. Intuitively it is clear that at certain strength of the magnetic field the pair creation become energetically unfavorable.

Let us estimate the critical magnetic field which makes the string breaking impossible. Consider a static distantly separated quark-antiquark pair $Q\bar Q$ in the $\mathbf{B}$-transverse plane (i.e., with $\mathbf{R} \equiv \mathbf{R}_\perp$ and $\mathbf{R}_\parallel = 0$). The energy of the unbroken state is

$$E_{\rm string} = 2m_Q + \sigma(B) R\,, \tag{14}$$

where the string tension is, generally, a function of the strength of the magnetic field B.

The emerging $q\bar q$ pair removes a segment of the string of the length $l_q(B)$, and the energy of the broken state is

$$E_{\rm broken} = 2m_Q + 2m_q(B) + \sigma(B)[R - l_q(B)]\,, \tag{15}$$

where m_q is the mass of the light quark.

The condition for the string breaking not to occur is given by the inequality $E_{\rm broken} \geqslant E_{\rm string}$. Using Eqs. (14) and (15), one gets

$$2m_q(B) \geqslant \sigma(B) l_q(B) \qquad \text{[no string breaking]}\,. \tag{16}$$

The equality in (16) defines the lowest possible magnetic field ("critical no-breaking field") for which the string breaking does not occur.

Let us consider the condition (16) at zero temperature. The dynamical masses of $q = u, d$ quarks in the QCD vacuum without a background magnetic field are

$$m_q^{\rm dyn} \simeq 300\,{\rm MeV}\,. \tag{17}$$

The external magnetic field of the practical scale (1) makes negligible contribution to the quarks' masses (a significant contribution to the masses of the quarks is expected at the strength $eB \gtrsim (10\,{\rm TeV})^2$, Ref. [12]). Assuming that the critical no-breaking magnetic field is lower than the crossover strength (2), and taking into account the fact that at the crossover all observables are smooth, we ignore an essentially nonperturbative B-dependence of the string tension. We set the string tension to its phenomenological value at $B = 0$,

$$\sigma(B) \approx \sigma(B=0) = (440\,{\rm MeV})^2\,. \tag{18}$$

Using Eqs. (16), (17) and (18) one finds that the QCD string fails to break via creation of $u\bar u$ and $d\bar d$ pairs if the magnetic fields reach the following values, respectively

$$eB^{(u)}_{\rm cr} \simeq 8m_\pi^2\,, \qquad eB^{(d)}_{\rm cr} = 16m_\pi^2\,. \tag{19}$$