Ψ_{block} may be written as

$$C(\Psi_{\text{block}}) = \text{dens}(\Theta \cup Y) + O(d^3/\text{AI}) + O(d^2).$$

Timing Comparison

We have seen that $C(\Psi_{\text{block}})$ is at least quadratic in d, and technically cubic but with a small leading coefficient. Depending on the distributional structure of Θ, the density evaluations comprising $C(\Psi_{\text{scalar}})$ may be unwieldy. The relative magnitude of these competing terms is difficult to intuitively gauge, so to gain practical insight, we perform a timing study of the Ψ_{scalar} (All Scalar) and Ψ_{block} (All Blocked) algorithms. Three models involving no likelihood components are considered, with prior structures on Θ given as:

- $\theta_i \sim \text{N}(\mu, \sigma)$ for each $\theta_i \in \Theta$
- $\theta_i \sim \text{Gamma}(\alpha, \beta)$ for each $\theta_i \in \Theta$
- $\Theta \sim \text{N}_d(\mu, \Sigma)$

Figure 2 presents timing results measured in seconds per 10,000 iterations, plotted against dimension d, without consideration of algorithmic efficiency (section 2.2). There are a number of interesting features, which we briefly summarize. $C(\Psi_{\text{scalar}})$ is $O(d)$ when each θ_i independently follows a univariate distribution, and therefore $\sum_{i=1}^{d} \text{dens}(\theta_i) = \text{dens}(\Theta) \leq d \cdot K$, where $K = \max_{\theta \in \Theta} \text{dens}(\theta)$. For all practical purposes, it appears that $C(\Psi_{\text{block}})$ is $O(d^2)$, as the cubic coefficient $1/\text{AI} = 1/200$ is relatively small. $C(\Psi_{\text{block}})$ is largely resilient to changes in the underlying distribution of Θ; univariate gamma densities are more costly than normal densities, as we would expect, and as for $C(\Psi_{\text{scalar}})$; and the multivariate normal structure even slightly more so. Perhaps most striking, $C(\Psi_{\text{scalar}})$ is $O(d^3)$ when the underlying distribution of Θ is multivariate, since each multivariate normal density evaluation is $O(d^2)$, which occurs d times for each iteration of Ψ_{scalar}. Although both are cubic in d, $C(\Psi_{\text{scalar}})$ dwarfs $C(\Psi_{\text{block}})$ due to the difference in the leading cubic coefficients.