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An Analysis Of Interpolation Methods for Super Resolution Images

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Abstract— The image such as CT scan, x – ray image, CCTV videos and hand phone's camera is kind of low resolution image producers. Digital camera captured the continuous scenes and transform into discrete presentation in term of space and intensity. In sampling process it may create aliasing and information lost at frequency below the Nyquist sampling rates. Therefore the image suffered with an ill-posed problem by aliasing and loss of frequency. The problem ill-pose problem could be solved by applying Super Resolution (SR) techniques. The SR process contains of image registration, interpolation and image reconstruction. However this paper is focus on an analysis the best performance offered by interpolation techniques. An analysis procedure requires interpolation kernel inspection into frequency domain plotting to determine the best kernel response in pass and stop band. Otherwise use Peak Signal to Noise Ratio as indicator the similarity simulated with original image. In this study found the cubic spline interpolation is provided the smoother function frequency response with less ripples in stop band and good pass response. Besides that, it shows a superiorly in lead the highest PSNR for all type image tests with several of upscale. The best response and less distortion effect generated by kernel is preferable candidate to produce an efficient image application with low maintenances.

Keywords— interpolation, cubic, bilinear, nearest neighbor, quadratic.

I. INTRODUCTION

Normally a digital images exist when the camera captured a continuous scenes and transform into a discrete presentation. The digital images visualized on any screen after the processor read the image in digital format. However, during discretization process the intensity and space of real images tend to reproduce an aliasing and some lost information. That contamination may occur when the sampling frequency is above the Nyquist rate. This phenomenon is normal in digital images processing. Which lead to the appearing ill posed problem in low resolution image visualization. The solution to recovers the low resolution images are called as super resolution reconstruction or just call super resolution images.

The super resolution image is capable to solve problem with reconstructing and preserving the highest frequencies images. At the same time it eliminates an aliasing problem from low resolution images samples. In this studies area, there are numerous methods available to perform the super resolution method. However most of them were computationally expensive but able reproduced degraded and noisy image. Theoretically super resolution consist of three parts sequentially dependent each other. The three essence step is image registration, interpolation and de- blurring.

The image registration capable to determine relationship between a number of input low resolution images. Moreover image registration is used to determine the offset the images with accuracy down to a small fraction of a pixel [1]. All those pixels that have been matched can be combined to create one form a new composite image. The interpolation step is to produce acceptable image at new size resolution from a single low resolution image. Generally resolution image can define as the total number of pixel in one image.

Details about problems and how interpolation kernel works is addressed in this paper. Normally de-blurring process is last part for super resolution. De-blurring responsible to restore high frequencies that has been suppressed during the low resolution imaging process. However this paper only focus on interpolation techniques.

II. INTERPOLATION

A. Introduction to Interpolation

The interpolation term is always interchangeably with resampling process. In the reality interpolation and sampling is part of resampling process and they uses in zooming or enlarge size of original image. In another words resampling process refer to transforming from one set of coordinate location discrete image to new set of coordinate location. Interpolation is process of determining the values that laying between samples. In normal cases, interpolation function process has to create a continuous function between two discrete samples. The new samples point could be determines by resampling the continuous function with performing sufficient frequency sampling. By following Nyquist sampling theory, the sampling frequency should be more than twice frequency of continuous function.

B. Interpolation Kernel

Interpolation kernel directly refers to numerical accuracy and computational cost of interpolation algorithm. The computational accuracy of interpolation algorithm depends on selection of proper interpolation kernel. Therefore the best kernel interpolation will provide good accuracy and efficiency onto output reconstructed image. Commonly the Interpolation kernel function in digital image functional as mapping the intervals point between two discrete samples s(k,l) point to transform onto continuous signal s(x,y). The analysis and calculation always constructed in one dimensional case. For two dimensional interpolations is an extensional from one dimensional case.

III. TECHNIQUES

A. Nearest Neighbor Interpolation

The most easiest and simple interpolation algorithm to implement in calculation is the nearest neighbor interpolation. It offers basic interpolation algorithm with manipulates pixel values which the closest new location pixels has to copied same value with his neighbor. In term of filtering concept, this method applied rectangular function low pass filter by convolving with spatial images. The convolving image spatial with rectangular function is same as multiplying the signal by a sinc (sin(x)/x) function in frequency domain. However, in among of low pass filter the sinc function is poorest filter since it has no convergence side lobes. Consequently, the nearest neighbor interpolation has poor filter in frequency domain response. Furthermore, this algorithm consider unsuccessfully when performing resampling to a large matrix size since the new pixel was replicated from his neighbor. As a result, it produces images with a blocky appearance. Therefore this problem make this techniques is inappropriate to apply when accuracy edge are required. This interpolation algorithm was known as point shift function where the images convolving with one pixel width rectangular in the spatial domain. The nearest neighbor interpolation kernel equation can illustrated as equation (1).

$$f = \begin{cases} 1, & 0 \leq |x| \geq 0.5 \\ 0, & elsewere \end{cases} \tag{1}$$

f = amplitude,
x = x axis coordinate.

