Discrete Optimization

Local Search: Part VI

Goals of the lecture

- Escaping local minima
- Connectivity

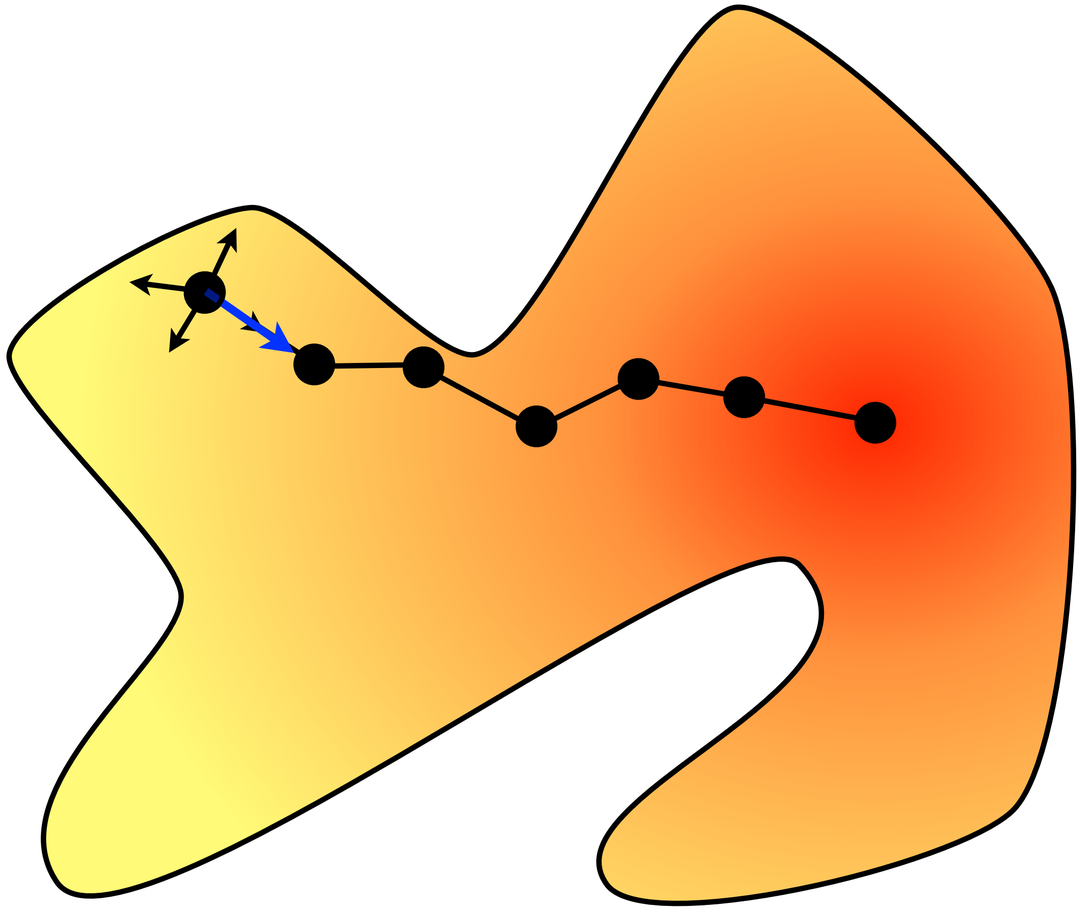
If you want guarantees, buy a toaster (C. Eastwood)

- Local minima
  - a configuration c is a local minima with respect to neighborhood N if

