AI - FOUNDATION AND APPLICATION

Instructor:
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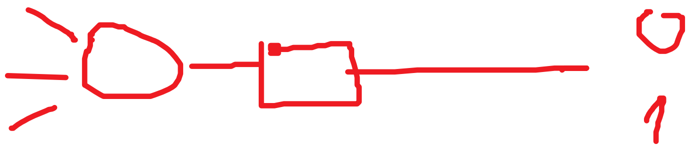
Chapter 2
Back Propagation

Outline

binary classification



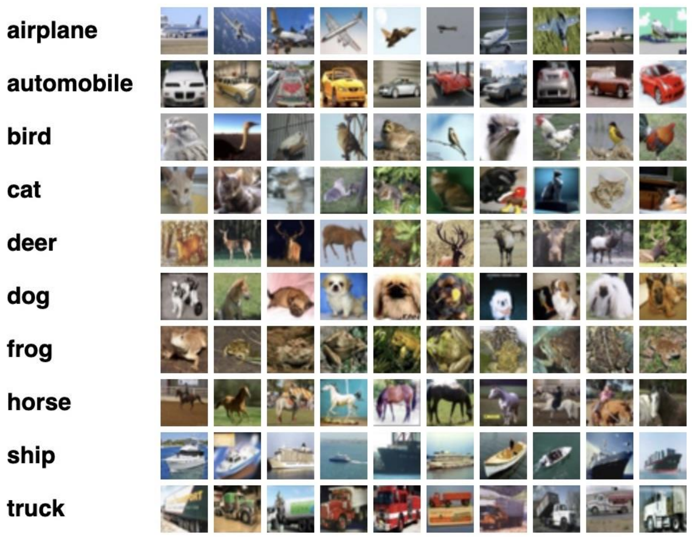
- ❑ Multiclass Classification With Softmax regression
- ❑ Lost function – cross entropy
- ❑ Stochastic gradient descent – batch and mini-batch gradient descent
- ❑ Translating math into code

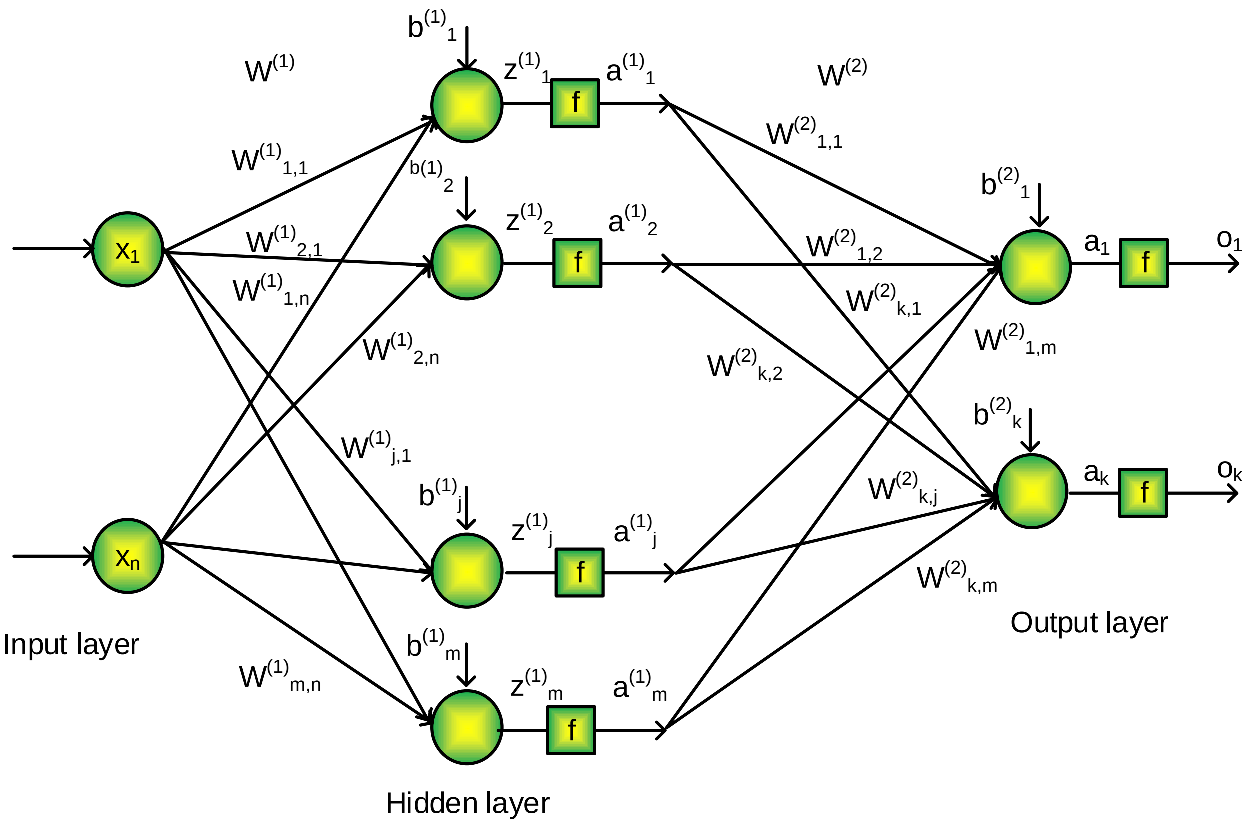
Multiclass Classification With Softmax regression

