

- **P (A): Probability that event A happen**

- $(A \cup B) = P(A) + P(B) - P(A \cap B)$

- **Expected Value**

## Expected value

- The mean of possible values of a R.V, weighted with the probability of each value. It's denoted as  $E(X)$ .

- $E(x)$

- If we regard the pips of a dice as a R.V., its expected value is

- $E(X) = 1 \cdot (1/6) + \dots + 6 \cdot (1/6) = 3.5$

- Discrete R.V.

- $E(X) = \sum x \cdot f(x)$

- Continuous R.V.

- $E(X) = \int x \cdot f(x) dx$

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45

## Calculation of expected value

- $E(c) = c$

- $E(X+c) = E(X) + c$

- $E(cX) = cE(X)$

- $E(X+Y) = E(X) + E(Y)$  : Addition formula

- Now, let us compare the expected values of the pip of a dice and the mean of the pips of two dices.

- $E(X) = 3.5$

- $E(Y) = E\{(X_1 + X_2)/2\} = \{E(X_1) + E(X_2)\}/2 = 3.5$

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47

- **Variance**

- Variance: the scale of variation of a R.V. around its expected value.

## Variance

- Let us denote the expected value and variance as  $\mu = E(X)$  and  $V(X)$ , respectively.

➤  $V(X) = E\{(X - \mu)^2\}$

- For discrete R.V.s,

➤  $V(X) = \sum (x - \mu)^2 f(x)$

- For continuous R.V.s,

➤  $V(X) = \int (x - \mu)^2 f(x) dx$

The following formula is frequently used.

➤  $V(X) = E(X^2) - \{E(X)\}^2$

(Expected value of  $X^2$ )  
- (squared expected value)

- $V(c) = 0$
- $V(X + c) = V(X)$
- $V(cX) = c^2 V(X)$

## Standard deviation and z-variable

### Standard deviation and z-variable

- Standard deviation is the square root of variance.
- It is denoted as  $D[X]$ .

$$D[X] = \sqrt{V[X]}$$

- Normalization of R.V.

$$Z = \frac{(X - E[X])}{D[X]}$$

- Every R.V. can be transformed to another R.V.  $Z$  that satisfies
- $E[Z] = 0$ ,  $V[Z] = 1$
- This  $Z$  is called as **the normalized R.V.**

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## Calculate Variance (And Standard Deviation)

$$S^2 = \frac{n_A \{S_A^2 + (\bar{x} - \bar{x}_A)^2\} + n_B \{S_B^2 + (\bar{x} - \bar{x}_B)^2\}}{n}.$$

And

$$V[X] = E[X^2] - \bar{x}^2 = \frac{n_A(S_A^2 + \bar{x}_A^2) + n_B(S_B^2 + \bar{x}_B^2)}{n} - \bar{x}^2$$

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