Discrete Optimization

Constraint Programming: Part X

Goals of the lecture

- Search in constraint programming
  - introduction
  - active research area

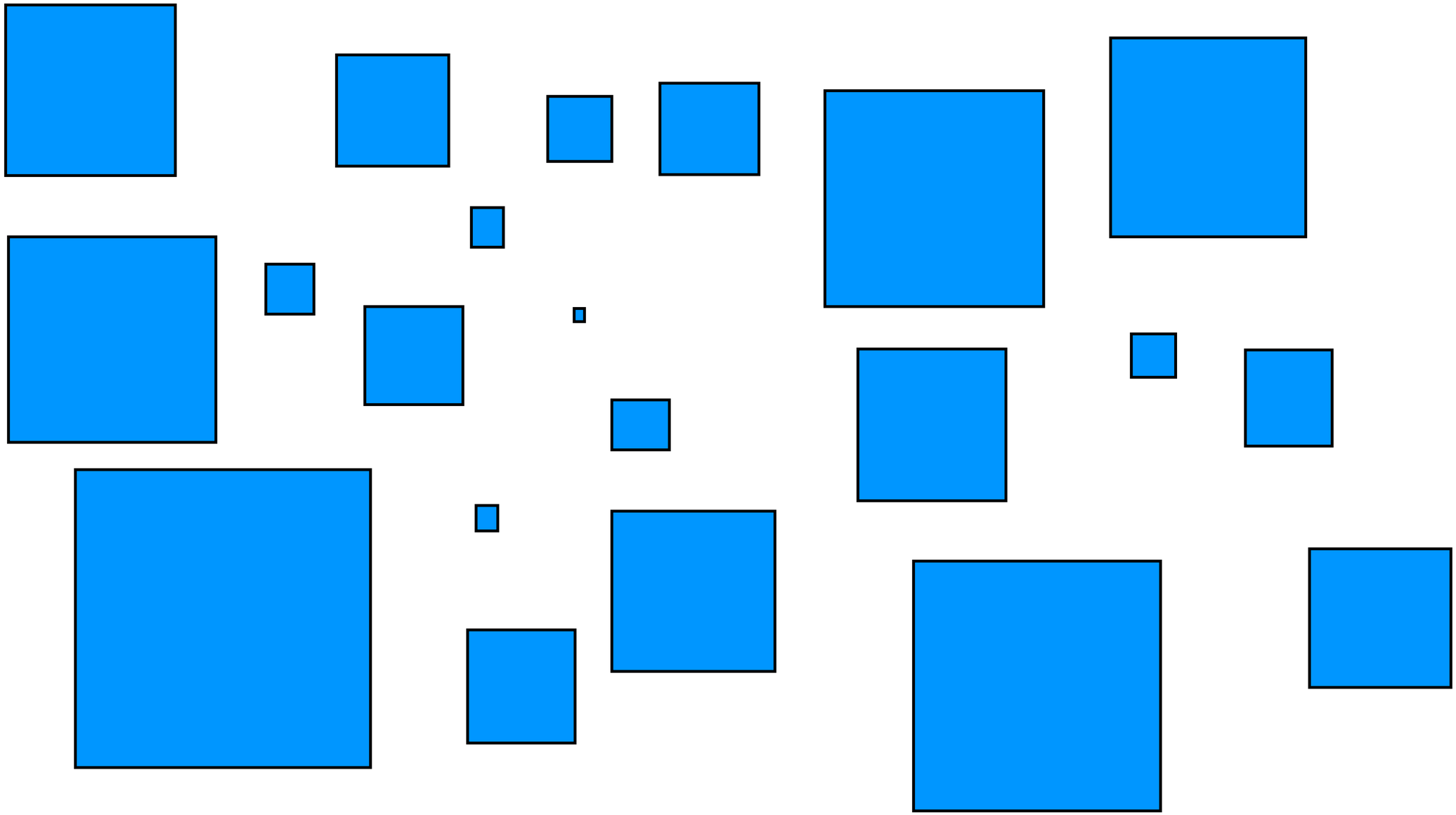
Searching in constraint programming

- variable/value labeling
- focusing on the objective
- value/variable labeling
- domain splitting
- symmetry breaking during search
- randomization and restarts

