Modeling Earnings Processes and Consumption Insurance at the Household Level

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Abstract

The study of income and consumption linkage and analysis of permanent and transitory income shocks is limited by model misspecification and availability of data. The misspecification arises from ignoring unemployment risk while estimating income shocks. I employ Heckman two step regression model to consistently estimate income shocks. Moreover, to deal with data sparsity, I propose identifying the partial consumption insurance and income and consumption volatility heterogeneities at the household level using Least Absolute shrinkage and Selection Operator (LASSO). Using PSID data, I estimate partial consumption insurance against permanent shock of 63% and 49% for white and black household heads, respectively; the white and black household heads self-insure against 100% and 90% of the transitory income shocks, respectively. Moreover, I find income and consumption volatilities and partial consumption insurance parameters vary across time.

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1. Introduction

Idiosyncratic earnings risk encompasses any shock worker encounters over his or her lifecycle. Unemployment is the main contributor to this idiosyncratic earnings risk (Lawrence Costa (2022)). However, while estimating income and consumption shocks numerous studies include employment and unemployment status as a deterministic component of earnings (BPP, Chatterjee et al (2021)). To incorporate unemployment risk, this study proposes using Heckman two step regression. Heckman two step regression estimates earnings by simultaneously modeling the probability of being employed. The model captures and consistently estimates income shocks.

Moreover, one of the focuses of analyzing the link between income and consumption shocks is to explore the nature of these correlations across different household characteristics (Blundell et al (2008), Chatterjee et al (20202) and Ganong et al (2020)). However, lack of single source with good quality income and consumption data, complicates the process. Blundell et al (2008) use imputation to build new panel series of income and consumption that combines information from the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX). Then, to estimate the various group-wise heterogeneity across individuals, the data is split into sub-samples resulting in thinly splitted data, which affects the consistent estimation of the parameters. Furthermore, separately estimating the parameters of interest for each group will ignore the potential for group interaction effects. To deal with data sparsity, this paper proposes identifying the partial consumption insurance and income and consumption volatility heterogeneities at the household level using Least Absolute Shrinkage and Selection Operator (LASSO). The LASSO helps for predicting and characterizing the extent of group heterogeneity in the data. I argue that the use of model regularization and shrinkage improves model interpretability by robustly selecting the most important heterogeneity.

I allow the insurance parameters and income and consumption volatilities to vary by household types (specifically race) and time. Then, using regularization method, without splitting the data into sub-samples, I select and estimate the most important heterogeneities present in the data. To the best of my knowledge, this study is the first one to use GMM with LASSO to estimate consumption partial insurance and income and consumption volatility heterogeneities at the household level. One advantage of my approach relative to data splitting is that, LASSO will shrink some of the coefficient estimates exactly to zero and produces an interpretable model. Monte Carlo experiments, under different settings, suggest that GMM with LASSO estimation method recovers the true population parameters very well. Subsequently, using PSID data, I estimate partial consumption insurance against permanent shock of 63% and 49% for white and black household heads, respectively. Another interesting result is that while white household heads self insure against 100% of the transitory income shocks, black household heads can only self insure against 90% of the transitory shocks. Also the permanent income and consumption shock volatilities for black household heads are higher than their white counter parts.

Over the last four decades, income inequality has steadily risen in advanced economies (Guvenen and Kaplan (2017)). However, whether consumption inequality has risen is disputed. In recent decades, many studies concluded that consumption inequality has risen less than income inequality (Cutler and Katz (1991), Krueger and Perri (2006), Heathcote et al. (2010), Fisher, Johnson, and Smeeding (2013), Meyer and Sullivan (2013)). And yet, a significant number of studies have found that the rise in consumption and income inequalities have been more or less indistinguishable (Aguiar and Bils (2012), Attanasio, Hurst, and Pistaferri (2013)). Key to answering why and how consumption inequality has risen is the covariance of income and consumption. A crucial ingredient of this exercise is to determine the relative importance of income shocks of different persistence and correlation across agents.

Numerous studies address various questions regarding income and consumption shocks. The central theme of the questions is how to characterize income inequalities and analyze whether those inequalities are also exhibited by consumption. Methodologically, the standard approach is to decompose income shocks into permanent and transitory components, which will help to better understand how income inequalities evolved, and thereby analyze how the consumption process adjusted to these shocks and consequently consumption inequality has evolved. An intuitive justification for income shock decomposition is provided as a response to Deaton's paradox. Deaton(1986) argues sharp shocks to income did not seem to cause large shocks to consumption. Labor income can be represented by random walk process, while consumption is relatively smooth. However, Quah (1990), made progress towards resolving Deaton's paradox by decomposing labor income into permanent and transitory components. This helps to establish clear link between income and consumption inequalities.

An early study exploring heterogeneity is Meghir and Pistaferri (2004). They estimate an income process for individual annual earnings in the U.S., allowing for differences across education groups and taking into account changes over time. Another important work is the seminal paper by Blundell, Pistaferri and Preston (2008)(hereafter BPP). The importance of this study is underlined by the central role it plays in macroeconomic calibration and policy analysis. Before BPP there was a strong consensus that the marginal propensity to consume (MPC) out of permanent income is very close to one (Friedman (1957) and Quah(1990)). However, BPP argue consumers self-insure against some fraction of permanent income shocks. Using the whole sample they find households self-insure against 36% of the permanent income shocks. However, the degree of consumption partial insurance against permanent income shocks falls to 6% for non-college graduates and it increases to 58% for college graduates. In addition BPP find partial consumption insurance of 29% and 13% for younger and older household heads, respectively.

While Kaplan and Violante (2010) analyze the degree of consumption smoothing implicit in a calibrated life-cycle version of the standard incomplete-markets model, they do not explore the heterogeneities of these parameters at household level. But they show that the partial insurance parameters depend on the tightness of debt limits. The partial insurance against permanent income shocks is as low as 7% for households closer to the borrowing limits (i.e. when the borrowing constraints are binding); while the estimates rise to about 22% for households far from the borrowing limits.

On the other hand , using quasi maximum likelihood estimation (QMLE) method, Chatterjee et al (2020) produced a significantly higher estimate of consumption insurance at 55%. Also, the partial consumption insurance increases to 71% for household heads with college degree and it falls to 36% when the household heads have no college degree. Furthermore, they find a partial consumption insurance of 45% and 75% for younger and older household heads respectively. They argue their estimates are quite different and more precise than BPP's because QMLE is more robust to the non-normality of income distribution and it avoids the need to estimate a weighting matrix.

Finally, I estimate income and consumption volatility and partial consumption insurance heterogeneity across race and time. The time varying income and consumption shock volatility estimates are comparable to the ones reported in BPP, Chatterjee et al (2021) and Arellano et al (2018). However, my results are more robust to data sparsity.

Section 2 reviews the literature related to our study. Section 3 introduces the model and estimation technique. Section 4 provides a Monte Carlo study.

Section 5 provides data summary and discusses the empirical results. Section 6 concludes. The appendix contains derivation of the moment conditions and additional empirical results.

2. Extended Literature Review

There is a subtle distinction between consumption smoothing theory in the spirit of Friedman (1957), Modigliani and Brumberg (1954, 1980) and Quah (1989) and the risk sharing and insurance models of Cochrane (1991), Mace (1991), Baxter and Crucini (1995), Hayashi, Altonji and Kotlikoff (1996) and Krueger and Perri (2005, 2011b). While the former is about persistence of various shocks, the later focuses on the covariance across agents of the shocks. In our study we characterize both the persistence of the shocks and their covariances across agents.

The dichotomy of income shocks into permanent and transitory components is viewed as central to household consumption choices. The rational for this decomposition can be traced to Milton Friedman's permanent income theory of consumption (Friedman (1957)). The theory argues that the marginal propensity to consume from the annuitized value of an income shock is unity. Thus permanent shocks yield one-to-one consumption responses whereas purely transitory shocks yield a consumption permanent response proportional to the real interest rate.

Three decades after Friedman's seminal work, Deaton (1987) argued that when labor income is characterized by a unit root process, the permanent-income hypothesis fails to predict the smoothness of consumption. This irregularity came to be known as "*Deaton's paradox*". Quah (1989) provided one of the first attempts to reconcile income dynamics and Deaton's argument. He started by decomposing income disturbances into permanent and transitory effects. Then he argued the permanent income hypothesis prediction for smoothness properties of consumption depends on the relative importance of the permanent and transitory disturbances in labor income.

After the earlier works, one of the most important and novel directions of research in this area is the use of panel data on individual income. This is important because it allows both the exploration of individual consumption and income dynamics and their aggregate implications. It also open door to explore various heterogeneities at the individual and household level. One of the earlier works along this direction is Meghir and Pistaferri (2004) which reaffirmed that permanent and transitory shocks are each important components of the composite income shock. More importantly they found that earnings variances are heterogeneous across individuals and there is a strong state dependence in the variances of transitory and permanent income shocks.

While most of the previous studies focused on developing the right distributional framework to capture the moments implied by the data, Heathcote et al (2007) is one of the earliest studies to developed a model with partial insurance against idiosyncratic wage shocks. They quantified risk sharing and decomposed sources of inequality into either shocks during life-cycle or baseline heterogeneity in preferences and productivity. One of the distinguishing contributions of the study is derivation of closed-form solutions for equilibrium allocation and moments of the joint distribution of consumption, hours, and wages. They exploited the closed-form cross-sectional moments to prove identification of the model parameters and estimated the model with data from the CEX and PSID over the period 1967 to 2006. They estimated 40% of permanent wage shocks pass through to consumption. The seminal work by BPP explores the link between income and consumption inequality. Since there is no single source with good quality income and consumption data, they built a household income and consumption panel using two different datasets. Thus, one of their important contributions is a new panel series of income and consumption that combines information from the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX); which is being used in this study as well. While PSID has a good panel series on earnings, information on non-durable consumption is mainly missing. Their imputation process starts from a standard demand function for food (a consumption item available in both the PSID and CEX surveys). This demand function is estimated using the CEX data. They modeled food expenditure and total expenditure as jointly endogenous and allowed their relationship to be time-varying. Given the monotonicity of the estimated food demand, this function is inverted to obtain a measure of nondurable consumption in the PSID. However, this approach posed new estimation challenges. For instance, one of the challenges is precise identification of parameters of interest due to the noise in the imputed consumption data. They focused on the period between 1978 and 1992, when some of the largest changes in income inequality occurred. Particularly, the 1980s are the periods when large changes in income inequality are observed. They found some partial insurance of permanent shocks and full insurance of transitory shocks, except among poor households. They estimated that households self-insure against 36% and 95% of permanent and transitory income shocks, respectively. They explore heterogeneity of partial insurance parameters across age and education. Older and college educated individuals have higher partial insurance against permanent shocks than the younger and no college degree individuals, respectively. These estimates are calculated by grouping the sample into sub-samples based on the groups considered.

