

# Problem Set 4 - CPSC 326

## Solutions

Fernando, Dang, Raj, Eric M.

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### Problem 1

Consider the following Language:

$$L_1 = \{w \in \{a, b, c, d\}^* \mid (w \text{ contains the strings } abb \text{ and } bbc) \text{ or } (w \text{ contains the string } abc)\}$$

Develop a NFA for  $L_1$ .

**Answer:**

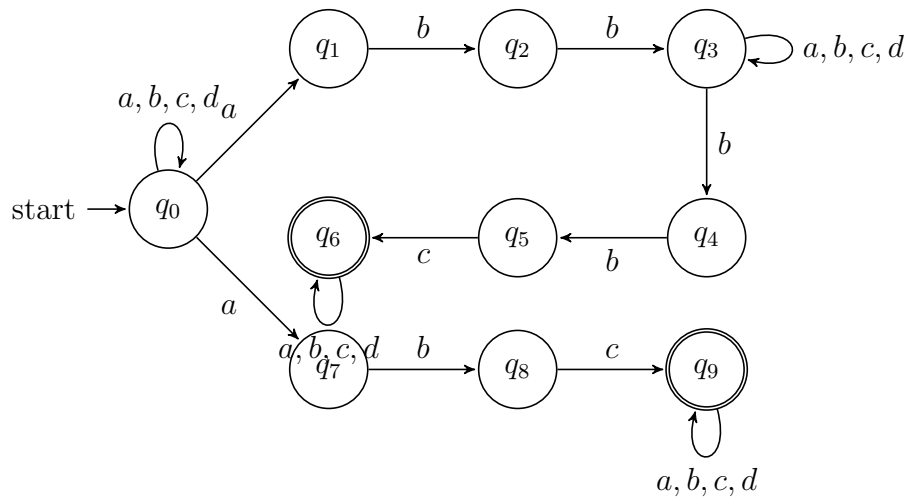
The NFA uses nondeterminism to check for either:

- Case 1: Both  $abb$  and  $bbc$  appear in  $w$
- Case 2:  $abc$  appears in  $w$

**States:**

- $q_0$ : Initial state
- $q_1, q_2, q_3$ : Track progress toward finding  $abb$
- $q_4, q_5, q_6$ : Track progress toward finding  $bbc$  (after finding  $abb$ )
- $q_7$ : Found both  $abb$  and  $bbc$  (accepting)
- $q_8, q_9, q_{10}$ : Track progress toward finding  $abc$
- $q_{11}$ : Found  $abc$  (accepting)

**NFA Diagram:**



**Explanation:** The NFA nondeterministically chooses between:

- Following the upper path to find  $abb$ , then continuing to find  $bbc$
- Following the lower path to find  $abc$

Once either accepting state is reached, the string is accepted.

## Problem 2

Let  $L_2 = \{w \in \{a, b, c\}^* \mid w = (abc)^*a^*\}$ . Design a NFA that accepts the language  $L_2$ .

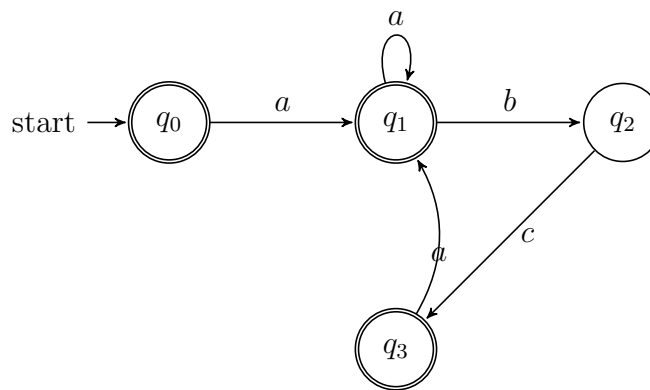
**Answer:**

The language consists of zero or more repetitions of  $abc$ , followed by zero or more  $a$ 's.

**States:**

- $q_0$ : Initial and accepting state
- $q_1$ : Just read  $a$  (could be start of  $abc$  or final  $a$ 's)
- $q_2$ : Read  $ab$  (continuing  $abc$ )
- $q_3$ : Read  $abc$  (back to accepting, or continue with more  $a$ 's)

**NFA Diagram:**



**Explanation:**

- From  $q_0$ , we can either accept immediately ( $\varepsilon$  is in the language) or read  $a$
- After reading  $a$ , we nondeterministically choose:
  - Continue with  $bc$  to complete an  $abc$  block
  - Stay in  $q_1$  reading more  $a$ 's (the final  $a^*$  part)
- After completing  $abc$  (reaching  $q_3$ ), we can start another  $abc$  block or accept
- States  $q_0$ ,  $q_1$ , and  $q_3$  are accepting to handle  $(abc)^*a^*$  properly

## Problem 3

Let  $L_3 = \{w \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* \mid \text{the final digit of the string } w \text{ has not appeared before in } w\}$ .

Design a NFA that accepts the language  $L_3$ .

