(1)

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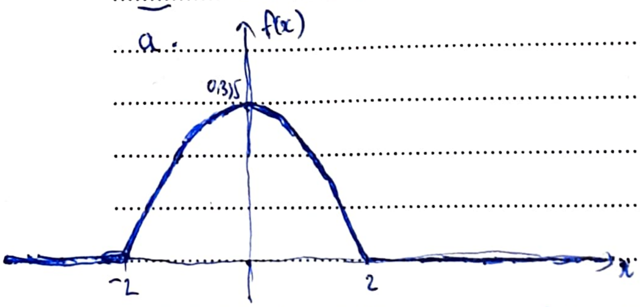
Lớp: 20CTT4 MSSV: 20120049

4.2

a.




b. $P(X>0) = \int_0^{+\infty} f(x)\,dx = \int_0^2 0{,}09375(4-x^2)\,dx = 0{,}5$

c. $P(-1<X<1) = \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} 0{,}09375(4-x^2)\,dx = 0{,}6875$

d. $P(X<-0{,}5$ hoặc $X>0{,}5) = \int_{-\infty}^{-0{,}5} f(x)\,dx + \int_{0{,}5}^{+\infty} f(x)\,dx$

$= \int_{-2}^{-0{,}5} 0{,}09375(4-x^2)\,dx + \int_{0{,}5}^{2} 0{,}09375(4-x^2)\,dx \approx 0{,}633$

4.3

Vì X là phân phối đều nên: $f(x) = \begin{cases} \frac{1}{10} & -5 \le x \le 5 \\ 0 & \text{chỗ khác} \end{cases}$

a. $P(X<0) = \int_{-\infty}^{0} f(x)\,dx = \int_{-5}^{0} \frac{1}{10}\,dx = 0{,}5$

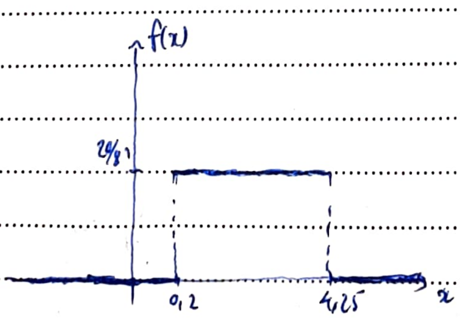
b. $P(-2{,}5<X<2{,}5) = \int_{-2{,}5}^{2{,}5} f(x)\,dx = \int_{-2{,}5}^{2{,}5} \frac{1}{10}\,dx = 0{,}5$

c. $P(-2\le X\le 3) = \int_{-2}^{3} f(x)\,dx = \int_{-2}^{3} \frac{1}{10}\,dx = 0{,}5$.

d. $P(k<X<k+4) = \int_{k}^{k+4} f(x)\,dx = \int_{k}^{k+4} \frac{1}{10}\,dx = \frac{1}{10}x\Big|_{k}^{k+4} = \frac{1}{10}(k+4-k) = \frac{4}{10} = 0{,}4$

4.4

a.




Vì X là phân phối đều nên

$f(x) = \begin{cases} \frac{20}{81} & 0{,}2 \le x \le 4{,}25 \\ 0 & \text{chỗ khác} \end{cases}$

b. $P(X\le 3) = \int_{-\infty}^{3} f(x)\,dx = \int_{0{,}2}^{3} \frac{20}{81}\,dx \approx 0{,}691$

(2)

c. $\mu_X = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{0,2}^{4,25} x \cdot \frac{20}{81}\,dx = 2,225$

Vậy độ kính thông vòng 1mm của độ kính TB là: $1,225 \le X \le 3,225$.