Multiclass classification example

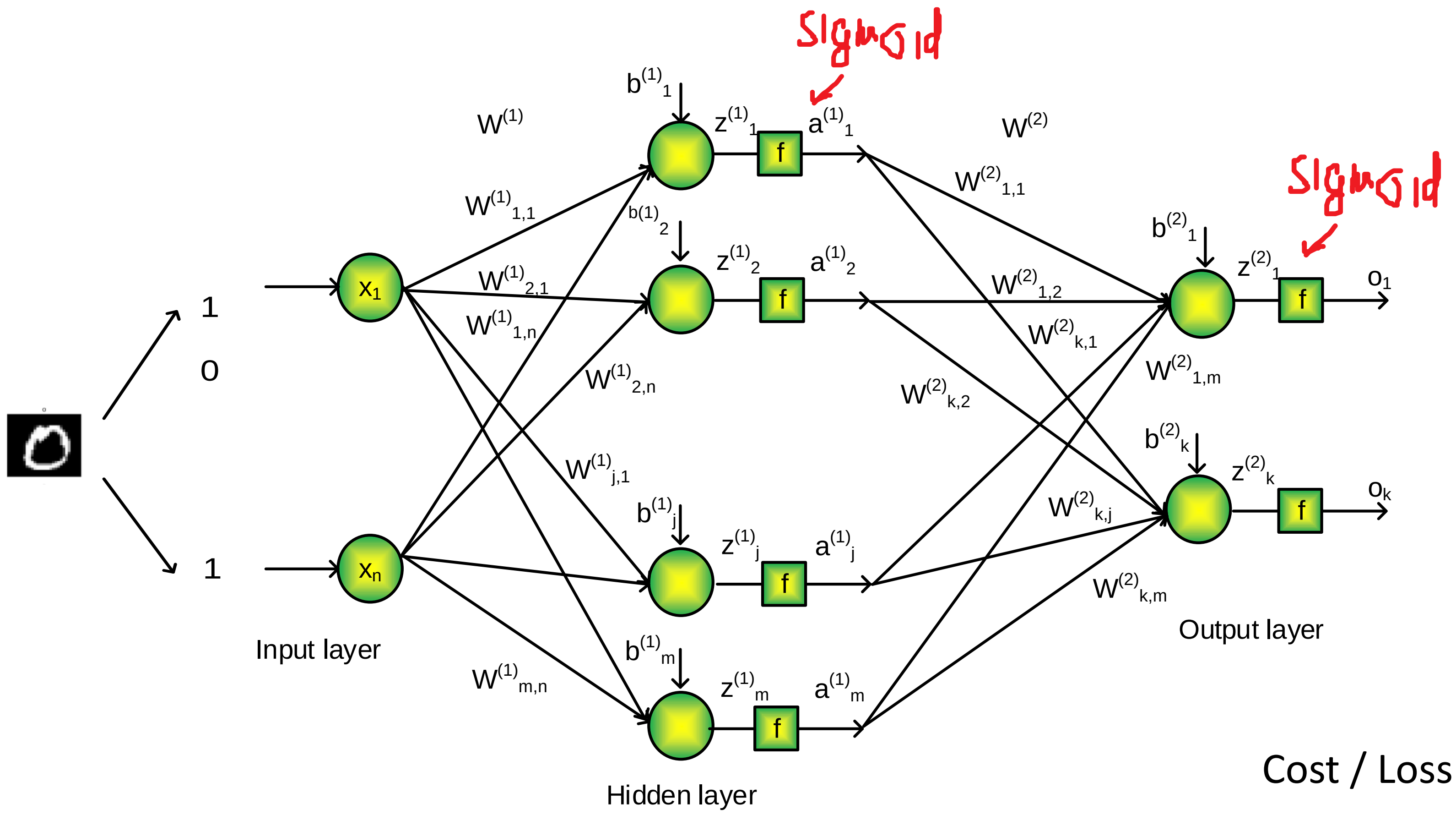
10 o/p

32×32





Multiclass classification example



Predictive output,o	Desired output, y
0.9	1
0.7	0
0.5	0

N samples, K outputs

clas

Cost / Loss $L = \frac{1}{\mathrm{N}} \sum_{t=1}^{\mathrm{N}} \sum_{k=1}^{\mathrm{K}} (y_k^t - o_k^t)^2$

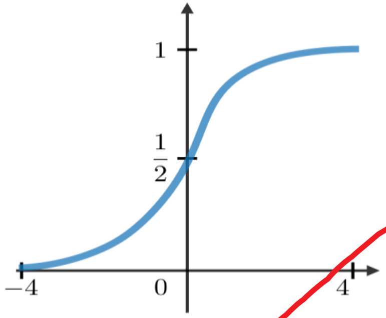
sample

MSE

Multiclass classification with logistic regression

Sigmoid function is used for the neurons at the output layer

$$\boxed{\sigma(x) = \frac{1}{1 + e^{-x}}}$$



- It is ideally for two-class classification
- The outputs are independent
- The sigmoid may produces high probability for all classes, some of them, or none of them

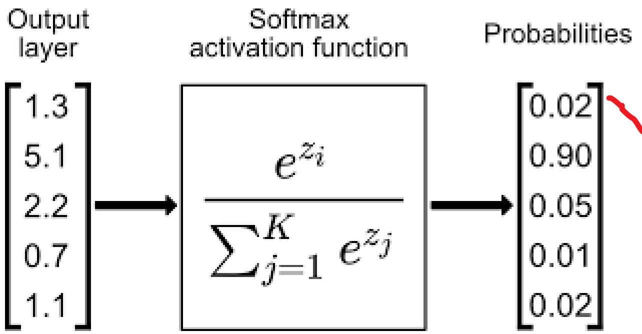
0.9	0.9	0.3
0.8	0.2	0.2
0.6	0.5	0.1

$$\frac{0,9}{0,9 + 0,8 + 0,6}$$

Multiclass classification with softmax regression

We expect that there is only one right answer, the outputs are mutually exclusive.

$$\sigma(z)_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}}$$



- ❑ The softmax will enforce that the sum of the probabilities of output classes are equal to one
- ❑ Softmax is used for multi-classification in the Logistic Regression model, whereas Sigmoid is used for binary classification in the Logistic Regression model

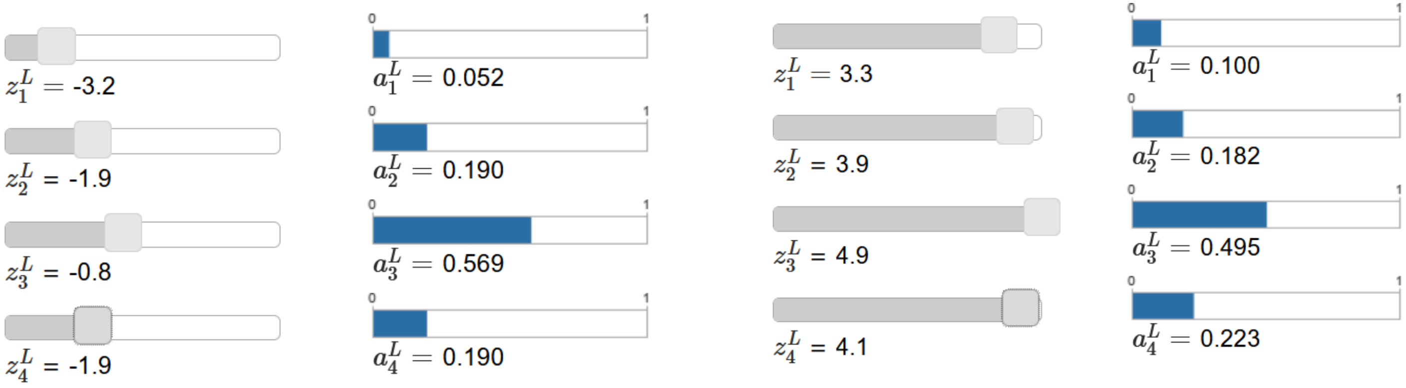
$$\frac{e^{1.3}}{e^{1.3}+e^{5.1}+e^{2.2}+e^{0.7}+e^{1.1}}$$

Multiclass classification with softmax regression

- Softmax function is mostly used in a final layer of Neural Network
- The outputs are probability distribution

sigmoid: hidden layer - làm ngõ ra độc lập vs nhau

$$\sigma(z)_j = \frac{e^{z_j}}{\sum_{k=1}^{K} e^{z_k}}$$



Lost function – cross entropy

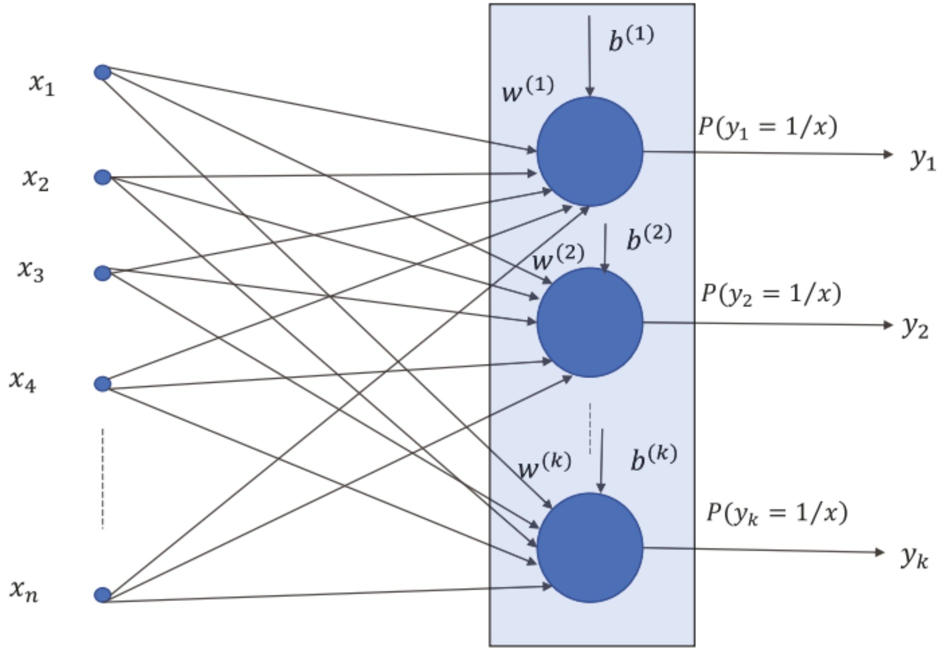
Cross-entropy takes the negative log likelihood of the predicted probability

