Discrete Optimization

Constraint Programming: Part V

Goal of the lecture

- Illustrating the modeling techniques in constraint programming
  - symmetry breaking

Symmetries

- Many problems naturally exhibit symmetries
  - exploring symmetrical parts of the search space is useless
- Many kinds of symmetries
  - variable symmetries
  - value symmetries
- The next slides
  - symmetry-breaking constraints

BIBDs

- Balanced Incomplete Block Designs (BIBDs)
  - Input: (v,b,r,k,l)
  - Output: a v by b 0/1 matrix with exactly r ones per row, k ones per column, and a scalar product of value l

- Why BIBDs?
  - example of combinatorial design
  - full of variable symmetries

(3,3,2,2,1)

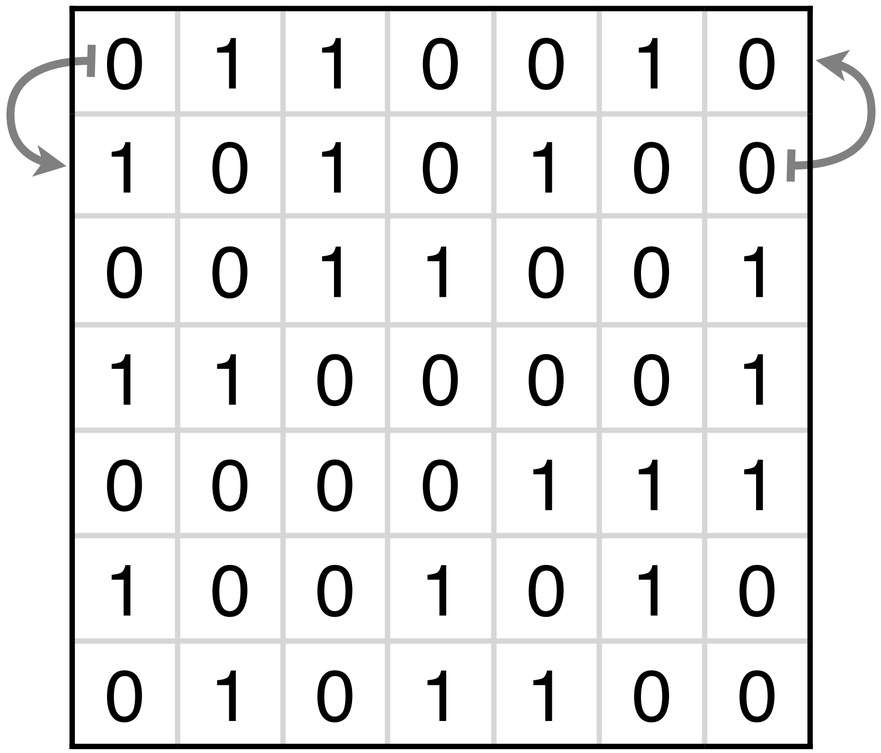
1	1	0
0	1	1
1	0	1

BIBDs

```
range Rows = 1..v;
range Cols = 1..b;
var{int} m[Rows,Cols] in 0..1;
solve {
   forall(i in Rows)
      sum(y in Cols) m[i,y] = r;
   forall(j in Cols)
      sum(x in Rows) m[x,j] = k;
   forall(i in Rows,j in Rows: j > i)
      sum(x in Cols) (m[i,x] & m[j,x]) = 1;
}
```

BIBDs

(7,7,3,3,1)

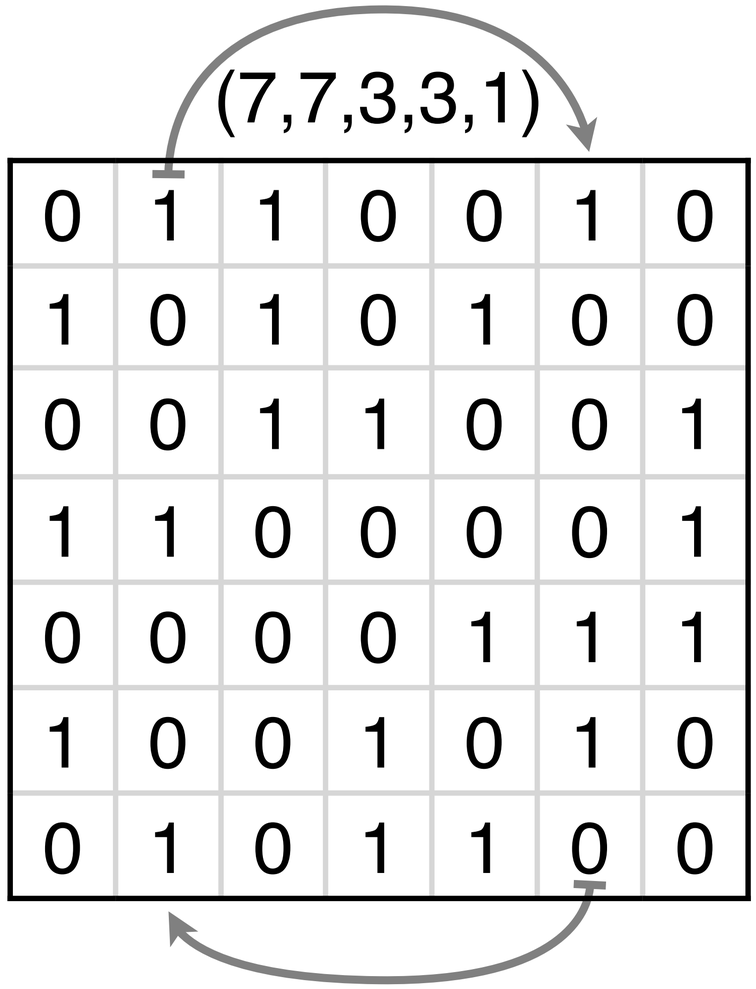


0	1	1	0	0	1	0
1	0	1	0	1	0	0
0	0	1	1	0	0	1
1	1	0	0	0	0	1
0	0	0	0	1	1	1
1	0	0	1	0	1	0
0	1	0	1	1	0	0

