Homework 2 - CPSC 326
Solutions

Dang Phung

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Question 1 (10 points)

Let $\Sigma = \{0, 1\}$. Let $L_1 = \{w \in \Sigma^* \mid |w|_0 \neq |w|_1\}$. Prove or disprove that $L_1^* = \Sigma^*$.

Answer:

We will **prove** that $L_1^* = \Sigma^*$.

To show $L_1^* = \Sigma^*$, we need to show that every string in Σ^* can be shown as a concatenation of strings from L_1.

First off,

- $0 \in L_1$ since $|0|_0 = 1 \neq 0 = |0|_1$
- $1 \in L_1$ since $|1|_1 = 1 \neq 0 = |1|_0$
- $\lambda \in L_1^*$ since $L_1^0 = \{\lambda\}$

For any non-empty string $w \in \Sigma^*$, you can write it as $w = \sigma_1\sigma_2 \cdots \sigma_n$ where each $\sigma_i \in \{0, 1\}$.

Since each single symbol (0 or 1) is in L_1, we have:

$$w = \sigma_1 \cdot \sigma_2 \cdot \ldots \cdot \sigma_n \in L_1^n \subseteq L_1^*$$

Therefore, every string in Σ^* including λ is in L_1^*.

This gives us $\Sigma^* \subseteq L_1^*$. Since $L_1^* \subseteq \Sigma^*$ by definition, we prove that

$$L_1^* = \Sigma^*$$

Question 2 (10 points)

Let $L_2 = \{w \in \{1, 2, 3\}^* \mid |w| \bmod 2 = 0\}$. Given the linear ordering $1 < 2 < 3$ on Σ, list the first 10 elements, in canonical order, in an enumeration of L_2.

Answer:

Canonical order means: order by length, and then lexicographically within each length.

L_2 contains all strings of even length over $\{1, 2, 3\}$ (mod $2 = 0$).

First 10 elements:

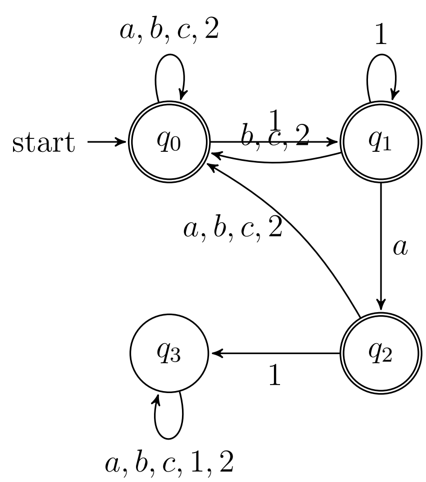
1. λ
2. 11
3. 12
4. 13
5. 21
6. 22
7. 23
8. 31
9. 32
10. 33

Question 3 (10 points)

Design a DFA for the following regular language:

$$L_3 = \{w \in \{a, b, c, 1, 2\}^* \mid w \text{ does not contain the substring } 1a1\}$$

Answer:

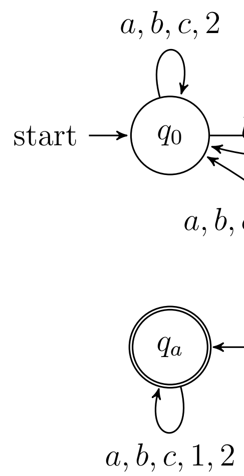


Question 4 (10 points)

Design a DFA for the following regular language:

$$L_4 = \{w \in \{a, b, c, 1, 2\}^* \mid w \text{ contains the substring } 1a1\}$$

Answer:

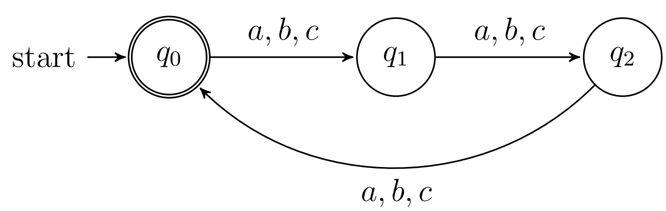


Question 5 (10 points)

Design a DFA for the following regular language:

$$L_5 = \{w \in \{a, b, c\}^* \mid |w| \bmod 3 = 0\}$$

Answer:

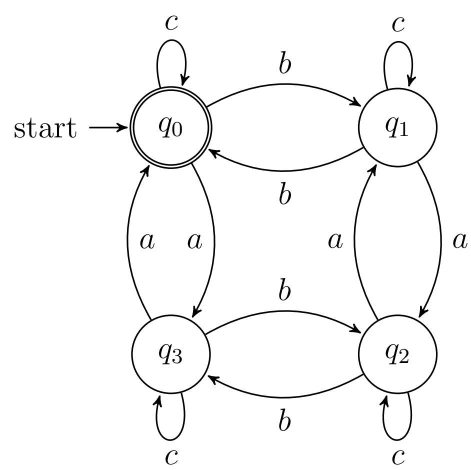


Question 6 (10 points)

Design a DFA for the following regular language:

$$L_6 = \{w \in \{a, b, c\}^* \mid |w|_a \bmod 2 = 0 \text{ and } |w|_b \bmod 2 = 0\}$$

