Quantum dynamics in the bosonic Josephson junction

Maya Chuchem[1], Katrina Smith-Mannschott[2,3], Moritz Hiller[4],
Tsampikos Kottos[2], Amichay Vardi[5,6], and Doron Cohen[1]
[1] *Department of Physics, Ben-Gurion University of the Negev, P.O.B. 653, Beer-Sheva 84105, Israel*
[2] *Department of Physics, Wesleyan University, Middletown, Connecticut 06459, USA*
[3] *MPI for Dynamics and Self-Organization, Bunsenstrasse10, D-37073Göttingen, Germany*
[4] *Physics Institute, Albert-Ludwigs-Universität, Hermann-Herder-Str. 3, D-79104 Freiburg, Germany*
[5] *Department of Chemistry, Ben-Gurion University of the Negev, P.O.B. 653, Beer-Sheva 84105, Israel*
[6] *ITAMP, Harvard-Smithsonian CFA, 60 Garden St., Cambridge, Massachusetts 02138, USA*

We employ a semiclassical picture to study dynamics in a bosonic Josephson junction with various initial conditions. Phase-diffusion of coherent preparations in the Josephson regime is shown to depend on the initial relative phase between the two condensates. For initially incoherent condensates, we find a universal value for the buildup of coherence in the Josephson regime. In addition, we contrast two seemingly similar on-separatrix coherent preparations, finding striking differences in their convergence to classicality as the number of particles increases.

I. INTRODUCTION

Bose-Einstein condensates (BECs) of dilute, weakly-interacting gases offer a unique opportunity for exploring non-equilibrium many-body dynamics, far beyond small perturbations of the ground state. Highly excited states are naturally produced in BEC experiments and their dynamics can be traced with great precision and control. The most interesting possibilities lie in strong correlation effects, which imply a significant role of quantum fluctuations.

The importance of correlations and fluctuations may be enhanced by introducing an optical lattice, that can be controlled by tuning its depth. This tight confinement decreases the kinetic energy contribution with respect to the interactions between atoms. In the tight-binding limit, such systems are described by a Bose-Hubbard Hamiltonian (BHH), characterized by the hopping frequency K between adjacent lattice sites, the on-site interaction strength U, and the total atom number N. The strong correlation regime is attained when the characteristic coupling parameter $u \equiv UN/K$ exceeds unity, as indicated by the quantum phase transition from a superfluid to a Mott-insulator [2, 3].

The simplest BHH is obtained for two weakly coupled condensates (dimer). Its dynamics is readily mapped onto a SU(2) spin problem and is closely related to the physics of superconductor Josephson junctions [4, 5]. To the lowest order approximation, it may be described by a Gross-Pitaevskii mean-field theory, accurately accounting for Josephson oscillations [6–8] and macroscopic self trapping [9], observed experimentally in Refs.[10, 11], as well as the equivalents of the ac and dc Josephson effect [12] observed in [13].

Both Josephson oscillations and macroscopic self trapping rely on coherent (Gaussian) preparations, with different initial population imbalance. The mean-field premise is that such states remain Gaussian throughout their evolution so that the relative phase φ between the two condensates remains defined. However, interactions between atoms lead to the collapse and revival of the relative phase in a process known as *phase diffusion* [14–16]. (the appropriate term is in fact *phase spreading*). Phase diffusion has been observed with astounding precision in an optical lattice in Refs.[17], in a double-BEC system in Refs.[18–20], and in a 1D spinor BEC in Ref. [21]. Typically, the condensates are coherently prepared, held for a varying duration ('hold time') in which phase-diffusion takes place, and are then released and allowed to interfere, thus measuring the relative coherence through the many-realization fringe visibility. In order to establish this quantity, the experiment is repeated many times for each hold period.

Phase-diffusion experiments focus on the initial preparation of a zero relative phase and its dispersion when no coupling is present between the condensates. However, in the presence of weak coupling during the hold time, the dynamics of phase diffusion is richer. It becomes sensitive to initial value of φ and the loss of coherence is most rapid for $\varphi = \pi$ [22–24]. Here, we expand on a recent letter [24], showing that this quantum effect can be described to excellent accuracy by means of a *semiclassical* phase-space picture. Furthermore, exploiting the simplicity of the dimer phase space, we derive analytical expressions based on the classical phase-space propagation [25].

Phase-space methods [26] have been extensively applied for the numerical simulation of quantum and thermal fluctuation effects in BECs [27–39]. Such methods utilize the semiclassical propagation of phase-space distributions with quantum fluctuations emulated via stochastic noise terms, and using a cloud of initial conditions that reflects the uncertainty of the initial quantum wave-packet. One particular example is the truncated Wigner approximation [28, 31, 34, 36] where higher order derivatives in the equation of motion for the Wigner distribution function are neglected, thus amounting to the propagation of an ensemble using the Gross-Pitaevskii equations.

Due to the relative simplicity of the classical phase-space of the two-site BHH, it is possible to carry out its