Analysis of Household Consumption Insurance

Daniel Gulti Kebede*

Purdue University

November 8, 2022 – Version 0.1
Job Market Paper

Abstract

The study of income and consumption linkage and analysis of permanent and transitory income shocks is limited by availability of data. To deal with data sparsity, I propose identifying the partial consumption insurance and income and consumption volatility heterogeneities at the household level using Least Absolute shrinkage and Selection Operator (LASSO). Using PSID data, I estimate partial consumption insurance against permanent shock of 63% and 49% for white and black household heads, respectively; the white and black household heads self-insure against 100% and 90% of the transitory income shocks, respectively. Moreover, I find income and consumption volatilities and partial consumption insurance parameters vary across time.

*I am grateful to my advisors. Any remaining errors are my responsibility.

1. Introduction

One of the focuses of analyzing the link between income and consumption shocks is to explore the nature of these correlations across different household characteristics (Blundell et al (2008), Chatterjee et al (20202) and Ganong et al (2020)). However, lack of single source with good quality income and consumption data complicates the process. Blundell et al (2008) use imputation to build new panel series of income and consumption that combines information from the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX). Then, to estimate the various heterogeneities, data is split into sub-samples resulting in thinly splitted data which affects the consistent estimation of the parameters. Moreover, separately estimating the parameters of interest for each groups will ignore the potential group interaction effects. To deal with data sparsity, this paper proposes identifying the partial consumption insurance and income and consumption volatility heterogeneities at the household level using Least Absolute Shrinkage and Selection Operator (LASSO). The LASSO helps for predicting and characterizing the true groups and patterns in our data. I argue that the use of model regularization and shrinkage improves model interpretability and select the most important heterogeneity. Also, by putting structure using the penalty term, I identify the true heterogeneities present in the data.

I allow the insurance parameters and income and consumption volatilities to vary by household types (specifically race) and time. Then, using regularization method, without splitting the data into sub-samples, I select and estimate the most important heterogeneities present in the data. To the best of my knowledge, this study is the first one to use GMM with LASSO to estimate consumption partial insurance and income and consumption volatility heterogeneities at the household level. One advantage of my approach relative to data splitting is that, LASSO will shrink some of the coefficient estimates exactly to zero and produces an interpretable model. Monte Carlo experiments, under different settings, suggest that GMM with LASSO estimation method recovers the true population parameters very well. Subsequently, using PSID data, I estimate partial consumption insurance against permanent shock of 63% and 49% for white and black household heads, respectively. Another interesting result is that while white household heads self insure against 100% of the transitory income shocks, black household heads can only self insure against 90% of the transitory shocks. Also the permanent income and consumption shock volatilities for black household heads are higher than their white counter parts.

Over the last four decades, income inequality has steadily risen in advanced economies (Guvenen and Kaplan (2017)). However, whether consumption inequality has risen is disputed. In recent decades, many studies concluded that consumption inequality has risen less than income inequality (Cutler and Katz (1991), Krueger and Perri (2006), Heathcote et al. (2010), Fisher, Johnson, and Smeeding (2013), Meyer and Sullivan (2013)). And yet, a significant number of studies have found that the rise in consumption and income inequalities have been more or less indistinguishable (Aguiar and Bils (2012), Attanasio, Hurst, and Pistaferri (2013)). Key to answering why and how consumption inequality has risen is the covariance of income and consumption. An important facet of this exercise is to determine the relative importance of income shocks of different persistence and correlation across agents.

Numerous studies address various questions regarding income and consumption shocks. The central theme of the questions is how to characterize income inequalities and analyze whether those inequalities are also exhibited by consumption. Methodologically, the standard approach is to decompose income shocks into permanent and transitory components, which will help to better understand how income inequalities evolved, and thereby analyze how the consumption process adjusted to these shocks and consequently consumption inequality has evolved. An intuitive justification for income shock decomposition is provided as a response to Deaton's paradox. Deaton(1986) argues sharp shocks to income did not seem to cause large shocks to consumption. Labor income can be represented by random walk process, while consumption is relatively smooth. However, Quah (1990), made progress towards resolving Deaton's paradox by decomposing labor income into permanent and transitory components. This helps to establish clear link between income and consumption inequalities.

An early study exploring heterogeneity is Meghir and Pistaferri (2004). They estimate an income process for individual annual earnings in the U.S., allowing for differences across education groups and taking into account changes over time. Another important work is the seminal paper by Blundell, Pistaferri and Preston (2008)(hereafter BPP). The importance of this study is underlined by the central role it plays in macroeconomic calibration and policy analysis. Before BPP there is a strong consensus that the marginal propensity to consume (MPC) out of permanent income is very close to one (Friedman (1957) and Quah(1990)). However, BPP argue consumers self-insure against some fraction of permanent income shocks. Using the whole sample they find households self-insure against 36% of the permanent income shocks. However, the degree of consumption partial insurance against permanent income shocks falls to 6% for non-college graduates and it increases to 58% for college graduates. In addition BPP find partial consumption insurance of 29% and 13% for younger and older household heads, respectively.

While Kaplan and Violante (2010) analyze the degree of consumption smoothing implicit in a calibrated life-cycle version of the standard incomplete-markets model, they do not explore the heterogeneities of these parameters at household level. But they show that the partial insurance parameters depend on the tightness of debt limits. The partial insurance against permanent income shocks is as low as 7% for households closer to the borrowing limits (i.e. when the borrowing constraints are binding); while the estimates rise to about 22% for households far from the borrowing limits.

On the other hand , using quasi maximum likelihood estimation (QMLE) method, Chatterjee et al (2020) produced a significantly higher estimate of consumption insurance at 55%. Also, the partial consumption insurance increases to 71% for household heads with college degree and it falls to 36% when the household heads have no college degree. Furthermore, they find a partial consumption insurance of 45% and 75% for younger and older household heads respectively. They argue their estimates are quite different and more precise than BPP's because QMLE is more robust to the non-normality of income distribution and it avoids the need to estimate a weighting matrix.

Finally, I estimate income and consumption volatility and partial consumption insurance heterogeneity across race and time. The time varying volatility estimates are comparable to the ones reported in BPP, Chatterjee et al (2021) and Arellano et al (2018). However, my results are more robust to data sparsity.

Section 2 review the literature related to our study. Section 3 introduces the model and estimation technique. Section 4 provides a Monte Carlo study. Section 5 provides data summary and discusses the empirical results. Section 6 concludes. The appendix contains derivation of the moment conditions and additional empirical results.

2. Literature Review

There is a subtle distinction between consumption smoothing theory in the spirit of Friedman (1957), Modigliani and Brumberg (1954, 1980) and Quah (1989) and the risk sharing and insurance models of Cochrane (1991), Mace (1991), Baxter and Crucini (1995), Hayashi, Altonji and Kotlikoff (1996) and Krueger and Perri (2005,2011b). While the former is about persistence of various shocks, the later focuses on the covariance across agents of the shocks. In our study we characterize both the persistence of the shocks and their covariances across agents.

The dichotomy of income shocks into permanent and transitory components is viewed as central to household consumption choices. The rational for this decomposition can be traced to Milton Friedman's permanent income theory of consumption (Friedman (1957)). The theory argues that the marginal propensity to consume from the annuitized value of an income shock is unity. Thus permanent shocks yield one-to-one consumption responses whereas purely transitory shocks yield a consumption permanent response proportional to the real interest rate.

Three decades after Friedman's seminal work, Deaton (1987) argued that when labor income is characterized by a unit root process, the permanent-income hypothesis fails to predict the smoothness of consumption. This irregularity came to be known as "*Deaton's paradox*". Quah (1989) provided one of the first attempts to reconcile income dynamics and Deaton's argument. He started by decomposing income disturbances into permanent and transitory effects. Then he argued the permanent income hypothesis prediction for smoothness properties of consumption depends on the relative importance of the permanent and transitory disturbances in labor income.

After the earlier works, one of the most important and novel directions of research in this area is the use of panel data on individual income. This is important because it allows both the exploration of individual consumption and income dynamics and their aggregate implications. It also open door to explore various heterogeneities at the individual and household level. One of the earlier works along this direction is Meghir and Pistaferri (2004) which reaffirmed that permanent and transitory shocks are important components of income shock. More importantly they found that earnings variances are heterogeneous across individuals and there is a strong state dependence in the variances of transitory and permanent income shocks.

