Nhat (Nick) Pham

Lecture 3: *"ROS, Positioning, and Navigation"*

(With grateful thanks to Andrew Markham and Johan Wahlstrom)

ROS
(Robot Operating System)

What is ROS?

ROS

"Robot Operating System (ROS or ros) is robotics middleware (i.e. collection of software frameworks for robot software development). Although ROS is not an operating system, it provides services designed for a heterogeneous computer cluster such as hardware abstraction, low-level device control, implementation of commonly used functionality, message-passing between processes, and package management."

--Wikipedia

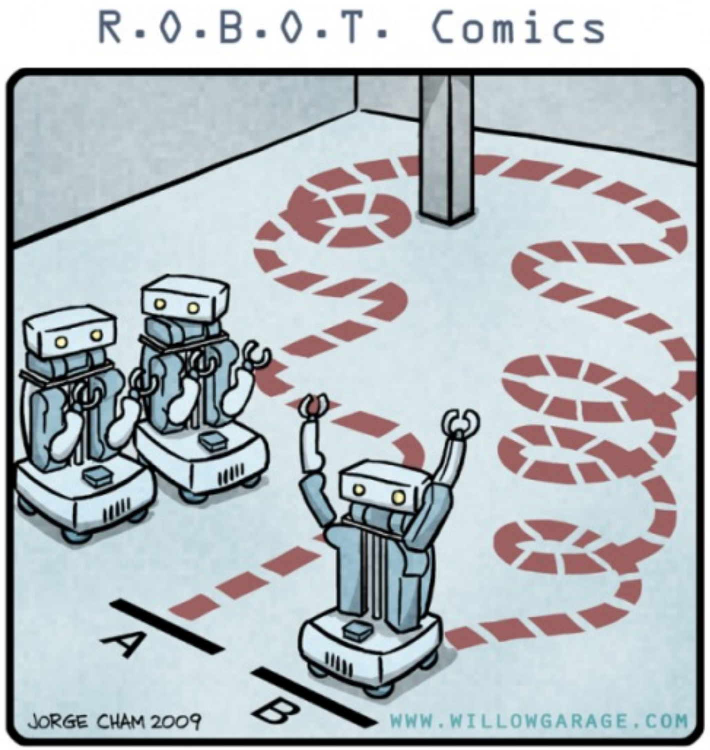
ROS.org

ROS Definitions

- **Nodes:** A process that performs computation or simply provides information to other nodes.
- **Topics:** Nodes can *publish* information to a *topic* and every node that *subscribes* to that topic will receive the same information.
- **Services:** Some nodes might need their data to be processed by other nodes (think of HTTP request) and this is done through *services*.
- **Parameter Server:** Nodes use this server to store and retrieve parameters at runtime.
- **Master:** One node to rule them all. ROS Master is a special node that keeps tabs on all topics, services and nodes that are available on the ROS Network. Communication between nodes are direct, but ROS Master tells each node useful information like IP, etc...

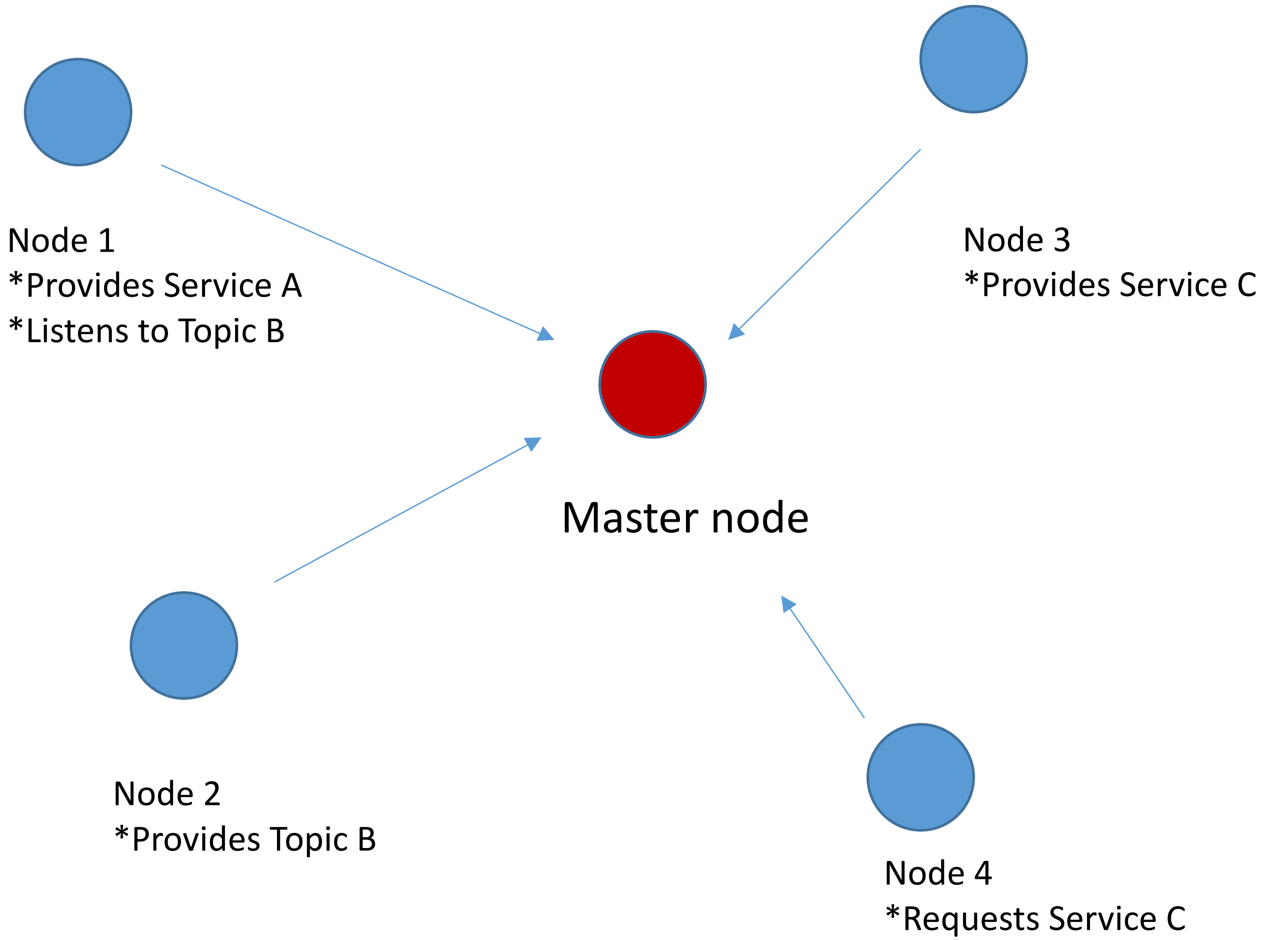
ROS Definitions

- Nodes talk to each other through *messages*. There are standard messages, but you can create your own.
- A ROS *package* will contain the necessary messages, nodes and service definitions to complete a specific task.
- You can develop your nodes in C++, Python and Lisp. MATLAB now contains a ROS Toolbox which allows you to interact with the whole ROS interface. A bit slow to be honest.

