E. Scale uncertainty

Because in this paper we only compute dimensionless ratios, the uncertainty in the determination of the lattice spacing only enters implicitly through the uncertainty in the light-quark masses and in the renormalization factors. We estimate the systematic error due to the truncation of lattice perturbation theory in the previous subsection and due to the light-quark mass determinations in the following subsection.

F. Light- and strange-quark mass uncertainties

We obtain the physical decay constants and mixing matrix elements by setting the light-quark masses to their physical values in the linear plus $SU(2)$ HMχPT chiral extrapolation formulae, once the low-energy constants have been determined from fits to numerical lattice data. We use the bare-quark mass value determined from fits to the light pseudoscalar meson masses [34]:

$$am_{ud} + am_{\rm res} \;=\; 0.001300(85), \tag{82}$$

where m_{ud} is the average of the up and down quark masses. The quoted error includes both statistics and the systematic uncertainties from the chiral extrapolation, finite-volume effects, discretization effects, and the unphysical strange sea quark mass. In order to estimate the systematic uncertainty in the ratios f_{B_s}/f_{B_d} and ξ, we vary the bare light-quark masses within their stated uncertainties. We then take the maximal difference from the central value to be the systematic error. From this method, we find that the systematic error in f_{B_s}/f_{B_d} due to the uncertainty in the light-quark mass determination is 0.2% for both smearings and the systematic error in ξ is 0.2% regardless of the smearing used for the heavy quark.

Because the strange-quark mass does not explicitly appear in the $SU(2)$ HMχPT extrapolation formulae, we cannot estimate the errors in f_{B_s}/f_{B_d} and ξ due to the simulated valence and sea strange-quark masses with the simple method used above for the light quarks. We must instead address the errors due to the uncertainty in the strange valence-quark mass (which is set to the physical value of m_s) separately from those due to the choice of strange sea-quark mass (which is not tuned to the physical m_s).

We calculate the decay constants and mixing matrix elements with the strange valence-quark mass tuned to the physical value obtained from fits of the light pseudoscalar meson