While most of the previous studies focused on developing the right distributional framework to capture the moments implied by the data, Heathcote et al (2007) is one of the earliest studies to developed a model with partial insurance against idiosyncratic wage shocks. They quantified risk sharing and decomposed sources of inequality into either shocks during life-cycle or baseline heterogeneity in preferences and productivity. One of the distinguishing contributions of the study is derivation of closed-form solutions for equilibrium allocation and moments of the joint distribution of consumption, hours, and wages. They exploited the closed-form cross-sectional moments to prove identification of the model parameters and estimated the model with data from the CEX and PSID over the period 1967 to 2006. They estimated 40% of permanent wage shocks pass through to consumption. The seminal work by BPP explores the link between income and consumption inequality. Since there is no single source with good quality income and consumption data, they built a household income and consumption panel using two different datasets. Thus, one of their important contributions is a new panel series of income and consumption that combines information from the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX); which is being used in this study as well. While PSID has a good panel series on earnings, information on non-durable consumption is mainly missing. Their imputation process starts from a standard demand function for food (a consumption item available in both the PSID and CEX surveys). This demand function is estimated using the CEX data. They modeled food expenditure and total expenditure as jointly endogenous and allowed their relationship to be time-varying. Given the monotonicity of the estimated food demand, this function is inverted to obtain a measure of nondurable consumption in the PSID. However, this approach posed new estimation challenges. For instance, one of the challenges is precise identification of parameters of interest due to the noise in the imputed consumption data. They focused on the period between 1978 and 1992, when some of the largest changes in income inequality occurred. Particularly, the 1980s are the periods when large changes in income inequality are observed. They found some partial insurance of permanent shocks and full insurance of transitory shocks, except among poor households. They estimated that households self-insure against 36% and 95% of permanent and transitory income shocks, respectively. They explore heterogeneity of partial insurance parameters across age and education. Older and college educated individuals have higher partial insurance permanent shock estimates than the younger and no college degree individuals, respectively. These estimates are calculated by grouping the sample into sub-sample based on the groups considered.

There are few studies that confirmed the results of BPP, while numerous others argued against the validity of some their parameter estimates, particularly the partial insurance against permanent income shock parameter. Along this, Kaplan and Violante (2010) analyze the degree of consumption smoothing implicit in a calibrated life-cycle version of the standard incomplete-markets model and compared their results to the empirical estimates of BPP. In their version of BPP's model, the insurable fraction of permanent shocks varies between 7% and 22%, depending on the tightness of debt limits. The important contribution of this study is exploring the effect of borrowing constraint on the degree of self insurance. While the results of BPP hold for households further away from the borrowing limit, the model predicts a much smaller self-insurance to permanent income shocks for households closer to the borrowing limits. Besides, the study showed the BPP estimates are downward biased and the bias grows as borrowing limits become tighter.

Another study following in the footsteps of BPP is Arellano et al (2018). The authors outlined a framework for income dynamics and the nonlinear transmission of income shocks to consumption. They developed a non-linear model where the impact of past shocks can be altered by the size and sign of new shocks. Their framework allows for ”unusual” shocks to alter the trend of past shocks. They have showed that modeling the effect of ”unusual” shocks matches the data very well. Hence, nonlinear persistence and conditional skewness (which cannot be captured in the more traditional models of earnings dynamics) are important features of earnings processes. Also, they showed that the non-linearity observed in the earnings process have important implications for consumption choices. More importantly, using family earnings data from administrative records in the Norwegian registers, they validated the evidence of nonlinear persistence in family earnings uncovered based on the PSID data from the 1999 to 2009 surveys.

Furthermore, building on the work of Arellano et al (2018), Arellano et al (2021) estimated the response of consumption to persistent nonlinear income shocks in the presence of unobserved heterogeneity. They found substantial heterogeneity in consumption responses. A very interesting finding is the variation in consumption response by levels and the different stages of consumption cycles. They found that low-consumption types respond more strongly to income shocks at the beginning of the life cycle when their assets are low, whereas the high-consumption types respond less on average, which changes little with age or asset level.

Including BPP, most of the studies estimated almost full partial insurance against transitory income shocks. This evidence is in line with the Permanent Income Hypothesis (PIH) theory which states change in permanent income, rather than the change in temporary income, is what derives the change in a consumer's consumption pattern. However, in a very interesting study, Kaplan et al (2014), analyzed the cohort they dubbed "wealthy hand-to-mouth" and suggested these households have a high marginal propensity to consume out of transitory income changes. Likewise, Edmund Crawley (2019), in his study about the effect of time aggregation, argued that time aggregation of a random walk induces serial correlation in the first difference that is not present in the original series. Once this problem is corrected the estimated partial insurance to transitory shocks (estimated using same data as in BPP) decreases to 76% (it was originally estimated in BPP to be 95%).

The other direction of departure of the recent literature from BPP is in estimation methods. In the labor income processes literature, GMM estimators are almost always used. However, recently bayesian and likelihood based estimators are getting increased attention. Nakata and Tonetti (2014) examine the validity of using likelihood based estimation by comparing the small sample properties of a Bayesian estimator to those of GMM. They showed the Bayesian estimators demonstrate favorable bias and better efficiency. The study also covers various extensions such as time varying and heterogeneous parameters, non-normal errors, unbalanced panel and missing data models. Moreover, in a more recent study, Chatterjee et al (2021), produced a more precise and significantly higher estimates of consumption insurance of 55% employing quasi maximum likelihood estimator (QMLE), instead.

3. Model Specification and Methodology

3.1 Model Specification

Following the literature, after controlling for the known component, I decompose the income shock into a permanent component, $\boldsymbol{\tau}$, and a transitory component, $\boldsymbol{\epsilon}$. The transitory income shocks captures events such as surprise bonus or temporary leave due to illnesses, while the permanent income shocks include severe health shocks, promotion or other factors that result in permanent income change. For the benchmark model, I assume constant volatilities of the permanent and transitory factors.

The income process for household i is:

$$\log\left(Y_{i,t}\right) = \boldsymbol{X}'_{i,t}\boldsymbol{\beta}_t + \tau_{i,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_\epsilon^2\right) \tag{1}$$

where $\boldsymbol{X}_{i,t}$ is a set of income characteristics observed at time t for each household i. These household characteristics include demographic, education, gender, ethnicity and other variables.

The unexplained income process is:

$$y_{i,t} = \tau_{i,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_\epsilon^2\right) \tag{2}$$

where $y_{i,t} = \log\left(Y_{i,t}\right) - \boldsymbol{X}'_{i,t}\boldsymbol{\beta}_t$ denotes the log of real income net of predictable individual components.

The permanent component follows a random walk process of the form,

$$\tau_{i,t} = \tau_{i,t-1} + \eta_{i,t}, \quad \eta_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_\eta^2\right) \tag{3}$$

where $\nu_{i,t}$ is serially uncorrelated, and the transitory component $\epsilon_{i,t}$ is a White Noise. However, without loss of generality, $\epsilon_{i,t}$ can be modeled as an

$MA(q)$ process; where the order q is established empirically:

$$\epsilon_{i,t} = \sum_{j=0}^{q} \gamma_j \xi_{i,t-j}$$

with $\gamma_0 \equiv 1$.

Now, the growth of the income residual can be written as:

$$\Delta y_{i,t} = \eta_{i,t} + \Delta\epsilon_{i,t} \tag{4}$$

where $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$.

Following the same process, I similarly derive consumption shocks. First (see equation (5)), I regress consumption on vectors of household characteristics; I use similar sets of controls as in the income regression above (see equation (1)).

$$\log\left(C_{i,t}\right) = \boldsymbol{X}'_{i,t}\boldsymbol{\alpha}_t + u_{i,t}, \quad u_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_u^2\right) \tag{5}$$

where $\boldsymbol{X}_{i,t}$ is a set of consumption characteristics observed at time t for each household i.

In the partial insurance framework, consumption growth is modeled as a function of the income shocks. This approach allows to analyze the degree of correlation between income and consumption shocks.

$$\Delta c_{i,t} = \phi\eta_{i,t} + \psi\epsilon_{i,t} + \nu_{i,t}, \quad \nu_{i,t} \sim \textit{i.i.d.}\left(0, \sigma_\nu^2\right) \tag{6}$$

where $c_{i,t}$ is the log of real consumption net of its predictable components; i.e. $c_{i,t} = \log\left(C_{i,t}\right) - \boldsymbol{X}'_{i,t}\boldsymbol{\alpha}_t$ and $\Delta c_{i,t} = c_{i,t} - c_{i,t-1}$. The transitory consumption shock, $u_{i,t}$ could capture innovations in consumption that are independent of those in income. This may include measurement errors due to the imputation of non-durable consumption, preference shocks, innovation to higher moments of the income process, etc.

The component $\nu_{i,t}$ consists of a serially uncorrelated earnings shock plus a measurement error component. The variance of the two components cannot be separately identified from the earnings data alone. Studies that attempt to distinguish the two typically either draw on outside estimates of the variance of pure measurement error or estimate the earnings process jointly with a model of a choice variable that should respond only to true earnings shocks, such as consumption.