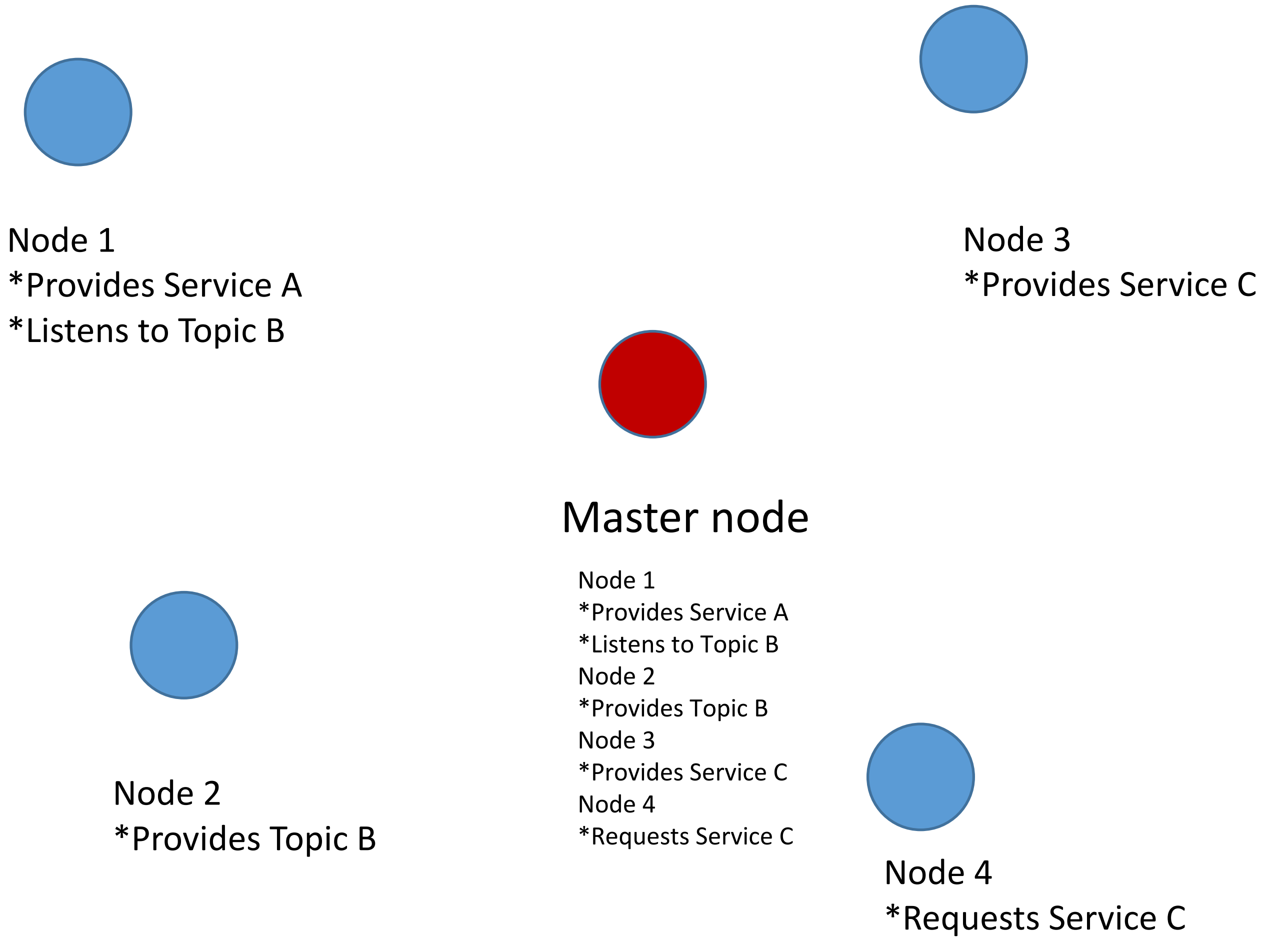


"HIS PATH-PLANNING MAY BE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

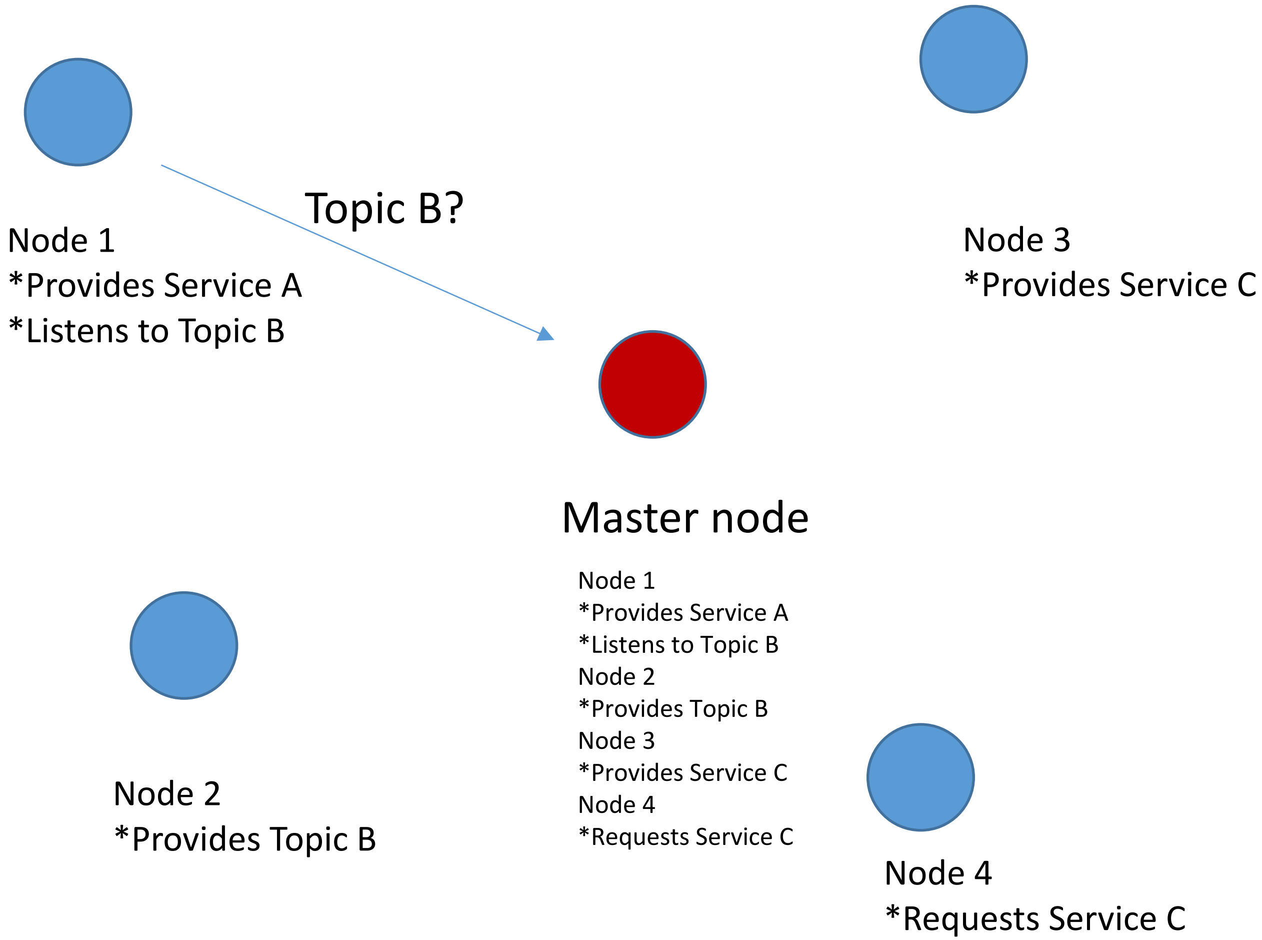
ROS Communication



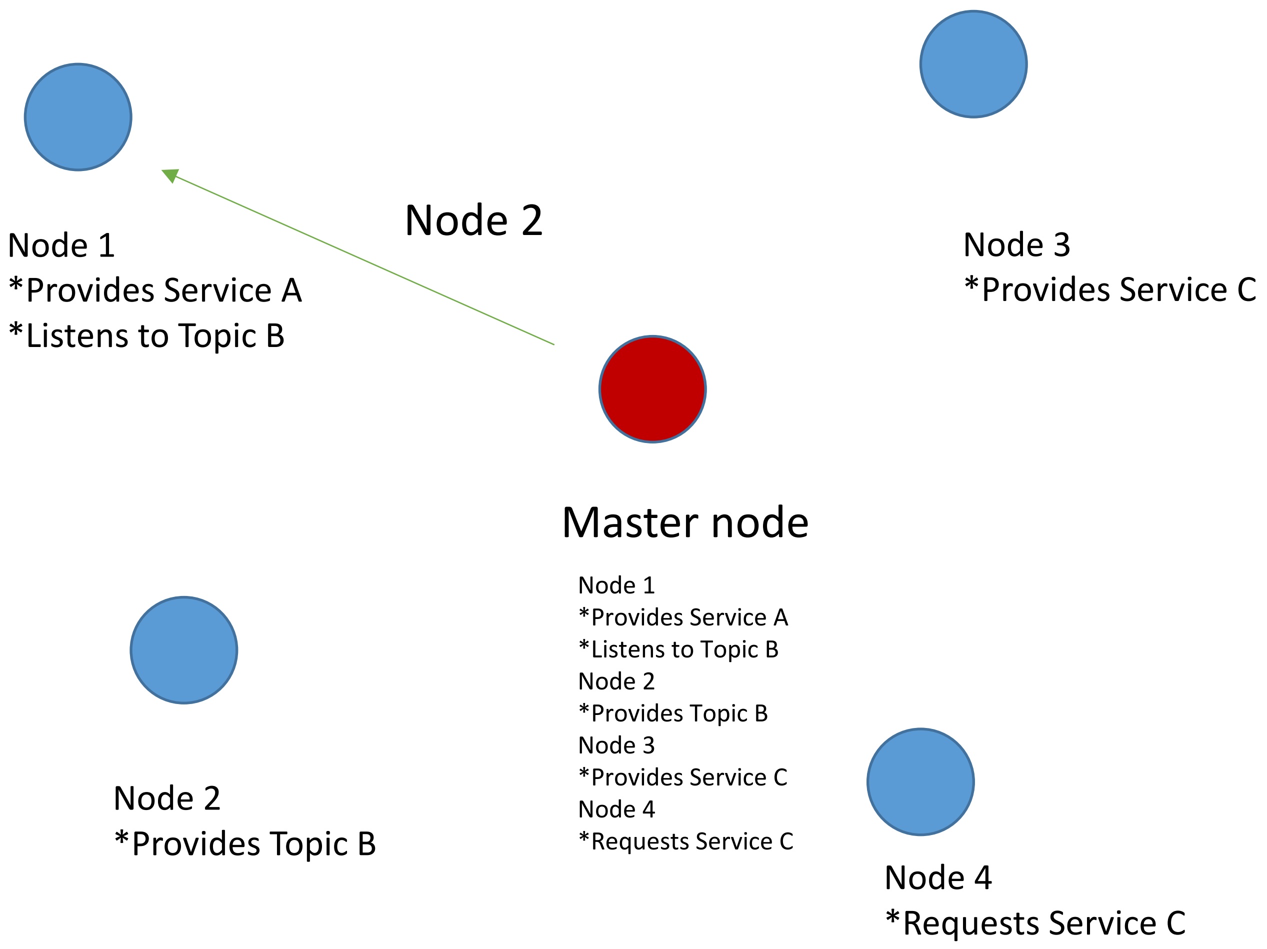
ROS Communication



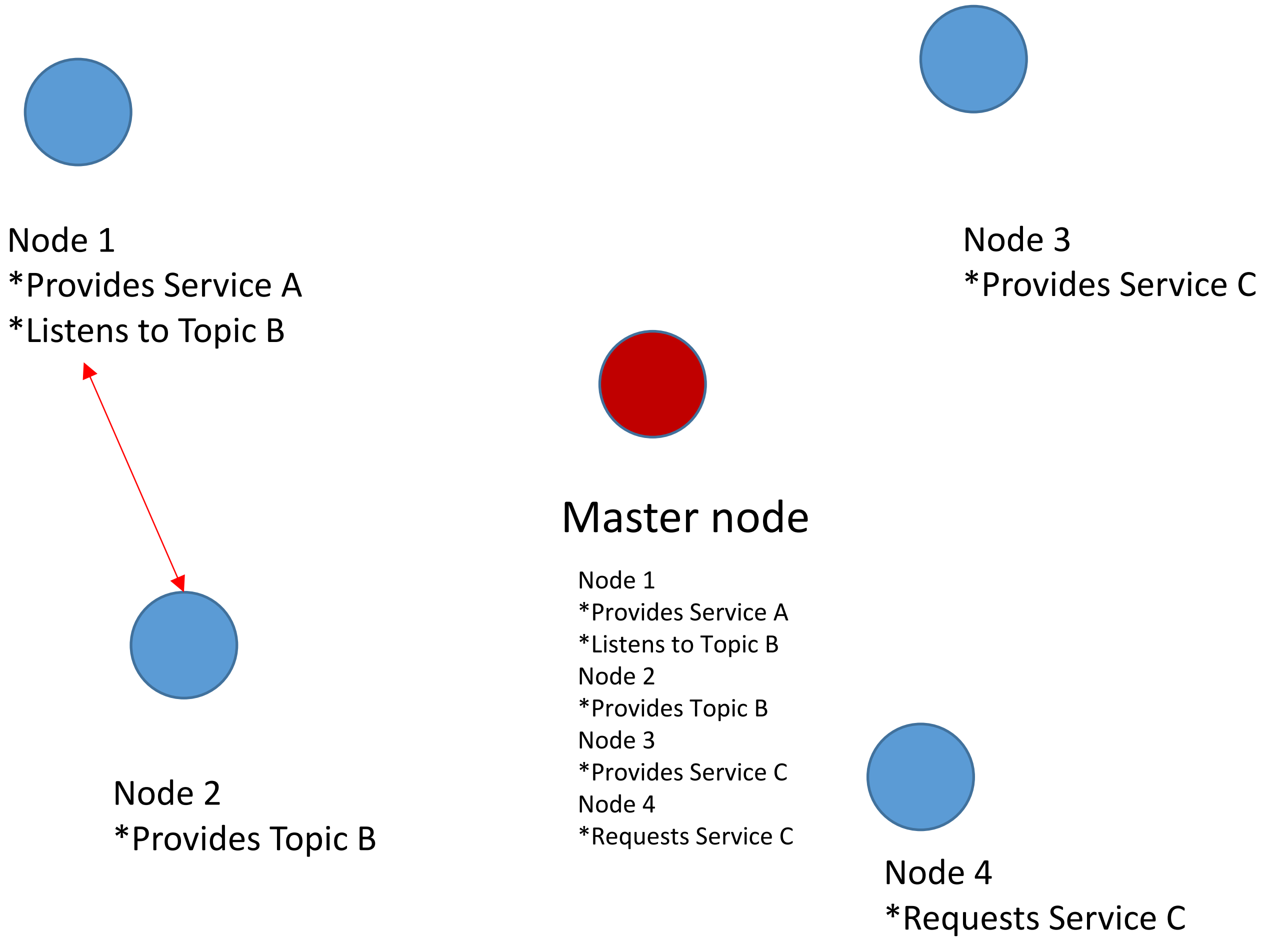
ROS Communication



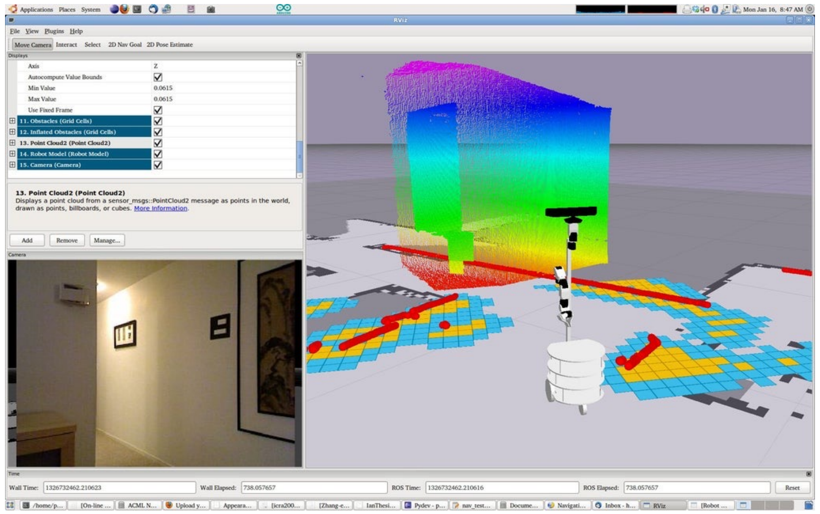
ROS Communication

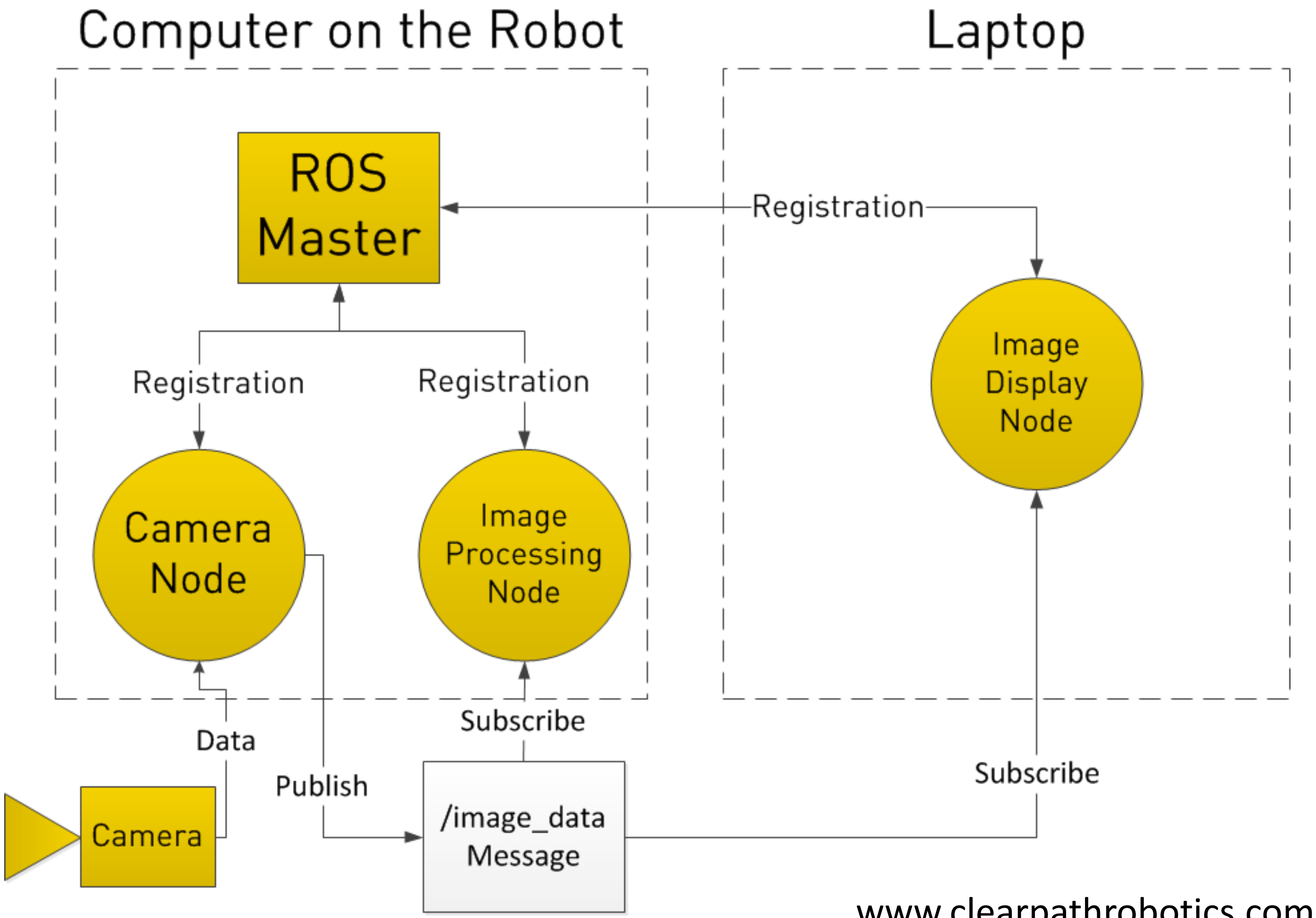


ROS Communication



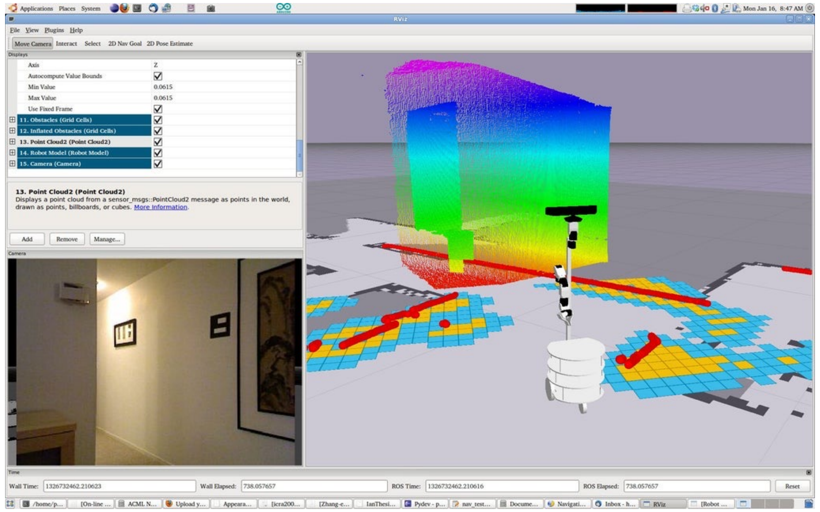
ROS - Typical Scenario

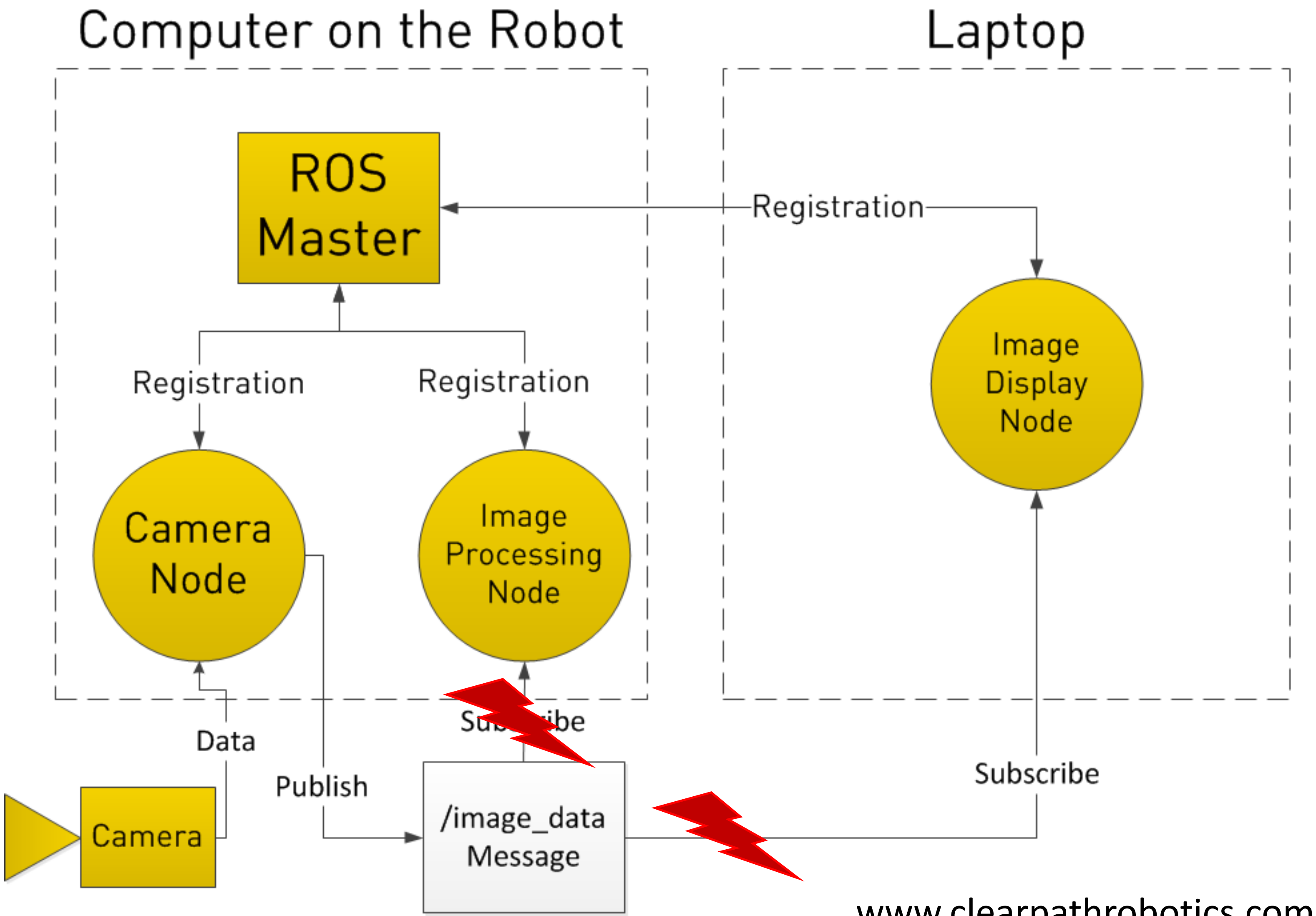




www.clearpathrobotics.com

ROS - Typical Scenario





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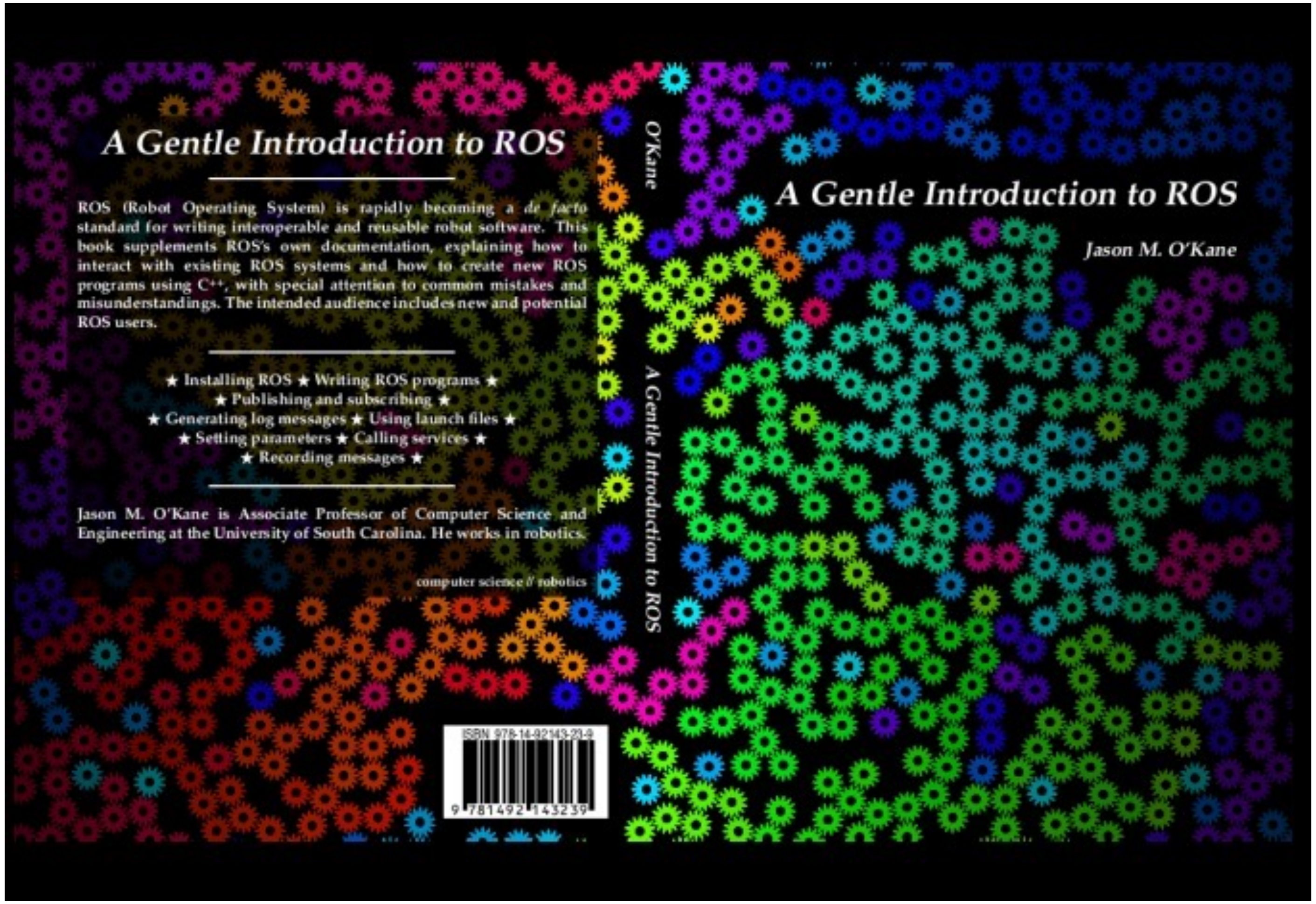
