6.1

a. Hình thang:

$$\int_{x_0}^{x_c} f(x)\,dx \approx \frac{x_c - x_0}{2}(y_0 + y_c)$$

$$\int_{x_0}^{x_n} f(x)\,dx = \sum_{i=1}^{n}\int_{x_{i-1}}^{x} f(x)\,dx \approx \sum_{i=1}^{n} \frac{x_i - x_{i-1}}{2}(y_{i-1} + y_i).$$

b. Simpson 1/3

$$\int_{x_0}^{x_c} f(x)\,dx \approx \frac{x_c - x_0}{6}(y_0 + 4y_g + y_c)$$

$$\int_{x_0}^{x_n} f(x)\,dx \approx \sum_{i=1}^{n} \frac{x_i - x_{i-1}}{6}(y_{i-1} + 4y_{i-1/2} + y_i).$$

c. Simpson 3/8

$$\int_{x_0}^{x_c} f(x)\,dx \approx \frac{(x_c - x_0)}{8}(y_0 + 3y_1 + 3y_2 + y_c)$$

$$\int_{x_0}^{x_n} f(x)\,dx \approx \sum_{i=1}^{n} \frac{(x_i - x_{i-1})}{8}(y_{i-1} + 3y_{i-2/3} + 3y_{i-1/3} + y_i)$$

d. Gauss:

$$\int f(x)\,dx \approx \sum_{k=1}^{n} w_k f(w_k) \quad (1)$$

Trong đó w_k là là các thọng số tương ứng với các vị trí x_k. Ta tìm các giá trị w_k sao cho (1) chính xác với các đa thức có bậc nhỏ hơn $2n$. Nghĩa là:

$$\begin{cases} b-a = w_1 + w_2 + \dots + w_n & f(x)=1 \\ \frac{b^2-a^2}{2} = w_1 x_1 + w_2 x_2 + \dots + w_n x_n & f(x)=x \\ \dots \\ \frac{b^n-a^n}{n} = w_1 x_1^{n-1} + w_2 x_2^{n-1} + \dots + w_n x_n^{n-1} & f(x)=x^{n-1} \end{cases}$$

6.2

a. ~~$\int_0^6 f(x)\,dx = \int_0^1 f(x)\,dx$~~

a - Hình thang:

$$\int_0^6 f(x)\,dx = \int_0^1 f(x)\,dx + \int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx + \int_4^5 f(x)\,dx + \int_5^6 f(x)\,dx$$

$$= 24,815 + 40,04 + 42,22 + 36,26 + 33,945 + 29,895$$

$$= \boxed{207,225}$$

- Simpson 1/3:

$$\int_0^6 f(x)\,dx = \int_0^2 f(x)\,dx + \int_2^4 f(x)\,dx + \int_4^6 f(x)\,dx$$

$$= \frac{2-0}{6}(15,42 + 4.34,21 + 45,88) + \frac{1}{3}(45,88 + 4.32,8 + 32,85) + \frac{1}{3}(32,85 + 4.35,04 + 24,25)$$

$$= \boxed{211,0968}$$

- Simpson 3/8:

$$\int_0^6 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^6 f(x)\,dx$$

$$= \frac{3-0}{8}(15,42 + 3.34,21 + 3.45,88 + 39,68) + \frac{3}{8}(39,68 + 3.32,85 + 3.35,04 + 24,25)$$

$$= \boxed{211,2825}$$

b. Hình thang:

$$\int_3^9 f(x)dx = \int_3^4 f(x)dx + \int_4^5 f(x)dx + \int_5^6 f(x)dx + \int_6^7 f(x)dx + \int_7^8 f(x)dx + \int_8^9 f(x)dx$$

$$= \frac{1}{2}(15,42 + 34,21) + \frac{1}{2}(34,21 + 45,87) + 42,77 + 36,2 + 33,945 + 29,895$$

$$= \boxed{207,825}.$$

- Simpson 1/3:

$$\int_3^9 f(x)dx = \int_3^5 f(x)dx + \int_5^9 f(x)dx$$

$$= \frac{5-3}{6}(15,42 + 4.34,21 + 45,87) + \frac{1}{3}(45,87 + 4.39,67 + 32,85) + \frac{1}{3}(32,85 + 4.35,04 + 24,25)$$

$$= 211,096$$

- Simpson 3/8:

$$\int_3^9 f(x)dx = \int_3^6 f(x)dx + \int_6^9 f(x)dx$$

$$= \frac{6-3}{8}(15,42 + 3.34,21 + 3.45,87 + 39,67) + \frac{9-6}{8}(39,67 + 3.32,85 + 3.35,04 + 24,25)$$

$$= \boxed{211,2825}.$$

b.3

a. Hình thang:

$$\int_0^2 x\,dx = \int_0^{1/3} x\,dx + \int_{1/3}^{2/3} x\,dx + \int_{2/3}^{1} x\,dx + \int_1^{4/3} x\,dx + \int_{4/3}^{5/3} x\,dx + \int_{5/3}^{2} x\,dx$$

$$= \frac{1}{6}\left(\frac{1}{3} + 1\right) + \frac{1}{6}\left(\frac{2}{3} + \frac{1}{3}\right) + \frac{1}{6}\left(1 + \frac{2}{3}\right) + \frac{1}{6}\left(\frac{4}{3} + 1\right) + \frac{1}{6}\left(\frac{5}{3} + \frac{4}{3}\right) + \frac{1}{6}\left(2 + \frac{5}{3}\right) = \frac{13}{6}$$

~~Simpson 1/3:~~

~~6.3~~

$$\int_2^3 \frac{x^3}{e^x+1}\,dx = \int_2^{13/6}$$

6.4

a. Đặt $f(x) = \frac{x^3}{e^x+1}$

HT: $\int_2^3 f(x)dx = \int_2^{13/6} f(x)dx + \int_{13/6}^{8/3} f(x)dx + \int_{8/3}^{5/2} f(x)dx + \int_{5/2}^{8/3} f(x)dx$
$+ \int_{8/3}^{13/6} f(x)dx + \int_{13/6}^{3} f(x)dx$

$= \frac{1}{12}(1,0454 + 0,2689) + \frac{1}{12}(1,123 + 1,0454)$
$+ \frac{1}{12}(1,1853 + 1,123) + \frac{1}{12}(1,232 + 1,1853)$
$+ \frac{1}{12}(1,2635 + 1,232) + \frac{1}{12}(1,2805 + 1,2635)$
$= 1,104$

SS 1/3: $\int_2^3 f(x)dx = \int_2^{8/3} f(x)dx + \int_{8/3}^{3} f(x)dx$
$= \frac{1}{18}(0,2689 + 4.1,0454 + 1,123)$
$+ \frac{1}{18}(1,123 + 4.1,1853 + 1,232)$
$+ \frac{1}{18}(1,2805 + 4.1,2635 + 1,232)$
$= 1,1242.$

SS 3/8: $\int_2^3 f(x)dx = \int_2^{5/2} f(x)dx + \int_{5/2}^{3} f(x)dx$
$= \frac{1}{16}(0,2689 + 3.1,0454 + 3.1,123 + 1,1853)$
$+ \frac{1}{16}(1,1853 + 3.1,232 + 3.1,2635 + 1,2805)$
$= 1,1195$

b. Đặt $g(x) = \frac{\ln(x+2)}{x+1}$

HT: $\int_1^3 g(x)dx = \int_1^{4/3} g(x)dx + \int_{4/3}^{5/3} g(x)dx + \int_{5/3}^{2} g(x)dx + \int_2^{7/3} g(x)dx$
$+ \int_{7/3}^{8/3} g(x)dx + \int_{8/3}^{3} g(x)dx$.

$= \frac{1}{6}(0,5116 + 0,5493) + \frac{1}{6}(0,4882 + 0,5116)$
$+ \frac{1}{6}(0,4621 + 0,4882) + \frac{1}{6}(0,4399 + 0,4621)$
$+ \frac{1}{6}(0,4201 + 0,4399) + \frac{1}{6}(0,4024 + 0,4201)$
$= \boxed{0,9337}$

SSTB: $\int_1^3 g(x)dx = \int_1^{5/3} g(x)dx + \int_{5/3}^{7/3} g(x)dx + \int_{7/3}^{3} g(x)dx$
$= \frac{1}{9}(0,4882 + 4.0,5116 + 0,5493)$
$+ \frac{1}{9}(0,4399 + 4.0,4621 + 0,4882)$
$+ \frac{1}{9}(0,4024 + 4.0,4201 + 0,4399)$
$= \boxed{0,9332}$

SSTB: $\int_1^3 g(x)dx = \int_1^2 g(x)dx + \int_2^3 g(x)dx$
$= \frac{1}{8}(0,5493 + 3.0,5116 + 3.0,4882 + 0,4621)$
$+ \frac{1}{8}(0,4621 + 3.0,4399 + 3.0,4201 + 0,4024)$
$= \boxed{0,9332}$.

