2019 Statistics and data analysis II

Week14

「Hypothesis test for mean」

2019. 16th. July

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Plan of this course

※ Might be changed.

On final exam

- Week12-15
- On MOOCS (+pdf)
- You can bring anything. (internet is not allowed)

第１回　ガイダンス、数学の準備
第２回　統計学とは，１次元のデータ(1)：度数分布とヒストグラム
第３回　１次元のデータ(2)：代表値，散らばりの尺度
第４回　２次元のデータ(1)：散布図と分割表
第５回　２次元のデータ(2)：相関係数，直線及び平面の当てはめ（単回帰分析）
確率(1)：ランダムネスと確率，標本空間と事象
第６回　確率(2)：確率の定義，加法定理，条件付確立と独立性
第７回　まとめ＋試験①
第８回　確率変数(1)：確率変数と確率分布，期待値と分散
第９回　確率変数(2)：チェビシェフの不等式，
確率分布(1)：二項分布，ポアソン分布
第１０回　確率分布(2)：正規分布，指数分布
第１１回　まとめ＋試験②
第１２回　記述統計から推測統計へ（標準正規分布表の見方、区間推定）
第１３回　仮説検定（１）～仮説検定の導入と統計量の基本的な分布（t分布）～
第１４回　仮説検定（２）～平均の検定～
第１５回　仮説検定（３）～平均の差の検定～
第１６回　まとめ＋試験③

2． Hypothesis test for mean

Agenda

- Hypothesis test for mean
- In case population S.D. is known
- In case population S.D. is unknown
- Exercises

Respon

2-1. Hypothesis test for mean

Hypothess test for mean

- In the inferential statistics, we estimate the characteristics of the population from observed samples.

- Especially, if we validate the value of the population mean, it is called as the hypothesis test for mean.

Usage

- By using samples, validate whether the product specification is correctly applied.

Take care…

①Population variance is known？ Unknown？

→ If known, apply z-dist., otherwise, t-dist.

②Two-sided？ One -sided？

→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

③The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

2-2. Hypothesis test for mean （In case the population S.D. is known）

Hypothesis test for mean①
（ In case the population S.D. is known ）

- Let the population mean and S.D. of a normal distribution be μ and σ, respectively. Then, the n samples extracted from them also follows the normal distribution.
  - The sample mean remains as μ, but the S.D. of the samples reduces to $\frac{\sigma}{\sqrt{n}}$

- 95 % C.I. of the normal distribution:

$$-1.96 \leq \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq 1.96$$

Hypothesis test for mean①
（ In case the population S.D. is known ）

・ In case σ^2 is known, we use the following quantity as the test statistic:

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- Then, what is the p.d.f. of the test statistic?

N(0,1)!

Hypothesis test for mean①
（ In case the population S.D. is known ）

- vi) For the significance level α, set the rejection region R that satisfies

$$P(T(X_1,X_2,\ldots,X_N)\in R) = \alpha.$$

・ In case of the two-sided test, it’s out of the upper / lower 2.5-percentiles (=±1.96) of z-distribution.

・ In case of the two-sided test, it’s out of the upper / lower 5-percentiles (=1.64).

Example①

In a certain maker of a part (named as “M”) of computer, its diameter is described as 1.54[cm] in its product specification. In a certain sample survey, they extracted 8 samples randomly, and observed the following data of measured diameter [cm]:

1.53 1.57 1.54 1.57 1.53 1.55 1.56 1.53

It’s known that the population variance is $\sigma^2=0.0001$. Then, can you say that this part follow its product specification? Test with the significance level of 5%.

Flow of hypothesis test

- i) Set the population （☚ Similar to confidence interval ）
- ii) Set the null hypothesis $H_{0.}$
- iii) Extract samples $x_1,x_2,\ldots,x_N$ from the population.
- iv) Find a statistics $T(x_1,x_2,\ldots,x_N)$ from the sample above.
- v) Calculate the probability density of the statistics $T(X_1,X_2,\ldots,X_N)$ for r.v.s $X_1,X_2,\ldots,X_N$.

Flow of hypothesis test

- vi) For a certain significance level α, find a region R, where

Usually, α=0.01 or 0.05

$$P(T(X_1,X_2,...,X_N)\in R) = \alpha$$

holds(This region R is called as the critical region)

- vii) If $T(x_1,x_2,...,x_N)\in R$, reject the null hypothesis H_0 / otherwise, H_0 cannot be rejected.

Example① 【 answer 】

Flow of hypothesis test.

- Null hypothesis H_0 ”The diameter of part “M” is1.54cm”

  Alternative hypothesis H_1:” The diameter of part “M” is not 1.54cm”

  （No good with too large nor too small results.）

- Find the test statistic $T(x_1, x_2, \ldots, x_N)$.

- Since the population variance is known,

we use

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Example① 【answer】

- Find the p.d.f. of the test statistic t.

Example① 【answer】

- If X follows N（μ， σ²), then

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

follows N(0,1).

Example① 【answer】

- Find the p.d.f. of the test statistic t.

  ⇒ z-distribution.

