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MSSV: 20120049 ; Lớp: 20CTT4

4.13

a. Gọi X là độ sâu của lớp sinh học. X có phân phối đều $X \sim U(7{,}5;20)$

$\Rightarrow f(x) = \frac{1}{20-7{,}5} = \frac{1}{12{,}5}$

b. Ta có hàm mật độ

$$f(x) = \begin{cases} 0 & \text{chỗ khác} \\ \frac{1}{12{,}5} & 7{,}5 \le x \le 20 \end{cases}$$

* $x < 7{,}5$: $F(x) = \int_{-\infty}^{7,5} f(x)\,dx = 0$

$7{,}5 \le x \le 20$: $F(x) = \int_{-\infty}^{7,5} f(x)dx + \int_{7,5}^{x} f(x)\,dx = \int_{7,5}^{x} \frac{1}{12{,}5}\,dx = \frac{1}{12{,}5}x - 0{,}6$

$x > 20$: $F(x) = \int_{-\infty}^{7,5} f(x)\,dx + \int_{7,5}^{20} f(x)\,dx + \int_{20}^{x} f(x)\,dx$

$= 1$

$$F(x) = \begin{cases} 0 & ; x < 7{,}5 \\ \frac{1}{12{,}5}x - 0{,}6 & ; 7{,}5 \le x \le 20 \\ 1 & ; x > 20 \end{cases}$$

c. $F(10) = \int_{7,5}^{10} = 0{,}2 = P(X \le 10)$

$P(10 < X < 15) = F(15) - F(10) = 0{,}6 - 0{,}2 = 0{,}4$

d. $\sigma^2 = \int_{-\infty}^{+\infty} (x - \mu_x)^2 f(x)\,dx = \int_{7,5}^{20} (x - 13{,}75)^2 \cdot \frac{1}{12{,}5}\,dx = 13{,}021$

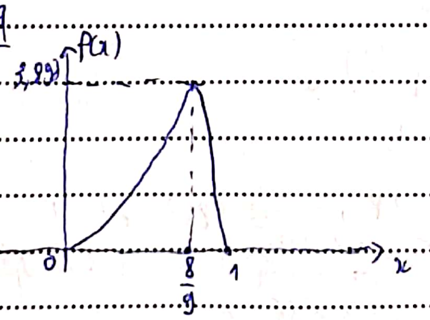
$\sigma = 3{,}608$

$E(X) = \mu_x = \int_{-\infty}^{+\infty} x\, f(x)\,dx = \int_{7,5}^{20} x \cdot \frac{1}{12{,}5}\,dx = 13{,}75$

$P(10{,}142 < X < 17{,}358) = F(17{,}358) - F(10{,}142) = 0{,}5773$

$P(6{,}534 < X < 20{,}966) = 1 - F(6{,}534) = 1$

4.19

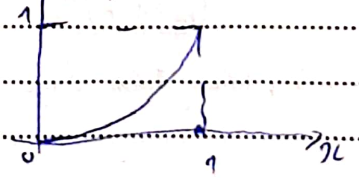
a. 

$x < 0$: $F(x) = 0$

$0 < x < 1$: $F(x) = \int_0^x 90x^8(1-x)\,dx$

$= \frac{90}{9}x^9 - 9x^{10}$

$x > 1$: $F(x) = 1$



②

b. $P(X \le 0,5) = \int_0^{0,5} 90x^8(1-x)\,dx = 0,0107$

$F(0,5) = 0,0107 = P(X \le 0,5)$

c. $P(0,25 \le X \le 0,5) = F(0,5) - 0,0107 = P(0,25 \le X \le 0,5)$

d. $P(X = x) = 0,25 \Leftrightarrow f(x) = 90x^8(1-x) = 0,25 \Leftrightarrow x = 0,62$

e. $E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^1 x\, 90x^8(1-x)\,dx = \frac{9}{11}$

