Bayesian Hierarchical MPT Modeling

(M-DG Seminar)

Daniel W. Heck

Philipps Universität Marburg

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Bayesian Hierarchical MPT Modeling

1) MPT models & heterogeneity
2) Hierarchical MPT models
3) Bayesian estimation with MCMC sampling
4) Advantages of MCMC

MPT models & heterogeneity

Traditional MPT Analysis

Limitations of traditional MPT models (Smith & Batchelder, 2008)

- Analysis on the group level
  - Frequencies are aggregated (summed) across persons
- Statistical assumption that responses are "i.i.d."
  - Observations are "independent and identically" (i.i.d.) distributed
  - ... people behave identically
  - ... items are similarly difficult

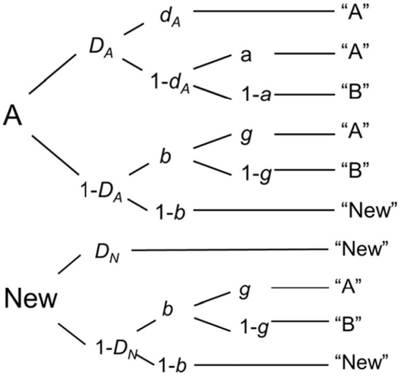
Application in psychology

- What about real data?
- What if we are interested in differences between people?

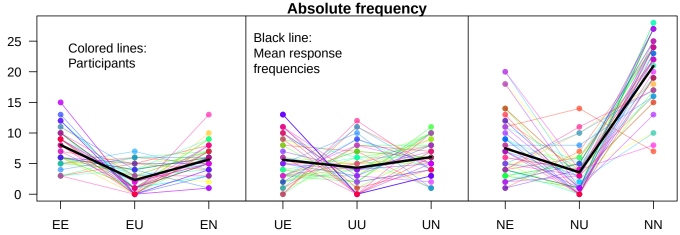
Source-Monitoring Model

Source-Monitoring

1) Study phase: List of words from Source A and B.
2) Test phase: Is the presented item from Source A/B/New?



People Behave Differently



Heterogeneity of participants

- Substantial variance in the choice patterns of participants
  - Differences in memory? Response bias?
- If we fit standard MPT models to aggregated data, these differences are ignored (treated as random, unsystematic noise)
- This can result in biased statistical inferences
  - Biased point estimates if parameter are correlated
  - Over-/underestimation of confidence intervals
  - Inflated model-fit statistics

How to Handle Heterogeneity?

a. **Analysis of aggregated frequencies** (complete pooling)
   - Ignores differences between persons
   - High power, but possibly biased statistical inference
b. **Separate analysis**: One MPT model per person (no pooling)
   - Low power, parameter estimates will have a large variance
   - Often, not enough data per participant
   - Problem: How to aggregate results across models?
c. **Hierarchical model** (partial pooling)
   - Account for differences AND similarities between persons jointly
   - Provides correct statistical inferences
   - Higher efficiency than separate analysis

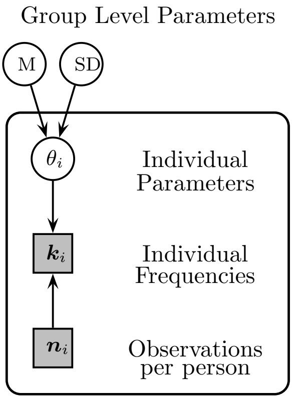
(Note: This classification is very general and not limited to MPT models.)

Hierarchical MPT models

Hierarchical MPT Models

Bayesian hierarchical MPT

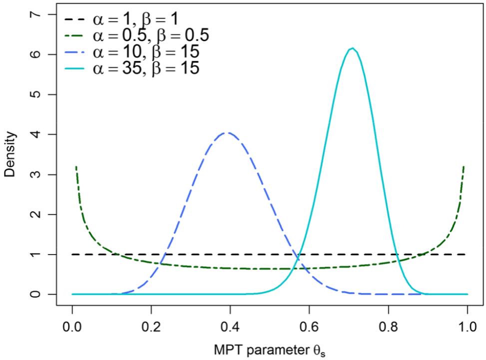
- Explicit assumptions about participant heterogeneity:
- MPT structure holds for each person, but with different parameters!
- One parameter vector $\theta_i = (D_i, d_i, g_i, \ldots)$ per person
- On the group level, the θ_i have a specific distribution
  a. Beta-MPT
  b. Latent-trait MPT



Beta-MPT

Beta distribution

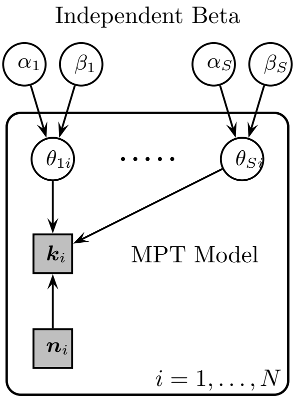
- Ideally suited to model the distribution of an MPT parameter:
  a. Allows values between 0 and 1
  b. Two shape parameters: α and β
- The group-level mean for the MPT parameter equals: $\alpha/(\alpha+\beta)$



