
M-DG Seminar: Multinomial Processing Tree Modeling

The Software multiTree

Summer semester 2020

Prof. Dr. Daniel Heck

M-DG: Multinomial Processing Tree Modeling

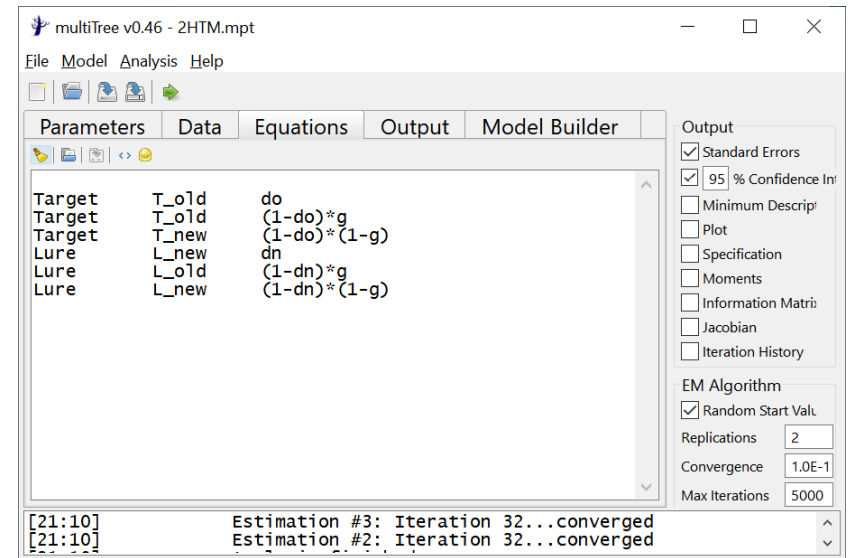
Part	Date	Topic	Literature
(A) Theory	Self study	A1) Introduction	Erdfelder et al. (2009)
		A2) Basics of MPT modeling	Batchelder & Riefer (1999)
		A3) The software multiTree	Moshagen (2010)
		A4) Hierarchical MPT modeling	Lee (2011) Heck et al. (2018)
(B) Application	15.5.*	B1) Questions & Practice with multiTree	Batchelder & Riefer (1986)
	20.5.*	B2) Workflow: Developing an MPT model	Jung et al. (2019)

* Web-Conference, 12:00 – 15:00, <https://webconf.hrz.uni-marburg.de/b/dan-fvk-ha6>

Software multiTree

multiTree (Moshagen, 2010)

- Specification, fitting and testing of MPT models
- Freeware
- Requires to install a Java Runtime Environment (JRE)
 - e.g., <https://www.java.com/en/download/>
- Download:
 - <https://www.sowi.uni-mannheim.de/erdfelder/forschung/software/multitree/>



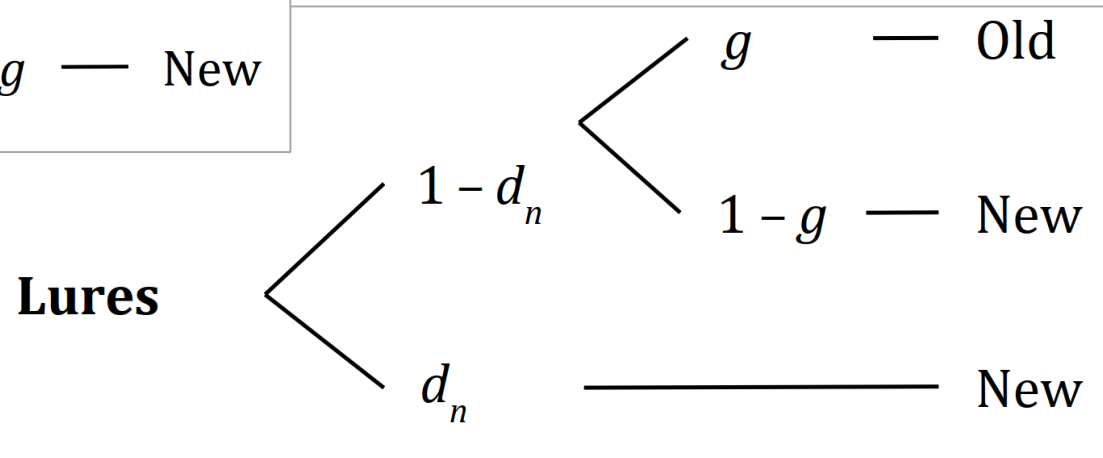
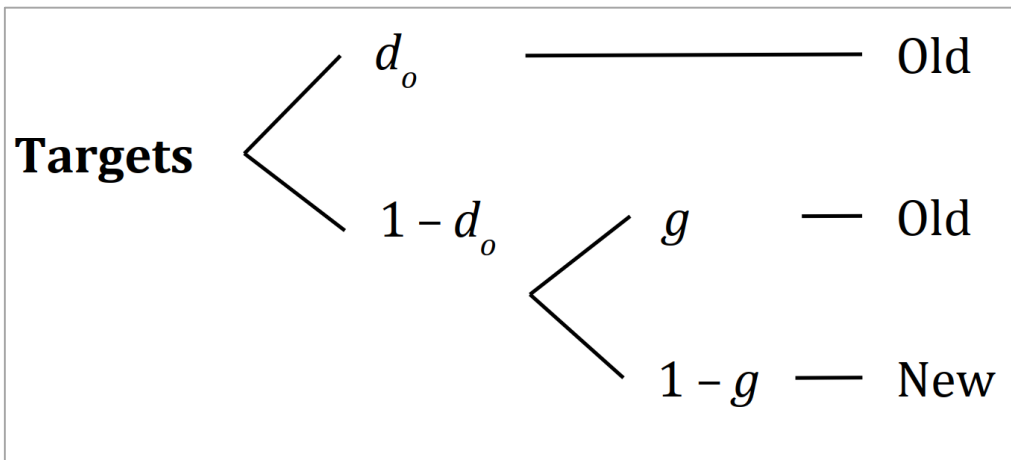
The Software multiTree

