locsitons with $\psi \approx \pm\pi/2$ (i. e., nearly transverse locsitons) may be exited at $\delta/\delta_{\rm LL} > 1$, and can be thus viewed as the "easiest to excite" on the Lorentz side of the band. On the opposite, "anti-Lorentz" side of the band, where $\delta/\delta_{\rm LL} \lesssim -1$, nearly longitudinal locsitons with $\psi \approx 0$ or π lie closer to the band edge and thus are easier to excite. The exact positions of the edges of the locsiton band cannot be found within the NRA, because minima and maxima of $D_2^{\rm NRA}(\mathbf{q})$ are reached at such $\mathbf{q}$ where the NRA may only be used for qualitative estimates. The case of larger q is addressed when we go beyond the NRA in the next subsection.

B. Locsitons in the first Brillouin zone

When going beyond the NRA, the orientation of $\mathbf{E}_{\rm in}$ within the lattice plane becomes an important factor, except for small q. Staying within the NNA, i. e., only taking into account the six nearest neighbors in Eq. (4) (but *individually*, instead of them being washed out over the ring, as in the NRA), we are still able to approach the problem analytically. The resulting equation is

$$\begin{aligned} \mathbf{E}_{\rm L}(\mathbf{r}) \; &= \; \mathbf{E}_{\rm in} - \frac{Q}{4} \sum_{\mathbf{u}_{\rm K}} \{ 3\mathbf{u}_{\rm K} [\mathbf{E}_{\rm L}(\mathbf{r} + l_a \mathbf{u}_{\rm K}) \cdot \mathbf{u}_{\rm K}] \\ & \qquad - \mathbf{E}_{\rm L}(\mathbf{r} + l_a \mathbf{u}_{\rm K}) \} \,, \end{aligned} \tag{19}$$

where $\mathbf{u}_{\rm K}$ denotes any of the six unit vectors pointing in the directions from the atom (located at $\mathbf{r}$) to one of its nearest neighbors.

The uniform, Lorentz, solution of Eq. (19) is still given by Eq. (8) and (9), which supports the above-mentioned convergence of the NRA and NNA results at $q \to 0$. Spatially varying *locsiton* solutions are found as in the previous subsection by using the ansatz (12) in Eq. (19). The corresponding dispersion relation $\delta(\mathbf{q})$ for locsitons in a 2D triangular lattice can be now written as

$$D_2^{\rm NNA}(\mathbf{q}) \equiv \sum_{n=0}^{2} \left(\cos 2\theta_n + \frac{1}{3} \right) \cos[q \cos(\theta_n - \psi)] = \frac{\delta + i}{\delta_{\rm LL}}. \tag{20}$$

where $\theta_n = \theta_0 + n\pi/3$, and $\mathbf{q}$ is represented by its polar coordinates q and ψ. The orientation of the lattice with respect to the incident field $\mathbf{E}_{\rm in}$ is described by θ_0, which is the angle that one of the vectors $\mathbf{u}_{\rm K}$ makes with $\mathbf{E}_{\rm in}$ (the fact that θ_0 is not unique does not affect the result). The ultimate proof that the results of the NRA and the more precise NNA converge