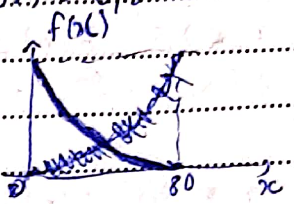
$\sigma^2 = \int_{-\infty}^{+\infty} (x-\mu_x)^2 f(x)\,dx = \int_0^1 \left(x - \frac{9}{11}\right)^2 90x^8(1-x)\,dx = 0,0124$

$\sigma = 0,1113$

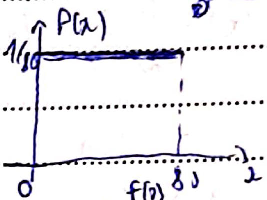
f. $P(X > 0,9295) = 1 - P(X \le 0,9295) = 1 - F(0,9295) = 0,8465$

4.15

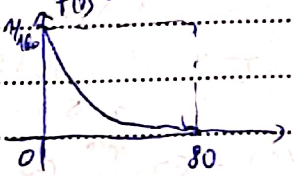
a. $f(x; 4; 80) = \begin{cases} \frac{4}{80}\left(1 - \frac{x}{80}\right)^3 & 0 \le x \le 80 \\ 0 & \text{chỗ} \ne \end{cases}$



$f(x; 1; 80) = \begin{cases} \frac{1}{80}\left(1 - \frac{x}{80}\right)^0 & 0 \le x \le 80 \\ 0 & \text{chỗ} \ne \end{cases}$



$f(x; 0,5; 80) = \begin{cases} \frac{1}{160}\left(1 - \frac{x}{80}\right)^{-0,5} & 0 \le x \le 80 \\ 0 & \text{chỗ} \ne \end{cases}$



b. $x < 0$: $F(x) = 0$

$0 \le x \le \tau$: $F(x) = 1 - \left(1 - \frac{x}{\tau}\right)^\theta$

$x > \tau$: $F(x) = 1$

c. $P(X \ge x) = P(X \le x) = 0,5$

$F(x) = 1 - \left(1 - \frac{x}{\tau}\right)^\theta = 0,5$

$\Leftrightarrow \left(1 - \frac{x}{\tau}\right)^\theta = 0,5 \Leftrightarrow \theta \ln\left(1 - \frac{x}{\tau}\right) = \ln 0,5$

$\Leftrightarrow \ln\left(1 - \frac{x}{\tau}\right) = \frac{\ln 0,5}{\theta} \Leftrightarrow 1 - \frac{x}{\tau} = \left(e^{\ln 0,5}\right)^{1/\theta}$

$\Leftrightarrow 1 - 0,5^{1/\theta} = \frac{x}{\tau} \Leftrightarrow x = \tau\left(1 - 0,5^{1/\theta}\right)$

d. $P(50 \le X \le 70) = F(70) - F(50) = 0,0195$

4.36 Gọi X là số lỗi: $X \sim (n, 10^{-5})$; $\mu = np$; $\sigma^2 = np(1-p)$, $n = 150 \cdot 10^5$

a. $P(X \le x) \approx P\left(Z < \frac{x + 0,5 - np}{\sqrt{np(1-p)}}\right) = P\left(Z < \frac{x + 0,5 - 150}{\sqrt{150}}\right) = P(Z < 0,75) = 0,227$

$(x = 150)$

Khó khăn: n quá lớn, không tính được C_n^k

4.37

3

Gọi X là số lượng hạt : $X \sim P(\lambda)$ với $P(X=x) = \frac{e^{-\lambda}.\lambda^x}{x!}$; $\mu_X = \lambda$, $\sigma_X^2 = \lambda$ ($\lambda = 1000$).