Overview:

1. Define MPT model
2. Provide data
3. Check identifiability
4. Comparing Nested Models
5. Preview: Exercises for the first interactive session
6. Advanced: Power analysis

Application: Define MPT Model

1.) Open multiTree and provide the model equations of the **Two-High-Threshold Model**:

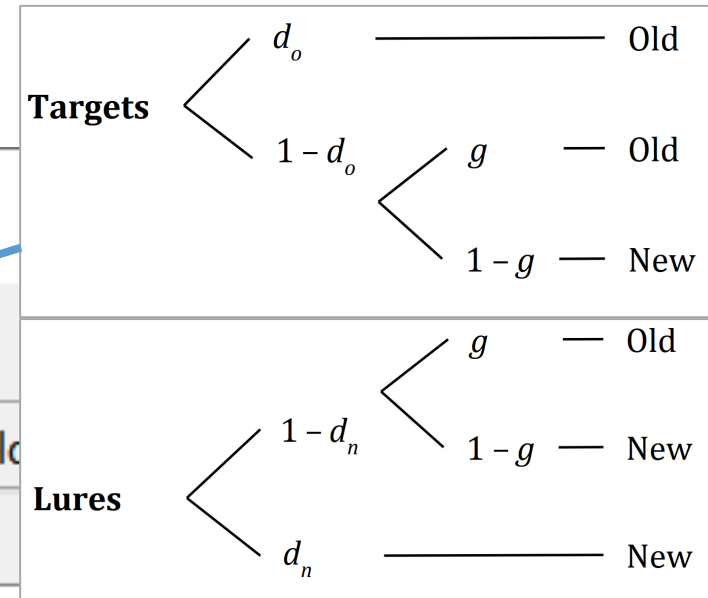


Two-High-Threshold Model (2HTM)

1.) Specification of an MPT model:

The screenshot shows the multiTree v0.46 interface with the 'Equations' tab selected. The interface includes a menu bar (File, Model, Analysis, Help), a toolbar with icons for file operations and execution, and a table for defining model components. The table has three columns: 'Condition', 'Response category', and 'Model equation'. The 'Equations' tab is highlighted with a blue arrow.

Condition	Response category	Model equation
Target	T_old	do
Target	T_old	$(1 - d_o) * g$
Target	T_new	$(1 - d_o) * (1 - g)$
Lure	L_new	dn
Lure	L_old	$(1 - d_n) * g$
Lure	L_new	$(1 - d_n) * (1 - g)$



condition

response category

model equation

The Software multiTree

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Application: Provide Data

2.) Provide the following data / observed frequencies:

	„old“	„new“
Old item	65	35
New item	13	87

Data

2.) Observed response frequencies:

	„old“	„new“
Old item	65	35
New item	13	87

The screenshot shows the multiTree v0.46 software interface. The title bar reads "multiTree v0.46 - two-high-threshold-model.mpt". The menu bar includes "File", "Model", "Analysis", and "Help". The toolbar contains icons for file operations and a green arrow pointing right. The "Data" tab is selected, showing a "Data Type" dropdown set to "Category Frequencies" and buttons for "Analyze" and "Batch Anal". The model equations are displayed in a text area:

```
Title: Example for MPT workshop
T_old      65
T_new      35
L_old      13
L_new      87
```

A blue arrow points from the "Data" tab to the table above. A red arrow points from the text "labels must be identical to the response categories in the model equations!" to the variable names in the equations.

labels must be identical to the response categories in the model equations!