**Answer:**

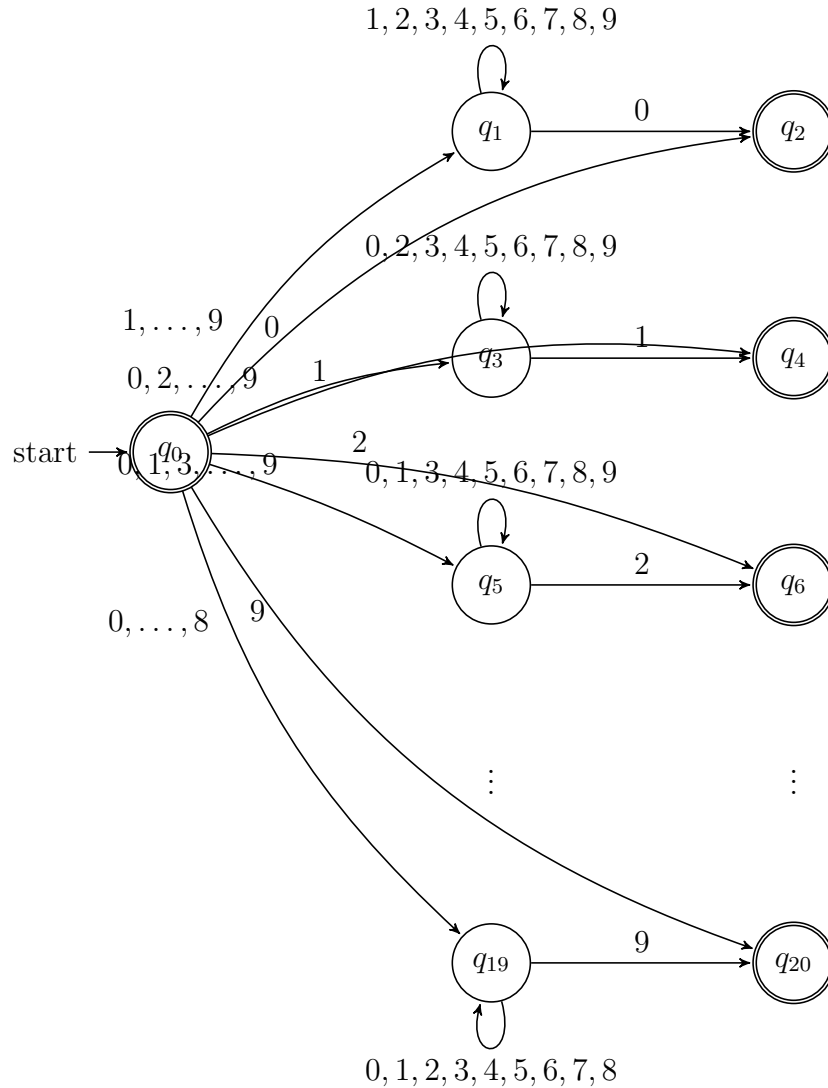
The NFA uses nondeterminism to guess which digit will be the final digit that hasn't appeared before.

**Strategy:** From the start state, when we read symbols, we nondeterministically choose which digit we're "watching for" as the final new digit. We have 10 branches (one for each digit 0-9), where each branch avoids seeing that specific digit until the very end.

**States:**

- $q_0$ : Initial state (accepting, for single-digit strings)
- Branches for each digit:
  - $q_1, q_2$ : Branch watching for digit 0
  - $q_3, q_4$ : Branch watching for digit 1
  - $q_5, q_6$ : Branch watching for digit 2
  - $q_7, q_8$ : Branch watching for digit 3
  - $q_9, q_{10}$ : Branch watching for digit 4
  - $q_{11}, q_{12}$ : Branch watching for digit 5
  - $q_{13}, q_{14}$ : Branch watching for digit 6
  - $q_{15}, q_{16}$ : Branch watching for digit 7
  - $q_{17}, q_{18}$ : Branch watching for digit 8
  - $q_{19}, q_{20}$ : Branch watching for digit 9
- Accepting states:  $q_0, q_2, q_4, q_6, q_8, q_{10}, q_{12}, q_{14}, q_{16}, q_{18}, q_{20}$

**NFA Diagram:**



### Explanation:

The NFA has 21 states total ( $q_0$  through  $q_{20}$ ), organized as follows:

- $q_0$ : Starting state (also accepting for single-digit strings)
- 10 branches, each with 2 states, one for each digit 0-9:
  - States  $q_1, q_3, q_5, q_7, q_9, q_{11}, q_{13}, q_{15}, q_{17}, q_{19}$ : "Avoiding" states where we read any digit except the one we're watching for
  - States  $q_2, q_4, q_6, q_8, q_{10}, q_{12}, q_{14}, q_{16}, q_{18}, q_{20}$ : "Found" states (accepting) where we just read the digit we were watching for

How it works:

1. From  $q_0$ , nondeterministically choose which digit will be the final new digit
2. For example, to watch for digit 0:

- Go to  $q_1$  by reading any digit 1-9
- Stay in  $q_1$  reading more digits 1-9 (avoiding 0)
- Read 0 and go to accepting state  $q_2$

3. Similarly for digits 1-9, using states  $(q_3, q_4), (q_5, q_6), \dots, (q_{19}, q_{20})$

**Example:**

For string  $w = 237$  (where 7 appears for the first time at the end):

- Read 2:  $q_0 \xrightarrow{2} q_{15}$  (nondeterministically choose to watch for 7)
- Read 3:  $q_{15} \xrightarrow{3} q_{15}$  (3 is not 7, stay in  $q_{15}$ )
- Read 7:  $q_{15} \xrightarrow{7} q_{16}$  (accept!)

For string  $w = 5$  (single digit):

- Read 5:  $q_0 \xrightarrow{5} q_{12}$  (accept immediately)
- Or stay in  $q_0$  which is also accepting