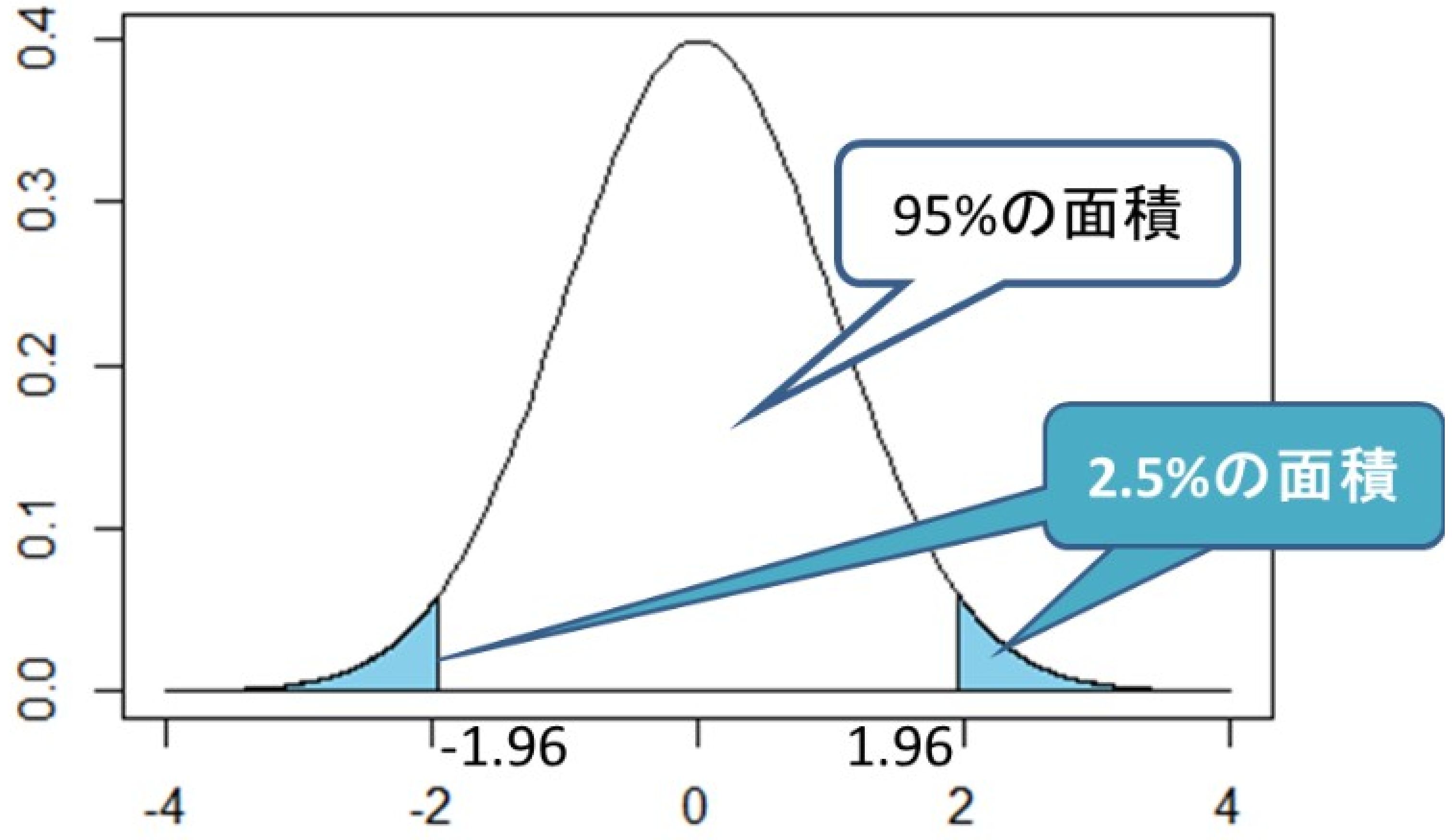
Example① 【answer】

- Determine the rejection region.

⇒ Two-sided test from the form of the alternative hypothesis.

The rejection region is as follows （out of upper /lower2.5-percentiles of z-distribution）.

- $|t|>1.96$ is the rejection region.



Example① 【answer】

- Find the value of $T(x_1,x_2,...,x_N)$:

$$\bar{x} = \frac{1.53 + 1.57 + \ldots + 1.53}{8} = 1.5475$$

$$\mu = 1.54 \qquad \sigma = \sqrt{0.0001} = 0.01$$

Thus, we have

$$t = \frac{1.5475 - 1.54}{\frac{0.01}{\sqrt{8}}} = 2.121$$

- Belong to the rejection region $|z|>1.96$!

Example① 【answer】

Reject H0 under the significance level of 5%. Employ the alternative hypothesis.

That is, “the diameter of the part M is not 1.54cm.”

Example① 【answer】： Using Python

①Population variance is $\boxed{\text{known}}$? Unknown？
→ If known, apply z-dist., otherwise, t-dist.

$\boxed{\text{②Two-sided}}$? One -sided？
→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

$\boxed{\text{③The row data is given}}$? Just some statistical indicators (sample mean, unbiased SD) are given?

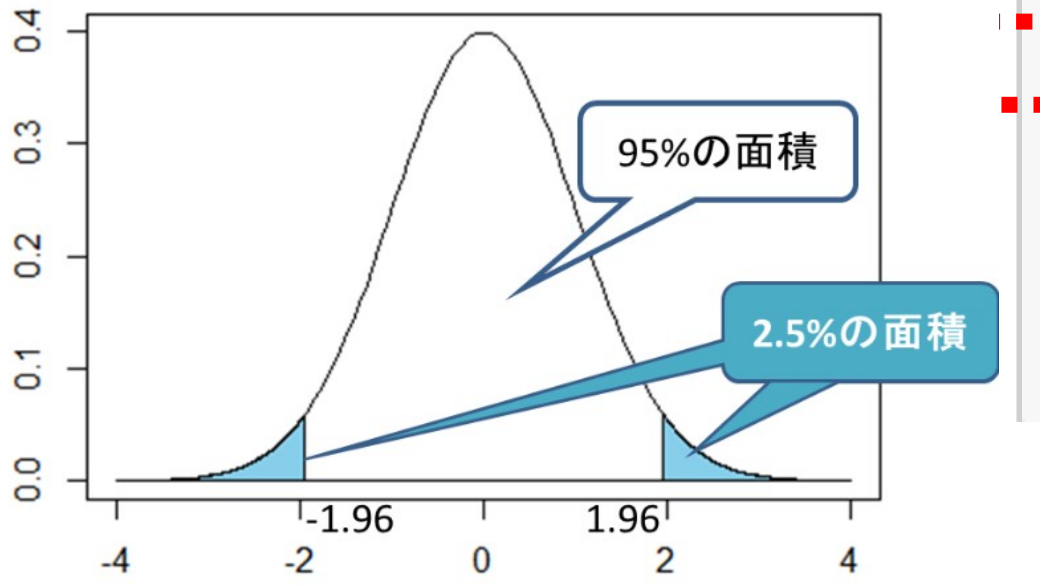
```
import numpy as np
from scipy import stats
from scipy.stats import norm

X = np.array([1.53, 1.57, 1.54,1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=1.54
avg = X.mean()
std = 0.01#X.std()
N=X.size
#print sample mean.
print(avg)
z = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(z)
p = norm.cdf(- np.abs(z), 0, 1) * 2
print(p)
```

Example① 【answer】： Using Python

```
import numpy as np
from scipy import stats
from scipy.stats import norm

X = np.array([1.53, 1.57, 1.54,1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=1.54
avg = X.mean()
std = 0.01#X.std()
N=X.size
#print sample mean.
print(avg)
z = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(z)
p = norm.cdf(- np.abs(z), 0, 1) * 2
print(p)
```

Null hypothesis

Z-statistic



- To find the p-value, you should make it twic in case of the two-sided test.

