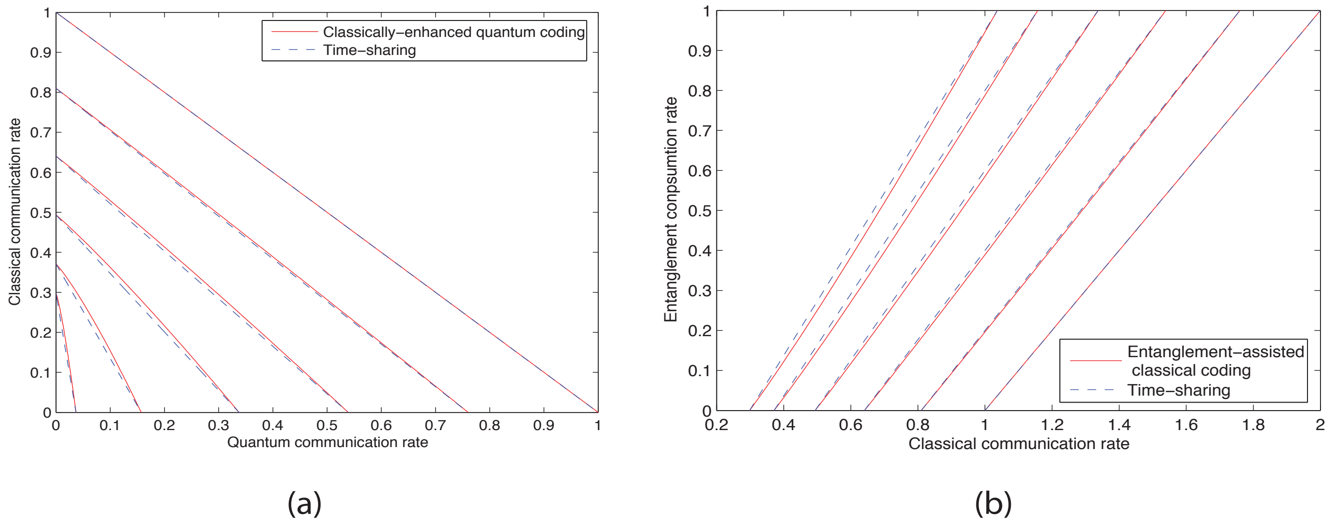

FIG. 4: (Color online) Plot of the CQ trade-off curve (a) and the CE trade-off curve (b) of an Unruh channel for $z = 0$, 0.2, 0.4, 0.6, 0.8, and 0.95. The trade-off curves for $z = 0$ correspond to those for the noiseless qubit channel and are the rightmost trade-off curves in each plot. In both plots, proceeding left from the $z = 0$ curve, we obtain trade-off curves for $z = 0.2$, 0.4, 0.6, 0.8, and 0.95 and notice that they all beat a time-sharing strategy by a larger relative proportion as z increases.

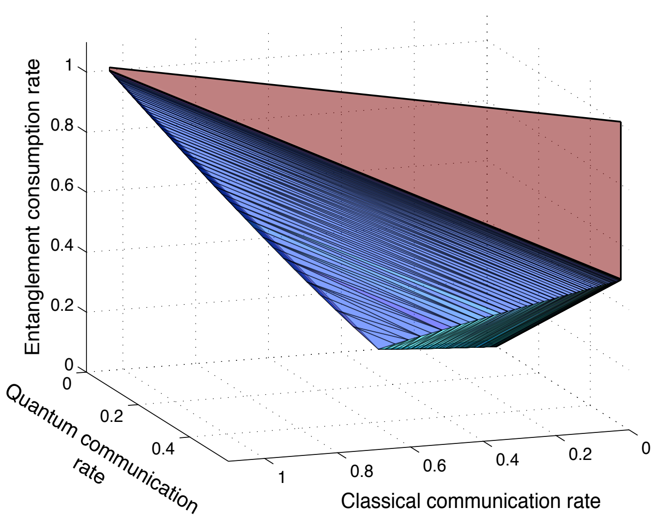

FIG. 5: (Color online) The figure plots the CQE capacity region for an Unruh channel with acceleration parameter $z = 0.95$. It features three distinct surfaces. The first is the flat vertical plane that arises from the bound $R + 2Q \leq \mathcal{C}_{\text{EAC}}$, where $\mathcal{C}_{\text{EAC}}$ is the entanglement-assisted classical capacity of an Unruh channel. The plane extends infinitely upward because we can always achieve these rate triples simply by wasting entanglement. The second surface is that below and to the left of the plane, formed by combining the CE trade-off curve with the inverse of super-dense coding, as described in Section III C. The final surface is that below and to the right of the plane, formed by combining the CQ trade-off curve with the inverse of entanglement distribution, as described in Section III C.

generalized dephasing channels and cloning channels, and we have computed exact formulas that specify their CQE capacity regions. Furthermore, we have obtained expressions for the CQE capacity region of an Unruh channel because of its close connection with the cloning channels. The classically-enhanced father protocol beats a simple time-sharing strategy for all of these channels, stressing the need for non-trivial coding techniques when trading multiple resources.

It is interesting to ponder the reason why a particular channel obtains an improvement over time-sharing. The relative improvements are most significant for the CQ trade-off, in which case the cloning and Unruh channels exhibit much more substantial gains than the dephasing channels. In retrospect, it is perhaps surprising that the dephasing channels exhibit any improvement at all. Since these channels can transmit classical data noiselessly, it would be natural to expect that any optimal strategy for sending classical bits would directly exploit this capability. For CQ trade-off coding, that would entail allocating some fraction of channel uses to noiseless classical data transmission and the rest to quantum, which is exactly the time-sharing strategy. The existence of a nontrivial CQ trade-off indicates that this strategy is actually not optimal. In contrast, the cloning and Unruh channels are incapable of sending classical data noiselessly when $N > 1$ or $z > 0$ so any communication strategy requires error correction with the attendant opportunity for nontrivial trade-off coding.

Some future directions for this work are in order. It would be desirable to discover other channels for which the full CQE capacity region single-letterizes, but it is