ROS - Concepts

- ❑Standard Message Definitions
- ❑Robot Geometry Library
- ❑Robot Description Language
- ❑Diagnostics
- ❑Pose Estimation
- ❑Localization
- ❑Mapping
- ❑Navigation

ROS



https://www.cse.sc.edu/~jokane/agitr/

Challenging Scenarios: Firefighter

Challenging Scenarios: Firefighter

Challenging Scenarios: Firefighter

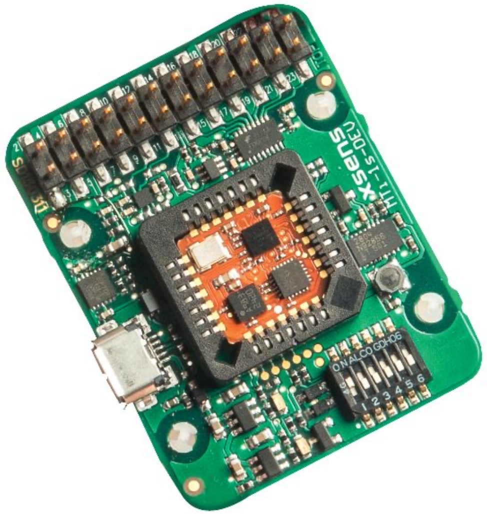
Firefighter positioning requires navigation systems that are

- Low-cost
- Independent of
  - pre-deployed infrastructure
  - fingerprinting databases
  - environmental conditions such as visibility

Inertial Navigation

(heads-up, heavy maths ahead!)

