hw6_report

December 13, 2022

1 MAS Final Homework Assignment

Daniel Engbert, Dec 2022

[ ]:
```python
# helper function for use in code code below
import random
import numpy as np

def set_seed(seed=42):
  rng = random.Random()
  if seed is None:
    seed = random.randint(0, 9999)
  rng.seed(seed)
  np.random.seed(seed)
```

1.1 Monte Carlo Estimation of Shapely Value

How to fairly split a taxi fare across $\{1, ...N\}$ players can be calculated using Shapely values, which treats this scenario as a cooperative game. For this problem we assume that all players live on the way of the Nth player, and that player A lives at distance of 1, player B at a distance of 2, etc.

1.1.1 Computing Shapely values for $n = 4$:

The code below computes all permutations (a.k.a. "coalitions") of the $N = 4$ players $\{A, B, C, D\}$, then for each permutation, `perm`, it assigns each player a share off the total payoff (the fare of total cost 4), by assuming player `perm[0]` showed up first at the taxi, followed by players `perm[1]`, `perm[2]`, `perm[3]`.

For example, for the permutation `perm = ['A', 'C', 'B', 'D']`, the respective payoffs (fare splits) will be `{'A': 1, 'B': 0, 'C': 2, 'D': 1}` (here player B pays 0 because by the time player he arrives at the taxi, it was already going to go by the his house anyways due to player C.)

The final computed Shapely values are simply the average payoffs of each player across all possible permutations.

[ ]:
```python
import random

def get_player_vals(N: int):
    """
```

```python
    Returns mapping of N player names (strings) to their distance along the␣
↪taxi route.
    e.g. for N=4: {'A': 1, 'B': 2, 'C': 3', 'D': 4}
    """
    if N <= 26:
        return {chr(ord('A') + n): n+1 for n in range(0, N)}
    return {n+1: n+1 for n in range(0, N)}

def get_perms(arr):
    """returns a list of the possible permutations of the entries in the␣
↪provided array."""
    all_perms = []
    for p in arr:
        other_elems = sorted(list(set(arr) - set([p])))
        sub_perms = get_perms(other_elems)
        if len(sub_perms) == 0:
            all_perms.append([p])
        else:
            all_perms = all_perms + [[p] + perm for perm in sub_perms]
    return all_perms

def get_shapley(N: int):
    #player_vals = {'A': 6, 'B': 12, 'C': 42} # should result in shapely values␣
↪{2, 5, 35}
    player_vals = get_player_vals(N)

    print(f"player_vals = {player_vals}")
    players = list(player_vals.keys())

    # list of permutations of coalitions of size len(players)
    perms = get_perms(players)
    print(f"there are {len(perms)} total permutations of {len(players)} players:
↪")
    display(perms[:5]) # print first few rows
    print('(only the first 5 rows of permutations are shown above)')

    running_payoffs = {p: 0 for p in players}
    total_payoff = max(player_vals.values())
    for perm in perms:
        cur = {p: 0 for p in players}
        for p in perm: # compute share of payoff for each player in this␣
↪permutation
            cur[p] = max(0, player_vals[p] - sum(cur.values()))
        running_payoffs = {k: v+cur[k] for (k,v) in running_payoffs.items()}

    shapely_values = {k: v/len(perms) for (k,v) in running_payoffs.items()}
    print(f"\nshapely_values: (for N = {N})")
```

```
    print(shapely_values)

    #print('percent of payoff:')
    #print({k: v/total_payoff for (k,v) in shapely_values.items()})

get_shapley(4)
```

```
player_vals = {'A': 1, 'B': 2, 'C': 3, 'D': 4}
there are 24 total permutations of 4 players:

[['A', 'B', 'C', 'D'],
 ['A', 'B', 'D', 'C'],
 ['A', 'C', 'B', 'D'],
 ['A', 'C', 'D', 'B'],
 ['A', 'D', 'B', 'C']]

(only the first 5 rows of permutations are shown above)

shapley_values: (for N = 4)
{'A': 0.25, 'B': 0.5833333333333334, 'C': 1.0833333333333333, 'D':
2.0833333333333335}
```

As can be seen from the output, the shapely values for $n = 4$ players is `{'A': 0.25, 'B': 0.583, 'C': 1.083, 'D': 2.083}`

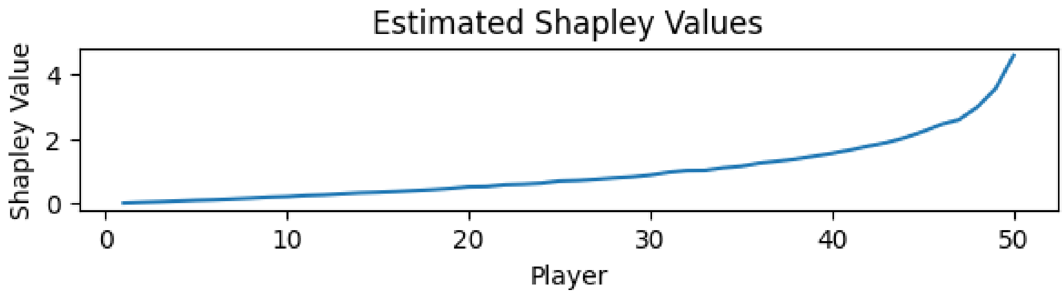
1.1.2 Estimating Shapley values for $n = 100$