Cross-entropy loss

$$Loss = -\sum_{i=1}^{M} y_i \log(o_i)$$

$$P(y_i = 1/x) = \frac{e^{w^{(i)T}x + b^{(i)}}}{\sum_{j=1}^{k} e^{w^{(j)T}x + b^{(j)}}}$$

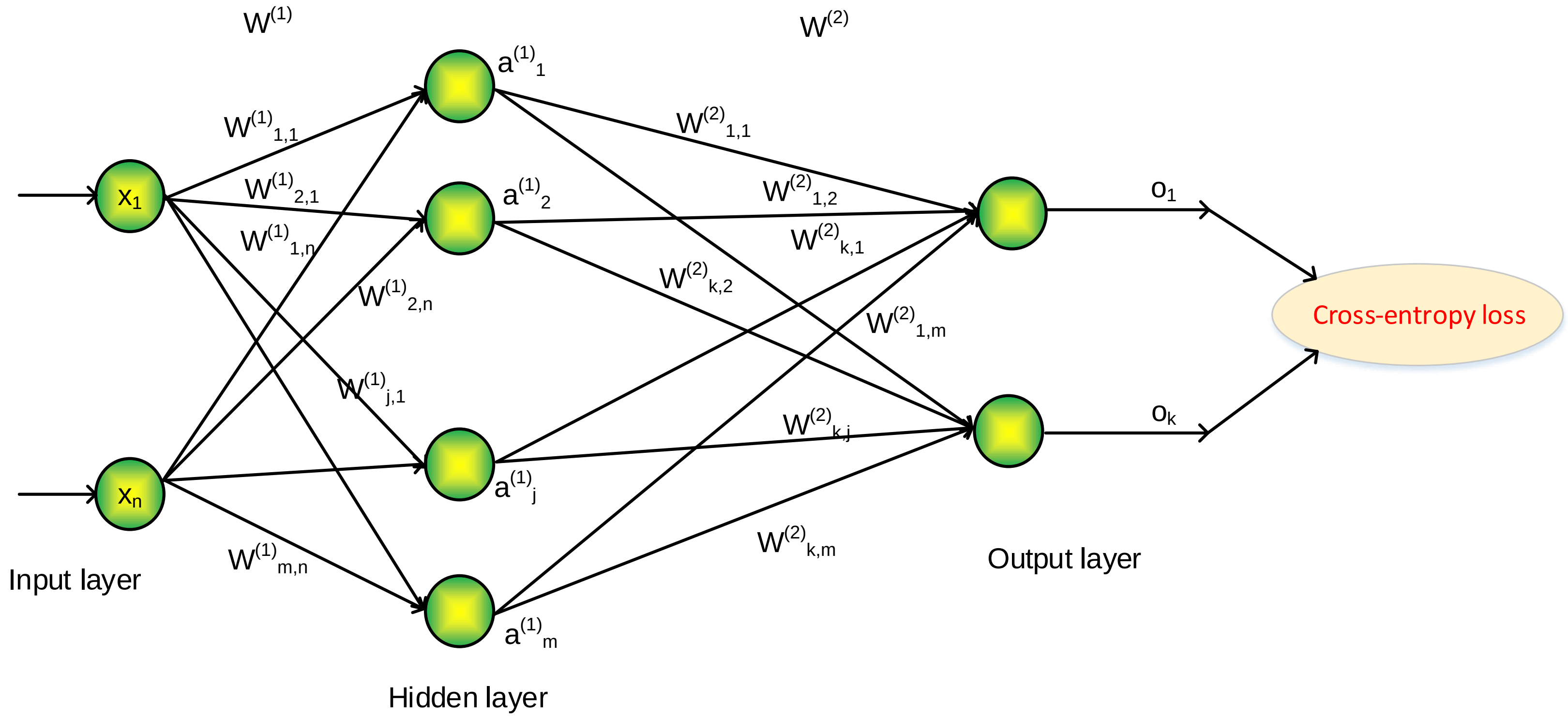
$$C = \sum_{i=1}^{k} -y_i \log P(y_i = 1/x)$$



M – number of classes
y: class label
o: predicted probability observation

Back propagation

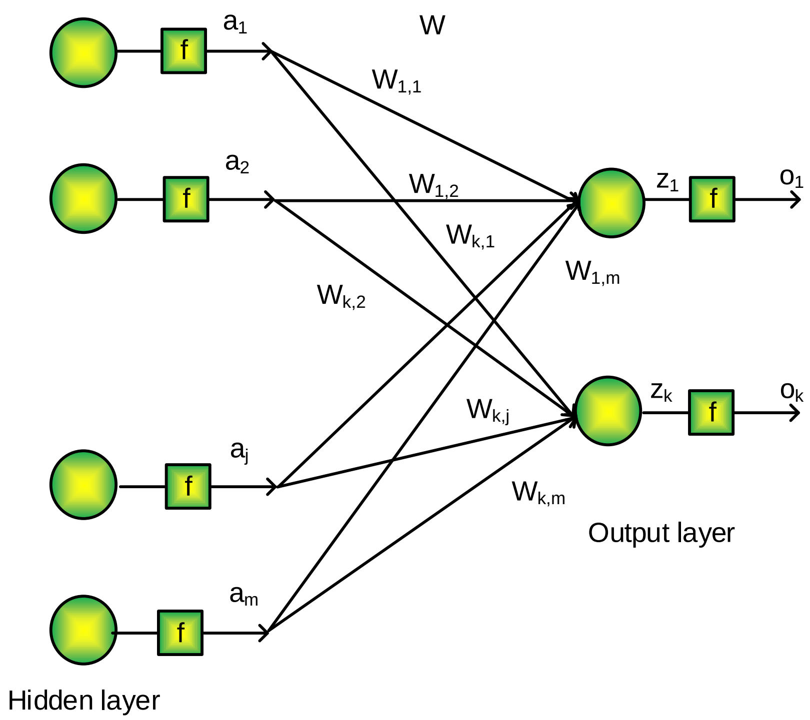
Feedforward propagation with softmax activation



$$Loss = -\sum_{k=1}^{K} y_k \log(o_k)$$

$$z_j^{(1)} = \sum_{i=1}^{M} x_i w_{j,i}^{(1)} + b_j^{(1)} \qquad a_j^{(1)} = \frac{1}{1+e^{-z_j^{(1)}}} \qquad z_k^{(2)} = \sum_{j=1}^{K} a_j^{(1)} w_{k,j}^{(2)} + b_k^{(2)} \qquad o_k = \frac{e^{z_k}}{\sum_{j=1}^{K} e^{z_j^{(2)}}}$$

Feedforward propagation with softmax activation



Gradient descent $$w_{k,j} = w_{k,j} - \eta \frac{\partial L}{\partial w_{k,j}}$$

$$L(y, o) = -\sum_{k=1}^{K} y_k \log(o_k)$$

$$o_k = \frac{e^{z_k}}{\sum_{j=1}^{K} e^{z_j^{(2)}}}$$

Apply the chain rule

$$\frac{\partial L}{\partial w_{k,j}} = \frac{\partial L}{\partial z_k} \frac{\partial z_k}{\partial w_{k,j}}$$

$$\frac{\partial L}{\partial z_k} = \frac{\partial L}{\partial o_k} \frac{\partial o_k}{\partial z_k}$$

Error of k[th] neuron of the output layer $$\delta_k = \frac{\partial L}{\partial z_k}$$