3.1.1 Heterogeneous Partial Insurance Parameters

Now, I extend the benchmark model by sequentially introducing heterogeneities in partial consumption insurance parameters and income and consumption volatilities. First, we allow the partial consumption insurance parameters to vary by household types. Hence, I consider income and consumption dynamic model with constant variance but heterogeneous partial insurance parameters.

$$\phi_i = \bar{\phi} + \phi^{type} D_i \tag{7}$$

$$\psi_i = \bar{\psi} + \psi^{type} D_i \tag{8}$$

where, D_i is a dummy variable that takes value 1 for individuals with the specific type and zero otherwise. In this specific case, $\{type\}$ is $\{race\}$. Hence, the base categories $\bar{\phi}$ and $\bar{\psi}$ represent the permanent and transitory partial consumption insurance parameters of white household heads respectively. Now, the permanent and transitory partial consumption insurance coefficient for black household heads will be:

$$\phi_{type} = \bar{\phi} + \phi^{type}$$

$$\psi_{type} = \bar{\psi} + \psi^{type}$$

3.1.2 Heterogeneous Partial Insurance and Volatility Parameters

Next, in addition to the partial consumption insurance parameters, I explore heterogeneity in income and consumption shock volatilities. Similar to the partial consumption insurance parameters, I allow the volatility parameters to depend on race.

$$\sigma^2_{\eta_i} = \sigma^2_{\eta} + \sigma^{type}_{\eta} D_i \tag{9}$$

$$\sigma^2_{\epsilon_i} = \sigma^2_{\epsilon} + \sigma^{type}_{\epsilon} D_i \tag{10}$$

$$\sigma^2_{\nu_i} = \sigma^2_{\nu} + \sigma^{type}_{\nu} D_i \tag{11}$$

Again the base categories represent the volatility parameters of white household heads; while the base categories plus the respective slope coefficients capture the volatility measures of black household heads.

3.1.3 Time Varying Partial Insurance and Volatility Parameters

The time varying parameters is the most extensive explored heterogeneity. I model both the variance and partial insurance as time varying parameters. Hence, the transitory and permanent income shocks and the the transitory consumption shock variances can be re-specified as:

$$\epsilon_{i,t} \sim \textit{i.i.d.}\left(0, \sigma^2_{\epsilon_t}\right)$$

$$\eta_{i,t} \sim \textit{i.i.d.}\left(0, \sigma^2_{\eta_t}\right)$$

$$\nu_{i,t} \sim \textit{i.i.d.}\left(0, \sigma^2_{\nu_t}\right)$$

BPP argue the partial consumption insurance parameters are constant overtime. However, I let the transitory and permanent partial consumption insurance vary across time:

$$\phi_t = \phi + \phi_2 D_2 + ... + \phi_T D_T \tag{12}$$

$$\psi_t = \psi + \psi_2 D_2 + ... + \psi_T D_T \tag{13}$$

where $D_2, ..., D_T$ are year dummy variables taking value 1 for the particular year and zero for all other years.

3.2 Methodology and Model Identification

However, the econometrician does not know the true data generating process. Hence, model selection in the face of limited data is an important issue in econometric modeling. Model selection can be framed as an exercise to decide which elements of the parameter vector should be set to zero. I propose the LASSO type GMM estimator; hereafter known as penalized-GMM (PGMM).

Let's start by stacking the variables of interest:

$$\Delta \boldsymbol{c}_i = \begin{pmatrix} \Delta c_{i,1} \\ \Delta c_{i,2} \\ \vdots \\ \Delta c_{i,T} \end{pmatrix}, \quad \Delta \boldsymbol{y}_i = \begin{pmatrix} \Delta y_{i,1} \\ \Delta y_{i,2} \\ \vdots \\ \Delta y_{i,T} \end{pmatrix}$$

Then,

$$\boldsymbol{x}_i = \begin{pmatrix} \boldsymbol{\Delta c}_i \\ \boldsymbol{\Delta y}_i \end{pmatrix}$$

Now we can derive

$$\boldsymbol{m} = vech\left\{\frac{1}{N}\sum_{i=1}^{N}\boldsymbol{x_i}\boldsymbol{x_i'}\right\}$$

The vector $\boldsymbol{m}$ contains the estimates of $T(2T+1)$ unique moments. Let θ be a k-dimensional parameter vector. Assume the true value of θ is θ_0, which belongs to the interior set of the compact parameter space $\Theta \subset \mathbb{R}^k$. Moreover, the population moments are $\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right)$, where in our case, the $\boldsymbol{\theta}$ is the vector of variances of the permanent shock and transitory shock, the partial insurance parameters and moving average and autoregressive parameters.

The $T(2T+1)$ dimension population orthogonality conditions are:

$$\mathbb{E}\left[\boldsymbol{\Phi}\left(\boldsymbol{\theta_0}\right) - \boldsymbol{m}\right] = 0$$

The GMM estimator, $\hat{\theta}_{GMM}$, minimizes the objective function $F^0\left(\theta\right)$ over $\theta \subset \Theta$:

$$F^0\left(\theta\right) = \left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right) - \boldsymbol{m}\right]' \boldsymbol{W}\left(\boldsymbol{\theta}\right)\left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right) - \boldsymbol{m}\right]$$

where $\boldsymbol{W}\left(.\right)$ is an $Op(1)$ and a positive definite $T(2T+1) \times T(2T+1)$ weighting matrix. The penalized-GMM estimator, $\hat{\theta}_{PGMM}$, minimizes the following objective function over the compact set Θ:

$$F\left(\theta\right) = \left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right) - \boldsymbol{m}\right]' \boldsymbol{W}\left(\boldsymbol{\theta}\right)\left[\boldsymbol{\Phi}\left(\boldsymbol{\theta}\right) - \boldsymbol{m}\right] + \lambda_N \sum_{j=1}^{k}\left|\theta_j\right| \tag{14}$$

for a given positive regularization parameter λ_N. While the limit for the penalized-GMM estimator is nonstandard, $\hat{\theta}_{PGMM}$ is a consistent estimator for θ_0 as long as $\lambda_N = o\left(N\right)$. Besides, when λ_N grows slowly, the limit of the penalized-GMM estimator converges to the limit of the regular GMM estimator (Caner (2009)). Also, the LASSO-type GMM estimator is powerful in that it endogenously selects subsets of features, without a need to build and compare large number of different models with subsets of the feature. It shrinks the estimates of the redundant variables to zero with positive probability. I harness this feature to uncover the most important partial insurance and consumption and income volatility parameters by household types. Below I conduct a Monte Carlo experiment to demonstrate how well our method works. But first, Table 1 summarizes the identifying equations linking the data with the unknown parameters to be estimated.

I present the data and moment condition to estimate income and consumption volatility parameters and partial consumption insurance parameters for different type of households. Although the equations estimate heterogeneity across one household type, the identification conditions can be easily extended to multiple household types. Also, it can be extended without loss of generality to capture the parameter identification condition for the time varying parameter models.

Table 1: Population parameters and the identifying equations

Data	Structural parameters
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1})\,\lvert D_i=0]$	σ_η^2
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1})\,\lvert D_i=1]$	$\sigma_\eta^2+\sigma_\eta^{type}$
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1})\,\lvert D_i=0]$	-σ_ϵ^2
$\mathbb{E}[\Delta y_{i,t}\,(\Delta y_{i,t-1})\,\lvert D_i=1]$	-$\sigma_\epsilon^2-\sigma_\epsilon^{type}$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta c_{i,t})\,\lvert D_i=0]$	$\phi^2\sigma_\eta^2+\psi^2\sigma_\epsilon^2+\sigma_\nu^2$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta c_{i,t})\,\lvert D_i=1]$	$(\phi+\phi^{type})^2(\sigma_\eta^2+\sigma_\eta^{type})+(\psi+\psi^{type})^2(\sigma_\epsilon^2+\sigma_\epsilon^{type})+\sigma_\nu^2+\sigma_\nu^{type}$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t})\,\lvert D_i=0]$	$\phi\sigma_\eta^2+\psi\sigma_\epsilon^2$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t})\,\lvert D_i=1]$	$(\phi+\phi^{type})(\sigma_\eta^2+\sigma_\eta^{type})+(\psi+\psi^{type})(\sigma_\epsilon^2+\sigma_\epsilon^{type})$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1})\,\lvert D_i=0]$	$\phi\sigma_\eta^2$
$\mathbb{E}[\Delta c_{i,t}\,(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1})\,\lvert D_i=1]$	$(\phi+\phi^{type})(\sigma_\eta^2+\sigma_\eta^{type})$
$\mathbb{E}[\Delta c_{i,t-1}\,(\Delta y_{i,t})\,\lvert D_i=0]$	-$\psi\sigma_\epsilon^2$
$\mathbb{E}[\Delta c_{i,t-1}\,(\Delta y_{i,t})\,\lvert D_i=1]$	-$(\psi+\psi^{type})(\sigma_\epsilon^2+\sigma_\epsilon^{type})$

‡ This table summarizes the identifying equations for the parameters $\phi, \phi^{type}, \psi, \psi^{type}, \sigma_\eta^2, \sigma_\eta^{type}, \sigma_\epsilon^2, \sigma_\epsilon^{type}, \sigma_\nu^2, \sigma_\nu^{type}$

4. Monte Carlo Simulation

In this section I first conduct a series of simulated experiments to assess how well the PGMM performs in recovering consumption and income volatilities and partial consumption insurance parameters in the data generating process. I then document the Root Mean Squared Error (RMSE) to assess how the PGMM estimator fairs against the usual GMM estimator.