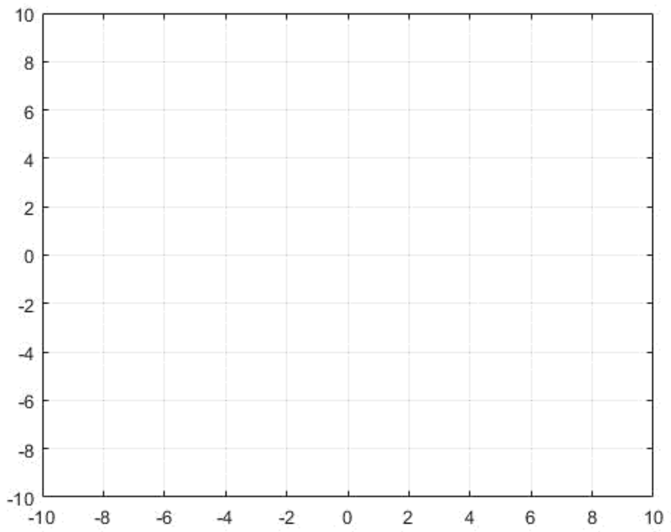
Inertial Sensors



Sensor	Measured quantity	Unit
Accelerometer	Specific force	meter/second^2
Gyroscope	Angular velocity	degree/second

IMU = inertial measurement unit (typically a three-axis accelerometer and a three-axis gyroscope)

Navigation using Foot-mounted Inertial Sensors



- Not dependent on environmental conditions
- Does not require external infrastructure
- Low-cost technology

See also:
Pedestrian Tracking with Shoe-Mounted Inertial Sensors [Foxlin 2005]
Fifteen Years of Progress at Zero Velocity: A Review [Wahlström and Skog, 2021]

Navigation using Foot-mounted Inertial Sensors

State-space Model

Key observation: when a person's walking, their feet alternate between a stationary stance phase and a moving stride phase, each lasting about 0.5 seconds.

The navigation equations $$x_{k+1} = f(x_k, u_k, \varepsilon_k)$$

The zero-velocity updates $$0 = h(x_k, w_k)$$

- x : Navigation state
- f : Navigation equations
- u : Inertial measurements
- ε : Inertial sensor errors
- h : Zero-velocity model
- w : Zero-velocity model errors

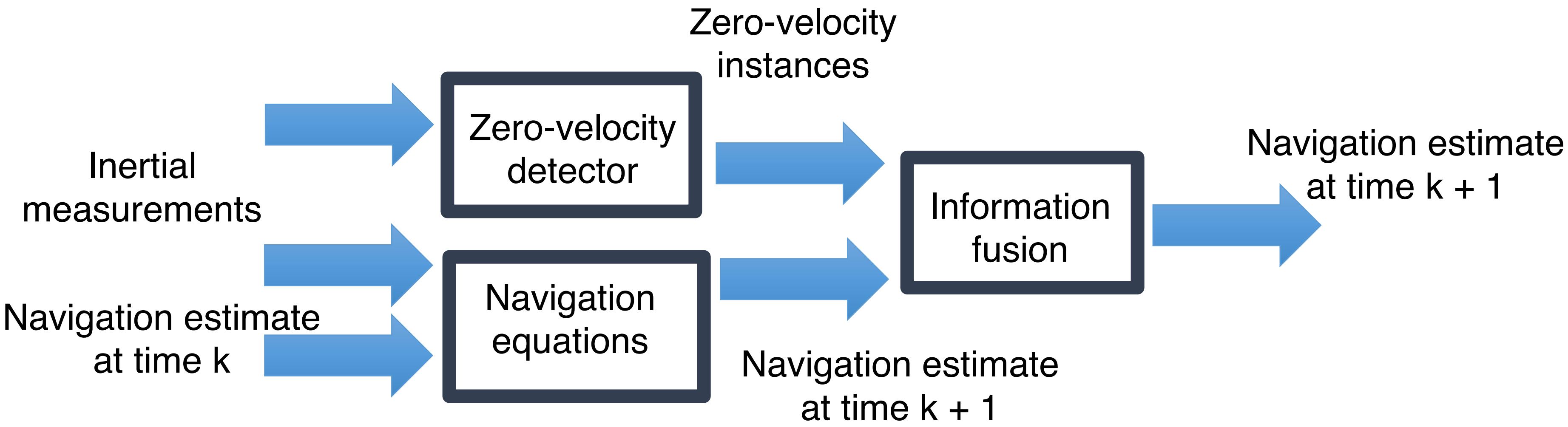
Solve the state-space model using some nonlinear filter or smoother, e.g., an extended Kalman filter.

Navigation Equations

$$p_{k+1} = p_k + \Delta t \cdot v_k$$
$$C_{k+1} = C_k(I_3 + \Delta t[\omega]^{\times})$$
$$v_{k+1} = v_k + \Delta t(C_k s + g)$$

- p : Position
- v : Velocity
- C : Orientation (matrix)
- Δt : Sampling interval
- s : Accelerometer measurements
- ω : Gyroscope measurements
- g : Gravity vector
- I_3 : 3×3 Identity matrix
- $[\,]^{\times}$: Skew symmetric matrix

Zero-velocity-aided Inertial Navigation



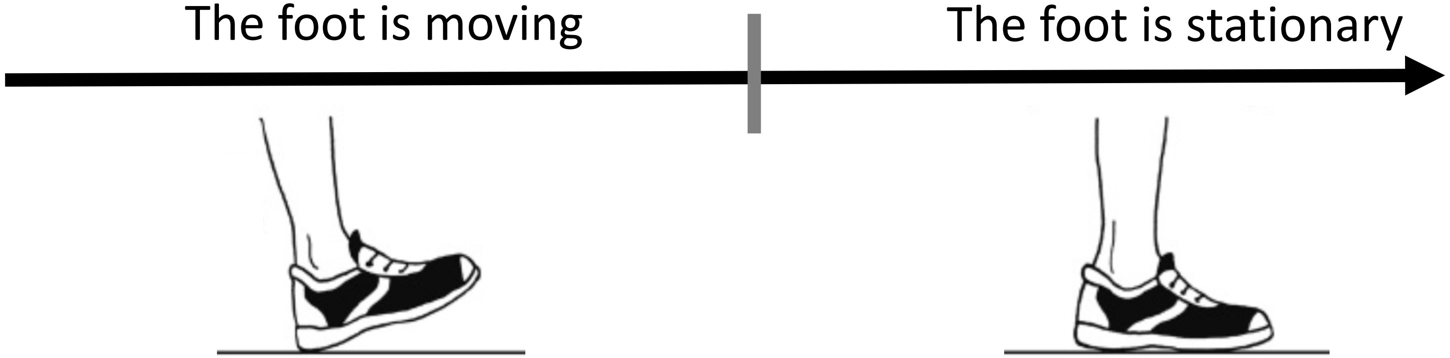
Zero-velocity Detection

Compute likelihood ratio: $L(\mathbf{z}_n) = \frac{p(\mathbf{z}_n|H_1)}{p(\mathbf{z}_n|H_0)}$

$L(\mathbf{z}_n) < \gamma$ The foot is moving

γ

$L(\mathbf{z}_n) \geq \gamma$ The foot is stationary



E.g., (1) Angular Rate Energy Detector (ARED)

$$y_k = \begin{cases} 1, & \text{if } \frac{1}{N}\sum_{n=k}^{k+N-1} \|\boldsymbol{\omega}_n\|^2 < \gamma_\omega, \\ 0, & \text{otherwise.} \end{cases}$$

(2) Stance Hypothesis Optimal Estimation (SHOE) Detector

$$y_k = \begin{cases} 1, & \text{if } \frac{1}{N}\sum_{n=k}^{k+N-1} \left(\frac{1}{\sigma_a^2} \left\| \mathbf{a}_n - g\frac{\bar{\mathbf{a}}}{\|\bar{\mathbf{a}}\|} \right\|^2 + \frac{1}{\sigma_\omega^2} \|\boldsymbol{\omega}_n\|^2 \right) < \gamma, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

Adaptive Thresholding

We show that $\gamma = \frac{1-p(H_1)}{p(H_1)} \cdot \eta$

where

- $p(H_1)$ is the prior probability that the foot is stationary
- η is a loss factor, dependent on the risk of a missed detection and a false alarm

$\Longrightarrow$ Set γ by modeling $p(H_1)$ and η.

E.g., (1) prior probability

$$p(\mathcal{H}_1) = \frac{1}{1 + e^{\beta_1 \xi_k + \beta_2}}$$

where β1 and β2 are design parameters. For ξk to be a reliable measure of how close the system is to have zero velocity.

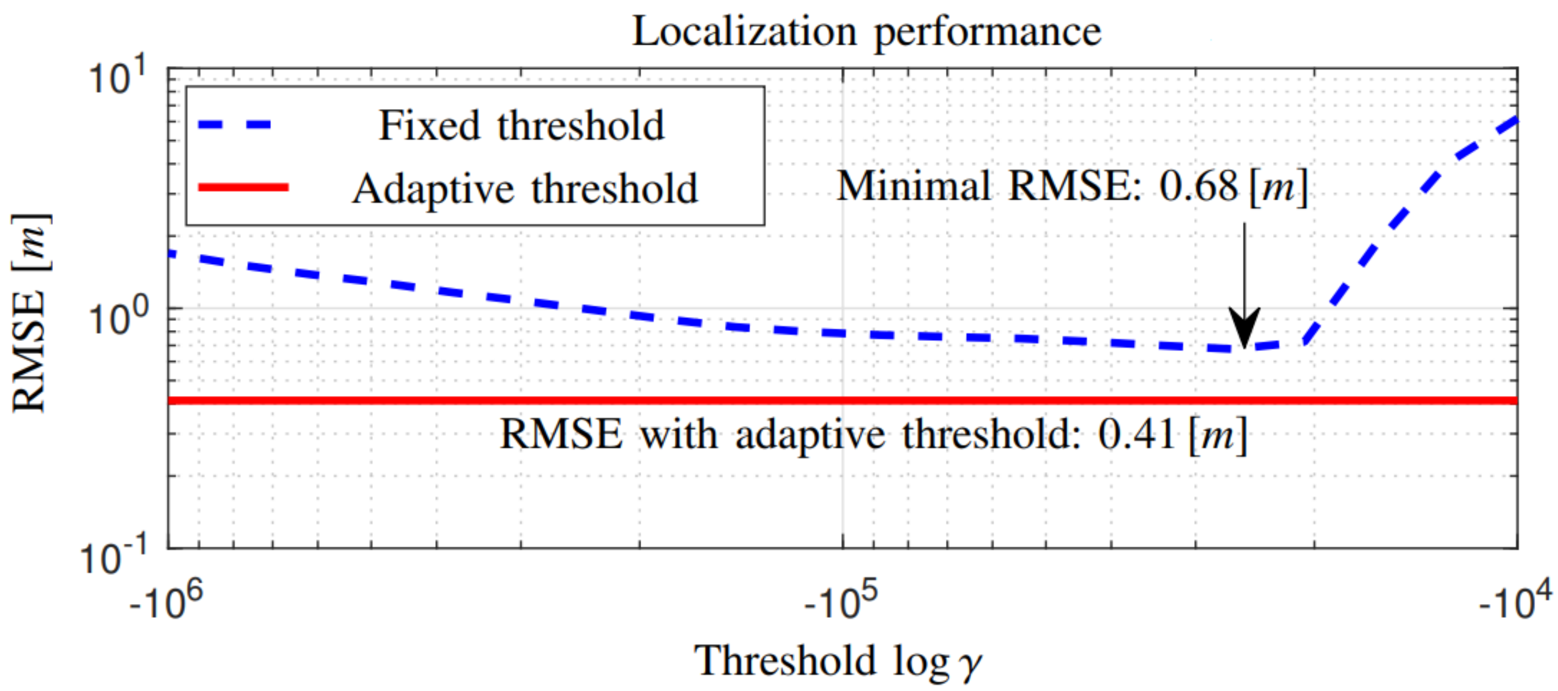
(2) Loss factor,

$$\eta = \alpha e^{-\theta \Delta t_k}$$

α and θ are design parameters, and tk is the time since the last zero-velocity instance.

