Basic Math for Physics

Mar/22/2025

Mục lục

1 Quadratic Equation - Phương trình bậc 2

Với phương trình $ax^2 + bx + c = 0$ $(a \neq 0)$.
Ta có $\Delta = b^2 - 4ac$.
TH1. $\Delta < 0$ phương trình vô nghiệm
TH2. $\Delta = 0$ phương trình có nghiệm kép $x_1 = x_2 = -\frac{b}{2ac}$
TH3. $\Delta > 0$ phương trình có hai nghiệm phân biệt $x_{1;2} = \frac{-b \pm \sqrt{\Delta}}{2a}$

2 Common Formulae of Algebra - Hằng đẳng thức đáng nhớ

(i) $(a+b)^2 = a^2 + b^2 + 2ab$

(ii) $(a-b)^2 = a^2 + b^2 - 2ab$

(iii) $(a+b)(a-b) = a^2 - b^2$

(iv) $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

(v) $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

(vi) $(a+b)^2 - (a-b)^2 = 4ab$

(vii) $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

(viii) $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

(ix) $a^3 + b^3 = (a+b)(a^2 + b^2 - ab)$

(x) $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(xi) $(a-b-c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$

(xii) $(a+b-c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

(xiii) $(a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$

3 Arithmetic Progression - Tiến trình số học

(i) n^{th} term of arithmetic progression

$$a_n = a_0 + (n-1)d$$

a_0 = First term,
n = Number of terms,
d = Common difference
$= (a_1 - a_0)$ or $(a_2 - a_1)$ or $(a_3 - a_2)$

(ii) Sum of arithmetic progression

$$S_n = \frac{n}{2}[2a_0 + (n-1)d] = \frac{n}{2}[a_0 + a_n]$$

a_n = last term

4 Geometric Progression - Tiến trình hình học

Here a = first term, r = common ratio (tỷ lệ chung)

(i) Sum of $'n'$ terms of G.P

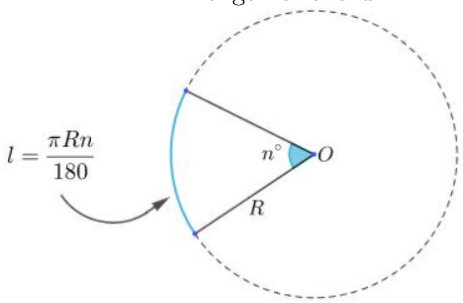
$$S_n = \frac{a(1-r^n)}{1-r} \quad \text{if} \quad r < 1$$
$$S_n = \frac{a(r^n-1)}{r-1} \quad \text{if} \quad r > 1$$

(ii) Sum of infinite terms of G.P

$$S_\infty = \frac{a}{1-r} \quad \text{if} \quad r < 1$$
$$S_\infty = \frac{a}{r-1} \quad \text{if} \quad r > 1$$

5 Trigonometry Angle

Hình 1: Length of chord



$$Angle(\theta) = \frac{ArcLength}{Radius} = \frac{L}{r}$$

(formula true for radian only)

6 Trigonometry Ratio

$$sin(\theta) = \frac{Perpendicular}{Hypotenuse} = \frac{p}{h} = \text{đi học}$$
$$cos(\theta) = \frac{Base}{Hypotenuse} = \frac{b}{h} = \text{không hư}$$
$$tan(\theta) = \frac{sin(\theta)}{cos(\theta)} = \frac{Perpendicular}{Base} = \frac{p}{b} = \text{đoàn kết}$$
$$cosec(\theta) = \frac{1}{sin(\theta)}; \qquad sec(\theta) = \frac{1}{cos(\theta)}; \qquad cot(\theta) = \frac{1}{tan(\theta)} = \frac{\cos(x)}{\sin(x)}$$

7 Trigonometry values of Larger Angles

$$sin(180^o - \theta) = sin(\theta) \qquad sin(360^o - \theta) = -sin(\theta)$$
$$cos(180^o - \theta) = -cos(\theta) \qquad cos(360^o - \theta) = cos(\theta)$$
$$tan(180^o - \theta) = -tan(\theta) \qquad tan(360^o - \theta) = -tan(\theta)$$
$$sin(180^o + \theta) = -sin(\theta) \qquad sin(360n^o + \theta) = sin(\theta)$$
$$cos(180^o + \theta) = -cos(\theta) \qquad cos(360n^o + \theta) = cos(\theta)$$
$$tan(180^o + \theta) = tan(\theta) \qquad tan(360n^o + \theta) = tan(\theta)$$

8 Trigonometrical Indentities

(i) $sin^2(\theta) + cos^2(\theta) = 1$

(ii) $sec^2(\theta) - tan^2(\theta) = 1$

(iii) $cosec^2(\theta) + cot^2(\theta) = 1$

9 Trigonometry Formulae

$$sin(A+B) = sin(A)cos(B) + cos(A)sin(B)$$
$$sin(A-B) = sin(A)cos(B) - cos(A)sin(B)$$
$$cos(A+B) = cos(A)cos(B) - sin(A)sin(B)$$
$$cos(A-B) = cos(A)cos(B) + sin(A)sin(B)$$
$$tan(A+B) = \frac{tan(A)+tan(B)}{1-tan(A)tan(B)}$$
$$tan(A-B) = \frac{tan(A)-tan(B)}{1+tan(A)tan(B)}$$
$$sin(2\theta) = 2sin(\theta)cos(\theta)$$
$$cos(2\theta) = cos^2(\theta) - sin^2(\theta) = 2cos^2(\theta-1) = 1 - 2sin^2(\theta)$$
$$tan(2\theta) = \frac{2tan(\theta)}{1-tan^2(\theta)}$$