The Software multiTree

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Check Identifiability

3.) Is the model **identifiable**?

multiTree v0.46 - two-high-threshold-model.mpt

File Model Analysis Help

Parameters Data Equations Output Model Builder

Hierarchical Model Families

- ☐ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).
- ☐ Compare current model against baseline model

Specification		
Number of trees		2
Number of categories		4
Number of free categories		2
Number of parameters		3
Number of constrained parameters		0
Degrees of freedom		-1

Output

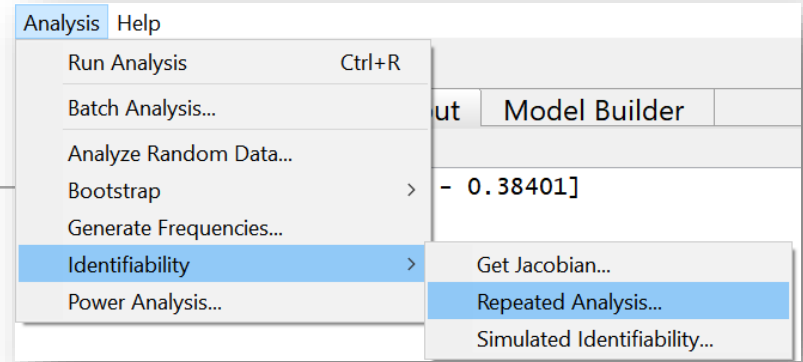
- ☒ Standard Errors
- ☒ 95 % Confidence Intervals
- ☐ Minimum Description Length
- ☐ Plot
- ☐ Specification
- ☐ Moments
- ☐ Information Matrix
- ☐ Jacobian
- ☐ Iteration History

EM Algorithm

- ☒ Random Start Values

Replications 2

Identifiability Checks in multiTree



- “Get Jacobian”

- Check whether the rank of the Jacobian matrix (for a random parameter θ) is equal to the number of free parameters

- “Repeated Analysis”

1. Estimate parameters multiple times for the same data
2. Check whether parameter estimates are stable/identical

- “Simulated Identifiability”

1. Draw random parameter vectors θ in Ω
2. Get expected category frequencies based on MPT model
3. Estimate the parameters of the MPT model
4. Check whether estimated & true parameters are identical

Application: Identifiability

- Example: Check identifiability via **repeated analysis**

multiTree v0.46 - 2HTM.mpt

File Model Analysis Help

Run Analysis Ctrl+R

Batch Analysis...

Analyze Random Data...

Bootstrap >

Generate Frequencies...

Identifiability >

Power Analysis...

Get Jacobian...

Repeated Analysis...

Simulated Identifiability...

Repeated Analysis

Run	dn	do	g	Fit	AIC	BIC	Delta AIC	Delta BIC
1	0.20114	0.58197	0.16273	0.00000	212.76667	222.66162	2.00000	5.29832
2	0.57178	0.49743	0.30359	0.00000	212.76667	222.66162	2.00000	5.29832
3	0.50071	0.52679	0.26037	0.00000	212.76667	222.66162	2.00000	5.29832
4	0.62998	0.46043	0.35134	0.00000	212.76667	222.66162	2.00000	5.29832
5	0.70466	0.37482	0.44016	0.00000	212.76667	222.66162	2.00000	5.29832
6	0.71662	0.35336	0.45874	0.00000	212.76667	222.66162	2.00000	5.29832
7	0.62593	0.46358	0.34753	0.00000	212.76667	222.66162	2.00000	5.29832
8	0.67560	0.41594	0.40074	0.00000	212.76667	222.66162	2.00000	5.29832
9	0.78259	0.12949	0.59794	0.00000	212.76667	222.66162	2.00000	5.29832
10	0.62316	0.46567	0.34497	0.00000	212.76667	222.66162	2.00000	5.29832

Application: Identifiability

5.) Obtaining an identifiable model by equality constraints:

Parameters Data Equations Output Model Builder

Hierarchical Model Families

☐ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).