Example① 【answer】： Using Python

[Output]

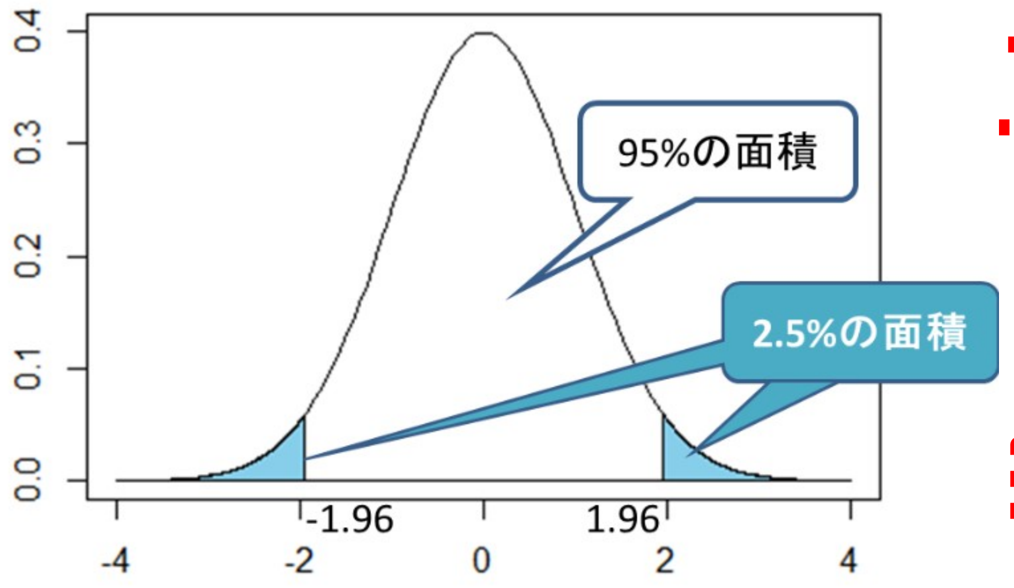
```
import numpy as np
from scipy import stats
from scipy.stats import norm

X = np.array([1.53, 1.57, 1.54,1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=1.54
avg = X.mean()
std = 0.01#X.std()
N=X.size
#print sample mean.
print(avg)
z = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(z)
p = norm.cdf(- np.abs(z), 0, 1) * 2
print(p)
```

Null hypothesis

Z-statistic

```
1.5475
2.1213203435596606
0.033894853524687726
```

P-val.<5%
In the rejection region



Answer.

“The diameter is not 1.54cm”

or

“Reject H0.”

※Assuming you can answer H0 correctly!

2-3. Hypothesis test for mean (In case population S.D. is unknown)

Test for mean②(population SD is unknown)

- In method ①, the population SD σ was known
  What about σ is unknown 、、?

- You should replace σ by the unbiased SD, S.

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}}$$

Follows the t-distribution of df = (N-1)

Test for mean②(population SD is unknown)

- vi) For α, find a rejection region that meets

$$P(T(X_1,X_2,\dots,X_N)\in R) = \alpha.$$

- For two-sided test, out of upper/ lower 2.5-percentiles(= $\pm t_{N-1}(0.025)$)
- For one-sided test, above the upper 5-percentile (= $t_{N-1}(0.05)$) or below the lower 5-percentile(= $-t_{N-1}(0.05)$)

Example②

In a certain factory, they make a component of a compter, named A. After we measured the length of 10 samples of this A, the mean and unbiased variance were 7.2[cm] and 0.04, respectively.

If the length of A follows the normal distribution, can we state that the length of A is 7.0cm on average? Conduct the hypothesis test with the significance level of 5%.

Flow of hypothesis test

- i) Set the population （☚ Similar to confidence interval ）
- ii) Set the null hypothesis H_0.
- iii) Extract samples $x_1,x_2,\ldots,x_N$ from the population.
- iv) Find a statistics $T(x_1,x_2,\ldots,x_N)$ from the sample above.
- v) Calculate the probability density of the statistics $T(X_1,X_2,\ldots,X_N)$ for r.v.s $X_1,X_2,\ldots,X_N$.

Flow of hypothesis test

- vi) For a certain significance level α, find a region R, where

$$P(T(X_1,X_2,\ldots,X_N)\in R) = \alpha$$

Usually, α=0.01 or 0.05

holds(This region R is called as the critical region)

- vii) If $T(x_1,x_2,\ldots,x_N)\in R$, reject the null hypothesis H_0 / otherwise, H_0 cannot be rejected.

Example② 【Answer】

Flow of hypothesis test:

- Null hypothesis H_0:”The length of A is 7.0cm.”

  Alternative hypothesis H_1:”The length of A is not 7.0cm.”

- Find the test statistic $T(x_1,x_2,\ldots,x_N)$.

- Since the population variance is unknown, we take

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}}$$

Example② 【Answer】

- Find the p.d.f. of the test statistic t.

Example② 【Answer】

- If X follows N（μ， σ²） , then, the test statistic

$$t = \frac{X - \mu}{\frac{s}{\sqrt{n}}}$$

follows the t-distribution with df=(n-1). Here,

$$s = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \ldots + (X_n - \bar{X})^2}{n-1}}$$

Example② 【Answer】

- Find the p.d.f. of the test statistic t.
- ⇒ t-distribution with df=9.