10 Logarithm

If $a^x = N$ then $log_a(N) = x$
Conversion of natural log into common log:

$$log_e(x) = ln(x) = 2.3026 log_{10}(x)$$

11 Important formulae of logarithm

(i) $log_a(m.n) = log_a(m) + log_a(n)$

(ii) $log_a(m/n) = log_a(m) - log_a(n)$

(iii) $log_a(m^n) = nlog_a(m)$

12 Differential Calculus

$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$	$y = sin(x)$	$\frac{dy}{dx} = cos(x)$
$y = kx$	$\frac{dy}{dx} = k\frac{dx}{dx} = k$	$y = cos(x)$	$\frac{dy}{dx} = -sin(x)$
$y = k$	$\frac{dy}{dx} = 0$	$y = tan(x)$	$\frac{dy}{dx} = sec^2(x)$
$y = \frac{1}{x}$	$\frac{dy}{dx} = -\frac{1}{x^2}$	$y = coses(x)$	$\frac{dy}{dx} = -cot(x)cosec(x)$
$y = \sqrt{x}$	$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$	$y = sec(x)$	$\frac{dy}{dx} = sec(x)tan(x)$
$y = ln(x)$	$\frac{dy}{dx} = \frac{1}{x}$	$y = cot(x)$	$\frac{dy}{dx} = -cosec^2(x)$
$y = log_a(x)$	$\frac{dy}{dx} = \frac{1}{x}log_a(e)$		
$y = e^x$	$\frac{dy}{dx} = e^x$		
$y = a^x$	$\frac{dy}{dx} = a^x log_e(a)$		

$(u+v)' = u' + v'$	$(u-v)' = u' - v'$	$(u.v)' = u'v + v'u$	$(\frac{u}{v})' = \frac{u'.v - v'.u}{v^2}$
$(e^u)' = u'.e^u$	$ln(u)' = \frac{u'}{u}$	$(u^\alpha)' = \alpha.u^{\alpha-1}.u'$	$(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$
$(\sqrt[n]{u})' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$	$(\sin(u))' = u'.\cos(u)$	$(\cos(u))' = -u'.\sin(u)$	$(a^u)' = u'.a^u.ln(a)$
$(log_a(u))' = \frac{u'}{u.ln(a)}$			

$$(\tan(u))' = \frac{u'}{\cos^2(u)}$$
$$(\cot(u))' = -\frac{u'}{\sin^2(u)}$$
$$\Big[(x^m)^n\Big]' = (m \geq n) \quad ? \quad m(m-1)(m-2)(m-n+1)x^{m-n} : 0$$
$$(\log_a(x))^n = (-1)^{n-1}.\frac{(n-1)!}{\ln(a)}\frac{1}{x^n}$$
$$(\ln(x))^n = (-1)^{n-1}.(n-1)!.x^{-n}$$
$$(e^{kx})^n = k^n.e^{kx}$$
$$(a^x)^n = (\ln(a))^n.a^x$$
$$(\sin(ax))^n = a^n.\sin(ax + n.\pi/2)$$
$$(\cos(ax))^n = a^n.\cos(ax + n.\pi/2)$$
$$\Big(\frac{1}{ax+b}\Big)^n = (-1)^n.a^n.n!.\frac{1}{(ax+b)^{n+1}}$$
$$(\arcsin(x))' = \frac{1}{\sqrt{1-x^2}}$$
$$(\arccos(x))' = \frac{-1}{\sqrt{1-x^2}}$$
$$(\arctan(x))' = \frac{1}{1+x^2}$$

Chain rule

$$\Big[f\Big(g(x)\Big)\Big]' = f'\Big(g(x)\Big).g'(x)$$

13 Integral Calculus

$y = x^n$	$\int x^n dx = \frac{x^{n+1}}{n+1} + c$	$y = sin(x)$	$\int sin(x)dx = -cos(x)$
$y = k$	$\int kdx = kx + c$	$y = cos(x)$	$\int cos(x)dx = sin(x)$
$y = 1$	$\int dx = x + c$	$y = sec^2(x)$	$\int sec^2(x)dx = tan(x)$
$y = \frac{1}{x}$	$\int \frac{1}{x}dx = ln(x) + c$	$y = sec(x)tan(x)$	$\int sec(x)tan(x)dx = sec(x)$
$y = e^x$	$\int e^x dx = e^x + c$	$y = cosec^2(x)$	$\int cosec^2(x)dx = -cot(x)$
$y = a^x$	$\int a^x dx = \frac{a^x}{log_e(a)} + c$	$y = cosec(x)cot(x)$	$\int cosec(x)cot(x)dx = -cosec(x)$

14 Circumferences

1. Circumderence of circle: $C = 2\pi R$
2. Circumderence of triangle: $P = a + b + c$

15 Formulae for Area and Volume

1. Area of square = $(side)^2$
2. Area of rectangle = $length \times length$
3. Area of triangle = $\frac{1}{2} \times base \times height$
4. Area of equilateral triangle = $\frac{\sqrt{3}}{4}a^2$
5. Are enclosed by a circle = πr^2
6. Surface area of sphere = $4\pi r^2$
7. Surface area of cube = $6L^2$
8. Surface area of cuboid
   $= 2[L \times b + b \times h + h \times L]$
9. Area of curved surface of cylinder = $2\pi rl$
10. Volume of cube = L^3
11. Volume of cuboid = $L \times b \times h$
12. Volume of sphere = $\frac{4}{3}\pi r^3$
13. Volume of cylinder = $\pi r^3 l$
14. Volume of cone = $\frac{1}{2}\pi r^2 h$