c. Đặt $h(x) = \sin(x^2+1)$

HT: $\int_0^1 h(x)dx = \int_0^{1/6} h(x)dx + \int_{1/6}^{1/3} h(x)dx + \int_{1/3}^{1/2} h(x)dx + \int_{1/2}^{2/3} h(x)dx$
$+ \int_{2/3}^{5/6} h(x)dx + \int_{5/6}^{1} h(x)dx$

$= \frac{1}{12}(0,8562 + 0,8415) + \frac{1}{12}(0,8962 + 0,8562)$
$+ \frac{1}{12}(0,949 + 0,8962) + \frac{1}{12}(0,992 + 0,949)$
$+ \frac{1}{12}(0,9924 + 0,992) + \frac{1}{12}(0,9093 + 0,9924)$
$= \boxed{0,9267}$

SS 1/3: $\int_0^1 h(x)dx = \int_0^{1/3} h(x)dx + \int_{1/3}^{2/3} h(x)dx + \int_{2/3}^1 h(x)dx$
$= \frac{1}{18}(0,8962 + 4.0,8562 + 0,8415)$
$+ \frac{1}{18}(0,992 + 4.0,949 + 0,8962)$
$+ \frac{1}{18}(0,9093 + 4.0,9924 + 0,992)$
$= \boxed{0,9288}$

SS 3/8: $\int_0^1 h(x)dx = \int_0^{1/2} h(x)dx + \int_{1/2}^1 h(x)dx$
$= \frac{1}{16}(0,949 + 3.0,8962 + 3.0,8562 + 0,8415)$
$+ \frac{1}{16}(0,9093 + 3.0,9924 + 3.0,992 + 0,949)$
$= \boxed{0,9288}$

(d) Đặt $t(x) = \ln(x^2+1)$

HT: $\int_{-2}^{3} t(x)dx = \int_{-2}^{-1} t(x)dx + \int_{-1}^{-1/3} t(x)dx + \int_{-1/3}^{1/3} t(x)dx + \int_{1/3}^{1} t(x)dx + \int_{1}^{5/3} t(x)dx$
$+ \int_{5/3}^{3/3} t(x)dx + \int_{3/3}^{3} t(x)dx$

$= \frac{1}{3}(0,1054 + 0,6931) + \frac{1}{3}(0,1054 + 0,1054)$
$+ \frac{1}{3}(0,6931 + 0,1054) + \frac{1}{3}(1,3291 + 0,6931)$
$+ \frac{1}{3}(1,8632 + 1,3291) + \frac{1}{3}(2,3026 + 1,8632)$
$= \boxed{3,7294}$

SS 1/3: $\int_{-1}^{3} f(x)dx = \int_{-1}^{1/3} f(x)dx + \int_{1/3}^{5/3} f(x)dx + \int_{5/3}^{3} f(x)dx$

$= \frac{2}{9}(0,1054 + 4.0,1054 + 0,6931)$

$+ \frac{2}{9}(1,3291 + 4.0,6931 + 0,1054)$

$+ \frac{2}{9}(2,3026 + 4.1,8632 + 1,3291)$

$= \boxed{3,6232}$

SS 3/8: $\int_{-1}^{3} f(x)dx = \int_{-1}^{1} f(x)dx + \int_{1}^{3} f(x)dx$

$= \frac{1}{4}(0,6931 + 3.0,1054 + 3.0,1054 + 0,6931)$

$+ \frac{1}{4}(2,3026 + 3.1,8632 + 3.1,3291 + 0,6931)$

$= \boxed{3,6438}$

6.6 Dùng luỹng tập tin:

$\int_{0}^{15} f(x)dx = \int_{0}^{3} f(x)dx + \int_{3}^{5} f(x)dx + \int_{5}^{9} f(x)dx + \int_{9}^{10} f(x)dx$

$+ \int_{10}^{13} f(x)dx$

$= \frac{3}{2}(35,6219 + 19,2261) + 1(33,9505 + 35,5219)$

$+ 2(25,2486 + 33,9505) + \frac{1}{2}(41,2512 + 35,2416)$

$+ \frac{3}{2}(32,5018 + 41,2512)$

$= \boxed{415,5695}$

6.8 8h20 = $\frac{25}{3}$ h, 13h30 = 13,5 h, 15h30 = 15,5 h

$\int_{6}^{18} f(x)dx = \int_{6}^{25/3} f(x)dx + \int_{25/3}^{11} f(x)dx + \int_{11}^{12} f(x)dx + \int_{12}^{13,5} f(x)dx$

