LID-DRIVEN CAVITY FLOW: SOLVING THE NAVIER-STOKES EQUATION

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LID-DRIVEN CAVITY

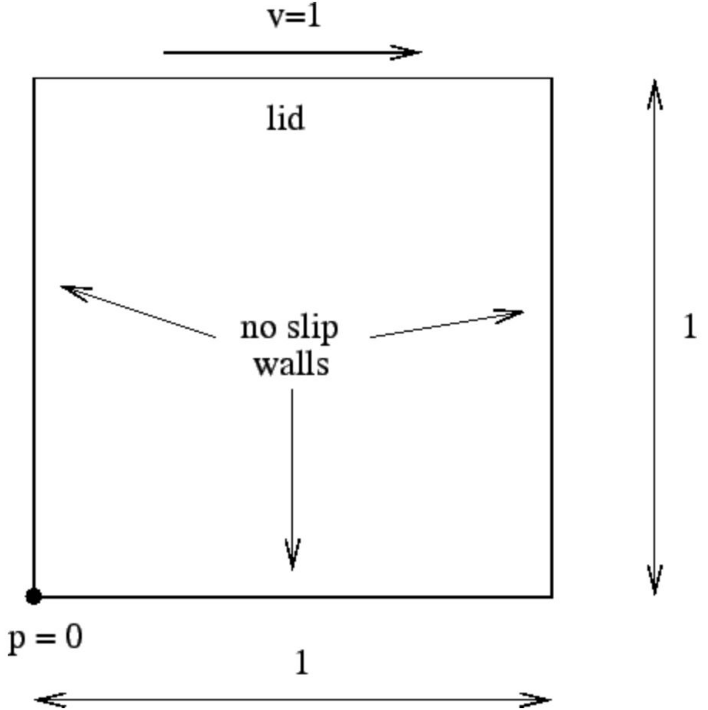


Figure 27: Geometry of the lid-driven cavity

- Well-known CFD problem for modelling viscous incompressible fluid flow.
- Geometry: square cavity filled with fluid, with three rigid walls (no-slip, velocity=0).
- At top, a tangential velocity is applied to drive fluid in the cavity.

THE NAVIER-STOKES EQUATION

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u}$$

$$\nabla \cdot \overline{u} = 0$$

Incompressible Navier-Stokes (momentum) equation.

$$(1) \qquad \frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -(\mathbf{u}^n \cdot \nabla)\mathbf{u}^n + \nu \nabla^2 \mathbf{u}^n$$

$$(2) \qquad \frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho}\nabla p^{n+1}$$

Chorin's projection method. Eq. (2), the **projection step,** shows operator splitting approach where viscous forces are considered in first half step and pressure forces in the second (separately).

- These equations are PDEs that describe motion of viscous fluids.
- Chorin's projection lets us numerically solve time-dependent incompressible fluid-flow problems by decoupling the computation of the velocity and pressure fields.
- Solving numerically involves using the **Finite Difference Method** (involving Central Difference method) to discretize PDEs .

METHODS: FINITE DIFFERENCE

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2} = \frac{\phi(x+a,y) + \phi(x-a,y) + \phi(x,y+a) + \phi(x,y-a) - 4\phi(x,y)}{a^2}$$

- Finite Difference methods are used for solving PDE’s, involving the discretization of points into a grid (in this case into a unit square grid).
- Laplacian operator (shown above) is used directly in this work and is very similar to the example problem for class involving electric potential.
- Specifically, this is a boundary condition PDE.

CHORIN'S PROJECTION STEPS

Step 1: Solve momentum equation without pressure with boundary conditions.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u}$$

$$(1) \qquad \frac{\mathbf{u}^* - \mathbf{u}^n}{\Delta t} = -(\mathbf{u}^n \cdot \nabla)\mathbf{u}^n + \nu \nabla^2 \mathbf{u}^n$$

Step 2: Solve pressure Poisson equation at next point in time.