$P(1,225 \le X \le 3,225) = \int_{1,225}^{3,225} f(x)\,dx \approx 0,494$

d.

d. $P(a < X < a+1) = \int_a^{a+1} f(x)\,dx = \int_a^{a+1} \frac{20}{81}\,dx = \frac{20}{81} x \Big|_a^{a+1} = \frac{20}{81}(a+1-a) = \frac{20}{81}$

4.6

a. $f(x;\theta) \ge 0 \;\forall x, \theta > 0$ (1)

$\int_{-\infty}^{+\infty} f(x;\theta)\,dx = \int_0^{+\infty} \frac{x}{\theta^2} e^{\frac{-x^2}{2\theta^2}}\,dx$

$= \lim_{t \to +\infty} \int_0^t e^{\frac{-x^2}{2\theta^2}}\, d\left(\frac{x^2}{2\theta^2}\right)$

$= \lim_{t \to +\infty} \left(-e^{\frac{-x^2}{2\theta^2}} \Big|_0^t\right) = 0 - (-1) = 1$ (2)

Từ (1)(2) $\Rightarrow f(x;\theta)$ là hàm mật độ xác suất.

b. $P(X \le 200) = \int_0^{200} \frac{x}{100^2} e^{\frac{-x^2}{2\cdot 100^2}}\,dx \approx 0,865$

$P(X < 200) = P(X \le 200) \approx 0,865$

$P(X > 200) = \int_{200}^{+\infty} f(x)\,dx = \int_{200}^{+\infty} \frac{x}{100^2} e^{\frac{-x^2}{2\cdot 100^2}}\,dx$

$= \lim_{t \to +\infty} \left(-e^{\frac{-x^2}{2\cdot 100^2}} \Big|_{200}^t\right) \approx 0,135$

c. $P(100 \le X < 200) = \int_{100}^{200} f(x)\,dx = \int_{100}^{200} \frac{x}{100^2} e^{\frac{-x^2}{2\cdot 100^2}}\,dx \approx 0,471$

d. $P(X \le x) = \int_{-\infty}^{x} f(t)\,dx$

* $x \le 0$ thì $P(X \le x) = 0$
* $x > 0$ thì $P(X \le x) = \int_0^x \frac{t}{\theta^2} e^{\frac{-t^2}{2\theta^2}}\,dt = -e^{\frac{-t^2}{2\theta^2}} \Big|_0^x = -e^{\frac{-x^2}{2\theta^2}} + 1$

4.7 $f(x) = \begin{cases} 0,15 e^{-0,15(x-1)} & x \ge 1 \\ 0 & \text{chỗ khác} \end{cases}$

a. $P(X \le 5) = \int_{-\infty}^{5} f(x)\,dx = \int_1^5 0,15\, e^{-0,15(x-1)}\,dx$

$= 0,15 \int_1^5 e^{-0,15(x-1)}\, d(x-1) = 0,15 \cdot \frac{-1}{0,15} e^{-0,15(x-1)} \Big|_1^5$

$= -e^{-0,15(x-1)} \Big|_1^5 \approx 0,451$

$P(X > 5) = 1 - P(X \le 5) \approx 0,549$

③

b. $P(2 < X < 5) = \int_2^5 f(x)dx = \int_2^5 0{,}15\, e^{-0{,}15(x-1)} dx \approx 0{,}312$

4.8

a. $c = 3.10^{-9}$

b. $E(X) = \mu_x = \int_{-\infty}^{+\infty} x f(x)\, dx = \int_0^{100} x \cdot 3.10^{-9} x^2 (100 - x)^2 dx = 50$

$V(X) = \sigma_x^2 = \int_{-\infty}^{+\infty} x^2 f(x)\, dx - \mu_x^2 = \int_0^{100} x^2 \, 3.10^{-9} \cdot x^2 (100 - x)^2 dx - 50^2$

$\approx 357{,}143$

c. $P(X \geq 60) = \int_{60}^{+\infty} f(x)\, dx = \int_{60}^{100} 3.10^{-9}\, x^2 (100 - x)^2 dx = 0{,}31774$

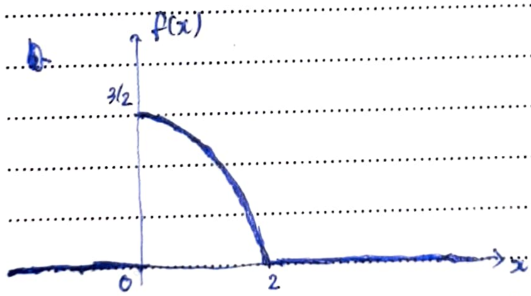
~~d. Gọi A là không gian những người đã 50 tuổi~~

d. $P(X \geq 60 \mid X \geq 50) = \dfrac{P(X \geq 60 \cap X \geq 50)}{P(X \geq 50)} = \dfrac{P(X \geq 60)}{P(X \geq 50)} = \dfrac{0{,}31774}{\int_{50}^{100} f(x)\, dx} = 0{,}63548$