☐ Compare current model against baseline model

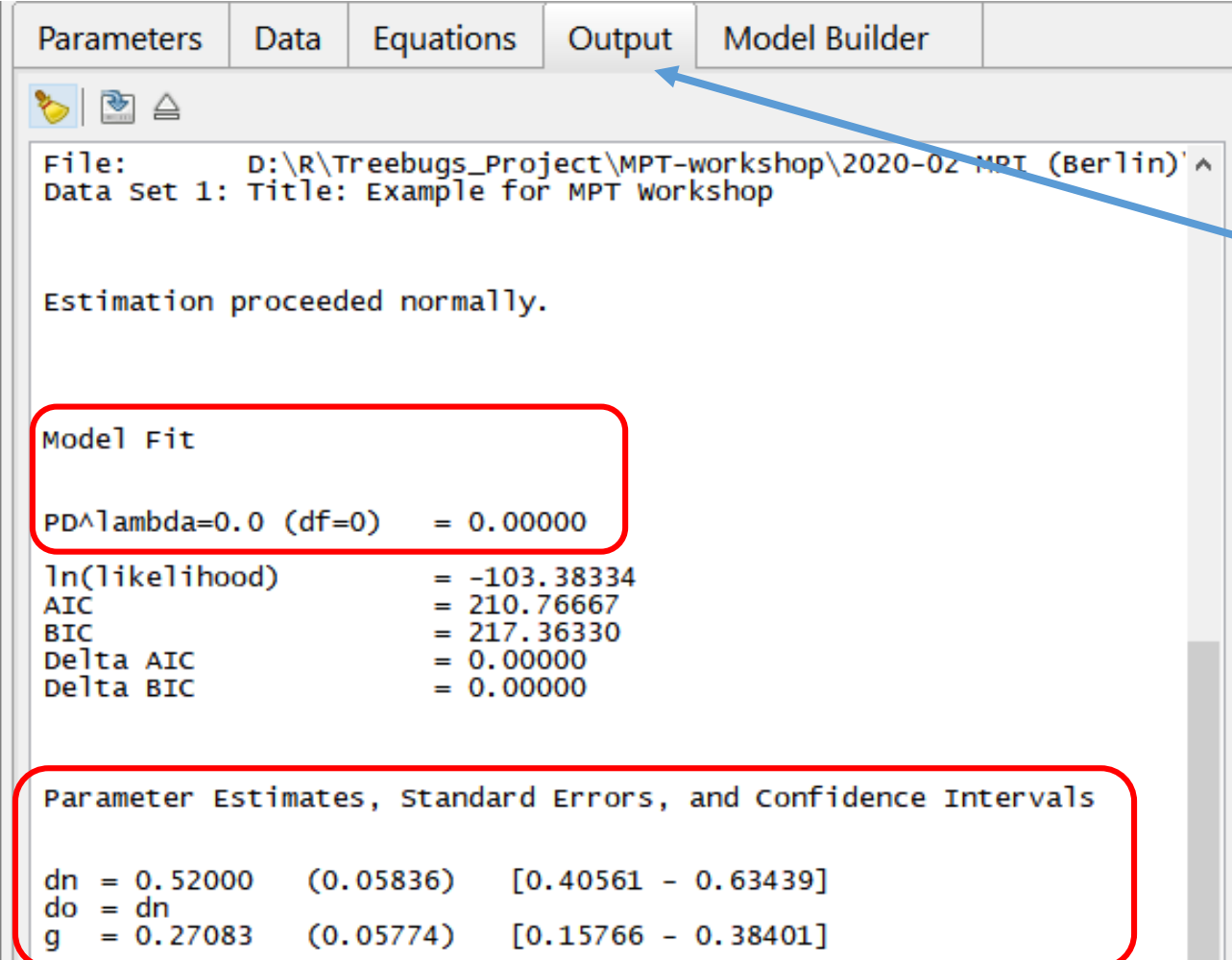
dn	free	0.52
do	= dn	0.52
g	free	0.27083

Specification

Number of trees	2
Number of categories	4
Number of free categories	2
Number of parameters	3
Number of constrained parameters	1
Degrees of freedom	0

Model Fitting: Output

6.) Results: Model fit and parameter estimates



Parameters Data Equations **Output** Model Builder

File: D:\R\Treebugs_Project\MPT-workshop\2020-02-MPT (Berlin)'^
Data Set 1: Title: Example for MPT workshop

Estimation proceeded normally.

