CMSC 313 — Spring 2017
Homework 3 — Digital Logic I

Assigned	Thursday, May 4th
Due	in class, Tuesday, May 9st (circuit due by 11:59pm)
Updates:	▪ None yet.

Paper submissions are due in class. Logisim circuits can be submitted online up to 11:59pm of the due date using the `submit` command on GL. Make a copy of your paper submission, if you need it to construct your Logisim circuit.

Problem #1 (30 points)

Using the Basic Identities of Boolean Algebra (covered in lecture and the textbook), simplify the following formulas. Show all of your work and justify each step.

a. bc + (a b')'(a'b')'

b. (c'a')' ((a + b) + c)

Problem #2 (30 points)

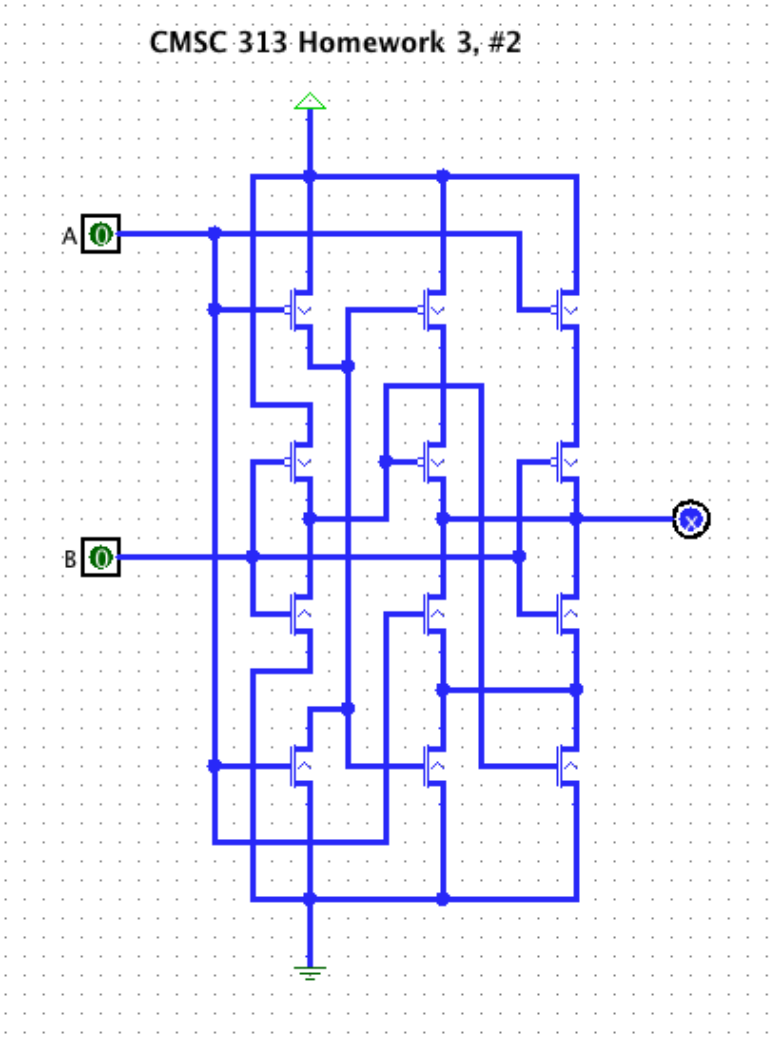
For the CMOS circuit shown below,

a. (10 points) Provide a truth table for the circuit's function. You may use the Logisim circuit, hw3-2.circ, to determine the truth table. (You must download the circuit on your machine and run Logisim. There is a link under the "Resources" tab for getting a copy of Logisim.) After loading the circuit into Logisim, select the "Hand" tool, then click on the inputs to toggle them on and off.

b. (5 points) Write down the Sum-of-Products (SOP) Boolean formula for the truth table.