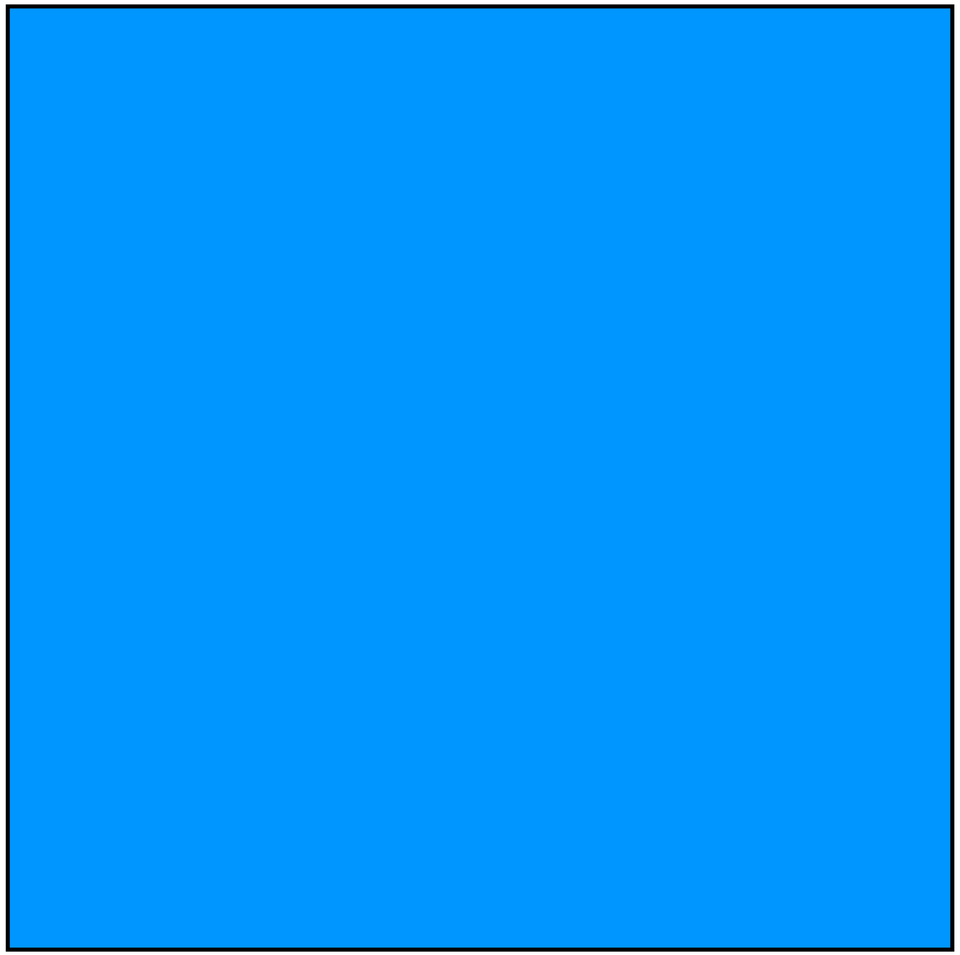
Value/variable labeling

- Two steps
  - choose the value to assign next
  - choose the variable to assign to this value
- Why it is useful?
  - you may know that a value must be assigned
  - often the case in scheduling and resource allocation problems