Beta-MPT

Beta-MPT (Smith & Batchelder, 2010)

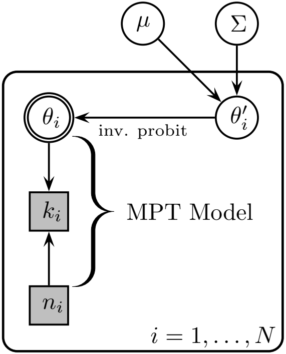
- Group level:
  - person parameters follow independent beta distributions
  - Shape parameters α_s and β_s
- Individual level:
  - MPT model with separate parameters per person
  - MPT parameters: θ_{si} of person i
- Notation for data:
  - k_i: Individual choice frequencies
  - n_i: Number responses per person



Latent-Trait MPT

Latent-trait MPT model (Klauer, 2010)

- Group level:
  - person parameters follow a multivariate normal distribution
  - Mean μ
  - Covariance matrix Σ
- Individual level:
  - MPT model with separate parameters θ_i per person
  - probit-values are transformed to probabilities
  - Probit-scaled parameters: $\theta_i' = \Phi^{-1}(\theta_{si})$
- Notation for data:
  - k_i: Individual choice frequencies
  - n_i: Number responses per person



The Probit-Transformation

Working with probability parameters

- We need to transform the probability parameters (d, D, ...)
- We want parameters between $(-\infty, +\infty)$ (to work with normal distributions)
- Solution: Transform parameters using the cumulative density function Φ of the standard-normal distribution (similar as in logistic regression)

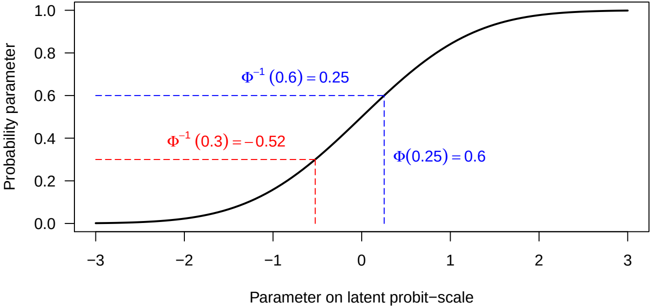
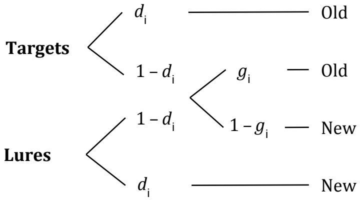


Illustration: Separate MPT Structure for each Person

Individual level: Each person has different parameters d_i amd g_i

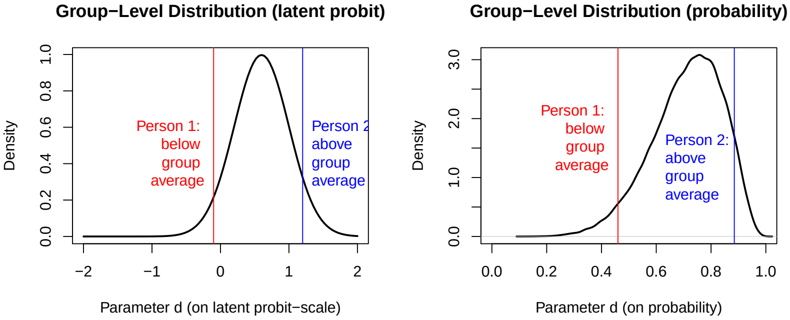


- We assume symmetric and identical guessing for everybody ($g_i = .50$)
- **Person 1** with a probit score: $\theta'_1 = -.10$
  1) $d_1 = \Phi(-.10) = .46$
  2) $P(hit) = d_1 + (1 - d_1)g = .46 + (1 - .46).50 = .73$
- **Person 2** with a probit score: $\theta'_2 = 1.20$
  1) $d_2 = \Phi(1.20) = .88$
  2) $P(hit) = d_2 + (1 - d_2)g = .88 + (1 - .88).50 = .94$

Group Level: Normal Distribution

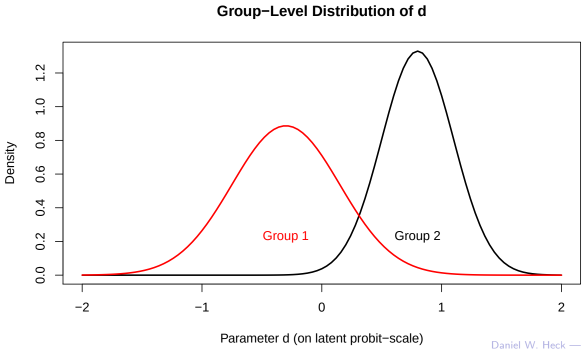
Assumption: Normal distribution of probit parameters

- Illustration: Normal distribution with mean $\mu_d = .80$ and standard deviation $\sigma_d = .3$
- For interpretation, it matters whether parameters are on the probit or the probability scale