B. Bilinear Interpolation

Linear interpolation algorithm offers improvement interpolation techniques better than nearest neighbor. In the time or space domain, linear interpolation interpreted as triangle function kernel. This method passes a straight line between two consecutive pixel locations. For example, given two pixels x_0 and $x_1$which have an amplitude f_0 and f_1. In polynomial equation the output f(x) in linear interpolation can express like equation (2).

$$f(x) = f_0 + \left(\frac{x - x_0}{x_1 - x_0}\right) \cdot (f_1 - f_0) \tag{2}$$

The theory mentions that the linear interpolation performed better than the nearest neighbor interpolation. To examine the performance of linear interpolation, we must transform linear interpolation into frequency response function. In frequency domain it considerably a good low pass filter where the side lobes in the stop band much less prominent, so it made improvement from nearest neighbor in the stop band. However this kernel contributed some aliasing since continued pass a significant amount of spurious high frequency around the cut off frequency. In additional, it produced image smoothing when passband is moderately attenuated.

C. Quadratic Interpolation

The algebraic polynomial is famous concept in way to creates interpolator kernel as seem look like Sinc function. The reason is this concept is suitable to form a finite interval. Besides that, it is easy to determine and uniform approximation of continuous function. The quadratic function is derived from algebraic polynomial but it always disregarded largely because they believed this method has been introduces phase distortion. However, Dodson has proved that quadratic derived (3) in the quadratic interpolator capable to show better performance [2].

$$f(x) = \begin{cases} |x|^2 + \frac{1}{4}, & 0 \leq x \leq \frac{1}{2} \\ -2|x|^2 + 2|x| + \frac{3}{4}, & \frac{1}{2} \leq x \leq \frac{3}{2} \\ 0, & elsewhere \end{cases} \tag{3}$$

D. Cubic Interpolation

Previously, the linear interpolation method basically used two nearest point to create interpolation function. However this method has some improvement from nearest neighbor method where requires extend into four number of pixel neighbor. From the discussion above, both bilinear and nearest neighbor have a significant deviation from ideal low pass filter model. The shape in pass band and stop band was not meet good quality shape[3][4]. However, the quadratic method implemented uses three point for interpolation and would result in to one point at center and another two points at each side. Therefore cubic method extending over four point pixels nearby, which the function required two nearest point in each side.

The mathematical expression is shown in equation (4) for spline concept was introduced by Robert Keys [5]. The Spline was derived from convolution from several rectangular functions. In order to create a cubic B-spline, it requires four rectangular combination functions have been convolved [6]. So Spline extended number of pixel into four points nearest pixel, which two nearest pixel for each side axis direction [7]. Therefore Cubic Spline has better performance in band stop and band pass area as show in frequency domain comparison in figure 5.

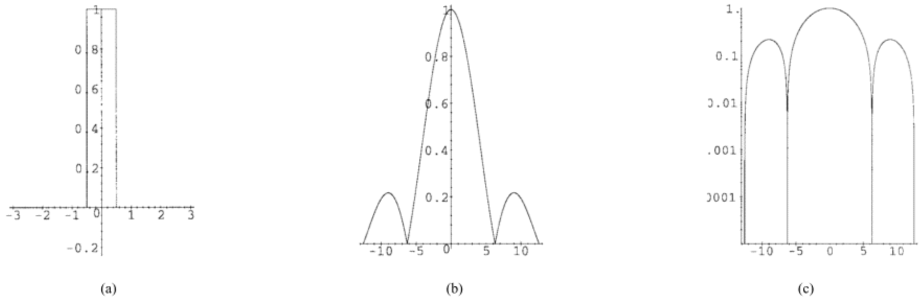

Figure 1: Nearest Neighbor a) kernel in pixel domain b) Magnitude for Fourier transform c) logarithmic plot of magnitude

