I. INTRODUCTION

The time-independent form of Schrödinger's equation in the presence of a potential V can be written as[1]

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi, \tag{1}$$

where ψ is the wave function, m is the mass of the particle, and E is the eigenenergy. Equation (1) can be solved exactly only for a few potentials. A particle in a box and a particle in a delta function potential are two well-known and instructive examples.[1] The former can be used to describe quantum dots and quantum wells at low temperatures,[2,3] and the latter can be used as a model for atoms and molecules.[4]

The solution for a particle in a box with a delta function potential has been investigated using a perturbative expansion in the strength of the delta function potential λ.[5] Exact solutions have been obtained for the weak ($\lambda \to 0$) and the strong ($1/\lambda \to 0$) coupling limits.[6]

In this paper we discuss the solution for a particle in a box with a delta function potential using the factorization method and show that the presence of the delta function simplifies the factorization procedure. In this way we find the full spectrum of the Hamiltonian in the first step of the factorization method. We also show that this result applies in the weak coupling limit $\lambda \to 0$. Note that if we put $\lambda = 0$ from the beginning, we need to continue the factorization procedure to find each eigenvalue in each step.

II. PARTICLE IN A BOX WITH A DELTA FUNCTION POTENTIAL

Consider a particle in a one-dimensional box of size a with the delta function potential $V(x) = \lambda\,\delta(x - x_0) = \lambda\,\delta(x - pa)$, where $0 < p < 1$. In this case Eq. (1) takes the form

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_n(x)}{dx^2} + \lambda\delta(x - pa)\psi_n(x) = E_n\psi_n(x), \tag{2}$$

where $\psi_n(x)$ and E_n are the corresponding eigenfunctions and eigenvalues, respectively. Because of the boundary conditions, $\psi_n(x) = 0$ for $x \le 0$ or $x \ge a$, the form of the eigenfunctions inside the box is

$$\psi_n(x) = \begin{cases} A\sin(k_n x) & (0 \le x \le pa) \\ B\sin[k_n(x - a)] & (pa \le x \le a), \end{cases} \tag{3}$$