Candidate CHARTS Model Update

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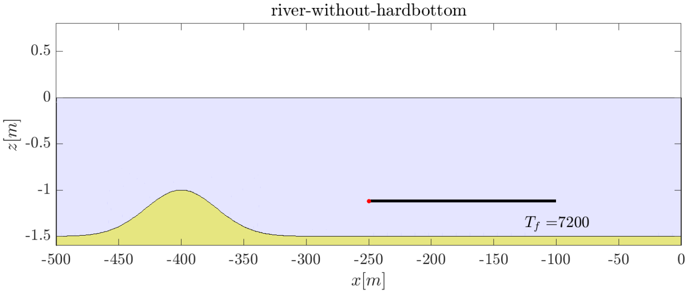
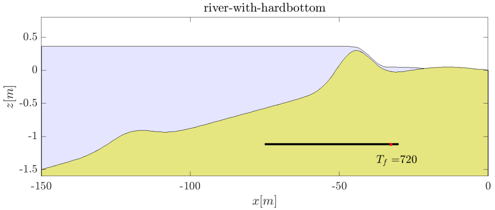


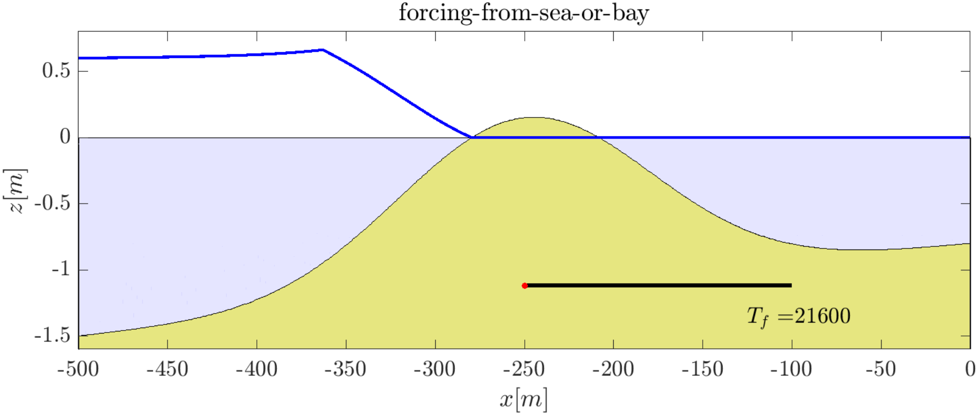
Model Review

Previous presentation introduced simple, stable, and computationally efficient hydrodynamics framework:

- One-dimensional
- Phase-averaged but low-frequency resolving
- Based on NLSW
- Heuristic wet/dry
- Emphasis on simple and efficient
- Limited options for 'ocean' and 'landward' BC
- No transport



Model now permits the specification of η on both ocean and bay sides, here with phase-lagged 'tide' and waves propagating in $+x$



Characterizing Bottom Shear

In keeping with the simple/efficient model objective, the first transport is formulated as an equilibrium energetics type, requiring shear estimates.
Instantaneous skin friction is expressed with quadratic shear

$$\tau_s = \rho c_f |u| u$$

where total velocity, $u = U_{NLSW} + U_r + \tilde{u}$ is comprised of a NLSW model current, a depth-averaged wave mass flux return current, and a wave component, respectively. The return current is expressed as a balance of the linear mass flux:

$$U_r = \frac{gH^2}{8ch}$$

Wave-Related Velocities

The impact of a skewed wave-form is admitted in the model with a near-bed velocity time-series represented as a fundamental sinusoidal component and the phase-locked first harmonic,

$$\tilde{u} = U_0 \cos \omega t + U_1 \cos 2\omega t$$

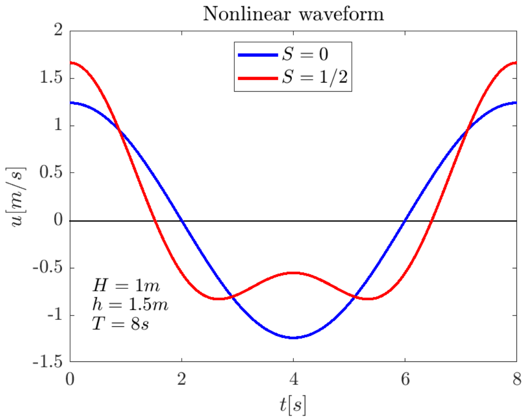
where

$$\sigma_u = \frac{\sigma_\eta \omega}{\sinh kh} = \frac{H\omega}{\sqrt{8} \sinh kh}$$

$$U_0 = \frac{\sqrt{2}\sigma_u}{\sqrt{1+S^2}}$$

$$U_1 = S U_0$$

where S is an empirical skewness param ranging from $0 \rightarrow \ 1/2$.