$P(X \le 950) = P(Z \le \frac{950 - 1000 + 0,5}{\sqrt{1000}}) = P(Z \le -1,581) = 0,0580$

4.38 ~~Số người từ 5 tuổi trở lên là 95 922 100 0~~

Gọi X là số người bị khuyết tật $X \sim B(1000 ; 0,193)$

$P(X \ge 200) = P(X \le 200 + 0,5) = P(Z \le \frac{200 + 0,5 - 0,193 . 1000}{\sqrt{0,193 . 1000 (1 - 0,193)}}) = P(Z \le 0,6) = 0,7258$

$\Rightarrow 1 - P(X \le 200) = 0,2743$

$P(180 \le X \le 300) = P(X \le 300) - P(X \le 180) = P(Z \le 8,61) - P(Z \le -1,002)$

$= 1 - 0,15810 = 0,8419$

4.39 Gọi X là số thiết bị hỏng : $X \sim B(5000 ; 0,001)$

$P(X \ge 10) = P(Z \ge \frac{10 - 0,5 - 0,001 . 5000}{\sqrt{0,001 . 5000 . 0,999}}) = P(Z \ge 2,013) = 1 - P(Z \le 2,013)$

$= 1 - 0,978 = 0,022$.

4.40 $X \sim P(\lambda)$; $P(X = x) = \frac{e^{-\lambda}.\lambda^x}{x!}$; $\mu = \lambda$, $\sigma^2 = \lambda$; $\lambda = \frac{10000}{10} = 1000$

$P(X \ge 1000) = P(Z \ge \frac{1000 - 1000 + 0,5}{\sqrt{1000}}) = P(Z \ge 0,02) = 1 - P(Z \le 0,02) \approx 0,5$

4.41 $X \sim P(10.000)$; $\lambda = 10000$

a. $P(X \ge 20.000) = P(Z \ge \frac{20000 - 10000 - 0,5}{\sqrt{10000}}) = P(Z \ge 99,995) = 1 - P(Z \le 99,995)$.

$= 0$

b. $P(X \le 9900) = P(X \le 9900 + 0,5) = P(Z \le \frac{9900 + 0,5 - 10000}{\sqrt{10000}}) = P(Z \le -0,995)$

$= 0,159$.

c. $P(X \ge x) = 0,01 = 1 - P(Z \le z)$.

$\Rightarrow z = 2,32 = \frac{x + 0,5 - 10000}{\sqrt{10000}} \Rightarrow x = 10233$.

d. $P(X \ge 10200) = P(Z \ge \frac{10200 - 0,5 - 10000}{\sqrt{10000}}) = P(Z \ge 1,995) = 1 - P(Z \le 1,995)$

$= 0,023$.

$\Rightarrow \approx 8,3$ ngày / năm

5.5

Sửi: $\cancel{\sum_x \sum_y f_{XY}(x,y) = 1 \Leftrightarrow \sum_{x=1}^{3} \sum_{y=0}^{x} f_{XY}(x,y) = 1 \Leftrightarrow \sum_{x=1}^{3} (c(x+1) + c(x+2) + c(x+3))} = 1$

$\cancel{\Leftrightarrow \sum_{x=1}^{3} (c(3x+6)) = 1 \Leftrightarrow 9c + 12c + 15c = 1 \Leftrightarrow c = \frac{1}{36}}$

$\cancel{\sigma_{XY}} =$

5.5

(4) $\int_0^3\int_0^x cxy\,dy\,dx = \int_0^3\left(\frac{cxy^2}{2}\right)\Big|_0^x dx = \int_0^3\left(\frac{cx^3}{2}\right)dx = c\left(\frac{x^4}{8}\right)\Big|_0^3 = 1$

$\Leftrightarrow \frac{81}{8}c = 1 \Leftrightarrow c = \frac{8}{81}$

$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \int_0^x cxy\,dy = \left(\frac{cxy^2}{2}\right)\Big|_0^x = \frac{8}{162}x^3$

$f_Y(y) = \frac{cy^3}{3}\Big|_0^3 = \frac{8}{9}$

~~$\mu_{x+y} = \int_0^3 x f_x(x)\,dx = \int_0^3 x\frac{4}{81}x^3\,dx = \frac{12}{5}$~~

