Next we prove σ_1 is a G-extendible set for ρ_1. Given $\psi_1 : \sigma_1 \to G$ we let $\psi_2 : \sigma_2 \to G$ be the function given by $\psi_2 = \psi_1 \circ \theta^*$. There is also a homomorphism $\Psi_2 : \rho_2 \to G$ such that $\Psi_2|_{\sigma_2} = \psi_2$ since σ_2 is a G-extendible set for ρ_2. There is a homomorphism $\Psi_1 : \rho_1 \to G$ such that for all $a, b \in X_1$ such that $a\rho_1 b$ we have $\Psi_1(a,b) = \Psi_2\left((\theta^*)^{-1}(a,b)\right)$ if $a \neq b$ and$\Psi_1(a,b) = 1$ if $a = b$. Given $a, b \in X_1$ such that $(a,b) \in \sigma_1$ there exist $x, y \in X_1$ such that $a = \theta(x)$, $b = \theta(y)$, and $(x,y) \in \sigma_2$. We have $\theta^*(x,y) = (a,b)$ and $\Psi_2(x,y) = \Psi_1(a,b)$ by construction, $\Psi_2(x,y) = \psi_2(x,y)$ since $(x,y) \in \sigma_2$, and $\psi_2(x,y) = \psi_1(a,b)$ since $\psi_2 = \psi_1 \circ \theta^*$. This shows $\Psi_1|_{\sigma_1} = \psi_1$ and σ_1 is a G-extendible set for ρ_1. ■

Definition 4.4 *Let ρ be a reflexive relation on a set X.*

1. *(X, ρ) is* stable *if ρ is balanced and if the relations $a\rho b$, $a\rho c$, $b\rho c$, $b\rho d$, and $c\rho d$ imply $a\rho d$ for all distinct $a, b, c, d \in X$.*

2. *An element $x \in X$ is a* clasp *if there exist $w, y \in X \setminus \{x\}$ such that $w\rho x$, $x\rho y$, and $(w,y) \notin \rho$.*

3. *$x \in X$ is a* locked clasp *if there exist $u, v, w, y \in X \setminus \{x\}$ such that $(w,y) \notin \rho$ and $(u,x,y), (u,x,v), (w,x,v) \in \mathrm{Trans}(X)$.*

4. *An* unlocked clasp *is a clasp which is not locked.*

It is easy to see a preorder is stable. The balanced relation determined by (d) in Figure 1 is not stable. Neither a balanced relation which is not stable nor a stable relation which contains a locked clasp can be the compression of a preorder by [8, Theorem 2.4 and Lemma 3.4].