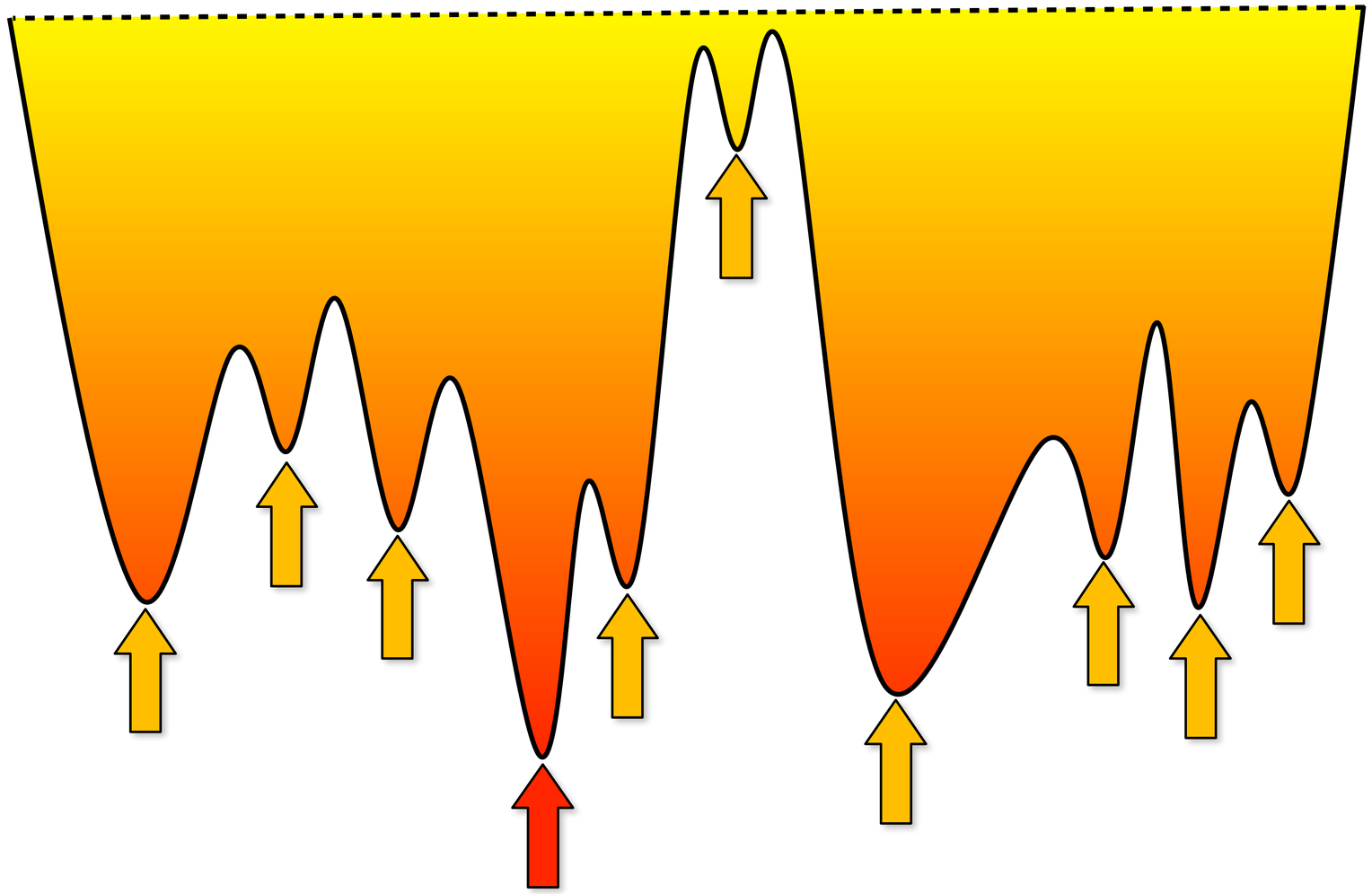
$$\forall\, n \in N(c) : f(n) \geq f(c)$$

- No guarantees for global optimality
  - escaping local optima is a critical issue in local search