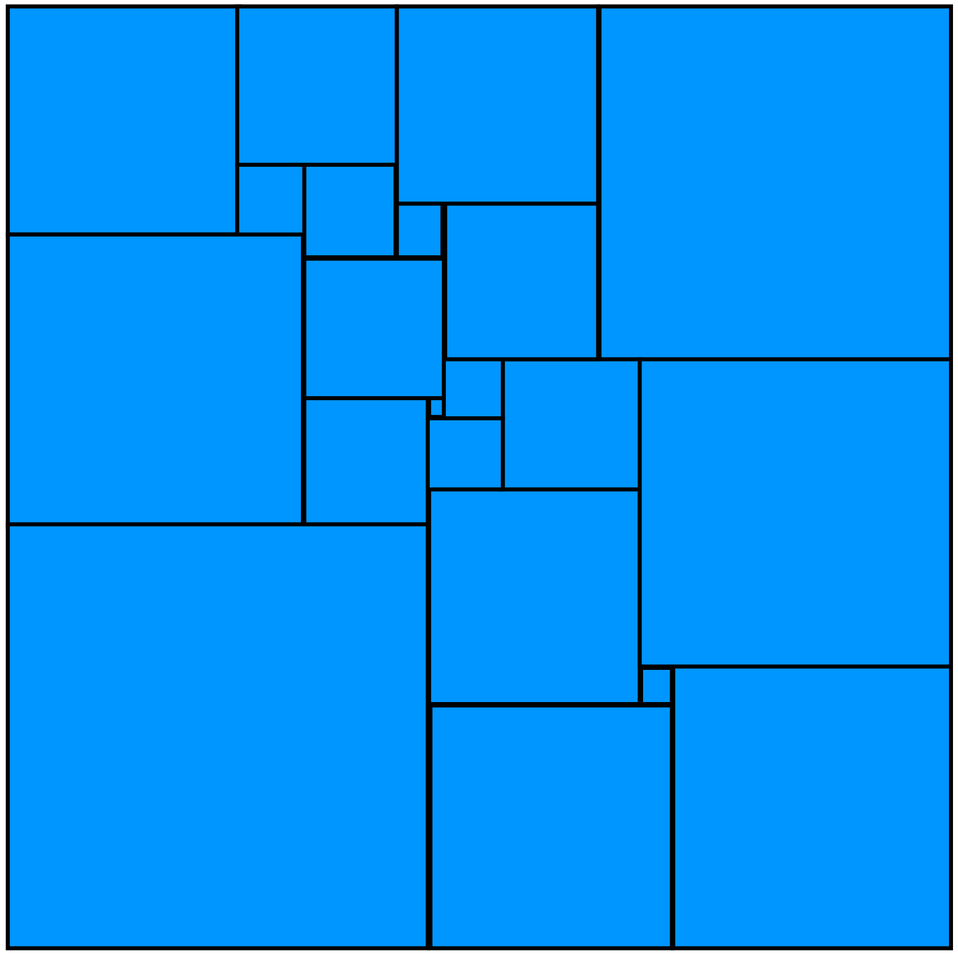
The perfect square problem



The perfect square problem



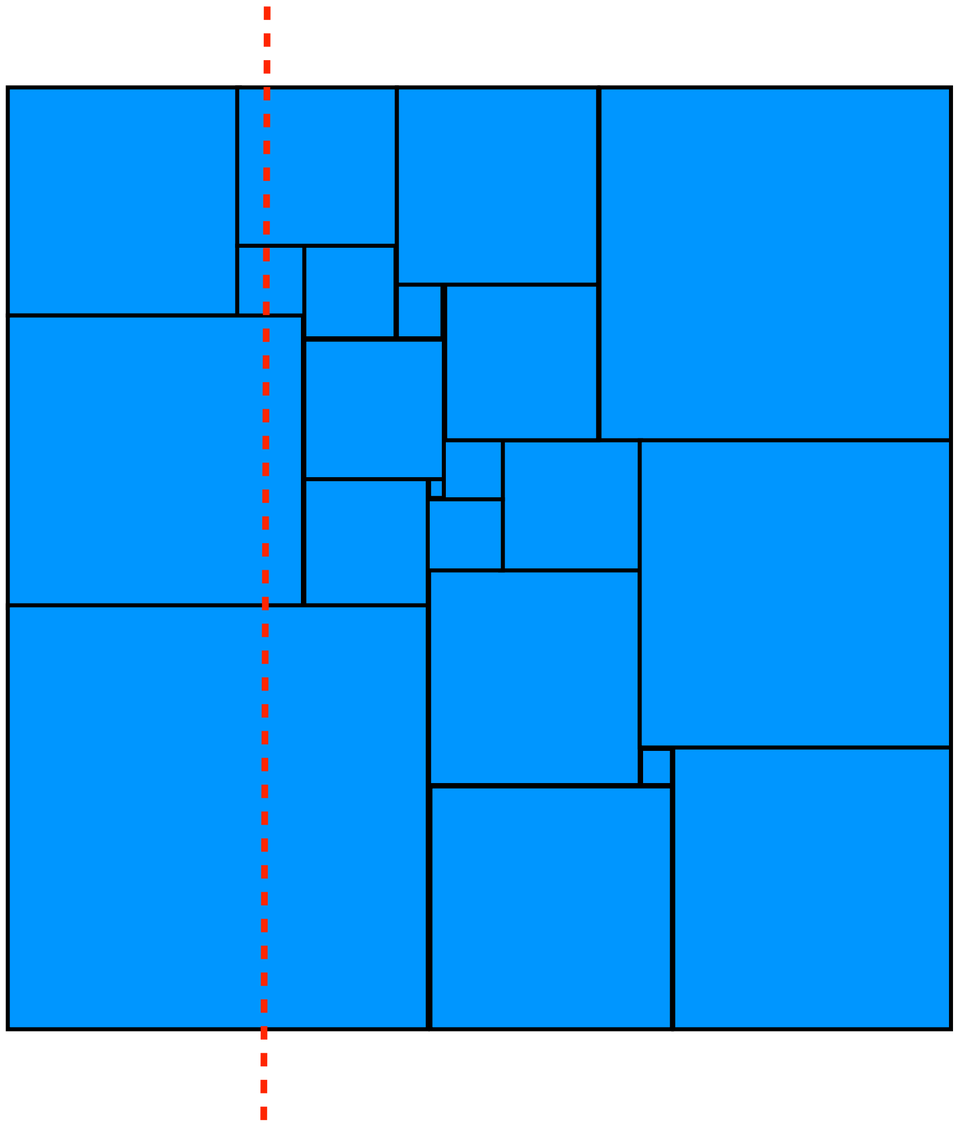
The perfect square problem



The perfect square problem

- What are the decision variables?
  - x and y-coordinates of the bottom-left corner of every square
- What are the constraints?
  - the squares fit in the larger square
  - the squares do not overlap
- There are also redundant constraints
  - here is the intuition

The perfect square problem



The perfect square problem

```
range R = 1..8;
int s = 122; range Side = 1..s; range Square = 1..21
int side[Square] = [50,42,37,35,33,29,27,25,24,19,18,17,16,15,11,9,8,7,6,4,2];
var{int} x[Square] in Side;
var{int} y[Square] in Side;

solveall {
  forall(i in Square) {
 x[i]<=s-side[i]+1;
   y[i]<=s-side[i]+1;
 }
  forall(i in Square,j in Square: i<j)
    x[i]+side[i]<= x[j] || x[j]+side[j]<=x[i] ||  y[i]+side[i]<=y[j] || y[j]+side[j]<=y[i];

  forall(p in Side) {
     sum(i in Square) side[i]*((x[i]<=p) && (x[i]>=p-side[i]+1)) = s;
     sum(i in Square) side[i]*((y[i]<=p) && (y[i]>=p-side[i]+1)) = s;
  }
}
```

