Mục lục

1 Beginner Angles

1.1 Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°

We have:

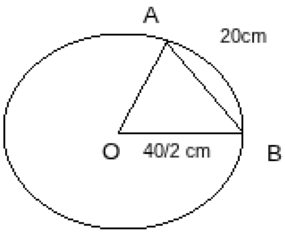
$$\theta = \frac{L}{r}$$
$$\rightarrow L = \theta r = \frac{15\pi 5}{180} = 1.31cm$$

1.2 Find in degrees the angle subtended at the center of a circle of diameter 50 cm by an arc of length 11 cm

We have (use with degree):

$$\theta = \frac{L}{r} = \frac{11}{50/2}.\frac{\pi}{180} = 25^o 12'$$

1.3 In a circle of diameter 40 cm the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord.

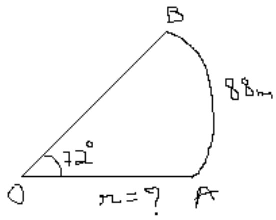


From formular we have

$$l = \frac{\pi r \theta}{180} \rightarrow \theta^o = \frac{l.180}{\pi r}$$
$$\theta = \frac{L}{r} \rightarrow L_{AB} = \frac{l.180}{\pi r}.\frac{\pi}{180}.r = 20cm$$

2 Expert Angles

2.1 A horse is tied to a post by a rope, if the horse moves along a circular path always keeping the rope tight and describe 88 meters when it has traced out 72° at the center, find the length of the rope



we have

$$l = \frac{\pi r \theta}{180} \rightarrow r = \frac{l.180}{\pi \theta^o}$$
$$r = \frac{88 \times 180}{\pi .72} = 70.03m$$

3 Beginner Trigonometry Ratio

3.1 If $\sqrt{3}\tan(\theta) = 3\sin(\theta)$, then find the value of $\sin^2(\theta) - \cos^2(\theta)$

We have

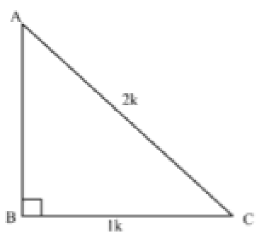
$$\sqrt{3}\tan(\theta) = 3\sin(\theta) \rightarrow \frac{\tan(\theta)}{\sin(\theta)} = \frac{3}{\sqrt{3}}$$
$$\rightarrow \frac{1}{\sin(\theta)}.\frac{\sin(\theta)}{\cos(\theta)} = \frac{3}{\sqrt{3}} \rightarrow \frac{1}{\cos(\theta)} = \frac{3}{\sqrt{3}}$$
$$\rightarrow \theta = \cos^{-1}\frac{\sqrt{3}}{3}$$
$$\rightarrow \sin^2(\theta) - \cos^2(\theta) = \frac{1}{3}$$

4 Expert Trigonometry Ratio

4.1 In triangle ABC, right angled at B if $\sin A = 1/2$, find the value of

(a) $\sin C \cos A - \cos C \sin A$

(b) $\cos A \cos C + \sin A \sin C$



$\sin A = BC/AC = 1/2.SetBC = 1k, AC = 2k.$
So BA $= \sqrt{(2k)^2 - k^2} = \sqrt{3k}$
$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ $\quad \cos C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$
$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$

(a) $\sin C \cos A - \cos C \sin A$

$$\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} - \frac{1}{2}.\frac{1}{2} = \frac{1}{2}$$

(b) $\cos A \cos C + \sin A \sin C$

$$\frac{\sqrt{3}}{2}.\frac{1}{2} + \frac{1}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$$