Equilibrium Transport Model

Equilibrium models are appropriate for regions of slowly varied bed and hydrodynamic forcing, and the following is based on an excess-shear energetics approach.

$$q_{eq} = 8B\sqrt{g(s-1)d_{50}^3}(\theta - \theta_{cr})^{3/2} \quad \text{for} \quad \theta > \theta_{cr}$$

where

$$\theta = \frac{\tau_s}{\rho g(s-1)d_{50}} \quad ; \quad \theta_{cr} \simeq 0.05$$

and B is an empirical param with $B = 1$ corresponding to the Meyer-Peter and Müller model.

Equilibrium Transport Model

Recall that the shear has both wave and current components, so time-averaging is done with a synthetic wave-current time-series and numerical summation

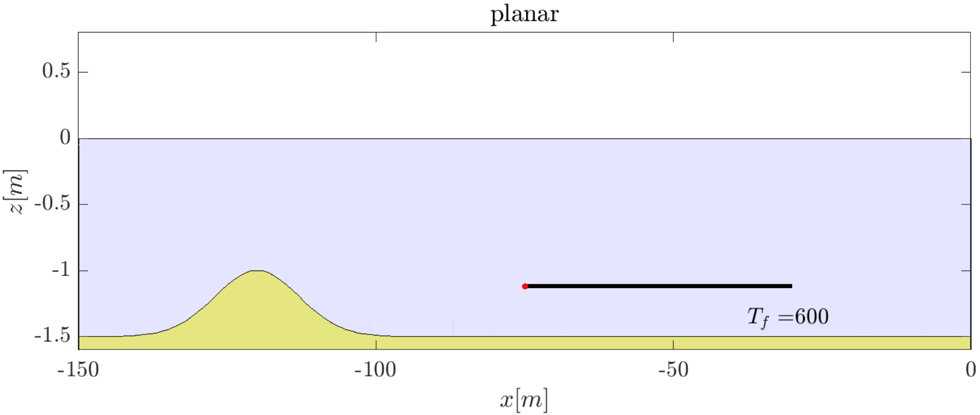
$$\overline{q_{eq}} = \frac{8B}{T}\sqrt{g(s-1)d_{50}^3}\int_0^T (\theta(t) - \theta_{cr})^{3/2} dt$$

Bottom evolution is dictated by conservation of sand

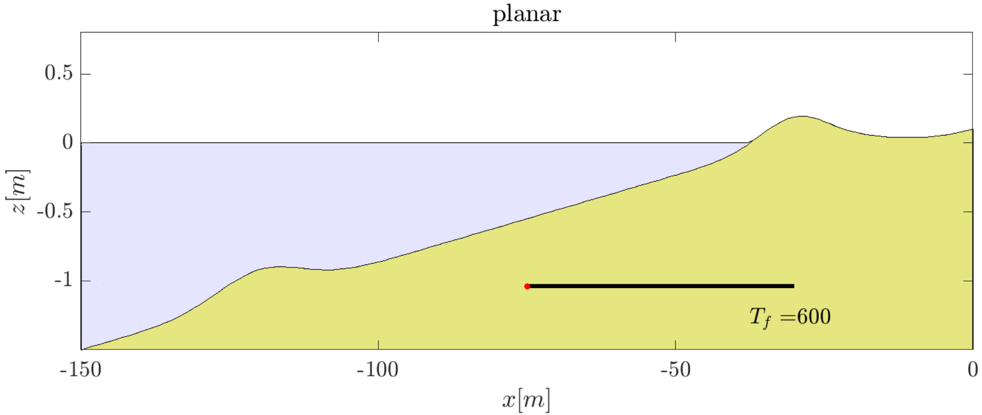
$$\frac{\partial z_b}{\partial t} = -\frac{1}{1-n}\frac{\partial \overline{q_{eq}}}{\partial x}$$

where n is the bed porosity, which is solved numerically for the bed evolution in time with an first-order time-explicit scheme with second-order central differences in space.

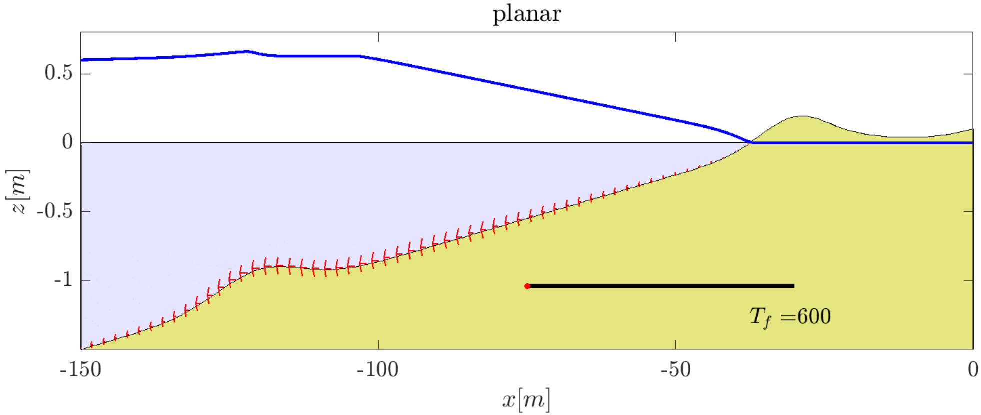
Equilibrium Transport Model for River



Equilibrium Transport Model Without Waves



Equilibrium Transport Model With Waves



Non-Equilibrium Transport Model

Of course, it is not always realistic to assume that the transport is in balance with the forcing owing to gradients in