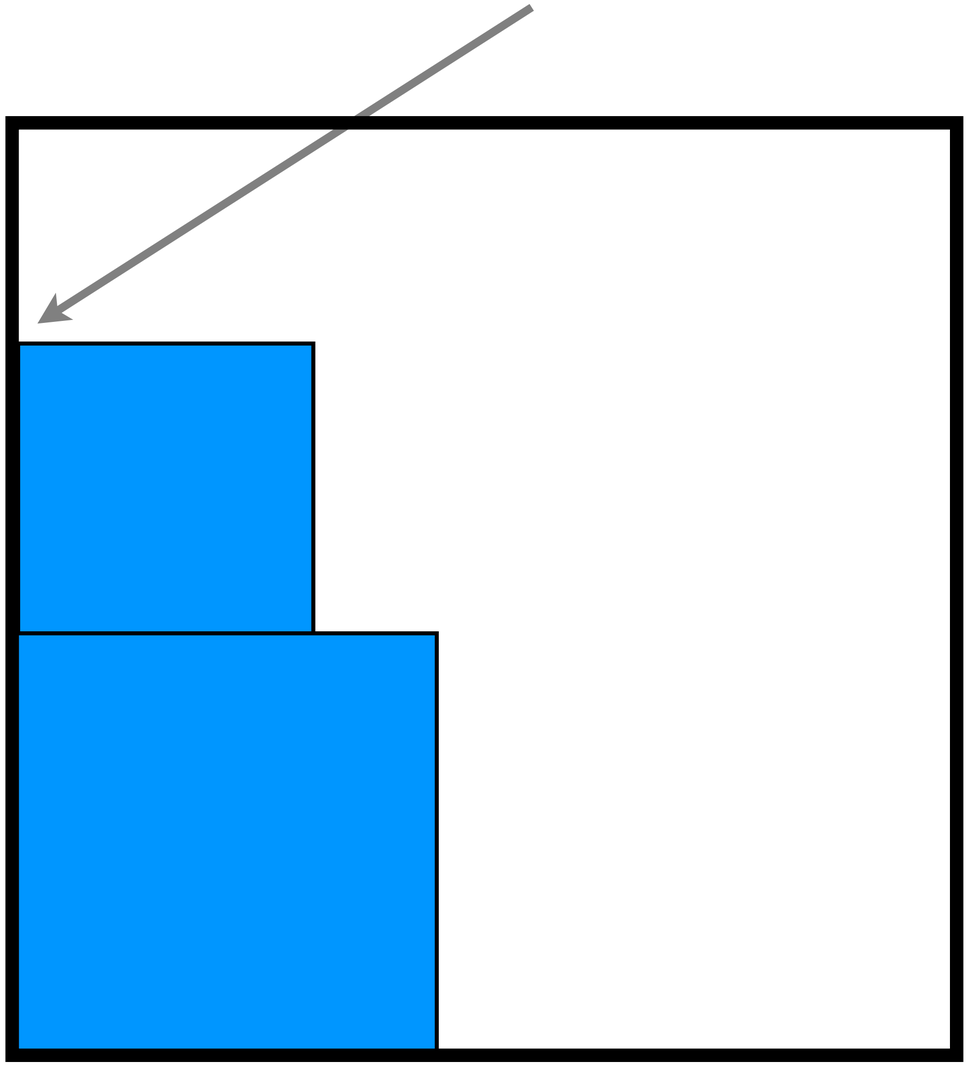
non-overlapping constraints

redundant constraints

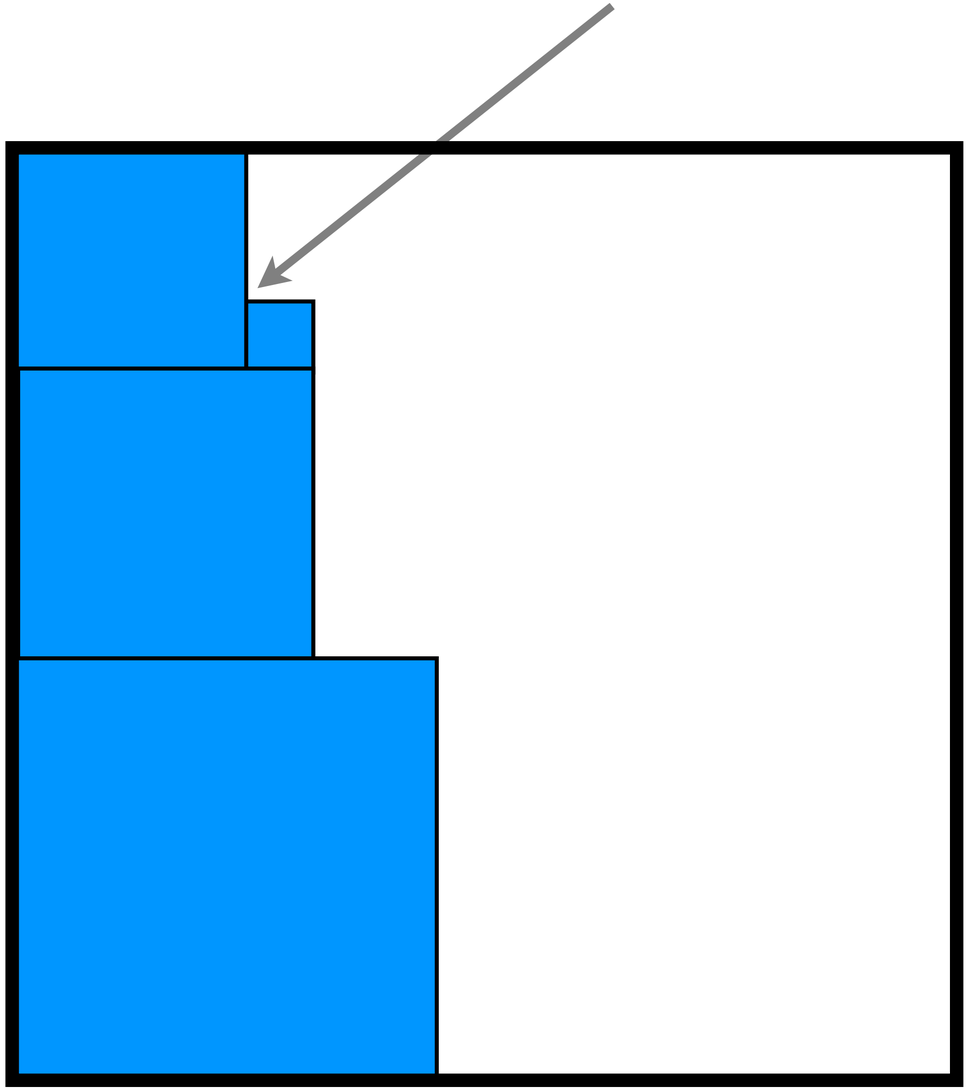
The value/variable labeling

- Why a value/variable labeling
  - we know that there is no empty space in the square to be filled.

The perfect square problem



The perfect square problem



The value/variable labeling

- What is the labeling doing?
  - choose a x-coordinate p
  - for all square i, decide whether to place i at coordinate p
    - that is, whether the bottom-left corner of i has x-coordinate p
  - repeat for all x-coordinates
  - repeat for all y-coordinates

The value/variable labeling

- What is the labeling doing?
  - choose a x-coordinate p
  - for all square i, decide whether to place i at coordinate p
    - that is, whether the bottom-left corner of i has x-coordinate p
  - repeat for all x-coordinates
  - repeat for all y-coordinates

The perfect square problem

```
using {

  forall(p in Side)
    forall(i in Square)
      try
         x[i] = p;
      |
       x[i] != p;

  forall(p in Side)
    forall(i in Square)
      try
         y[i] = p;
      |
         y[i] != p;

}
```