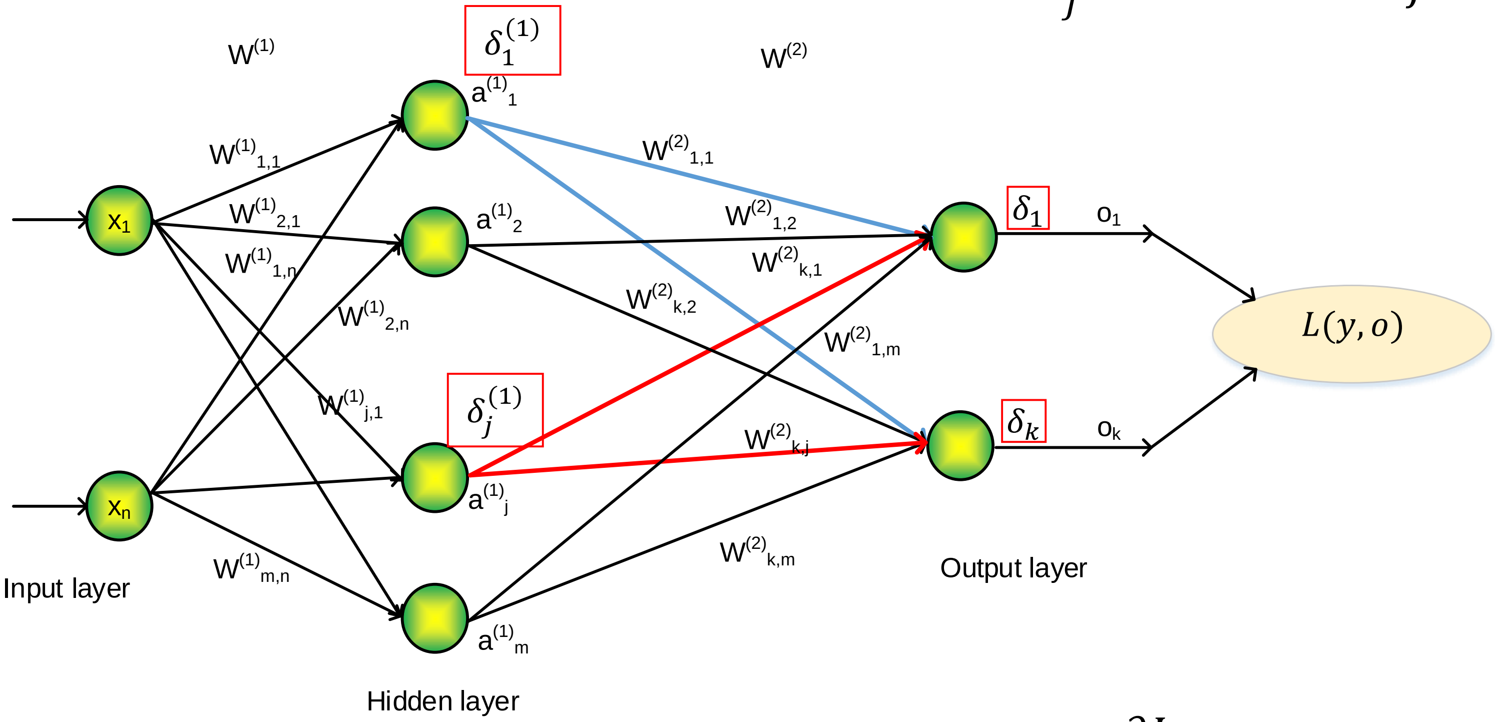
Using calculus, we obtain $$\frac{\partial L}{\partial z_k} = o_k - y_k$$

$$\delta_k = o_k - y_k$$

o_k *is the kth component of the neuron's prediction and* y_k *is the kth component of the label*

Back-propagation

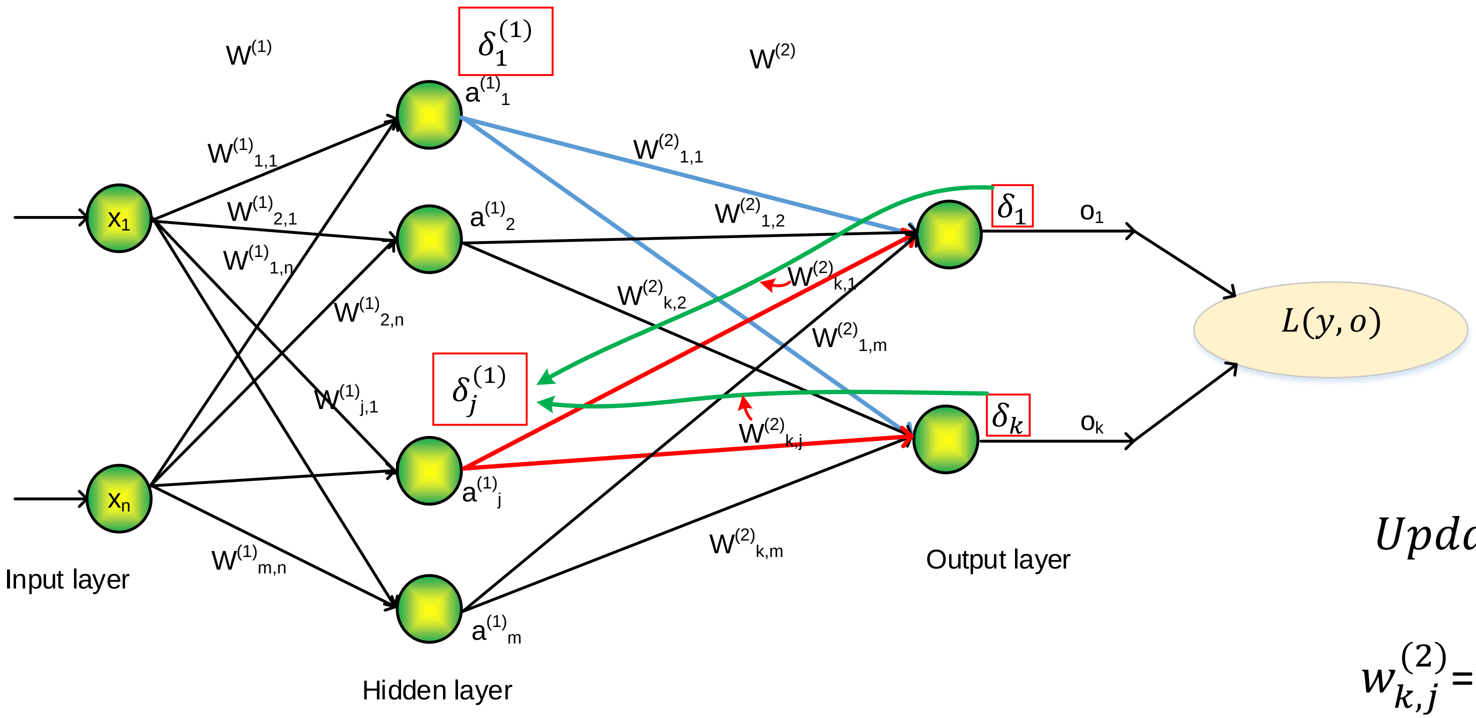
$\delta_j^{(1)}$ *is the error of the ith neuron in hidden layer*



$$\frac{\partial L}{\partial a_j^{(1)}} = \delta_j^{(1)}$$

$\delta_j^{(1)}$affects all neurons of next layer via the weights

Back-propagation



Back propagate error

$$\delta_1^{(1)} = \delta_1 w_{1,1}^{(2)} + \cdots + \delta_k w_{k,1}^{(2)}$$

$$\delta_j^{(1)} = \sum_{k=1}^{K} \delta_k w_{k,j}^{(2)}$$

Back propagate through sigmoid function

$$\frac{\partial L}{\partial z_j^{(1)}} = \delta_j^{(1)} a_j^{(1)} (1 - a_j^{(1)})$$