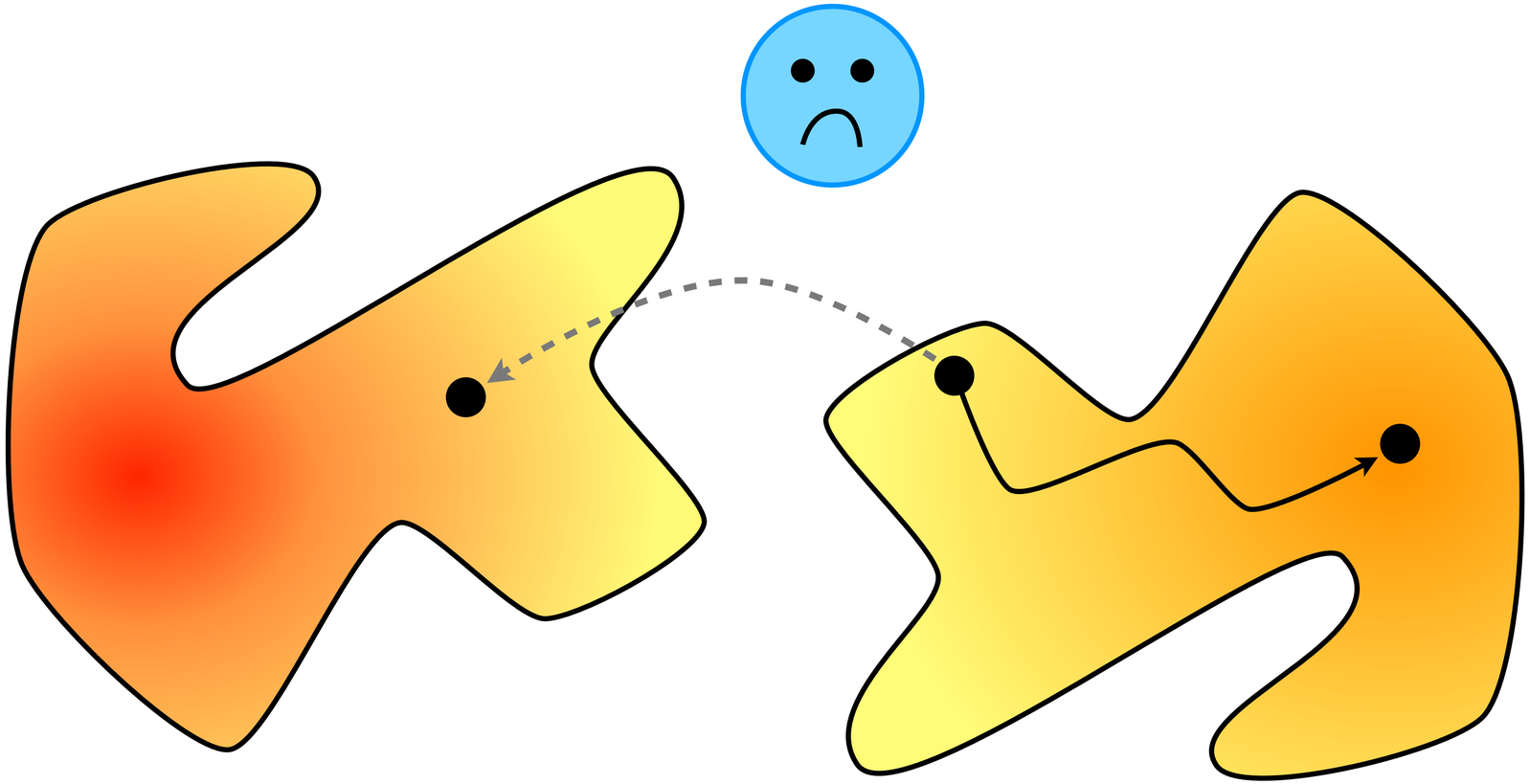
Local Search



How to Find High-Quality Minima?



How to Find High-Quality Minima?



Connectivity

- A neighborhood N is connected if, from every configuration S, some optimal solution O can be reached by a sequence of moves, i.e.,