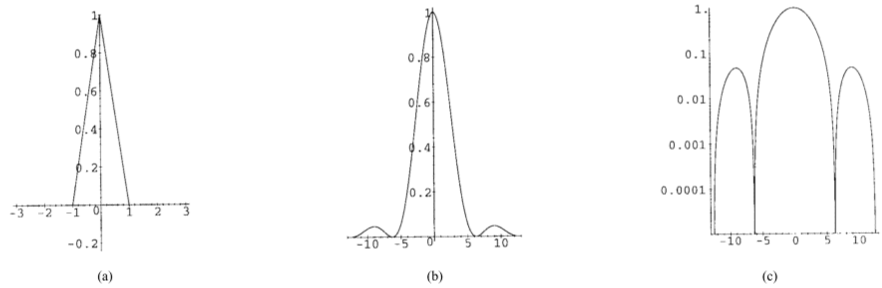

Figure 2: Bilinear interpolation a) kernel in pixel doain b) Magnitude for Fourier transform c) logarithmic plot of magnitude

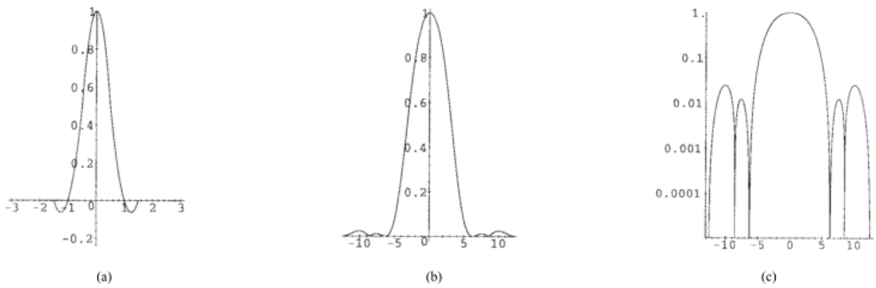

Figure 3: Quadratic interpolation a) kernel in pixel domain b) Magnitude for Fourier transform c) logarithmic plot of magnitude

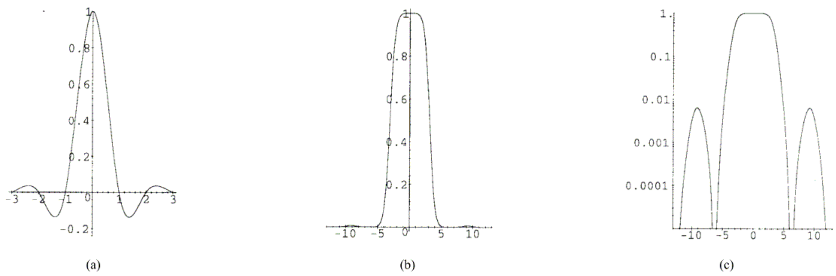

Figure 4: Cubic Spline interpolation with a = -1 a) kernel in pixel domain b) Magnitude for Fourier transform c) logarithmic plot of magnitude

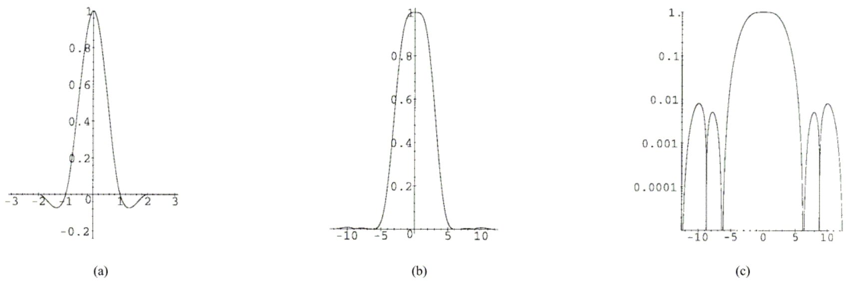

Figure 5: Cubic Spline interpolation with a = -0.5 a) kernel in pixel domain b) Magnitude for Fourier transform c) logarithmic plot of magnitude

The cubic B Spline are only need an interval range between (0,2) because the calculation procedures consider in symmetrically. However, the function is positive over the whole interval in the space domain. Thus in frequency domain it has more deviation from the constant gain within the pass band. Additionally, the cubic spline has higher in complexity than other previous techniques. Cubic Spline interpolation kernel also known as Parzen Window.

$$f(x) = \begin{cases} \frac{3}{2}|x|^3 - \frac{5}{2}|x|^2 + 1 & 0 \le |x| < 1 \\ -\frac{1}{2}|x|^3 + \frac{5}{2}|x|^2 - 4|x| + 2 & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases} \quad (4)$$