Update weights $\quad w_{j,i} = w_{j,i} - \eta \frac{\partial L}{\partial w_{j,i}}$

$$w_{k,j}^{(2)} = w_{k,j}^{(2)} - \eta \frac{\partial L}{w_{k,j}^{(2)}} = w_{k,j}^{(2)} - \eta \delta_k \frac{\partial \delta_k}{\partial w_{k,j}^{(2)}}$$

$$w_{k,j}^{(2)} = w_{k,j}^{(2)} - \eta \delta_k a_j^{(1)}$$

$$\delta_k = o_k - y_k$$

$$\delta_j^{(1)} = \sum_{k=1}^{K} \delta_k w_{k,j}^{(2)}$$

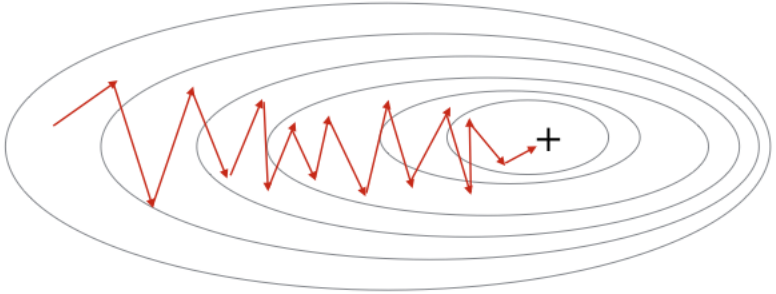
$$w_{j,i}^{(1)} = w_{j,i}^{(1)} - \eta \delta_j^{(1)} a_j^{(1)} (1 - a_j^{(1)}) x_i$$

you should spend time to master them

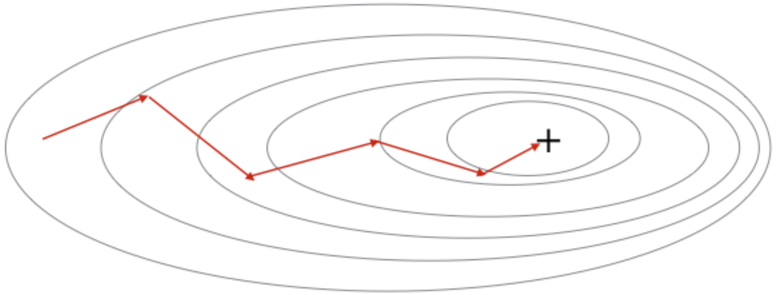
Stochastic gradient descent – batch and mini-batch gradient descent

- Stochastic gradient descent (SGD): use 1 sample in each iteration
- Batch gradient descent (GD): use all samples in each iteration
- Mini-batch gradient descent (Mini-batch GD): use *b* sample in each iteration, in this case, *b* is the batch size
- Mini-batch stochastic gradient descent (Mini-batch SGD): use *b* sample in each iteration, the batch of training samples is randomly selected