There are few studies that confirmed the results of BPP, while numerous others argued against the validity of some their parameter estimates, particularly the partial insurance against permanent income shock parameter. Along these, Kaplan and Violante (2010) analyze the degree of consumption smoothing implicit in a calibrated life-cycle version of the standard incomplete-markets model and compared their results to the empirical estimates of BPP. In their version of BPP's model, the insurable fraction of permanent shocks varies between 7% and 22%, depending on the tightness of debt limits. The important contribution of this study is exploring the effect of borrowing constraint on the degree of self insurance. While the results of BPP hold for households further away from the borrowing limit, the model predicts a much smaller self-insurance to permanent income shocks for households closer to the borrowing limits. Besides, the study showed the BPP estimates are downward biased and the bias grows as borrowing limits become tighter.

Another study following in the footsteps of BPP is Arellano et al (2018). The authors outlined a framework for income dynamics and the nonlinear transmission of income shocks to consumption. They developed a nonlinear model where the impact of past shocks can be altered by the size and sign of new shocks. Their framework allows for "unusual" shocks to alter the trend of past shocks. They have showed that modeling the effect of "unusual" shocks matches the data very well. Hence, nonlinear persistence and conditional skewness (which cannot be captured in the more traditional models of earnings dynamics) are important features of earnings processes. Also, they showed that the non-linearity observed in the earnings process have important implications for consumption choices. More importantly, using family earnings data from administrative records in the Norwegian registers, they validated the evidence of nonlinear persistence in family earnings uncovered based on the PSID data from the 1999 to 2009 surveys.

Furthermore, building on the work of Arellano et al (2018), Arellano et al (2021) estimated the response of consumption to persistent nonlinear income shocks in the presence of unobserved heterogeneity. They found substantial heterogeneity in consumption responses. A very interesting finding is the variation in consumption response by levels and the different stages of consumption cycles. They found that low-consumption types respond more strongly to income shocks at the beginning of the life cycle when their assets are low, whereas the high-consumption types respond less on average, which changes little with age or asset level.

Including BPP, most of the studies estimated almost full partial insurance against transitory income shocks. This evidence is in line with the Permanent Income Hypothesis (PIH) theory which states change in permanent income, rather than the change in temporary income, is what derives the change in a consumer's consumption pattern. However, in a very interesting study, Kaplan et al (2014), analyzed the cohort they dubbed "wealthy hand-to-mouth" and suggested these households have a high marginal propensity to consume out of transitory income changes. Likewise, Edmund Crawley (2019), in his study about the effect of time aggregation, argued that time aggregation of a random walk induces serial correlation in the first difference that is not present in the original series. Once this problem is corrected the estimated partial insurance to transitory shocks (estimated using same data as in BPP) decreases to 76% (it was originally estimated in BPP to be 95%).

The other direction of departure of the recent literature from BPP is in estimation methods. In the labor income processes literature, GMM estimators are almost always used. However, recently Bayesian and likelihood based estimators are getting increased attention. Nakata and Tonetti (2014) examine the validity of using likelihood based estimation by comparing the small sample properties of a Bayesian estimator to those of GMM. They showed the Bayesian estimators demonstrate favorable bias and better efficiency. The study also covers various extensions such as time varying and heterogeneous parameters, non-normal errors, unbalanced panel and missing data models. Moreover, in a more recent study, Chatterjee et al (2021), produced a more precise and significantly higher estimates of consumption insurance of 55% employing quasi maximum likelihood estimator (QMLE), instead.

3. Model Specification and Methodology

3.1 Model Specification

What are the factors determining earnings and employment of individuals over the life cycle?

Altonji et al. (2013) constructed a semi-structural earnings dynamics model with endogenous job mobility and employment, and estimated this model on PSID data using indirect inference. Their modeling framework is similar to that of Low et al. (2010), who analyzed wage dynamics by combining an earning dynamics model with a model of endogenous job mobility and employment based on a search-theoretic framework. Low et al. (2010) found that the size of permanent shocks becomes smaller when allowing for endogenous transitions, and that wage shocks are to a large extent as a result of individuals changing jobs to firms offering better matches. More recently Holmberg (2021) present a model of earnings dynamics that includes transitions into and out of employment as well as business cycle fluctuations.

Here the earnings model is a simplified version of Altonji et al. (2013) and Holmberg (2021). Earnings and labor market participation are simultaneously modeled. I endogenously model labor market transitions, which depend on different sets of observable variables including years of education, potential experience, tenure, employment duration, unemployment duration, dummies for region, race dummies and interaction terms. The unobserved variables include individual heterogeneity such as individual ability, propensity to change employer and other unobserved job match process.

I estimate simple version of Altonji et al. (2013) and Holmberg (2021) earnings model. While Altonji et al. (2013) and Holmberg (2021) model earnings, hours, job matches and employment, I only focus on earnings with endogenous selection to employment.

$$log(Y_{it}) = \boldsymbol{X}_{it}\boldsymbol{\beta} + y_{it} \tag{1}$$

where i and t represent the household unit and time variable, respectively.

The observed household earnings (Y_{it}) is the realization of latent earnings $\left(Y_{it}^{lat}\right)$ through employment $(empl_{it})$, which assumes the value 1 when the individual is employed. The first part of the latent earnings $(\boldsymbol{X_{it}})$ includes a set of observable variables including current tenure, current employment duration, and a set of dummies whether the worker has received unemployment benefits or has been on parental leave at some point during current or previous year. The error term, y_{it}, captures the part of earnings unexplained by the included controls. I model the residuals as income shocks and decompose them into transitory and permanent shocks.

3.2 Estimation of Income and Consumption Shocks

3.2.1 Heckman two step selection model

The Heckman two stage selection model fits a panel-data model with endogenous sample selection. If ignored the endogenous sample selection creates selection bias. Heckman (1979) accounts for the within-panel correlation by using panel-level random effects. The outcome of interest, household earning, is modeled as:

$$log(Y_{it}) = \boldsymbol{X}_{it}\boldsymbol{\beta} + \lambda_{1i} + y_{it} \tag{2}$$

where $log(Y_{it})$ is log of household earnings and $\boldsymbol{X_{it}}$ are the covariates modeling the outcome including education, year of birth, union membership, living in big city, age, age^2 and race. Furthermore, λ_{1i} is the panel-level random effect and y_{it} is the observation-level error.

Next the selection process for the outcome is modeled as;

$$empl_{it} = 1\left(\boldsymbol{Z_{it}\alpha} + \lambda_{2i} + \epsilon_{2it} > 0\right) \tag{3}$$

where $empl_{it} = 1$ if we observe $log\left(y_{it}\right)$ and 0 otherwise, $\boldsymbol{Z}_{it}$ are the covariates modeling selection including education level and number of dependents in the family unit. Also, λ_{2i} is the panel-level random effect for the selection, and ϵ_{2it} is the observation-level selection error.

I assume that the random effects λ_{1i} and λ_{2i} are bivariate normal with mean 0 and variance

$$\boldsymbol{\Sigma_{\lambda}}= \begin{bmatrix} \sigma^2_{1\lambda} & \rho_{\lambda}\sigma_{1\lambda}\sigma_{2\lambda} \\ \rho_{\lambda}\sigma_{1\lambda}\sigma_{2\lambda} & \sigma^2_{2\lambda} \end{bmatrix}$$

Next, the observation-level errors y_{it} and ϵ_{2it} are bivariate normal, with mean zero and variance

$$\boldsymbol{\Sigma}= \begin{bmatrix} \sigma^2_{y} & \rho\sigma_1 \\ \rho\sigma_1 & 1 \end{bmatrix}$$

The observation-level errors are independent of the random effects. The variance of ϵ_{2it} is normalized to 1 for identification.

The earning process in equation 2 can be estimated using two approaches. The first method is known as the two-step method. First I run a Probit on the employment model (specified in equation 3). Then, I recover estimated Inverse Mills Ratio (IMR),

$$IMR = \frac{\phi\left(\boldsymbol{Z}_{it}\hat{\alpha}\right)}{\Phi\left(\boldsymbol{Z}_{it}\hat{\alpha}\right)}$$

Finally, using Ordinary Least Squares (OLS) estimate the model,

$$log(Y_{it}) = \boldsymbol{X}_{it}\boldsymbol{\beta} + \rho IMR + y_{it}$$

The Inverse Mills Ratio picks up the expected value of the error in the earning equation conditional on employment. This will reflect the idea that the households with large negative earning errors are not working, so the expected value of the earning error is no longer zero for some of the households who do work. The coefficient on the IMR is used to test for selection, since it represents the covariance between the errors in the earnings and the employment equation under the assumptions of the model.

The unconditional predictor of average earnings, for the case where we do not know whether or not either the household head or the spouse is employed, is given by equation (4) below.

$$\mathbb{E}[\boldsymbol{log}(Y_{it})] = log\left(\hat{Y}_{it}\right) = \boldsymbol{X}'_{it}\hat{\boldsymbol{\beta}} \tag{4}$$

While the conditional predicted earnings for employed and unemployed households are given by equations 5 and 6 respectively.

$$\mathbb{E}[\boldsymbol{log}(Y_{it})|empl_{it} = 1] = log\left(\hat{Y}^{e}_{it}\right) = \boldsymbol{X}'_{it}\hat{\boldsymbol{\beta}} + \hat{\rho}\frac{\phi\left(\boldsymbol{Z}_{it}\hat{\alpha}\right)}{\Phi\left(\boldsymbol{Z}_{it}\hat{\alpha}\right)} \tag{5}$$

$$\mathbb{E}[\boldsymbol{log}(Y_{it})|empl_{it} = 0] = log\left(\hat{Y}^{un}_{it}\right) = \boldsymbol{X}'_{it}\hat{\boldsymbol{\beta}} + \hat{\rho}\frac{-\phi\left(\boldsymbol{Z}_{it}\hat{\alpha}\right)}{1-\Phi\left(\boldsymbol{Z}_{it}\hat{\alpha}\right)} \tag{6}$$

Note that if $\hat{\rho} = 0$ there is no selection bias.

The second method uses Maximum Likelihood Estimator (MLE) over the full parameter set. Conditioning on the random effects λ_{1i} and λ_{2i}, we can write the joint density of $log(Y_{it})$ and $empl_{it}$.

$$f\left(Y_{it}, empl_{it}|\lambda_{1i}, \lambda_{2i}\right) = \begin{cases} \Phi\left(\frac{\boldsymbol{Z_{it}\alpha}+\lambda_{2i}+\frac{\rho}{\sigma_y}(log(Y_{it})-\boldsymbol{X_{it}\beta}-\lambda_{1i})}{\sqrt{1-\rho^2}}\right) + \phi\left(\frac{log(Y_{it})-\boldsymbol{X_{it}\beta}}{\sigma_y}\right) & \text{if} \quad empl_{it}=1 \\ \Phi\left(\frac{-\boldsymbol{Z_{it}\alpha}-\lambda_{2i}}{\sqrt{1-\rho^2}}\right) & \text{if} \quad empl_{it}=0 \end{cases}$$

So the likelihood for panel i is

$$L_i = \int_{\mathbb{R}^2} \left[\prod_{t=1}^{N_i} f\left(log(Y_{it}), empl_{it}|\lambda_{1i}, \lambda_{2i}\right) \phi_2\{(\lambda_{1i}, \lambda_{2i}), \Sigma_\lambda\} d\lambda_{1i} d\lambda_{2i}\right]$$

Since this multivariate integral is generally not tractable, we use a change of variables technique to transform it into a set of nested univariate integrals. Then, the univariate integral can be approximated using Gauss-Hermite quadrature (GHQ).

