Solution 1

(a) $\exists x \in X$ such that $S(x) \land A(x)$ is true.

(b) $\forall x \in X, T(x) \land S(x) \implies A(x)$

(c) $\nexists x \in X$ such that $(T(x) \land S(x)) \land \neg A(x)$

(d) $S = \{\forall x \in X : T(x) \land \neg S(x)\}$. Then, $|S| \geq 3$

Solution 2

TODO: It's easy enough. So, I'll skip it for later.

Solution 3

(a) (i) $\neg(A \text{ nand } B)$

(ii) $(A \text{ nand } A)$ nand $(B \text{ nand } B)$

(iii) $(\neg A \text{ nand } \neg A)$ nand $(B \text{ nand } B)$

(b) $\neg A = A$ nand A.

(c) true is A nand $(A$ nand $A)$. Similarly, false is $(A$ nand $(A$ nand $A))$ nand $(A$ nand $(A$ nand $A))$

Solution 4

Split the 12 coins in two groups of 6. Weigh the two groups in the balance scale. Since one of the coin is heavier the balance scale will tilt towards the heavy side. Now split the 6 coins in the heavy side into two groups of 3. Weigh the two groups in the balance scale. Since one of the coin is heavier the balance scale will tilt towards the heavy side. Now split the 3 coins in the heavy side into three groups of 1. Put any of the two coins on the balance scale. Now only two cases can arise:

(i) Balance scale is balanced. This implies the coin that was not kept in the balance scale is the heavier coin.

(ii) Balance scale tilts towards one of the sides. The heavier side has the heavy coin.

Solution 5

By Contrapositive: If $r^{1/5}$ is not irrational then r is not irrational.

Proof

Let, $r^{1/5} = (a/b)$. Then, $r = (a/b)^5 = a^5/b^5$. $a, b \in \mathbb{Z} \implies a^5, b^5 \in \mathbb{Z}$. Thus $r \in \mathbb{Q}$.

Solution 6

$w^2 + x^2 + y^2 = z^2$.

Proof

For the forward implication: Consider w, x, y are even. Then, $w = 2i_1, x = 2i_2, y = 2i_3$. So, $w^2 + x^2 + y^2 = 2^2(i_1^2 + i_2^2 + i_3^2) = z^2$. Since, $z \in \mathbb{Z}^+$, any integer solution to $2^2(i_1^2 + i_2^2 + i_3^2) = z^2$ will be divisible by 2. Thus z is even.

For the backward implication: If $z = 2j$. Then $2^2j^2 = z^2 = w^2 + x^2 + y^2$ gives $j^2 = (w/2)^2 + (x/2)^2 + (y/2)^2$.

Since $j \in \mathbb{Z}^+ \implies j^2 \in \mathbb{Z}^+$. This implies that w, x, y has to be divisible by 2 so they are even.