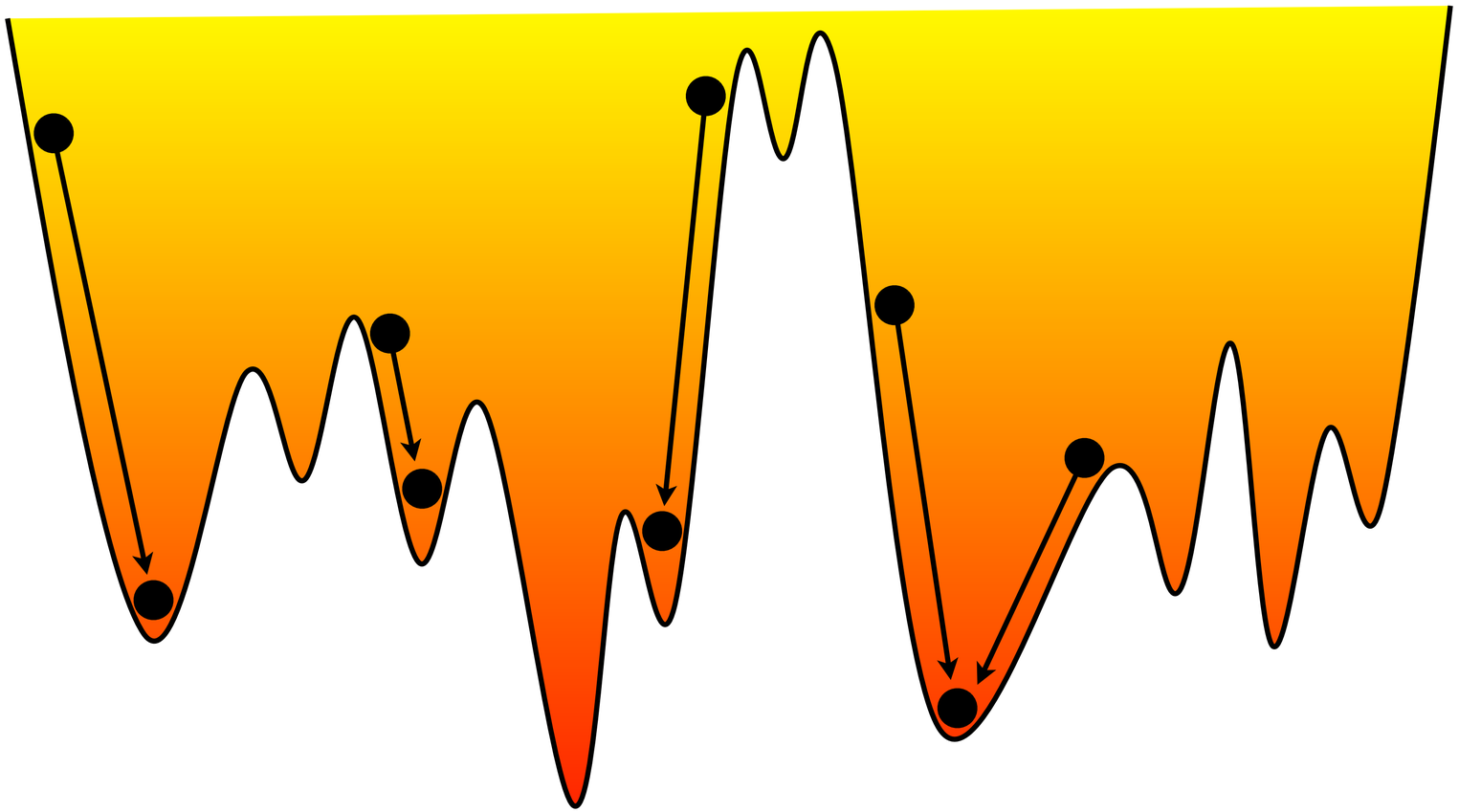
$$S = S_0 \rightarrow S_1 \rightarrow S_2 \rightarrow .... \rightarrow S_n = O$$

where

$$S_i \in N(S_{i-1}).$$