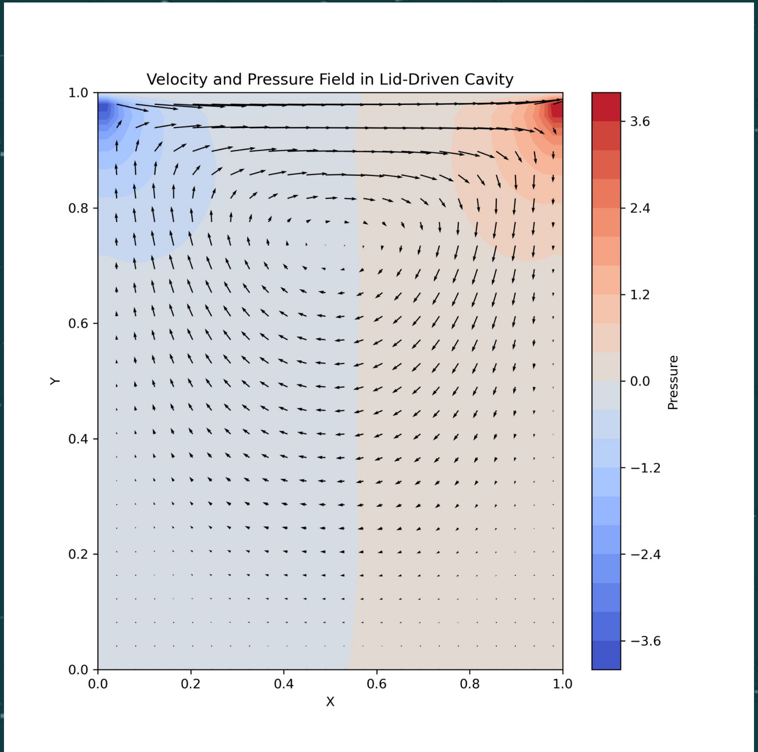
$$(2) \qquad \frac{\mathbf{u}^{n+1} - \mathbf{u}^*}{\Delta t} = -\frac{1}{\rho}\nabla p^{n+1}$$

