hw6_report

December 27, 2022

1 MAS Final Homework Assignment

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[ ]:
```
# helper function for use in code code below
import random
import numpy as np

def set_seed(seed=42):
  rng = random.Random()
  if seed is None:
    seed = random.randint(0, 9999)
  rng.seed(seed)
  np.random.seed(seed)
```

1.1 Monte Carlo Estimation of Shapely Value

How to fairly split a taxi fare across $\{1, ...N\}$ players can be calculated using Shapely values, which treats this scenario as a cooperative game. For this problem we assume that all players live on the way of the Nth player, and that player A lives at distance of 1, player B at a distance of 2, etc.

1.1.1 Computing Shapely values for $n = 4$:

The code below computes all permutations (a.k.a. "coalitions") of the $N = 4$ players $\{A, B, C, D\}$, then for each permutation, `perm`, it assigns each player a share off the total payoff (the fare of total cost 4), by assuming player `perm[0]` showed up first at the taxi, followed by players `perm[1]`, `perm[2]`, `perm[3]`.

For example, for the permutation `perm = ['A', 'C', 'B', 'D']`, the respective payoffs (fare splits) will be `{'A': 1, 'B': 0, 'C': 2, 'D': 1}` (here player B pays 0 because by the time he arrives at the taxi, it was already going to go by his house anyways due to player C.)

The final computed Shapely values are simply the average payoffs of each player across all possible permutations.

[ ]:
```
import random
import shapley

def get_player_vals(N: int):
```

```
    """
    Returns mapping of N player names (strings) to their distance along the␣
↪taxi route.
    e.g. for N=4: {'A': 1, 'B': 2, 'C': 3', 'D': 4}
    """
    if N <= 26:
        return {chr(ord('A') + n): n+1 for n in range(0, N)}
    return {n+1: n+1 for n in range(0, N)}

def get_shapley(N: int):
    #player_vals = {'A': 6, 'B': 12, 'C': 42} # should result in shapely values␣
↪{2, 5, 35}
    player_vals = get_player_vals(N)
    print(f"player_vals = {player_vals}")
    players = list(player_vals.keys())

    # list of permutations of coalitions of size len(players)
    perms = shapley.get_perms(players)
    print(f"there are {len(perms)} total permutations of {len(players)} players:
↪")
    display(perms[:5]) # print first few rows
    print('(only the first 5 rows of permutations are shown above)')

    running_payoffs = {p: 0 for p in players}
    total_payoff = max(player_vals.values())
    for perm in perms:
        cur = {p: 0 for p in players}
        for p in perm: # compute share of payoff for each player in this␣
↪permutation
            cur[p] = max(0, player_vals[p] - sum(cur.values()))
        running_payoffs = {k: v+cur[k] for (k,v) in running_payoffs.items()}

    shapely_values = {k: v/len(perms) for (k,v) in running_payoffs.items()}
    print(f"\nshapley_values: (for N = {N})")
    print(shapely_values)

    #print('percent of payoff:')
    #print({k: v/total_payoff for (k,v) in shapely_values.items()})

get_shapley(4)
```

```
player_vals = {'A': 1, 'B': 2, 'C': 3, 'D': 4}
there are 24 total permutations of 4 players:
```

```
[['A', 'B', 'C', 'D'],
 ['A', 'B', 'D', 'C'],
 ['A', 'C', 'B', 'D'],
 ['A', 'C', 'D', 'B'],
```

```
 ['A', 'D', 'B', 'C']]
```

```
(only the first 5 rows of permutations are shown above)

shapley_values: (for N = 4)
{'A': 0.25, 'B': 0.5833333333333334, 'C': 1.0833333333333333, 'D':
2.0833333333333335}
```

As can be seen from the output, the shapely values for $n = 4$ players is `{'A': 0.25, 'B': 0.583, 'C': 1.083, 'D': 2.083}`