Connectivity

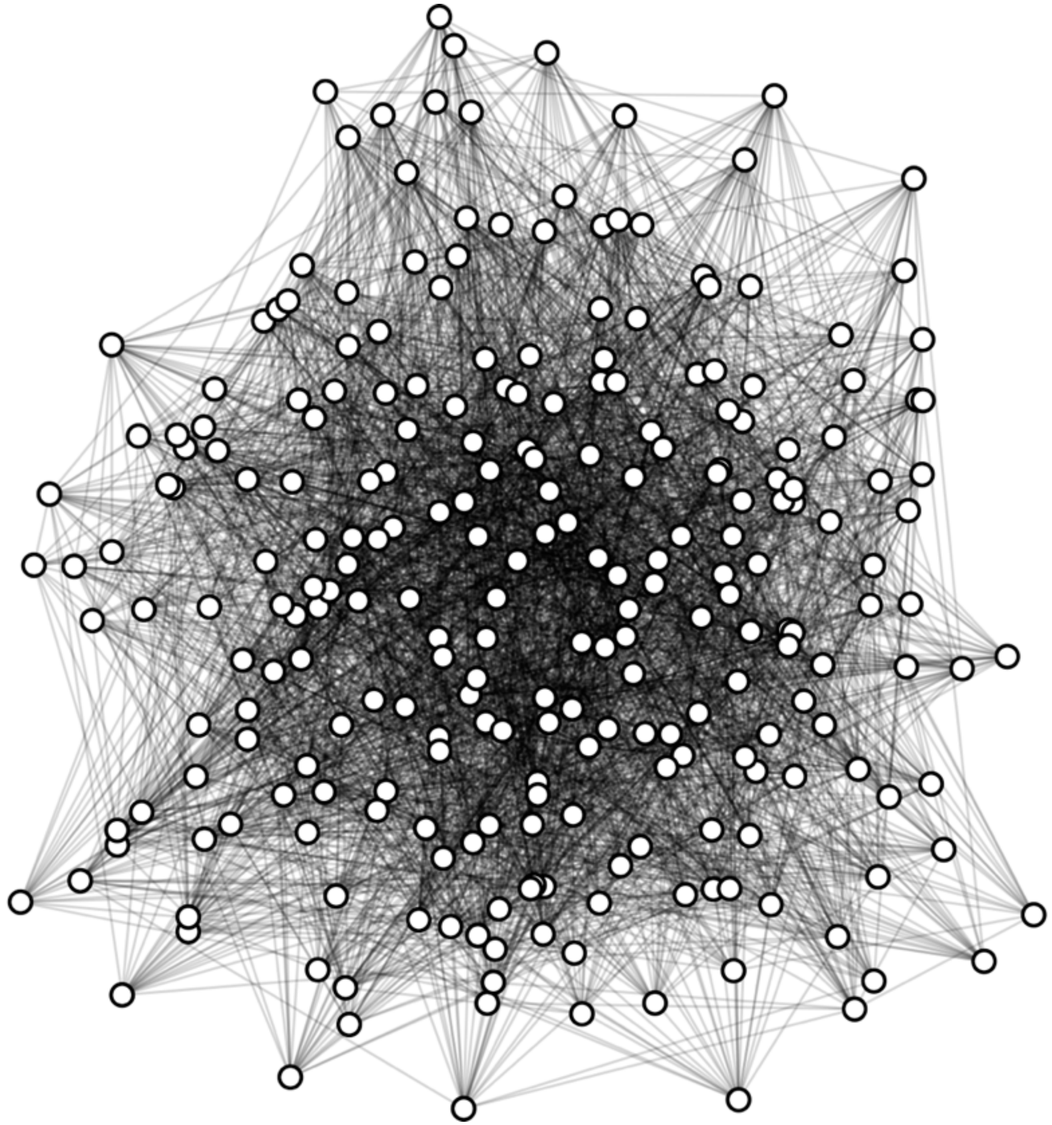
- Connectivity does not guarantee optimality
  - our local searches have been greedy



