Three Body Problem

HW 5: Thursday, Feb. 28, 2013
DUE: Thursday, Mar. 7, 2013
READ: *Numerical Recipes in C++*, Section 16.2 pages 910-920
OPTIONAL: Gerald and Wheatley, 7th edit., Runge-Kutta-Fehlberg method, p.343-347

The numerical calculation of many interacting objects is discussed further in: R.W.Hockney, J.W.Eastwood, *Computer Simulation Using Particles*, McGraw-Hill 1981,1989. This may also be a good source of ideas for your final project

PROBLEM (10 points):

This homework will numerically calculate the behavior of three objects in the Earth/Moon system interacting via gravity. The Earth/Moon coordinate system is defined as shown in Fig. 1.

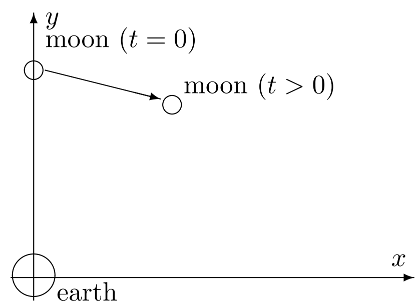

Figure 1: Earth-Moon coordinate system.

The goal will be to test the orbital stability of an arbitrary second moon. Use a Cartesian coordinate system with the moon orbiting the earth in the xy plane (for simplicity, ignore motion in the z direction), with the earth near the origin. You should also allow for the center of mass motion of the Earth. For this problem assume that the moon is traveling in a clockwise circular orbit and on the vertical y axis at time t =0. The name moon will refer to our normal moon and moon-2 will be the extra moon. The initial conditions for position (x, y) and velocity (v_x, v_y) in mks units are:

	Earth	Moon	Moon-2
x	0.0	0.0	-6.0e8
y	0.0	3.84e8	4.05e8
v_x	-12.593	1019.0	500.0
v_y	0.0	0.0	50.0

Some relevant constants are[1]:

M_e = mass of the Earth = 5.976 $\times 10^{24}$ kg
M_m = mass of the Moon = 0.0123 $\times M_e$
M_{m2} = mass of the second Moon = 0.20 $\times M_m$

T_m = orbital period of Moon = 648 hrs.
G = gravitational constant = 6.6726 $\times 10^{-11}$ Nm^2/kg^2
R_e = radius of the Earth = 6,378 km
R_m = radius of the Moon = 3,476 km
R_{m2} = radius of the second Moon = 0.5 $\times R_m$

These objects interact under the influence of gravity following the equations below (using mks units and Cartesian coordinates). There are really $2 \times 2 \times N$ coupled equations (where N is the number of interacting objects).

$$\frac{d\vec{v}_i}{dt} = \frac{1}{m_i}\vec{F}_i = G\sum_{j\neq i} m_j \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|^3} \tag{1}$$

$$\frac{d\vec{x}_i}{dt} = \vec{v}_i \tag{2}$$

where i, j=1,2,3,... for each of the objects, and $\vec{x}_i$ and $\vec{v}_i$ are the position and velocity respectively of the i^{th} object. You should map the variables $\vec{v}_i$ and $\vec{x}_i$ into a single array y[i] for use in the Runge-Kutta method. It is best to map them in a systematic way as $x_1, x_2, \cdots, y_1, y_2, \cdots$ (or similar) so that it is easier to program. Try toprogram this for an arbitrary number (N) of objects, not just this small number of objects (Hint: the main calculation can be done with a pair of embedded for loops and a variable mapping and offset constants such as:

```
y[i+xo]  = x pos. of ith object         xo = 0
y[i+yo]  = y pos. of ith object         yo = N
y[i+vxo] = vx velocity of ith object    vxo = 2*N
y[i+vyo] = vy velocity of ith object    vyo = 3*N
```

You should use the automatic step size Runge-Kutta with the embedded 5th/4th order solution as described in section 17.2 of Numerical Recipes[2] and the Dormand-Prince coefficients (in the table on page 913). To use these coefficient, equation 17.2.5 of the text should really read:

$$y^*_{n+1} = y_n + b^*_1 k_1 + b^*_2 k_2 + \cdots + b^*_6 k_6 + b^*_7 h f(x_n + h, y_{n+1}) + \mathcal{O}(h^5)$$

You may use the function Odeint() and driver StepperDopr5() from Numerical Recipes and the functions it calls or you may write your own version. This code does more than is really necessary and you may find that it is actually easier to write your own than figure out Odeint() including the routines it calls. You should use double precision and you may also want to write the data to a disk file instead of a storing it in a large array. Calculate the trajectories of all objects.

You should first test your program with the harmonic oscillator equation or equivalently track the moon's orbit for several orbits without the second moon (or just give the second moon a negligible mass). Next track the Earth, Moon and Moon-2 for 200 days or until they collide. Produce an xy graph of the trajectories of all three objects and plot the time step h versus time t to see what the auto-step size code is actually doing. Also make an expanded view of the Earth to see its small motion. Is this a stable orbit for the extra moon?

OPTIONAL: try different initial conditions to see what happens.

References

[1] Jay M. Pasachoff, in: *AIP Physics Desk Reference* E. Richard Cohen, David R. Lide, George J. Trig (editors), (Springer-Verlag, 2003), section 4.1.

[2] W. H. Press, S. A, Teukolsky, W. T. Vetterling and B. P. Flannery *Numerical Recipes, The Art of Scientific Computing, 3rd edit.*, Camb. Univ. Press 2007.