$$
\begin{aligned}
&= -\sqrt{n}(\hat\theta_n - \theta_0)'\mathbb{E}\left[g(X,\theta_0)\frac{\partial g(X,\theta_0)}{\partial\theta'}\right]\sqrt{n}(\hat\theta_n-\theta_0) + o_p(1) \\
&= O_p(1),
\end{aligned}
$$

following similar arguments in proving the negligibility of C_{2n}. Hence $C_{3n} = O_p(n^{-1/2}) = o_p(1)$. This ends the proof of Lemma A.5. $\square$

The next two lemmas establish the (uniform) convergence of $G_n(u,\hat\theta_n)$ and $\Delta_n^{-1}(\hat\theta_n)$ to $G(u,\theta_0)$ and $\Delta^{-1}(\theta_0)$, respectively.

Lemma A.6 *Under Assumptions 3.1-3.3, we have*

$$
\sup_{u\in\Pi}\left|G_n(u,\hat\theta_n) - G(u,\theta_0)\right| = o_p(1).
$$

Proof of Lemma A.6: The proof follows directly from the ULLN of Newey and McFadden (1994). $\square$

Lemma A.7 *Under Assumptions 3.1-3.2, we have*

$$
\Delta_n^{-1}(\hat\theta_n) = \Delta^{-1}(\theta_0) + o_p(1).
$$

Proof of Lemma A.7: The proof follows from the ULLN of Newey and McFadden (1994) and the continuous mapping theorem. $\square$

Now, we are ready to proceed with the proofs of our main theorems.

Proof of Theorem 1: By a straightforward decomposition, we have

$$
\begin{aligned}
\hat R_n^p(u) &= \frac{1}{\sqrt{n}}\sum_{i=1}^n \varepsilon_i(\hat\theta_n)\left(1\left\{q(X_i,\hat\theta_n)\le u\right\} - g'(X_i,\hat\theta_n)\Delta_n^{-1}(\hat\theta_n)G_n(u,\hat\theta_n)\right) \\
&= \hat R_n(u) - G_n'(u,\hat\theta_n)\Delta_n^{-1}(\hat\theta_n)\frac{1}{\sqrt{n}}\sum_{i=1}^n \varepsilon_i(\hat\theta_n)g(X_i,\hat\theta_n) \\
&:= \hat R_n(u) - G_n'(u,\hat\theta_n)\Delta_n^{-1}(\hat\theta_n)\hat S_n.
\end{aligned}
$$

By Lemmas A.4-A.7, we have that

$$
\begin{aligned}
\hat R_n^p(u) =& \frac{1}{\sqrt{n}}\sum_{i=1}^n \varepsilon_i(\theta_0)1\left\{q(X_i,\theta_0)\le u\right\} - G'(u,\theta_0)\sqrt{n}(\hat\theta_n-\theta_0) \\
&- G'(u,\theta_0)\Delta^{-1}(\theta_0)\left[\frac{1}{\sqrt{n}}\sum_{i=1}^n \varepsilon_i(\theta_0)g(X_i,\theta_0) - \Delta(\theta_0)\sqrt{n}(\hat\theta_n-\theta_0)\right] + o_p(1) \\
=& \frac{1}{\sqrt{n}}\sum_{i=1}^n \varepsilon_i(\theta_0)\left(1\left\{q(X_i,\theta_0)\le u\right\} - G'(u,\theta_0)\Delta^{-1}(\theta_0)g(X_i,\theta_0)\right) + o_p(1)
\end{aligned}
$$