First, I simulate consumption and income shocks for $N = 1500$ households across $T = 15$ years; this mimics the BPP data set I use in the study. I estimate the volatility and partial consumption insurance parameters using GMM and PGMM estimators with identity weighting matrix. The parameter estimates with the corresponding true values summarized in tables 2 and 3 below. Additionally, using 100 Monte Carlo draws, I estimate the RMSE of the two estimators. This enable us to compare the performances of the GMM and PGMM estimators. The results show the PGMM estimator is as good or better than the GMM estimator in the RMSE sense.

Most of the data sets available for estimating labor income processes are unbalanced. Hence, it is imperative to assess the estimators' performance in the face of missing data. I conduct the Monte Carlo experiment for both balanced data and unbalanced data set with missing values. Note that in the simulation experiment I allow the partial consumption insurance parameters and the variances of income and consumption shocks to vary with race. Moreover, for the PGMM estimator we chose the tuning parameter, λ.

Table 2: Heterogeneous partial consumption insurance and volatility estimates

Parameter	symbol	True value	GMM	RMSE	P-GMM	RMSE
		N=1500	$\lambda = 0.0029$			
base var of trans. y	σ_ϵ^2	0.1	0.0799	0.0054	0.0802	0.0053
base var of perm. y	σ_η^2	0.1	0.1139	0.0069	0.1139	0.0069
base var of trans. c	σ_ν^2	0.075	0.0786	0.0017	0.0775	0.0016
base c to a perm. y	ϕ	0.2	0.1194	0.0267	0.1347	0.0256
type c to a perm. y	ϕ^{type}	0.2	0.2931	0.0376	0.2598	0.0336
base c to a trans. y	ψ	0.05	0.0874	0.0214	0.047	0.018
type c to a trans. y	ψ^{type}	0	-0.0361	0.015	0	0.021
type var of trans. y	σ_ϵ^{type}	0.1	0.0831	0.0112	0.0845	0.011
type var of perm. y	σ_η^{type}	0.1	0.1348	0.0146	0.1334	0.0149
type var of trans. c	σ_ν^{type}	0	-0.0034	0.003	0	0.003

‡ This table summarizes the Monte Carlo experiment result comparing GMM and PGMM estimators. Based on the RMSE, PGMM estimator recovers the true population parameters better.

Table 2 above summarizes the Monte Carlo experiment results for the balanced panel data case. The results show GMM and PGMM estimators recover the population parameters very well. The PGMM estimator has slightly smaller RMSE values; hence, PGMM estimates the parameters more accurately (i.e. with lower bias and variance).

However, when using real data, I need to modify the model to address unique features of the dataset. Most panel datasets used for estimating the consumption insurance parameters are unbalanced. Attrition is a common problem and, most of the time, household data are available for different time lengths. The dataset contains different cohorts and will contain many missing observations. I assign a 5% probability of being missing to each of the $(T \times N)$ observations. The GMM and Penalized GMM estimators still recover the true population parameters very well even with the prevalence of missing observations.

Moreover, it is clear from the boxplots that the PGMM has lower Mean Squared Errors (MSEs). For example, the median MSE for PGMM is 0.0078, while that of GMM is 0.0083.

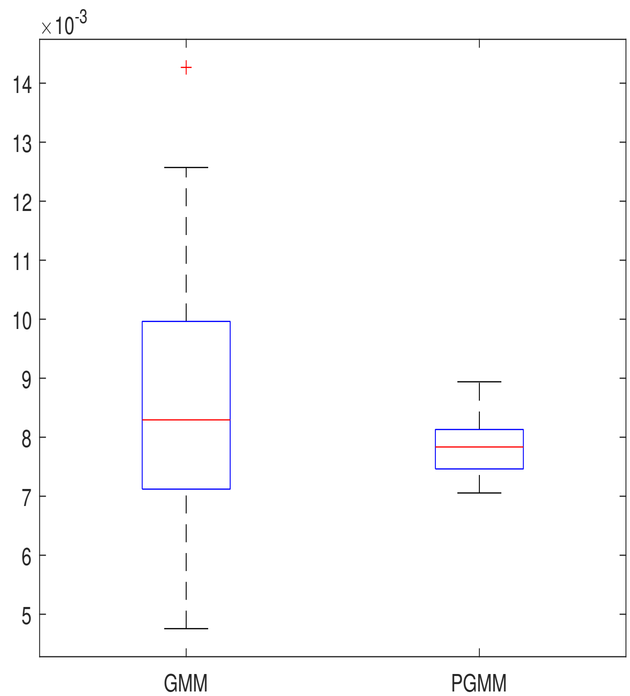

Figure 1: Boxplots of the mean squared errors for the GMM and the PGMM estimators.The central mark indicates the median, whereas the bottom and the top edges of the box indicate the 25th and 75th percentiles, respectively. The whiskers extend to the minimum and the maximum.

Table 3: Heterogeneous partial consumption insurance and volatility model with missing values

Parameter	symbol	True value	GMM	RMSE	P-GMM	RMSE
		N=1500	$\lambda = 0.0029$			
base var of trans. y	σ^2_ϵ	0.1	0.0799	0.01	0.0802	0.01
base var of perm. y	σ^2_η	0.1	0.1139	0.017	0.1139	0.017
base var of trans. c	σ^2_ν	0.075	0.0786	0.003	0.0775	0.003
base c to a perm. y	ϕ	0.2	0.1194	0.071	0.1347	0.063
type c to a perm. y	ϕ^{type}	0.2	0.2931	0.097	0.2598	0.085
base c to a trans. y	ψ	0.05	0.0874	0.048	0.047	0.036
type c to a trans. y	ψ^{type}	0	-0.0361	0.06	0	0.032
type var of trans. y	σ^{type}_ϵ	0.1	0.0831	0.04	0.0845	0.039
type var of perm. y	σ^{type}_η	0.1	0.1348	0.022	0.1334	0.022
type var of trans. c	σ^{type}_ν	0	-0.0034	0.008	0	0.006

‡ This table summarizes the Monte Carlo experiment result comparing GMM and PGMM estimators, for panel data with missing observations. Based on the RMSE, PGMM estimator recovers the true population parameters better.

5. Data and Empirical Results

5.1 Data

To explore the effect of income shocks on consumption, I need a panel series on both consumption and income. However, in US such comprehensive data sets do not exist. But two data sets come close. The first one is the Panel Study of Income Dynamics (PSID), which collects longitudinal annual data. The main shortcomings of PSID, related to this study, is that it collects data only for a subset of consumption items, mainly food expenditures. The second data source is the Consumer Expenditure Survey (CEX). CEX provides comprehensive information on the spending habits of US households; but households are followed only for a maximum of four quarters. While various studies investigate the link between the evolution of income and consumption inequality using either the PSID or CEX datasets (Krueger and Perri (2006)), few studies combined the PSID data with data from repeated cross-sections of the CEX (BPP, Ziliak (1998) and Jonathan Skinner (1987)).

The measure of consumption captures the flow of consumption services to households at a given period of time. However, for large durable goods such as cars and houses, current expenditure may not reflect the flow of services at a given period of time. Hence, consumption expenditure is mainly spending on nondurable goods and services.

What constitutes nondurable expenditure? Following Attanasio and Weber (1995) nondurable consumption includes food, alcoholic beverages and tobacco, services, heating fuel, transports (including gasoline), personal care, clothing and footwear, and rents. While PSID has data on food expenditure, it lacked information on other nondurable expenditures. However, the dynamics of food expenditure greatly differs from those of other nondurable expenditures. Being more of a necessity, food expenditure has preference elasticity and expenditure volatility less likely to be generalized to total nondurable consumption. Since food expenditures are very inelastic, using them as a proxy for other nondurable expenditures will greatly underestimate the volatility of total nondurable consumption.

One way around this challenge will be to combine information from the CEX and the PSID and impute a measure of consumption to the PSID households. This approach is first tried by Skinner (1987). He imputed total consumption in the PSID data base using the estimated coefficients of a regression of total consumption on common controls present in both the PSID and CEX data bases. The regression is first estimated using CEX data and later a corresponding PSID panel of consumption series is constructed using the estimated coefficients. Although, this approach technically resembles to the idea of matching based on observed characteristics, the quality of the imputed data depends on the reliability of the estimated coefficients. Moreover, the regression model can be ridden by biases mainly induced by initial differences in the input and target data sets and measurement errors in consumption.