Connectivity

- Connectivity does not guarantee optimality
  - our local searches have been greedy

Graph Coloring



Graph Coloring

- Neighborhood
  - Change the color of a node
- The neighborhood is connected
  - simple algorithm
  - S_n is the color of node n in configuration S
  - O is the optimal configuration

```
S := some configuration
for each node n
  if Sn != On
    Sn := On
```

Car Sequencing

- Cars on an assembly line
- Cars require specific options
  - leather seats, moonroof
- Capacity constraints on the production units
  - at most 2 out of 5 successive cars can require a moonroof
- Sequence all the cars such that the capacity constraints are satisfied

Car Sequencing

Slots	1	2	3	4	5	6	7	8	9	10	Demand
Class 1	■										1
Class 2		■									1
Class 3			■	■							2
Class 4					■	■					2
Class 5							■	■			2
Class 6									■	■	2

Options	1	2	3	4	5	Demand
Class 1	yes		yes	yes		1
Class 2				yes		1
Class 3		yes			yes	2
Class 4		yes		yes		2
Class 5	yes		yes			2
Class 6	yes	yes				2
Capacity	1/2	2/3	1/3	2/5	1/5	

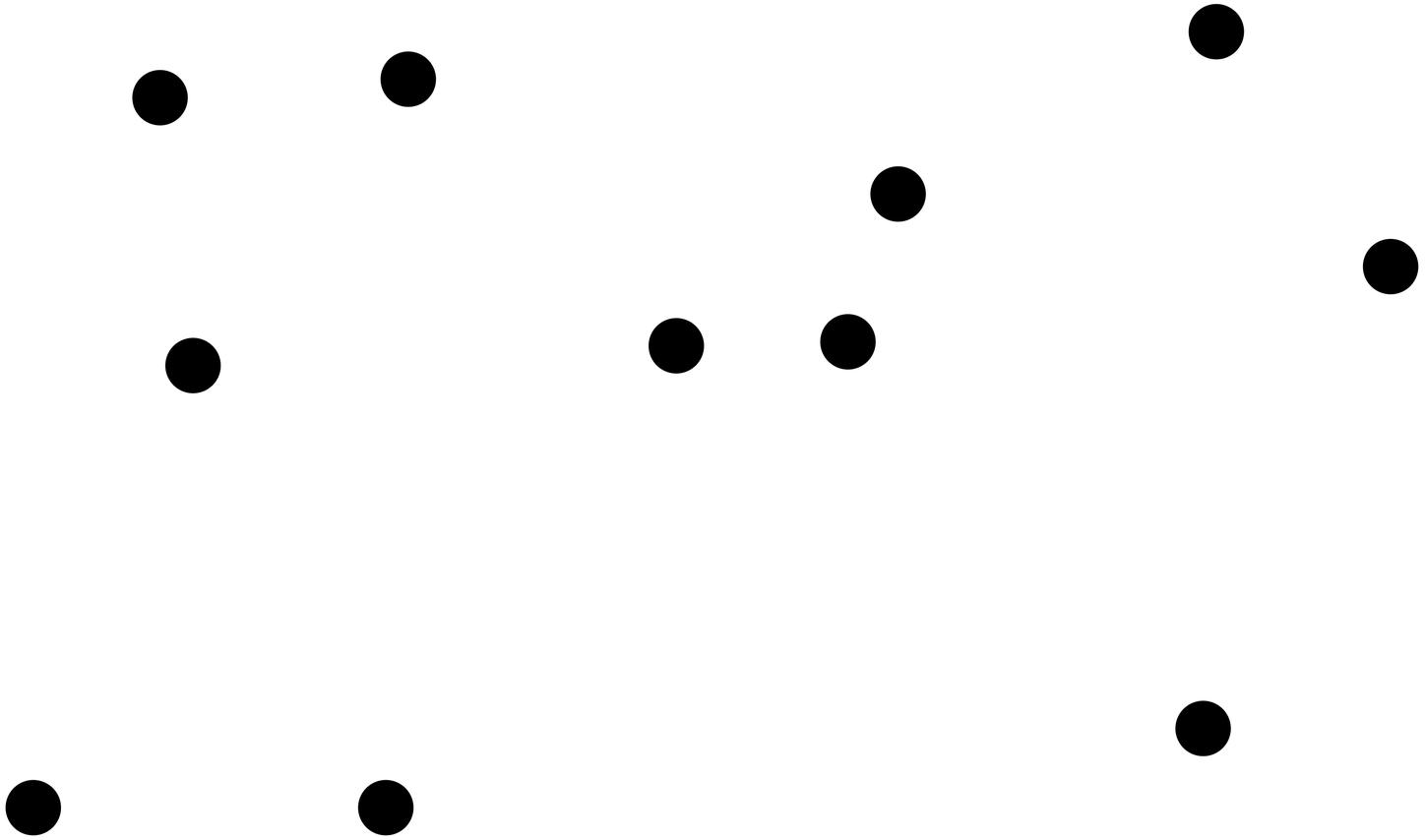
Setup	1	2	3	4	5	6	7	8	9	10	Capacity	
Option 1	■						✕	✕	✕	✕	1/2	3
Option 2			✕	✕	✕	✕			■	■	2/3	2
Option 3	■						✕	✕			1/3	2
Option 4	✕	✕			✕	✕					2/5	2
Option 5			✕	✕							1/5	3

