Mục lục

1 Angles

1.1 Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°

We have:

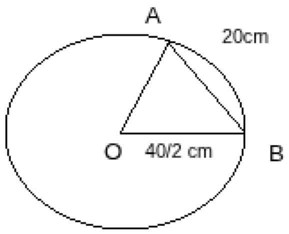
$$\theta = \frac{L}{r}$$
$$\rightarrow L = \theta r = \frac{15\pi 5}{180} = 1.31cm$$

1.2 Find in degrees the angle subtended at the center of a circle of diameter 50 cm by an arc of length 11 cm

We have (use with degree):

$$\theta = \frac{L}{r} = \frac{11}{50/2}.\frac{\pi}{180} = 25^o12'$$

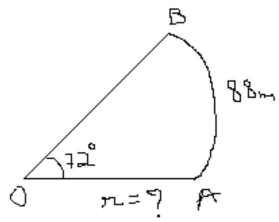
1.3 In a circle of diameter 40 cm the length of a chord is 20 cm. Find the length of minor arc corresponding to the chord.



From formular we have

$$l = \frac{\pi r \theta}{180} \rightarrow \theta^o = \frac{l.180}{\pi r}$$
$$\theta = \frac{L}{r} \rightarrow L_{AB} = \frac{l.180}{\pi r}.\frac{\pi}{180}.r = 20cm$$

1.4 A horse is tied to a post by a rope, if the horse moves along a circular path always keeping the rope tight and describe 88 meters when it has traced out 72° at the center, find the length of the rope



we have

$$l = \frac{\pi r \theta}{180} \rightarrow r = \frac{l.180}{\pi \theta^o}$$
$$r = \frac{88 \times 180}{\pi.72} = 70.03m$$

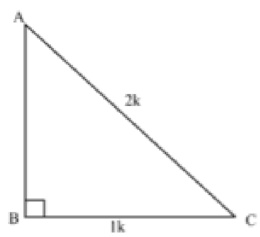
2 Trigonometry Ratio

2.1 If $\sqrt{3}\tan(\theta) = 3\sin(\theta)$, then find the value of $\sin^2(\theta) - \cos^2(\theta)$

We have

$$\sqrt{3}\tan(\theta) = 3\sin(\theta) \rightarrow \frac{\tan(\theta)}{\sin(\theta)} = \frac{3}{\sqrt{3}}$$
$$\rightarrow \frac{1}{\sin(\theta)}.\frac{\sin(\theta)}{\cos(\theta)} = \frac{3}{\sqrt{3}} \rightarrow \frac{1}{\cos(\theta)} = \frac{3}{\sqrt{3}}$$
$$\rightarrow \theta = \cos^{-1}\frac{\sqrt{3}}{3}$$
$$\rightarrow \sin^2(\theta) - \cos^2(\theta) = \frac{1}{3}$$

2.2 In triangle ABC, right angled at B if $\sin A = 1/2$, find the value of
(a) $\sin C \cos A - \cos C \sin A$
(b) $\cos A \cos C + \sin A \sin C$



$\sin A = BC/AC = 1/2. Set BC = 1k, AC = 2k.$
So BA $= \sqrt{(2k)^2 - k^2} = \sqrt{3k}$
$\sin C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ $\cos C = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$
$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$
$(a) \sin C \cos A - \cos C \sin A$

$$\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}-\frac{1}{2}.\frac{1}{2}=\frac{1}{2}$$

(b) $\cos A\cos C+\sin A\sin C$

$$\frac{\sqrt{3}}{2}.\frac{1}{2}+\frac{1}{2}.\frac{\sqrt{3}}{2}=\frac{\sqrt{3}}{2}$$

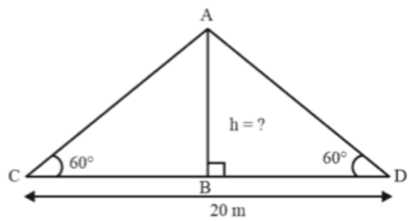
2.3 If θ is an acute angle and $\frac{\sin\theta+1}{\sin\theta+1}=\frac{\sqrt{3}+2}{\sqrt{3}-2}$

