

FIG. 11: The evolution of the optical integral in the NS (top) and the SCS (bottom) in the original MFLI model. Parameters are the same as above. Note that only  $\sim 75 - 80\%$  of the spectral weight is recovered up to  $1\text{eV}$ .

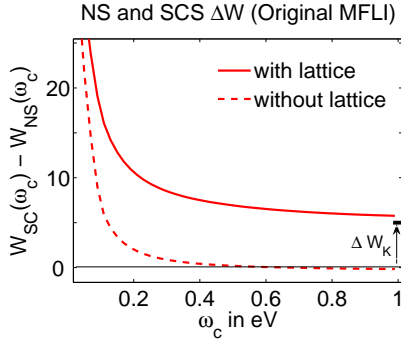


FIG. 12: Evolution of the difference of the optical integrals in the SCS and the NS with the upper cut-off  $\omega_c$ . Parameters are the same as before. Observe that the optical sum in the SCS is larger than in the NS and that  $\Delta W$  has not yet reached  $\Delta W_K$  up to the bandwidth. The dashed line is the FGT result.

This clearly affects  $n_k$  because it is expressed via the full Green's function and competes with the conventional effect of the gap opening. The distribution function from this model, which we show in Fig.2b brings this point out by showing that in a MFLI model, at  $\epsilon < 0$ ,  $n_k$  in a superconductor is larger than  $n_k$  in the normal state, in clear difference with the BCSI case.

We analyzed the original MFLI model for various parameters and found that the behavior presented in Fig. 12, where  $\Delta W(\omega_c) > 0$  for all frequencies, is typical but

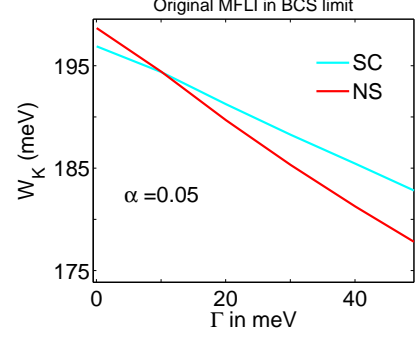


FIG. 13: Behavior of  $W_K$  with  $\Gamma$  for the original MFLI model at very small  $\alpha = 0.05$ . We set  $\omega_1 = \Delta = 32\text{meV}$ . Observe the inconsistency with  $W_K$  in the BCSI model in Fig 4.

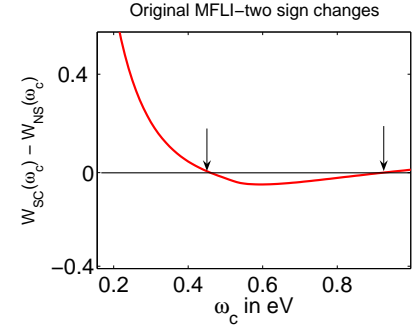


FIG. 14: The special case of  $\alpha = 1.5, \Gamma = 5\text{meV}$ , other parameters the same as in Fig. 10. These parameters are chosen to illustrate that two sign changes (indicated by arrows in the figure) are also possible within the original MFLI model.

not not a generic one. There exists a range of parameters  $\alpha$  and  $\Gamma$  where  $\Delta W_K$  is still positive, but  $\Delta W(\omega_c)$  changes the sign twice and is negative at intermediate frequencies. We show an example of such behavior in Fig.14. Still, for most of the parameters, the behavior of  $\Delta W(\omega_c)$  is the same as in Fig. 12.

On more careful looking we found the problem with the original MFLI model. We recall that in this model the self-energy in the SCS state was obtained by just cutting the NS self energy at  $\omega_1$  (see Eq.18). We argue that this phenomenological formalism is not fully consistent, at least for small  $\alpha$ . Indeed, for  $\alpha = 0$ , the MFLI model reduces to BCSI model for which the behavior of the self-energy is given by Eq. (12). This self-energy evolves with  $\omega$  and  $\Sigma''$  has a square-root singularity at  $\omega = \Delta + \omega_o$  (with  $\omega_o = 0$ ). Meanwhile  $\Sigma''$  in the original MFLI model in Eq. (18) simply jumps to zero at  $\omega = \omega_1 = \Delta$ , and this happens for all values of  $\alpha$  including  $\alpha = 0$  where the MFLI and BCSI model should merge. This inconsistency is reflected in Fig 13, where we plot the near-BCS limit of MFLI model by taking a very small  $\alpha = 0.05$ . We see that the optical integral  $W_K$  in the SCS still remains larger than in the NS over a wide range of  $\Gamma$ , in clear difference with the exactly known behavior in the BCSI