Furthermore, BPP constructed a new panel data set with household information on income and nondurable consumption. Since, I am basing my analysis on their dataset, I provide short summary of the procedure. As explained above CEX has enough information to construct consumption regression model. However, PSID is missing some of the variables in the statistical relationship. Hence, the procedure extracts information from CEX to impute the missing variables in the PSID data set, exploiting the common statistical relationship among the variables across the two data sets. Assume c index observations from the CEX (the input data set) and p index an observation from PSID (the target data set). Now, consider the following food-demand equation:

$$D\left(f_{i,c}\right) = \boldsymbol{X'_{i,c}} \boldsymbol{\beta}^c + \alpha^c \psi\left(z_{i,c}\right) + \nu_{i,c}$$

where f is food expenditure variable available in both the PSID and CEX data sets. While $\boldsymbol{X}$ contains vectors of individual characteristics and aggregate variables which are available in both data sets, z is the total nondurable expenditure variable only available in the CEX data set. Lastly, the error term ν captures the unobserved heterogeneity in the demand for food regression model and $D\left(.\right)$ and $\psi\left(.\right)$ are food and nondurable demand functions respectively. Assuming the functions D and ψ are monotonically increasing and augmenting the standard OLS estimator to tackle the presence of measurement error in total consumption, Blundell et al (2006) estimated the above food demand equation parameters $\hat{\boldsymbol{\beta}}^c$ and $\hat{\alpha}^c$. Now, using these estimated parameter values, the imputed consumption variable in the PSID is calculated as:

$$\hat{z}_{i,p} = \psi^{-1}\left(\frac{D\left(f_{i,p}\right) - \boldsymbol{X}'_{i,p}\hat{\boldsymbol{\beta}}^c}{\hat{\alpha}^c}\right)$$

(assume $\hat{\alpha}^c > 0$).

For this study we used the BPP data set with imputed consumption observation. The BPP data focused on male headed stable households. Hence, I cannot explore the insurance and volatility heterogeneity across gender and, for now, I will focus on uncovering the heterogeneities across race only.

The panel data runs from 1978 to 1992. After the final cleaning we are left with a total of 17,612 households of which about 5.5% have black household heads.

5.2 The Covariance of Income and Consumption

The estimation proceeds in two steps. In the first stage we project income, log $\left(Y\right)$, and consumption, log $\left(C\right)$, onto household characteristics. After deriving the residuals from the first stage regression, I decompose the income residuals into permanent and transitory income shocks and further project them onto the residuals from the consumption regression. From the second stage estimation we derive the partial consumption insurances and consumption and income volatilities.

Table 4 below presents the first stage regression result for a subset of regressors. For the level of education completed by household head variable, none high school graduates are the base category. Both the high school graduates and college graduates earn significantly higher than the base category. While college graduates earn 25.6% more, high school graduates earn about 11.4% more than the base category. However, the consumption differentials are not statistically significant.

Moreover, black household heads earn and consume significantly less than their white counter parts. They earn about 22.6% less and end up consuming 24.3% less as well. For family size, the base category is households with two family members. The maximum family size reported in our sample is 10. We have omitted the results for other family sizes and only include households with 3 and 9 family sizes. Both households earn significantly more than the base category. Households with 9 family members earn about 96% more than those with 2 family members. While this differential seems large, it makes sense given the difference in the member of families supported by the households. However, the results on consumption are not statistically significant.

Furthermore, employed household heads earn and consume 43.7% and 23.4% more than unemployed household heads, respectively. Also the results for both income and consumption differentials are statistically significant. The retired household heads earn 26.5% less than the unretired household heads and consume 17.3% less. While these estimates are in line with the life cycle consumption theory, they are not statistically significant for our particular sample. Maybe this is due to the small number of retired household heads in our sample. Only 2% of the total household heads are retired.

Table 4: Income and consumption regression results

	(1)		(2)	
variable	log(y)	t-stat	log(c)	t-stat
high school	0.114***	2.772	-0.023	-0.567
	(0.041)		(0.041)	
college	0.256***	6.447	0.055	1.416
	(0.04)		(0.039)	
black	-0.226**	-2.009	-0.243**	-2.207
	(0.113)		(0.11)	
big city	0.138***	4.491	0.136***	4.500
	(0.031)		(0.03)	
family size(==3)	0.111*	1.758	0.074	1.194
	(0.063)		(0.062)	
family size(==9)	0.968**	2.308	-0.054	-0.131
	(0.42)		(0.412)	
employed	0.437***	6.126	0.234***	3.308
	(0.0714)		(0.071)	
retired	-0.265	-1.359	0.173	0.897
	(0.195)		(0.19)	
N	17571		14992	

Note: *** p$<$0.01, ** p$<$0.05, * p$<$0.1. The standard errors are provided in parenthesis.

‡ This table summarizes the first stage regression result of consumption and income. I projected log income and log consumption on the same sets of household characteristics. The residuals from the first stage regression are used to analyze the link between income and consumption inequalities.

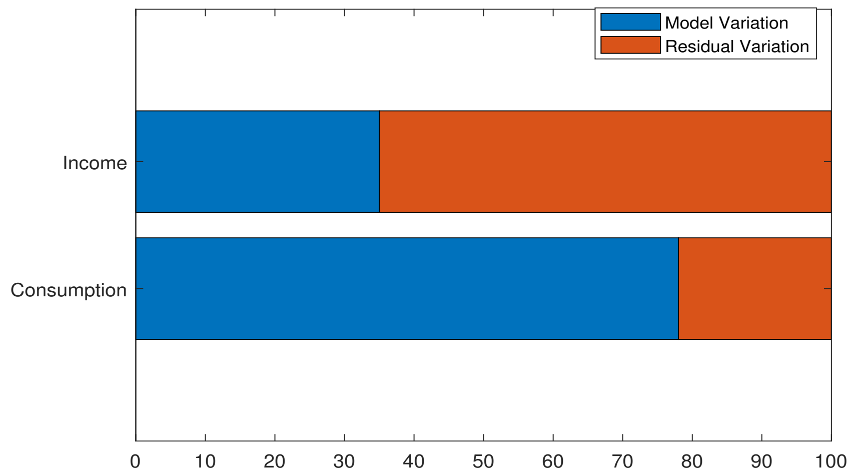

Figure 2: The variance decomposition of income and consumption[1]

To estimate the partial consumption insurance we are projecting the consumption residual onto the income residual. Hence, we need to mind how much of the variations in income and consumption we took out during the first stage regression. We took out 35% and 78% of income and consumption variation, respectively.

Before analyzing the heterogeneities of the consumption insurance and volatility parameters, we plotted (see Figure 2) the log income volatilities for the black household heads, white household heads and the combined sample. As we can see the log income of black household heads is much more volatile than both the whole sample and white household heads. The volatility plot for the white household heads appears to closely track the whole sample. This is not surprising given about 95% o the households in the sample have white household heads.

Moreover, similar to income volatilities, households with black heads have

[1]I decompose the income and consumption variations into model variation (i.e. captured by the first stage regression controls) and residual variation (capturing the income and consumption shocks).

a much more volatile income shock (see Figure 2). Here the income shock captures the none deterministic part of log income and it appears to be more volatile for households with black heads compared to both the ones with white heads and the combined sample. Also, as can be seen from Figure 2, the average income of households with black heads is significantly smaller than those with white household heads for the entire period of our sample.

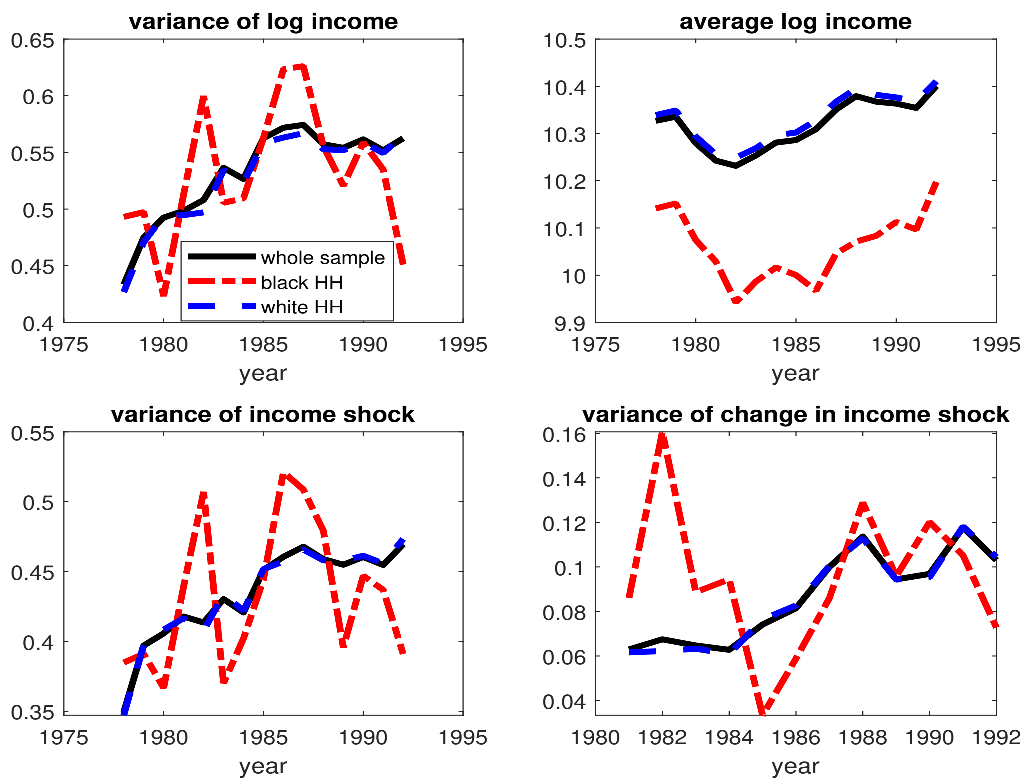

Figure 3: Mean and variance of log income and income shocks[2]

Table 6 below shows the correlations of income and consumption for the current, lagged and lead time periods.