$$\frac{\sin\theta\cos 1+\cos\theta\sin 1}{\sin\theta\cos 1-\cos\theta\sin 1}=\frac{\sqrt{3}+2}{\sqrt{3}-2}$$
$$\Big(\sin\theta\cos 1+\cos\theta\sin 1\Big)\Big(\sqrt{3}-2\Big)=\Big(\sqrt{3}+2\Big)\Big(\sin\theta\cos 1-\cos\theta\sin 1\Big)$$
$$\sqrt{3}\cos 1\sin\theta+\sqrt{3}\sin 1\cos\theta-2\cos 1\sin\theta-2\sin 1\cos\theta=\sqrt{3}\cos 1\sin\theta-\sqrt{3}\sin 1\cos\theta+2\cos 1\sin\theta-2\sin 1\cos\theta$$
$$\Big(\sqrt{3}\cos 1-2\cos 1\Big)\sin\theta+\Big(\sqrt{3}\sin 1-2\sin 1)\cos\theta=\Big(\sqrt{3}\cos 1+2\cos 1\Big)\sin\theta-\Big(\sqrt{3}\sin 1+2\sin 1\Big)\cos\theta$$
$$\Bigg[\Big(\sqrt{3}\cos 1-2\cos 1\Big)-\Big(\sqrt{3}\cos 1+2\cos 1\Big)\Bigg]\sin\theta=-\Bigg[\Big(\sqrt{3}\sin 1+2\sin 1\Big)+\Big(\sqrt{3}\sin 1-2\sin 1\Big)\Bigg]\cos\theta$$

We have: $ax=by\rightarrow\frac{x}{y}=\frac{b}{a}$

$$\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{-\Big(\sqrt{3}\sin 1+2\sin 1\Big)-(\sqrt{3}\sin 1-2\sin 1\Big)}{\Big(\sqrt{3}\cos 1-2\cos 1\Big)-\Big(\sqrt{3}\cos 1+2\cos 1\Big)}$$
$$\rightarrow\theta=53^{o}26'$$

2.4 Find the height of the triangle



We have

$$\tan(60)=\frac{h}{20/2}$$
$$\rightarrow h=\tan(60)*10=10\sqrt{3}$$

3 Trigonometry Angles

3.1 If $\sin(\theta)-\cos(\theta)=1$, then $\sin(\theta)\cos(\theta)$ equal

$$\frac{d}{h}-\frac{k}{h}=1;\qquad\frac{d}{h}\frac{k}{h}=\frac{dk}{h^2}=?$$
$$\frac{d}{h}=1+\frac{k}{h}\text{ and }\frac{k}{h}=\frac{d}{h}-1$$
$$\sin\theta\cos\theta=\Big(1+\frac{k}{h}\Big)\Big(\frac{d}{h}-1\Big)=?$$
$$\frac{d}{h}-1+\frac{kd}{h^2}-\frac{k}{h}=?$$
$$\frac{kd}{h^2}=\sin\theta\cos\theta=\frac{-d}{h}+\frac{k}{h}+1=-1+1=0$$

3.2 If $\sin(\theta)=\mathbf{e^x}$ then $\cos(\theta)$ equal

$$\sin(\theta)=e^x\rightarrow\cos^2(\theta)+e^{2x}=1$$
$$\cos(\theta)=\sqrt{1-e^{2x}}$$

4 Trigonometry Formula

4.1 If $\mathbf{x} = \mathbf{sec}(\theta) + \tan(\theta)$, then $\mathbf{x} + \frac{\mathbf{1}}{\mathbf{x}} = ?$

$$
\begin{aligned}
x + \frac{1}{x} &= sec(\theta) + \tan(\theta) + \frac{1}{sec(\theta) + \tan(\theta)} \\
&= \frac{sec(\theta)\Big(sec(\theta) + \tan(\theta)\Big) + \tan(\theta)\Big(sec(\theta) + \tan(\theta)\Big) + 1}{sec(\theta) + \tan(\theta)} \\
&= sec^2(\theta) + sec(\theta)\tan(\theta) + \tan^2(\theta) + sec(\theta)\tan(\theta) + 1 \\
&= sec^2(\theta) + sec(\theta)\tan(\theta) + \tan^2(\theta) + sec(\theta)\tan(\theta) + sec^2(\theta) - \tan^2(\theta) \\
&= 2sec^2(\theta) + 2sec(\theta)\tan(\theta) \\
&= \frac{2sec(\theta)\Big(sec(\theta) + \tan(\theta)\Big)}{sec(\theta) + \tan(\theta)} \\
&= 2sec(\theta)
\end{aligned}
$$

4.2 $\sqrt{\frac{\mathbf{1}-\sin(\theta)}{\mathbf{1}+\sin(\theta)}}$ equal?

$$
\begin{aligned}
\sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} &= \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} \times 1 = \sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} \times \sqrt{\frac{1-\sin(\theta)}{1-\sin(\theta)}} \\
&= \frac{\Big[1-\sin(\theta)\Big]^{1/2} \times \Big[1-\sin(\theta)\Big]^{1/2}}{\Big[1+\sin(\theta)\Big]^{1/2} \times \Big[1-\sin(\theta)\Big]^{1/2}} = \frac{1-\sin(\theta)}{\Bigg[\Big(1+\sin(\theta)\Big)\Big(1-\sin(\theta)\Big)\Bigg]^{1/2}} \\
&= \frac{1-\sin(\theta)}{\Big[1-\sin(\theta)+\sin(\theta)-\sin^2(\theta)\Big]} \\
&= \frac{1-\sin(\theta)}{\sqrt{1-\sin^2(\theta)}} = \frac{1-\sin(\theta)}{\sqrt{\cos^2(\theta)}} = \frac{1-\sin(\theta)}{\cos(\theta)} \\
&= \frac{1}{\cos(\theta)} - \tan(\theta) = \sec(\theta) - \tan(\theta)
\end{aligned}
$$