$+ \int_{13,5}^{15,5} f(x)dx + \int_{15,5}^{17} f(x)dx + \int_{17}^{18} f(x)dx$

$= \frac{1}{6}(12,5 + 19,2) + \frac{4}{3}(19,2 + 29,1) + \frac{1}{2}(29,1 + 35,8)$

$+ \frac{3}{4}(35,8 + 30,3) + 1(25,6 + 30,3) + \frac{3}{4}(25,6 + 15,4)$

$+ \frac{1}{2}(15,4 + 8,2)$

= 283,1083

6.10

$$\int_1^{31} f(x)dx = \int_1^{7} f(x)dx + \int_7^{15} f(x)dx + \int_{15}^{22} f(x)dx + \int_{22}^{31} f(x)dx$$

$$= 3(538+498) + 4(498+605) + 3,5(605+521) + 4,5(521+553)$$

$$= 16694$$

⇒ Doanh thu: 16694 * 25000 (đ) = 417 350 000 (đ)

6.11

a. Số kết bia: $\int_1^{31} f(x)dx = \frac{30}{8}(32 + 3.28 + 3.42 + 35) = 1052,5$

Số kết nước ngọt: $\int_1^{31} g(x)dx = \frac{30}{8}(46 + 3.50 + 3.42 + 40) = 1352,5$.

⇒ Doanh thu: 1052,5 * 280 000 (đ) + 1352,5 * 190 000 = 554 025 000 (đ)

b. Lợi nhuận của mỗi kết bia: 1052,5 * 25000 = 26 312 500 (đ)

Kết nước ngọt: 1352,5 * 32000 = 43 440 000 (đ)

⇒ Tổng lợi nhuận: 69 752 500 (đ)

⇒ Lợi nhuận từ việc bán nước ngọt cao hơn

6.12

$$\int_1^{13} f(x)dx = \int_1^{8} f(x)dx + \int_8^{13} f(x)dx$$

$$= \frac{8}{2}(609 + 4.842 + 2.716) + \frac{5}{2}(2.716 + 4.395 + 2.763)$$

$$= 18844.$$

⇒ Chi phí thuê: 18844 * 4 (triệu đ) = 75 376 (triệu đ).

6.14

a) $\int_{-1}^{1} x \, dx = \frac{5}{9}$

b) Đặt $f(x) = \frac{1}{x^2+1}$

$\int_{-1}^{1} f(x)\,dx = \frac{5}{9} f\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} f(0) + \frac{5}{9} f\left(\sqrt{\frac{3}{5}}\right)$

$= \frac{19}{12} = \boxed{1,5833}$

c) Đặt $g(x) = \frac{e^x}{x^2+e^x}$

$\int_{-1}^{1} g(x)\,dx = \frac{5}{9} g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} g(0) + \frac{5}{9} g\left(\sqrt{\frac{3}{5}}\right)$

$= 1,5654$

d) Đặt $h(x) = \frac{\sin x + 1}{\cos x + 1}$

$\int_{-1}^{1} h(x)\,dx = \frac{5}{9} h\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} h(0) + \frac{5}{9} h\left(\sqrt{\frac{3}{5}}\right)$

$= 1,0924.$

e) Đặt $t(x) = \frac{x - \sin x}{x+2}$

$\int_{-1}^{1} t(x)\,dx = \frac{5}{9} t\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9} t(0) + \frac{5}{9} t\left(\sqrt{\frac{3}{5}}\right)$

$= -0,019$

6.16

$E(2) = \frac{2}{\sqrt{\pi}} \int_0^2 e^{-t^2} dt$

Đổi biến $t = x + 1$

$\Rightarrow E(2) = \frac{2}{\sqrt{\pi}} \int_0^2 e^{-t^2} dt = \frac{2}{\sqrt{\pi}} \int_{-1}^{1} e^{-(x+1)^2} \cdot \frac{2-0}{2} dx$

$= \frac{2}{\sqrt{\pi}} \left(\frac{5}{9} e^{-\left(-\sqrt{\frac{3}{5}}+1\right)^2} + \frac{8}{9} e^{-(0+1)^2} + \frac{5}{9} e^{-\left(\sqrt{\frac{3}{5}}+1\right)^2} \right)$

$= 0,99 + \gamma$