3.2.2 Permanent and Transitory Shocks

Following the literature building on Friedman (1958) permanent income theory, the income shock is decomposed into permanent component, $\boldsymbol{\tau}$, and transitory component, $\boldsymbol{\epsilon}$. The transitory income shocks captures events such as surprise bonus or temporary leave due to illnesses, while the permanent income shocks include severe health shocks, promotion or other factors that result in permanent income change.

The income shock is the residual of the earning model given by equation 2 above.

$$y_{it} = \log\left(Y_{i,t}\right) - \boldsymbol{X}'_{i,t}\boldsymbol{\beta}$$

Next the income shock (or unexplained income process) is decomposed into a permanent and transitory components:

$$y_{it} = \tau_{i,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_{\epsilon}^{2}\right) \tag{7}$$

The permanent component follows a random walk process of the form,

$$\tau_{i,t} = \tau_{i,t-1} + \eta_{i,t}, \quad \eta_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_{\eta}^{2}\right) \tag{8}$$

where $\eta_{i,t}$ is serially uncorrelated, and the transitory component $\epsilon_{i,t}$ is a white noise (WN). However, without loss of generality, the transitory component can be modeled as an $MA(q)$ process; where the order q is established empirically:

$$\epsilon_{i,t} = \sum_{j=0}^{q} \gamma_j \xi_{i,t-j}$$

with $\gamma_0 \equiv 1$.

Now, the growth of the income residual can be written as:

$$\Delta y_{i,t} = \eta_{i,t} + \Delta\epsilon_{i,t} \tag{9}$$

where $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$ and $\Delta\tau_{i,t} = \eta_{i,t}$.

3.2.3 Consumption Partial Insurance

Estimating consumption shock, I follow BPP with a slight modification. While BPP include employment and unemployment status as deterministic controls, I exclude both variables from the controls of first stage consumption regression. First, (see equation 10 below) I regress consumption on vectors of household characteristics excluding employment and unemployment status; I use similar sets of controls as in the income regression above (see equation 2).

$$\log\left(C_{i,t}\right) = \boldsymbol{X}_{i,t}'\boldsymbol{\gamma} + c_{i,t}, \quad c_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_{c}^{2}\right) \tag{10}$$

where $\boldsymbol{X}_{i,t}$ is a set of consumption characteristics observed at time t for each household i. In the partial insurance framework, consumption growth is modeled as a function of the income shocks. This approach allows to analyze the degree of correlation between income and consumption shocks.

$$\Delta c_{i,t} = \phi \eta_{i,t} + \psi \epsilon_{i,t} + \nu_{i,t}, \quad \nu_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_\nu^2\right) \tag{11}$$

where $c_{i,t}$ is the log of real consumption net of its predictable components; i.e. $c_{i,t} = \log\left(C_{i,t}\right) - \boldsymbol{X}'_{i,t}\boldsymbol{\alpha}_t$ and $\Delta c_{i,t} = c_{i,t} - c_{i,t-1}$.

The component $\nu_{i,t}$ consists of a serially uncorrelated earnings shock plus a measurement error component. The variance of the two components cannot be separately identified from the earnings data alone. Studies that attempt to distinguish the two typically either draw on outside estimates of the variance of pure measurement error or estimate the earnings process jointly with a model of a choice variable that should respond only to true earnings shocks, such as consumption.

3.3 Estimation of Insurance Parameters and Volatilities

3.3.1 Benchmark Model

To recap, the benchmark model is given:

$$\Delta c_{i,t} = \phi \eta_{i,t} + \psi \epsilon_{i,t} + \nu_{i,t}, \quad \nu_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_\nu^2\right)$$

For the benchmark model I estimate the partial insurance and volatility parameters using Generalized Method of Moments (GMM). Let's start by stacking the variables of interest:

$$\Delta \boldsymbol{c}_i = \begin{pmatrix} \Delta c_{i,1} \\ \Delta c_{i,2} \\ \vdots \\ \Delta c_{i,T} \end{pmatrix}, \quad \Delta \boldsymbol{y}_i = \begin{pmatrix} \Delta y_{i,1} \\ \Delta y_{i,2} \\ \vdots \\ \Delta y_{i,T} \end{pmatrix}$$

Then,

$$\boldsymbol{x}_i = \begin{pmatrix} \boldsymbol{\Delta c}_i \\ \boldsymbol{\Delta y}_i \end{pmatrix}$$

Now we can derive

$$\boldsymbol{m} = vech \left\{ \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{x_i x_i'} \right\}$$

The vector $\boldsymbol{m}$ contains the estimates of $T(2T+1)$ unique moments. Let θ be a k-dimensional parameter vector. Assume the true value of θ is θ_0, which belongs to the interior set of the compact parameter space $\Theta \subset \mathbb{R}^k$. Moreover, the population moments are $\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right)$, where in our case, the $\boldsymbol{\theta}$ is the vector of variances of the permanent shock and transitory shock, the partial insurance parameters and moving average and autoregressive parameters.

The $T(2T+1)$ dimension population orthogonality conditions are:

$$\mathbb{E}\left[\boldsymbol{\Phi}\left(\boldsymbol{\theta_0}\right) - \boldsymbol{m}\right] = 0$$

The GMM estimator, $\hat{\theta}_{GMM}$, minimizes the objective function $F^0\left(\theta\right)$ over $\theta \subset \Theta$:

$$F^0\left(\theta\right) = \left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right) - \boldsymbol{m}\right]' \boldsymbol{W}\left(\boldsymbol{\theta}\right)\left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right) - \boldsymbol{m}\right]$$

where $\boldsymbol{W}\left(\boldsymbol{.}\right)$ is an $Op(1)$ and a positive definite $T(2T+1) \times T(2T+1)$

weighting matrix.

3.3.2 Heterogeneous Parameters Model

Now, I extend the benchmark model by sequentially introducing heterogeneities in partial consumption insurance parameters and income and consumption volatilities. First, we allow the partial consumption insurance parameters to vary by household types. Hence, I consider income and consumption dynamic model with constant variance but heterogeneous partial insurance parameters.

$$\phi_i = \bar{\phi} + \phi^{type} D_i \tag{12}$$

$$\psi_i = \bar{\psi} + \psi^{type} D_i \tag{13}$$

where, D_i is a dummy variable that takes value 1 for individuals with the specific type and zero otherwise. In this specific case, $\{type\}$ is $\{race\}$. Hence, the base categories $\bar{\phi}$ and $\bar{\psi}$ represent the permanent and transitory partial consumption insurance parameters of white household heads respectively. Now, the permanent and transitory partial consumption insurance coefficient for black household heads will be:

$$\phi_{type} = \bar{\phi} + \phi^{type}$$

$$\psi_{type} = \bar{\psi} + \psi^{type}$$

Next, in addition to the partial consumption insurance parameters, I explore heterogeneity in income and consumption shock volatilities. Similar to the partial consumption insurance parameters, I allow the volatility parameters to depend on race.

$$\sigma^2_{\eta_i} = \sigma^2_{\eta} + \sigma^{type}_{\eta} D_i \tag{14}$$

$$\sigma^2_{\epsilon_i} = \sigma^2_{\epsilon} + \sigma^{type}_{\epsilon} D_i \tag{15}$$

$$\sigma^2_{\nu_i} = \sigma^2_{\nu} + \sigma^{type}_{\nu} D_i \tag{16}$$

Again the base categories represent the volatility parameters of white household heads; while the base categories plus the respective slope coefficients capture the volatility measures of black household heads.

The time varying parameters is the most extensive explored heterogeneity. I model both the variance and partial insurance as time varying parameters. Hence, the transitory and permanent income shocks and the the transitory consumption shock variances can be re-specified as:

$$\epsilon_{i,t} \sim \textit{i.i.d.}\left(0, \sigma^2_{\epsilon_t}\right)$$

$$\eta_{i,t} \sim \textit{i.i.d.}\left(0, \sigma^2_{\eta_t}\right)$$

$$\nu_{i,t} \sim \textit{i.i.d.}\left(0, \sigma^2_{\nu_t}\right)$$

BPP argue the partial consumption insurance parameters are constant overtime. However, I let the transitory and permanent partial consumption insurance vary across time:

$$\phi_t = \phi + \phi_2 D_2 + ... + \phi_T D_T \tag{17}$$

$$\psi_t = \psi + \psi_2 D_2 + ... + \psi_T D_T \tag{18}$$

where $D_2, ..., D_T$ are year dummy variables taking value 1 for the particular year and zero for all other years.

However, the econometrician does not know the true data generating process. Hence, model selection in the face of limited data is an important issue in econometric modeling. Model selection can be framed as an exercise to decide which elements of the parameter vector should be set to zero. I propose the LASSO type GMM estimator; hereafter known as penalized-GMM (PGMM).

The penalized-GMM estimator, $\hat{\theta}_{PGMM}$, minimizes the following objective function over the compact set Θ:

$$F(\theta) = [\boldsymbol{\Phi}(\boldsymbol{\theta}) - \boldsymbol{m}]' \boldsymbol{W}(\boldsymbol{\theta}) [\boldsymbol{\Phi}(\boldsymbol{\theta}) - \boldsymbol{m}] + \lambda_k \sum_{j=1}^{k} |\theta_j| \tag{19}$$

for a given positive regularization parameter λ_k. While the limit for the penalized-GMM estimator is nonstandard, $\hat{\theta}_{PGMM}$ is a consistent estimator for θ_0 as long as $\lambda_k = o(N)$. Besides, when λ_k grows slowly, the limit of the penalized-GMM estimator converges to the limit of the regular GMM estimator (Caner (2009)). Also, the LASSO-type GMM estimator is powerful in that it endogenously selects subsets of features, without a need to build and compare large number of different models with subsets of the feature. It shrinks the estimates of the redundant variables to zero with positive probability. I harness this feature to uncover the most important partial insurance and consumption and income volatility parameters by household types. Below I conduct a Monte Carlo experiment to demonstrate how well our method works. But first, Table 1 summarizes the identifying equations linking the data with the unknown parameters to be estimated.