4.2 If θ is an acute angle and $\frac{\sin\theta+1}{\sin\theta+1} = \frac{\sqrt{3}+2}{\sqrt{3}-2}$

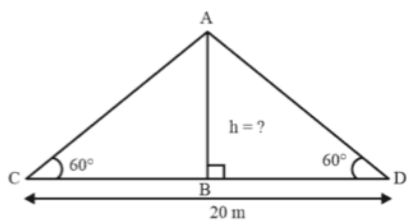
$$\frac{\sin\theta\cos 1 + \cos\theta \sin 1}{\sin\theta\cos 1 - \cos\theta\sin 1} = \frac{\sqrt{3}+2}{\sqrt{3}-2}$$
$$\Big(\sin\theta\cos 1 + \cos\theta\sin 1\Big)\Big(\sqrt{3}-2\Big) = \Big(\sqrt{3}+2\Big)\Big(\sin\theta\cos 1 - \cos\theta\sin 1\Big)$$
$$\sqrt{3}\cos 1\sin\theta + \sqrt{3}\sin 1\cos\theta - 2\cos 1\sin\theta - 2\sin 1\cos\theta = \sqrt{3}\cos 1\sin\theta - \sqrt{3}\sin 1\cos\theta + 2\cos 1\sin\theta - 2\sin 1\cos\theta$$
$$\Big(\sqrt{3}\cos 1 - 2\cos 1\Big)\sin\theta + \Big(\sqrt{3}\sin 1 - 2\sin 1)\cos\theta = \Big(\sqrt{3}\cos 1 + 2\cos 1\Big)\sin\theta - \Big(\sqrt{3}\sin 1 + 2\sin 1\Big)\cos\theta$$
$$\Bigg[\Big(\sqrt{3}\cos 1 - 2\cos 1\Big) - \Big(\sqrt{3}\cos 1 + 2\cos 1\Big)\Bigg]\sin\theta = -\Bigg[\Big(\sqrt{3}\sin 1 + 2\sin 1\Big) + \Big(\sqrt{3}\sin 1 - 2\sin 1\Big)\Bigg]\cos\theta$$

We have: $ax = by \rightarrow \frac{x}{y} = \frac{b}{a}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{-\Big(\sqrt{3}\sin 1 + 2\sin 1\Big) - (\sqrt{3}\sin 1 - 2\sin 1\Big)}{\Big(\sqrt{3}\cos 1 - 2\cos 1\Big) - \Big(\sqrt{3}\cos 1 + 2\cos 1\Big)}$$

$\rightarrow \theta = 53^o 26'$

4.3 Find the height of the triangle



We have

$$\tan(60) = \frac{h}{20/2}$$
$$\rightarrow h = \tan(60) * 10 = 10\sqrt{3}$$

5 Trigonometry Angles

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6 Beginner Trigonometry Formula

6.1 If $\sin(\theta) - \cos(\theta) = 1$, then $\sin(\theta)\cos(\theta)$ equal

$$\frac{d}{h} - \frac{k}{h} = 1; \qquad \frac{d}{h}\frac{k}{h} = \frac{dk}{h^2} = ?$$
$$\frac{d}{h} = 1 + \frac{k}{h} \text{ and } \frac{k}{h} = \frac{d}{h} - 1$$
$$\sin\theta\cos\theta = \Big(1 + \frac{k}{h}\Big)\Big(\frac{d}{h} - 1\Big) = ?$$
$$\frac{d}{h} - 1 + \frac{kd}{h^2} - \frac{k}{h} = ?$$
$$\frac{kd}{h^2} = \sin\theta\cos\theta = \frac{-d}{h} + \frac{k}{h} + 1 = -1 + 1 = 0$$

6.2 If $\sin(\theta) = \mathbf{e^x}$ then $\cos(\theta)$ equal

$$\sin(\theta) = e^x \rightarrow \cos^2(\theta) + e^{2x} = 1$$
$$\cos(\theta) = \sqrt{1 - e^{2x}}$$

7 Expert Trigonometry Formula

7.1 If $\mathbf{x} = \mathbf{sec}(\theta) + \tan(\theta)$, then $\mathbf{x} + \frac{\mathbf{1}}{\mathbf{x}} = ?$

$$\begin{aligned}
x + \frac{1}{x} &= sec(\theta) + \tan(\theta) + \frac{1}{sec(\theta) + \tan(\theta)} \\
&= \frac{sec(\theta)\Big(sec(\theta) + \tan(\theta)\Big) + \tan(\theta)\Big(sec(\theta) + \tan(\theta)\Big) + 1}{sec(\theta) + \tan(\theta)} \\
&= sec^2(\theta) + sec(\theta)\tan(\theta) + \tan^2(\theta) + sec(\theta)\tan(\theta) + 1 \\
&= sec^2(\theta) + sec(\theta)\tan(\theta) + \tan^2(\theta) + sec(\theta)\tan(\theta) + sec^2(\theta) - \tan^2(\theta) \\
&= 2sec^2(\theta) + 2sec(\theta)\tan(\theta) \\
&= \frac{2sec(\theta)\Big(sec(\theta) + \tan(\theta)\Big)}{sec(\theta) + \tan(\theta)} \\
&= 2sec(\theta)
\end{aligned}$$