Previously, the cubic B spline function is one kind of cubic spline function. By definition, cubic splines are piecewise for third order function. Mostly most all function should in symmetrically, where the center of symmetrical about on zero. Therefore the computational consider starting at point zero until point two (0,2). Generally the cubic spline expressed in equation (5).

$$\begin{aligned} f(x) &= a_{31}\,x^3 + a_{21}\,x^2 + a_{11}\,x + a_{01} \quad (0,1) \\ f(x) &= a_{31}\,x^3 + a_{21}\,x^2 + a_{11}\,x + a_{01} \quad (1,2) \end{aligned} \quad (5)$$

In order to generate the final cubic spline kernel interpolation, it should meet some properties that show the real cubic characteristics. The properties design cubic requires the kernel function to provide value 1 (f(0) = 1) when at location 0 and at location 1 and 2 it should provide value zeros (f(1) and f(2) = 0). Three more constraints can be obtained by apply first differentiation upon the equation (5) as $f'(x)$ to obtain the function is continuous at nodes 0,1 and 2. All the constraint imposed on $f(x)$ generated in seven equations. However the equation above gave eight unknown coefficient. Thus one more conditions need be obtain as a unique solution. Let assumes, $A_2 = a$, so the remaining seven coefficients can be solved in term of a and as result equation (6) is generated.

$$\begin{aligned} f(x) &= (a+2)x^3 - (a+3)\,x^2 + 1 \quad (0,1) \\ f(x) &= a\,x^3 - 5ax^{\,2} + 8ax - 4a \quad (1,2) \end{aligned} \quad (6)$$

The constant a value give some implication on the curve shape of the cubic kernel function, the constant a was in negative values so the cubic function in interval 0 and 1 was positive and falling to negative values in interval between 1 and 2. The depth of lobes curve negative will increase in interval 1 and 2 when the value constant a increased. The general cubic spline function is look like the general form of window Sinc function. This function has better performance in high frequency rather than the Cubic B Spline. Furthermore, [8][9] choose the constant a = -1 to match up the kernel function with the slop of Sinc function. However Robert Keys[10] selected a = -1/2 , the frequency spectrum was flat in the low frequency and fall off toward the cut off frequency. By refer Robert Keys[10] the cubic interpolation kernel should obey with first three term of Taylor series expansion, so the constraint a should equal -1/2. In order to achieve third order precision the constraint a should equal -1/2 otherwise the result produced in at most first order approximation.

IV. Methodology

The discussion about interpolation techniques is continuously researched and an every year new techniques are introduced. However, this paper focus on comparison between common techniques that have been used in broad image application super resolution techniques such as nearest neighbor, bilinear interpolator, quadratic, newton, Gaussian and cubic interpolator. In this investigation, the images T-shirt, Book, Rack, Table and Surau images were took from digital camera with different resolution and used them as ground truth images. All experiment and analysis about kernel interpolation images is done toward images in gray scale level.

However in the image reconstruction analysis is hard to describe the quality of techniques applied. As already knows the quality image is subjective and complicated to interpret how good images had been enhanced. Thus researcher introduced PSNR method as images measurement quality tool. PSNR stand for peak signal noise ratio and it requires Means Square Error (MSE) as control variable. PSNR capable to express the ratio between the maximum possible power of benchmark signal versus tester or distortion signal that change the quality of presentation. Usually value of PSNR expressed as equation (7) and the unit in term of logarithmic decibel scale. MSE represents an average of square of deviation between the simulated image and ground truth images. In fact, the best techniques will produce minimum MSE between images to indicate the highest PSNR. MSE represent average deviation image intensity between two images and can express like equation (8).

$$PSNR = 20log_{10}\left(\frac{MAXf}{\sqrt{MSE}}\right) \quad (7)$$

$$MSE = \frac{1}{m*n}\sum_0^{m-1}\sum_0^{n-1}\|f(i,j) - g(i,j)\|^2 \quad (8)$$

In order to trace which particular algorithms give a better result, the systematic comparative system is required. Same test image databases are tested on image interpolation method. Therefore PSNR uses as indicator to highlight compromise enhancement techniques.

V. Result

They have some factor as roles to measured performance of the kernel techniques. The comparison is accomplished by spatial analysis upon kernel function trough visual quality assessments, computational complexity and run time measurement. The fastest processing technique is nearest neighbor interpolator but in same time, it incurs many of errors as table 1, table 2 and table 3 show the nearest neighbor convey the lowest PSNR. Additional an image produced contaminated by staircase problem.