Model Fit

PD λ lambda=0.0 (df=0) = 0.00000

ln(likelihood) = -103.38334
AIC = 210.76667
BIC = 217.36330
Delta AIC = 0.00000
Delta BIC = 0.00000

Parameter Estimates, Standard Errors, and Confidence Intervals

dn = 0.52000 (0.05836) [0.40561 - 0.63439]
do = dn
g = 0.27083 (0.05774) [0.15766 - 0.38401]

G² likelihood
ratio test

The Software multiTree

Overview:

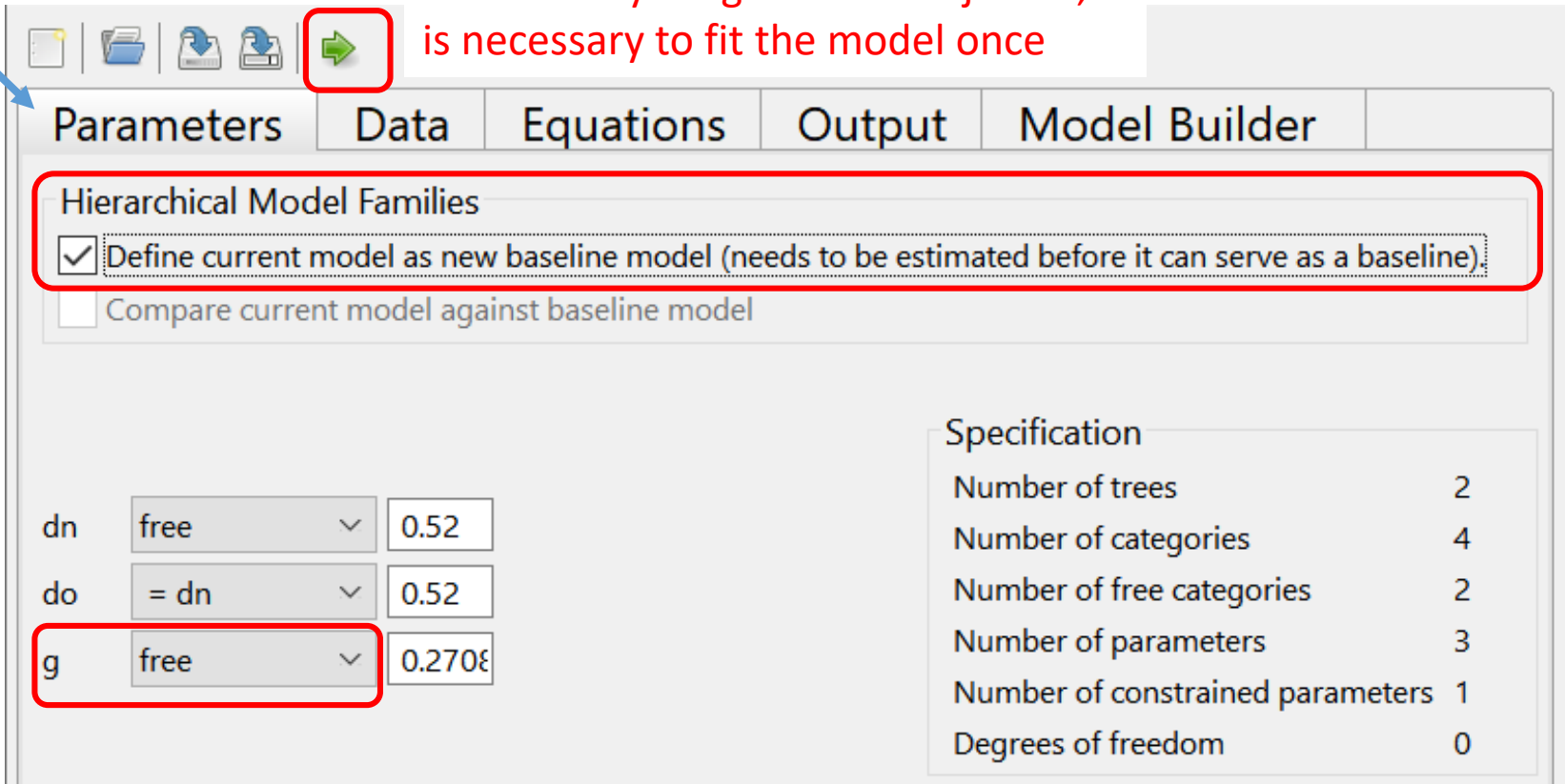
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Comparing Nested Models

7.) Hypothesis testing: Does the **constraint $g = .50$** hold?