[2]Black household heads, on average, earn significantly less than white household heads but they are exposed to higher income and income shock volatilities.

Table 5: Income and consumption shock covariance (whole sample)

variables	Δy_t	Δy_{t-1}	Δy_{t+1}	$\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}$	Δc_t	Δc_{t-1}	Δc_{t+1}	$\Delta c_{t-1}+\Delta c_t+\Delta c_{t+1}$
Δy_t	0.07	-0.021	-0.02	0.024	0.012	-0.002	-0.003	0.007
Δy_{t-1}	-0.021	0.07	-0.006	0.04	-0.002	0.011	6.63E-06	0.0082
Δy_{t+1}	-0.02	-0.006	0.08	0.052	-0.003	-0.0001	0.013	0.01
$\Delta y_{t-1}+\Delta y_t+\Delta y_{t+1}$	0.024	0.04	0.052	0.12	0.007	0.009	0.009	0.025
Δc_t	0.012	-0.002	-0.003	0.007	0.15	-0.064	-0.067	0.016
Δc_{t-1}	-0.002	0.011	-0.0001	0.009	-0.064	0.15	-0.003	0.08
Δc_{t+1}	-0.003	6.63E-06	0.013	0.009	-0.067	-0.003	0.15	0.08
$\Delta c_{t-1}+\Delta c_t+\Delta c_{t+1}$	0.007	0.0082	0.01	0.025	0.016	0.08	0.08	0.18

‡ This table summarizes the income and consumption covariances (for the whole sample). Tables 9 and 10 summarizes the income and consumption variances for black and white household heads respectively. The black household heads encounter higher income and consumption volatilities compared to white household heads and the whole sample.

5.3 Income and Consumption Volatilities and the partial insurance parameter Estimates

Now we estimate the partial consumption insurance and volatility parameters using the penalized-GMM method. The partial consumption insurance against permanent income shocks is estimated to be 63% and 36% for households with white heads and households with black heads, respectively. Ironically, the partial consumption insurance against permanent income shocks estimated by BPP is 36%.

Another interesting result is that households with white heads self insure against 100% of the transitory shocks, while the self insurance for the ones with black heads is 90%. This is interesting given most studies argued households self insure against more than 96% of the transitory income shocks(BPP, Chatterjee et al (2021) and Kaplan and Violante (2014)).

In addition the households with black heads have more volatile permanent income shock and consumption shock parameter estimates. However, the transitory income shock volatility estimates of the two groups are similar.

Table 6: Heterogeneous partial consumption insurance and income and consumption volatility estimates

Parameter	Symbol	GMM	Bootstrap SE	P-GMM	Bootstrap SE
		N=17,612	$\lambda = 0.0029$		
base var of trans. y	σ^2_ϵ	0.027	0.002	0.027	0.002
base var of perm. y	σ^2_η	0.029	0.002	0.029	0.002
base var of trans. c	σ^2_ν	0.15	0.0061	0.15	0.007
base c to a perm. y	ϕ	0.38	0.1	0.37	0.083
black c to a perm. y	ϕ^{black}	0.34	0.47	0.27	0.222
base c to a trans. y	ψ	-0.024	0.08	0	0.042
black c to a trans. y	ψ^{black}	0.14	0.22	0.10	0.188
black var of trans. y	σ^{black}_ϵ	-0.004	0.012	0	0.010
black var of perm. y	σ^{black}_η	0.008	0.0079	0.009	0.009
black var of trans. c	σ^{black}_ν	0.031	0.026	0.033	0.027

‡ This table summarizes empirical results using both GMM and PGMM estimators. I have also reported the bootstrap standard errors.

Tables 7 and 8 report the PGMM estimates of the time varying variances and partial insurance parameter estimates. While various studies explored the heterogeneity of income and consumption volatilities across time, very few analyzed whether the partial consumption parameters vary across time (Meghir and Pistaferri (2004), BPP, Chatterjee et al (2021)).

I allow the permanent and transitory partial insurance parameters vary across time. The partial consumption insurance to permanent shock estimates supports the time varying parameter model. However, the out of range estimates the partial consumption insurance to the transitory income shocks provide an evidence against the time varying parameter model.

Furthermore, the transitory and permanent income shocks and transitory consumption shock time varying volatility estimates fall in comparable range of the results reported by BPP and Chatterjee et al (2021).

Table 7: PGMM estimates of time-varying variances of income and consumption shocks

year	variance perm. shock $\left(\sigma^2_{\eta_t}\right)$	variance trans. shock $\left(\sigma^2_{\epsilon_t}\right)$	variance cons. shock $\left(\sigma^2_{\nu_t}\right)$
1979	0.025	0.021	0.161
1980	0.022	0.016	0.126
1981	0.026	0.020	0.104
1982	0.028	0.020	0.127
1983	0.028	0.015	0.132
1984	0.020	0.029	0.168
1985	0.034	0.032	0.183
1986	0.038	0.034	0.130
1987	0.015	0.040	NA
1988	0.028	0.037	NA
1989	0.053	0.032	NA
1990	0.025	0.033	0.202
1991-92	0.056	0.046	0.166

‡ This table summarizes the empirical PGMM estimates of the time varying transitory income shock, permanent income shock and consumption shock volatilities.

Table 8: PGMM estimates of time-varying variances of partial consumption insurance to permanent and transitory income shocks and the corresponding noise variances

year	partial insurance perm. shock (ϕ_t)	partial insurance trans. shock (ψ_t)
1979	0.122	-0.151
1980	0.262	0.011
1981	0.234	0.328
1982	0.260	0.343
1983	0.559	0.136
1984	0.288	0.051
1985	0.290	0.054
1986	0.625	-0.297
1987	NA	NA
1988	NA	NA
1989	NA	NA
1990	0.682	-0.320
1991-92	0.056	0.078

‡ This table summarizes the empirical PGMM estimates of the time varying partial consumption insurance to permanent and transitory income shocks.

6. Conclusions

The main objective of this study is to better explore the heterogeneities in income and consumption volatilities and partial consumption insurance parameter in the face of limited availability of data. This is achieved by using GMM LASSO, a popular method for shrinkage and estimation.

My analysis show that there is a significant heterogeneity by race; specifically, black household heads self-insure, both against transitory and permanent income shocks, less than white household heads. I also find that the amount of self-insurance against the permanent income shock for white household heads is much higher than reported by previous studies (BPP, Chatterjee et al (2021) and Kaplan and Violante (2010)). Also, my study suggests that black household heads face higher permanent income shock and transitory consumption volatilities.

Moreover, while my estimates of the time varying transitory and permanent income shock volatilities are in the ballpark of previous studies (BPP and Chatterjee et al (2021)), I have showed there is an evidence for time-varying parameter representation for the partial consumption insurance against permanent income shock parameter.

References

[1] Arellano, M., Blundell, R. and Bonhomme, S. (2017), *Earnings and Consumption Dynamics: A Nonlinear Panel Data Framework*. Econometrica, Volume 85, Issue 3, Pages 693-734.

[2] Attanasio, O., Pavoni, N.(2006), *Risk Sharing in Private Information Models with Asset Accumulation: Explaining the Excess Smoothness of Consumption*. Econometrica, Volume 79, Issue 4, Pages 1027-1068.

[3] Attanasio, O., Weber, G.(1995), *Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey*. Journal of Political Economy, 103(6): 1121–57.

[4] Baxter, M. and Crucini, J. M. (1995), Business Cycles and the Asset Structure of Foreign Trade. International Economic Review 36, N0.4.

[5] Blundell, R., Pistaferri, L. and Preston, I. (2006), *Imputing Consumption in the PSID using Food Demand Estimates from the CEX*. The Institute for Fiscal Studies, WP04/27.

[6] Blundell, R., Pistaferri, L. and Preston, I. (2008), *Consumption Inequality and Partial Insurance*. American Economic Review 98, 1887-1921.

[7] Brown, S., Ghosh, P., Su, L. and Taylor, K. (2015), *Modelling Household Finances: A Bayesian Approach to a Multivariate Two-Part Model*. Journal of Empirical Finance 33, 190-207.

[8] Chatterjee, A., Morley, J. and Singh, A. (2019), *Estimating Household Consumption Insurance*. University of Sydney, School of Economics.

[9] Chan, J. (2017), *Notes on Bayesian Macroeconomics*. Purdue University.

[10] Chan, J. and Jeliazkov, I. (2009), *Efficient simulation and integrated likelihood estimation in state space models.*International Journal of Mathematical Modelling and Numerical Optimisation 1(1/2), 101-120.

[11] Frühwirth-Schnatter, S. and Wagner, H. (2010), *Stochastic model specification search for Gaussian and partial non-Gaussian state space models.* Journal of Econometrics 154, 85-100.