choose a x-coordinate p

consider a square i

decide whether to place i at position p

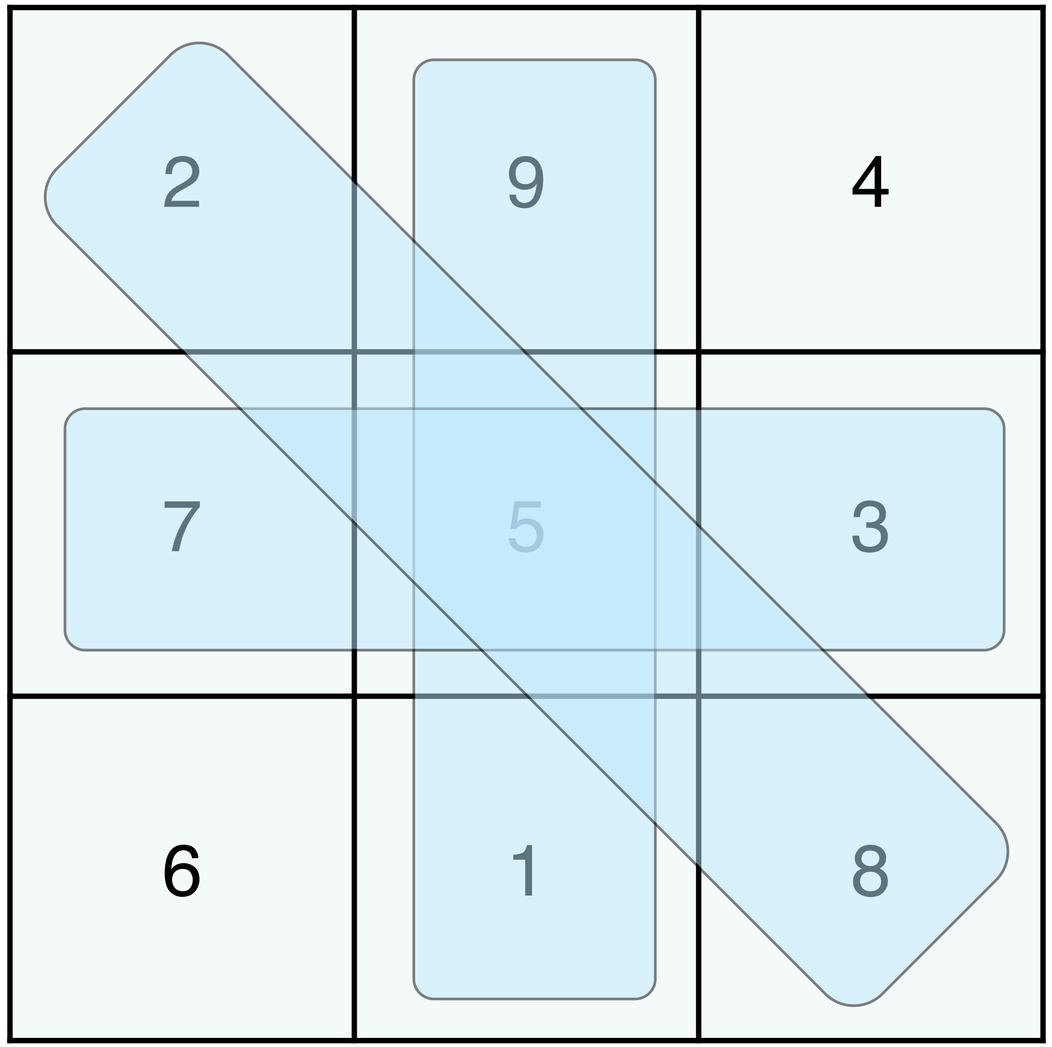
Searching in constraint programming

- variable/value labeling
- focusing on the objective
- value/variable labeling
- domain splitting
- symmetry breaking during search
- randomization and restarts

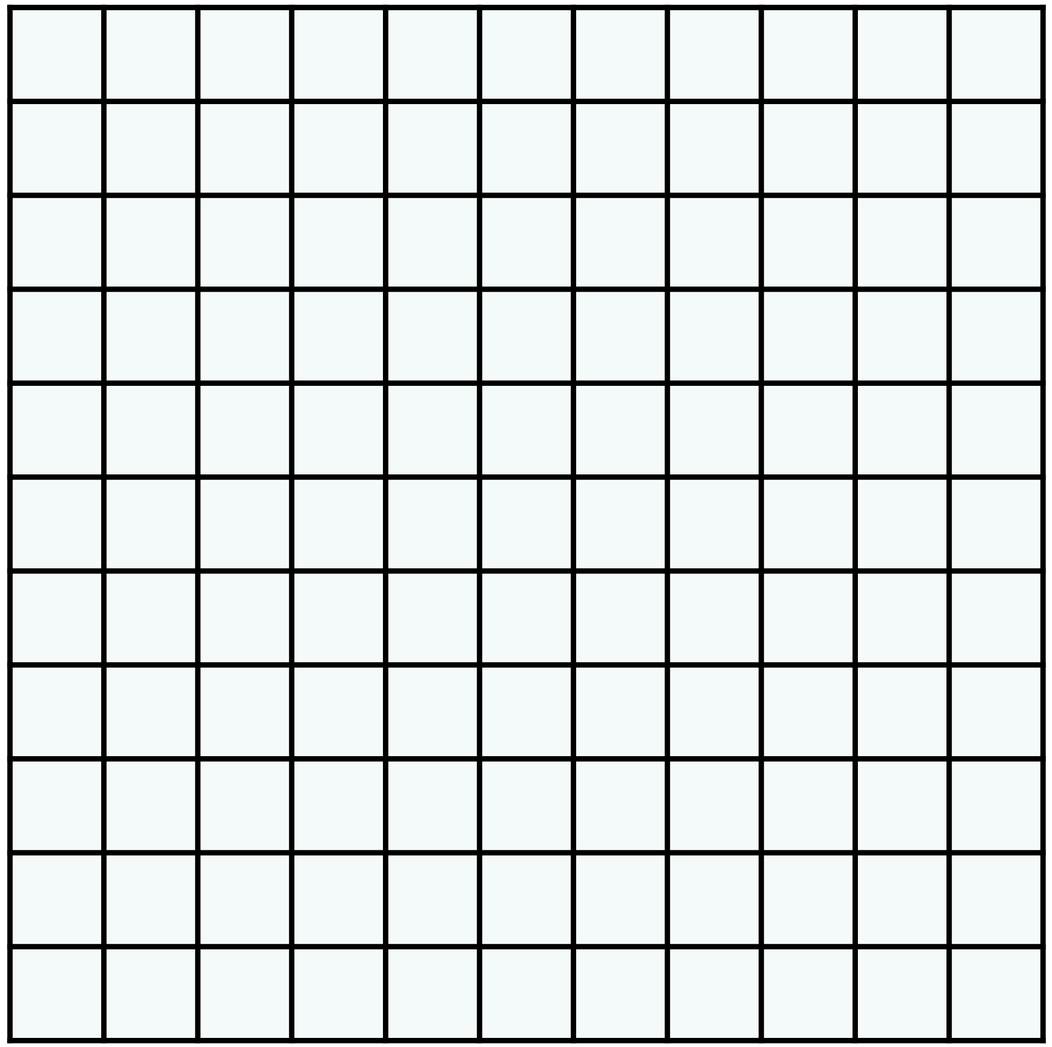
The magic square problem

- Place numbers in a square
  - all numbers are different
  - all rows, columns, and diagonals sum to the same number