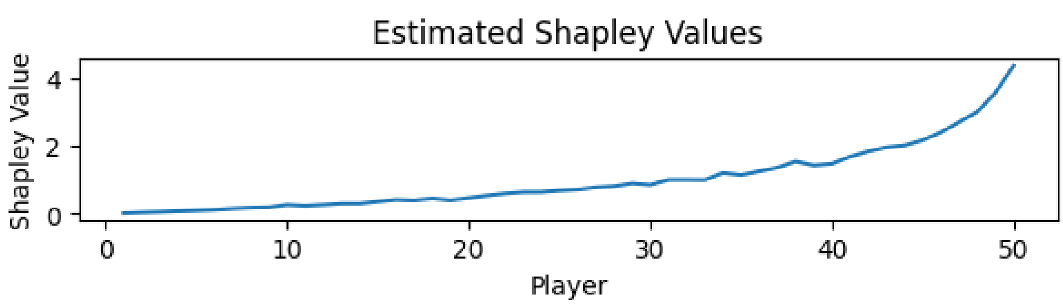
1.1.2 Estimating Shapley values for $n = 100$

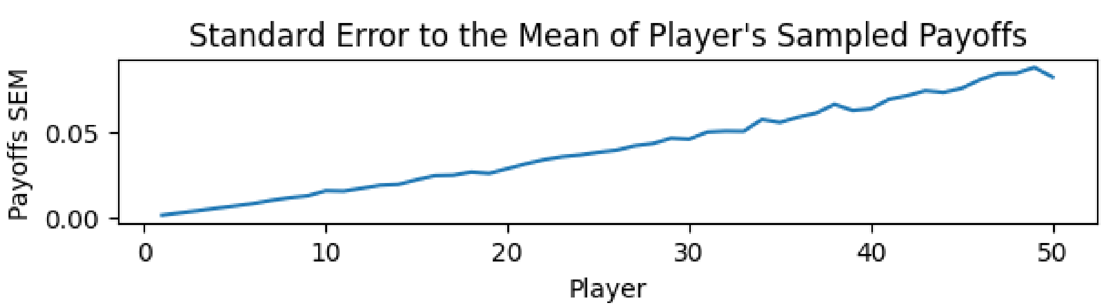
Here we use Monte Carlo sampling to approximiate the Shapely values for the taxi fare problem when $n = 100$.

```
[ ]: import matplotlib.pyplot as plt
import scipy

def estimate_shapley(N: int, samples: int):
    player_vals = get_player_vals(N)

    players = list(player_vals.keys())
    players_set = set(players)

    # store list of sampled payoffs for each player
    running_payoffs = {p: [] for p in players}
    total_payoff = max(player_vals.values())
    for _ in range(samples):
        perm = shapley.get_random_perm(players_set)
        cur = {p: 0 for p in players}
        for p in perm: # compute share of payoff for each player in this␣
 ↪permutation
            cur[p] = max(0, player_vals[p] - sum(cur.values()))
            running_payoffs[p].append(cur[p])
        #running_payoffs = {k: v+cur[k] for (k,v) in running_payoffs.items()}

    shapely_values = {k: sum(pays)/samples for (k,pays) in running_payoffs.
 ↪items()}
    print(f"\nestimated shapley values: (for N = {N} players, and {samples} of␣
 ↪permutations):")
    print(shapely_values)
    print(f"\nsum of estimated shapley values: {sum(shapely_values.values())}")

    fig, axs = plt.subplots(2)
    fig.tight_layout(pad=6.0)
    fig.set_size_inches(8, 5)
    axs[0].set_title(f"Estimated Shapley Values")
    axs[0].set_xlabel(f'Player')
```

```
    axs[0].set_ylabel(f"Shapley Value")
    axs[0].plot(players, [shapely_values[p] for p in players])

    axs[1].set_title(f"Standard Error to the Mean of Player's Sampled Payoffs")
    axs[1].set_xlabel(f'Player')
    axs[1].set_ylabel(f"Payoffs SEM")
    sems = [scipy.stats.sem(pays) for player, pays in running_payoffs.items()]
    axs[1].plot(player_vals.keys(), sems)



estimate_shapley(50, 10000)
```

```
estimated shapley values: (for N = 50 players, and 10000 of permutations):
{1: 0.0222, 2: 0.0423, 3: 0.0582, 4: 0.0802, 5: 0.0982, 6: 0.1196, 7: 0.1539, 8:
0.1795, 9: 0.1917, 10: 0.2644, 11: 0.2372, 12: 0.2659, 13: 0.3015, 14: 0.3006,
15: 0.3614, 16: 0.4169, 17: 0.3982, 18: 0.4525, 19: 0.4001, 20: 0.4685, 21:
0.5361, 22: 0.5968, 23: 0.6383, 24: 0.6406, 25: 0.6851, 26: 0.7129, 27: 0.7791,
28: 0.8111, 29: 0.8952, 30: 0.8529, 31: 1.0053, 32: 1.006, 33: 1.0008, 34:
1.2031, 35: 1.1424, 36: 1.2506, 37: 1.3584, 38: 1.5389, 39: 1.4244, 40: 1.4711,
41: 1.6773, 42: 1.8337, 43: 1.9543, 44: 2.0115, 45: 2.1639, 46: 2.3938, 47:
2.6948, 48: 2.9949, 49: 3.5573, 50: 4.3564}

sum of estimated shapley values: 49.99999999999999
```





