Numerical Integration

HW 2: Thursday, Jan. 31, 2013
DUE: Thursday, Feb. 7, 2013
READ: Numerical Recipes in C++, section 4.0-4.4 pages 155-172
(optional: section 4.6, 7,8 pages 179-200)
OPTIONAL:
Gerald and Wheatley, section 5.2, 5.3, 5.6

You should continue reading the book you have chosen on C/C++ programming.

PROBLEM:

Elliptic integrals occur in physics and engineering but cannot be done analytically. The complete elliptic integral of the first kind is:

$$K(x) = \int_0^{\pi/2} \frac{1}{\sqrt{1 - x\sin^2\theta}} d\theta \tag{1}$$

for $0 \leq x \leq 1$.

Write a program that uses both the trapezoid rule and Simpson's rule to numerically evaluate this integral. Test the convergence of each methods by tabulating the value of $K(x = 0.5)$ and $K(x = 0.9999)$ calculated by your program starting with N =4 intervals (i.e. the number of sampled points = 5, must be odd to use Simpson's rule). Double N several times to get $N = 4, 8, 16, \cdots, 4096$ and present your results in a short table of numbers. Both $K(x)$ and the integrand are well behaved except near x =1. Therefore $K(0.5)$ should converge quickly but $K(0.9999)$ should be more interesting. Note that $K(1.0)$ is infinite.

Next chose an appropriate N to give about 4-5 significant digits accuracy and make a graph of $K(x)$ from x =0 to x =0.9999 with about 100 points in between.

OPTIONAL: Try automating the search for a proper number of points (using the Trapezoid rule or Simpson's rule, as in section 4.2 of Numerical Recipes[2]) for the final graph.

NOTE-1: You may use the value $K(0.5) = 1.854075$ to debug your program. This is tabulated on page 608 of Abraomowitz and Stegun[1].

NOTE-2: In C/C++ the value of π can be found from: `pi = 4.0 * atan( 1.0 );`

References

[1] M. Abramowitz and I. A. Stegun, editors, *Handbook of Mathematical Functions*, National Bureau of Standards, 1964, and Dover 1965.

[2] W. H. Press, S. A, Teukolsky, W. T. Vetterling and B. P. Flannery *Numerical Recipes, The Art of Scientific Computing, 3rd edit.*, Camb. Univ. Press 2007.

An efficient recursive form of the trapezoid rule in algorithmic form, to a tolerance of tol is:

$n \leftarrow 1$ (number of intervals NOT points)
$S \leftarrow 0.5[f(x_{min}) + f(x_{max})]$
$I_{NEW} \leftarrow (x_{max} - x_{min}) \times S$
repeat
    $n \leftarrow 2n$
    $\Delta x \leftarrow (x_{max} - x_{min})/n$
    $I_{OLD} \leftarrow I_{NEW}$
    $S \leftarrow S + \sum_{i=1,3,5,\cdots,(n-1)} f(x_{min} + i\Delta x)$
    $I_{NEW} \leftarrow S\Delta x$
until $|I_{NEW} - I_{OLD}| < |tol|$ and $n > n_{min} \sim 8$
I_{NEW} is the integral

An efficient recursive form of Simpson's rule in algorithmic form, to a tolerance of tol is:

$n \leftarrow 2$ (number of intervals NOT points)
$S_1 \leftarrow f(x_{min}) + f(x_{max})$
$S_2 \leftarrow 0$
$S_4 \leftarrow f(\frac{1}{2}(x_{min} + x_{max}))$
$I_{NEW} \leftarrow 0.5(x_{max} - x_{min})(S_1 + 4S_4)/3$
repeat
    $n \leftarrow 2n$
    $\Delta x \leftarrow (x_{max} - x_{min})/n$
    $S_2 \leftarrow S_2 + S_4$
    $I_{OLD} \leftarrow I_{NEW}$
    $S_4 \leftarrow \sum_{i=1,3,5,\cdots,(n-1)} f(x_{min} + i\Delta x)$
    $I_{NEW} \leftarrow \Delta x(S_1 + 2S_2 + 4S_4)/3$
until $|I_{NEW} - I_{OLD}| < |tol|$ and $n > n_{min} \sim 8$
I_{NEW} is the integral