The bilinear interpolator has improved from nearest neighbor method, where requires further multiplication and has limited features. From the all table recorded, the bilinear kernel has good performance and all PSNR close with cubic interpolation kernel result. Sometime in table 1 the bilinear provided the less error than others kernel. Thus, this method frequently used in many publications after discovered by [11].

In this study the cubic kernel interpolator is superior kernel compare with another common interpolator kernel. In all table experimental result show the cubic kernel provided the highest PSNR for various type of magnify scale. In additional the B-Spline interpolator produces the best similarity output images with ground truth images and in same time this method runs faster during calculating process and practically result meet theoretical concept as state in [12] found that, the cubic Spline is sufficient for several image application.

TABLE I. Interpolation enlargement with scale 2.

	T-Shirt 320x240	**Book 320x240**	**Library 320x240**	**Rack 320x240**	**Table 320x240**	**Surau 320x240**
Spline Cubic	33.9914	20.8863	29.0582	28.0697	32.8028	27.95
Cubic Lagrange	**34.2317**	20.8653	**29.1781**	**28.1072**	**32.939**	**27.9897**
Quadratic Spline	32.4822	20.111	28.744	27.2103	31.3953	27.6331
quadratic	33.7259	20.283	28.8551	27.5476	32.2062	27.7129
Newton	32.4374	19.1034	27.7556	26.0218	30.7477	26.7288
Gaussian	34.0802	19.1034	29.109	28.0137	32.7854	27.9205
Bilinear	34.0992	**20.901**	29.0966	28.0484	32.8337	27.8915
Nearest Neighbor	30.791	17.8281	26.4638	24.1148	28.8593	25.1169
Lanczos	30.8965	20.7432	26.9252	27.0295	30.2274	26.8259

TABLE II. Interpolation enlargement with scale 4.

	T-Shirt 640x480	**Book 640x480**	**Library 640x480**	**Rack 640x480**	**Table 640x480**	**Surau 640x480**
Spline Cubic	33.8509	**20.8417**	29.0103	27.9207	32.3822	25.6527
Cubic Lagrange	**34.1613**	20.836	**29.1465**	**27.9634**	**32.5369**	**25.6774**
Quadratic Spline	31.3161	19.8048	28.555	26.6826	30.1506	25.3982
quadratic	33.5696	20.1634	28.7984	27.3567	31.5631	25.5004
Newton	31.8746	18.8824	27.5135	25.5426	30.1388	24.8013
Gaussian	34.0086	20.8039	29.0652	27.8723	32.3971	25.6273
Bilinear	33.9568	20.8657	28.9937	27.8165	32.3508	25.5562
Nearest Neighbor	29.2421	16.4785	25.0377	22.3882	26.812	22.6979
Lanczos	30.8188	20.7534	26.7664	26.8495	29.8954	24.8453

TABLE III. Interpolation enlargement with scale 4.

	T-Shirt 1280x960	**Library 1280x960**	**Rack 1280x960**	**Table 1280x960**
Spline Cubic	33.4985	28.7585	27.5307	31.3513
Cubic Lagrange	**33.7793**	**28.9059**	**27.5933**	**31.5212**
Quadratic Spline	30.3648	27.969	25.8663	28.9261
Quadratic Lagrange	32.8998	28.2576	26.629	30.3844
Newton	31.6844	27.6781	25.5778	29.6973
Gaussian	33.6521	28.8265	27.5147	31.4279
Bilinear	33.6161	28.7944	27.4641	31.4243
Nearest Neighbor	27.9465	23.8406	21.0422	25.4594
Lanczos	30.7992	26.5452	26.5096	29.1972

VI. CONCLUSION

The nearest neighbor function does moderately in pass band but it has highest side lobes in stop band. The bilinear function does some improvement from nearest neighbor function which provided better respond in stop band. However it has expense such amount of smoothing in the pass band. Cubic interpolation has better performance than nearest neighbor and bilinear but it has wider pass band. The cubic spline offered good performance in pass band and stop band. The achievement cubic spline depend on values of constant a , the maximum performance reached when the constant a = -0.5 then function will attenuated when constant decreased toward -2. Although cubic spline provided the good performance than others but it has computational expensive, where it required extend four pixel neighbors during estimating new pixel but the nearest neighbor and bilinear only required two pixel neighbors. The cubic spline is good estimator for interpolation techniques however it still incapable to recovers some missing information image and still produced blurry and unshapen images. Thus it requires sharpening techniques to recovery the techniques weakness that will be focus on further research.

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