Example② 【Answer】

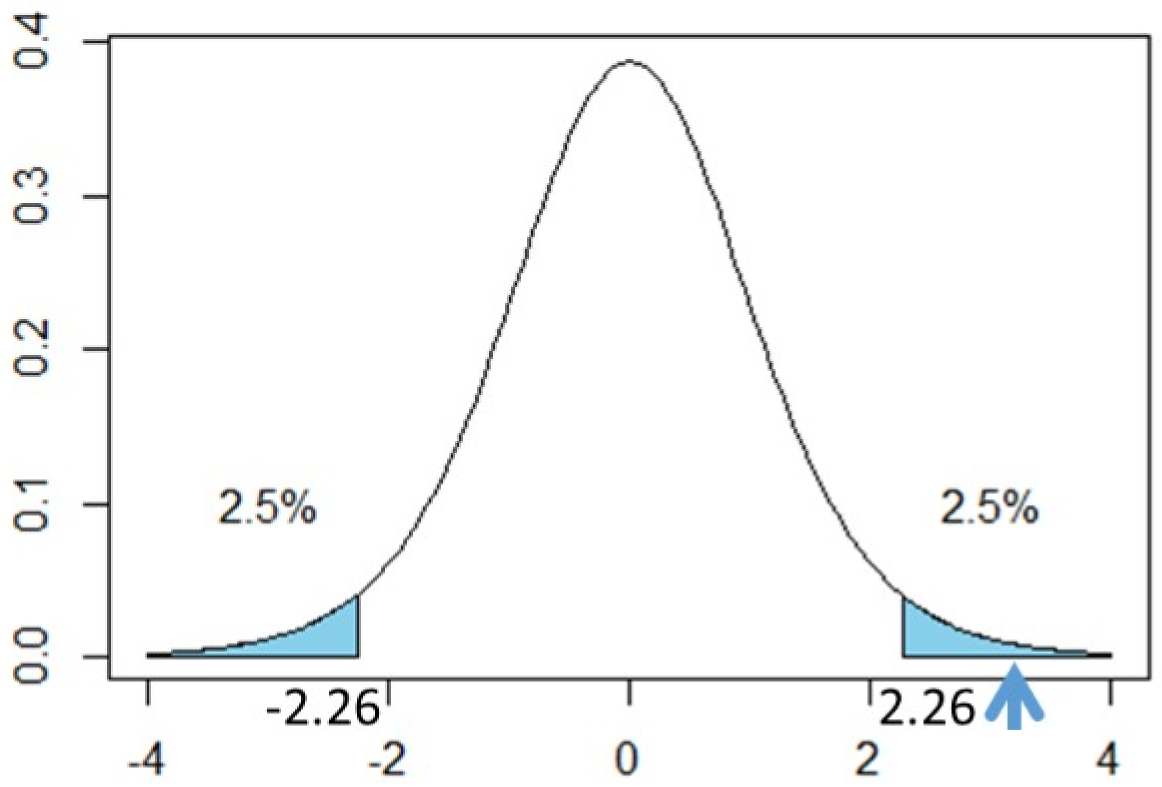
- Determine the rejection region.

  ⇒ Two-sided test.

The rejection region is outside of upper/lower 2.5-percentiles of the t-distribution.

$$t_9(0.05/2) = 2.26$$

- The region $|t|>2.26$ is the rejection region.

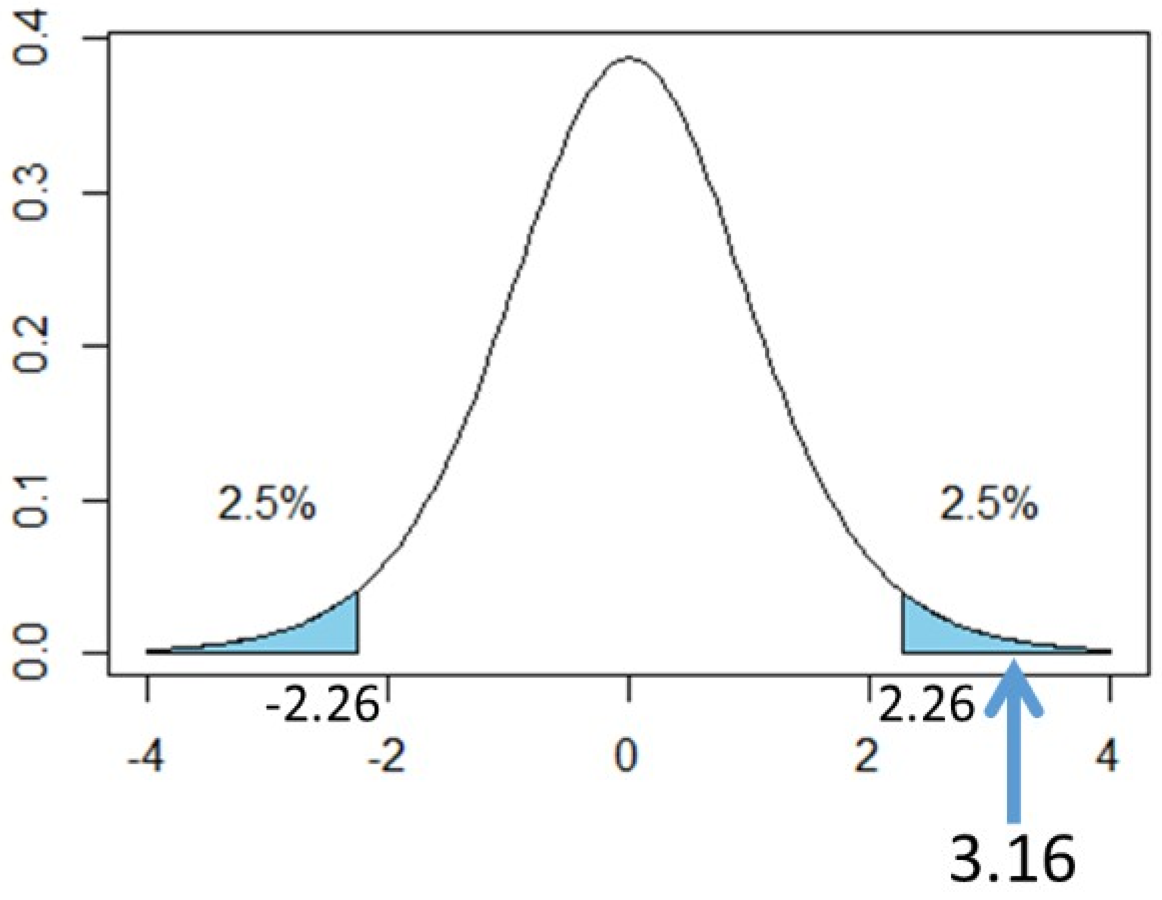


Example② 【Answer】

- Find the value of the test statistic $T(x_1,x_2,...,x_N)$:

$$t = \frac{7.2 - 7.0}{\frac{0.2}{\sqrt{10}}} = 3.16$$

- Belongs to the rejection region $|t|>2.26$!



Example② 【Answer】

Reject H0 under the significance level of 5%.

"The length of A is not 7.0cm."

Example② 【Answer】 Using Python

```
import numpy as np
from scipy import stats
from scipy.stats import t

mu_0=7.0
avg = 7.2
std = 0.2#X.std()
N=10 #X.size
#print sample mean.
stat_t = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(stat_t)
p = t.cdf(-np.abs(stat_t), df=N-1) * 2
print(p)
```

① Population variance is known ? Unknown ?
→ If known, apply z-dist., otherwise, t-dist.