- Temporal: Waves, in particular, can generate transport in disequilibrium owing to the rapid variation of the wave orbital velocity components. This is particularly true of the suspended transport that can exhibit temporal lag between forcing and response. I have not approached this issue at present as the previously introduced transport formulation is more consistent with a bedload algorithm, which, conventionally, is assumed to react instantaneously.
- Spatial: In the presence of pronounced variation in either forcing or sediment characteristics, transport can demonstrate significant departures from the equilibrium estimates. Of particular importance are cases that have limitations in sediment availability such as domains that have a combination of sand bed and hard-bottom.

Non-Equilibrium Transport Model

Accounting for spatial gradients is achieved by equating gradients in transport q and bed-pickup P and fallout, F

$$\frac{\partial \overline{q}}{\partial x} = P - F$$

Fallout F can be expressed in terms of near-bed concentration, c as $F = w_f c = \frac{w_f}{u\delta}\overline{q}$ which makes use of $q = u\delta c$ where δ is a bedload layer thickness $\sim 0.01m$. Similarly, the pickup, $P = A\frac{w_f}{u\delta}\overline{q_{eq}}$ where A is a modifier equal to 0 or 1 indicating the availability of sand in the bed to be suspended. Note that the formulation can also be represented with a disequilibrium model

$$\frac{\partial \overline{q}}{\partial x} = \frac{A\overline{q_{eq}} - \overline{q}}{L} \quad \text{where} \quad L = \frac{u\delta}{w_f}$$

and L has the physical interpretation of a horizontal advection length that a particle travels while falling through the boundary layer.

Non-Equilibrium Transport Model

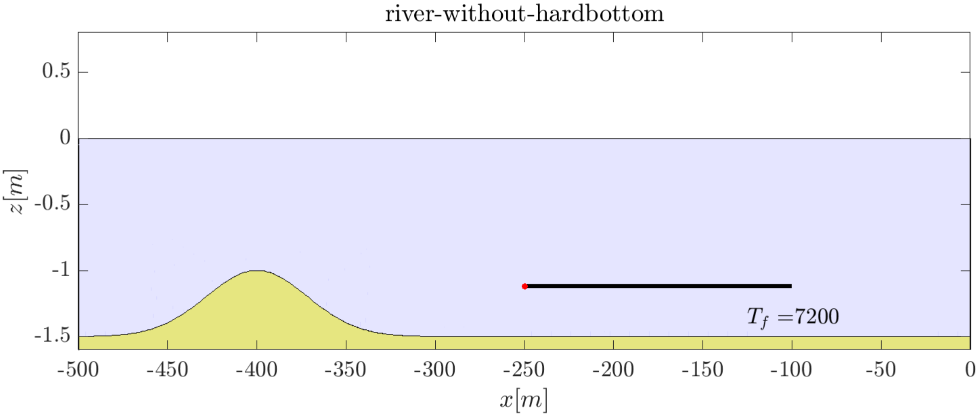
To demonstrate the numerical solution for the previously introduced expression for non-equilibrium transport, a FD statement is provided at node i:

$$\frac{q_{i+1} - q_{i-1}}{2\Delta x} = \frac{w_f}{u_i \delta} \{q_{eq_i} - q_i\}$$

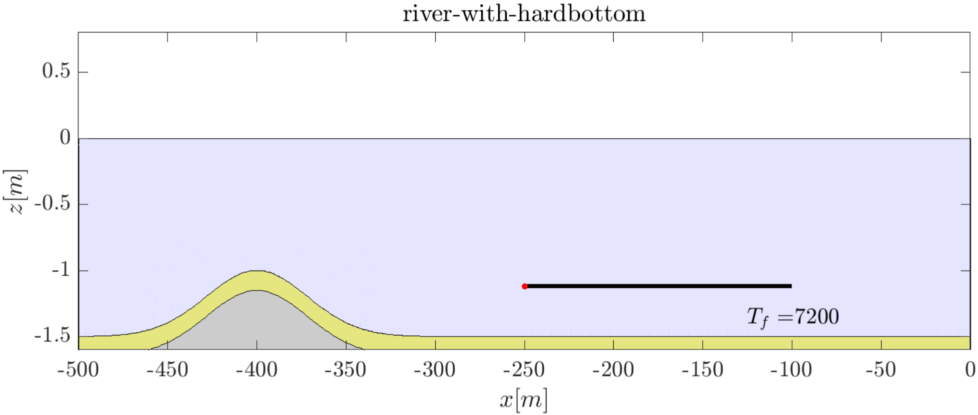
constituting a linear expression in q for nodes 2...N-1. Presently, boundaries are treated with a zero gradient with $q_1 = q_2$ and $q_N = q_{N-1}$