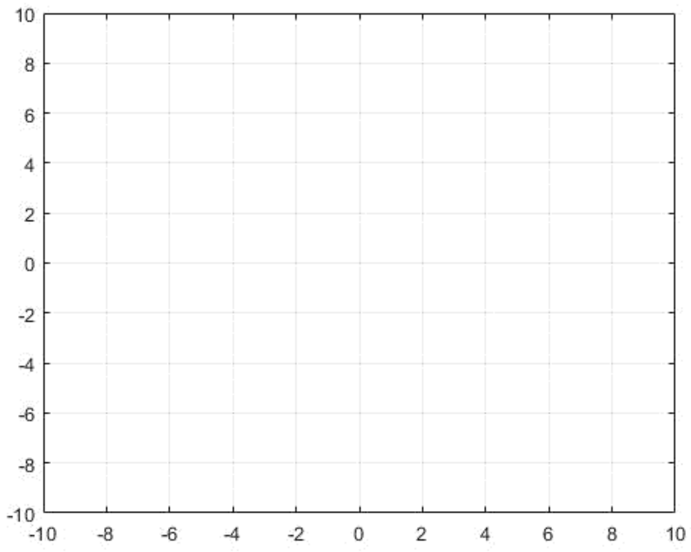
Performance Evaluation

After walking along a closed-loop trajectory
with an approximate length of 84 meter:

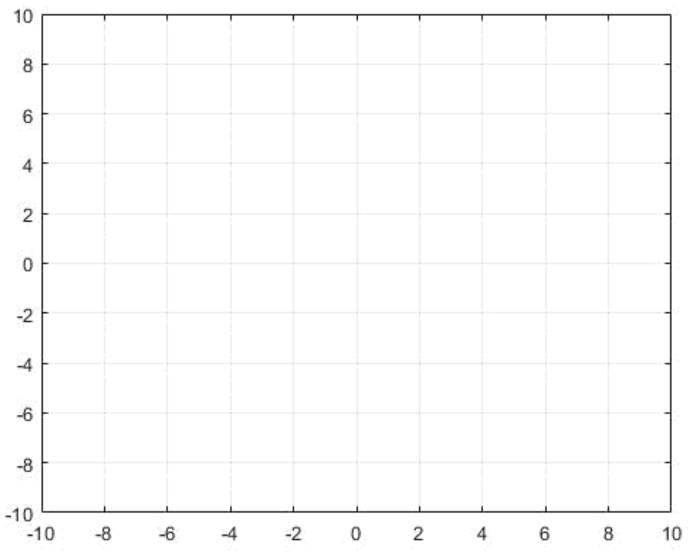


Zero-velocity-aided Inertial Navigation

<u>With</u> zero-velocity updates



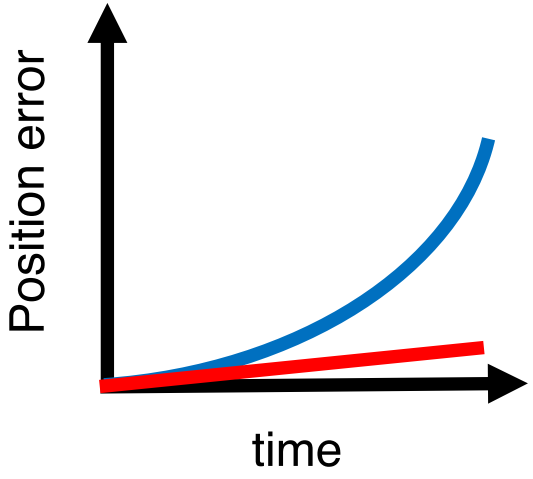
<u>Without</u> zero-velocity updates



Position Error Growth

Without zero-velocity updates:
Cubic position error growth.

With zero-velocity updates:
Linear position error growth.

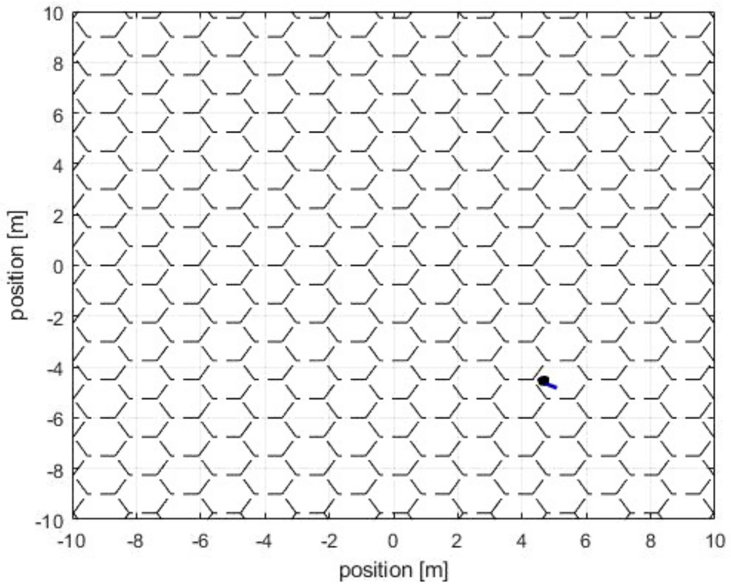


Navigation	Navigation time
Stand-alone inertial navigation	A few seconds
Zero-velocity-aided inertial navigation	Several minutes

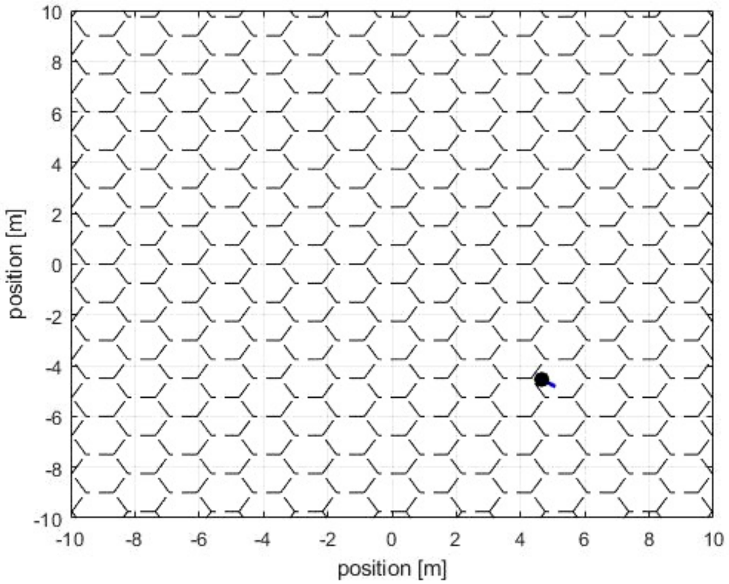
FootSLAM

Eventually the position error will grow so large that the estimates become useless
-> (1) incorporating position measurements or map information
or (2) divide the navigation area into a grid of hexagons and then learn the probability of transitioning from one hexagon to an adjacent one

FootSLAM



Inertial odometry



Magneto-Inductive Navigation

(heads-up, heavy maths ahead!)

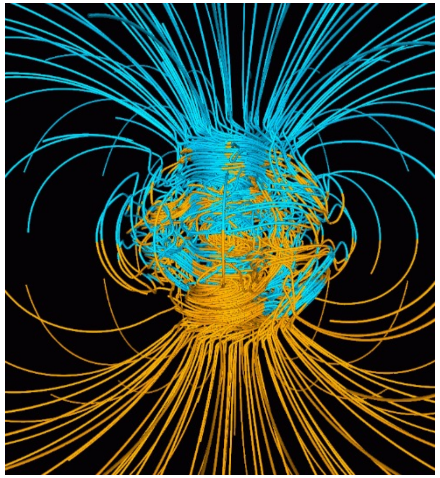
Positioning with Magnetic Field

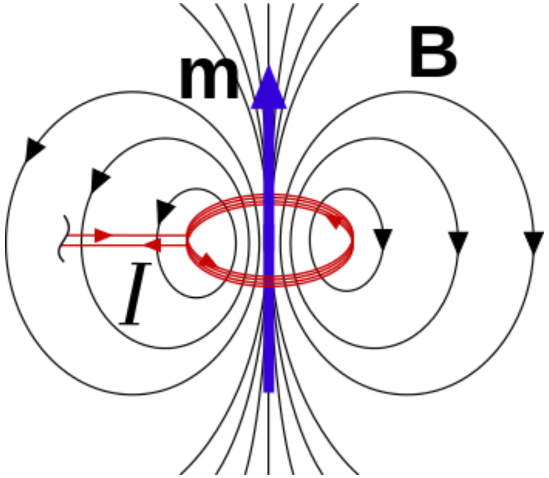
- People have been using the Earth's magnetic field for navigation since at least the 11^{th} century A.D.

- Animals too! They can sense the geomagnetic field (magnetoception)