→ Step 1: **Define baseline** model & fit model

When everything below is adjusted, it is necessary to fit the model once



Parameters | Data | Equations | Output | Model Builder

Hierarchical Model Families

- ☒ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).
- ☐ Compare current model against baseline model

Parameter	Estimate
dn	free 0.52
do	= dn 0.52
g	free 0.2708

Specification

- Number of trees: 2
- Number of categories: 4
- Number of free categories: 2
- Number of parameters: 3
- Number of constrained parameters: 1
- Degrees of freedom: 0

Comparing Nested Models

7.) Hypothesis testing: Does the constraint $g = .50$ hold?

→ Step 2: Compare current model to baseline

When everything below is adjusted, it is necessary to fit the model a *second* time

Parameters Data Equations Output Model Builder

Hierarchical Model Families

☐ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).

☒ Compare current model against baseline model

Specification




dn	free	0.52
do	= dn	0.52
g	constant	0.50

Comparing Nested Models

- 7.) Hypothesis testing: Does the constraint $g = .50$ hold?

Likelihood
ratio test with
 ΔG^2 statistic

Parameters | Data | Equations | Output | Model Builder

Difference to Baseline Model (Difference = Current - Baseline)

PD^{lambda}=0.0 (df=1) = 13.66530 p = 0.00022

AIC difference = 11.66530

BIC difference = 8.36699

Ratio of AIC weights = 0.00292

Ratio of BIC weights = 0.01502

	Baseline	Current
dn	free	free
do	= dn	= dn
g	free	0.50

what are the parameters
that are tested?

Parameter Estimates, Standard Errors, and Confidence Intervals

dn	= 0.52000	(0.06040)	[0.40162 - 0.63838]
do	= dn		
g	= 0.50000	(constant)	

what are the parameters
that are tested?

The Software multiTree

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Preview:

Exercises for the first interactive session

1. Extend the 2HTM to **two base-rate conditions**:

		„old“	„new“
30% Targets	Target	65	35
	Lure	13	87
70% Targets	Target	83	17
	Lure	43	57

2. What are the **parameters** of the extended model?
3. Is the model **identifiable** with separate d_n and d_o ?
4. Fit the model to **both conditions jointly**.
5. **Test whether g differs** significantly between conditions.

The Software multiTree

Overview:

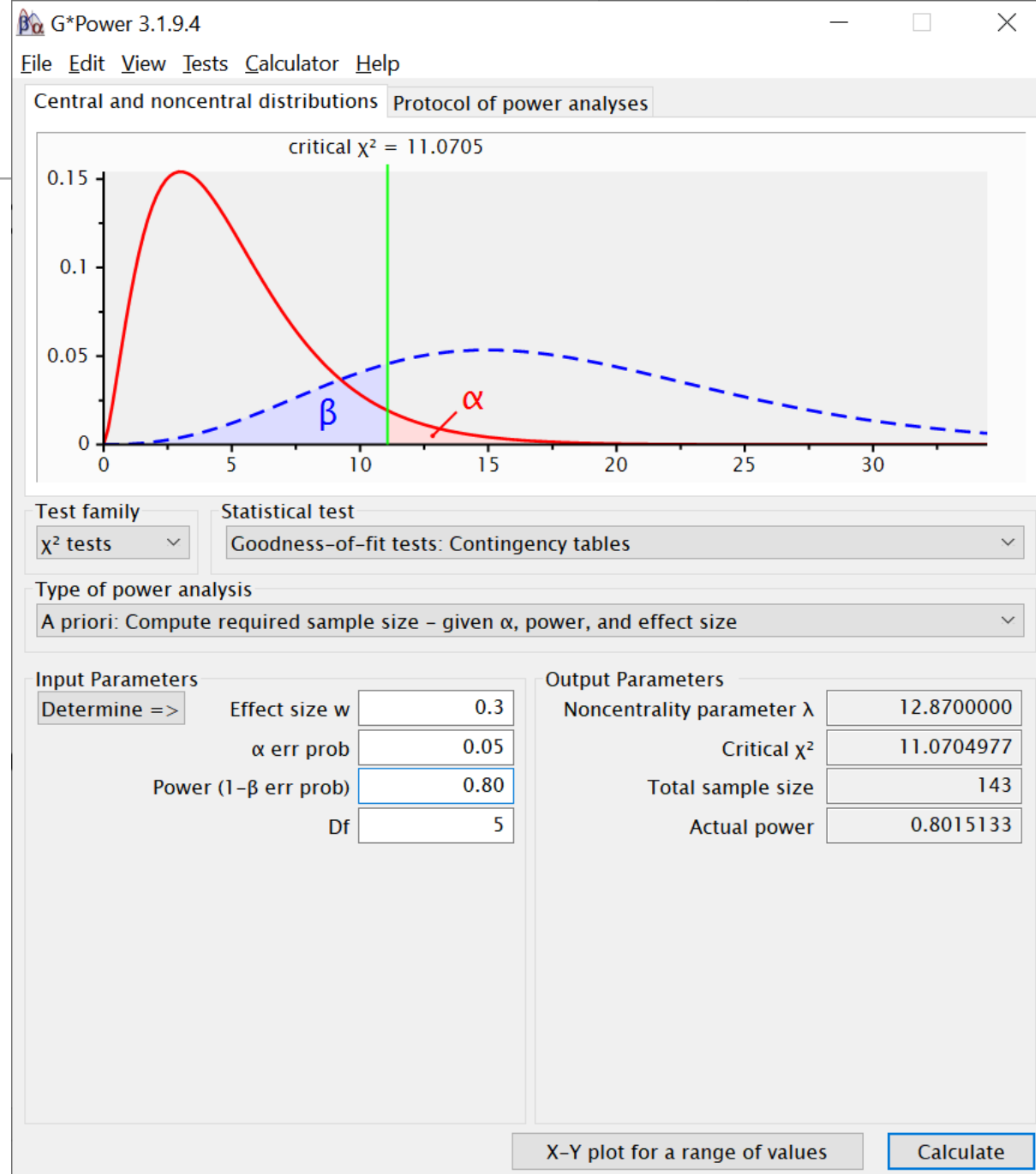
1. Define MPT model
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Traditional Power Analysis