The magic square problem



The magic square problem



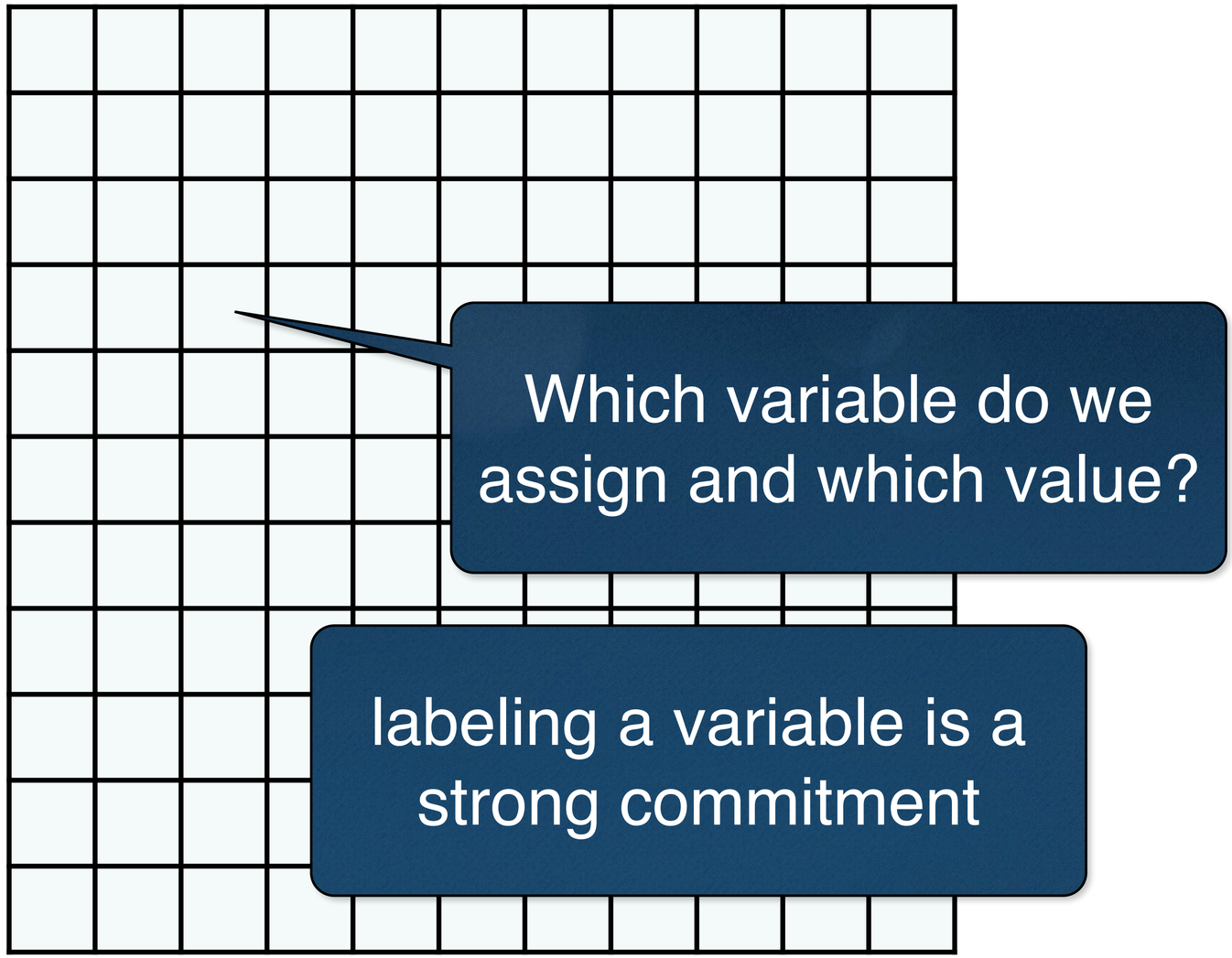
The magic square problem

1	90	18	14	119	112	117	118	20	54	8
93	2	99	16	29	92	94	31	68	56	91
39	75	3	5	12	87	95	101	57	97	100
21	83	103	4	86	78	84	58	89	15	50
79	70	98	107	55	51	59	28	25	76	23
24	106	63	32	109	60	9	108	104	34	22
69	49	105	110	61	26	82	7	13	72	77
73	46	102	62	40	36	38	113	81	33	47
71	42	64	114	41	27	44	53	115	52	48
80	65	6	111	74	67	19	37	11	116	85
121	43	10	96	45	35	30	17	88	66	120

The magic square problem

```
range R = 1..n;
range D = 1..n^2;
int T = n*(n^2+1)/2;
var{int} s[R,R] in D;
solve {
   forall(i in R) {
      sum(j in R) s[i,j] = T;
      sum(j in R) s[j,i] = T;
   }
   sum(i in R) s[i,i] = T;
   sum(i in R) s[i,n-i+1] = T;
   alldifferent(all(i in R,j in R) s[i,j]);
}
```

The magic square problem



Domain splitting

- Select a variable
- Split its domain in two or more sets
  - much weaker commitment