1.2 Monte Carlo Tree Search (MCTS)

[ ]:
```
import math
from collections import defaultdict
import matplotlib.pyplot as plt
import networkx as nx
import numpy as np
import random
import statistics
import importlib
import tree_search as ts
importlib.reload(ts)

MCTS = ts.MCTS

###### experiment params
set_seed(None)
#depth = 10
depth = 4   # depth of tree to search
B = 25      # for computing values of leaf nodes
c = 2       # hyperparam
# TODO: define computational budget
######

#mc = MCTS(depth, c=c, B=B, draw=True)

def make_graph(data, show=True, save=False, samples=None):
  plt.clf()
  fig, ax = plt.subplots(1, 1)
  ax.set_xlabel("UCB c Value")
  ax.set_ylabel("Average Payoff Received (%)")
  ax.set_title(
      f"Effect of UCB c Value on Payoffs (MCTS with Depth {depth}, Samples =␣
 ↪{samples})"
  )
  #ax.plot(data.keys(), data.values())
  # show standard deviation as total length of error bar
  ax.errorbar(data.keys(), [sum(d)/len(d) for d in data.values()],␣
 ↪yerr=[statistics.stdev(d)/2 for d in data.values()])
  #ax.errorbar(data.keys(), [sum(d)/len(d) for d in data.values()], yerr=[50␣
 ↪for d in data.values()])
  if save:
    plt.savefig("c_graphs.pdf", dpi=400)
    #print("wrote c_graphs.pdf")
  if show:
```

```python
    plt.show()


data = defaultdict(list)
depth = 12
NUM_SAMPLES = 5
for c in np.linspace(0, 5, num=10):
  if c % 0.25 == 0:
    print(f"testing with c = {c}")
  for _ in range(NUM_SAMPLES):
    mc = MCTS(depth, c=c, B=B)
    tra = mc.run()
    value = mc.tree.nodes[tra[-1]]["value"]
    target_value = mc.tree.nodes[mc.target_name]["value"]
    data[c].append(100 * value / target_value)
  make_graph(data, samples=NUM_SAMPLES, show=False, save=True)
make_graph(data, samples=NUM_SAMPLES, show=True, save=True)


#tree = ts.create_tree(depth, draw=True)
#nx.draw(tree, with_labels=True, node_size=300)



# now do MCTS:
# store in each node:
# {'values': []} # (also tells you num visits)
```

```
testing with c = 0.0
testing with c = 5.0
```

```
<Figure size 640x480 with 0 Axes>

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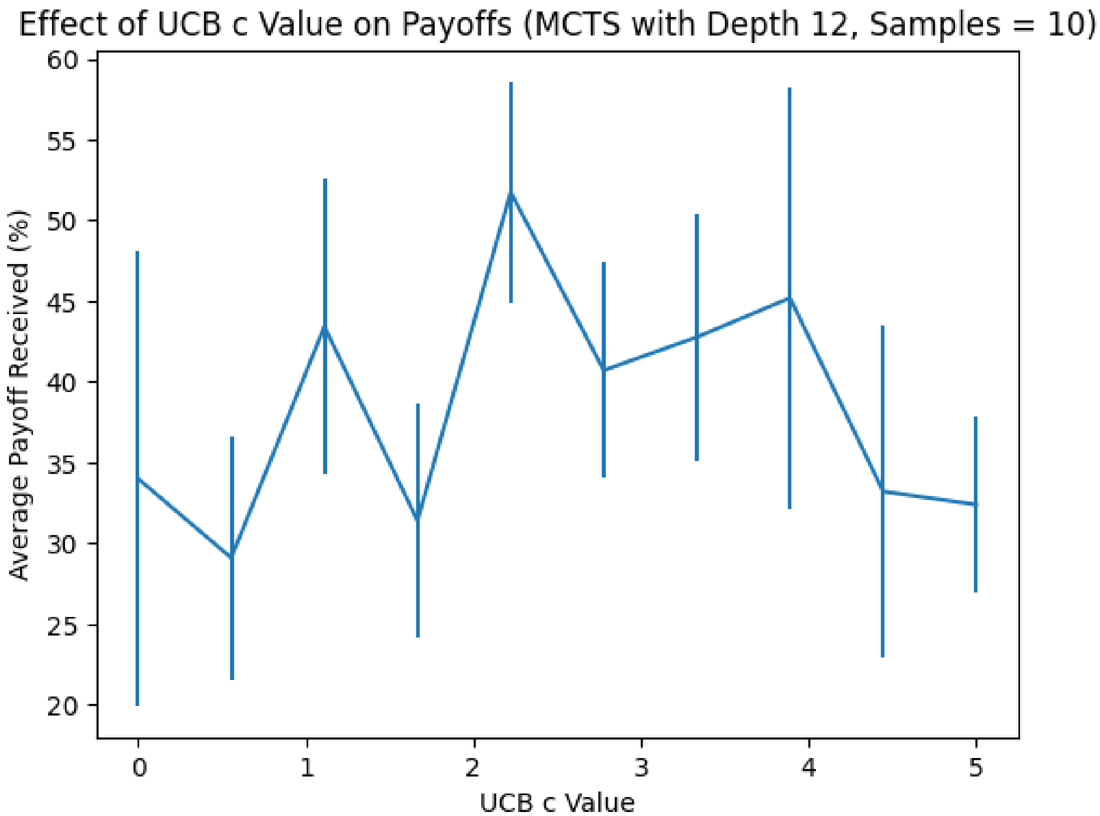


1.3 RL: SARSA and Q-Learning

1.3.1 Strategy 1: SARSA with Greedification

For the first experiment, SARSA is used in combination with greedification to search for an optimal policy.

[ ]:
```
import importlib
import matplotlib.pyplot as plt
import qlearn as ql
importlib.reload(ql)