The Swap Neighborhood is Connected

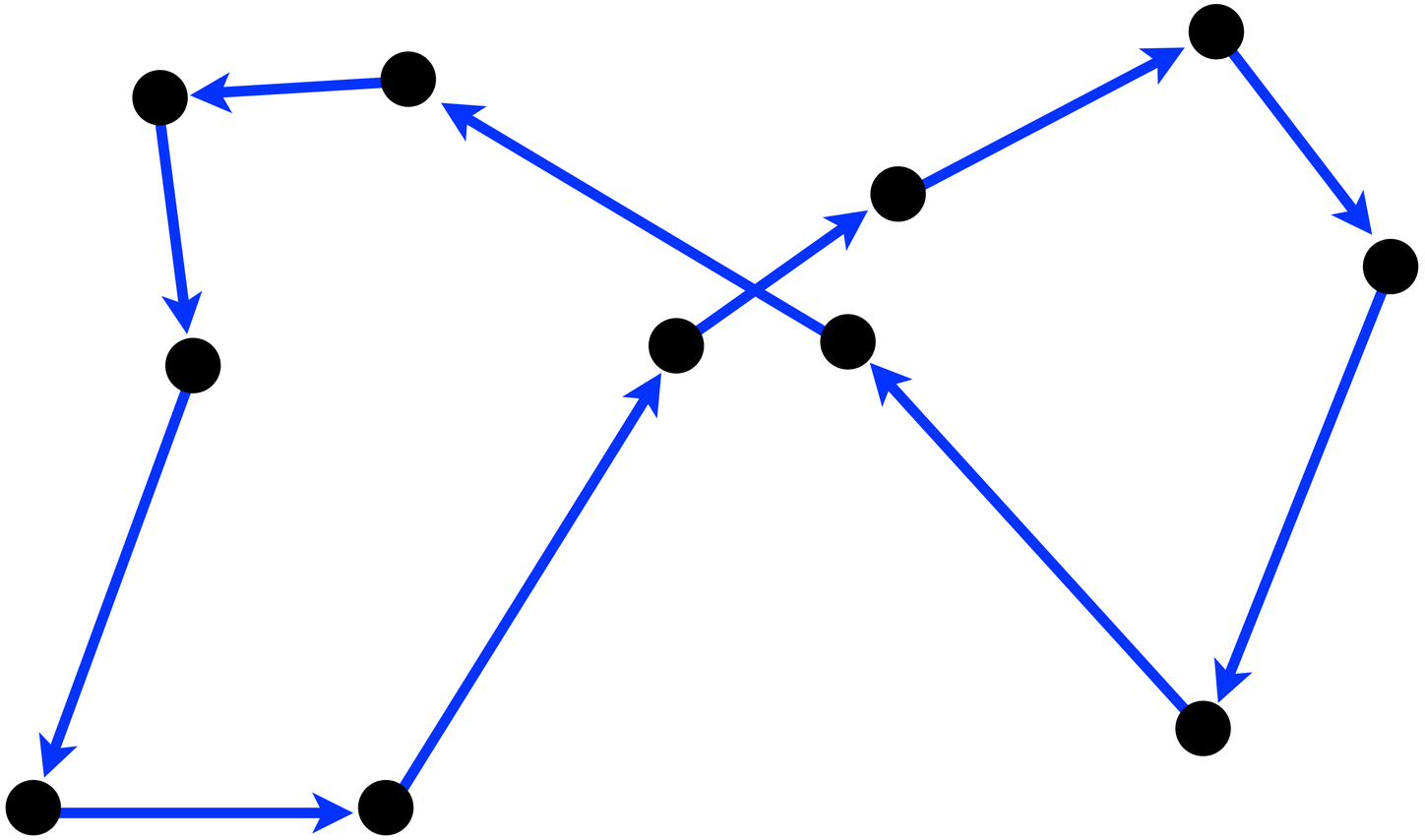
- Neighborhood
  - Swap two slots in the assembly
- The neighborhood is connected
  - simple algorithm

```
S := some configuration
for(int i = 1; i <= n; i++)
   if (Si != Oi)
      let Sj = Oi (j > i)
          Si <-> Sj
```