- Under H_1 , the G^2 statistic of H_0 follows a **noncentral χ^2 distribution**
 - noncentrality parameter $\gamma = N \cdot w^2$
 - w denotes the effect size ($w^2 = G^2(H_0 \mid H_1 \text{ holds given } \theta) / N$)
- Effect size conventions
 - $w = .10$ (“small effect”)
 - $w = .30$ (“medium effect”)
 - $w = .50$ (“large effect”)
- Types of power analysis
 - **A priori**: Compute required N as a function of w , α , and $1 - \beta$
 - **Post hoc**: Compute $1 - \beta$ as a function of w , α , and N

Traditional Power Analysis

- Example with **G*Power** for a “medium” effect size ($w = 0.3$)



Traditional Power Analysis: Limitations

- Problems:
 1. How does effect size translate into **parameter values**?
 2. Same meaning of effect size labels in **different models**?
 3. **Relative size of groups**/conditions is ignored.
- Cohen (1988, p. 244) on w effect sizes conventions:
 - “**Their use requires particular caution**, since, apart from their possible inaptness in a particular substantive context, what is subjectively the same degree of departure or degree of correlation (...) may yield varying w , and conversely. The investigator is best advised to use the conventional definitions as a general frame of reference (...) and **not to take them too literally.**”

Approach 2: Meaningful Power Analysis

Power as a function of the model parameters θ under H_1 :

- 1) Specify a **specific H_1 model** with all parameter values θ fixed at „plausible values“
- 2) Specify a **nested H_0 model**
- 3) Choose number of observations **N_k for each tree k** and calculate the expected frequencies under H_1
- 4) Fit the H_0 model to the H_1 expected frequencies by minimizing G^2
- 5) Use minimum G^2 value as noncentrality parameter γ
- 6) Compute the power $1 - \beta = P(\chi^2(\gamma, df) \geq c_{(df, \alpha)})$

Power Analysis in multiTree

H_1 : “true state of the world”
→ power may depend on the values of *all* parameters θ !

Power Analysis

Statistical power of a test is defined as the probability of rejecting a null hypothesis if it is in fact false, and depends on the alpha error, effect size, and sample size. To perform a power analysis, 'true' population values of the parameters, a H1 model and a H0 model need to be specified. The H1 model is

Type of power analysis

☒ A-priori: Compute required sample size given power, alpha, and effect size.

☐ Post-hoc: Compute achieved power given sample size, alpha, and effect size.

Alpha error probability: 0.05

Desired power (ignored in post-hoc power analysis): 0.8

Please specify the parameter values in the population as well as the H1 and the H

dn	0.3	free	0	free	0
do	0.5	= dn	0	= dn	0
g30	0.40	free	0	free	0
g70	0.60	free	0	= g30	0

Please enter the number of observations (represent weights in a-priori power analysis)