sim = ql.QLearn()
#sim.visualize_qtable()

# TODO:
```

1.3.2 Q-Learning

Strategy 2a: Direct Updates Here we directly update the Q-table while rolling out each sample path.

Specifically we simulate `ITERATIONS` episodes (starting at a random initial position each time) and perform q-learning with ϵ-greedy selection) until each episode terminates.

[ ]:
```
import importlib
import matplotlib.pyplot as plt
import qlearn as ql
importlib.reload(ql)

ITERATIONS = 150

sim = ql.QLearn()
#world = ql.create_world()
print(f"initial Q table:")
display(sim.qtable)
sim.visualize_qtable(title="Initial Random Q table (Showing Greedy Actions)")

s0 = (0,0)
rewards = []
for _ in range(ITERATIONS):
  rewards.append(sim.run_episode()[0])
  #rewards.append(sim.run_episode(s0=s0))

plt.title(f'Total Rewards Across {ITERATIONS} Learning Episodes (With Random␣
↪Initial States)')
plt.plot(range(0, len(rewards)), [sum(r) for r in rewards])
plt.xlabel('Episode')
plt.ylabel('Total Reward')

print(f"new Q table:")
display(sim.qtable)
sim.visualize_qtable(title="Q Table Post Learning")

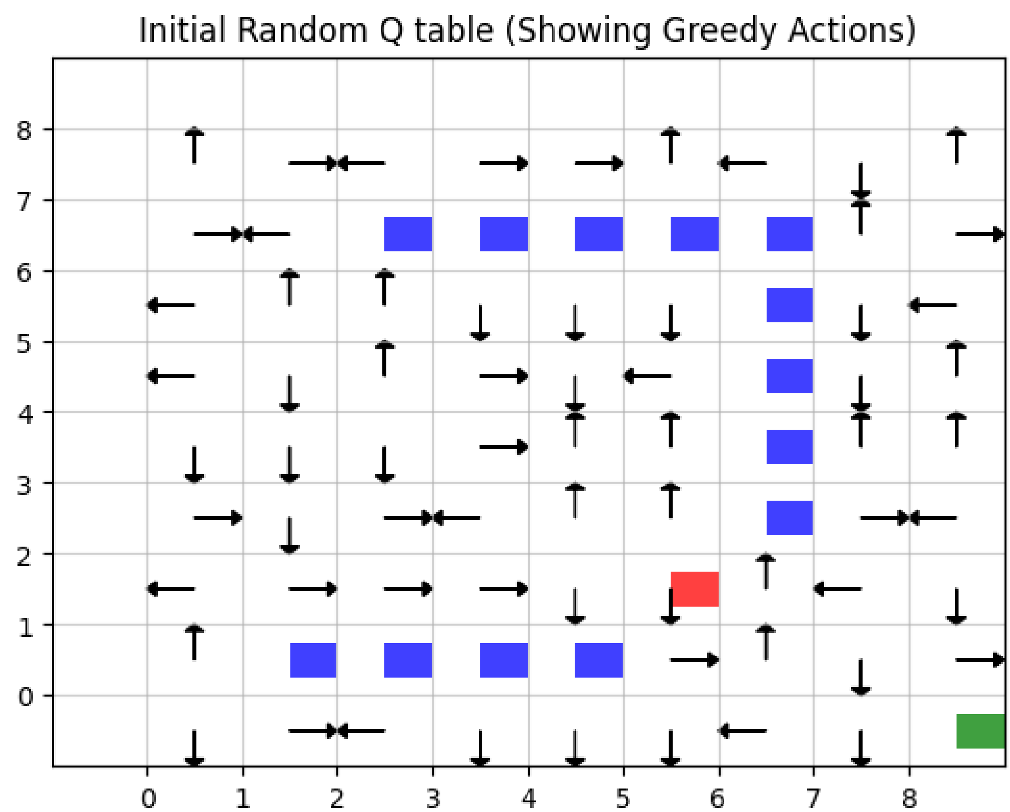

plt.title(f'Reward Across Each Step of Single Episode (With Greedy Selection)')
step_rewards = sim.run_episode(s0=s0, epsilon=1.0)[0]
plt.plot(range(0, len(step_rewards)), step_rewards)
plt.xlabel('Time Step')
plt.ylabel('Reward')
```

```
initial Q table:

         s    a     r         q
0   (0, 0)  3.0  -1.0  1.352051
1   (0, 0)  1.0  -1.0  0.259216
```

```
2     (0, 0)  0.0  -1.0  1.820377
3     (0, 0)  2.0  -1.0  0.390505
4     (0, 1)  3.0  -1.0  0.106118
..       …   …    …         …
319   (8, 7)  2.0  -1.0  1.450301
320   (8, 8)  3.0  50.0  0.000000
321   (8, 8)  1.0  50.0  0.000000
322   (8, 8)  0.0  50.0  0.000000
323   (8, 8)  2.0  50.0  0.000000