I present the data and moment condition to estimate income and consumption volatility parameters and partial consumption insurance parameters for different type of households. Although the equations estimate heterogeneity across one household type, the identification conditions can be easily extended to multiple household types. Also, it can be extended without loss of generality to capture the parameter identification condition for the time varying parameter models.

Table 1: Population parameters and the identifying equations

Data	Structural parameters
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1})\,\vert D_i = 0]$	σ_η^2
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1})\,\vert D_i = 1]$	$\sigma_\eta^2 + \sigma_\eta^{type}$
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1})\,\vert D_i = 0]$	$-\sigma_\epsilon^2$
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1})\,\vert D_i = 1]$	$-\sigma_\epsilon^2 - \sigma_\epsilon^{type}$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta c_{i,t})\,\vert D_i = 0]$	$\phi^2\sigma_\eta^2 + \psi^2\sigma_\epsilon^2 + \sigma_\nu^2$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta c_{i,t})\,\vert D_i = 1]$	$(\phi + \phi^{type})^2(\sigma_\eta^2 + \sigma_\eta^{type}) + (\psi + \psi^{type})^2(\sigma_\epsilon^2 + \sigma_\epsilon^{type}) + \sigma_\nu^2 + \sigma_\nu^{type}$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t})\,\vert D_i = 0]$	$\phi\sigma_\eta^2 + \psi\sigma_\epsilon^2$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t})\,\vert D_i = 1]$	$(\phi + \phi^{type})(\sigma_\eta^2 + \sigma_\eta^{type}) + (\psi + \psi^{type})(\sigma_\epsilon^2 + \sigma_\epsilon^{type})$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1})\,\vert D_i = 0]$	$\phi\sigma_\eta^2$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t-1} + \Delta y_{i,t} + \Delta y_{i,t+1})\,\vert D_i = 1]$	$(\phi + \phi^{type})(\sigma_\eta^2 + \sigma_\eta^{type})$
$\mathbb{E}[\Delta c_{i,t-1}\,(\Delta y_{i,t})\,\vert D_i = 0]$	$-\psi\sigma_\epsilon^2$
$\mathbb{E}[\Delta c_{i,t-1}\,(\Delta y_{i,t})\,\vert D_i = 1]$	$-(\psi + \psi^{type})(\sigma_\epsilon^2 + \sigma_\epsilon^{type})$

‡ This table summarizes the identifying equations for the parameters $\phi, \phi^{type}, \psi, \psi^{type}, \sigma_\eta^2, \sigma_\eta^{type}, \sigma_\epsilon^2, \sigma_\epsilon^{type}, \sigma_\nu^2, \sigma_\nu^{type}$

4. Monte Carlo Simulation

In this section I first conduct a series of simulated experiments to assess how well the PGMM performs in recovering consumption and income volatilities and partial consumption insurance parameters in the data generating process. I then document the Root Mean Squared Error (RMSE) to assess how the PGMM estimator fairs against the usual GMM estimator.

First, I simulate consumption and income shocks for $N = 1,500$ households across $T = 15$ years; this mimics the BPP data set I use in the study. I estimate the volatility and partial consumption insurance parameters using GMM and PGMM estimators with the identity weighting matrix. The parameter estimates with the corresponding true values summarized in tables 2 and 3 below. Additionally, using 100 Monte Carlo draws, I estimate the RMSE of the two estimators. This enables me to compare the performances of the GMM and PGMM estimators. The results show the PGMM estimator is as good or better than the GMM estimator in the RMSE sense.

Most of the data sets available for estimating labor income processes are unbalanced. Hence, it is imperative to assess the estimators' performance in the face of missing data. I conduct the Monte Carlo experiment for both balanced data and unbalanced data set with missing values. Note that in the simulation experiment I allow the partial consumption insurance parameters and the variances of income and consumption shocks to vary with race. Moreover, for the PGMM estimator we chose the tuning parameter, λ.

Table 2: Panel A: Heterogeneous partial consumption insurance

Parameter	symbol	True value	GMM	RMSE	PGMM	RMSE
		N=1500	$\lambda = 0.0029$			
base c to a perm. y	ϕ	0.2	0.1194	0.0267	0.1347	0.0256
type c to a perm. y	ϕ^{type}	0.2	0.2931	0.0376	0.2598	0.0336
base c to a trans. y	ψ	0.05	0.0874	0.0214	0.047	0.018
type c to a trans. y	ψ^{type}	0	-0.0361	0.015	0	0.021

‡ This table summarizes the Monte Carlo experiment result comparing GMM and PGMM estimators. Based on the RMSE, PGMM estimator recovers the true population parameters better.

Table 3: Panel B: Heterogeneous income and consumption volatility

Parameter	symbol	True value	GMM	RMSE	PGMM	RMSE
		N=1500	$\lambda = 0.0029$			
base var of trans. y	σ_ϵ^2	0.1	0.0799	0.0054	0.0802	0.0053
base var of perm. y	σ_η^2	0.1	0.1139	0.0069	0.1139	0.0069
base var of trans. c	σ_ν^2	0.075	0.0786	0.0017	0.0775	0.0016
type var of trans. y	σ_ϵ^{type}	0.1	0.0831	0.0112	0.0845	0.011
type var of perm. y	σ_η^{type}	0.1	0.1348	0.0146	0.1334	0.0149
type var of trans. c	σ_ν^{type}	0	-0.0034	0.003	0	0.003

‡ This table summarizes the Monte Carlo experiment result comparing GMM and PGMM estimators. Based on the RMSE, PGMM estimator recovers the true population parameters better.

Table 2 above summarizes the Monte Carlo experiment results for the balanced panel data case. The results show GMM and PGMM estimators recover the population parameters very well. The PGMM estimator has slightly smaller RMSE values; hence, PGMM estimates the parameters more accurately (i.e. with lower bias and variance).

However, when using real data, I need to modify the model to address unique features of the dataset. Most panel datasets used for estimating the consumption insurance parameters are unbalanced. Attrition is a common problem and, most of the time, household data are available for different lengths of time. The dataset contains different cohorts and will contain many missing observations. I assign a 5% probability of being missing to each of the $(T \times N)$ observations. The GMM and Penalized GMM estimators still recover the true population parameters very well even with the prevalence of missing observations.

Moreover, it is clear from the boxplots that the PGMM has lower Mean Squared Errors (MSEs). For example, the median MSE for PGMM is 0.0078, while that of GMM is 0.0083.

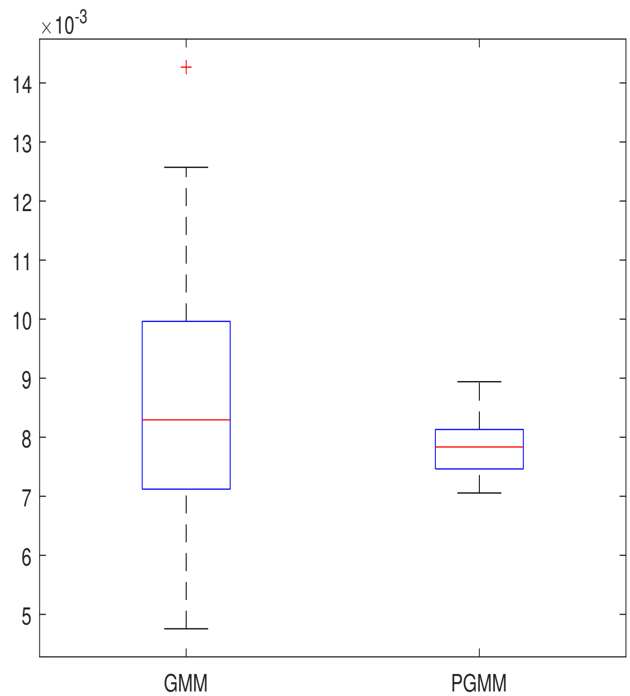

Figure 1: Boxplots of the mean squared errors for the GMM and the PGMM estimators.The central mark indicates the median, whereas the bottom and the top edges of the box indicate the 25th and 75th percentiles, respectively. The whiskers extend to the minimum and the maximum.

Table 4: Heterogeneous partial insurance with missing values

Parameter	symbol	True value	GMM	RMSE	PGMM	RMSE
		N=1500	$\lambda = 0.0029$			
base c to a perm. y	ϕ	0.2	0.1194	0.071	0.1347	0.063
type c to a perm. y	ϕ^{type}	0.2	0.2931	0.097	0.2598	0.085
base c to a trans. y	ψ	0.05	0.0874	0.048	0.047	0.036
type c to a trans. y	ψ^{type}	0	-0.0361	0.06	0	0.032

‡ This table summarizes the Monte Carlo experiment result comparing GMM and PGMM estimators, for panel data with missing observations. Based on the RMSE, PGMM estimator recovers the true population parameters better.

Table 5: Heterogeneous volatility with missing values

Parameter	symbol	True value	GMM	RMSE	PGMM	RMSE
		N=1500	$\lambda = 0.0029$			
base var of trans. y	σ^2_ϵ	0.1	0.0799	0.01	0.0802	0.01
base var of perm. y	σ^2_η	0.1	0.1139	0.017	0.1139	0.017
base var of trans. c	σ^2_ν	0.075	0.0786	0.003	0.0775	0.003
type var of trans. y	σ^{type}_ϵ	0.1	0.0831	0.04	0.0845	0.039
type var of perm. y	σ^{type}_η	0.1	0.1348	0.022	0.1334	0.022
type var of trans. c	σ^{type}_ν	0	-0.0034	0.008	0	0.006

‡ This table summarizes the Monte Carlo experiment result comparing GMM and PGMM estimators, for panel data with missing observations. Based on the RMSE, PGMM estimator recovers the true population parameters better.

5. Data and Empirical Results

5.1 Data

To explore the effect of income shocks on consumption, I need a panel series on both consumption and income. However, in US such comprehensive data sets do not exist. But two data sets come close. The first one is the Panel Study of Income Dynamics (PSID), which collects longitudinal annual data. The main shortcomings of PSID, related to this study, is that it collects data only for a subset of consumption items, mainly food expenditures. The second data source is the Consumer Expenditure Survey (CEX). CEX provides comprehensive information on the spending habits of US households; but households are followed only for a maximum of four quarters. While various studies investigate the link between the evolution of income and consumption inequality using either the PSID or CEX datasets (Krueger and Perri (2006)), few studies combined the PSID data with data from repeated cross-sections of the CEX (BPP, Ziliak (1998) and Jonathan Skinner (1987)).

The measure of consumption captures the flow of consumption services to households at a given period of time. However, for large durable goods such as cars and houses, current expenditure may not reflect the flow of services at a given period of time. Hence, consumption expenditure is mainly spending on nondurable goods and services.