7.2 $\sqrt{\frac{\mathbf{1}-\sin(\theta)}{\mathbf{1}+\sin(\theta)}}$ equal?

$$\begin{aligned}
\sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} &= \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} \times 1 = \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} \times \sqrt{\frac{1-\sin(\theta)}{1-\sin(\theta)}} \\
&= \frac{\Big[1-\sin(\theta)\Big]^{1/2} \times \Big[1-\sin(\theta)\Big]^{1/2}}{\Big[1+\sin(\theta)\Big]^{1/2} \times \Big[1-\sin(\theta)\Big]^{1/2}} = \frac{1-\sin(\theta)}{\Bigg[\Big(1+\sin(\theta)\Big)\Big(1-\sin(\theta)\Big)\Bigg]^{1/2}} \\
&= \frac{1-\sin(\theta)}{\Big[1-\sin(\theta)+\sin(\theta)-\sin^2(\theta)\Big]} \\
&= \frac{1-\sin(\theta)}{\sqrt{1-\sin^2(\theta)}} = \frac{1-\sin(\theta)}{\sqrt{\cos^2(\theta)}} = \frac{1-\sin(\theta)}{\cos(\theta)} \\
&= \frac{1}{\cos(\theta)} - \tan(\theta) = \sec(\theta) - \tan(\theta)
\end{aligned}$$

7.3 $\frac{\sin(\theta)}{\mathbf{1}-\cot(\theta)} + \frac{\cos(\theta)}{\mathbf{1}-\tan(\theta)}$ equal?

$$\begin{aligned}
\frac{\sin(\theta)}{1-\cot(\theta)} + \frac{\cos(\theta)}{1-\tan(\theta)} &= \frac{\sin(\theta)[1-\tan(\theta)] + \cos(\theta)[1-\cot(\theta)]}{[1-\cot(\theta)][1-\tan(\theta)]} \\
&= \frac{\sin(\theta) - \sin(\theta)\tan(\theta) + \cos(\theta) - cos(\theta)\cot(\theta)}{[1-\cot(\theta)][1-\tan(\theta)]} \\
&= \frac{\sin(\theta)[1-\tan(\theta)] + \cos(\theta)[1-\cot(\theta))]}{[1-\cot(\theta)][1-\tan(\theta)]} \\
&= \sin(\theta) + \cos(\theta)
\end{aligned}$$

8 Differentiation

8.1 $\mathbf{f(x) = 6x^3 - 9x + 4}$

$$f'(x) = 18x^2 - 9$$

8.2 $\mathbf{f(x)} = \sqrt{\mathbf{x}} + \mathbf{8}\sqrt[3]{x} - 2\sqrt[4]{x}$

$$\begin{aligned}
f'(x) &= \frac{1}{2}x^{-1/2} + \frac{8}{3}x^{-2/3} - \frac{1}{2}x^{-3/4} \\
&= \frac{1}{2\sqrt{x}} + \frac{8}{3\sqrt[3]{x^2}} - \frac{1}{2\sqrt[4]{x^3}}
\end{aligned}$$

8.3 $\mathbf{f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3}$

$$\begin{aligned} f'(x) &= 6x^{-2/5} - \frac{7}{2}x^{5/2} + 16x^{5/3} \\ &= \frac{6}{\sqrt{x^5}} - \frac{7\sqrt{x^5}}{2} + 16\sqrt[3]{x^5} \end{aligned}$$

8.4 $\mathbf{g(x) = \frac{4x^3-7x+8}{x}}$

Way 1: $(\frac{u}{v})^2 = \frac{u'.v-v'u}{v^2}$

$$\begin{aligned} &u = 4x^3 - 7x + 8 \quad u' = 12x^2 - 7 \\ &v = x \quad v' = 1 \end{aligned}$$

$$\begin{aligned} g'(x) &= \frac{(12x^2-7)(x) - (4x^3-7x+8)}{x^2} \\ &= \frac{12x^3 - 7x - 4x^3 + 7x - 8}{x^2} \\ &= \frac{8x^3-8}{x^2} = \frac{8x^3}{x^2} - \frac{8}{x^2} = 8x - \frac{8}{x^2} \end{aligned}$$