[12] Gu, J. and Koenker, R. (2014), *Unobserved heterogeneity in income dynamics: an empirical Bayes perspective*. Cemmap Working Paper CWP43/14

[13] Geweke, J. and Keane, M. (2000), *An Empirical Analysis of Earnings Dynamics Among Men in the PSID: 1968-1989*. Journal of Econometrics, 96, 293–356.

[14] Hall, R. and Mishkin, F. (1982), *The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households*. Econometrica, Volume 50, Issue 2, Pages 461-81.

[15] Heathcote, J., Storesletten, K. and Violante, G. (2014), *Consumption and Labour Supply with Partial Insurance: An Analytical Framework*. American Economic Review, 104(7): 2075–2126.

[16] Hirano, K. (1998), *A Semiparametric Model for Labor Earnings Dynamics*. in Practical Nonparametric and Semiparametric Bayesian Statistics, ed. by D. D. Dey, P. Muller, and D. Sinha, chap. 20, pp. 355–367. Springer.

[17] Kaplan, G., and Violante, G. (2010), *How Much Consumption Insurance beyond Self-Insurance?*. American Economic Journal: Macroeconomics, Volume 2, No. 4, Pages 53-87.

[18] Kaplan, G., and Violante, G. (2014), *A Model of the Consumption Response to Fiscal Stimulus Payments*. Econometrica, Volume 82, Issue 4, Pages 1199-1239.

[19] Krueger, P., and Perri, F. (2006), *Does Income Inequality Lead to Consumption Inequality? Evidence and Theory*. Review of Economic Studies 73, 163-193.

[20] McCausland, W. J., Miller, S. and Pelletier, D. (2011), *Simulation smoothing for state-space models: A computational efficiency analysis*. Computational Statistics and Data Analysis 55(1), 199-212.

[21] Nakata, T. and Tonetti, C. (2015), *Small Sample Properties of Bayesian Estimators of Labor Income Processes*. Journal of Applied Economics, Vol. 18(1):122-148.

[22] Rue, H., Martino, S. and Chopin, N. (2009), *Approximate Bayesian inference for latent Gaussian models by using integrated nested Laplace*. Journal of the Royal Statistical Society Series B 71(2), 319-392.

[23] Skinner, J. (1987), *A Superior Measure of Consumption from the Panel Study of Income Dynamics*. Economic Letters, 23(2): 213–16.

[24] Smith, M. and Tonetti, C. (2014), *A Bayesian Approach to Imputing a Consumption-Income Panel Using the PSID and CEX*. Working Paper.

[25] Tobias, J., and Chan, J. (2019), *An alternate Parametrization for Bayesian Nonparametric/Semiparametric Regression*. Advances in Econometrics, Volume 40B, 47-64.

A. Moment Conditions

A.1 Heterogeneous Partial Insurance Parameters

Below are the moment conditions to identify the population parameters.

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)\right)=\sigma_\eta^2 \tag{15}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}\right)=-\sigma_\epsilon^2 \tag{16}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(1-gender_{it})\right)=(\bar{\phi}^m)^2\sigma_\eta^2+(\bar{\psi}^m)^2\sigma_\epsilon^2+\sigma_\nu^2 \tag{17}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^b)^2\sigma_\eta^2+(\bar{\psi}^m+\bar{\psi}^b)^2\sigma_\epsilon^2+\sigma_\nu^2 \tag{18}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^w)^2\sigma_\eta^2+(\bar{\psi}^m+\bar{\psi}^w)^2\sigma_\epsilon^2+\sigma_\nu^2 \tag{19}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^m\sigma_\eta^2+\bar{\psi}^m\sigma_\epsilon^2 \tag{20}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^b)\sigma_\eta^2+(\bar{\psi}^m+\bar{\psi}^b)\sigma_\epsilon^2 \tag{21}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^w)\sigma_\eta^2+(\bar{\psi}^m+\bar{\psi}^b)\sigma_\epsilon^2 \tag{22}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^m\sigma_\eta^2 \tag{23}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(race_{it})(1-gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^b)\sigma_\eta^2 \tag{24}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(gender_{it})\right)=(\bar{\phi}^m+\bar{\psi}^w)\sigma_\eta^2 \tag{25}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(1-gender_{it})\right)=-\bar{\psi}^m\sigma_\epsilon^2 \tag{26}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(race_{it})(1-gender_{it})\right)=-(\bar{\psi}^m+\bar{\psi}^b)\sigma_\epsilon^2 \tag{27}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(gender_{it})\right)=-(\bar{\psi}^m+\bar{\psi}^w)\sigma_\epsilon^2 \tag{28}$$

Equations 15-28 identifies the parameters $\sigma_\eta^2, \sigma_\epsilon^2, \sigma_\nu^2, \bar{\phi}^b, \bar{\psi}^w, \bar{\psi}^b, \bar{\phi}^w, \bar{\phi}^m, \bar{\psi}^m$.

A.2 Heterogeneous Partial Insurance and Volatility Parameters

For the heterogeneous partial insurance and volatility model, the moment conditions will be:

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(1-gender_{it})\right)=\bar{\sigma_\eta}^m \tag{29}$$

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(race_{it})(1-gender_{it})\right)=\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^b \tag{30}$$

$$\mathbb{E}\left(\Delta y_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(gender_{it})\right)=\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^w \tag{31}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}(1-race_{it})(1-gender_{it})\right)=-\bar{\sigma_\epsilon}^m \tag{32}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}(race_{it})(1-gender_{it})\right)=-\bar{\sigma_\epsilon}^m-\bar{\sigma_\epsilon}^b \tag{33}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta y_{i,t-1}(1-race_{it})(gender_{it})\right)=-\bar{\sigma_\epsilon}^m-\bar{\sigma_\epsilon}^w \tag{34}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(1-gender_{it})\right)=(\bar{\phi}^m)^2\bar{\sigma_\eta}^m+(\bar{\psi}^m)^2\bar{\sigma_\epsilon}^m+\bar{\sigma_\nu}^m \tag{35}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^b)^2(\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^b)+(\bar{\psi}^m+\bar{\psi}^b)^2(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^b)+\bar{\sigma_\nu}^m+\bar{\sigma_\nu}^b \tag{36}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta c_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^w)^2(\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^w)+(\bar{\psi}^m+\bar{\psi}^w)^2(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^w)+\bar{\sigma_\nu}^m+\bar{\sigma_\nu}^w \tag{37}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^m\bar{\sigma_\eta}^m+\bar{\psi}^m\bar{\sigma_\epsilon}^m \tag{38}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(race_{it})(1-gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^b)(\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^b)+(\bar{\psi}^m+\bar{\psi}^b)(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^b) \tag{39}$$

$$\mathbb{E}\left(\Delta c_{i,t}\Delta y_{i,t}(1-race_{it})(gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^w)(\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^w)+(\bar{\psi}^m+\bar{\psi}^w)(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^w) \tag{40}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(1-gender_{it})\right)=\bar{\phi}^m\bar{\sigma_\eta}^m \tag{41}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(race_{it})(1-gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^b)(\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^b) \tag{42}$$

$$\mathbb{E}\left(\Delta c_{i,t}\left(\Delta y_{i,t-1}+\Delta y_{i,t}+\Delta y_{i,t+1}\right)(1-race_{it})(gender_{it})\right)=(\bar{\phi}^m+\bar{\phi}^w)(\bar{\sigma_\eta}^m+\bar{\sigma_\eta}^w) \tag{43}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(1-gender_{it})\right)=-\bar{\psi}^m\bar{\sigma_\epsilon}^m \tag{44}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(race_{it})(1-gender_{it})\right)=-(\bar{\psi}^m+\bar{\psi}^b)(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^b) \tag{45}$$

$$\mathbb{E}\left(\Delta y_{i,t}\Delta c_{i,t-1}(1-race_{it})(gender_{it})\right)=-(\bar{\psi}^m+\bar{\psi}^w)(\bar{\sigma_\epsilon}^m+\bar{\sigma_\epsilon}^w) \tag{46}$$

Equations 29-46 identify the partial insurance and volatility parameters:

$$\{\bar{\sigma_\eta}^m,\bar{\sigma_\epsilon}^m,\bar{\sigma_\nu}^m,\bar{\phi}^b,\bar{\phi}^w,\bar{\psi}^b,\bar{\psi}^w,\bar{\phi}^m,\bar{\psi}^m,\bar{\sigma_\eta}^b,\bar{\sigma_\eta}^w,\bar{\sigma_\epsilon}^b,\bar{\sigma_\epsilon}^w,\bar{\sigma_\nu}^b,\bar{\sigma_\nu}^w\}$$

.

B. Results

[4] Black household heads, on average, consume less than white household heads but they are exposed to higher consumption and consumption shock volatilities.