What constitutes nondurable expenditure? Following Attanasio and Weber (1995) nondurable consumption includes food, alcoholic beverages and tobacco, services, heating fuel, transports (including gasoline), personal care, clothing and footwear, and rents. While PSID has data on food expenditure, it lacked information on other nondurable expenditures. However, the dynamics of food expenditure greatly differs from those of other nondurable expenditures. Being more of a necessity, food expenditure has preference elasticity and expenditure volatility less likely to be generalized to total nondurable consumption. Since food expenditures are very inelastic, using them as a proxy for other nondurable expenditures will greatly underestimate the volatility of total nondurable consumption.

One way around this challenge will be to combine information from the CEX and the PSID and impute a measure of consumption to the PSID households. This approach is first tried by Skinner (1987). He imputed total consumption in the PSID data base using the estimated coefficients of a regression of total consumption on common controls present in both the PSID and CEX data bases. The regression is first estimated using CEX data and later a corresponding PSID panel of consumption series is constructed using the estimated coefficients. Although, this approach technically resembles to the idea of matching based on observed characteristics, the quality of the imputed data depends on the reliability of the estimated coefficients. Moreover, the regression model can be ridden by biases mainly induced by initial differences in the input and target data sets and measurement errors in consumption.

BPP also constructed a new panel data set with household information on income and nondurable consumption. Since, I am basing my analysis on their dataset, I provide short summary of the procedure. As explained above CEX has enough information to construct consumption regression model. However, PSID is missing some of the variables in the statistical relationship. Hence, the procedure extracts information from CEX to impute the missing variables in the PSID data set, exploiting the common statistical relationship among the variables across the two data sets. Assume c index observations from the CEX (the input data set) and p index an observation from PSID (the target data set). Now, consider the following food-demand equation:

$$D\left(f_{i,c}\right) = \boldsymbol{X'_{i,c}}\boldsymbol{\beta}^{c} + \alpha^{c}\psi\left(z_{i,c}\right) + \nu_{i,c}$$

where f is food expenditure variable available in both the PSID and CEX data sets. While $\boldsymbol{X}$ contains vectors of individual characteristics and aggregate variables which are available in both data sets, z is the total nondurable expenditure variable only available in the CEX data set. Lastly, the error term ν captures the unobserved heterogeneity in the demand for food regression model and $D\left(.\right)$ and $\psi\left(.\right)$ are food and nondurable demand functions respectively. Assuming the functions D and ψ are monotonically increasing and augmenting the standard OLS estimator to tackle the presence of measurement error in total consumption, Blundell et al (2006) estimated the above food demand equation parameters $\hat{\boldsymbol{\beta}}^c$ and $\hat{\alpha}^c$. Now, using these estimated parameter values, the imputed consumption variable in the PSID is calculated as:

$$\hat{z}_{i,p} = \psi^{-1}\left(\frac{D\left(f_{i,p}\right) - \boldsymbol{X}'_{i,p}\hat{\boldsymbol{\beta}}^c}{\hat{\alpha}^c}\right)$$

(assume $\hat{\alpha}^c > 0$).

For this study we used the BPP data set with imputed consumption observation. The BPP data focused on male headed stable households. The panel data runs from 1978 to 1992. After the final cleaning we are left with a total of 17,612 households of which about 5.5% have black household heads. Hence, I cannot explore the insurance and volatility heterogeneity across gender and, for now, I will focus on uncovering the heterogeneities across race only.

The main variables of interest are husband and spouse earnings and non-durable consumption spending. I use real after tax earnings and real non-durable consumption spending.

$$log\left(\tilde{Y}_{it}\right) = log\left(\frac{Y_{it} - tax_{it}}{CPI_t}\right)$$

where tax_{it} and Y_{it} are federal income tax and nominal husband and wife earnings, respectively. The CPI base is $1982 - 1984 = 100$.

$$log\left(\tilde{C}_{it}\right) = log\left(\frac{C_{it}}{CPI_t}\right)$$

where C_{it} is imputed nominal consumption spending.

5.2 Benchmark Model

In the BPP earning model, employment status is included as an exogenous control. However, the sample exhibits frequent in and out of employment transitions by both head of households and spouses. Furthermore, the unemployment incidences are not randomly distributed in the sample. Particular fraction of the households contribute to the majority of the unemployment incidences. For instance, 4% of the household heads are unemployed for the entire sample period. On the other hand, 30% of the household heads are employed over the entire period. Moreover, 45% of the household heads are responsible for 70% of the unemployment incidence (see Figure 2).

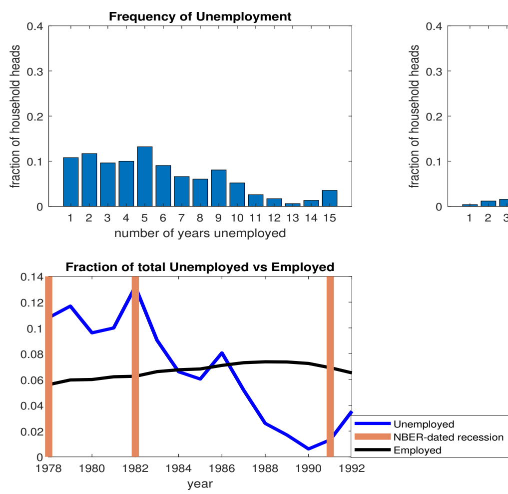

Figure 2: The Frequency of unemployment and employment incidences[1]

Although, employment and unemployment transitions appear to be frequent in the sample, previous studies (including BPP, Chatterjee et al (2021)) consider employment and unemployment status as predictable part of earn-

[1]Figure 2 summarizes the employment and unemployment incidences in the sample. The first and second panels present the frequency of unemployment and employment respectively. The bottom third panel represents the evolution of fractions of employed and unemployed. The fractions of employed and unemployed are calculated out of the total employed and unemployed in the sample. The plot shows that the highest fractions of unemployed are observed during the early period of the sample.

ings. Figure 3 below compares the volatility of income shock with and without unemployment risk. The red and blue lines represent volatilities of income shock with and without unemployment risk, respectively. The bar plots show the rate of the difference between the two plots. Unemployment risk contributes between 4% and 13% to income shock volatility.

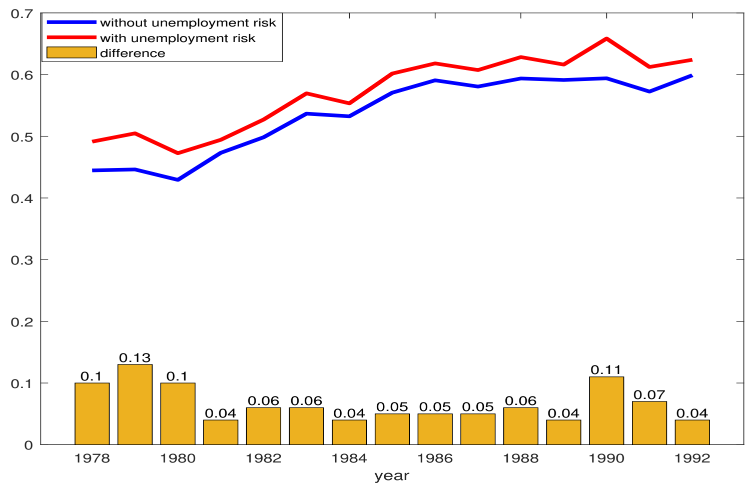

Figure 3: The variance income shock with and without unemployment risk[2]

The selection to employment equation (equation 3) is estimated by maximum likelihood as an independent probit model to determine the decision to work using information from the whole sample of employed and unemployed. Then the vector of inverse Mills ratios, which is an estimated expected error, is generated from the parameter estimates (Greene, 1993). The level of real log earnings, $log(Y_{it})$, is observed only when the employment equation equals 1 and is then regressed on the exogenous controls, $\boldsymbol{X}_{i,t}$, and the vector of inverse Mills ratios from the selection equation by ordinary least squares (OLS). Hence, the second stage runs the regression with the estimated expected error included as an extra explanatory variable, after removing the part of the error term correlated with the explanatory variable

[2] Figure 3 captures the evolution of income shock volatilities with and without unemployment risk. The bar plots represent the rate of the difference.

to avoid the bias. Thus, sample selection bias is corrected by the selection equation, which determines whether an observation makes it into the non-random sample.

Table 6: Income and consumption regression results

	(1)		(2)	
variable	log(y)	t-stat	log(c)	t-stat
high school	0.39***	21.27	-0.023	-0.567
	(0.018)		(0.041)	
college	0.61***	35	0.055	1.416
	(0.018)		(0.039)	
black	-0.295**	-8.69	-0.243**	-2.207
	(0.034)		(0.11)	
big city	0.037***	3.71	0.136***	4.500
	(0.01)		(0.03)	
family size(==3)	-0.074***	-5.91	0.074	1.194
	(0.012)		(0.062)	
family size(==9)	0.23	1.35	-0.054	-0.131
	(0.17)		(0.412)	
retired	-0.265***	-11	0.173	0.897
	(0.022)		(0.19)	
N	17612		14992	

Note: *** p<0.01, ** p<0.05, * p<0.1. The standard errors are provided in parenthesis.

‡ This table summarizes the first stage regression result of consumption and income. I projected log income and log consumption on the same sets of household characteristics. The residuals from the first stage regression are used to analyze the link between income and consumption inequalities.

Table 6 above presents the first stage regression result for a subset of regressors. For the level of education completed by household head variable, non-completion of high-school graduates are the base category. Both the high school graduates and college graduates earn significantly higher than the base category. While college graduates earn 61% more, high school graduates earn about 39% more than the base category. However, the consumption differentials are not statistically significant.

Moreover, black household heads earn and consume significantly less than their white counter parts. They earn about 30% less and end up consuming 24% less as well. For family size, the base category is households with two family members. The maximum family size reported in our sample is 10. We have omitted the results for other family sizes and only include households with 3 and 9 family sizes. While households with 3 family size earns marginally less than the base category, households with 9 family members earn significantly more than the base category. Households with 9 family members earn about 23% more than those with 2 family members. While this differential seems large, it makes sense given the difference in the member of families supported by the households. However, the results on consumption are not statistically significant.

Furthermore, the retired household heads earn 26.5% less than the yet to be retired household heads and consume 17.3% more. These estimates are in line with the life cycle consumption theory.

The correlations of interest from the Heckman two step regression are reported in Table 5. The first correlation is the correlation of the residuals in the earnings and employment (selection) equation, i.e. the correlation between $\boldsymbol{y}$ and $\boldsymbol{\epsilon_2}$. The second is the correlation of the random effects and time invariant unobservables, i.e. $\boldsymbol{\lambda_1}$ and $\boldsymbol{\lambda_2}$. Selection will be an issue if either of the correlations are significant. In our case, for both models, the correlation between the time invariant unobservables are significant at 5% level. Thus, selection is an issue and it needs to be endogenously modeled.

A classic potential random effect and time invariant unobservable that can be correlated across the earning and selection equation is ability. Households with high ability household heads and/or spouses tend to earn more and also have a higher probability of selecting into employment. Thus, the correlation coefficient estimate is positive.