Way 2:

$$\begin{aligned} g(x) &= 4x^2 - 7 + 8x^{-1} \\ g'(x) &= 8x - \frac{8}{x^2} \end{aligned}$$

8.5 $\mathbf{y = e^{\sqrt{2x+17}}}$

We have:

$$(e^u)' = u'e^u \qquad (\sqrt[n]{u})' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$$

So we have:

$$\begin{aligned} \frac{dx}{dy} &= (\sqrt{2x+17})'e^{\sqrt{2x+17}} = \frac{2}{2\sqrt{2x+17}}e^{\sqrt{2x+17}} \\ &= \frac{e^{\sqrt{2x+17}}}{\sqrt{2x+17}} \end{aligned}$$

8.6 $\mathbf{y = e^x \ln(5x^3 + x^2)}$

We have:

$$(uv)' = u'v + v'u \qquad (e^u)' = u'e^u \qquad (\ln(u))' = \frac{u'}{u}$$

So,

$$\frac{dy}{dx} = e^x \ln(5x^3 + x^2) + \frac{(15x^2+2x)e^x}{5x^3+x^2}$$

8.7 $\mathbf{y = \frac{x^2}{\ln(1-4x^2)}}$

We have:

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{u^2} \qquad (x^n)' = nx^{n-1} \qquad (\ln(u))' = \frac{u'}{u}$$

So,

$$\begin{aligned} \frac{dy}{dx} &= \frac{2x.\ln(1-4x^2) - \frac{-8x.x^2}{(1-4x^2)}}{\ln^2(1-4x^2)} \\ &= \frac{2x(1-4x^2)\ln(1-4x^2) + 8x^3}{(1-4x^2)\ln^2(1-4x^2)} \end{aligned}$$

8.8 $\mathbf{y = \frac{\tan(x)+\cot(x)}{\tan(x)-\cot(x)}}$

We have:

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \qquad (\tan(u))' = \frac{u'}{\cos^2(u)} \qquad (\cot(u))' = \frac{-u'}{\sin^2(u)}$$

So, now we have

$$\begin{aligned} \frac{dy}{dx} &= \frac{\left(\frac{1}{\cos^2(x)} + \frac{-1}{\sin^2(x)}\right)\Big(\tan(x) - \cot(x)\Big) - \left(\frac{1}{\cos^2(x)} - \frac{-1}{\sin^2(x)}\right)\Big(\tan(x) + \cot(x)\Big)}{\Big(\tan(x) - \cot(x)\Big)^2} \\ &= \frac{\Big(\sec^2(x) - cosec^2(x)\Big)\Big(\tan(x) - \cot(x)\Big) - \Big(\sec^2(x) + cosec^2(x)\Big)\Big(\tan(x) + \cot(x)\Big)}{\Big(\tan(x) - \cot(x)\Big)^2} \\ &= \frac{-2\tan(x)cosec^2(x)}{\Big(\tan(x) - \cot(x)\Big)^2} \end{aligned}$$

9 Beginer Implicit Differentiation

9.1 Find $\mathbf{\frac{dy}{dx}}$ for $x^2 + y^3 = 3$

Differentiationg both size and aplly chain rule

$$\begin{aligned} &\frac{dy}{dx}x^2 + \frac{dy}{dx}y^3 = \frac{dy}{dx}3 \\ &\to 2x + \frac{dy}{dx}3y^2 = 0 \\ &\to \frac{dy}{dx} = -\frac{2x}{3y^2} \end{aligned}$$

9.2 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $x^2 + y^2 = 2$

Differentiationg both size and aplly chain rule

$$\begin{aligned}
&(x^2 + y^2 = 2)' \\
&\to 2x + 2y\frac{dy}{dx} = 0 \\
&\to \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}
\end{aligned}$$

9.3 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $2y^3 + 4x^2 - y = x^6$

Differentiationg both size and aplly chain rule

$$\begin{aligned}
&(2y^3 + 4x^2 - y = x^6)' \\
&\to 6y^2\frac{dy}{dx} + 8x - \frac{dy}{dx} = 6x^5 \\
&\to \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}
\end{aligned}$$

9.4 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $7y^2 + \sin(3x) = 12 - y^4$

