

CMSC 478 — Spring 2017 — C. S. Marron

Lab 2: Simple and Multiple Regression

Collinearity in Simulated Data

Exercise 1: Perform the following commands in R:

```
> set.seed(1)
> x1 = runif(100)
> x2 = 0.5*x1 + rnorm(100)/10
> y = 2 + 2*x1 + 0.3*x2 + rnorm(100)
```

The last line corresponds to creating a linear model in which y is a function of x_1 and x_2 .

- a. Write out the form of the linear model.
- b. What are the regression coefficients?

Exercise 2: What is the correlation between x_1 and x_2 ? Create a scatterplot displaying the relationship between the two variables.

Exercise 3: Fit a least-squares regression to predict y using x_1 and x_2 . Describe the results obtained.

- a. What are $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$?
- b. How do these relate to the true β_0 , β_1 , and β_2 ?
- c. Can you reject the null hypothesis $H_0 : \beta_1 = 0$? Can you reject the null hypothesis $H_0 : \beta_2 = 0$?

Exercise 4: Now you will fit two least-squares regressions, one using only x_1 as a predictor, the other using only x_2 .

- a. Fit a least-squares regression to predict y using only x_1 . Comment on your results. Can you reject the null hypothesis $H_0 : \beta_1 = 0$?
- b. Fit a least-squares regression to predict y using only x_2 . Comment on your results. Can you reject the null hypothesis $H_0 : \beta_1 = 0$?
- c. Explain why your results for this question do *not* contradict the results from Exercise 3.

Multiple Linear Regression

For the following exercises, you will use the Boston data set, which is part of the MASS library. Load the library with the command

```
> library(MASS)
```

The variables in the Boston data set are described in [the appendix](#).

Exercise 5: Fit a multiple regression model to predict `crim` using all other variables as predictors. Describe your results. For which predictors can we reject the hypothesis $H_0 : \beta_j = 0$?

Exercise 6: For each of the predictors `zn`, `indus`, `nox`, and `medv`, fit a simple linear regression model to predict the response. Describe your results.

- In which models is there a statistically significant association between the predictor and the response?
- Explain any discrepancies between the simple linear regression models and the results from Exercise 5.

Exercise 7: Is there evidence of non-linear association between any of the predictors `zn`, `indus`, `nox`, or `medv` and the response? To answer this question, for each predictor, fit a model of the form

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon.$$

Appendix: Boston Data Set

crim

per capita crime rate by town.

zn

proportion of residential land zoned for lots over 25,000 sq.ft.

indus

proportion of non-retail business acres per town.

chas

Charles River dummy variable (= 1 if tract bounds river; 0 otherwise).

nox

nitrogen oxides concentration (parts per 10 million).

rm

average number of rooms per dwelling.

age

proportion of owner-occupied units built prior to 1940.

dis

weighted mean of distances to five Boston employment centres.

rad

index of accessibility to radial highways.

tax

full-value property-tax rate per \$10,000.

prratio

pupil-teacher ratio by town.

black

$1000(B_k - 0.63)^2$ where B_k is the proportion of blacks by town.

lstat

lower status of the population (percent).

medv

median value of owner-occupied homes in \$1000s.

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