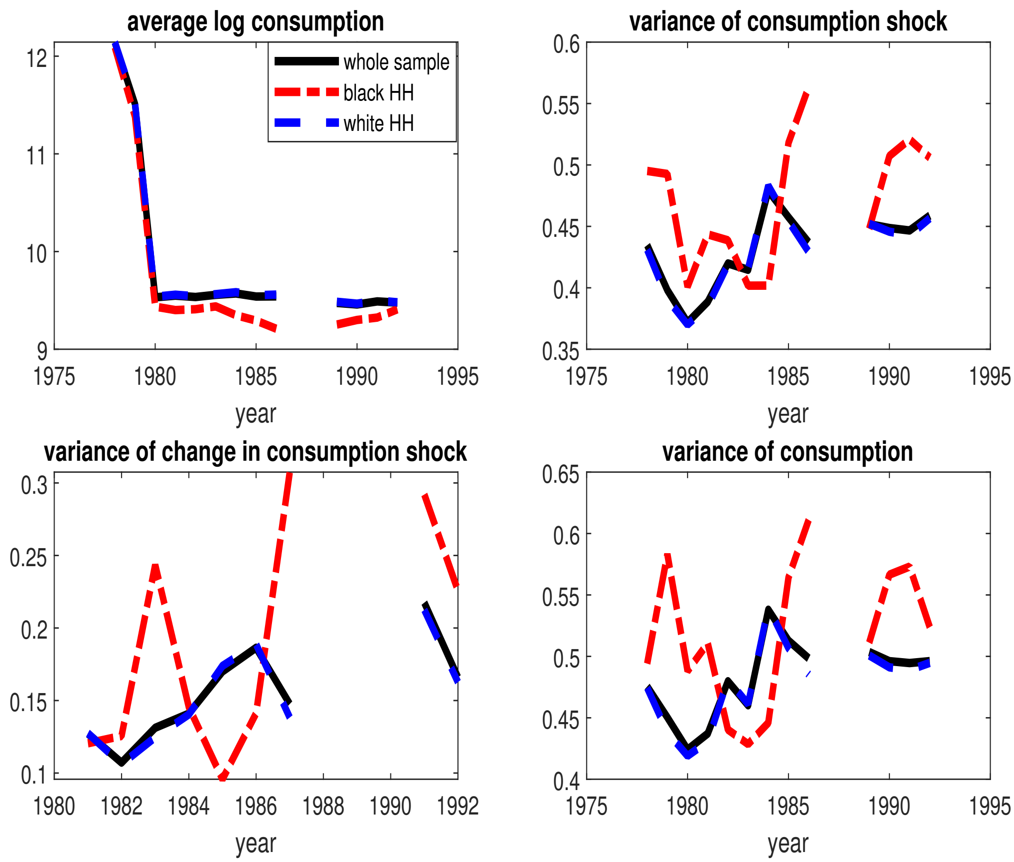

Figure 4: Mean and variance of log consumption and consumption shock (by race)[4]

[5]Income is more volatile than consumption. However, consumption shocks have higher volatility than income shocks. This might be the result of imputing consumption data, which end up introducing more noise in the consumption expenditure observations.

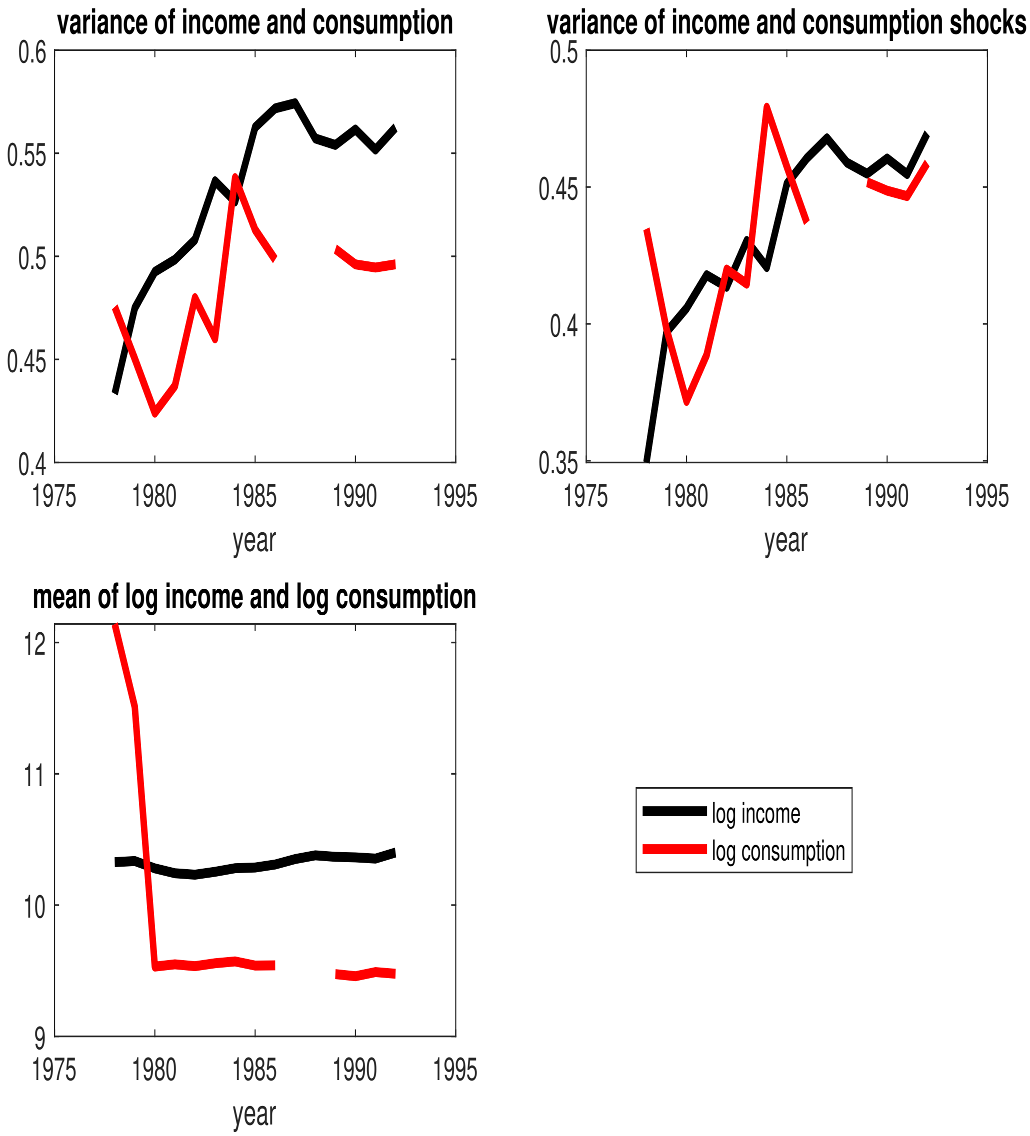

Figure 5: Income and consumption mean and variance[5]

Table 9: Income and consumption shock covariance (Black household heads)

variables	Δy_t	Δy_{t-1}	Δy_{t+1}	$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	Δc_t	Δc_{t-1}	Δc_{t+1}	$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$
Δy_t	0.079	-0.026	-0.024	0.029	0.030	-0.010	-0.017	0.003
Δy_{t-1}	-0.026	0.081	-0.014	0.040	-0.021	0.036	0.014	0.029
Δy_{t+1}	-0.024	-0.014	0.090	0.053	0.003	-0.011	0.023	0.015
$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	0.029	0.040	0.053	0.122	0.012	0.016	0.019	0.046
Δc_t	0.030	-0.021	0.003	0.012	0.160	-0.069	-0.066	0.024
Δc_{t-1}	-0.010	0.036	-0.011	0.016	-0.069	0.182	-0.015	0.098
Δc_{t+1}	-0.017	0.014	0.023	0.019	-0.066	-0.015	0.186	0.104
$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$	0.003	0.029	0.015	0.046	0.024	0.098	0.104	0.226

‡ This table summarizes the income and consumption covariances for black household heads. The black household heads encounter higher income and consumption volatilities compared to white household heads and the whole sample.

Table 10: Income and consumption shock covariance (White household heads)

variables	Δy_t	Δy_{t-1}	Δy_{t+1}	$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	Δc_t	Δc_{t-1}	Δc_{t+1}	$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$
Δy_t	0.069	-0.021	-0.025	0.023	0.012	-0.001	-0.003	0.008
Δy_{t-1}	-0.021	0.069	-0.006	0.041	-0.001	0.009	-0.001	0.007
Δy_{t+1}	-0.025	-0.006	0.082	0.052	-0.003	0.000	0.012	0.009
$\Delta y_{t-1} + \Delta y_t + \Delta y_{t+1}$	0.023	0.041	0.052	0.116	0.007	0.008	0.009	0.024
Δc_t	0.012	-0.001	-0.003	0.007	0.146	-0.064	-0.067	0.015
Δc_{t-1}	-0.001	0.009	0.000	0.008	-0.064	0.146	-0.002	0.080
Δc_{t+1}	-0.003	-0.001	0.012	0.009	-0.067	-0.002	0.147	0.078
$\Delta c_{t-1} + \Delta c_t + \Delta c_{t+1}$	0.008	0.007	0.009	0.024	0.015	0.080	0.078	0.173

‡ This table summarizes the income and consumption covariances for white household heads. The white household heads have about the same income and consumption volatilities as the whole sample.