② Two-sided ? One -sided ?
→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

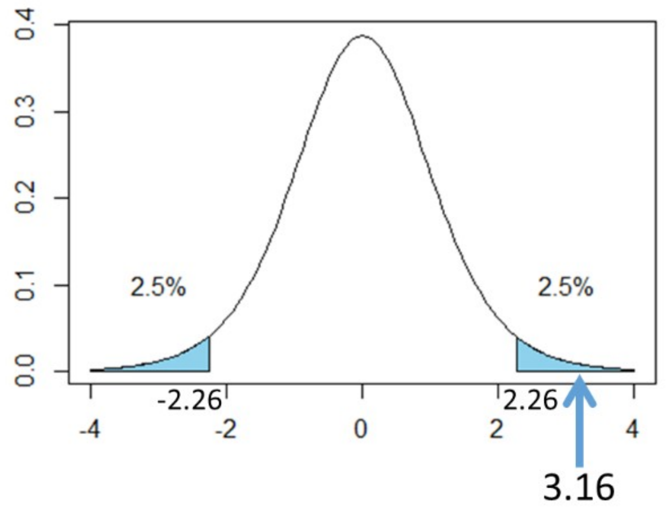
③ The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

Example② 【Answer】 Using Python

```
import numpy as np
from scipy import stats
from scipy.stats import t

mu_0=7.0
avg = 7.2
std = 0.2#X.std()
N=10 #X.size
#print sample mean.
stat_t = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(stat_t)
p = t.cdf(-np.abs(stat_t), df=N-1) * 2
print(p)
```

Null hypothesis

Test statistic



- To find the p-value, you should make it twice in case of the two-sided test.

Answer

“The diameter is not 7.0cm /significantly different from 7.0cm”

Or

“Reject H0”

※Assuming you can answer H0 correctly!

Hypothesis test for mean
(In case the population S.D. is unknown / one-sided test)

Example ③

A certain maker states that the lifetime of their light bulb is 2000 hours. To validate this statement, we bought 15 samples and tested their lifetime.

Then, the mean and unbiased S.D. were 1900 and 150 hours, respectively.

If the lifetime follows the normal distribution, can we say that the statement of this maker is correct?
Conduct the hypothesis test with the significance level of 5%.

Flow of hypothesis test

- i) Set the population （☚ Similar to confidence interval ）
- ii) Set the null hypothesis H_0.
- iii) Extract samples $x_1,x_2,\ldots,x_N$ from the population.
- iv) Find a statistics $T(x_1,x_2,\ldots,x_N)$ from the sample above.
- v) Calculate the probability density of the statistics $T(X_1,X_2,\ldots,X_N)$ for r.v.s $X_1,X_2,\ldots,X_N$.

Flow of hypothesis test

- vi) For a certain significance level α, find a region R, where

$$P(T(X_1,X_2,...,X_N)\in R) = \alpha$$

holds(This region R is called as the critical region)

Usually, α=0.01 or 0.05

- vii) If $T(x_1,x_2,...,x_N)\in R$, reject the null hypothesis H_0 / otherwise, H_0 cannot be rejected.

Example ③ 【Answer】

- Null hypothesis H_0:”The lifetime is 2000 hours.”

Alternative hypothesis H_1:”The lifetime is less than 2000 hours.” （No problem if it’s longer）

- Find the test statistic $T(x_1,x_2,\ldots,x_N)$.
  - Since the population variance is unknown, we take:

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}}$$

Example ③ 【Answer】

- Find the p.d.f. of the test statistic t.

Example③ 【Answer】

- If X follows N（μ， σ²） , then the test statistic

$$t = \frac{X - \mu}{\frac{s}{\sqrt{n}}}$$

follows the t-distribution with df=(n-1). Here,

$$s = \sqrt{\frac{(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + \ldots + (X_n - \bar{X})^2}{n-1}}$$

Example ③ 【Answer】

- Find the pd.f. of the test statistic t.

  ⇒ follows the t-distribution with df= 14.

Example ③ 【Answer】

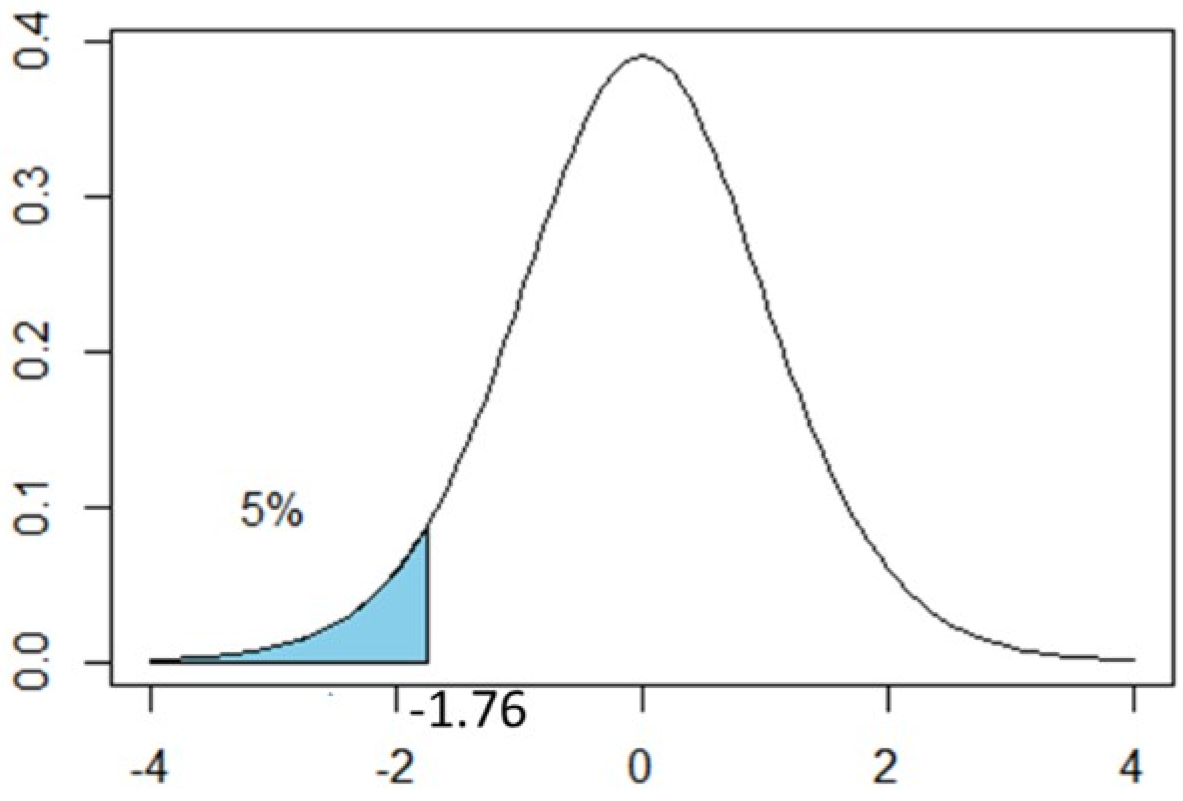
- Determine the rejection region.