$\mu_X = \int_0^3\int_0^x x\cdot\frac{8}{81}xy\,dy\,dx = 2{,}401$

$\mu_Y = \int_0^3\int_0^x y\cdot\frac{8}{81}xy\,dy\,dx = 1{,}601$

$\sigma_{XY} = E(XY) - \mu_X\mu_Y$

$= \int_0^3\int_0^x xy\frac{8}{81}xy\,dy\,dx - 2{,}401\cdot 1{,}601 = 0{,}156$

$\rho_{XY} = \frac{\sigma_{XY}}{\sigma_X\sigma_Y} = \frac{0{,}156}{\sqrt{\int_0^3\int_0^x (y-\mu_y)^2 f_Y(x)\,dy\,dx \cdot \int_0^3\int_0^x (x-\mu_x) f_X(x)\,dy\,dx}}$

$= \frac{0{,}156}{0{,}44\cdot 0{,}24} = 1{,}477$

~~5.6 $\int_0^5\int_{x-1}^{x+1} c\,dy\,dx = c\int_0^5[(x+1)-(x-1)]\,dx = c\int_0^5 2\,dx = c(2x)|_0^5$~~

~~$= 10c = 1 \Leftrightarrow c = \frac{1}{10}$~~

~~$\mu_X = \int_0^5\int_{x-1}^{x+1} x\cdot\frac{1}{10}\,dy\,dx = \frac{1}{10}\int_0^5 (xy)\Big|_{x-1}^{x+1} = \frac{1}{10}\int_0^5[x(x+1)-x(x-1)]\,dx = \frac{5}{2} = 2{,}5$~~

~~$\mu_Y = \int_0^5\int_{x-1}^{x+1} y\cdot\frac{1}{10}\,dy\,dx = \frac{1}{10}\int_0^5\left(\frac{y^2}{2}\right)\Big|_{x-1}^{x+1}dx = \frac{1}{10}\int_0^5\left[\frac{(x+1)^2}{2} - \frac{(x-1)^2}{2}\right]dx = 2{,}5$~~

~~$\sigma_{XY} = E(XY) - \mu_X\mu_Y$~~

~~$= \int_0^5\int_{x-1}^{x+1} xy\frac{1}{10}\,dy\,dx - 2{,}5^2 = \frac{1}{10}\int_0^5\left(\frac{xy^2}{2}\right)\Big|_{x-1}^{x+1}dx - 2{,}5^2 = \frac{1}{10}\int_0^5\left[\frac{x(x+1)^2 - x(x-1)^2}{2}\right]dx - 2{,}5^2$~~

~~$= 5{,}83$~~

~~$\rho_{XY} = \frac{5{,}83}{\left(\int_0^5\int_{x-1}^{x+1} y^2\frac{1}{10}\,dy\,dx - 2{,}5\right)\left(\int_0^5\int_{x-1}^{x+1} x^2\frac{1}{10}\,dy\,dx - 2{,}5\right)} = \frac{5{,}83}{(-0{,}5)(70{,}7)} = 16{,}66$~~

Bài 5.6: trang sau cùng

⑤ 5.2. $\int_0^\infty \int_0^\infty e^{-x-y}\,dy\,dx = \int_0^\infty \left(-e^{-x-y}\right)\Big|_0^\infty dx = \int_0^\infty -e^{-x}\,dx = e^{-x} = 1$

$\mu_x = \int_0^\infty \int_0^\infty x e^{-x-y}\,dy\,dx = 1$

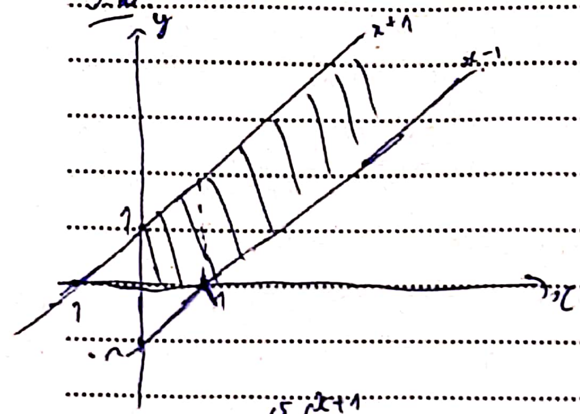
$\mu_y = 1$.

