More on the renormalization of the horizon function of the Gribov-Zwanziger action and the Kugo-Ojima Green function(s)

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In this paper we provide strong evidence that there is no ambiguity in the choice of the horizon function underlying the Gribov-Zwanziger action. We show that there is only one correct possibility which is determined by the requirement of multiplicative renormalizability. As a consequence, this means that relations derived from other horizon functions cannot be given a consistent interpretation in terms of a local and renormalizable quantum field theory. In addition, we also discuss that the Kugo-Ojima functions $u(p^2)$ and $w(p^2)$ can only be defined after renormalization of the underlying Green function(s).

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I. INTRODUCTION

In 1977, Gribov [1] showed, in a saddle point approximation, that the restriction of the Euclidean functional integral to the Gribov region Ω has far reaching implications for the infrared behavior of the ghost and the gluon propagator. We recall that the region Ω is defined as the set of field configurations fulfilling the Landau gauge condition and for which the Faddeev-Popov operator,

$$\mathcal{M}^{ab} \;=\; -\partial_\mu \left(\partial_\mu \delta^{ab} + g f^{acb} A^c_\mu\right) , \tag{1}$$

is strictly positive. Therefore,

$$\Omega \;\equiv\; \{A^a_\mu,\, \partial_\mu A^a_\mu = 0,\, \mathcal{M}^{ab} > 0\} . \tag{2}$$

Later on, in a series of works, Zwanziger [2–5] elaborated on Gribov's approximation, being able to extend the previous results order by order at the quantum level. This resulted in an improvement of the Faddeev-Popov action which is now called the Gribov-Zwanziger action. In particular, the Gribov-Zwanziger action leads to a ghost propagator which is enhanced in the infrared region, a feature which has been confirmed by explicit two loop calculations in [6, 7], which constitute a non-trivial check of the predictions of the Gribov-Zwanziger formalism.

Recently, it has been claimed [8–10] that the Gribov-Zwanziger action is plagued by a certain ambiguity. Depending on the choice of the so called horizon function [2], different results for the ghost propagator might be found, namely: an enhanced or a non-enhanced ghost, according to [8–10]. Also, the recent lattice results seem to point towards a non-enhanced ghost[42] [11–15]. A natural conclusion would seem to be that one should take the horizon function which leads to the non-enhanced ghost. However, this does not agree with the original results by Gribov and Zwanziger and therefore asks for an explanation.

In this paper, we would like to clear the situation. We shall show that there is no ambiguity in the choice of the horizon function. The correct form of the horizon function is the one originally constructed by Zwanziger [4], and is clearly dictated by the renormalization properties of the Gribov-Zwanziger action. We stress that renormalization is of paramount importance for defining meaningful Green functions. A dynamical improvement of this Gribov-Zwanziger action, consistent with the renormalization, consequently allows to obtain the non-enhanced ghost, as discussed in previous work [21, 22], giving results compatible with other analytical approaches [18, 23–26], based on the Schwinger-Dyson formalism, see also [27].

As a corollary of the present analysis, we shall elaborate on the meaning of the Kugo-Ojima functions $u(p^2)$ and $w(p^2)$ defined as follows [28, 29]

$$\int \mathrm{d}^d x \mathrm{d}^d y e^{ip(x-y)} \left\langle (g f_{abc} A^b_\mu c^c)(x)(g f_{ak\ell} A^k_\nu c^\ell)(y)\right\rangle_{1PI} = g_{\mu\nu} u(p^2) + \frac{p_\mu p_\nu}{p^2} w(p^2) \tag{3}$$

and their meaning at the level of renormalization, shedding more light on certain claims in [8–10]. We shall also discuss about several results obtained in the literature [8–10], where the other choice of the horizon function was investigated. As we shall show that this particular

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