Table 7: Correlations between income and selection equations

correlations	estimate	standard error	p_{value}
corr($\boldsymbol{y_i}$,$\boldsymbol{\epsilon_i}$)	-0.064	0.03	0.036
corr($\boldsymbol{\lambda_1}$,$\boldsymbol{\lambda_2}$)	0.3	0.02	0

‡ Summarizes the correlation between the idiosyncratic error terms and unobserved random effects of the earning and selection equations. Since, both are statistically significant, selection is an issue.

5.2.1 The Covariance of Income and Consumption

The estimation proceeds in two steps. In the first stage we project real after tax income, log $\left(\tilde{Y}\right)$, and real non-durable consumption spending, log $\left(\tilde{C}\right)$, onto household characteristics. After deriving the residuals from the first stage regression, I decompose the income residuals into permanent and transitory income shocks and further project them onto the residuals from the consumption regression. From the second stage estimation we derive the partial consumption insurances and consumption and income volatilities. Table 6 below shows the correlations of income and consumption for the current, lagged and lead time periods. The GMM estimator identifies the permanent and transitory partial consumption insurance parameters and the income and consumption volatilities using the covariances of income and consumption shocks.

Moreover, Figure 4 compares the first and second moments of income shocks and the predicted earnings with and with out unemployment risk. For most of the sample periods, the average predicted income is higher without including unemployment risk. However, the income shock volatilities are higher for the Heckman model (where unemployment is endogenously modeled).

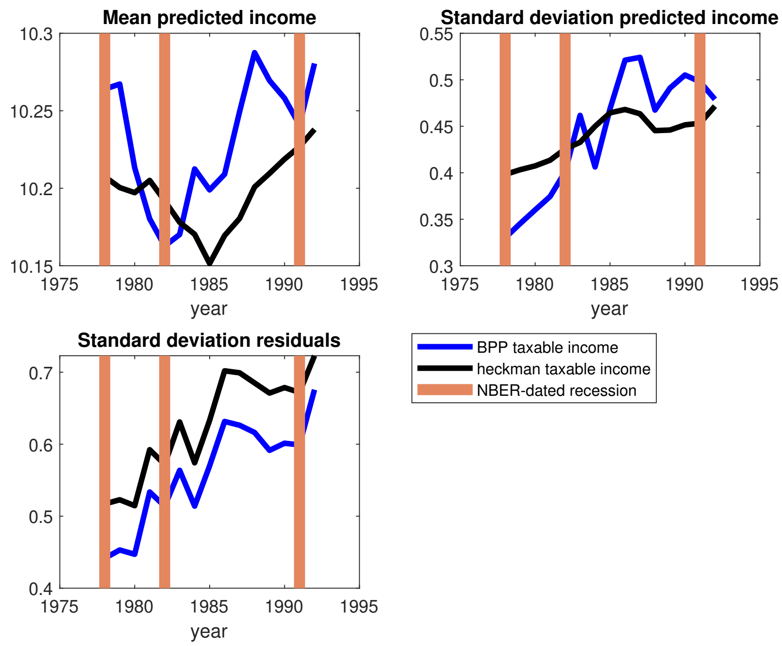

Figure 4: Comparing the volatility of income shock between BPP and Heckman[4]

Table 7 summarizes the transitory and permanent partial consumption insurance estimates with and without unemployment risk. Since, Heckman two step regression estimates shocks of the employed households, it reports higher partial insurance parameter estimates compared to the ones using BPP approach.

[4]Figure 4 compares the first and second moments of predicted income and residual income for BPP (OLS) and Heckman two step selection model. When unemployment risk is considered, the volatility of income shock is significantly higher.

Table 8: The partial insurance parameters with and without unemployment risk

Shock	Model	Symbol	GMM	Bootstrap SE
Permanent	BPP	ϕ^b	0.27	0.05
Permanent	Heckman	ϕ^h	0.22	0.035
Transitory	BPP	ψ^b	0.08	0.032
Transitory	Heckman	ψ^h	-0.015	0.047

‡ Summarizes the permanent and transitory partial insurance parameters with and without unemployment risk. The corresponding bootstrap standard errors are also reported.

5.3 Heterogeneous Parameters Model

To estimate the partial consumption insurance I am projecting the consumption residual onto the income residual. Hence, I need to make note of how much of the variations in income and consumption we took out during the first stage regression. We took out 35% and 78% of income and consumption variation, respectively.

Before analyzing the heterogeneities of the consumption insurance and volatility parameters, I examine (see Figure 2) the log income volatilities for the black household heads, white household heads and the combined sample. As we can see the log income of black household heads is much more volatile than both the whole sample and white household heads. The volatility plot for the white household heads appears to closely track the whole sample. This is not surprising given about 95% o the households in the sample have white household heads.

Similar to income volatilities, households with black heads have a much more volatile income shock (see Figure 2). Here the income shock captures the unpredictable part of log income and it appears to be more volatile for households with black heads compared to both the ones with white heads and the combined sample. Also, as can be seen from Figure 2, the average income of households with black heads is significantly lower than those with white household heads for the entire period of our sample.

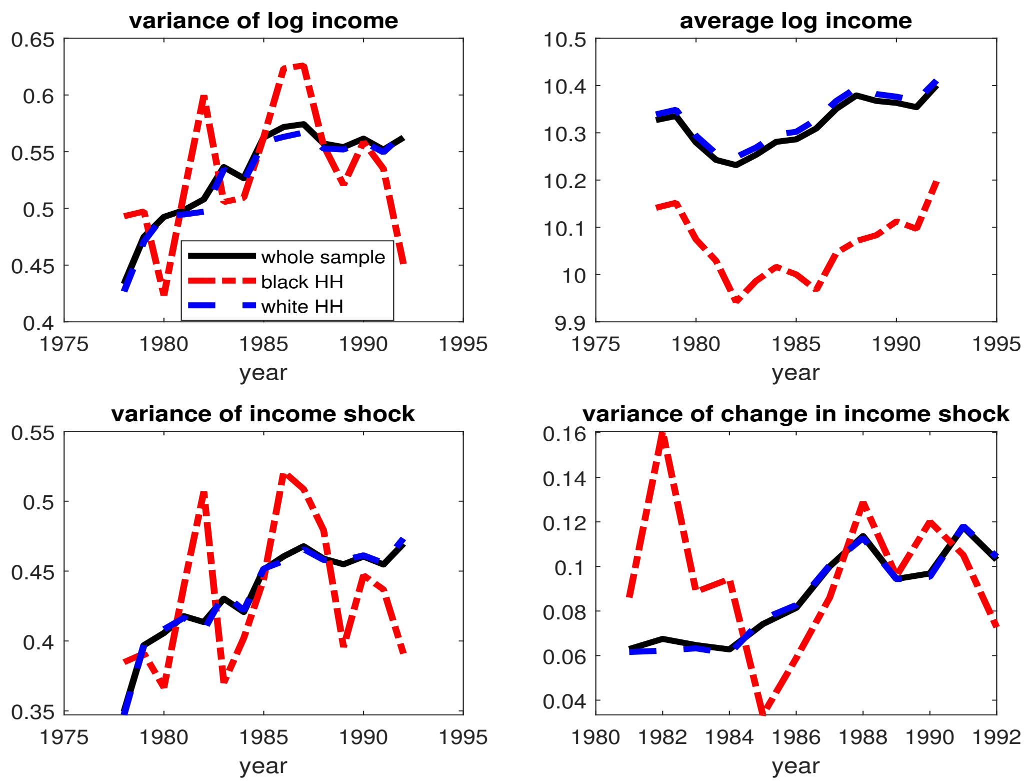

Figure 5: Mean and variance of log income and income shocks[5]

5.3.1 Income and Consumption Volatilities and the partial insurance parameter Estimates

Now we estimate the partial consumption insurance and volatility parameters using the penalized-GMM method. The partial consumption insurance against permanent income shocks is estimated to be 63% and 36% for households with white heads and households with black heads, respectively. The partial consumption insurance against permanent income shocks estimated by BPP is 36%.

Another interesting result is that households with white heads self insure

[5]Black household heads, on average, earn significantly less than white household heads but they are exposed to higher income and income shock volatilities.

Table 9: Panel A: Heterogeneous partial consumption insurance

Parameter	Symbol	GMM	Bootstrap SE	P-GMM	Bootstrap SE
		N=17,612	$\lambda = 0.0029$		
base c to a perm. y	ϕ	0.38	0.1	0.37	0.083
black c to a perm. y	ϕ^{black}	0.34	0.47	0.27	0.222
base c to a trans. y	ψ	-0.024	0.08	0	0.042
black c to a trans. y	ψ^{black}	0.14	0.22	0.10	0.188

‡ This table summarizes empirical results using both GMM and PGMM estimators. I have also reported the bootstrap standard errors.

against 100% of the transitory shocks, while the self insurance for the ones with black heads is 90%. This is consistent with most studies argued households self insure against more than 96% of the transitory income shocks(BPP, Chatterjee et al (2021) and Kaplan and Violante (2014)).

In addition the households with black heads have more volatile permanent income shock and consumption shock parameter estimates. However, the transitory income shock volatility estimates of the two groups are similar.

Table 10: Heterogeneous income and consumption volatility

Parameter	Symbol	GMM	Bootstrap SE	P-GMM	Bootstrap SE
		N=17,612	$\lambda = 0.0029$		
base var of trans. y	σ^2_ϵ	0.027	0.002	0.027	0.002
base var of perm. y	σ^2_η	0.029	0.002	0.029	0.002
base var of trans. c	σ^2_ν	0.15	0.0061	0.15	0.007
black var of trans. y	σ^{black}_ϵ	-0.004	0.012	0	0.010
black var of perm. y	σ^{black}_η	0.008	0.0079	0.009	0.009
black var of trans. c	σ^{black}_ν	0.031	0.026	0.033	0.027

‡ This table summarizes empirical results using both GMM and PGMM estimators. I have also reported the bootstrap standard errors.

Tables 11 and 12 report the PGMM estimates of the time varying variances and partial insurance parameter estimates. While various studies explored the heterogeneity of income and consumption volatilities across time, very few analyzed whether the partial consumption parameters vary across time (Meghir and Pistaferri (2004), BPP, Chatterjee et al (2021)).

I allow the permanent and transitory partial insurance parameters vary across time. The partial consumption insurance to permanent shock estimates supports the time varying parameter model. However, the out of range estimates the partial consumption insurance to the transitory income shocks provide an evidence against the time varying parameter model.

Furthermore, the transitory and permanent income shocks and transitory consumption shock time varying volatility estimates fall in comparable range of the results reported by BPP and Chatterjee et al (2021).