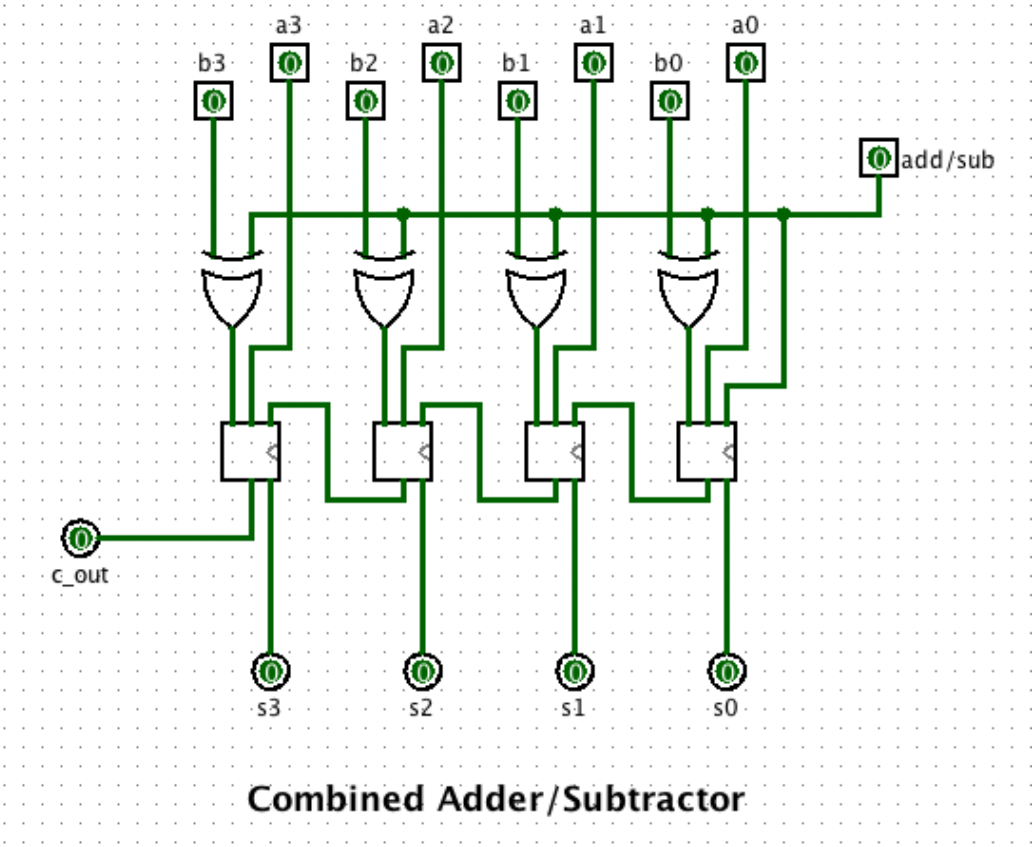
c. (5 points) Write down the Product-of-Sums (POS) Boolean formula for the truth table.

d. (10 points) Draw the logic diagram of the POS formula using AND, OR and NOT gates (do not simplify).

