Vector basics

vector: indexed list of numbers

elements

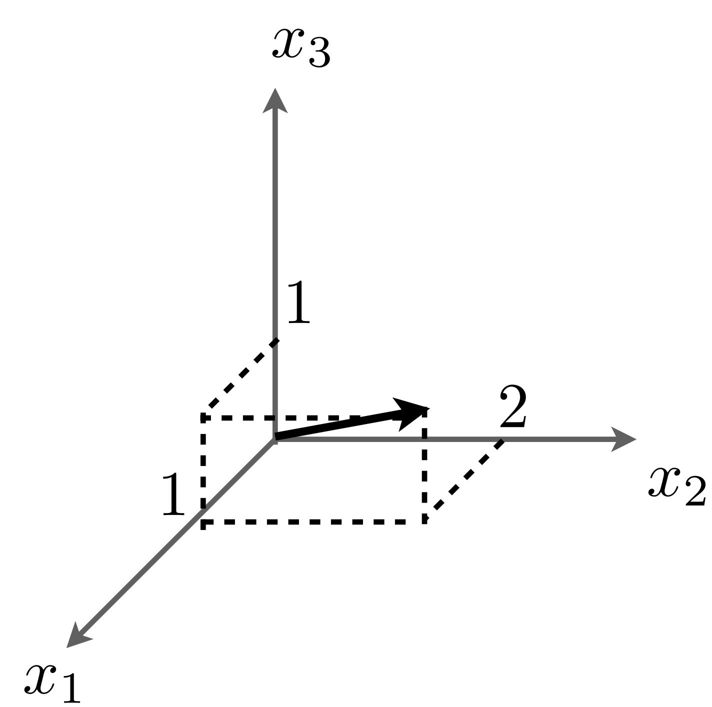
$$\begin{array}{cccc} \begin{bmatrix} 3 & 6 & 2 & -3 \end{bmatrix} \\ \end{array} \qquad \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \qquad \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}$$

indices

if three elements or fewer, can draw as arrow

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \quad =$$



Vector basics

if more than three elements, hard to draw, but can still think of as point in high-dimensional space

$\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}$: arbitrary point in 4-D space

Vector basics

simple operations

dot product: $\mathbf{x} \cdot \mathbf{y} = x_1 y_1 + x_2 y_2 + ... + x_n y_n$

takes two n-D vectors and outputs scalar

norm: $\|\mathbf{x}\| = \sqrt{x_1^2 + x_2^2 + ... + x_n^2}$

takes one n-D vector and outputs scalar

Vector basics

simple operations

finding a unit vector: $\hat{\mathbf{x}} = \dfrac{\mathbf{x}}{\|\mathbf{x}\|}$

takes in n-D vector, outputs n-D vector with unit norm

Vector basics

the dot product of y with unit vector $\hat{\mathrm{x}}$ tells you how much of y lies in the direction of $\hat{\mathrm{x}}$

we often call $\mathrm{y} \cdot \hat{\mathrm{x}}$ the projection of y onto the direction defined by $\hat{\mathrm{x}}$

if $\mathrm{y} \cdot \hat{\mathrm{x}} = 0$, we say that y is *orthogonal* to $\hat{\mathrm{x}}$

in two or three dimensions, orthogonal is synonymous with perpendicular

Functions are vectors

almost everything you can do with vectors you can also do with functions

to see the equivalence, remember that a function is essentially an indexed list of numbers also

when talking about functions we use the word *argument* instead of *index* and *function value* instead of *element*

Functions are vectors

example: writing $x(t) = 2t + 3$ like a vector

function value (element)

$$
\begin{array}{llllllllll}
x(t): & [\ldots & -7.2 & -7.0 & -6.8 & \ldots & 2.8 & 3.0 & 3.2 & \ldots] \\
t: & \ldots & -5.1 & -5.0 & -4.9 & \ldots & -0.1 & 0.0 & 0.1 & \ldots
\end{array}
$$

argument (index)

in the real world (pun intended) the values that t can take on are infinitely close together, but intuitively it's easier to think of them as having some small spacing

Functions are vectors

since functions are basically really high dimensional vectors, we can define vector-like operations on them

the dot product becomes the *inner product*:

$$\langle x(t)|y(t)\rangle = \int_a^b x(t)y(t)dt$$

takes in two functions and outputs a scalar

is generalization of multiplying elements pairwise and adding them all up

two functions are *orthogonal* if their inner product is zero

Functions are vectors

function norm: $\|x(t)\| = \sqrt{\int_a^b x(t)^2 dt}$

takes in function and outputs scalar

finding a unit function: $\hat{x}(t) = \frac{x(t)}{\|x(t)\|}$

takes in function and outputs function with unit norm

projections: $\langle y(t)|\hat{x}(t)\rangle = \int_a^b y(t)\hat{x}(t)dt$

we say this is the projection of $y(t)$ onto the direction defined by $\hat{x}(t)$