Stochastic Gradient Descent



Mini-Batch Gradient Descent

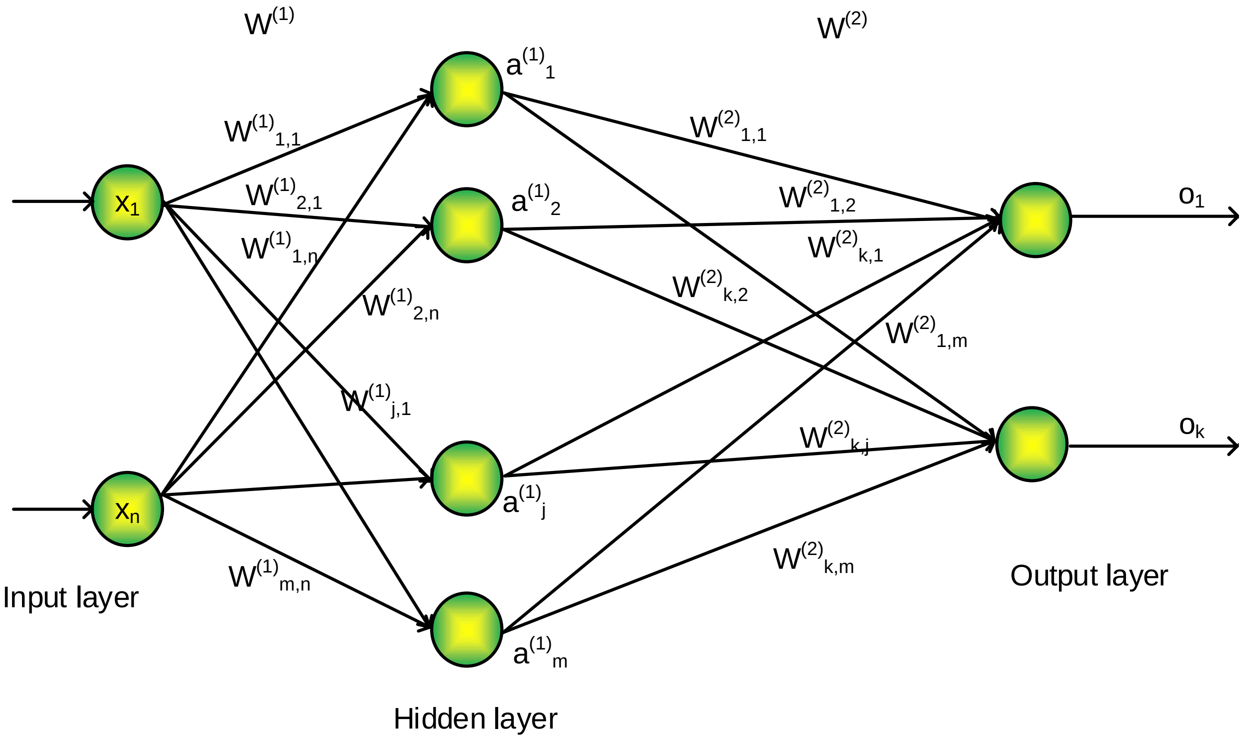


PYTHON CODE

Translating Math into Code

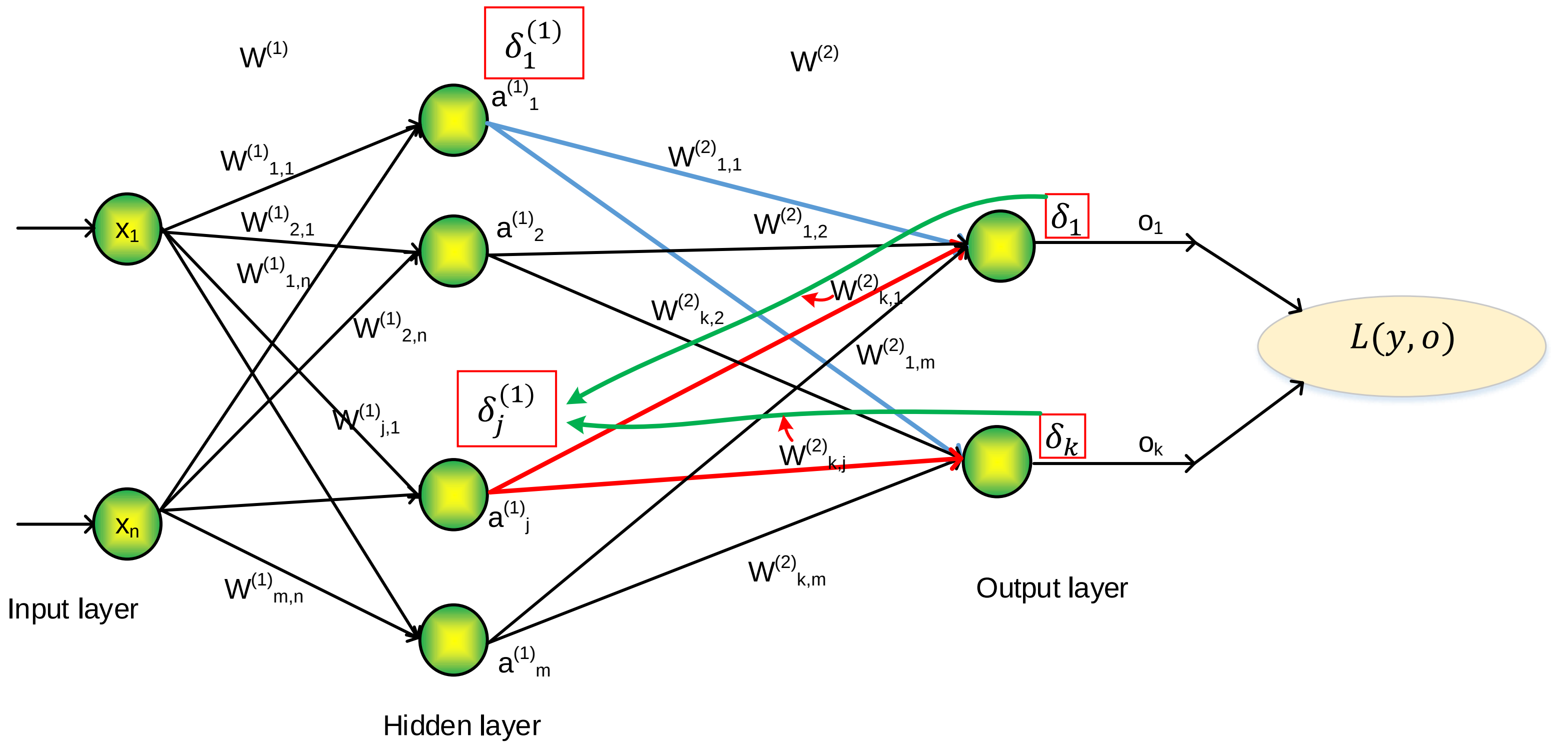
Translating mathematics into code

MNIST Dataset



Feed forward propagation

$$z2 = z^1$$
$$Wh = W^{(1)}$$
$$Wo = W^{(2)}$$



$$Define:$$
$$z2 = z^1$$
$$Wh = W^{(1)}$$
$$Wo = W^{(2)}$$

$$Fordward$$
$$a = XWh^T$$
$$z1 = \sigma(z1) = \frac{1}{1 + e^{-z1}} \qquad z2 = XWo^T \qquad o_k = \frac{e^{z2_k}}{\sum_{j=1}^{K} e^{z2_k}}$$

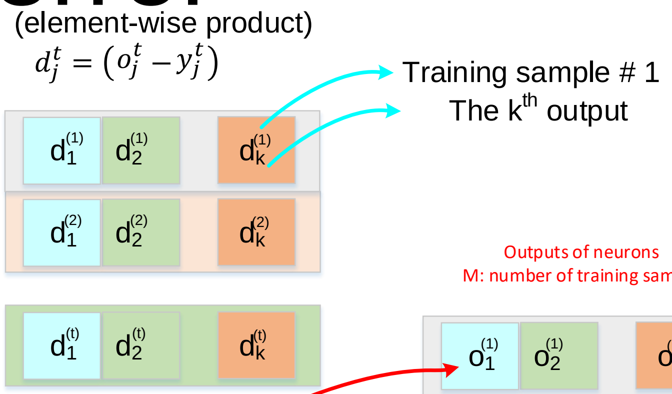
Back-propagation error

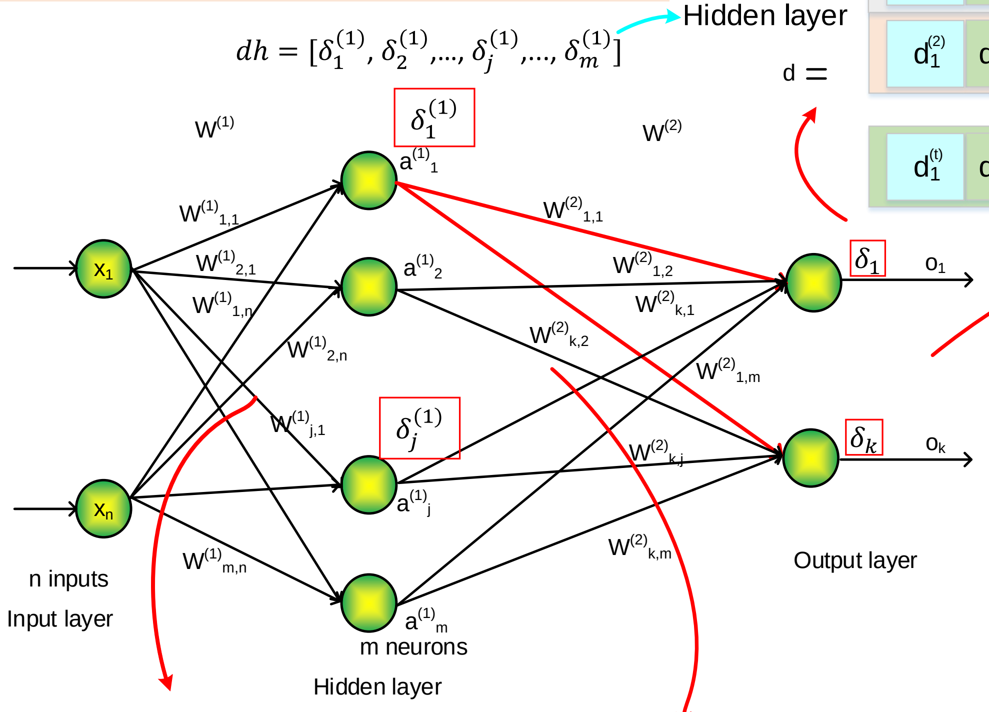
Parameters
n: number of inputs
m: number of neurons in hidden layer
k: number of neurons in output layer
t: number of training sample (batch_size)

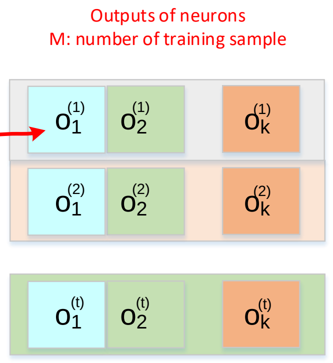
Back propagate error

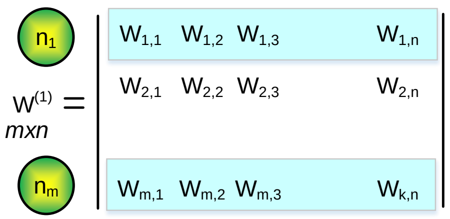
$$\delta_1^{(1)} = \delta_1 w_{1,1}^{(2)} + \cdots + \delta_k w_{k,1}^{(2)}$$