Earth’s Magnetic Field

Earth’s magnetic field can be well approximated by a magnetic dipole

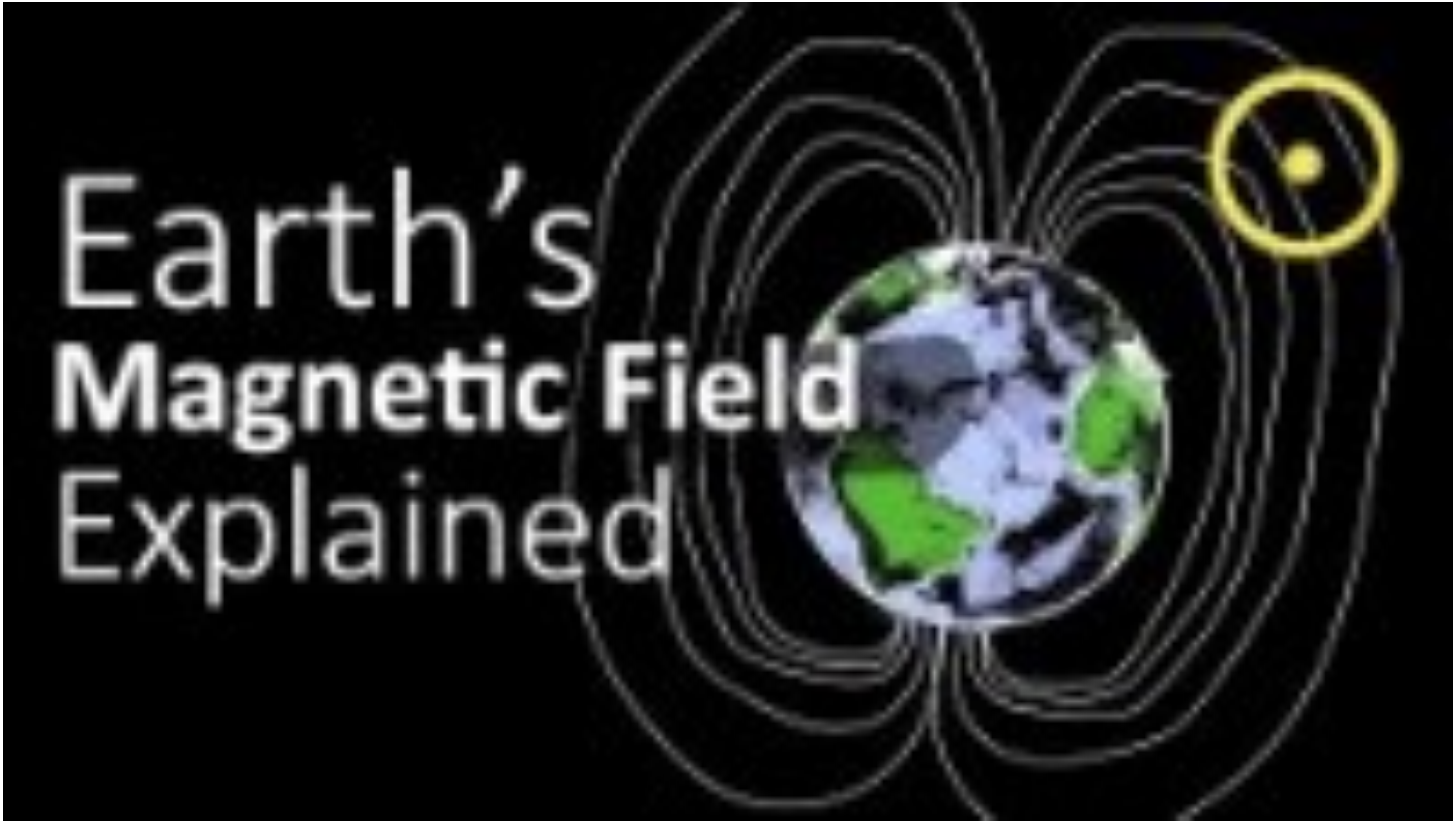




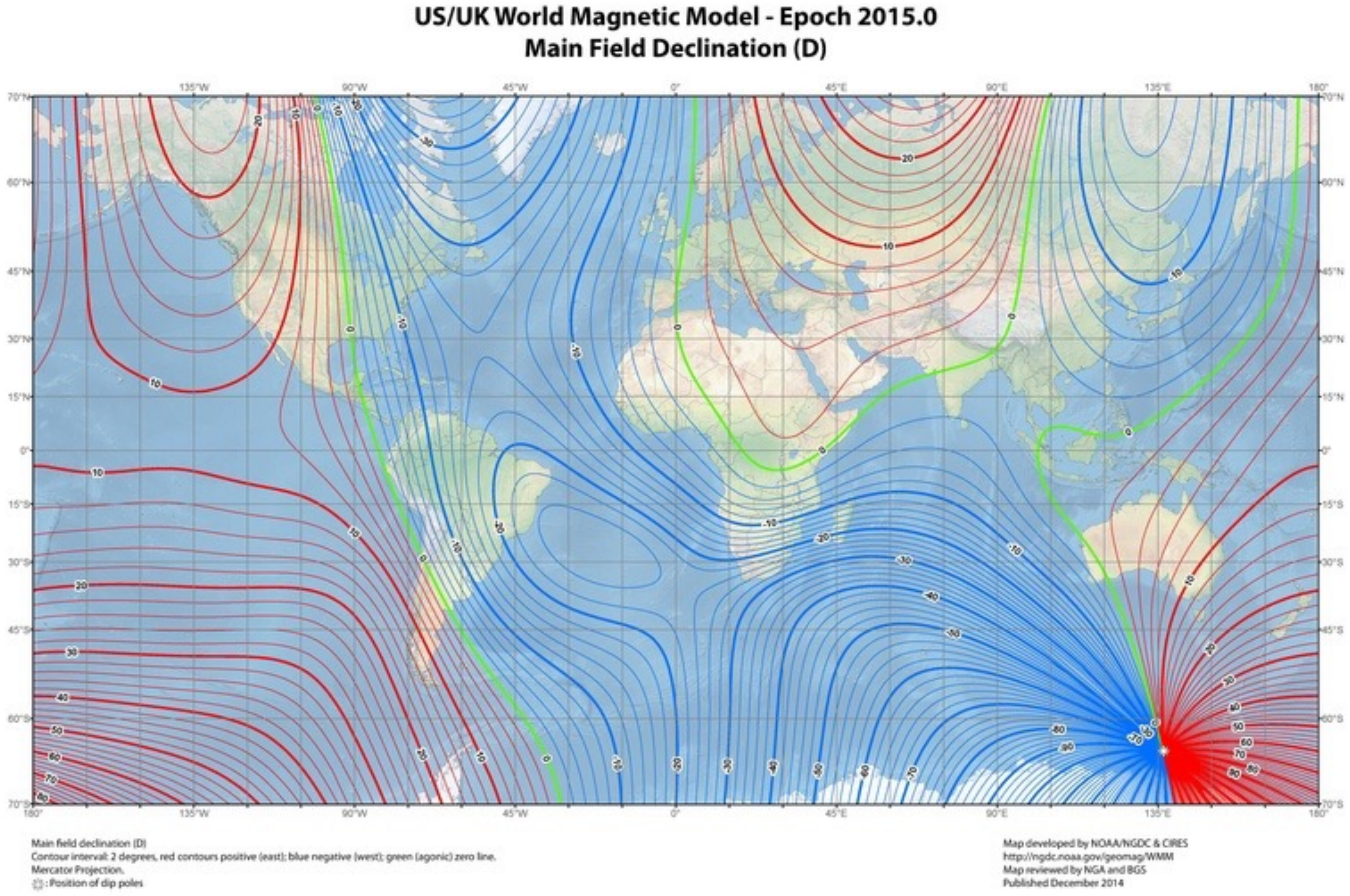
The intensity of the magnetic field varies from 0.25 Gauss at the Equator to 0.65 Gauss at the poles

Have we ever wondered why Earth has magnetic field?

Why does Earth has magnetic field?



Earth’s Magnetic Field



Earth's Magnetic Field

- The angle of the field alters over the surface of the Earth

- Submarines in particular have exploited these microvariations to position themselves without suffering from Gyro drift

- Accuracy is relatively coarse (kilometres)

Earth's Magnetic Field

Indoors, ferromagnetic material, especially in reinforced concrete, distorts the Earth's magnetic field.

Earth's Magnetic Field

- A spatial map of these distortions can be built, using techniques very similar to HORUS
- The distortions themselves are not spatially unique
- However, for someone moving through the area, the sequence of distortions can be exploited to provide positioning
- Typical accuracy is around 2-3 m and various startups (IndoorAtlas) are exploiting this technique

Earth's Magnetic Field

- Advantages:
  - No infrastructure needs to be deployed
  - Virtually all smart-phones have a magnetometer
  - Adds absolute location to traditional IMU

Earth's Magnetic Field

- Disadvantages:
  - Map has to be built and maintained
  - Users can only be tracked when they move
  - Accuracy is not good enough for many applications

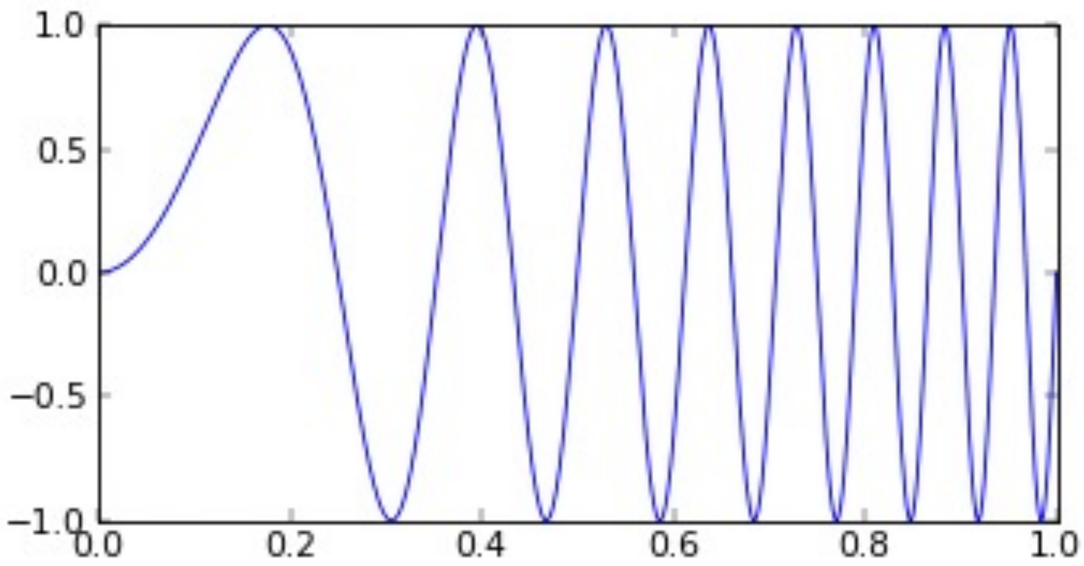
Creating your own Magnetic Field

- We can produce magnetic fields using coils of wire
  - $B \propto INA$
  - I = current, N = number of turns, A = cross-sectional area

- We can also sense a time-varying field using a coil of wire
  - $V \propto B \cos(\theta)$

Modulation

- Modulate the signal to make it easier to detect, especially compared with the strong Earth's magnetic field



RSSI vs MI: Decay

- Magnetic fields fall off more rapidly

RSSI	**MI**
RSSI $\propto 1/r^2$	IBI $\propto 1/r^3$
40 dB/decade	60 dB/decade

RSSI vs MI: Decay

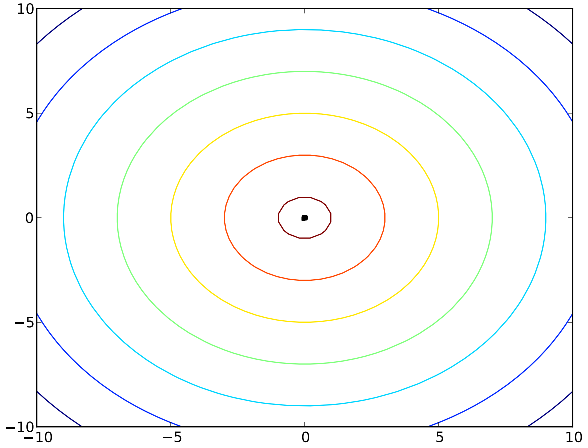
- Magnetic fields fall off more rapidly
- Range less than the traditional radio

RSSI	**MI**
RSSI $\propto 1/r^2$	IBI $\propto 1/r^3$
40 dB/decade	60 dB/decade

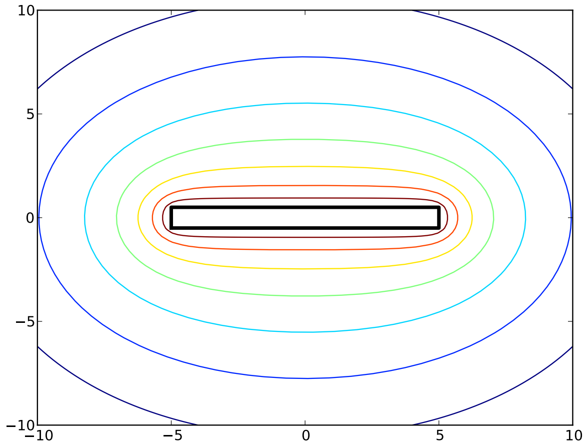
RSSI vs MI: Antennas

- Magnetic field controlled by size and shape of generating coil

RSSI



MI



RSSI vs MI: Antennas

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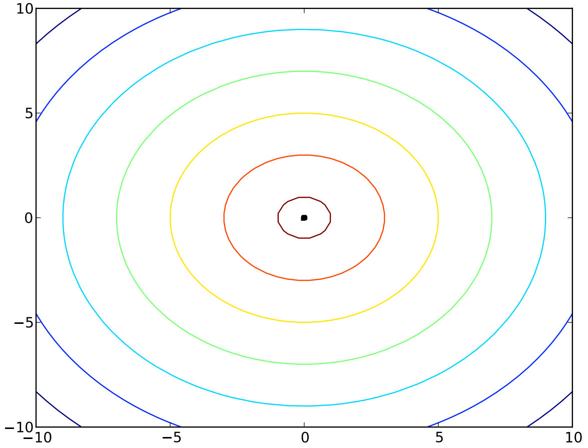
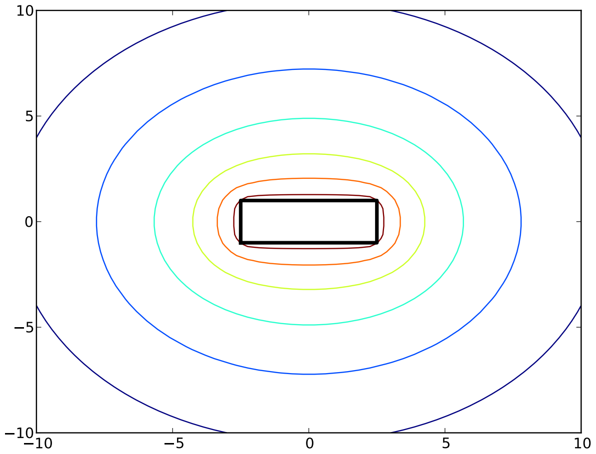
RSSI



MI



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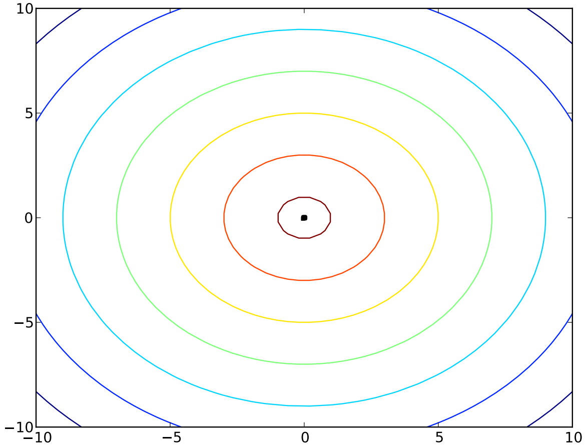
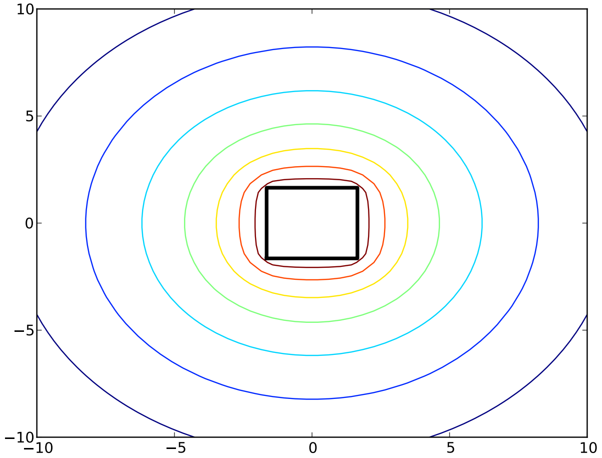
RSSI



MI



RSSI vs MI: Antennas

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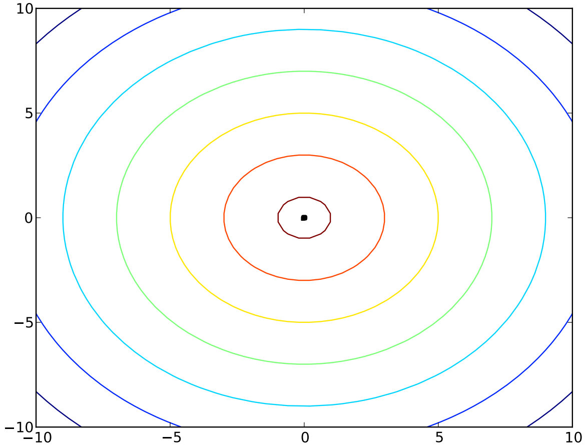
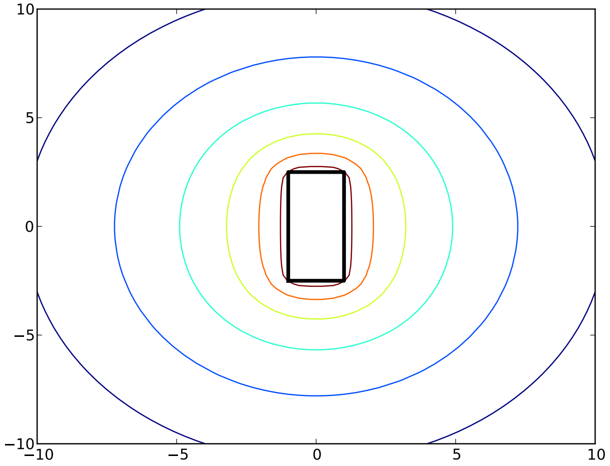
RSSI