⇒ one-sided test.

The rejection region is left side of the lower 5-percentile of the ti-distribution.

$$t_{14}(0.05) = -1.76$$

- The region of t<-1.76 is the rejection region.

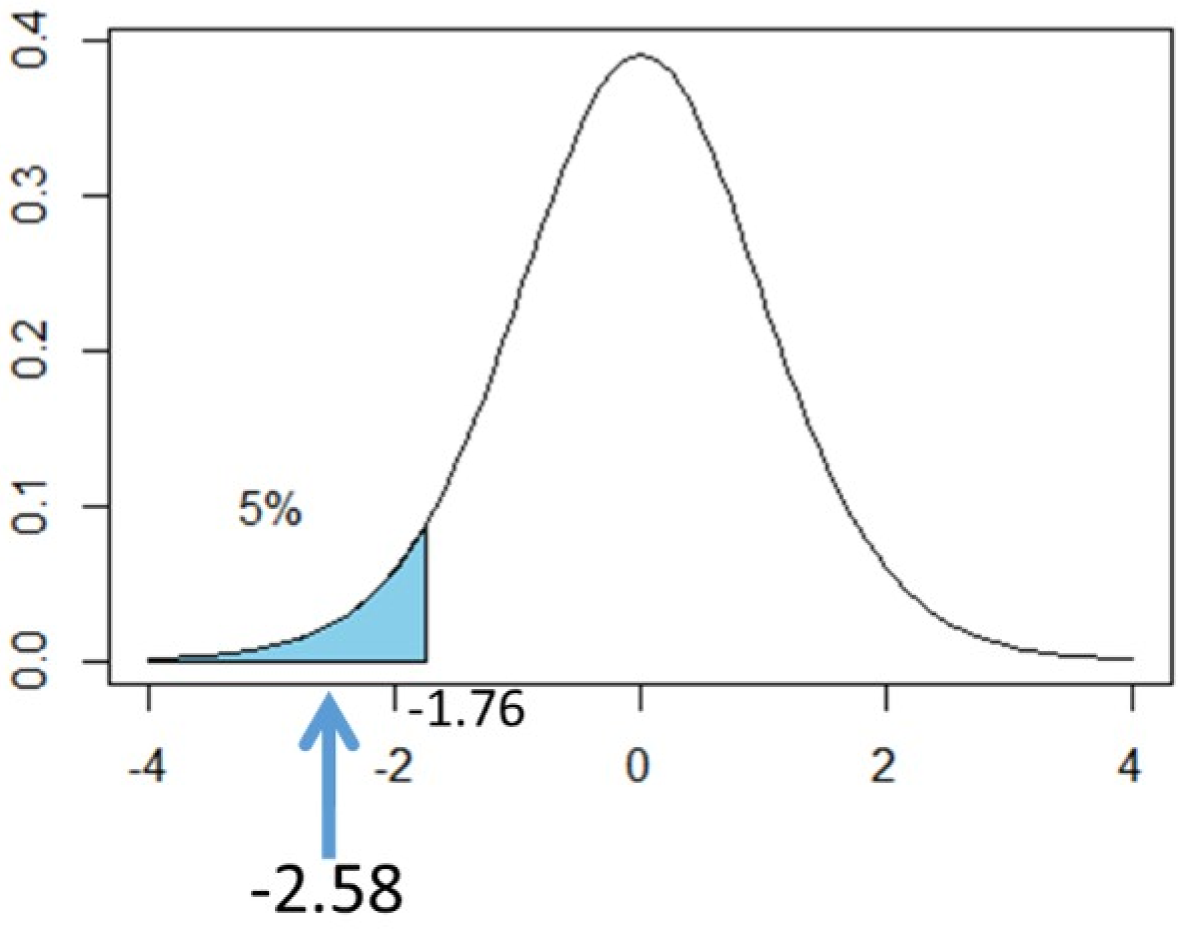


Example ③ 【Answer】

- Find the value of the test statistic $T(x_1,x_2,...,x_N)$:

$$t = \frac{1900 - 2000}{\frac{150}{\sqrt{15}}} = -2.58$$

- Belong to the rejection region t<-1.76 !



Example ③ 【Answer】

We reject H0 under the significance level of 5%.

Thus, “The lifetime of ther light bulb is shorter than 2000 hours.
Their statement should be corrected.”

①Population variance is known? Unknown?

→ If known, apply z-dist., otherwise, t-dist.

②Two-sided? One-sided?

→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

③The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

Example ③ 【Answer】 Using Python

```
import numpy as np
from scipy import stats
from scipy.stats import t

mu_0=2000
avg = 1900
std = 150 #X.std()
N=15 #X.size
#print sample mean.
stat_t = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(stat_t)
p = t.cdf(-np.abs(stat_t), df=N-1)
print(p)
```