$$\delta_j^{(1)} = \sum_{k=1}^{K} \delta_k w_{k,j}^{(2)}$$

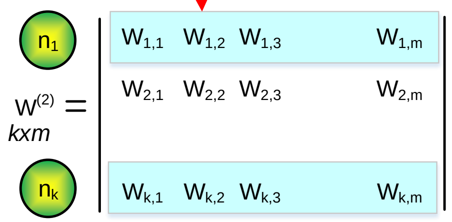




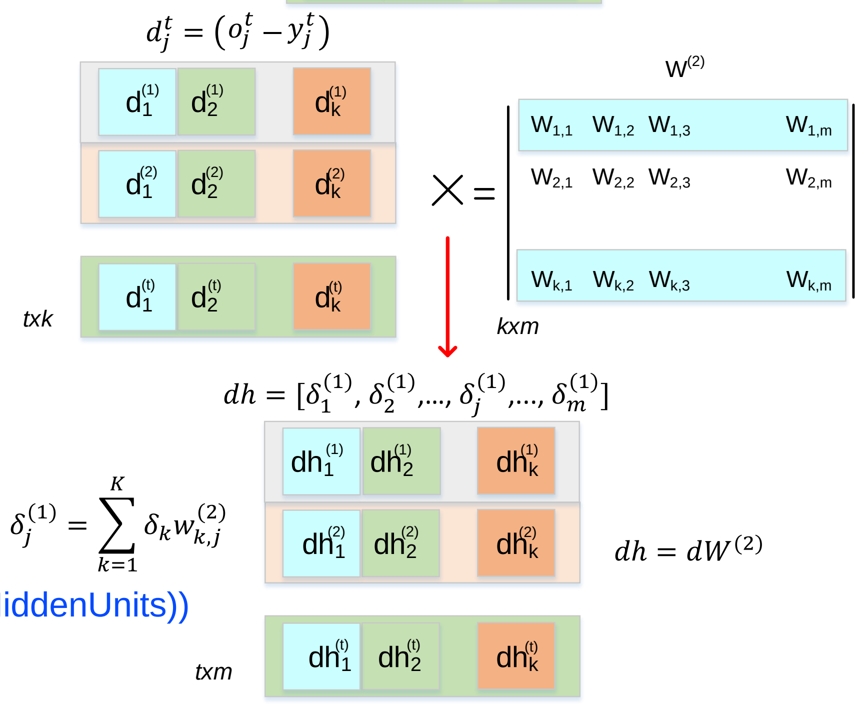




Wh = np.matrix(np.random.uniform
(-0.5,0.5,(NumHiddenUnits,NumInputs)))



Wo = np.random.uniform
(-0.5,0.5,(NumClasses,NumHiddenUnits))



Update weights

output layer

$$L(y,o) = -\sum_{k=1}^{K} y_k \log(o_k)$$

loss.append(-np.sum(np.multiply(y,np.log10(o))))

$d = o - y$ *softmax*   d = o - y

$$\Delta Wo = -\eta \frac{2}{t} d^T a$$ dWo = np.matmul(np.transpose(d),a)

$$Wo = \mathrm{Wo} + \Delta \mathrm{Wo}$$

output layer

back propagate through weights

$dh = dWo$ dh = d@Wo

back propagate through signmoid

$dhs = dh(a(1-a))$

dhs = np.multiply(np.multiply(dh,a),(1-a))

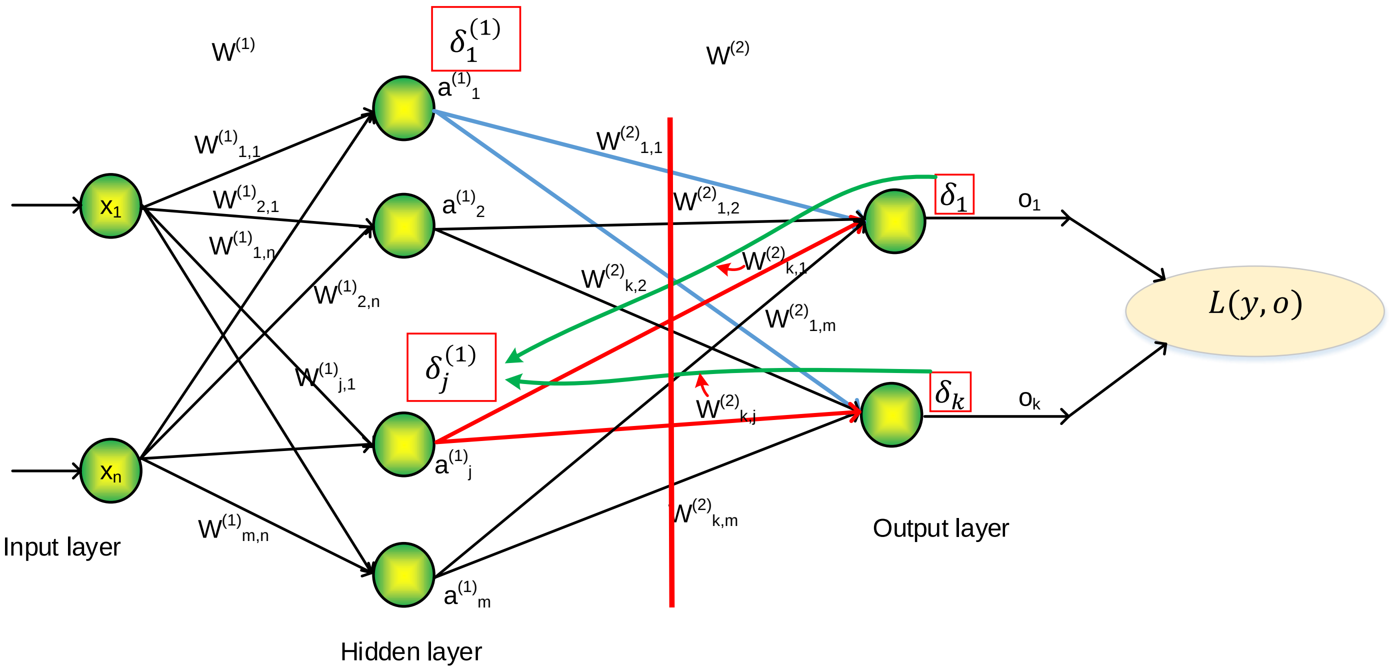
$$\Delta Wo = -\eta \frac{2}{t} dhs^T X$$

t: number of training sample

Update weights

$$w_{j,i} = w_{j,i} - \eta \frac{\partial L}{\partial w_{j,i}}$$

$$w_{k,j}^{(2)} = w_{k,j}^{(2)} - \eta \frac{\partial L}{w_{k,j}^{(2)}} = w_{k,j}^{(2)} - \eta \delta_k \frac{\partial \delta_k}{\partial w_{k,j}^{(2)}}$$



PYTHON CODE

Python code

Load dataset

Batch Gradient Descent

```
import numpy as np
import tensorflow as tf
#load datashet
print("Load MNIST Database")
mnist = tf.keras.datasets.mnist
(x_train,y_train),(x_test,y_test)= mnist.load_data()
x_train=np.reshape(x_train,(60000,784))/255.0
x_test= np.reshape(x_test,(10000,784))/255.0
y_train = np.matrix(np.eye(10)[y_train])
y_test = np.matrix(np.eye(10)[y_test])
print("----------------------------------")
print(x_train.shape)
print(y_train.shape)
```

Python code

Define functions

```
def sigmoid(x):
    return 1./(1.+np.exp(-x))

def softmax(x):
    return np.divide(np.matrix(np.exp(x)),np.mat(np.sum(np.exp(x),axis=1)))

def Forwardpass(X,Wh,bh,Wo,bo):
    zh = X@Wh.T + bh
    a = sigmoid(zh)
    z=a@Wo.T + bo
    o = softmax(z)
    return o
def AccTest(label,prediction):    # calculate the matching score
    OutMaxArg=np.argmax(prediction,axis=1)
    LabelMaxArg=np.argmax(label,axis=1)
    Accuracy=np.mean(OutMaxArg==LabelMaxArg)
    return Accuracy
```

Python code

Define network architecture, initialize weights

```
learningRate = 0.5
Epoch=50
NumTrainSamples=60000
NumTestSamples=10000

NumInputs=784
NumHiddenUnits=512
NumClasses=10
#inital weights
#hidden layer
Wh=np.matrix(np.random.uniform(-0.5,0.5,(NumHiddenUnits,NumInputs)))
bh= np.random.uniform(0,0.5,(1,NumHiddenUnits))
dWh= np.zeros((NumHiddenUnits,NumInputs))
dbh= np.zeros((1,NumHiddenUnits))
#Output layer
Wo=np.random.uniform(-0.5,0.5,(NumClasses,NumHiddenUnits))
bo= np.random.uniform(0,0.5,(1,NumClasses))
dWo= np.zeros((NumClasses,NumHiddenUnits))
dbo= np.zeros((1,NumClasses))
```