The permanent and transitory income shocks vary with the business cycle. In the late 1970s and early 1980s (see Figure 6) consumption response to both transitory and permanent income shocks increased. This is mainly due to the recession in 1978 and 1982 (as dated by NBER). However, until 1986, the sensitive of consumption shock to the permanent income shock

Table 11: PGMM estimates of time-varying variances of income and consumption shocks

year	variance perm. shock $\left(\sigma^2_{\eta_t}\right)$	variance trans. shock $\left(\sigma^2_{\epsilon_t}\right)$	variance cons. shock $\left(\sigma^2_{\nu_t}\right)$
1979	0.025	0.021	0.161
1980	0.022	0.016	0.126
1981	0.026	0.020	0.104
1982	0.028	0.020	0.127
1983	0.028	0.015	0.132
1984	0.020	0.029	0.168
1985	0.034	0.032	0.183
1986	0.038	0.034	0.130
1987	0.015	0.040	NA
1988	0.028	0.037	NA
1989	0.053	0.032	NA
1990	0.025	0.033	0.202
1991-92	0.056	0.046	0.166

‡ This table summarizes the empirical PGMM estimates of the time varying transitory income shock, permanent income shock and consumption shock volatilities.

increased very slowly (on average), while the responsiveness of consumption shock to the transitory income shock declined more quickly. I attribute this to the relative nature of permanent income shocks; they tend to persist. Hence, the negative or positive permanent income shocks tend to linger longer compared to the transitory income shocks. This can be seen from the -0.33 correlation between the estimates of responsiveness of consumption shock to the transitory and permanent income shocks.

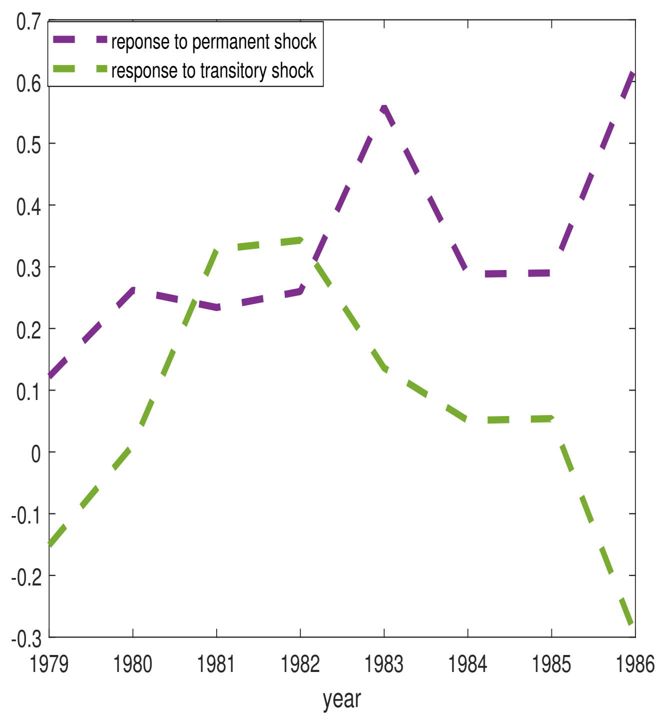

Figure 6: The evolution of consumption shock responses to permanent and transitory income shocks [7]

[7]Figure 6 summarizes the response of consumption shock to the permanent and transitory income shocks. Early 1980s consumption shock appears more responsive to both permanent and transitory income shocks. The responsiveness of consumption shock to the permanent income shock increased overtime. However, the responsiveness of consumption shock to the transitory income shock declines gradually.

Table 12: PGMM estimates of time-varying variances of partial consumption insurance to permanent and transitory income shocks and the corresponding noise variances

year	partial insurance perm. shock (ϕ_t)	partial insurance trans. shock (ψ_t)
1979	0.122	-0.151
1980	0.262	0.011
1981	0.234	0.328
1982	0.260	0.343
1983	0.559	0.136
1984	0.288	0.051
1985	0.290	0.054
1986	0.625	-0.297
1987	NA	NA
1988	NA	NA
1989	NA	NA
1990	0.682	-0.320
1991-92	0.056	0.078

‡ This table summarizes the empirical PGMM estimates of the time varying partial consumption insurance to permanent and transitory income shocks.

6. Conclusions

The main objective of this study is to consistently estimate income and consumption shocks and better explore the heterogeneities in income and consumption volatilities and partial consumption insurance parameter in the face of limited availability of data. This is achieved by using Heckman two step regression for estimating income and consumption shocks and the GMM LASSO, a popular method for shrinkage and estimation, for uncovering the true heterogeneity.

My analysis show that the Heckman two step estimates are different from the OLS estimates. Properly modeling earnings by endogenizing employment address the issue of selection bias. Moreover, there is a significant heterogeneity by race; specifically, black household heads self-insure, both against transitory and permanent income shocks, less than white household heads. I also find that the amount of self-insurance against the permanent income shock for white household heads is much higher than reported by previous studies (BPP, Chatterjee et al (2021) and Kaplan and Violante (2010)). Also, my study suggests that black household heads face higher permanent income shock and transitory consumption volatilities.

Furthermore, while my estimates of the time varying transitory and permanent income shock volatilities are in the ballpark of previous studies (BPP and Chatterjee et al (2021)), I have showed there is an evidence for time-varying parameter representation for the partial consumption insurance against permanent income shock parameter.

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A. Moment Conditions

A.1 Heterogeneous Partial Insurance Parameters

Below are the moment conditions to identify the population parameters.

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)\right)=\sigma_{\eta}^{2} \tag{20}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}\right)=-\sigma_{\epsilon}^{2} \tag{21}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m})^{2}\sigma_{\eta}^{2}+(\bar{\psi}^{m})^{2}\sigma_{\epsilon}^{2}+\sigma_{\nu}^{2} \tag{22}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{b})^{2}\sigma_{\eta}^{2}+(\bar{\psi}^{m}+\bar{\psi}^{b})^{2}\sigma_{\epsilon}^{2}+\sigma_{\nu}^{2} \tag{23}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{w})^{2}\sigma_{\eta}^{2}+(\bar{\psi}^{m}+\bar{\psi}^{w})^{2}\sigma_{\epsilon}^{2}+\sigma_{\nu}^{2} \tag{24}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^{m}\sigma_{\eta}^{2}+\bar{\psi}^{m}\sigma_{\epsilon}^{2} \tag{25}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{b})\sigma_{\eta}^{2}+(\bar{\psi}^{m}+\bar{\psi}^{b})\sigma_{\epsilon}^{2} \tag{26}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{w})\sigma_{\eta}^{2}+(\bar{\psi}^{m}+\bar{\psi}^{b})\sigma_{\epsilon}^{2} \tag{27}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^{m}\sigma_{\eta}^{2} \tag{28}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{b})\sigma_{\eta}^{2} \tag{29}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(gender_{it})\right)=(\bar{\phi}^{m}+\bar{\psi}^{w})\sigma_{\eta}^{2} \tag{30}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(1-gender_{it})\right)=-\bar{\psi}^{m}\sigma_{\epsilon}^{2} \tag{31}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(race_{it})(1-gender_{it})\right)=-(\bar{\psi}^{m}+\bar{\psi}^{b})\sigma_{\epsilon}^{2} \tag{32}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(gender_{it})\right)=-(\bar{\psi}^{m}+\bar{\psi}^{w})\sigma_{\epsilon}^{2} \tag{33}$$

Equations 15-28 identifies the parameters $\sigma_{\eta}^{2},\sigma_{\epsilon}^{2},\sigma_{\nu}^{2},\bar{\phi}^{b},\bar{\psi}^{w},\bar{\psi}^{b},\bar{\phi}^{w},\bar{\phi}^{m},\bar{\psi}^{m}$.

A.2 Heterogeneous Partial Insurance and Volatility Parameters

For the heterogeneous partial insurance and volatility model, the moment conditions will be:

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(1-gender_{it})\right)=\bar{\sigma_{\eta}}^{m} \tag{34}$$

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(race_{it})(1-gender_{it})\right)=\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{b} \tag{35}$$

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(gender_{it})\right)=\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{w} \tag{36}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}(1-race_{it})(1-gender_{it})\right)=-\bar{\sigma_{\epsilon}}^{m} \tag{37}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}(race_{it})(1-gender_{it})\right)=-\bar{\sigma_{\epsilon}}^{m}-\bar{\sigma_{\epsilon}}^{b} \tag{38}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}(1-race_{it})(gender_{it})\right)=-\bar{\sigma_{\epsilon}}^{m}-\bar{\sigma_{\epsilon}}^{w} \tag{39}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m})^{2}\bar{\sigma_{\eta}}^{m}+(\bar{\psi}^{m})^{2}\bar{\sigma_{\epsilon}}^{m}+\bar{\sigma_{\nu}}^{m} \tag{40}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{b})^{2}(\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{b})+(\bar{\psi}^{m}+\bar{\psi}^{b})^{2}(\bar{\sigma_{\epsilon}}^{m}+\bar{\sigma_{\epsilon}}^{b})+\bar{\sigma_{\nu}}^{m}+\bar{\sigma_{\nu}}^{b} \tag{41}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{w})^{2}(\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{w})+(\bar{\psi}^{m}+\bar{\psi}^{w})^{2}(\bar{\sigma_{\epsilon}}^{m}+\bar{\sigma_{\epsilon}}^{w})+\bar{\sigma_{\nu}}^{m}+\bar{\sigma_{\nu}}^{w} \tag{42}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^{m}\bar{\sigma_{\eta}}^{m}+\bar{\psi}^{m}\bar{\sigma_{\epsilon}}^{m} \tag{43}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{b})(\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{b})+(\bar{\psi}^{m}+\bar{\psi}^{b})(\bar{\sigma_{\epsilon}}^{m}+\bar{\sigma_{\epsilon}}^{b}) \tag{44}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{w})(\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{w})+(\bar{\psi}^{m}+\bar{\psi}^{w})(\bar{\sigma_{\epsilon}}^{m}+\bar{\sigma_{\epsilon}}^{w}) \tag{45}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^{m}\bar{\sigma_{\eta}}^{m} \tag{46}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(race_{it})(1-gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{b})(\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{b}) \tag{47}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(gender_{it})\right)=(\bar{\phi}^{m}+\bar{\phi}^{w})(\bar{\sigma_{\eta}}^{m}+\bar{\sigma_{\eta}}^{w}) \tag{48}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(1-gender_{it})\right) = -\bar{\psi^m}\bar{\sigma_\epsilon}^m \tag{49}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(race_{it})(1-gender_{it})\right) = -(\bar{\psi}^m+\bar{\psi}^b)(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^b) \tag{50}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(gender_{it})\right) = -(\bar{\psi}^m+\bar{\psi}^w)(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^w) \tag{51}$$

Equations 29-46 identify the partial insurance and volatility parameters:

$$\{\bar{\sigma_\eta}^m, \bar{\sigma_\epsilon}^m, \bar{\sigma_\nu}^m, \bar{\phi}^b, \bar{\phi}^w, \bar{\psi}^b, \bar{\psi}^w, \bar{\phi}^m, \bar{\psi}^m, \bar{\sigma_\eta}^b, \bar{\sigma_\eta}^w, \bar{\sigma_\epsilon}^b, \bar{\sigma_\epsilon}^w, \bar{\sigma_\nu}^b, \bar{\sigma_\nu}^w\}$$

.

