of view, the benefits of using our procedure can outweigh the costs in many relevant situations.

3.2.2 Power against local alternatives

Next, we study the performance of our projection-based tests under a sequence of local alternative hypotheses converging to the null at the parametric rate $n^{-1/2}$ given by

$$H_{1n} : E\left[D - q(X, \theta_0) | q(X, \theta_0)\right] = \frac{r\left(q(X, \theta_0)\right)}{\sqrt{n}} \quad a.s. \tag{3.4}$$

for some $\theta_0 \in \Theta$, where $r\left(q(X, \theta_0)\right)$ represents directions of departure from H_0, and $n^{-1/2}$ indicates the rate of convergence of H_{1n} to H_0. The function $r : [0, 1] \to \mathbb{R}$ is required to satisfy the following assumption.

Assumption 3.4 *The function $r(q)$ is continuous in q and satisfies $\mathbb{E}|r(q(X, \theta_0))| < \infty$.*

Theorem 3 *Suppose Assumptions 3.1 -3.4 hold. Then, under the local alternatives H_{1n} given by (3.4), we have*

$$\hat{R}_n^p(u) \Rightarrow R_\infty^p + \Delta_r,$$

where R_∞^p is the same Gaussian process as defined in Theorem 1, and Δ_r is a deterministic shift function given by

$$\Delta_r(u) \equiv \mathbb{E}\left[r\left(q\left(X, \theta_0\right)\right)\mathcal{P}1\left\{q(X, \theta_0) \leq u\right\}\right].$$

Note that, in general, the deterministic shift function $\Delta_r\left(u\right) \neq 0$ for at least some $u \in \Pi$, implying that tests based on continuous even functionals of $\hat{R}_n^p(\cdot)$ will have non-trivial power against local alternatives of the form in (3.4). A situation in which our tests will have trivial local power against such alternatives is when directions $r(q\left(x, \theta_0\right))$ are a linear combination of score function $g(x, \theta_0)$, i.e. $r(q(x, \theta_0)) = \beta g(x, \theta_0)$ for some β. In such a case, the limiting distribution of $\hat{R}_n^p(u)$ under H_0 and H_{1n} is the same so that H_{1n} cannot be detected. On the other hand, note that tests based on the local smoothing approach such as Shaikh et al. (2009) are not able to detect alternatives of the form (3.4).

3.3 Computation of critical values

From the above theorems, we see that the asymptotic distribution of continuous functionals $\Gamma\left(\hat{R}_n^p\right)$ depend on the underlying data generating process and of course on $\Gamma\left(\cdot\right)$ itself. Furthermore, the complicated covariance structure of $K^p(\cdot, \cdot)$ given in (3.1) does not allow for a