[324 rows x 4 columns]
```

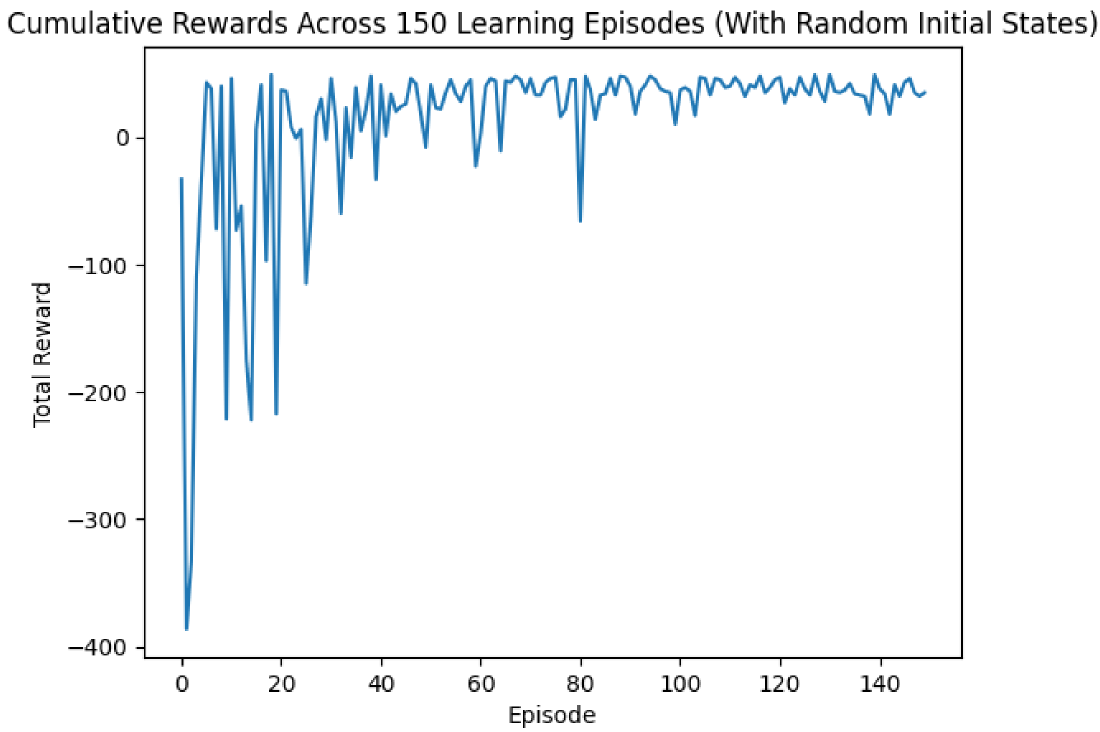


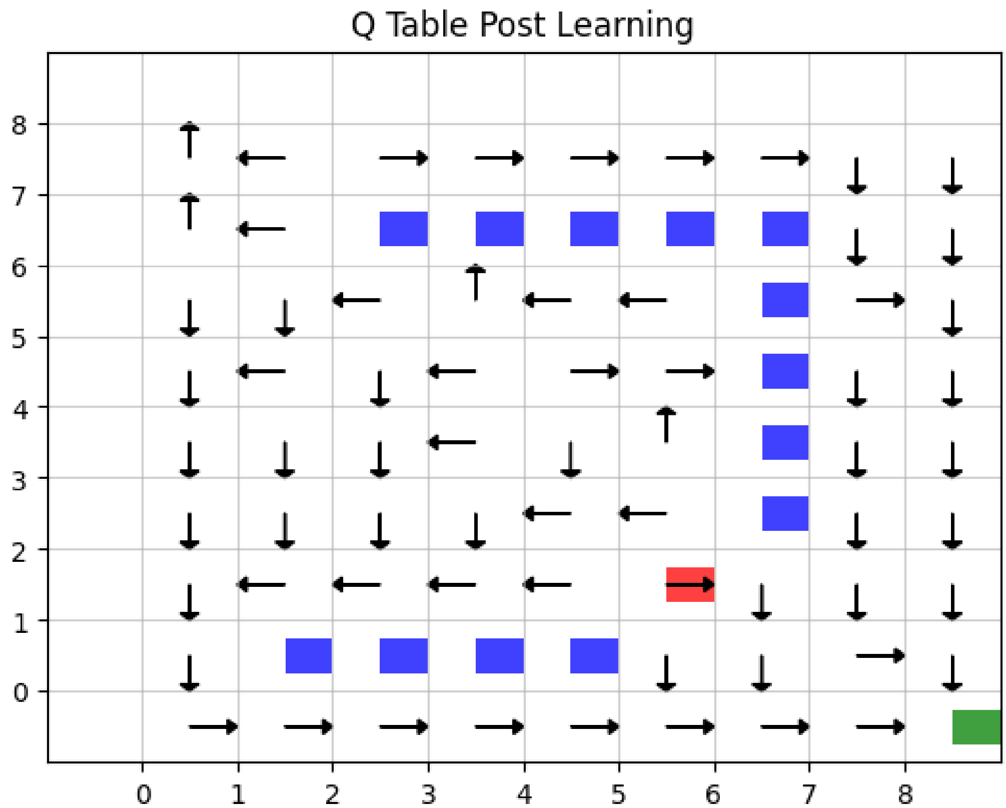

```
new Q table:

          s     a     r          q
0     (0, 0)  3.0  -1.0 -2.990689
1     (0, 0)  1.0  -1.0 -2.991275
2     (0, 0)  0.0  -1.0 -2.990615
3     (0, 0)  2.0  -1.0 -2.991360
4     (0, 1)  3.0  -1.0 -2.992087
..       …   …    …         …
319   (8, 7)  2.0  -1.0 -1.473547
```

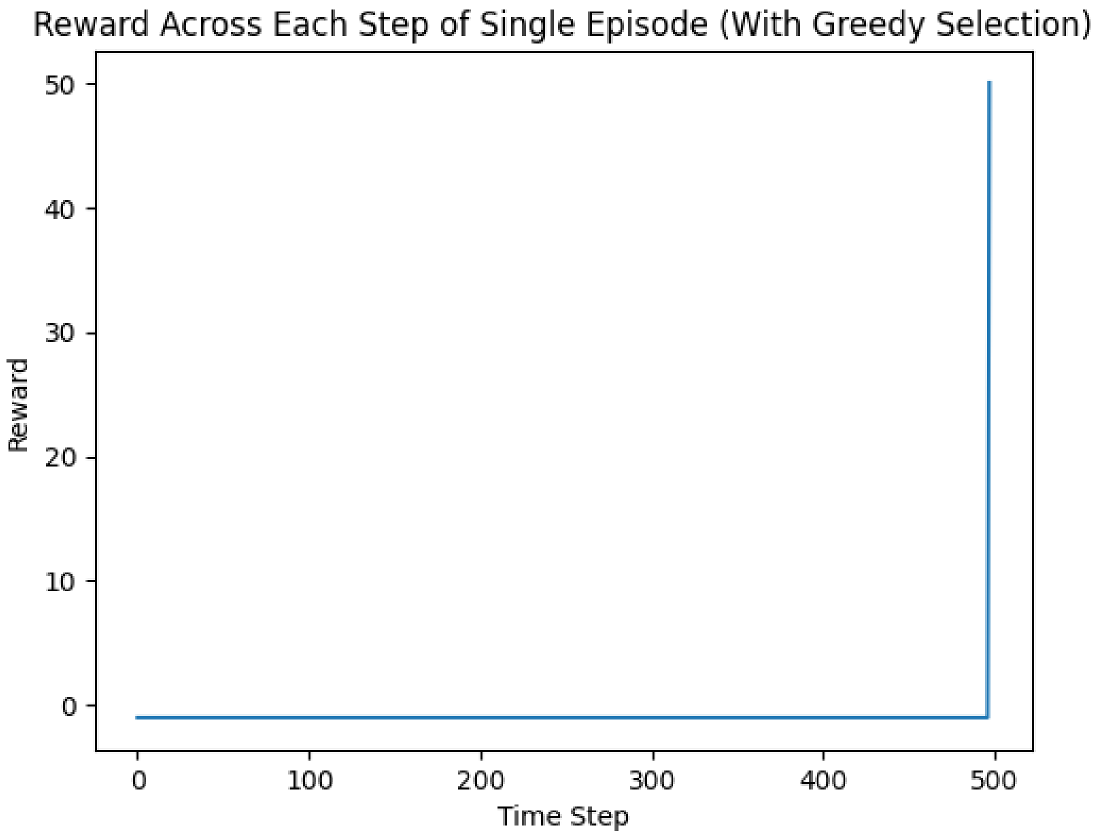
```
320  (8, 8)  3.0  50.0  0.000000
321  (8, 8)  1.0  50.0  0.000000
322  (8, 8)  0.0  50.0  0.000000
323  (8, 8)  2.0  50.0  0.000000