Differentiationg both size and aplly chain rule

$$\begin{aligned}
&(7y^2 + \sin(3x) = 12 - y^4)' \\
&\to 14y\frac{dy}{dx} + 3\cos(3x) = -4y^3\frac{dy}{dx} \\
&\to \frac{dy}{dx}(14y + 4y^3) = -3\cos(3x) \\
&\to \frac{dy}{dx} = \frac{-3\cos(3x)}{14y + 4y^3}
\end{aligned}$$

9.5 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $xy = 1$

Apply product rule

$$\begin{aligned}
&(xy = 1)' \\
&\to x'y + xy' = 0 \\
&\to \frac{dy}{dx}y = -x \\
&\to \frac{dy}{dx} = -\frac{x}{y}
\end{aligned}$$

9.6 Find $y' = \frac{dy}{dx}$ for each of the follwing

a) $x^3y^5 + 3x = 8y^3 + 1$
b) $x^2\tan(y) + y^{10}\sec(x) = 2x$
c) $e^{2x+3y} = x^2 - \ln(xy^3)$

<**Solution**>
a) $x^3y^5 + 3x = 8y^3 + 1$
We have

$$\begin{aligned}
&(u.v)' = u'v + v'u \\
&\to 3x^2y^5 + 5x^3y^4\frac{dy}{dx} + 3 = 24y^2\frac{dy}{dy} \\
&\leftrightarrow \frac{dy}{dy}(5x^3y^4 - 24y^2) = -3 - 3x^2y^5 \\
&\to \frac{dy}{dx} = \frac{-3 - 3x^2y^5}{5x^3y^4 - 24y^2}
\end{aligned}$$

b) $x^2\tan(y) + y^{10}\sec(x) = 2x$
We have

$$\begin{aligned}
&(uv)' = u'v + v'u \\
&\Big(\tan(u)\Big)' = \frac{u'}{\cos^2(u)} \\
&\Big(\sec(x)\Big)' = \sec(x).\tan(x) \\
\to&\Big(x^2\tan(y)\Big)' = 2x\tan(y) + \frac{dy/dx}{\cos^2(y)}x^2 \\
\to&\Big(y^{10}\sec(x)\Big)' = 10y^9\frac{dy}{dx}\sec(x) + y^{10}\sec(x)\tan(x)
\end{aligned}$$

So,

$$\begin{aligned}
&2x\tan(y)+\frac{dy/dx}{\cos^2(y)}x^2+10y^9\frac{dy}{dx}\sec(x)+y^{10}\sec(x)\tan(x)=0\\
\rightarrow&\frac{dy/dx}{\cos^2(y)}x^2+10y^9\frac{dy}{dx}\sec(x)=-y^{10}\sec(x)\tan(x)-2x\tan(y)\\
\rightarrow&\frac{dy}{dx}=\frac{2-y^{10}\sec(x)\tan(x)-2x\tan(y)}{x^2/\cos^2(y)+10y^9\sec(x)}\\
\rightarrow&\frac{dy}{dx}=\frac{2-2x\tan(y)-y^{10}\sec(x)\tan(x)}{x^2\sec^2(y)+10y^9\sec(x)}
\end{aligned}$$

c) $e^{2x+3y}=x^2-\ln(xy^3)$
We have

$$\begin{aligned}
\Big(e^{2x+3y}\Big)'&=\Big(2+3\frac{dy}{dx}\Big)e^{2x+3y}\\
&=2e^{2x+3y}+3e^{2x+3y}\frac{dy}{dx}\\
\Big(\ln(xy^3)\Big)'&=\frac{1}{xy^3}\Big(y^3+3xy^2\frac{dy}{dx}\Big)\\
&=\frac{y^3}{xy^3}+\frac{3xy^2(dy/dx)}{xy^3}\\
&=\frac{1}{x}+\frac{3}{y}\frac{dy}{dx}
\end{aligned}$$

Now we have

$$\begin{aligned}
2e^{2x+3y}+3e^{2x+3y}\frac{dy}{dx}&=2x-\frac{1}{x}-\frac{3}{y}\frac{dy}{dx}\\
\frac{dy}{dx}\Big(3e^{2x+3y}-3y\Big)&=2x-\frac{1}{x}-2e^{2x+3y}\\
\rightarrow\frac{dx}{dy}&=\frac{2x-1/x-2e^{2x+3y}}{3e^{2x+3y}-3/y}
\end{aligned}$$