$$\nabla^2 p^{n+1} = \frac{\rho}{\Delta t}\nabla \cdot \mathbf{u}^*$$

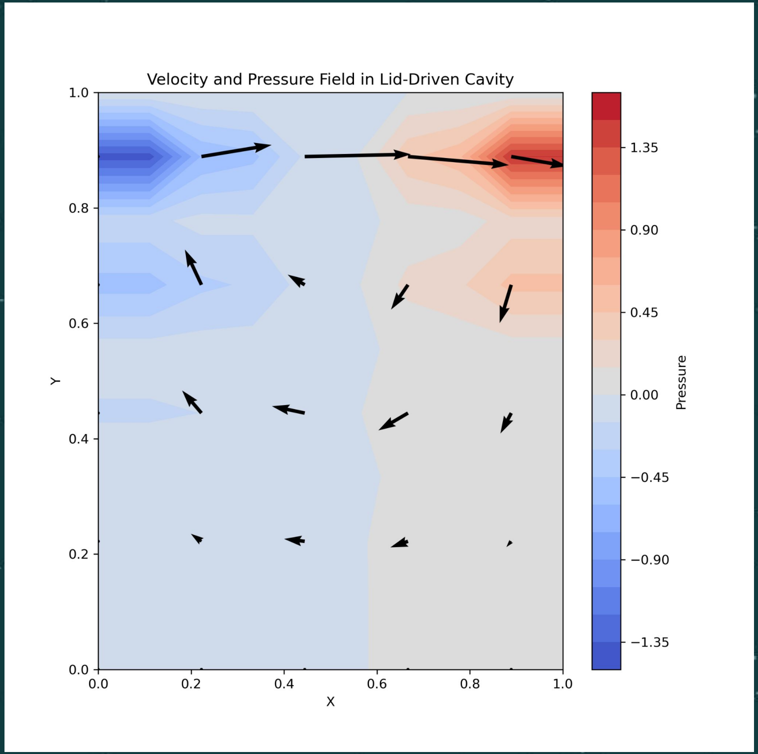
Step 3: Correct velocities

$$\mathbf{u}^* = \mathbf{u}^{n+1} + \frac{\Delta t}{\rho}\nabla p^{n+1}$$

RESULTS

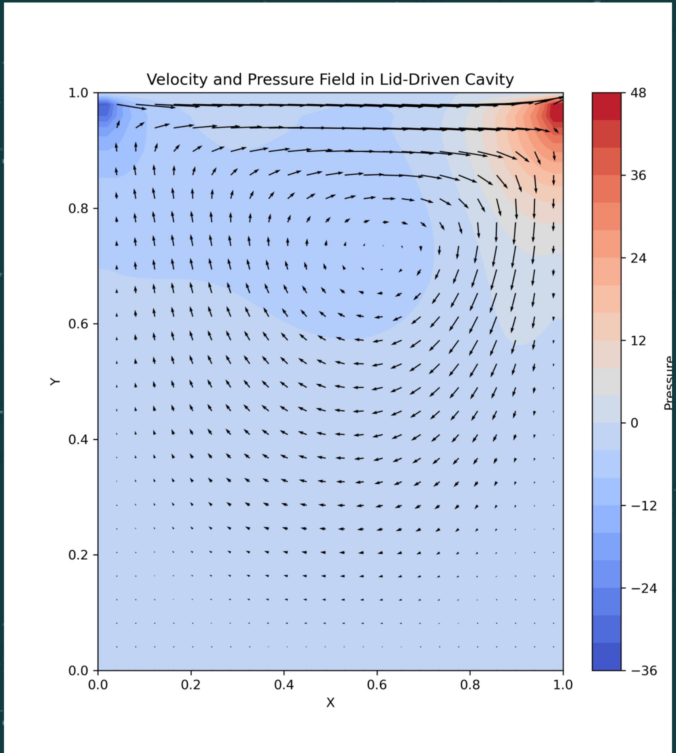


# of points along each axis = 50

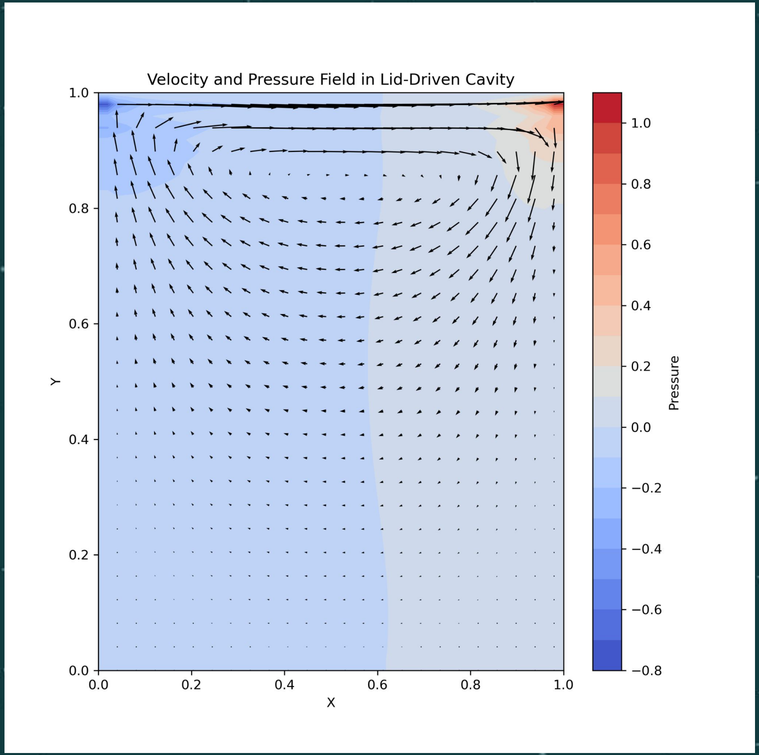


# of points along each axis = 10

RESULTS



Horizontal velocity = 10 (instead of 1)



Kinematic viscosity = 0.01 (instead of 0.1)

CHALKBOARD BACKGROUND INFOGRAPHICS

MERCURY
It's the closest planet to the Sun

JUPITER
It's the biggest planet of them all

EARTH
It's the planet we live on

NEPTUNE
The farthest planet from the Sun

SATURN
It's a gas giant and has several rings