THE ART OF COMPUTER PROGRAMMING

VOLUME 4   PRE-FASCICLE 8B

CLIQUES
(ridiculously preliminary draft)

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Internet
Stanford GraphBase
`MMIX`

Internet page `http://www-cs-faculty.stanford.edu/~knuth/taocp.html` contains
current information about this book and related books.

See also `http://www-cs-faculty.stanford.edu/~knuth/sgb.html` for information
about *The Stanford GraphBase*, including downloadable software for dealing with
the graphs used in many of the examples in Chapter 7.

See also `http://www-cs-faculty.stanford.edu/~knuth/mmixware.html` for down-
loadable software to simulate the `MMIX` computer.

See also `http://www-cs-faculty.stanford.edu/~knuth/programs.html` for various
experimental programs that I wrote while writing this material (and some data files).

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PREFACE

But that is not my point.
I have no point.
— DAVE BARRY (2002)

THIS BOOKLET contains draft material that I'm circulating to experts in the field, in hopes that they can help remove its most egregious errors before too many other people see it. I am also, however, posting it on the Internet for courageous and/or random readers who don't mind the risk of reading a few pages that have not yet reached a very mature state. *Beware:* This material has not yet been proofread as thoroughly as the manuscripts of Volumes 1, 2, 3, and 4A were at the time of their first printings. And alas, those carefully-checked volumes were subsequently found to contain thousands of mistakes.

Given this caveat, I hope that my errors this time will not be so numerous and/or obtrusive that you will be discouraged from reading the material carefully. I did try to make the text both interesting and authoritative, as far as it goes. But the field is vast; I cannot hope to have surrounded it enough to corral it completely. So I beg you to let me know about any deficiencies that you discover.

To put the material in context, this portion of fascicle 8 previews Section 7.2.2.5 of *The Art of Computer Programming*, entitled "Cliques." I haven't had time to write much of it yet—not even this preface!

* * *

The explosion of research in combinatorial algorithms since the 1970s has meant that I cannot hope to be aware of all the important ideas in this field. I've tried my best to get the story right, yet I fear that in many respects I'm woefully ignorant. So I beg expert readers to steer me in appropriate directions.

Please look, for example, at the exercises that I've classed as research problems (rated with difficulty level 46 or higher), namely exercises ...; I've also implicitly mentioned or posed additional unsolved questions in the answers to exercises .... Are those problems still open? Please inform me if you know of a solution to any of these intriguing questions. And of course if no solution is known today but you do make progress on any of them in the future, I hope you'll let me know.

I urgently need your help also with respect to some exercises that I made up as I was preparing this material. I certainly don't like to receive credit for

things that have already been published by others, and most of these results are
quite natural "fruits" that were just waiting to be "plucked." Therefore please
tell me if you know who deserves to be credited, with respect to the ideas found
in exercises . . . . Furthermore I've credited exercises . . . to unpublished work of
. . . . Have any of those results ever appeared in print, to your knowledge?

<div align="right">Knuth</div>

<div align="center">*   *   *</div>

Special thanks are due to . . . for their detailed comments on my early attempts
at exposition, as well as to numerous other correspondents who have contributed
crucial corrections.

<div align="center">*   *   *</div>

I happily offer a "finder's fee" of $2.56 for each error in this draft when it is first
reported to me, whether that error be typographical, technical, or historical.
The same reward holds for items that I forgot to put in the index. And valuable
suggestions for improvements to the text are worth 32¢ each. (Furthermore, if
you find a better solution to an exercise, I'll actually do my best to give you
immortal glory, by publishing your name in the eventual book:–)

Cross references to yet-unwritten material sometimes appear as '00'; this
impossible value is a placeholder for the actual numbers to be supplied later.

Happy reading!

Stanford, California   D. E. K.
99 Umbruary 2020

> *The worst cliques are those which consist of one man.*
> — GEORGE BERNARD SHAW, *Back to Methuselah* (1921)

7.2.2.5. Cliques. Blah blah de blah blah blah.

<p style="text-align:center">*   *   *</p>

Who knows what I might eventually say here?

<p style="text-align:center">*   *   *</p>

Several of our examples will be based on subgraphs of the "π graph" Π, which is an infinite graph on the nonnegative integers based on π's binary digits $\pi_0 \pi_1 \pi_2 \pi_3 \pi_4 \ldots = 11001\ldots$: Vertex j is adjacent to vertex k, for $j < k$, if and only if $\pi_{j+\binom{k}{2}} = 1$. The π graph of order n is $\Pi_n = \Pi \mid \{0, 1, \ldots, n-1\}$; for example,

$$\Pi_8 \quad = \quad \text{}. \tag{11}$$

EXERCISES

50. [*M22*] For which n is Π_n planar? (See (11).)

99. [*00*] this is a temporary dummy exercise

999. [*M00*] this is a temporary exercise (for dummies)

Π_n
planar

> *After [this] way of Solving Questions, a man may steale a Nappe,*
> *and fall to worke again afresh where he left off.*
> — JOHN AUBREY, *An Idea of Education of Young Gentlemen* (c. 1684)

SECTION 7.2.2.5

50. $n \leq 9$. (When $n = 10$, identify $\{0, 1, 3, 7\}$ and $\{8, 9\}$, get a $K_{3,3}$.)

99. ...

999. ...

March 26, 2020

INDEX AND GLOSSARY

Index-making has been held to be the driest
as well as lowest species of writing.
We shall not dispute the humbleness of it;
but the task need not be so very dry.
— LEIGH HUNT, in *The Indicator* (1819)

When an index entry refers to a page containing a relevant exercise, see also the *answer* to that exercise for further information. An answer page is not indexed here unless it refers to a topic not included in the statement of the exercise.

Nothing else is indexed yet (sorry). Preliminary notes for indexing appear in the upper right corner of most pages.

If I've mentioned somebody's name and forgotten to make such an index note, it's an error (worth $2.56).