

defined by:

$$\frac{d\sigma^{exp}}{d\Omega}(\theta_{cm}, E_x) = \sum_{L=0}^7 a_L(E_x) \times \frac{d\sigma_L^{cal}}{d\Omega}(\theta_{cm}, E_x) \quad (1)$$

where  $\frac{d\sigma_L^{cal}}{d\Omega}(\theta_{cm}, E_x)$  is the calculated distorted-wave Born approximation (DWBA) cross section corresponding to 100% energy-weighted sum rule (EWSR) for the L-th multipole. The fractions of the EWSR,  $a_L(E_x)$ , for various multipole components were determined by minimizing  $\chi^2$ . This procedure is justified since the angular distributions are well characterized by the transferred angular momentum  $L$ , according to the DWBA calculations for  $\alpha$  scattering. It was confirmed that the MDA fits were not affected by including  $L > 7$ .

The DWBA calculations were performed following the method of Satchler and Khoa [45], using the density-dependent single-folding model for the real part, obtained with a Gaussian  $\alpha$ -nucleon potential, and a phenomenological Woods-Saxon potential for the imaginary term. Therefore, the  $\alpha$ -nucleus interaction is given by:

$$U(r) = V_F(r) + iW/(1 + \exp((r - R_I)/a_I)) \quad (2)$$

where  $V_F(R)$  is the real single-folding potential obtained by folding the ground-state density with the density-dependent  $\alpha$ -nucleon interaction:

$$v_{DDG}(r, r', \rho) = -v(1 - \beta\rho(r')^{2/3})\exp(-|r - r'|^2/t^2) \quad (3)$$

where  $v_{DDG}(r, r', \rho)$  is the density-dependent  $\alpha$ -nucleon interaction,  $|r - r'|$  is the distance between the center of mass of the  $\alpha$  particle and a target nucleon,  $\rho(r')$  is the ground-state density of the target nucleus at the position  $r'$  of the target nucleon,  $\beta=1.9 \text{ fm}^2$ , and  $t=1.88 \text{ fm}$ .  $W$  is the depth of the Woods-Saxon type imaginary part of the potential, with the reduced radius  $R_I$  and diffuseness  $a_I$ .

These calculations were performed with the computer code PTOLEMY [46, 47], with the input values modified [48] to take into account the correct relativistic kinematics. The shape of the real part of the potential and the form factor for PTOLEMY were obtained using the codes SDOLFIN and DOLFIN [49]. We use the transition densities and sum rules for various multipolarities described in Refs. [13, 50, 51]. The radial moments were obtained by numerical integration of the Fermi mass distribution with the parameter values from Ref. [52] (listed in Table I).