Null hypothesis

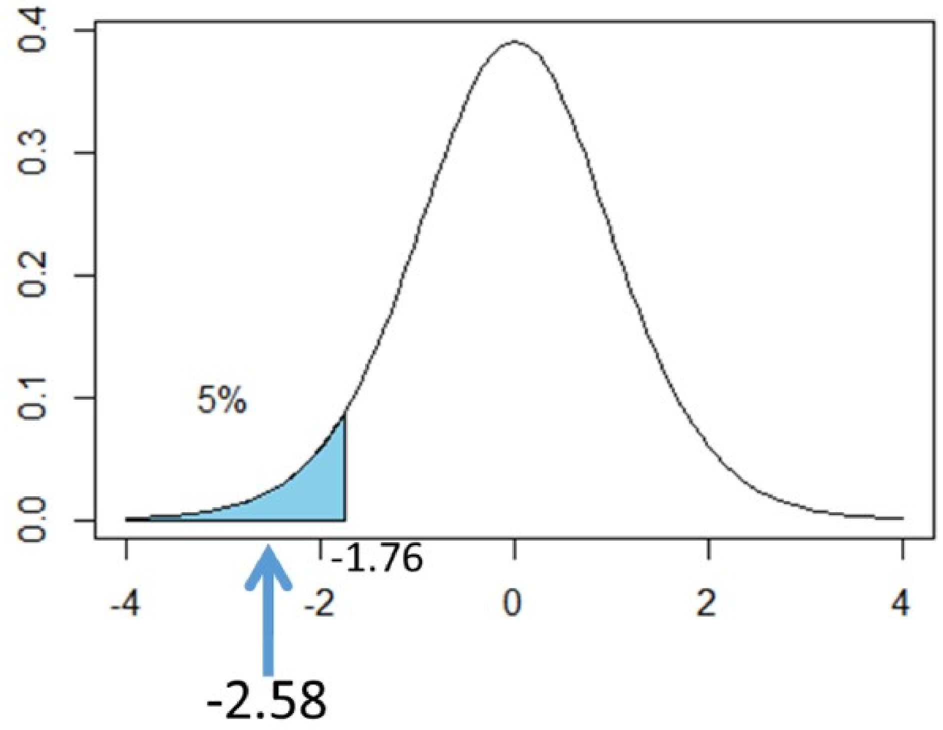
Test statistic

- Find the p-value. One-sided test this case! (No need to make twice).

Example ③ 【Answer】 Using Python

[出力]

```
import numpy as np
from scipy import stats
from scipy.stats import t

mu_0=2000
avg = 1900
std = 150 #X.std()
N=15 #X.size
#print sample mean.
stat_t = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(stat_t)
p = t.cdf(-np.abs(stat_t), df=N-1)
print(p)
```




```
-2.581988897471611
0.010862621599305880
```

P-value < 5%
In the rejection region.

Cautions in one-sided test

- In case of the one-sided test, if the sample mean belongs to the reverse side of the null hypothesis, we can promptly stop test by concluding “we cannot reject H_0”.

- ※In case of the prevous example, if the samples mean is less than 2000, we can promptly terminate the test.

Exercises

Exercise①

In a certain farm, they developed a new fertilizer. Then, after we measured the their yield in 6 farms where they used this new fertilizer, we obtained the following data of the yield per unit area.

42.9 43.7 43.2 40.8 42.8 44.2   [kg]

The average yield with the conventional fertilizer per unit area was 41.4kg. Then, can we say that this new fertilizer improves the yield? Do the hypothesis test with known S.D. $\sigma=3.5$[kg] under the significance level of 5%.

Exercise① 【Answer】

Let us denote the average yield with the new fertilizer as μ.

Null hypothesis H_0:"μ=41.4[kg]"

Alternative hypothesis H_1:" μ>41.4[kg]"

Since the population variance is known, we take:

$$t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Exercise① 【Answer】

We have: $\bar{x} = 42.93$

Thus,

$$t = \frac{42.93 - 41.4}{\frac{3.5}{\sqrt{6}}} = 1.071$$

Does not satisfy t>1.64. It does not belong to the significance level. So we can't reject H_0.

⇒ " We can't say that the new fertilizer has a significant effect"

Exercise① 【Answer】 Using Python

```
#Exercise 4.
import numpy as np
from scipy import stats
from scipy.stats import norm

X = np.array([42.9, 43.7, 43.2, 40.8, 42.8, 44.2])
mu_0=41.4
avg = X.mean()
std = 3.5 #X.std()
N=X.size
#print sample mean.
print(avg)
z = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(z)
p = norm.cdf( -np.abs(z), 0, 1)
print(p)
```

① Population variance is known ? Unknown ?
→ If known, apply z-dist., otherwise, t-dist.

② Two-sided ? One -sided ?
→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

③ The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

Exercise① 【Answer】 Using Python

```
#Exercise 4.
import numpy as np
from scipy import stats
from scipy.stats import norm

X = np.array([42.9, 43.7, 43.2, 40.8, 42.8, 44.2])
mu_0=41.4
avg = X.mean()
std = 3.5 #X.std()
N=X.size
#print sample mean.
print(avg)
z = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(z)
p = norm.cdf( -np.abs(z), 0, 1)
print(p)
```

- Find the p-value. One-sided test this case! (No need to make twice).

Exercise① 【Answer】 Using Python

[Output]

```
#Exercise 4.
import numpy as np
from scipy import stats
from scipy.stats import norm

X = np.array([42.9, 43.7, 43.2, 40.8, 42.8, 44.2])
mu_0=41.4
avg = X.mean()
std = 3.5 #X.std()
N=X.size
#print sample mean.
print(avg)
z = (avg -mu_0)/ (std / np.sqrt(N))
#print z-value.
print(z)
p = norm.cdf( -np.abs(z), 0, 1)
print(p)
```

```
42.933333333333334
1.0731097920764434
0.14161092902072725
```

Exercise②

In a certain maker of a part (named as “M”) of computer, its diameter is described as 1.54[cm] in its product specification. In a certain sample survey, they extracted 8 samples randomly, and observed the following data of measured diameter [cm]:

1.53 1.57 1.54 1.57 1.53 1.55 1.56 1.53

The population variance is unknown. Then, can you say that this part follow its product specification? Test with the significance level of 5%.

Exercise② 【Answer】

Null hypothesis H_0:"μ=1.54[cm]"

Alternative hypothesis H_1:" μ≠1.54[cm]"

Since the population variance is known, we take:

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}}$$

Exercise② 【Answer】

We have $\bar{x} = 1.5475$, and the unbiased S.D. is

$$S^2 = \frac{(1.53-1.5475)^2 + (1.57-1.5475)^2 + \ldots + (1.53-1.5475)^2}{8-1} = 0.000307$$

$$t = \frac{1.5475 - 1.54}{\frac{0.0175}{\sqrt{8}}} = 1.212$$

The upper 2.5-percentile of the t-distribution with df=(8-1=7) is

$$t_7(\frac{0.05}{2}) = 2.365$$

Since |t| < 2.365, it doesn't belong to the rejection region. We cannot reject H0.

"We cannot recognize the different under the significance level of 5%."

Exercise② 【Answer】 Using Python

①Population variance is known？Unknown？
→ If known, apply z-dist., otherwise, t-dist.