Tree: old30_Target 100

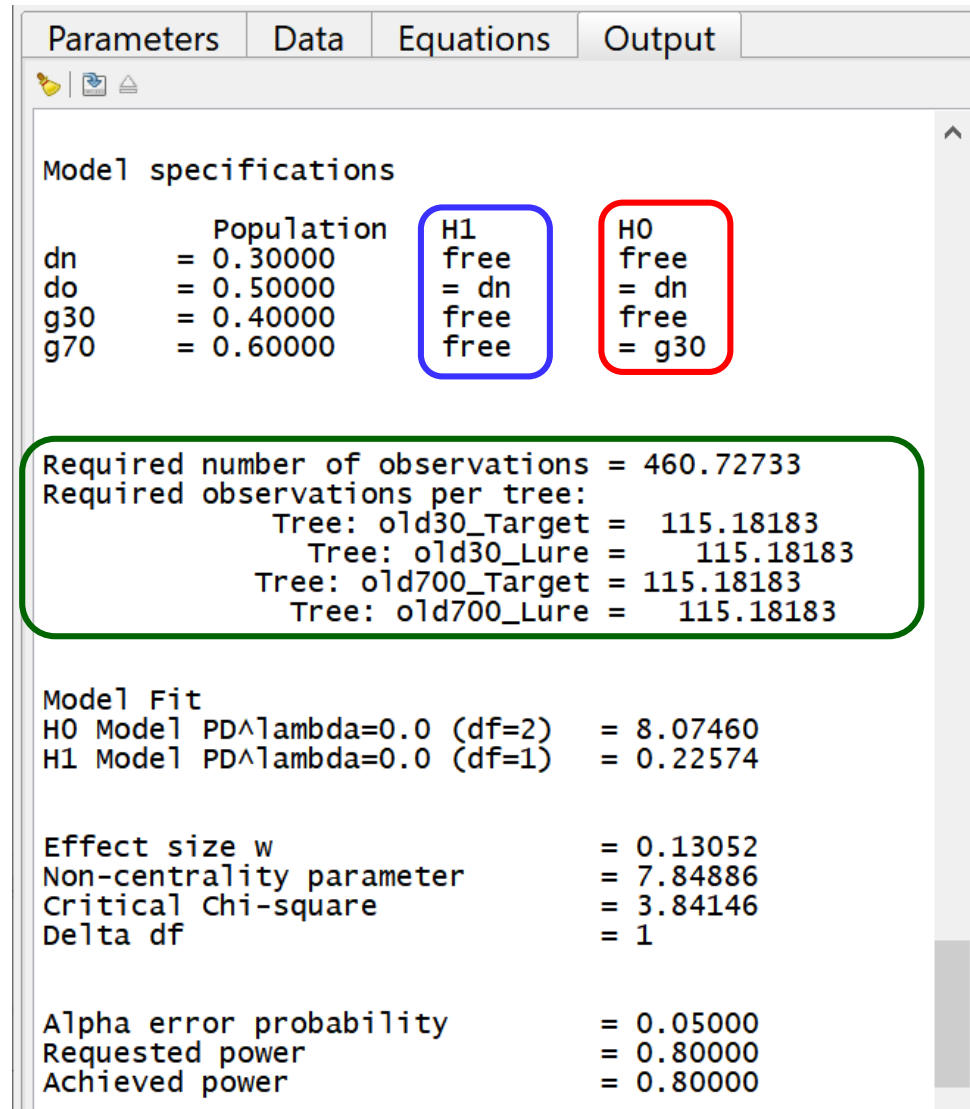
Tree: old30_Lure 100

OK Cancel

H_0 : nested model
→ which parameter constraints are tested
→ here: $g30 = g70$

N per tree

Power Analysis in multiTree: Output



The screenshot shows the 'Output' tab of the multiTree software. It displays model specifications, required observations, model fit statistics, effect size, and power analysis results. Key elements are highlighted with colored boxes: a blue box around the H1 model specifications, a red box around the H0 model specifications, and a green box around the required number of observations and per-tree observations.

Model specifications		H1	H0
dn	= 0.30000	free	free
do	= 0.50000	= dn	= dn
g30	= 0.40000	free	free
g70	= 0.60000	free	= g30

Required number of observations = 460.72733	
Required observations per tree:	
Tree: old30_Target	= 115.18183
Tree: old30_Lure	= 115.18183
Tree: old700_Target	= 115.18183
Tree: old700_Lure	= 115.18183

Model Fit	
H0 Model PD λ lambda=0.0 (df=2)	= 8.07460
H1 Model PD λ lambda=0.0 (df=1)	= 0.22574

Effect size w	
Effect size w	= 0.13052
Non-centrality parameter	= 7.84886
Critical Chi-square	= 3.84146
Delta df	= 1

Alpha error probability	
Alpha error probability	= 0.05000
Requested power	= 0.80000
Achieved power	= 0.80000

Result:

→ How many observations are required?