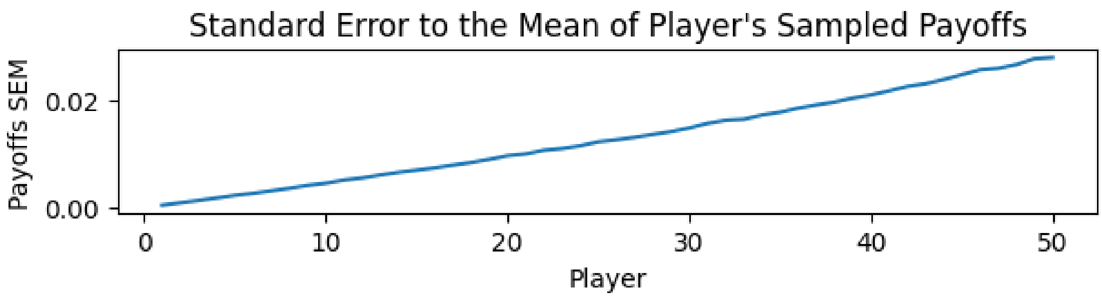
Here we use Monte Carlo sampling to approximiate the Shapely values for the taxi fare problem when $n = 100$.

[ ]:
```
import matplotlib.pyplot as plt
import scipy

def get_random_perm(vals):
    """returns a random permutation of the entries in the provided set."""
    assert len(vals) > 0

    item = random.choice(list(vals))
    if len(vals) > 1:
        return [item] + get_random_perm(vals - set([item]))
    return [item]

def estimate_shapley(N: int, samples: int):
    player_vals = get_player_vals(N)

    players = list(player_vals.keys())
    players_set = set(players)

    # store list of sampled payoffs for each player
```

```
    running_payoffs = {p: [] for p in players}
    total_payoff = max(player_vals.values())
    for _ in range(samples):
        perm = get_random_perm(players_set)
        cur = {p: 0 for p in players}
        for p in perm: # compute share of payoff for each player in this␣
 ↪permutation
            cur[p] = max(0, player_vals[p] - sum(cur.values()))
            running_payoffs[p].append(cur[p])
        #running_payoffs = {k: v+cur[k] for (k,v) in running_payoffs.items()}

    shapely_values = {k: sum(pays)/samples for (k,pays) in running_payoffs.
 ↪items()}
    print(f"\nestimated shapley values: (for N = {N} players, and {samples} of␣
 ↪permutations):")
    print(shapely_values)

    fig, axs = plt.subplots(2)
    fig.tight_layout(pad=6.0)
    fig.set_size_inches(8, 5)
    axs[0].set_title(f"Estimated Shapley Values")
    axs[0].set_xlabel(f'Player')
    axs[0].set_ylabel(f"Shapley Value")
    axs[0].plot(players, [shapely_values[p] for p in players])

    axs[1].set_title(f"Standard Error to the Mean of Player's Sampled Payoffs")
    axs[1].set_xlabel(f'Player')
    axs[1].set_ylabel(f"Payoffs SEM")
    sems = [scipy.stats.sem(pays) for player, pays in running_payoffs.items()]
    axs[1].plot(player_vals.keys(), sems)



estimate_shapley(50, 100000)
```

```
estimated shapley values: (for N = 50 players, and 100000 of permutations):
{1: 0.02054, 2: 0.04157, 3: 0.05971, 4: 0.08312, 5: 0.10704, 6: 0.12341, 7:
0.14642, 8: 0.17153, 9: 0.20036, 10: 0.21856, 11: 0.25418, 12: 0.2726, 13:
0.30493, 14: 0.33462, 15: 0.35186, 16: 0.37605, 17: 0.40556, 18: 0.43444, 19:
0.47223, 20: 0.52456, 21: 0.53544, 22: 0.58973, 23: 0.60535, 24: 0.63494, 25:
0.7026, 26: 0.72011, 27: 0.75406, 28: 0.79576, 29: 0.82959, 30: 0.8871, 31:
0.97181, 32: 1.02784, 33: 1.03342, 34: 1.11018, 35: 1.16159, 36: 1.25415, 37:
1.3153, 38: 1.38205, 39: 1.4687, 40: 1.55524, 41: 1.66011, 42: 1.78025, 43:
1.88723, 44: 2.03769, 45: 2.22738, 46: 2.45, 47: 2.59895, 48: 2.98686, 49:
3.55325, 50: 4.58003}
```