B. Results

[9] Black household heads, on average, consume less than white household heads but they are exposed to higher consumption and consumption shock volatilities.

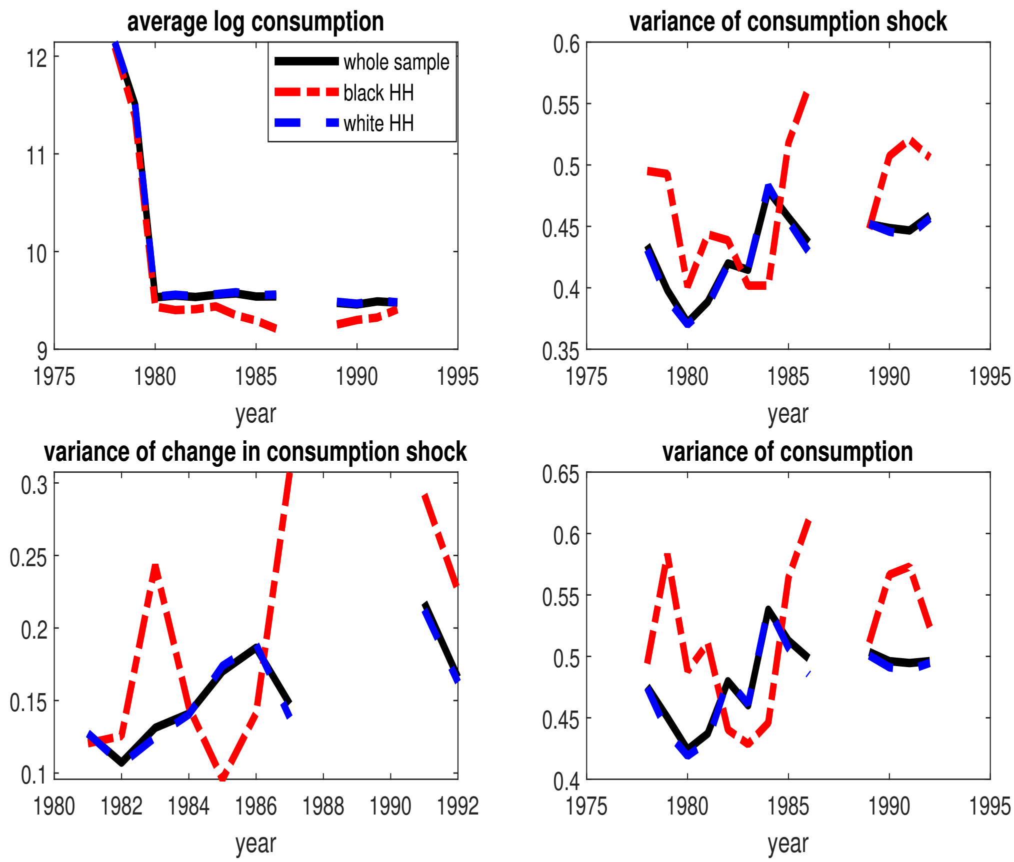

Figure 7: Mean and variance of log consumption and consumption shock (by race)[9]

[10]Income is more volatile than consumption. However, consumption shocks have higher volatility than income shocks. This might be the result of imputing consumption data, which end up introducing more noise in the consumption expenditure observations.

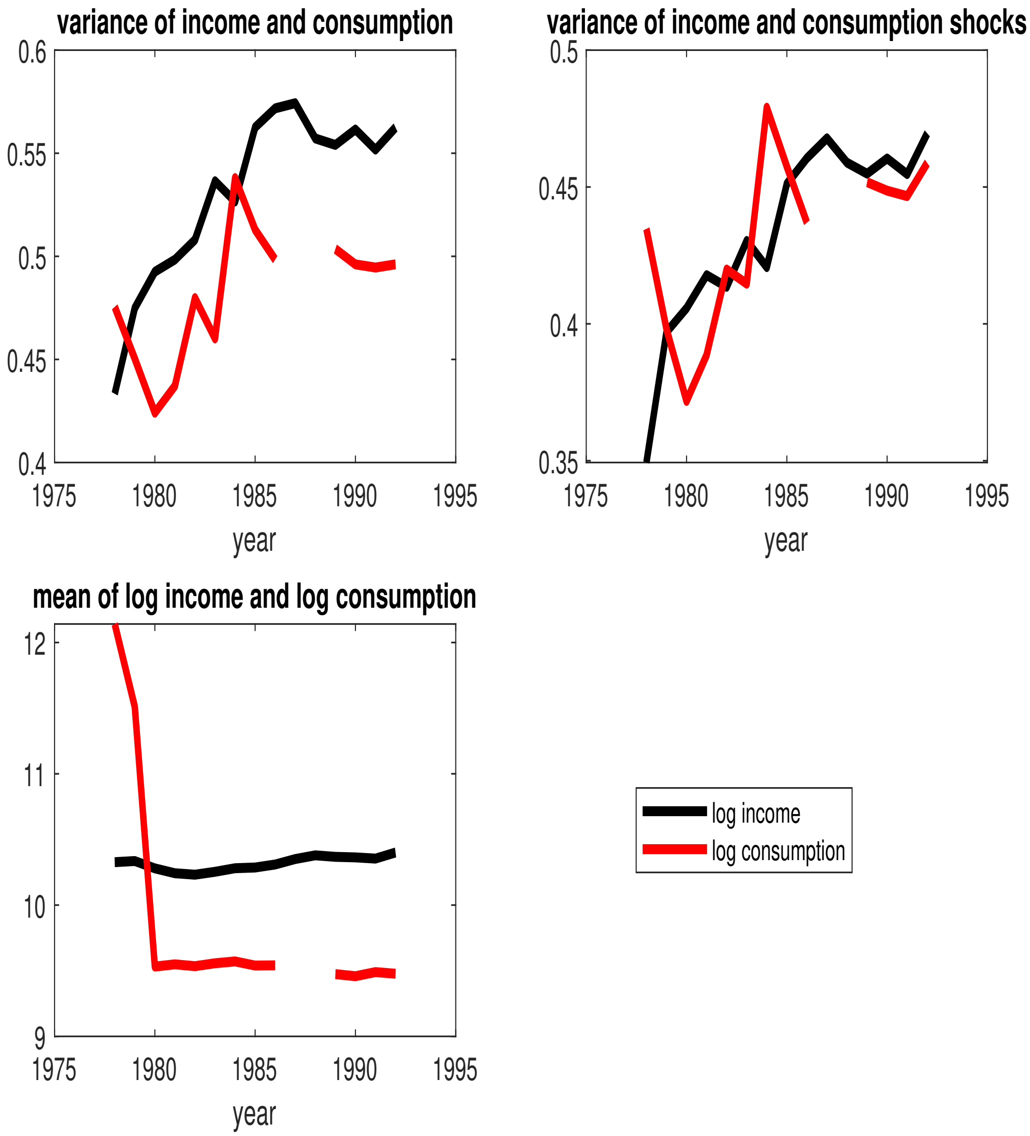

Figure 8: Income and consumption mean and variance[10]

Table 13: Income and consumption shock covariance (whole sample)

variables	Δy_t	Δy_{t-1}	Δy_{t+1}	$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	Δc_t	Δc_{t-1}	Δc_{t+1}	$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$
Δy_t	0.07	-0.021	-0.02	0.024	0.012	-0.002	-0.003	0.007
Δy_{t-1}	-0.021	0.07	-0.006	0.04	-0.002	0.011	6.63E-06	0.0082
Δy_{t+1}	-0.02	-0.006	0.08	0.052	-0.003	-0.0001	0.013	0.01
$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	0.024	0.04	0.052	0.12	0.007	0.009	0.009	0.025
Δc_t	0.012	-0.002	-0.003	0.007	0.15	-0.064	-0.067	0.016
Δc_{t-1}	-0.002	0.011	-0.0001	0.009	-0.064	0.15	-0.003	0.08
Δc_{t+1}	-0.003	6.63E-06	0.013	0.009	-0.067	-0.003	0.15	0.08
$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$	0.007	0.0082	0.01	0.025	0.016	0.08	0.08	0.18

‡ This table summarizes the income and consumption covariances (for the whole sample). Tables 9 and 10 summarizes the income and consumption variances for black and white household heads respectively. The black household heads encounter higher income and consumption volatilities compared to white household heads and the whole sample.

Table 14: Income and consumption shock covariance (Black household heads)

variables	Δy_t	Δy_{t-1}	Δy_{t+1}	$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	Δc_t	Δc_{t-1}	Δc_{t+1}	$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$
Δy_t	0.079	-0.026	-0.024	0.029	0.030	-0.010	-0.017	0.003
Δy_{t-1}	-0.026	0.081	-0.014	0.040	-0.021	0.036	0.014	0.029
Δy_{t+1}	-0.024	-0.014	0.090	0.053	0.003	-0.011	0.023	0.015
$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	0.029	0.040	0.053	0.122	0.012	0.016	0.019	0.046
Δc_t	0.030	-0.021	0.003	0.012	0.160	-0.069	-0.066	0.024
Δc_{t-1}	-0.010	0.036	-0.011	0.016	-0.069	0.182	-0.015	0.098
Δc_{t+1}	-0.017	0.014	0.023	0.019	-0.066	-0.015	0.186	0.104
$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$	0.003	0.029	0.015	0.046	0.024	0.098	0.104	0.226

‡ This table summarizes the income and consumption covariances for black household heads. The black household heads encounter higher income and consumption volatilities compared to white household heads and the whole sample.

Table 15: Income and consumption shock covariance (White household heads)

variables	Δy_t	Δy_{t-1}	Δy_{t+1}	$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	Δc_t	Δc_{t-1}	Δc_{t+1}	$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$
Δy_t	0.069	-0.021	-0.025	0.023	0.012	-0.001	-0.003	0.008
Δy_{t-1}	-0.021	0.069	-0.006	0.041	-0.001	0.009	-0.001	0.007
Δy_{t+1}	-0.025	-0.006	0.082	0.052	-0.003	0.000	0.012	0.009
$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	0.023	0.041	0.052	0.116	0.007	0.008	0.009	0.024
Δc_t	0.012	-0.001	-0.003	0.007	0.146	-0.064	-0.067	0.015
Δc_{t-1}	-0.001	0.009	0.000	0.008	-0.064	0.146	-0.002	0.080
Δc_{t+1}	-0.003	-0.001	0.012	0.009	-0.067	-0.002	0.147	0.078
$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$	0.008	0.007	0.009	0.024	0.015	0.080	0.078	0.173

‡ This table summarizes the income and consumption covariances for white household heads. The white household heads have about the same income and consumption volatilities as the whole sample.