$$P_{xy} = \frac{1}{\left(\int_0^\infty \int_0^\infty x^2 e^{-x-y}\,dy\,dx - 1\right)\left(\int_0^\infty \int_0^\infty y^2 e^{-x-y}\,dy\,dx - 1\right)} = \frac{1}{(2-1)(2-1)} = 1$$

$\sigma_{xy} = E(XY) - \mu_x \mu_y$

$= \int_0^\infty \int_0^\infty xy\,e^{-x-y}\,dy\,dx - 1 = 1 - 1 = 0$.

5.



Ta có:

$$\int_1^5\int_{x-1}^{x+1} c\,dy\,dx + \int_0^1\int_0^{x+1} c\,dy\,dx = 1$$

$$\Leftrightarrow \int_1^5 c(x+1-x+1)\,dx + \int_0^1 c(x+1)\,dx = 1$$

$$\Leftrightarrow 2\int_1^5 c\,dx + c\int_0^1 (x+1)\,dx = 1$$

$$\Leftrightarrow 2.4c + \frac{3}{2}c = 1 \Leftrightarrow 9,5c = 1 \Leftrightarrow c = \frac{2}{19}$$

$$\mu_x = \int_1^5\int_{x-1}^{x+1} x\cdot\frac{2}{19}\,dy\,dx + \int_0^1\int_0^{x+1} x\cdot\frac{2}{19}\,dy\,dx$$

$$= \frac{2}{19}\int_1^5 \left(x(x+1) - x(x-1)\right)dx + \frac{2}{19}\int_0^1 x(x+1)\,dx \approx 2,614$$

$$\mu_y = \int_1^5\int_{x-1}^{x+1} y\cdot\frac{2}{19}\,dy\,dx + \int_0^1\int_0^{x+1} y\cdot\frac{2}{19}\,dy\,dx$$

$$= \frac{2}{38}\int_1^5 \left((x+1)^2 - (x-1)^2\right)dx + \frac{2}{38}\int_0^1 (x+1)^2\,dx \approx 2,404$$

$$\sigma_{XY} = E(XY) - \mu_X\mu_Y$$

$$= \int_1^5\int_{x-1}^{x+1} xy\cdot\frac{2}{19}\,dy\,dx + \int_0^1\int_0^{x+1} xy\cdot\frac{2}{19}\,dy\,dx - 2,614\cdot 2,404$$

$$= \int_1^5 \left(\frac{2}{38}xy^2\right)\Big|_{x-1}^{x+1} dx + \int_0^1 \left(\frac{2}{38}xy^2\right)\Big|_0^{x+1} dx - 6,284$$

$$= \frac{2}{38}\int_1^5 \left(x(x+1)^2 - x(x-1)^2\right)dx + \frac{2}{38}\int_0^1 x(x+1)^2\,dx - 6,284$$

$$\approx 2,492$$

$$\rho_{XY} = \frac{2,492}{\left(\int_1^5\int_{x-1}^{x+1} x^2\frac{2}{19}\,dy\,dx + \int_0^1\int_0^{x+1} x^2\frac{2}{19}\,dy\,dx - 2,614^2\right)\left(\int_1^5\int_{x-1}^{x+1} y^2\frac{2}{19}\,dy\,dx + \int_0^1\int_0^{x+1} y^2\frac{2}{19}\,dy\,dx - 2,404^2\right)}$$

$$= \frac{2,492}{\left(\frac{2}{19}\int_1^5 \left(x^2(x+1) - x^2(x-1)\right)dx + \frac{2}{19}\int_0^1 x^2(x+1)\,dx - 2,614^2\right)\left(\frac{2}{57}\int_1^5 \left((x+1)^3 - (x-1)^3\right)dx + \frac{2}{57}\int_0^1 (x+1)^3\,dx - 2,404^2\right)}$$

$$\approx 0,388$$