[324 rows x 4 columns]
```





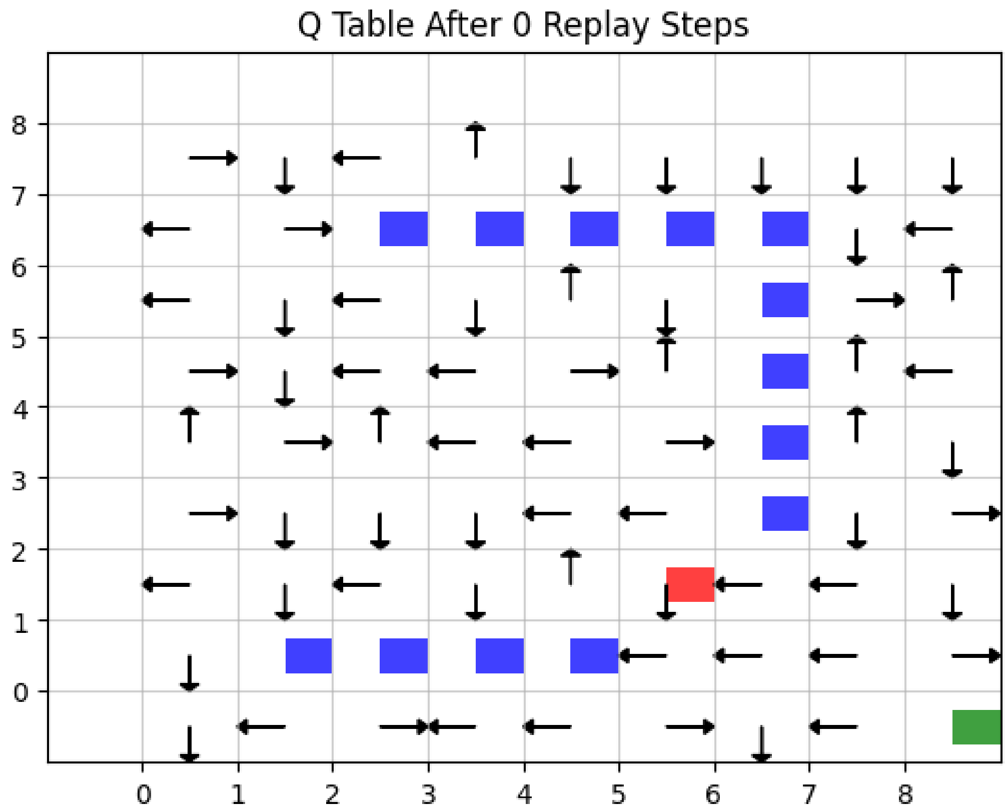

```
[ ]: Text(0, 0.5, 'Reward')
```



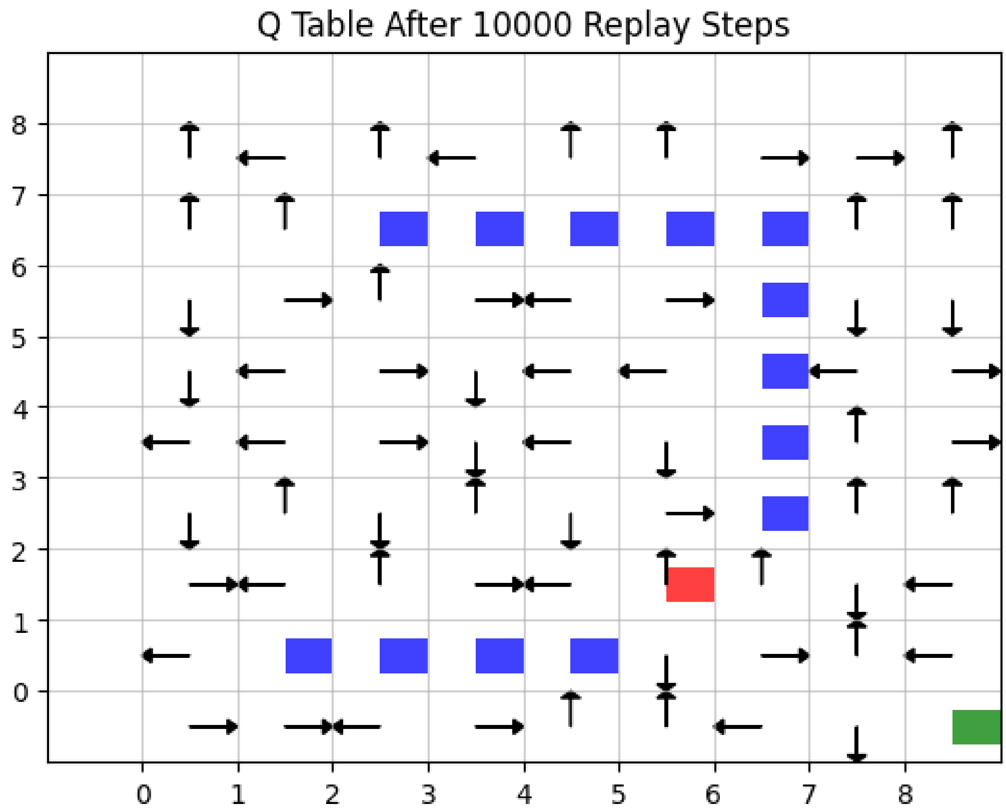
Strategy 2b: Replay Buffer Here we collect experiences by simulating episodes (without learning), and then sample from the stored experiences to perform Q-learning.

[ ]:
```
import qlearn as ql
importlib.reload(ql)
#sim = ql.QLearn()

ql.strategy2b()
```



collected 10000 experiences to sample from.



1.3.3 Discussion

TODO: Compare the solutions and the corresponding computational effort for the three solution strategies.

TODO: make a fitness func() for evaluating a qtable by evaluating it on N experiences from random locations…

Add up the number of (learning) steps needed for each strategy to reach a given fitness.