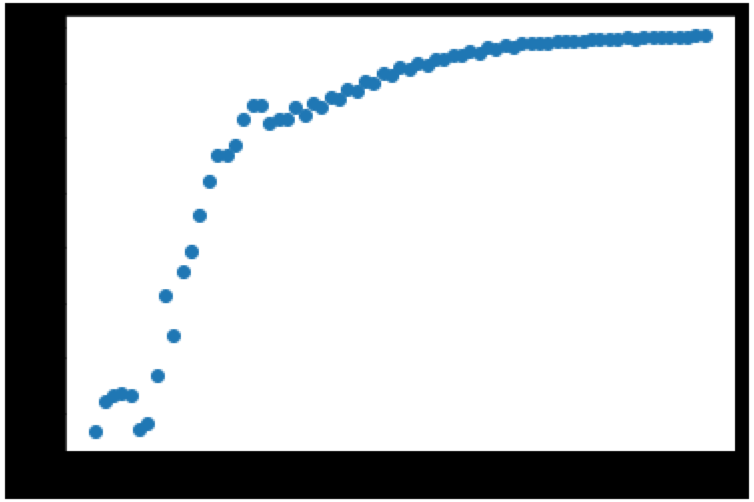
Python code

Batch Gradient Descent

Training the model

```
from IPython.display import clear_output
loss = []
Acc = []
for ep in range (Epoch):
  #feed fordware propagation
  x = x_train
  y=y_train
  zh = x@Wh.T + bh
  a = sigmoid(zh)
  z=a@Wo.T + bo
  o = softmax(z)
  #calculate loss
  loss.append(-np.sum(np.multiply(y,np.log10(o))))
  #calculate the error for the ouput layer
  d = o-y
  #Back propagate error
  dh = d@Wo
  dhs = np.multiply(np.multiply(dh,a),(1-a))
  #update weight
  dWo = np.matmul(np.transpose(d),a)
  dbo = np.mean(d)  # consider a is 1 for bias
  dWh = np.matmul(np.transpose(dhs),x)
  dbh = np.mean(dhs)  # consider a is 1 for bias
  Wo =Wo - learningRate*dWo/NumTrainSamples
  bo =bo - learningRate*dbo
  Wh =Wh-learningRate*dWh/NumTrainSamples
  bh =bh-learningRate*dbh
  #Test accuracy with random initial weights
  prediction = Forwardpass(x_test,Wh,bh,Wo,bo)
  Acc.append(AccTest(y_test,prediction))
  clear_output(wait=True)
  plt.plot([i for i, _ in enumerate(Acc)],Acc,'o')
  plt.show()
```

Python code



Test the model

```
prediction = Forwardpass(x_test,Wh,bh,Wo,bo)
Rate = AccTest(y_test,prediction)
Print(Rate)
```

Python code

Mini-Batch Gradient Descent

Training the model

```
from IPython.display import clear_output
loss = []
Acc = []
Batch_size = 200
Stochastic_samples = np.arange(NumTrainSamples)
for ep in range (Epoch):
  np.random.shuffle(Stochastic_samples)
  for ite in range (0,NumTrainSamples,Batch_size):
    #feed fordware propagation
    Batch_samples = Stochastic_samples[ite:ite+Batch_size]
    x = x_train[Batch_samples,:]
    y=y_train[Batch_samples,:]
    zh = x@Wh.T + bh
    a = sigmoid(zh)
    z=a@Wo.T + bo
    o = softmax(z)
    #calculate loss
    loss.append(-np.sum(np.multiply(y,np.log10(o))))
    #calculate the error for the ouput layer
    d = o-y
    #Back propagate error
    dh = d@Wo
    dhs = np.multiply(np.multiply(dh,a),(1-a))
    #update weight
```

```
#update weight
    dWo = np.matmul(np.transpose(d),a)
    dbo = np.mean(d)  # consider a is 1 for bias
    dWh = np.matmul(np.transpose(dhs),x)
    dbh = np.mean(dhs)  # consider a is 1 for bias
    Wo =Wo - learningRate*dWo/Batch_size
    bo =bo - learningRate*dbo
    Wh =Wh-learningRate*dWh/Batch_size
    bh =bh-learningRate*dbh
    #Test accuracy with random innitial weights
    prediction = Forwardpass(x_test,Wh,bh,Wo,bo)
    Acc.append(AccTest(y_test,prediction))
    clear_output(wait=True)
    plt.plot([i for i, _ in enumerate(Acc)],Acc,'o')
    plt.show()
  print('Epoch:', ep )
  print('Accuracy:',AccTest(y_test,prediction) )
```

Python code

Just calculate the loss and accuracy after each epoch

Mini-Batch Gradient Descent

```
from IPython.display import clear_output
loss = []
Acc = []
Batch_size = 200
Stochastic_samples = np.arange(NumTrainSamples)
for ep in range (Epoch):
  np.random.shuffle(Stochastic_samples)
  for ite in range (0,NumTrainSamples,Batch_size):
    #feed fordware propagation
    Batch_samples =
Stochastic_samples[ite:ite+Batch_size]
    x = x_train[Batch_samples,:]
    y=y_train[Batch_samples,:]
    zh = x@Wh.T + bh
    a = sigmoid(zh)
    z=a@Wo.T + bo
    o = softmax(z)
    #calculate loss
    loss.append(-np.sum(np.multiply(y,np.log10(o))))
    #calculate the error for the ouput layer
    d = o-y
    #Back propagate error
    dh = d@Wo
    dhs = np.multiply(np.multiply(dh,a),(1-a))
```

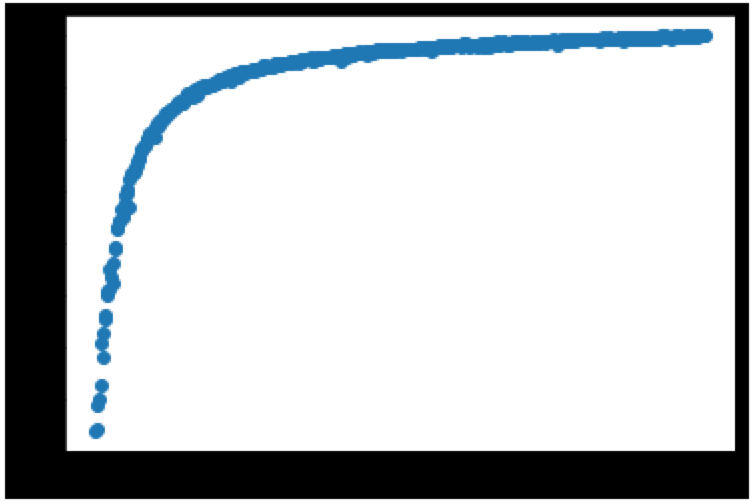
```
dWo = np.matmul(np.transpose(d),a)
    dbo = np.mean(d)  # consider a is 1 for bias
    dWh = np.matmul(np.transpose(dhs),x)
    dbh = np.mean(dhs)  # consider a is 1 for bias
    Wo =Wo - learningRate*dWo/Batch_size
    bo =bo - learningRate*dbo
    Wh =Wh-learningRate*dWh/Batch_size
    bh =bh-learningRate*dbh
  #Test accuracy with random innitial weights
  prediction = Forwardpass(x_test,Wh,bh,Wo,bo)
  Acc.append(AccTest(y_test,prediction))
  print('Epoch:', ep )
  print('Accuracy:',AccTest(y_test,prediction) )
```

```
Epoch: 0
Accuracy: 0.8762
Epoch: 1
Accuracy: 0.9013
Epoch: 2
Accuracy: 0.9136
Epoch: 3
Accuracy: 0.9165
Epoch: 4
Accuracy: 0.9251
```

Python code

Training the model

Mini-Batch Gradient Descent




Math