②Two-sided？One -sided？
→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

③The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

```
#Exercise 5.
import numpy as np
from scipy import stats
from scipy.stats import t

X = np.array([1.53, 1.57, 1.54,1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=1.54
avg = X.mean()
std = np.std(X, ddof=1)
N=X.size
#print sample mean.
#print(avg)
stats_t = (avg -mu_0)/ (std / np.sqrt(N))
#print stats_t-value.
print(stats_t)
p = t.cdf( -np.abs(stats_t), df=N-1) * 2
print(p)
```

To find test statistic, you should make it twice in case of the two-sided test.

Exercise② 【Answer】 Using Python

[Output]

```
#Exercise 5.
import numpy as np
from scipy import stats
from scipy.stats import t

X = np.array([1.53, 1.57, 1.54,1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=1.54
avg = X.mean()
std = np.std(X, ddof=1)
N=X.size
#print sample mean.
#print(avg)
stats_t = (avg -mu_0)/ (std / np.sqrt(N))
#print stats_t-value.
print(stats_t)
p = t.cdf( -np.abs(stats_t), df=N-1) * 2
print(p)
```

P-value >5%
Cannot reject the null hypothesis.

```
1.2104198771789023
0.2653980394260665
```

Exercise③

A catalog of a climbing shop I states that the breaking strength of their 15mm rope is 4500kg.
Now, after we conducted the sample measurement of its strength by using 50 samples, the mean and unbiased S.D. were 4450[kg] and 120[kg], respectively.

Then, can we state that the stated strength 4500[kg] is satisfied on average? Conduct the hypothesis test with the significance level of 5%.

Exercise③ 【Answer】

Let μ denote its mean.

Null hypothesis H_0:"μ=4500"

Alternative hypothesis H_1:"μ<4500" （No problem in case it's larger.）

Since the population variance is unknown, we take:

$$t = \frac{\bar{x} - \mu}{\frac{S}{\sqrt{N}}}$$

Exercise③ 【Answer】

Sample mean and unbiased S.D. were 4450 and 120, respectively. So,

$$t = \frac{4450 - 4500}{\frac{120}{\sqrt{50}}} = -2.946$$

This follows the t-distribution with df= 50-1=49. But its lower 5-percentile is $-t_{49}(0.05)$ =-1.68.

Now, since t < -1.68, it belongs to the regnificance region, and H0 is rejected.

We can say “The mean of samples are different from the one stated in their catalog under the significance level of 5%.”

Exercise③ 【Answer】 Using Python

①Population variance is known？ Unknown？
→ If known, apply z-dist., otherwise, t-dist.

②Two-sided？ One -sided？
→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

③The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

```
#Exercise 6.
import numpy as np
from scipy import stats
from scipy.stats import t

#X = np.array([1.53, 1.57, 1.54, 1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=4500
avg =4450# X.mean()
std = 120#np.std(X, ddof=1)
N=50 #X.size
stats_t = (avg -mu_0)/ (std / np.sqrt(N))
#print stats_t-value.
print(stats_t)
p = t.cdf( -np.abs(stats_t), df=N-1)
print(p)
```

```
-2.946278254943948
0.0024555744280253798
```

- Find the p-value. One-sided test this case! (No need to make twice).

Exercise③ 【Answer】 Using Python

[Output]

```
#Exercise 6.
import numpy as np
from scipy import stats
from scipy.stats import t

#X = np.array([1.53, 1.57, 1.54, 1.57, 1.53, 1.55, 1.56, 1.53])
mu_0=4500
avg =4450# X.mean()
std = 120#np.std(X, ddof=1)
N=50 #X.size
stats_t = (avg -mu_0)/ (std / np.sqrt(N))
#print stats_t-value.
print(stats_t)
p = t.cdf( -np.abs(stats_t), df=N-1)
print(p)
```

```
-2.946278254943948
0.0024555744280253798
```

P-value<5%
Reject the null hypothesis.

Exercise④

For a certain product, we measured its diameter of 5 samples, and observed:
36.3, 35.7, 35.9, 37.1, 36.1 [mm].

The spec of this product states that it should be 35.5m. Then, check if we can state that the actual diameter of this product is significantly larger than the spec or not.

Assume the normality of population and apply the hypothesis test for mean under the significance level of 5%.

演習④ 【解答】

```
import numpy as np
from scipy import stats
from scipy.stats import t

X=np.array([36.3, 35.7, 35.9, 37.1, 36.1])
mu_0=35.5
X_mean=X.mean()
X_sd=np.std(X,ddof=1)
N=X.size

stats_t = (X_mean -mu_0)/ (X_sd / np.sqrt(N))
p_val = t.cdf( -np.abs(stats_t),df=N-1)

print("p-value is")
print(p_val)

if p_val<0.05:
    print("帰無仮説棄却")
else:
    print("帰無仮説棄却できない")
```

```
p-value is
0.020381913442855257
帰無仮説棄却
```

①Population variance is known？Unknown？

→ If known, apply z-dist., otherwise, t-dist.

②Two-sided？One -sided？

→ The p-value python returns depends on cases.
Is it two-sided p-val.? Or one-sided?

③The row data is given? Just some statistical indicators (sample mean, unbiased SD) are given?

Course plan

Topic 1: Review of 1-2Q
Topic 2: Hypothesis testing(1)~distribution of statistical measures: t-distribution,chi-squared and F-distributions~
Topic 3: Hypothesis testing(2)~difference of averages and ratio of variance~
Topic 4: Hypothesis testing(3)~binomial test, Fisher's exact test, Wilcoxon signed-rank test~
Topic 5: ANalysis of variance, one-way ANOVA (1)~introduction of ANOVA, one-way ANOVA~
Topic 6: ANalysis of variance, one-way ANOVA (2)~test for homogeneity of variance~
Topic 7: ANalysis of variance, one-way ANOVA (3)~N-way ANOVA, multiple comparisons~
Topic 8: Review and exam(1)
Topic 9: Point estimation
Topic 10: (Review) regression analysis
Topic 11: Multiple Regression Analysis (1)~foundation of multiple regression~
Topic 12: Multiple Regression Analysis (2)~significance test, power~
Topic 13: Multiple Regression Analysis (3)~outliers, multicolinearity~
Topic 14: Other types of Regression analysis (Poisson regression, logistic regression)
Topic 15: Comprehensive review and exam(2)
Topic 16: exam(3)