Problem Set 1

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Problem 1

Statement: Using a proof by contradiction, prove for any integer n if $n^3 + 5$ is odd, then n is even.

Proof by Contradiction:
Assume for contradiction that $n^3 + 5$ is odd and n is odd.
Since n is odd, we can write $n = 2k + 1$ for some integer k.
Then:

$$\begin{aligned} n^3 &= (2k+1)^3 && (1) \\ &= 8k^3 + 12k^2 + 6k + 1 && (2) \\ &= 2(4k^3 + 6k^2 + 3k) + 1 && (3) \end{aligned}$$

Let $m = 4k^3 + 6k^2 + 3k$. Since k is an integer, m is also an integer.
Therefore, $n^3 = 2m + 1$, which means n^3 is odd.
Now, $n^3 + 5 = (2m + 1) + 5 = 2m + 6 = 2(m + 3)$.
Since $m + 3$ is an integer, $n^3 + 5$ is even.
Any integer times 2 is even. Therefore, $n^3 + 5$ is even.
This contradicts the assumption that $n^3 + 5$ is odd.
Therefore, if $n^3 + 5$ is odd, then n must be even.

Problem 2

Statement:

$$\overline{A} \cap \overline{B} = \overline{A \cup B}$$

Example: Let the universal set be

$$U = \{1, 2, 3, 4, 5\}$$

and let

$$A = \{1, 2\}, \quad B = \{3, 4\}.$$

Step 1: Find complements:

$$\overline{A} = U - A = \{3, 4, 5\}, \quad \overline{B} = U - B = \{1, 2, 5\}$$

Step 2: Solve left-hand side:

$$\overline{A} \cap \overline{B} = \{3, 4, 5\} \cap \{1, 2, 5\} = \{5\}$$

Step 3: Solve right-hand side:

$$A \cup B = \{1, 2\} \cup \{3, 4\} = \{1, 2, 3, 4\}$$

$$\overline{A \cup B} = U - (A \cup B) = \{5\}$$

Step 4: Compare both sides:

$$\overline{A} \cap \overline{B} = \{5\} = \overline{A \cup B}$$

Conclusion: The equality holds in this example, showing that it's true

$$\overline{A} \cap \overline{B} = \overline{A \cup B}$$

Problem 3

Problem 4