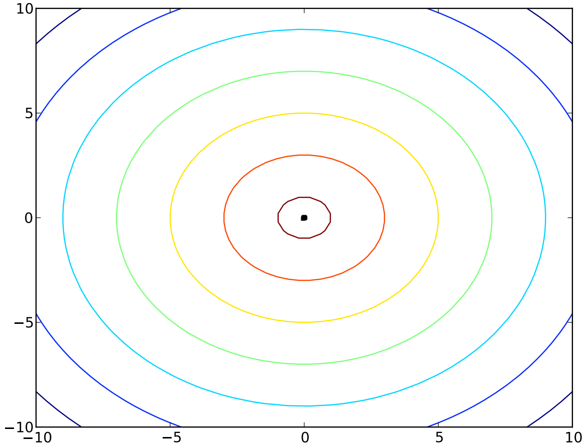
MI



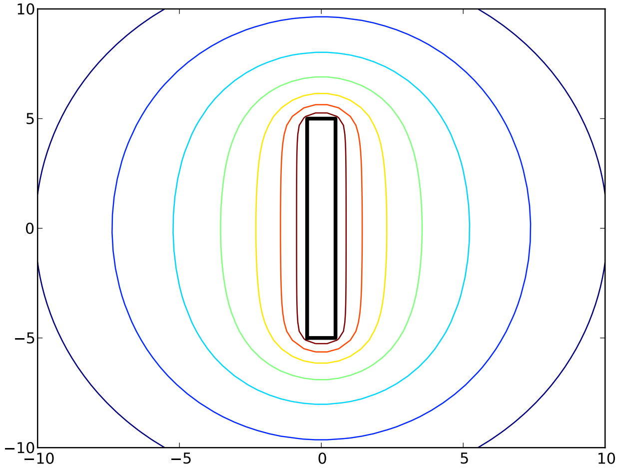
RSSI vs MI: Antennas

- Magnetic field controlled by size and shape of generating coil
- Simple to alter field patterns to optimize localization

RSSI



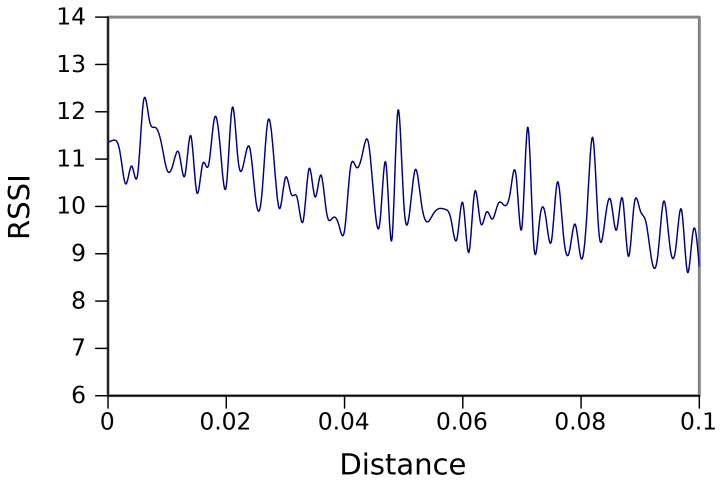
MI



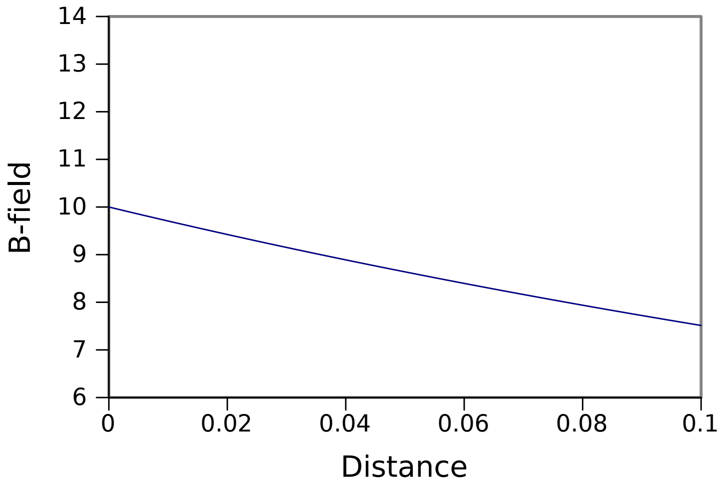
RSSI vs MI: Multipath and penetration

- MI penetrates any non-metallic objects and does not suffer from multipath

RSSI



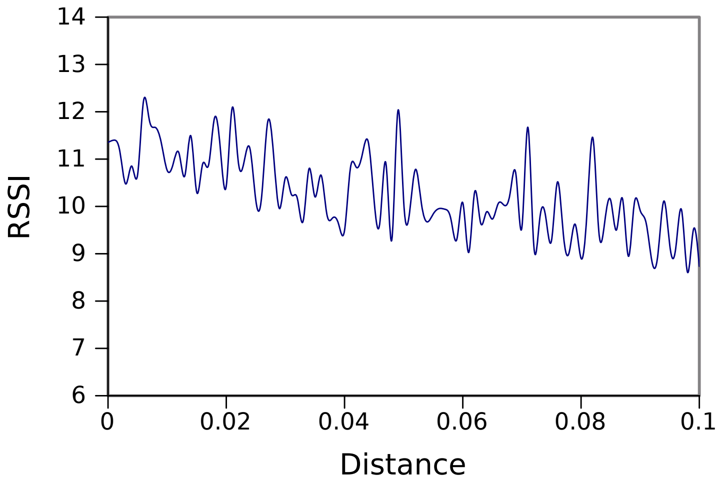
MI



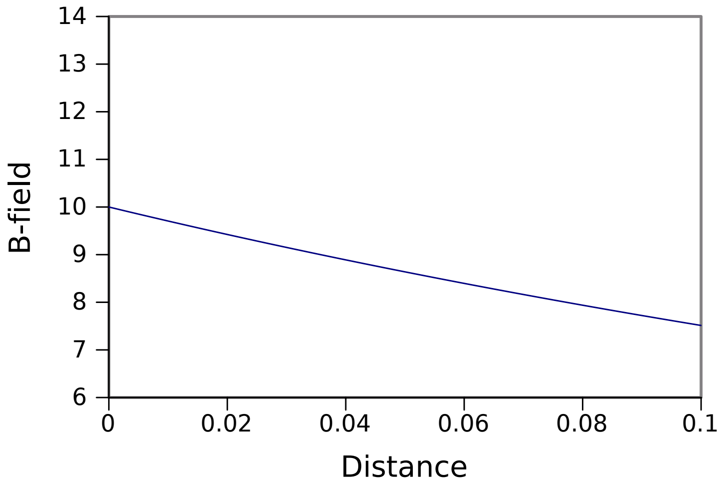
RSSI vs MI: Multipath and penetration

- MI penetrates any non-metallic objects and does not suffer from multipath
- Environmental obstacles do not affect MI localization accuracy

RSSI



MI

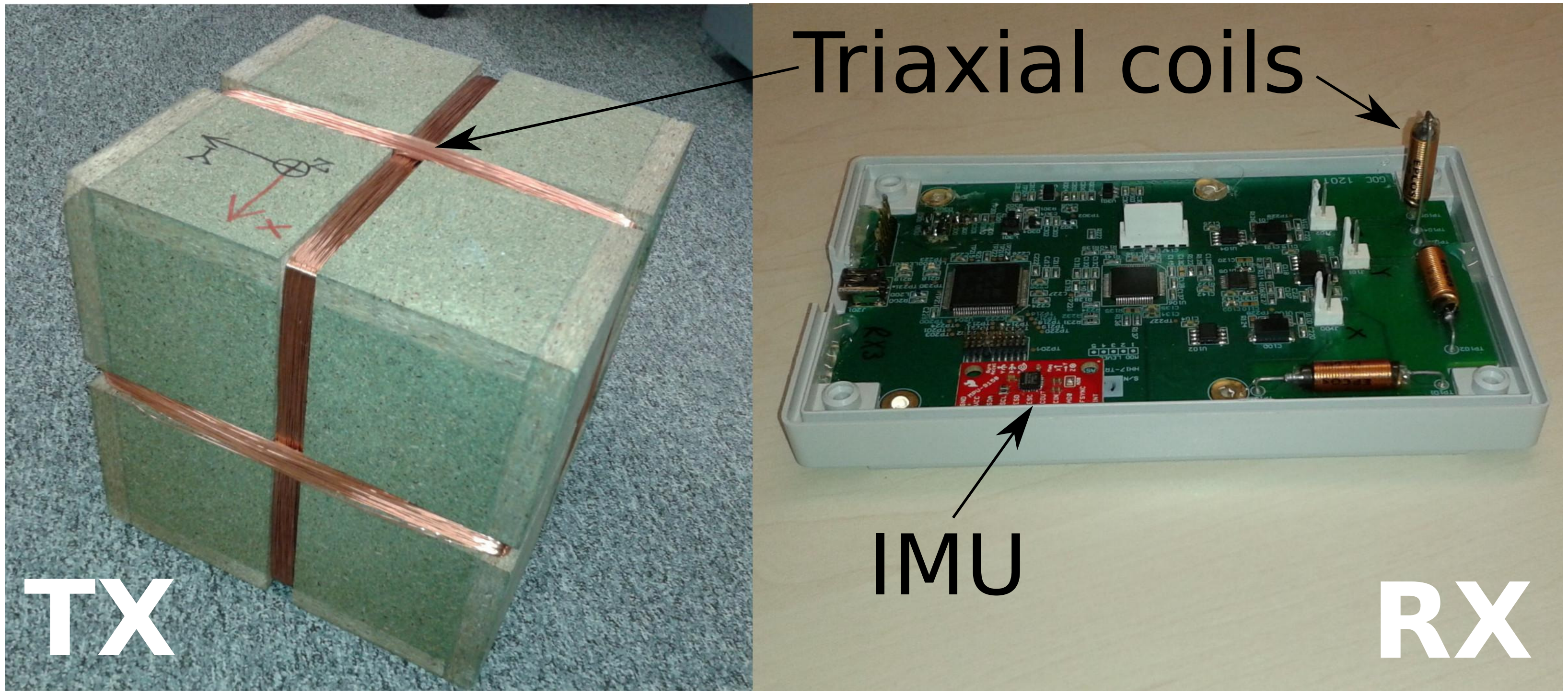


RSSI vs MI: Properties

- MI is a *vector field* i.e. it has magnitude and direction
- Using a triaxial sensor, we can make the device rotationally invariant

Magnetic Fields for Navigation

- We can electronically control the orientation and the magnitude of the magnetic moment by using 3 mutually perpendicular coils

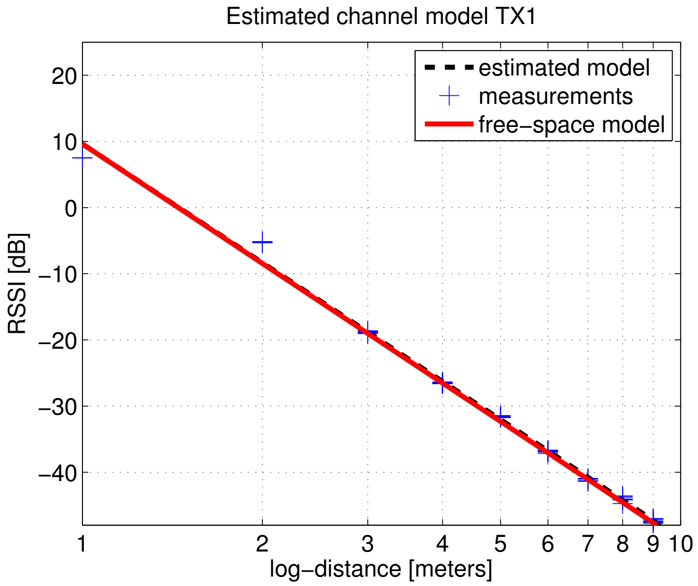


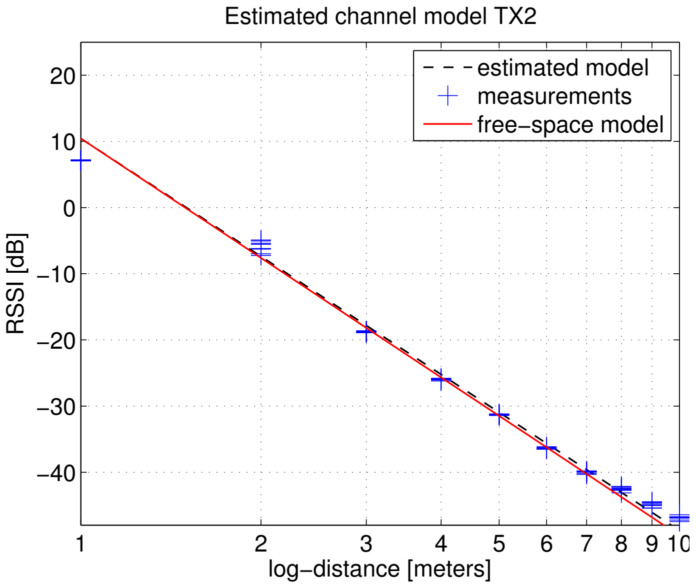
- The sensor is also equipped with a triaxial coil
- The total received power is invariant to TX and RX orientation