10 Expert Implicit Differentiation

10.1 If $x=exp\left[\tan^{-1}\left(\frac{y-x^2}{x^2}\right)\right]$ then $\frac{dy}{dx}$ equals

11 Beginner Application of Differentiation

11.1 The radius of a circle is increasing at the rate of 0.7 (cm/s). What is the rate of its circumderence (chu vi)?

We have

$$\begin{aligned}
C&=2\pi R\\
\rightarrow\frac{dC}{dR}&=2\pi R'=2\pi\times 0.7\\
\rightarrow\frac{dC}{dR}&=rate=1.4\pi\ \text{(cm/s)}
\end{aligned}$$

11.2 The sides of an equilateral triangle are changing length at the rate of 0.2 cm/s. At what rate is the area changing when the sides are 4 cm?

We have

$$\begin{aligned}
S&=\frac{\sqrt{3}}{4}x^2\\
\rightarrow\frac{dS}{dx}&=\frac{\sqrt{3}}{4}.2x.\frac{dx}{dt}=\frac{\sqrt{3}}{4}.2x.(0.2)
\end{aligned}$$

when the sides changing 4 cm

$$\rightarrow\frac{dS}{dx}=\frac{\sqrt{3}}{4}.2.4.(0.2)=\frac{2\sqrt{3}}{5}$$

11.3 What is the rate of change of the surface area of a shpere when the radius of the sphere is 3 cm and the radius is increasing at 6 cm/s?

With surface area of shpere, we have:

$$
\begin{aligned}
S &= 4\pi R^2 \\
\rightarrow \frac{dS}{dR} &= 4\pi.2.R.\frac{dR}{dt} \\
&= 4\pi.2.3.6 \\
&= 144\pi
\end{aligned}
$$

11.4 A metal ring is being heated so that at any instant of time t in second, its area is given by $A = 3t^2 + \frac{t}{3} + 2$. What will be the rate of increae of area at $t = 10(s)$

We have:

$$
\frac{dA}{dt} = 6t + \frac{1}{3}
$$

When t = 10s:

$$
\rightarrow \frac{dA}{dt} = 60 + \frac{1}{3} = \frac{181}{3}
$$

11.5 Find the slope of the tanget line to $g(x) = \frac{16}{x} - 4\sqrt{x}$ at x = 4

We have

$$
\begin{aligned}
\text{the slope} = g'(x) &= \left(16x^{-1} - 4x^{1/2}\right) \\
&= -\frac{16}{x^2} - \frac{2}{\sqrt{x}}
\end{aligned}
$$

When x = 4

$$
\rightarrow \text{the slope} = -\frac{16}{4^2} - \frac{2}{\sqrt{4}} = -2
$$

11.6 If the surface area of a spherical balloon is increasing at the rate of 900 cm^2/sec. Then the rate of change of radius of balloon at instant when radius is 15 cm [in cm/sec]

Aplly the formula surface area of a spherical.

$$
\begin{aligned}
S &= 4\pi R^2 \\
\frac{dS}{dt} &= 900 = 4\pi.2R\frac{dR}{dt}
\end{aligned}
$$

With R = 15 cm

$$
\rightarrow \frac{dR}{dt} = \frac{900}{8\pi.15} = 2.39 \text{ (cm/s)}
$$

12 Expert Application of Differentiation

12.1 The volume of a sphere is given by $V = \frac{4}{3}\pi R^3$. Where R is the radius of the sphere. The rate of change of volume with respect to R and the change in volume of the sphere as the radius is inscread from 20.0 cm to 20.1 cm respectively are (Assume that the rate does not appreciably change between R = 20.0 cm to R = 20.1 cm)

We have

$$
\begin{aligned}
V &= \frac{4}{3}\pi R^3 \\
\rightarrow \frac{dV}{dR} &= 4\pi R^2 \frac{dR}{dt}
\end{aligned}
$$

with R = 20 cm, we have

$$
\rightarrow \frac{dV}{dR} = 4\pi(20^2) = 1600\pi(cm^2)
$$

12.2 At the point (1, b) on the curve $y = 2x^3$, the gradient of the curve is increasing K times as fast as x. Then K = ?