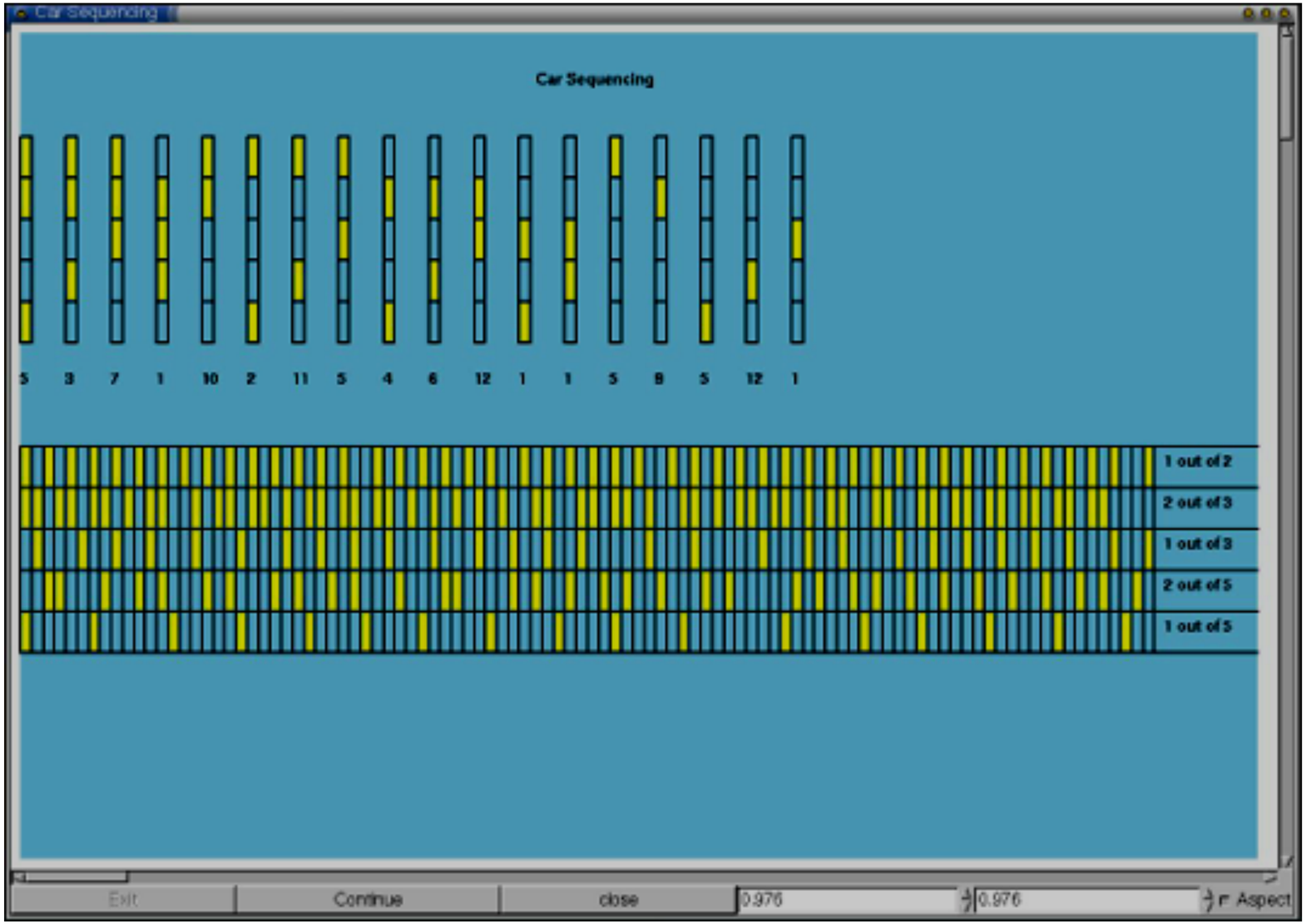
The magic square problem

```
using {
  var{int}[] x = all(i in R,j in R) s[i,j];
  range V = x.getRange();
  while (!bound(x)) {
    selectMin(i in V:!x[i].bound())(x[i].getSize()) {
      int mid = (x[i].getMin()+x[i].getMax())/2;
      try
        x[i] <= mid;
      |
        x[i] > mid;
    }
  }
}
```

Car sequencing

- Cars on an assembly line
- Cars require specific options
  - leather seats, moonroof
- Capacity constraints on the production units
  - at most 2 out of 5 successive cars can require a moonroof
- Sequence all the cars such that the capacity constraints are satisfied

Car sequencing



Domain splitting in car sequencing

- Motivation
  - focus on difficult options
  - decide which slots take these options
- Domain splitting
  - do not assign configurations to slots
  - decide whether the slot take the option

Domain splitting in car sequencing

```
using {
  forall(o in Options) by (slack[o])
    forall(i in Slots)
      try
        line[i] in options[o];
      |
        line[i] notin options[o];
}
```

start with the option that has the least slack

Force the slot to take the option

Force the slot not to take the option

Searching in constraint programming

- variable/value labeling
- focusing on the objective
- value/variable labeling
- domain splitting
- symmetry breaking during search
- randomization and restarts

Scene Allocation




- Shooting scenes for a movie
  - an actor plays in some of the scenes
  - at most k scenes can be shot per day
  - each actor is paid by the day
- Objective
  - minimize the total cost
- Symmetries
  - what kind of symmetries do we have here?

Scene allocation

- How can we eliminate these symmetries?
  - consider the first scene s_1. What are the days that we consider for this scene?

    Only one day, say day 1
  - consider the second scene s_2. Which days must be considered for s_2?

    Only two days, day 1 and day 2
  - In general the already used days and one additional new day

existing days

new day

$$D(s_k) = \{1, 2, \ldots, \max(s_1, ..., s_{k-1}) + 1\}$$

Scene allocation

```
range Scenes = 1..n;
range Days  = 1..m;
range Actor = …;
int fee[Actor] = …;
set{Actor} appears[Scenes] = …;
set{int} which[a in Actor] = setof(i in Scenes) member(a,appears[i]);
var{int} shoot[Scenes] in Days;

minimize
   sum(a in Actor) sum(d in Days)
      fee[a] * or(s in which[a]) (shoot[s]=d)
subject to {
   atmost(all(i in Days) 5,Days,shoot);
   scene[1] = 1;
   forall(s in Scenes: s > 1)
      scene[s] <= max(k in 1..s-1) scene[k] + 1;
}
```

Symmetry-breaking constraints

- Side effect
  - interferences with the search heuristic
- Can we avoid this?
  - symmetry-breaking during search
  - dynamically impose the symmetry-breaking constraints
- How?
  - same constraints
  - the order is different and discovered dynamically

Symmetry-breaking during search

- Choose a scene to shoot
  - use a good heuristic
    - first-fail
    - expensive scene
- Consider existing days + 1 new day
  - to label the scene
- Advantages
  - break symmetries
  - does not interfere with the search heuristic

