Problem Set 3 - Solutions
CPSC 326

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Problem 1

Problem: Consider the following Language:

$$L_1 = \{w \in \{a, b, c, d\}^* \mid |w|_a \geq |w|_c\}$$

Is L_1 a regular language? Prove your answer by either developing a DFA for L_1 or using the pumping lemma to show non-regularity.

Solution

L_1 is **NOT regular**. We prove this using the pumping lemma.

Assume for contradiction that L_1 is regular. Let p be the pumping length.

We can consider the string $w = c^p a^p \in L_1$ (since $|w|_a = p = |w|_c$, we have $|w|_a \geq |w|_c$).

We have $|w| = 2p \geq p$, so by the pumping lemma, we can write $w = w_1 w_2 w_3$ where:

1. $|w_1 w_2| \leq p$
2. $|w_2| \geq 1$
3. $w_1 w_2^i w_3 \in L_1$ for all $i \geq 0$

Since $|w_1 w_2| \leq p$ and $w = c^p a^p$, the substring $w_1 w_2$ consists entirely of c's. Therefore, $w_2 = c^k$ for some $k \geq 1$.

Now we can consider $w_1 w_2^2 w_3 = c^{p+k} a^p$.

For this string: $|w_1 w_2^2 w_3|_a = p$ and $|w_1 w_2^2 w_3|_c = p + k$.

Since $k \geq 1$, we have $|w_1 w_2^2 w_3|_a = p < p + k = |w_1 w_2^2 w_3|_c$.

This means $|w_1 w_2^2 w_3|_a < |w_1 w_2^2 w_3|_c$, which violates the condition $|w|_a \geq |w|_c$.

Therefore, $w_1 w_2^2 w_3 \notin L_1$, which contradicts the pumping lemma.

Thus, L_1 is not regular.

Problem 2

Problem: Consider the following Language:

$$L_2 = \{w \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}^* \mid \text{the number of odd digits in } w \text{ equals the number of even digits in } w\}$$

Is L_2 a regular language? Prove your answer by either developing a DFA for L_2 or using the pumping lemma to show non-regularity.

Solution

L_2 is **NOT regular**. We prove this using the pumping lemma.

Assume L_2 is regular with pumping length p.

Consider the string $w = 0^p 1^p \in L_2$ (since it has p even digits (all 0's) and p odd digits (all 1's)).

We have $|w| = 2p \geq p$. By the pumping lemma, we can write $w = w_1 w_2 w_3$ where:

1. $|w_1 w_2| \leq p$
2. $|w_2| > 0$
3. $w_1 w_2^i w_3 \in L_2$ for all $i \geq 0$

Since $|w_1 w_2| \leq p$ and $w = 0^p 1^p$, the substring $w_1 w_2$ consists entirely of 0's. Therefore, $w_2 = 0^k$ for some $k > 0$.

Consider $w_1 w_2^0 w_3 = w_1 w_3 = 0^{p-k} 1^p$.

In this string:

- Number of even digits $= p - k$
- Number of odd digits $= p$

Since $k > 0$, we have $p - k < p$, so the number of even digits does not equal the number of odd digits.

Therefore, $w_1 w_3 \notin L_2$, which contradicts the pumping lemma.

Thus, L_2 is not regular.

Problem 3

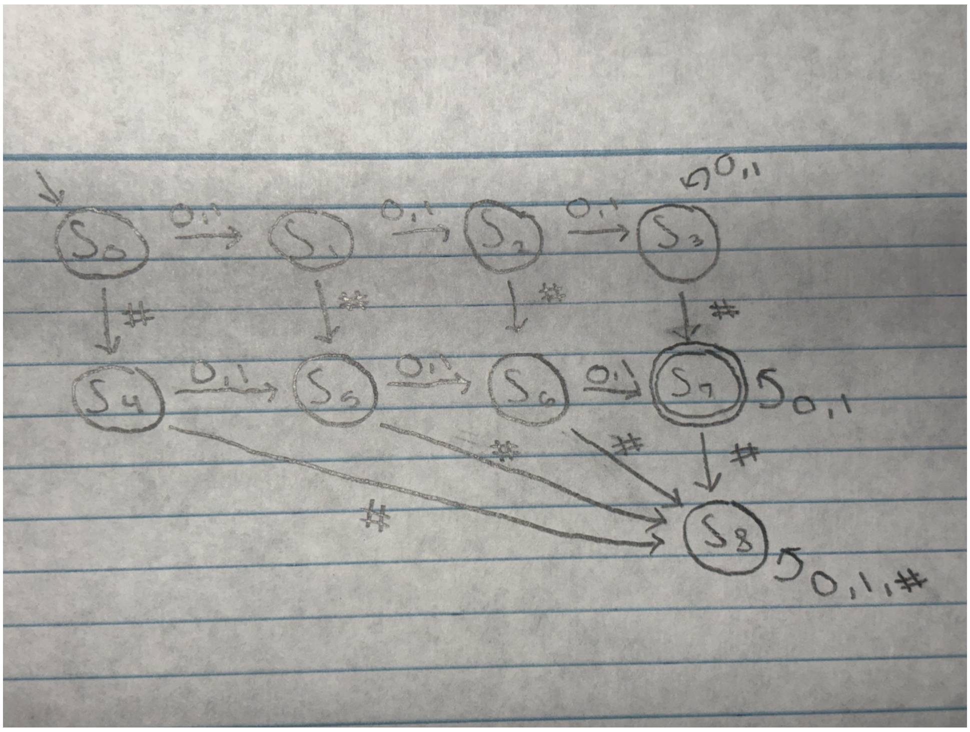
Problem: Consider the following Language:

$$L_3 = \{w \in \{0, 1, \#\}^* \mid w = w_1\#w_2, w_1, w_2 \in \{0, 1\}^*, |w_1| + |w_2| \geq 3\}$$

Is L_3 a regular language? Prove your answer by either developing a DFA for L_3 or using the pumping lemma to show non-regularity.

Solution

L_3 is a **regular language**. We can make a DFA with finitely many states.



Problem 4

Problem: Design a DFA for the following regular language:

$$L_4 = \{w \in \{0,1\}^* \mid \text{if } |w|_0 \bmod 2 = 0, \text{ then } |w|_1 \bmod 3 = 0$$
$$\text{or if } |w|_0 \bmod 2 = 1 \text{ then } |w|_1 \bmod 2 = 0\}$$

Solution