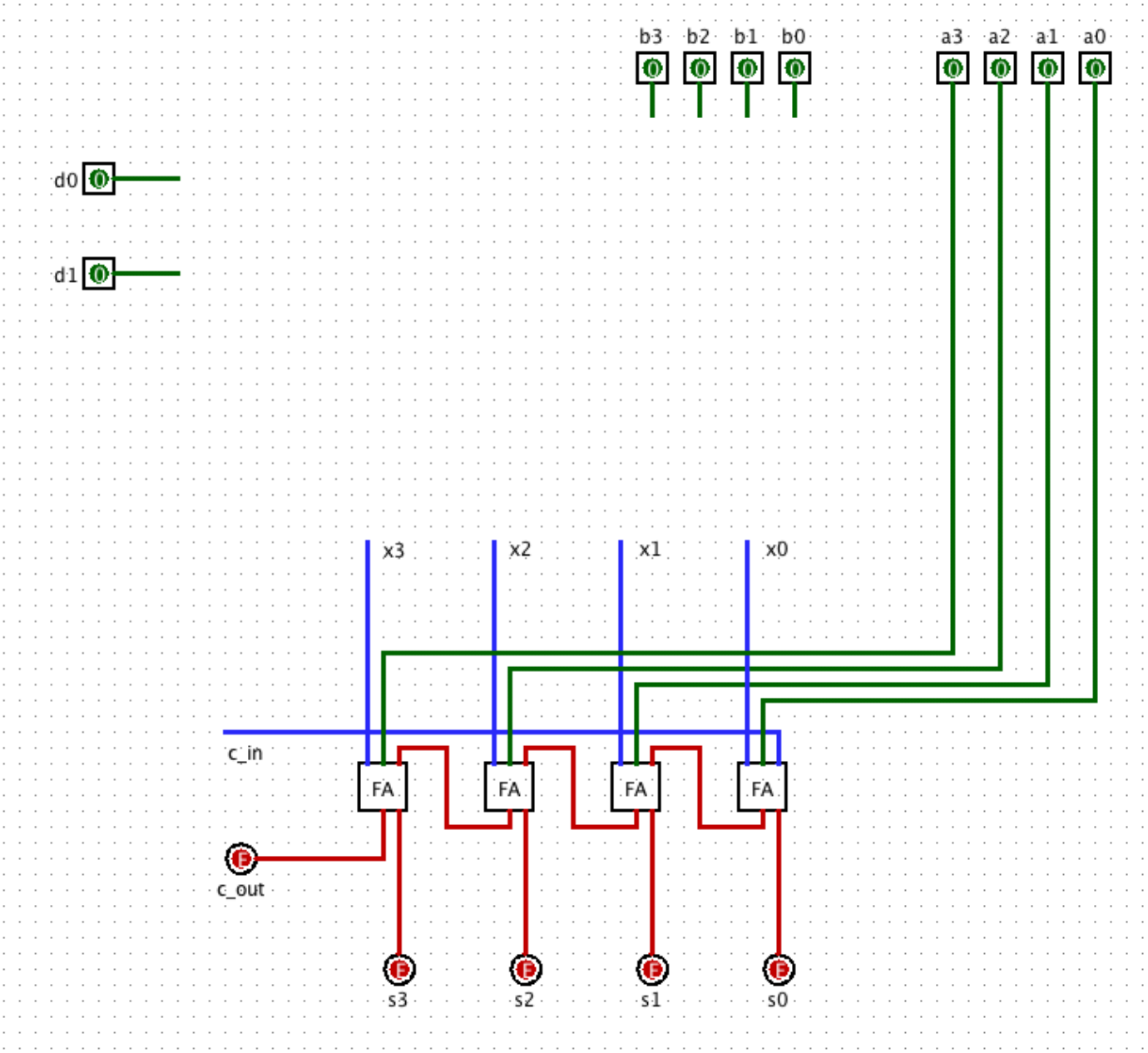


Problem #3 (40 points)

For this problem, you will build a 4-bit Arithmetic Logic Unit (ALU) that adds, subtracts, decrements and increments in Logisim. The design is based on the Combined Adder/Subtractor shown in class:



We can think of this circuit as having 1 control line that determines whether the circuit adds or subtracts. Our 4-bit ALU will have two control lines (d1 d0) that determines whether the ALU will add, subtract, decrement or increment:



This functionality can be built using just the 4-bit ripple-carry adder. Your problem is to figure out how to connect x3 x2 x1 x0 to d1 d0 and b3 b2 b1 b0. Follow these steps:

First, we write down what we want to accomplish:

- **d1 d0 = 00 (ADD)**
  We want to add b3 b2 b1 b0 to a3 a2 a1 a0 and provide the sum in s3 s2 s1 s0. This can be accomplished by passing each b_i unchanged thru to x_i.
- **d1 d0 = 01 (INC)**
  We want to add 1 to a3 a2 a1 a0 and provide the result in s3 s2 s1 s0. In this case, the values in b3 b2 b1 b0 are ignored. To add 1, we simply set each x_i to 0 and make c_in 1.
- **d1 d0 = 10 (SUB)**
  We want to subtract b3 b2 b1 b0 from a3 a2 a1 a0 and provide the difference in s3 s2 s1 s0. We can use the same

trick that the Combined Adder/Subtractor used: make x_i the complement of b_i and set the carry-in (c_in) to 1. That effectively adds the two's complement of b3 b2 b1 b0 to a3 a2 a1 a0.

- **d1 d0 = 11 (DEC)**
  We want to subtract 1 from a3 a2 a1 a0 and provide the result in s3 s2 s1 s0. As in the case for INC, we ignore b3 b2 b1 b0. We can subtract 1 by adding 1 1 1 1 to a3 a2 a1 a0, since 1111 is −1 in 4-bit two's complement. Thus, each x_i should simply be set to 1 in this case. The carry-in should be 0.

From the description above, we can produce a truth table for the values of x_i and c_in, given the inputs d1, d0 and b_i:

Inputs			**Outputs**	
d1	**d0**	**bi**	**xi**	**c_in**
0	0	0	0	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

What to do:

- On a piece of paper to be turned in with Problems 1 and 2, write down a Boolean formula for x_i in terms of d1 d0 and b_i. Simplify the formula as much as you can. Clearly indicate your final formula for x_i.
- Write down a formula for c_in. Simplify the formula as much as you can. Clearly indicate your final formula for c_in.
- Download the starting point for your circuit: hw3-3.circ.
- Launch Logisim, and load the circuit file. Then double-click on the "ALU" box to open that up to see the partial circuit inside (you have to click exactly on the middle). This is where you'll be adding your logic gate circuits.
- connect the ripple-carry adder's inputs, x3 x2 x1 x0, to d1 d0 and b3 b2 b1 b0 according to your formula from the previous step. (Yes, you will have four copies of the same circuit. You can use copy-paste to do this more quickly.)
- In Logisim, connect c_in to d1 d0 and b0 according to your formula for c_in from above.
- Test your ALU and make sure that it works as advertised.
- Save your circuit diagram, transfer the file to GL and submit from GL as usual:

```
submit cs313_park hw3 hw3-3.circ
```

- Submit the work done on paper in class along with your work for Problems #1 & #2.

Implementation notes:

- When you simplify your Boolean formula, you want to reduce the number of gates. This might not correspond to a "simpler" formula as defined in high school and middle school Algebra.
- Give some thought to the layout of your circuit. Neatness will count in grading.
- Logisim gates come in three sizes: narrow, medium and wide. Pick the appropriate size.
- Logsim gates can have negated inputs (bubbles). This can greatly simplify your circuit's layout because you do not need to make room for NOT gates. To negate a particular input, select the gate using the arrow tool. Then, in the panel on the lower left, choose "Yes" to negate a particular input line.