4.9

a. f(x) là hàm xác suất $\Leftrightarrow \int_{-\infty}^{+\infty} f(x)\, dx = 1$

$\Leftrightarrow \int_0^2 Kx^2 dx = K \frac{x^3}{3}\Big|_0^2 = K\left(\frac{8}{3} - 0\right) = K \cdot \frac{8}{3} = 1 \Leftrightarrow K = \frac{3}{8}$




b. $P(X \leq 1) = \int_{-\infty}^{1} f(x)dx = \int_0^1 \frac{3}{8} x^2 dx = \frac{1}{8}$

c. $P(1 < X < 1{,}5) = \int_1^{1{,}5} f(x)\, dx = \int_1^{1{,}5} \frac{3}{8} x^2 dx \approx 0{,}297$

d. $P(X \geq 1{,}5) = \int_{1{,}5}^{+\infty} f(x)\, dx = \int_{1{,}5}^{2} \frac{3}{8} x^2 dx \approx 0{,}297$

4.11

a. $P(X < 0) = F(0) = \frac{1}{2} + \frac{3}{32}\left(4.0 - \frac{0^3}{3}\right) = \frac{1}{2}$

b. $P(-1 < X < 1) = F(1) - F(-1) = \frac{27}{32} - \frac{5}{32} = 0{,}6875$

c. $P(X > 0{,}5) = 1 - P(X \leq 0{,}5) = 1 - F(0{,}5) \approx 0{,}684$

4

$\begin{cases} F'(x) = 0 & x < -2 \\ F'(x) = 0 & x \geq 2 \end{cases}$

d. $F'(x) = \frac{3}{32}(4 - x^2) = f(x) \quad \forall -2 \leq x < 2$

$\begin{cases} f(x) \geq 0 \quad \forall x \in \mathbb{R} \\ \int_{-\infty}^{+\infty} f(x)\,dx = \int_{-2}^{2} \frac{3}{32}(4-x^2)\,dx = 1 \end{cases}$

Vậy $f(x)$ là hàm xác suất

e. ~~$\bar{\mu} = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_{-2}^{2} x \frac{3}{32}(4-x^2)\,dx = 0$~~

$P(X \geq 0) = 0{,}5$ và $P(X \leq 0) = 0{,}5 \Rightarrow \bar{\mu} = 0$

4.12

a. $P(X \leq 1) = F(1) = \frac{1}{4}$

b. $P(0{,}5 \leq X \leq 1) = F(1) - F(0{,}5) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$

c. $P(X \geq 1{,}5) = 1 - P(X < 1{,}5) = 1 - F(1{,}5) = \frac{7}{16}$

d. $f(x) = F'(x) = \begin{cases} 0 & x < 0 \\ \frac{x}{2} & 0 \leq x < 2 \\ 0 & x \geq 2. \end{cases}$

~~e. $\bar{\mu} = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^2 x \frac{x}{2}\,dx = \frac{4}{3}$~~

e. $P(X \leq x) = 0$ nếu $x < 0$ hoặc $x \geq 2$

$P(X \leq x) = F(x)$

$P(X \geq x) = 1 - F(x)$

Vậy $\bar{\mu} = x$ thoả $F(x) = 1 - F(x)$

$\Leftrightarrow \frac{x}{2} = 1 - \frac{x}{2} \Leftrightarrow x = 1 \Leftrightarrow \bar{\mu} = 1$

f. $E(X) = \int_{-\infty}^{+\infty} x f(x)\,dx = \int_0^2 x \frac{x}{2}\,dx = \frac{4}{3}$

g. $V(X) = \int_{-\infty}^{+\infty} x^2 f(x)\,dx - \mu_x^2 = \int_0^2 x^2 \frac{x}{2}\,dx - \left(\frac{4}{3}\right)^2 = \frac{2}{9} = \sigma^2$

$\Rightarrow \sigma_x = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$

h. $E h(X) = \int_{-\infty}^{+\infty} h(x) f(x)\,dx$

$EX^2 = \int_{-\infty}^{+\infty} x^2 \cdot \frac{x}{2}\,dx = \frac{2}{9}$