Answer:

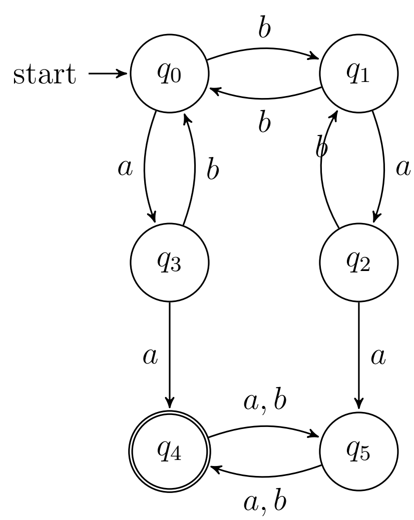


Question 7 (10 points)

Design a DFA for the following regular language:

$$L_7 = \{w \in \{a,b\}^* \mid \text{length of } w \text{ is even and } w \text{ contains the substring } aa\}$$

Answer:

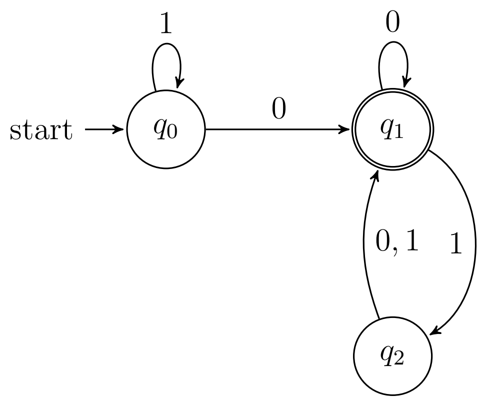


Question 8 (10 points)

Design a DFA for the following regular language:

$L_8 = \{w \in \Sigma^*_{\text{bool}} \mid w$ contains at least one '0' and an even number of 1s following the last 0$\}$

Answer:

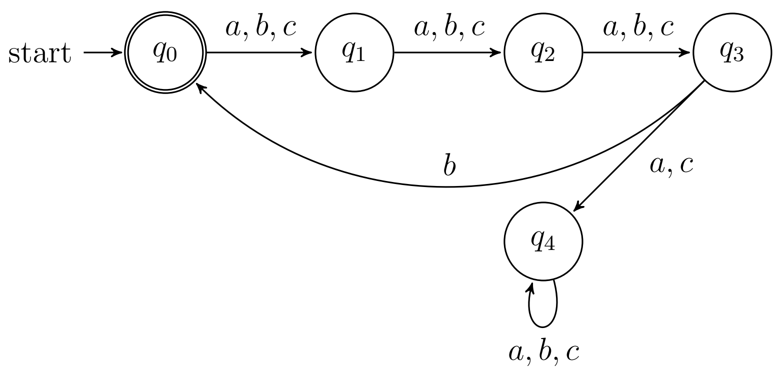


Question 9 (10 points)

Design a DFA for the following regular language:

$$L_9 = \{w \in \{a, b, c\}^* \mid \text{Every fourth symbol of } w \text{ is a 'b'}\}$$

Answer:

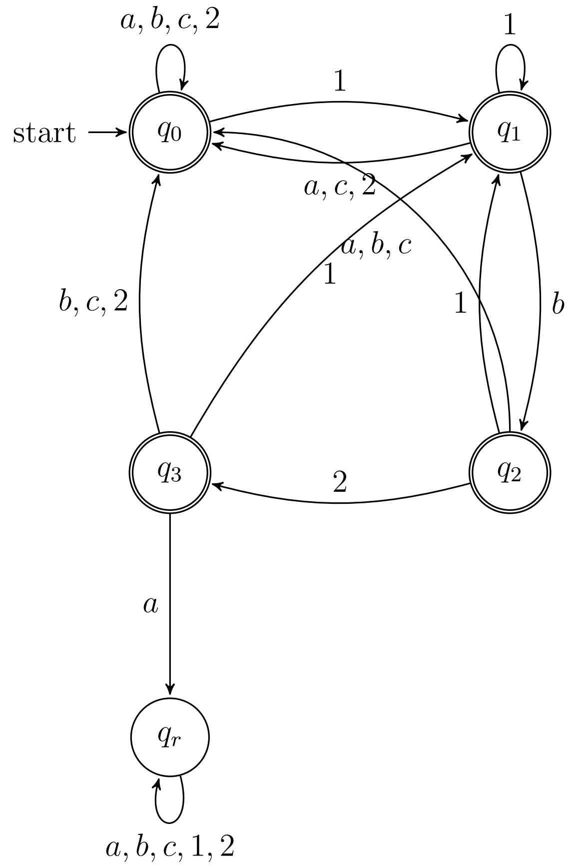


Question 10 (10 points)

Design a DFA for the following regular language:

$$L_{10} = \{w \in \{a, b, c, 1, 2\}^* \mid w \text{ does not contain } 1b2a\}$$

Answer:



$a, b, c, 2$
1
1
start
q_0
q_1
$a, c, 2$
a, b, c
1
$b, c, 2$
1
b
q_3
2
q_2
a
q_r
$a, b, c, 1, 2$