(7,7,3,3,1)

1	0	1	0	1	0	0
0	1	1	0	0	1	0
0	0	1	1	0	0	1
1	1	0	0	0	0	1
0	0	0	0	1	1	1
1	0	0	1	0	1	0
0	1	0	1	1	0	0

BIBDs



(7,7,3,3,1)

0	1	1	0	0	1	0
1	0	1	0	1	0	0
0	0	1	1	0	0	1
1	0	0	0	0	1	1
0	1	0	0	1	0	1
1	1	0	1	0	0	0
0	0	0	1	1	1	0

BIBDs

- How to break variable symmetries
  - impose an ordering on the variables
- Consider the row symmetries
  - impose a lexicographic constraint
- Lexicographic ordering
  - a: 0 1 1 0 0 1 0 1 1 1 0 0 1 0
  - b: 1 0 1 0 1 0 0 1 0 1 0 1 0 0

$a \leq b$ $a \geq b$

BIBDs

(7,7,3,3,1)

0	0	0	0	1	1	1
0	0	1	1	0	0	1
0	1	0	1	1	0	0
0	1	1	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	1	0	0
1	1	0	0	0	0	1

Lexicographic Ordering

(7,7,3,3,1)

0	0	0	0	1	1	1
0	0	1	1	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
1	0	0	1	1	0	0
1	0	1	0	0	1	0
1	1	0	0	0	0	1

BIBDs

```
range Rows = 1..v;
range Cols = 1..b;
var{int} m[Rows,Cols] in 0..1;
solve {
   forall(i in Rows)
      sum(y in Cols) m[i,y] = r;
   forall(j in Cols)
      sum(x in Rows) m[x,j] = k;
   forall(i in Rows,j in Rows: j > i)
      sum(x in Cols) (m[i,x] & m[j,x]) = 1;
   forall(i in 1..v-1)
      lexleq(all(j in Cols) m[i,j],all(j in Cols) m[i+1,j]);
   forall(j in 1..b-1)
      lexleq(all(i in Rows) m[i,j],all(i in Rows) m[i,j+1]);
}
```

Scene Allocation




- Shooting scenes for a movie
  - an actor plays in some of the scenes
  - at most k scenes can be shot per day
  - each actor is paid by the day
- Objective
  - minimize the total cost
- Symmetries
  - what kind of symmetries do we have here?

Scene Allocation

```
range Scenes = …;
range Days  = …;
range Actor = …;
int fee[Actor] = …;
set{Actor} appears[Scenes] = …;
set{int} which[a in Actor] = setof(i in Scenes) member(a,appears[i]);
var{int} shoot[Scenes] in Days;

minimize
   sum(a in Actor) sum(d in Days)
      fee[a] * or(s in which[a]) (shoot[s]=d)
subject to
   atmost(all(i in Days) 5,Days,shoot);
```

Scene Allocation

- Value symmetries
  - the days are interchangeable
  - I can swap all the scenes in day 1 and all the scenes in day 2 and I still have a solution
  - nothing can be used to distinguish the days: they are interchangeable
  - if *s* is a solution, then *p(s)* is a solution where the days of *s* have been permuted by permutation *p*
- How can we eliminate these symmetries?
  - Consider the first scene s_1. What are the days that we consider for this scene?

Scene Allocation

- Value symmetries
  - the days are interchangeable
  - if *s* is a solution, then *p(s)* is a solution where the days of *s* have been permuted by permutation *p*
- How can we eliminate these symmetries?
  - consider the first scene s_1. What are the days that we consider for this scene?

Only one day, say day 1

All days are interchangeable at this point

Only two days, day 1 and day 2

Scene Allocation

- How can we eliminate these symmetries?
  - consider the first scene s_1. What are the days that we consider for this scene?

    Only one day, say day 1
  - consider the second scene s_2. Which days must be considered for s_2?

    Only two days, day 1 and day 2
  - In general the already used days and one additional new day

existing days

new day

$$D(s_k) = \{1, 2, \ldots, \max(s_1, ..., s_{k-1}) + 1\}$$

Scene Allocation

```
range Scenes = 1..n;
range Days  = 1..m;
range Actor = …;
int fee[Actor] = …;
set{Actor} appears[Scenes] = …;
set{int} which[a in Actor] = setof(i in Scenes) member(a,appears[i]);
var{int} shoot[Scenes] in Days;

minimize
   sum(a in Actor) sum(d in Days)
      fee[a] * or(s in which[a]) (shoot[s]=d)
subject to {
   atmost(all(i in Days) 5,Days,shoot);
   scene[1] = 1;
   forall(s in Scenes: s > 1)
      scene[s] <= max(k in 1..s-1) scene[k] + 1;
}
```

Citations

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