We have

$$
\begin{aligned}
\frac{dy}{dx} &= 6x^2 \\
\rightarrow K &= 6
\end{aligned}
$$

13 Level 1 Indefinite Integration

13.1 Evaluate the indefinate integral $\int (6x^5 - 18x^2 + 7)dx$

We have

$$
\begin{aligned}
\int (6x^5 - 18x^2 + 7)dx &= \frac{6}{6}x^6 - \frac{18}{3}x^3 + 7x + c \\
&= x^6 - 6x^3 + 7x + c
\end{aligned}
$$

13.2 Evaluate the indefinite integral $\int(\mathbf{40x^3+12x^2-9x+14)dx}$

We have

$$\int(40x^3+12x^2-9x+14)dx$$
$$=10x^4+4x^3-4.5x^2+14x+c$$

13.3 Evaluate the indefinate integral $\int(\mathbf{12t^7-t^2-t+3)dt}$

We have

$$\int(12t^7-t^2-t+3)dt$$
$$=\frac{12}{8}t^8-\frac{t^3}{3}+\frac{t^2}{2}+3t+c$$

13.4 $\int(\mathbf{10w^4+9w^3+7w)dw}$

We have

$$\int(10w^4+9w^3+7w)dw$$
$$=2w^5+\frac{9}{4}w^4+\frac{7}{2}w^2+c$$

13.5 Find the indefinate integral of a function: (Use the basic indefinate integral formulas and rules)

a) $\int(3x^2-6x+3)dx$

$$\int(3x^2-6x+3)dx=x^3-3x^2+3x+c$$

b) $\int(8x^3-x^2+5x-1)dx$

$$\int(8x^3-x^2+5x-1)dx=2x^4-\frac{x^3}{3}+\frac{5}{2}x^2-x+c$$

c) $\int\left(x^5+\frac{1}{4}x^4+\frac{1}{3}x^3\right)dx$

$$\int\left(x^5+\frac{1}{4}x^4+\frac{1}{3}x^3\right)dx=\frac{x^6}{6}+\frac{x^5}{20}+\frac{x^4}{12}+c$$

d) $\int\left(-\frac{x^4}{2}-\frac{x^3}{3}-\frac{x^2}{6}\right)$

$$\int\left(-\frac{x^4}{2}-\frac{x^3}{3}-\frac{x^2}{6}\right)=-\frac{x^5}{10}-\frac{x^4}{12}-\frac{x^3}{18}+c$$

e) $\int(2x-6)^3dx$

$$(2x-6)^3=8x^3-6^3-36x(2x-6)$$
$$=8x^3-72x^2+216x-216$$

So, we have

$$\int(8x^3-72x^2+216x-216)dx$$
$$=2x^4-24x^3+108x^2-216x+c$$

f) $\int\left[(\sqrt{x}-5)-x\right]^2dx$

$$(\sqrt{x}-5)^2=x+25-10\sqrt{x}$$
$$\to(\sqrt{x}-5)^2-x=25-10\sqrt{x}$$
$$\to(25-10\sqrt{x})^2=625+100x-500\sqrt{x}$$

So, we have

$$\int(100x-500x^{1/2}+625)dx$$
$$=50x^2-\frac{1000}{3}x^{3/2}+625x+c$$

g) $\int(x^{10}-x^8+x^6-x^4)dx$

$$=\frac{x^{11}}{11}-\frac{x^9}{9}+\frac{x^7}{7}-\frac{x^5}{5}+c$$

h) $\int(x^{-5}+x^{-3}+x^{-1})dx$

$$=4x^{-4}-2x^{-2}+\ln(x)+c$$

i) $\int(\frac{16}{x^5}-\frac{9}{x^4}+\frac{4}{x^3})dx$

$$=-4x^{-4}+3x^{-3}-2x^{-2}+c$$

13.6 $\int\frac{1+\cos^2x}{\sin^2x}dx$

$$\frac{1+\cos^2x}{\sin^2x}=\frac{1}{\sin^2x}+\frac{\cos^2x}{\sin^2x}$$
$$=cosec^2x+\cot^2x$$

So, we have

$$\to\int(cosec^2x+\cot^2x)dx$$
$$=\int(2cosec^2x-cosec^2x+\cot^2x)dx$$
$$=\int(2cosec^2x-1)dx$$
$$=-2\cot x-x+c$$