Symmetry breaking during search

```
using {
  while (!bound(shoot)) {
    int eday = max(-1,maxBound(shoot));
    selectMin(s in Scenes: !shoot[s].bound())
        (shoot[s].getSize(),-sum(a in appears[s]) fee[a])
      tryall(d in 0..eday + 1)
        shoot[s] = d;
  }
}
```

dynamic ordering

existing days

new day

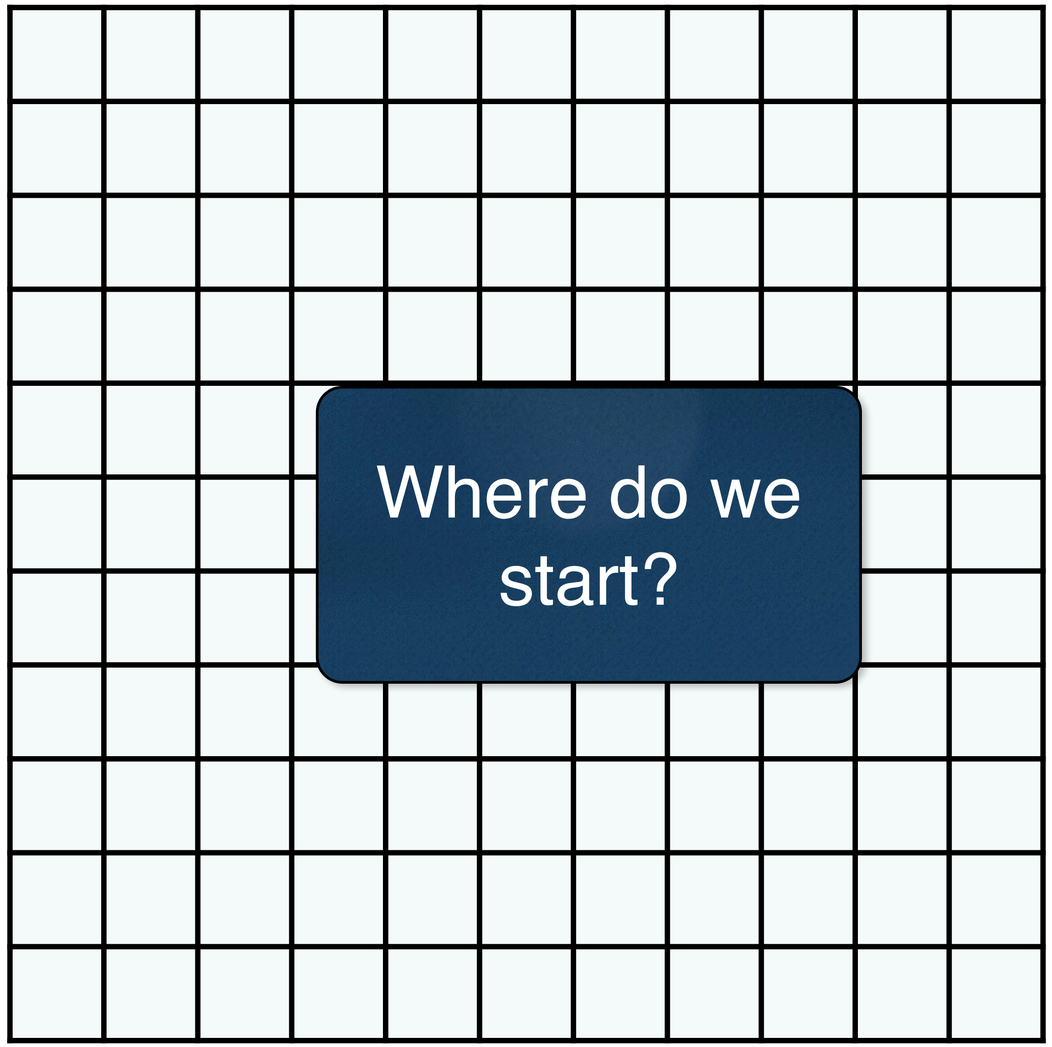
Searching in constraint programming

- variable/value labeling
- focusing on the objective
- value/variable labeling
- domain splitting
- symmetry breaking during search
- randomization and restarts

Randomization and restarts

- Sometimes there is no (obvious) search ordering
  - but there exists some good ones
- How to find them?
  - brute force
  - randomization and restarts
- Key idea
  - try a random ordering
  - if no solution is found after some limit, restart the search

The magic square problem



Randomization and restarts

- Key idea
  - apply a heuristic but with randomization
    - e.g., one of the three best variables
  - limit the time in the search
  - if the limit is reached, restart and possibly increase the limit

The magic square problem

```
using {
 timeLimit = 10;
  repeat {
 limitTime(timeLimit) {
    var{int}[] x = all(i in R,j in R) s[i,j];
    range V = x.getRange();
    while (!bound(x)) {
      selectMin[3](i in V:!x[i].bound())(x[i].getSize()) {
        int mid = (x[i].getMin()+x[i].getMax())/2;
        try x[i] <= mid; | x[i] > mid;
      }
    }
  }
  onFailure {
    timeLimit = 1.1 * timeLimit;
  }
}
```

Citations

Wolfsburg - Volkswagen Assembly Line (http://www.flickr.com/photos/24736216@N07/2994043188/) by Roger Wollstadt (http://www.flickr.com/photos/24736216@N07/) CC BY-SA 2.0 (http://creativecommons.org/licenses/by-sa/2.0/deed.en)

Geely assembly line in Beilun, Ningbo (http://commons.wikimedia.org/wiki/File:Geely_assembly_line_in_Beilun,_Ningbo.JPG) by Siyuwj (Own work) [CC-BY-SA-3.0 (http://creativecommons.org/licenses/by-sa/3.0)], via Wikimedia Commons

66ème Festival de Venise (Mostra)(http://www.flickr.com/photos/nicogenin/3902889281/) by Nicolas Genin (http://www.flickr.com/photos/nicogenin/) CC BY-SA 2.0 (http://creativecommons.org/licenses/by-sa/2.0/deed.en)

Julia Roberts 2011 Shankbone (http://www.flickr.com/photos/shankbone/5656729480/) by David Shankbone (http://www.flickr.com/people/shankbone/) CC BY 2.0 (http://creativecommons.org/licenses/by/2.0/deed.en)

Pride and Prejudice_004 (http://www.flickr.com/photos/tonyshek/7271339306/) by Tony Shek (http://www.flickr.com/photos/tonyshek/) CC BY-SA 2.0 (http://creativecommons.org/licenses/by-sa/2.0/deed.en)

Clive Owen, (http://www.flickr.com/photos/oneras/251106343/) by Mario Antonio Pena Zapateira (http://www.flickr.com/photos/oneras/) CC BY-SA 2.0 (http://creativecommons.org/licenses/by-sa/2.0/deed.en)