Traveling Salesman Problem

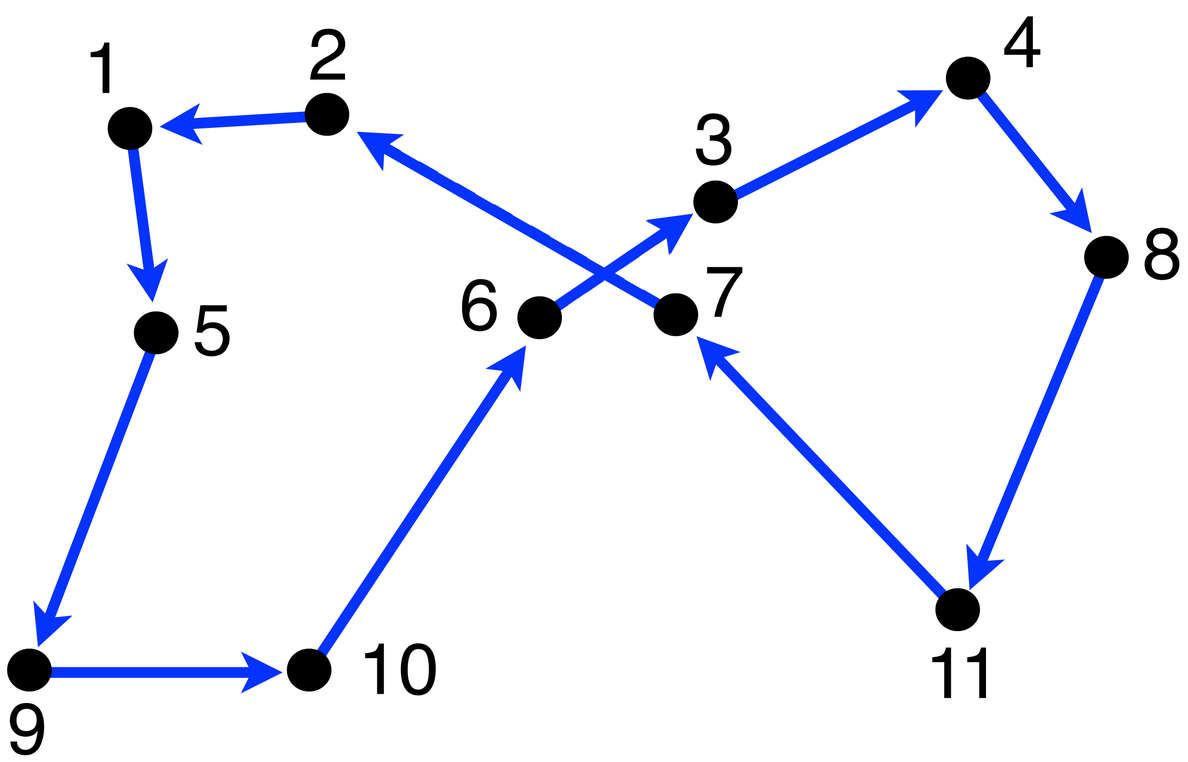


2-OPT

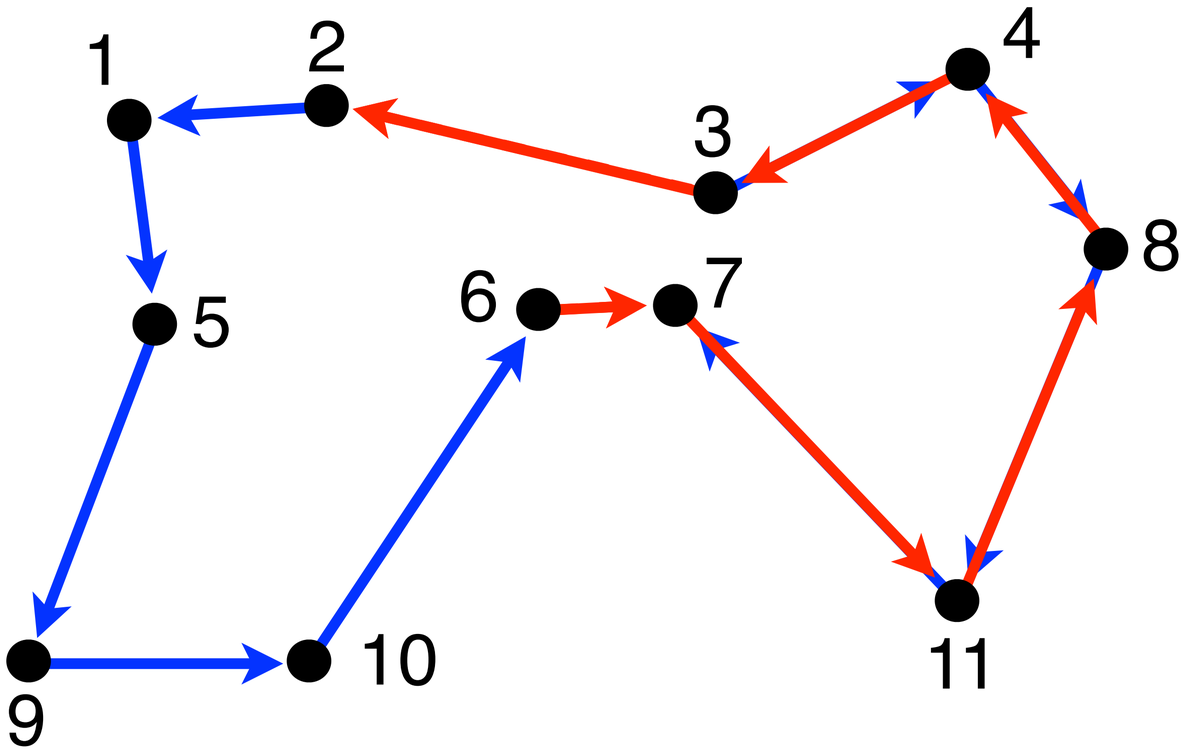


Is 2-OPT Connected?

2-OPT



1 - 5 - 9 - 10 - 6 - 3 - 4 - 8 - 11 - 7 - 2



1 - 5 - 9 - 10 - 6 - 7 - 11 - 8 - 4 - 3 - 2

Is 2-OPT Connected?

Is 2-OPT Connected?

- The neighborhood is connected
  - simple algorithm

```
T := some tour
for(int i = 1; i <= n; i++)
   if (Ti != Oi)
     find Si,...,Si+k such that Si+k = Oi
     S := (S1,...,Si-1,Si+k,Si+k-1,...,Si,Si+k+1,...,Sn)
```

Is the TPP Neighborhood Connected?

- Call me if you have the proof

Until Next Time

Citations

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