Distortion Rejecting Magneto-Inductive Three-Dimensional Localization (MagLoc) [Abrudan et al. 2015]

Typical Measurements

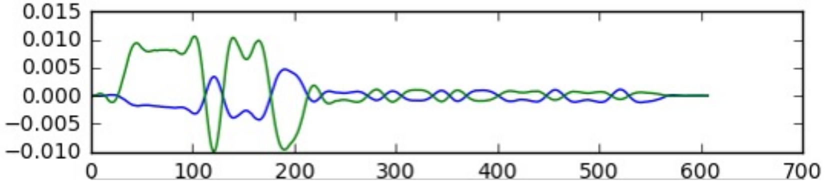
- The abrupt decay (60 dB/decade) of the magnetic field dB-magnitude corresponds to a path-loss exponent of 6 (i.e., 1 million time)
  - Limits the transmission range 😢
  - Enables easy detection of tiny changes in distance (few cm) 😊



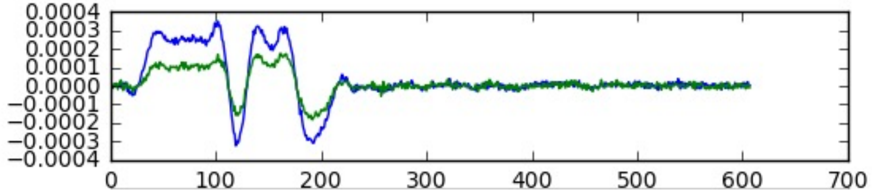


Typical Measurements

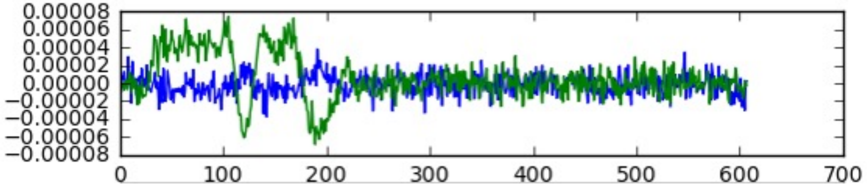
Rx at 3m



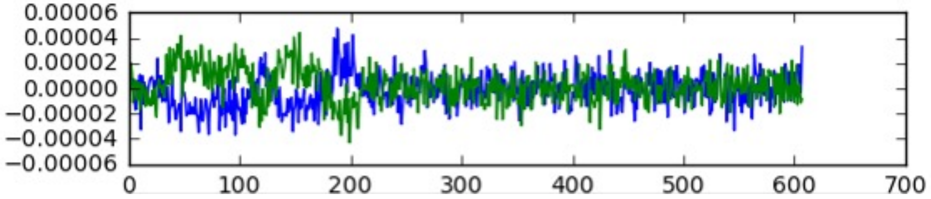
Rx at 10 m



Rx at 20 m



Rx at 30 m



Maths behind MI

- As previously mentioned we can:
  - Generate and steer a magnetic vector using 3 orthogonal transmitting coils
  - Sense and recover the magnetic vector using 3 orthogonal receiving coils
- This provides the *unique* ability to localize a receiver using a single transmitter

Maths behind MI

- With 3 transmitting antennas and 3 receiving antennas, we have a total of 9 measurements, of which 6 are unique

- Using some physics we can work out the 6 degrees of freedom in the receiver (position and orientation)

- Due to symmetry, we have a hemispherical ambiguity (receiver at (x,y,z) could also have been at (-x,-y,-z)), which can be solved with some prior knowledge (e.g., map information, IMU measurements).

Free-space Magnetic Channel Model

- Consider TX located at the origin $(x, y, z) = (0, 0, 0)$
- Let RX position in 3D be described by the position vector $\mathbf{r} = (x_r, y_r, z_r)$
- The range is $r = \|\mathbf{r}\|_2 = \sqrt{x_r^2 + y_r^2 + z_r^2}$
- TX is energized in each axis, and the corresponding magnetic moments are:

$$\mathbf{m}_i = N_{TX} I_{TX} A_{TX} \boldsymbol{e}_i$$

N_{TX}= number of turn of the TX coil
I_{TX}= the coil input TX current
A_{TX}= the area of the TX coil
$\mathbf{e}_i$= 1, 2, 3 are the standard Euclidean basis vectors

Free-space Magnetic Channel Model

- GOAL: Estimate the 3D position of RX

- The B-field at an arbitrary position $\boldsymbol{r}$, given an arbitrary magnetic moment $\mathbf{m}$ is:

$$\mathbf{B}(\mathbf{r},\mathbf{m}) = \frac{\mu_{TX}}{4\pi}\left[\frac{3\mathbf{r}\left(\mathbf{m}^{\mathbf{T}}\mathbf{r}\right)}{r^5} - \frac{\mathbf{m}}{r^3}\right]$$
$$= \frac{\mu_{TX}}{4\pi r^3}\left[\frac{3\mathbf{r}\mathbf{r}^{\mathbf{T}}}{r^2} - \mathbf{I}_3\right]\mathbf{m}$$

μ_{TX}= magnetic permeability of TX coil core
I_3 = 3x3 Identity matrix
$(.)^T$= Matrix Transpose

- For each TX magnetic moment $\mathbf{m}_i$,$i = 1,2,3$, we get a vector
- $\mathbf{b}_i = \mathbf{B}(\mathbf{r},\mathbf{m}_i)$

Free-space Magnetic Channel Model

- Define the matrix whose columns are $\boldsymbol{b}_i$

$$\mathrm{B}_{\mathbf{1,2,3}} \quad \stackrel{\text{def}}{=} [\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3] = \frac{\mu_{TX}}{4\pi r^3}\left[\frac{3\mathbf{r}\mathbf{r}^{\mathbf{T}}}{r^2} - \mathbf{I}_3\right][\mathbf{e}_{\mathbf{1}}, \mathbf{e}_{\mathbf{2}}, \mathbf{e}_{\mathbf{3}}]$$

- Let $\Omega \in SO(3)$ be an orthogonal matrix describing the orientation of the RX frame w.r.t. the TX frame.
- Then, the magnetic vector field described in the *RX frame* is

$$\Omega\, \mathrm{B}_{\mathbf{1,2,3}} \quad = \frac{\mu_{TX} N_{TX} I_{TX} A_{TX}}{4\pi r^3}\, \Omega \left[\frac{3\mathbf{r}\mathbf{r}^{\mathbf{T}}}{r^2} - \mathbf{I}_3\right]$$

Free-space Magnetic Channel Model

- The voltages induced in the RX (x, y, z)-axes due to TX excitations $(\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3})$ are described by the following ***channel matrix***

$$\mathrm{S} = \begin{bmatrix} S_{x1} & S_{x2} & S_{x3} \\ S_{y1} & S_{y2} & S_{y3} \\ S_{z1} & S_{z2} & S_{z3} \end{bmatrix} = 2\pi f \mu_{RX} N_{RX} A_{RX} \Omega B_{1,2,3}$$

f = frequency of the excitation
N_{RX}= number of turn of the RX coil
μ_{RX}= magnetic permeability of the RX coil core
A_{RX}= the area of the RX coil

Free-space Magnetic Channel Model

- Define the range-dependent scaling factor that also incorporates all the TX/RX coils specific constants

$$\mathrm{c} = \frac{1}{\mathrm{r}^3} 0.5 f \mu_{TX} \mu_{RX} N_{TX} N_{RX} I_{TX} A_{TX} A_{RX}$$

- Then we can write

$$S = c\Omega \left[\frac{3\mathbf{r}\mathbf{r}^{\mathbf{T}}}{r^2} - \mathbf{I}_3 \right]$$

- Note that the channel matrix depends on the position vector $\mathbf{r}$ we are interested in, and on the orientation Ω .

Free-space Magnetic Channel Model

- The channel matrix S containing the position information can be estimated at the RX using a known transmitted preamble

- The input-output relationship of the 3×3 channel is

$$P_{RX} = P_{TX}S^T + \text{(Gaussian noise)}$$

where P_{TX} and P_{RX} are the Nx3 transmitted and received preamble, respectively

- Therefore, the LS estimate (which in this case is also Maximum Likelihood estimate) of the channel matrix is

$$\hat{S} = \left[P_{TX}^{\dagger} P_{RX}\right]^T$$

† - stands for the Moore-Penrose Pseudoinverse

Free-space Magnetic Channel Model

- Let us analyze a bit the channel matrix and try to infer the range r

$$S = c\Omega\left[\frac{3\mathbf{r}\mathbf{r}^{\mathrm{T}}}{r^2} - \mathbf{I}_3\right]$$

- The scaling factor $\mathrm{c} \propto \frac{1}{r^3}$ and therefore contains the range information
- The latter factor depends only on the versor $\mathbf{r}/\|\mathbf{r}\|$ (i.e., only on the direction of the position vector, not on its magnitude).
- Therefore, in free-space, the ***Frobenius*** norm of S also decays with the cube of the range, and can be used for range estimation.

$$\|S\|_F \propto r^{-3}$$

- Since orthogonal matrices preserve the Frobenius norm, the range estimate is invariant w.r.t. TX/RX relative orientation

Free-space Magnetic Channel Model

- Define the overall RSSI (Received Signal Strength Indicator) measured in dB as

$$\rho = 20log\|S\|_F$$

- Since $\|S\|_F \propto r^{-3}$, the law describing the RSSI vs. distance in free-space is
  - *(This is why we have the propagation loss of 60db/decade)*

$$\rho = \rho_0 - 60\log(\frac{r}{r_0})$$

where ρ_0 is the RSSI measured at some reference distance r_0

- Therefore, the range estimate in free-space is

$$r = r_0 10^{(\rho_0 - \rho)/60}$$

When plotting the RSSI vs. the log-distance $\log(\frac{r}{r_0})$, the slope of the line is 60dB/decade

Free-space Magnetic Channel Model

- Our next goal is to determine the position vector
- Its modulus (range) $r = \|\mathbf{r}\|$ is known by now, we only need to determine its direction in 3D. Recall that

$$S = c\Omega\left[\frac{3\mathbf{r}\mathbf{r}^{\mathrm{T}}}{r^2} - \mathbf{I}_3\right]$$

- We first get rid of the arbitrary RX orientation $\Omega \in SO(3)$ as follows
- Define the *channel inner product*

$$C = S^T S$$

- From previous equations we have

$$C = c^2\left[\frac{3\mathbf{r}\mathbf{r}^{\mathrm{T}}}{r^2} - \mathbf{I}_3\right]^{\mathrm{T}} \Omega^T\Omega\left[\frac{3\mathbf{r}\mathbf{r}^{\mathrm{T}}}{r^2} - \mathbf{I}_3\right] = c^2\left[\frac{3\mathbf{r}}{\|\mathbf{r}\|}\frac{\mathbf{r}^{\mathrm{T}}}{\|\mathbf{r}\|} - \mathbf{I}_3\right]^2$$

* $\Omega^T\Omega = I_3$ Since Ω is orthogonal

Free-space Magnetic Channel Model

- Let $C = UDU^T$ be the eigen-decomposition of the channel inner product C. We get

$$\frac{\boldsymbol{r}}{\|\boldsymbol{r}\|}\frac{\boldsymbol{r}^T}{\|\boldsymbol{r}\|} = \frac{1}{3c}\mathrm{C}^{\frac{1}{2}} + \frac{1}{3}I_3 = U\left[\frac{1}{3c}D^{1/2} + \frac{1}{3}\mathrm{I}_3\right]U^T$$

Which is a rank-one matrix

- Consequently, the maximal eigenvector $\mathbf{u}_{max}$ of C is the position versor we are interested in:

$$\frac{\boldsymbol{r}}{\|\boldsymbol{r}\|} = \mathbf{u}_{max}$$

- Finally, the 3D position is given by

$$r\mathbf{u}_{max} = \mathbf{r}$$

* $C^{\frac{1}{2}} = UD^{\frac{1}{2}}U^T$ and $UU^T = I_3$

Summary

- If some property varies over space, then there is a strong chance that someone has or will use it for positioning!
- Magneto-Inductive positioning has some unique properties which make it an interesting alternative
- Positioning itself though is just another sensor - it is what we do with it that is important