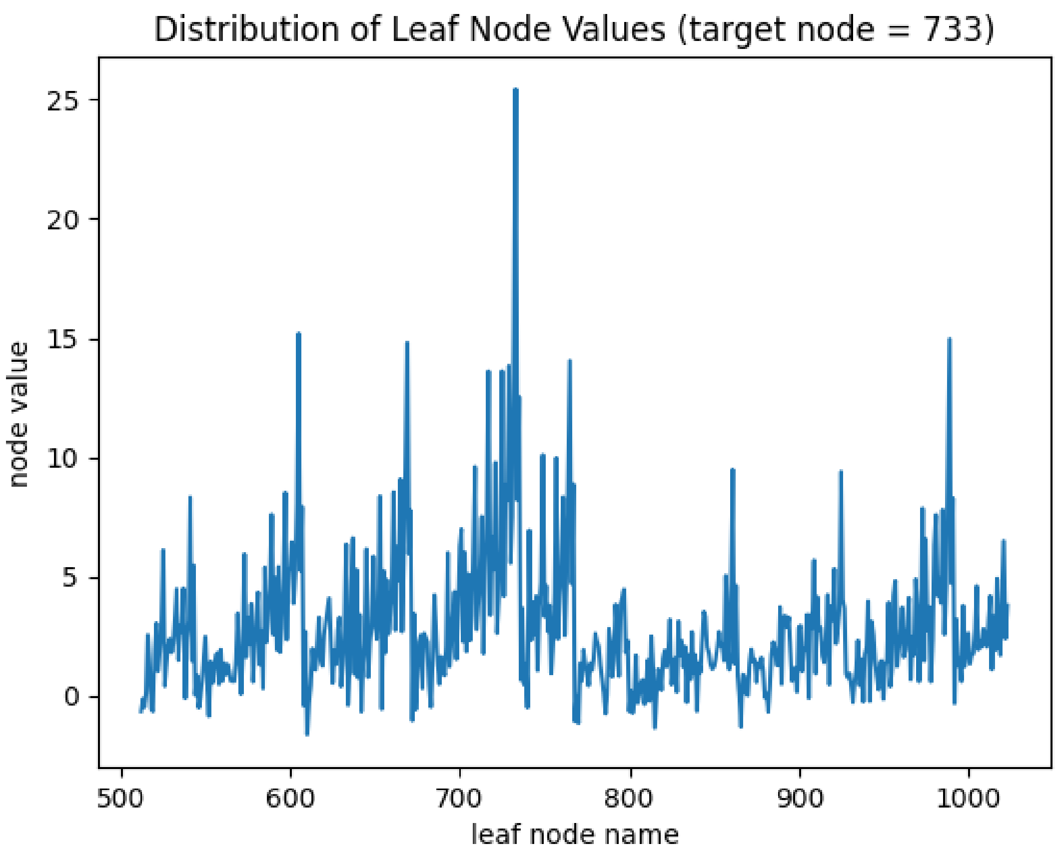
1.2 Monte Carlo Tree Search (MCTS)

[ ]:
```
import math
import matplotlib.pyplot as plt
import networkx as nx
import numpy as np
import random
import importlib
import tree_search as ts
importlib.reload(ts)

#### experiment params
set_seed(None)
depth = 10
B = 25       # for computing values of leaf nodes
####

tree = ts.create_tree(depth)
#nx.draw(tree, with_labels=True, node_size=300)

first_leaf_node = tree.number_of_nodes() - 2**(depth-1) + 1
leaf_node_names = list(range(first_leaf_node, tree.number_of_nodes()+1))
#print(f"leaf nodes: {leaf_node_names}")
```

```
target_name = random.choice(leaf_node_names)
dists = [ts.edit_distance(tree.nodes[n]['address'], tree.
 ↪nodes[target_name]['address']) for n in leaf_node_names]
dmax = max(dists)
#print(f"dists: {dists}")
print(f"target node = {target_name}, num leaf nodes = {len(leaf_node_names)},␣
 ↪max distance: {dmax}, min distance: {min(dists)}")
# compute values for each leaf node based on distance from target node
for idx, n in enumerate(leaf_node_names):
    tree.nodes[n]['value'] = B * math.pow(math.e, (-5 * dists[idx] / dmax)) +␣
 ↪np.random.normal()

leaf_vals = [tree.nodes[n]['value'] for n in leaf_node_names]
#print(f"vals: {leaf_vals}")

plt.xlabel('leaf node name')
plt.ylabel('node value')
plt.title(f'Distribution of Leaf Node Values (target node = {target_name})')
plt.plot(leaf_node_names, leaf_vals)
```

```
target node = 733, num leaf nodes = 512, max distance: 9, min distance: 0
```

[ ]:
```
[<matplotlib.lines.Line2D at 0x7fc820488df0>]
```



1.3 RL: SARSA and Q-Learning

[ ]:
```
import numpy as np

EMPTY = 0
WALL = 1
SNAKES = 2
TREASURE = 3

def create_world():
  world = np.zeros((9,9), dtype=int)
  world[1, 2:7] = WALL
  world[1:5, 6] = WALL
  world[7, 1:5] = WALL
  world[6:5] = SNAKES
  world[-1, -1] = TREASURE
  return world
```

```
world = create_world()
display(world)
```

```
array([[0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 1, 1, 1, 1, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 1, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 0],
       [0, 1, 1, 1, 1, 0, 0, 0, 0],
       [0, 0, 0, 0, 0, 0, 0, 0, 3]])
```

[ ]:

```

```