(3.1):

- Euler:
$$\begin{cases} x_{k+1} = x_k + h \\ y_{k+1} = y_k + f(x_k, y_k, z_k) \cdot h \\ z_{k+1} = z_k + g(x_k, y_k, z_k) \cdot h \end{cases}$$

- Euler cải tiến
$$\begin{cases} x_{k+1} = x_k + h_k \\ \tilde{y}_{k+1} = y_k + f(x_k, y_k, z_k) \cdot h_k \\ \tilde{z}_{k+1} = z_k + g(x_k, y_k, z_k) \cdot h_k \\ y_{k+1} = y_k + \dfrac{f(x_k, y_k, z_k) + f(x_{k+1}, \tilde{y}_{k+1}, \tilde{z}_{k+1})}{2} h_k \\ z_{k+1} = z_k + \dfrac{g(x_k, y_k, z_k) + g(x_{k+1}, \tilde{y}_{k+1}, \tilde{z}_{k+1})}{2} h_k \end{cases}$$

- Runge-Kutta:
$$\begin{cases} K_1 = h_i f(x_i, y_i, z_i) \\ I_1 = h_i g(x_i, y_i, z_i) \\ K_2 = h_i f\left(x_i + \tfrac{1}{2} h_i, y_i + \tfrac{1}{2} K_1, z_i + \tfrac{1}{2} I_1\right) \\ I_2 = h_i g\left(x_i + \tfrac{1}{2} h_i, y_i + \tfrac{1}{2} K_1, z_i + \tfrac{1}{2} I_1\right) \\ K_3 = h_i f\left(x_i + h_i, y_i - K_1 + 2K_2, z_i - I_1 + 2I_2\right) \\ I_3 = h_i g\left(x_i + h_i, y_i - K_1 + 2K_2, z_i - I_1 + 2I_2\right) \\ y_{i+1} \approx y_i + \tfrac{1}{6}\left(K_1 + 4K_2 + K_3\right) \\ z_{i+1} \approx z_i + \tfrac{1}{6}\left(I_1 + 4I_2 + I_3\right) \end{cases}$$

- Vi phân bậc 2:

Bài)
$$\begin{cases} y'' = f(x, y, y') \\ y(x_0) = y_0, \quad y'(x_0) = z_0 \end{cases}$$

Đặt $z = y' \Rightarrow z' = y'' \Rightarrow$
$$\begin{cases} y' = z & y(x_0) = y_0 \\ z' = f(x, y, z) & z(x_0) = z_0 \end{cases}$$

(8.2) $u' = f(x, u, v)$; $v' = g(x, u, v)$

a. $\begin{cases} x_1 = 0,5 \\ u_1 = 0 + f(0; 0; 1) \cdot 0,5 = 1 \quad ; \text{ N}_0 \text{ chính xác} = 0,98 \\ v_1 = 1 + g(0; 0; 1) \, 0,5 = 0,1 ; \text{ n}_0 \text{ chính xác} = 0,828 \end{cases}$

b. $\begin{cases} x_1 = 0,5 \\ u_1 = 1 + f(0; 1; 0) \cdot 0,5 = 1,5 ; \text{ N}_0 \text{ chính xác} = 1,15 \\ v_1 = 0 + g(0; 1; 0) = 0 \qquad\qquad ; \text{ N}_0 \text{ chính xác} = 0,125 \end{cases}$

(8.3)

a. Đặt $z = y' \Rightarrow z' = y''$

$\begin{cases} y' = z = f(x, y, z) \\ z' = e^{-x} - y - 2z = g(x, y, z) \\ y(0) = 1 ; z(0) = 2 \end{cases}$

$\begin{cases} x_1 = 0,25 \\ y_1 = 1 + f(0; 1; 2) \cdot 0,25 = 2,5 \Rightarrow \text{Sai số} = 0,8322 \\ z_1 = 2 + g(0; 1; 2) \cdot 0,25 = -2 \end{cases}$

b. Đặt $z = y' \Rightarrow z' = y''$

$\begin{cases} y' = z = f(x, y, z) \\ z' = x^2 - \dfrac{y}{2} + 3z = g(x; y; z) \\ y(0) = 1 ; y'(0) = 0 \end{cases}$

$$\begin{cases} z_1 = 0 + f(0; 1; 0) \cdot 0{,}5 = -0{,}25 \\ z_1 = 0{,}5 \\ y_1 = 1 + f(0; \underline{z}) \cdot 0{,}5 = 1 \Rightarrow \text{sai số} = 0{,}026 \end{cases}$$

(8.4) Bằng phương pháp Euler $\begin{cases} t = 3 ; t_0 = 0; y(0) = 1000; z(0) = 90 \\ y'(t) = f(t, y, z) ; z'(t) = g(t, y, z) \end{cases}$

$t_1 = 3$

$y_1 = 1000 + f(0; 1000; 40) \cdot 3 = \boxed{1120 \ (con)}$

$z_1 = 40 + g(0; 1000; 40) \cdot 3 = \boxed{40 \ (con)}$

(8.5) phương pháp Euler $\begin{cases} t_1 = 2; t_0 = 0; y(0) = 1000; z(0) = 200 \\ y'(t) = f(t, y, z); z'(t) = g(t, y, z) \end{cases}$

$t_1 = 2$

$y_1 = 1000 + f(0; 1000; 200) \cdot 2 = -400 \Rightarrow \boxed{0 \ (con)}$

$z_1 = 200 + g(0; 1000; 200) \cdot 2 = \boxed{120 \ (con)}$

(8.6) Đặt $z = p'(t) = y'; z' = p''(t) = y''$

$\begin{cases} y' = z = \\ y'' = 0{,}2 y' \left(1 - \dfrac{2y}{3000}\right) = f(x, y, z). \end{cases}$

a. $y(0) = 2400; y'(0) = 100$

$t_1 = 3$

$y_1 = 2400 + f(0; 2400; 100) \cdot 3 = 2300$

b. $y(0) = 3500; y'(0) = -120$

$t_1 = 3$

$y_1 = 3500 + f(0; 3500; -120) = 3500$

(8.I). $z = y' = \theta''(t)$; $z' = y'' = \theta''(t)$.

$$\begin{cases} y' = z \\ y'' = -9,8 \sin y = f(x, y, z) \end{cases}$$

a. $y(0) = 0,2$; $y'(0) = 1$

$t_1 = 5$

$y_1 = 0,2 + f(0; 0,2; 1) \cdot 5 = -9,535$

b. $y(0) = 0$; $y'(0) = 2$

$t_1 = 5$

$y_1 = 0 + f(0; 0; 2) \cdot 5 = 0$