$$\begin{bmatrix} -1 & 1 & 0 & & & & & \\ -1 & 2\Delta x \frac{w_f}{u_2 \delta} & 1 & 0 & & & & \\ 0 & -1 & 2\Delta x \frac{w_f}{u_3 \delta} & 1 & 0 & & & \\ & & & & . & & & \\ & & & & & . & & \\ & & & & & -1 & 2\Delta x \frac{w_f}{u_{N-1} \delta} & 1 \\ & & & & & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ . \\ . \\ q_{N-1} \\ q_N \end{bmatrix} = \begin{bmatrix} 0 \\ A_2 2\Delta x \frac{w_f}{u_2 \delta} q_{eq_2} \\ A_3 2\Delta x \frac{w_f}{u_2 \delta} q_{eq_3} \\ . \\ . \\ A_{N-1} 2\Delta x \frac{w_f}{u_2 \delta} q_{eq_{N-1}} \\ 0 \end{bmatrix}$$

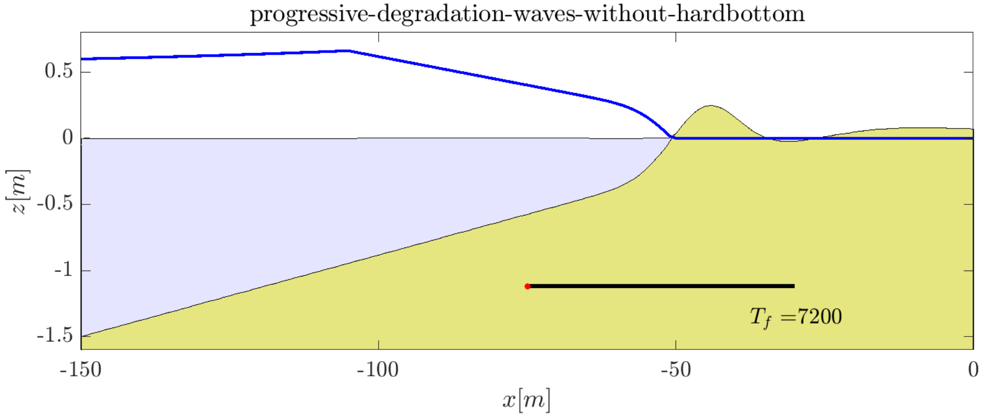
Non-Equilibrium Transport Model



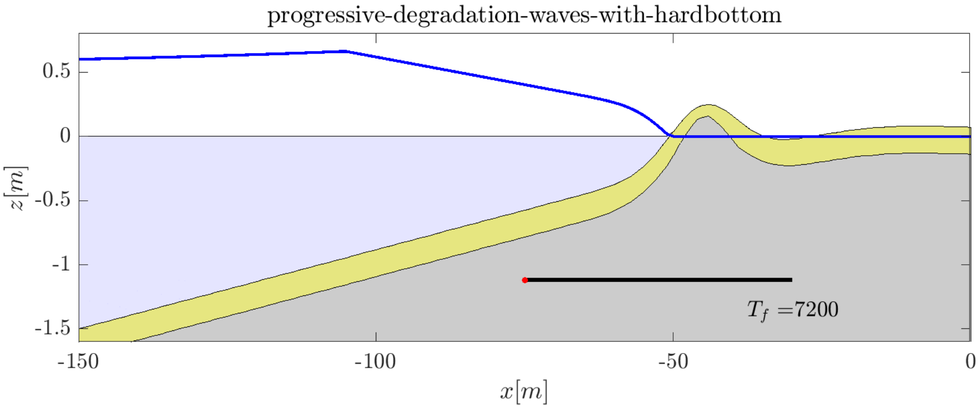
Non-Equilibrium Transport Model



Non-Equilibrium Transport Model



Non-Equilibrium Transport Model



Runtimes and Next Steps

Clock time for simulating 24 hrs

H	$50s$
$H+W$	$200s$
$H+S_{eq}$	$60s$
$H+S_{neq}$	$440s$

Kill or Continue?
If we proceed,

- Validate MSBC with Carrier and Greenspan?
- Develop suspended sed routine
- Validate sed transport or morphology (implement some sort of testbed?)
- Start considering the union of the Storm/Quiescent periods