4.3 $\frac{\sin(\theta)}{\mathbf{1}-\cot(\theta)} + \frac{\cos(\theta)}{\mathbf{1}-\tan(\theta)}$ equal?

$$
\begin{aligned}
\frac{\sin(\theta)}{1-\cot(\theta)} + \frac{\cos(\theta)}{1-\tan(\theta)} &= \frac{\sin(\theta)[1-\tan(\theta)] + \cos(\theta)[1-\cot(\theta)]}{[1-\cot(\theta)][1-\tan(\theta)]} \\
&= \frac{\sin(\theta) - \sin(\theta)\tan(\theta) + \cos(\theta) - cos(\theta)\cot(\theta)}{[1-\cot(\theta)][1-\tan(\theta)]} \\
&= \frac{\sin(\theta)[1-\tan(\theta)] + \cos(\theta)[1-\cot(\theta))]}{[1-\cot(\theta)][1-\tan(\theta)]} \\
&= \sin(\theta) + \cos(\theta)
\end{aligned}
$$

5 Differentiation

5.1 $\mathbf{f(x) = 6x^3 - 9x + 4}$

$$f'(x) = 18x^2 - 9$$

5.2 $\mathbf{f(x)} = \sqrt{\mathbf{x}} + \mathbf{8}\sqrt[3]{x} - 2\sqrt[4]{x}$

$$
\begin{aligned}
f'(x) &= \frac{1}{2}x^{-1/2} + \frac{8}{3}x^{-2/3} - \frac{1}{2}x^{-3/4} \\
&= \frac{1}{2\sqrt{x}} + \frac{8}{3\sqrt[3]{x^2}} - \frac{1}{2\sqrt[4]{x^3}}
\end{aligned}
$$

5.3 $\mathbf{f(x) = 10\sqrt[5]{x^3} - \sqrt{x^7} + 6\sqrt[3]{x^8} - 3}$

$$\begin{aligned} f'(x) &= 6x^{-2/5} - \frac{7}{2}x^{5/2} + 16x^{5/3} \\ &= \frac{6}{\sqrt{x^5}} - \frac{7\sqrt{x^5}}{2} + 16\sqrt[3]{x^5} \end{aligned}$$

5.4 $\mathbf{g(x) = \frac{4x^3-7x+8}{x}}$

Way 1: $(\frac{u}{v})^2 = \frac{u'.v-v'u}{v^2}$

$$\begin{aligned} &u = 4x^3 - 7x + 8 \quad u' = 12x^2 - 7 \\ &v = x \quad v' = 1 \end{aligned}$$

$$\begin{aligned} g'(x) &= \frac{(12x^2-7)(x) - (4x^3 - 7x + 8)}{x^2} \\ &= \frac{12x^3 - 7x - 4x^3 + 7x - 8}{x^2} \\ &= \frac{8x^3 - 8}{x^2} = \frac{8x^3}{x^2} - \frac{8}{x^2} = 8x - \frac{8}{x^2} \end{aligned}$$

Way 2:

$$\begin{aligned} g(x) &= 4x^2 - 7 + 8x^{-1} \\ g'(x) &= 8x - \frac{8}{x^2} \end{aligned}$$

5.5 $\mathbf{y = e^{\sqrt{2x+17}}}$

We have:

$$(e^u)' = u'e^u \qquad (\sqrt[n]{u})' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$$

So we have:

$$\begin{aligned} \frac{dx}{dy} &= (\sqrt{2x+17})'e^{\sqrt{2x+17}} = \frac{2}{2\sqrt{2x+17}}e^{\sqrt{2x+17}} \\ &= \frac{e^{\sqrt{2x+17}}}{\sqrt{2x+17}} \end{aligned}$$

5.6 $\mathbf{y = e^x \ln(5x^3 + x^2)}$

We have:

$$(uv)' = u'v + v'u \qquad (e^u)' = u'e^u \qquad (\ln(u))' = \frac{u'}{u}$$

So,

$$\frac{dy}{dx} = e^x \ln(5x^3 + x^2) + \frac{(15x^2 + 2x)e^x}{5x^3 + x^2}$$

5.7 $\mathbf{y = \frac{x^2}{\ln(1-4x^2)}}$

We have:

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{u^2} \qquad (x^n)' = nx^{n-1} \qquad (\ln(u))' = \frac{u'}{u}$$

So,

$$\begin{aligned} \frac{dy}{dx} &= \frac{2x.\ln(1-4x^2) - \frac{-8x.x^2}{(1-4x^2)}}{\ln^2(1-4x^2)} \\ &= \frac{2x(1-4x^2)\ln(1-4x^2) + 8x^3}{(1-4x^2)\ln^2(1-4x^2)} \end{aligned}$$

5.8 $\mathbf{y = \frac{\tan(x)+\cot(x)}{\tan(x)-\cot(x)}}$

We have:

$$\left(\frac{u}{v}\right)' = \frac{u'v - v'u}{v^2} \qquad (\tan(u))' = \frac{u'}{\cos^2(u)} \qquad (\cot(u))' = \frac{-u'}{\sin^2(u)}$$

So, now we have

$$\begin{aligned} \frac{dy}{dx} &= \frac{\Big(\frac{1}{\cos^2(x)} + \frac{-1}{\sin^2(x)}\Big)\Big(\tan(x) - \cot(x)\Big) - \Big(\frac{1}{\cos^2(x)} - \frac{-1}{\sin^2(x)}\Big)\Big(\tan(x) + \cot(x)\Big)}{\Big(\tan(x) - \cot(x)\Big)^2} \\ &= \frac{\Big(\sec^2(x) - cosec^2(x)\Big)\Big(\tan(x) - \cot(x)\Big) - \Big(\sec^2(x) + cosec^2(x)\Big)\Big(\tan(x) + \cot(x)\Big)}{\Big(\tan(x) - \cot(x)\Big)^2} \\ &= \frac{-2\tan(x)cosec^2(x)}{\Big(\tan(x) - \cot(x)\Big)^2} \end{aligned}$$

5.9 Find $\mathbf{\frac{dy}{dx}}$ for $x^2 + y^3 = 3$

Differentiationg both size and aplly chain rule

$$\begin{aligned} &\frac{dy}{dx}x^2 + \frac{dy}{dx}y^3 = \frac{dy}{dx}3 \\ &\to 2x + \frac{dy}{dx}3y^2 = 0 \\ &\to \frac{dy}{dx} = -\frac{2x}{3y^2} \end{aligned}$$

5.10 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $x^2 + y^2 = 2$

Differentiationg both size and aplly chain rule

$$\begin{aligned}
&(x^2 + y^2 = 2)' \\
&\to 2x + 2y\frac{dy}{dx} = 0 \\
&\to \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}
\end{aligned}$$

5.11 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $2y^3 + 4x^2 - y = x^6$

Differentiationg both size and aplly chain rule

$$\begin{aligned}
&(2y^3 + 4x^2 - y = x^6)' \\
&\to 6y^2\frac{dy}{dx} + 8x - \frac{dy}{dx} = 6x^5 \\
&\to \frac{dy}{dx} = \frac{-2x}{2y} = -\frac{x}{y}
\end{aligned}$$

5.12 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $7y^2 + \sin(3x) = 12 - y^4$

Differentiationg both size and aplly chain rule

$$\begin{aligned}
&(7y^2 + \sin(3x) = 12 - y^4)' \\
&\to 14y\frac{dy}{dx} + 3\cos(3x) = -4y^3\frac{dy}{dx} \\
&\to \frac{dy}{dx}(14y + 4y^3) = -3\cos(3x) \\
&\to \frac{dy}{dx} = \frac{-3\cos(3x)}{14y + 4y^3}
\end{aligned}$$

5.13 Find $\frac{\mathbf{dy}}{\mathbf{dx}}$ for $xy = 1$

Apply product rule

$$\begin{aligned}
&(xy = 1)' \\
&\to x'y + xy' = 0 \\
&\to \frac{dy}{dx}y = -x \\
&\to \frac{dy}{dx} = -\frac{x}{y}
\end{aligned}$$

5.14 Find y' for each of the follwing

a) $x^3y^5 + 3x = 8y^3 + 1$
b) $x^2\tan(y) + y^{10}\sec(x) = 2x$
c) $e^{2x+3y} = x^2 - \ln(xy^3)$

<**Solution**>
a) $x^3y^5 + 3x = 8y^3 + 1$
We have

$$\begin{aligned}
&(u.v)' = u'v + v'u \\
&\to 3x^2y^5 + 5x^3y^4\frac{dx}{dy} + 3 = 24y^2\frac{dx}{dy} \\
&\leftrightarrow \frac{dx}{dy}(5x^3y^4 - 24y^2) = -3 - 3x^2 + y^5 \\
&\to \frac{dx}{dy} = \frac{-3 - 3x^2 + y^5}{5x^3y^4 - 24y^2}
\end{aligned}$$