5.5.2 Inference and Bias Correction

Asymptotic properties of my estimators other than consistency have not been covered in this paper. The complete asymptotic theory for the estimators can be investigated by adopting arguments from Abadie et al. (2002), Koenker and Xiao (2002), Angrist et al. (2005), and Fan and Park (2010). Abadie et al. (2002) provided asymptotic properties for their weighted quantile regression coefficients for the fixed quantile level q, while Koenker and Xiao (2002) and Angrist et al. (2005) focused on the standard quantile regression *process*. Fan and Park (2010) derived asymptotic properties for the plug-in estimators of Makarov bounds. Since they estimated marginal distribution functions using empirical distributions in the context of randomized experiments, their arguments follow standard empirical process theory. To investigate asymptotic properties of the bounds estimators and the estimated maximizer or minimizer for the bounds, I am currently extending the asymptotic analysis on the quantile regression process presented by Koenker and Xiao (2002) and Angrist et al. (2005) to the quantile curves which are obtained from the weighted quantile regression of Abadie et al. (2002).

Canonical bootstrap procedures may be invalid for inference in this setting. Fan and Park (2010) found that asymptotic distributions of their plug-in estimators for Makarov bounds discontinuously change around the boundary where the true lower and upper Makarov bounds reach zero and one, respectively. Specifically, they estimated the Makarov lower bound $\sup\limits_y \max\left\{F_1(y) - F_0(y-\delta), 0\right\}$ using empirical distribution functions $\widehat{F}_0$ and $\widehat{F}_1$. They found that the asymptotic distribution of their estimator of the Makarov lower bound is discontinuous on the boundary where $\sup_y \left\{F_1(y) - F_0(y-\delta)\right\} = 0$. Since my improved lower bound under MTR is written as the supremum of the sum of $\max\left\{F_1(a_k) - F_0(a_{k-1}), 0\right\}$ over integers k, the asymptotic distribution of my plug-in estimator is likely to suffer discontinuities near multiple boundaries where $F_1(a_k) - F_0(a_{k-1}) = 0$ for each integer k. To avoid the failure of the standard bootstrap, I recommend subsampling or the fewer than n bootstrap procedure following Politis et al. (1999), Andrews (2000), Andrews and Han (2009).

Although the estimator $\widehat{F}_\Delta^{NL}$ is consistent, it may have a nonnegligible bias in small samples.[23] I suggest that one use a bias-adjusted estimator based on subsampling when the sample size is small in practice. Let

$$\widehat{F}^{NL}_{\Delta,n,b,j}(\delta) = \sup_{0\leq y\leq\delta} \sum_{k=\left\lfloor \frac{500-y}{\delta}\right\rfloor}^{\left\lfloor \frac{5500-y}{\delta}\right\rfloor+1} \max\left(\widehat{F}_1^{n,b,j}(y+k\delta) - \widehat{F}_0^{n,b,j}(y+(k-1)\delta), 0\right),$$

where for $d = 0, 1$, $\widehat{F}_d^{n,b,j}$ is an estimator of F_d from the jth subsample $\left\{(Y_{j_1}, D_{j_1}), ..., (Y_{j_b}, D_{j_b})\right\}$ with the

[23] Since $\max(x, 0)$ is a convex function, by Jensen's inequality my plug-in estimator is upward biased. This has been also pointed out in Fan and Park (2009) for their estimator of Makarov bounds.