Comparison of Groups

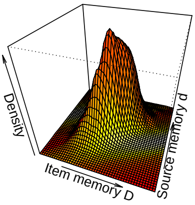
- Possible research questions: Does the group distribution differ across samples / parameters / manipulations?
- Example: Reduced memory for Group 1 (alcoholics) vs. Group 2 (controls)
- Similar as in simple t-test, but now we test theories about latent-probit parameters

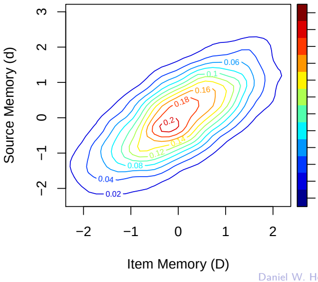


Multivariate Normal Distribution

Parameter correlations

- Item and source memory might be correlated (parameters g and d)
- "The more likely I remember the item, the more likely I also remember the source."
- Solution: Assumption that the vector θ_i' with probit-transformed MPT parameters follows a *multivariate* normal distribution
- Caveat: Correlation estimates are often very unprecise and require both large number of responses and large number of participants

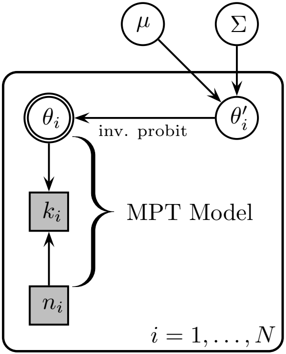




Summary: Hierarchical Models

Core ideas of hierarchical models

- Assume an MPT model with separate MPT parameters θ_i per person
- On the group-level, the parameters have a specific distribution
  1) Beta-MPT: Beta distribution
  2) Latent-trait MPT: multivariate normal distribution of probit-parameters with mean μ and covariance matrix Σ
  3) Other option (not discussed here): Discrete latent classes (Klauer, 2006)



Some Advantages of Hierarchical Models

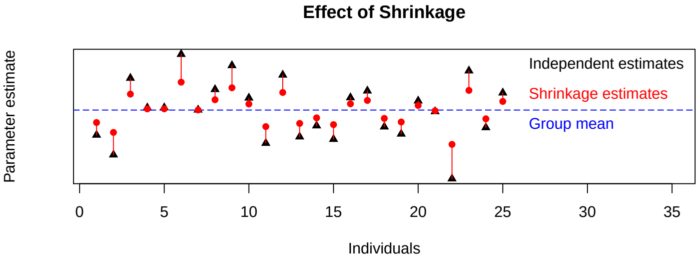
Substantive advantages

- We get estimates for each person (not only group-level estimates)
- Relevant for research questions such as: “Does memory ability decline over age?”
- The core idea of hierarchical models can easily applied to any other (cognitive/statistical) model
  1) Assume that a specific model holds for each person
  2) Specificy group-level distribution of parameters across persons

Some Advantages of Hierarchical Models

Statistical advantages

- Avoids aggregation errors
- "Shrinkage" of parameter estimates
  - Parameter estimates for each person are closer together compared to fitting each person separately
  - Hence, extreme estimates are less likely
  - Overall, this ensures that parameter estimates are better (i.e., closer to the true values)



Bayesian estimation with MCMC

Fitting Hierarchical MPT Models

Parameter estimation

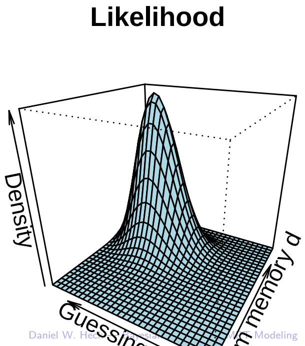
- How can we actually fit such models?
- Which are the "best" parameters given the data?
  - Standard MPT models: Maximum likelihood estimation
  - Not an option for hierarchical models (intractable likelihood function due to high-dimensional integrals)

Solution

- Hierarchical models are often fitted using Bayesian statistics

Maximum Likelihood

- Logic of parameter estimation with maximum-likelihood
  1) Define likelihood function $p(x \mid \theta)$
  2) Find parameters θ that maximize f
- Interpretation: "The estimator $\hat{\theta}$ has the highest likelihood."
- Computational solution: Algorithm searches for the "top of the mountain"

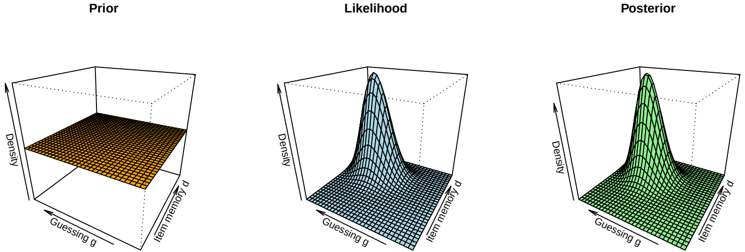


Bayesian Estimation

- Logic of Bayesian parameter estimation
  1) Define likelihood $p(x \mid \theta)$ and prior distribution $p(\theta)$
  2) Derive the posterior distribution of the parameters via Bayes' theorem:

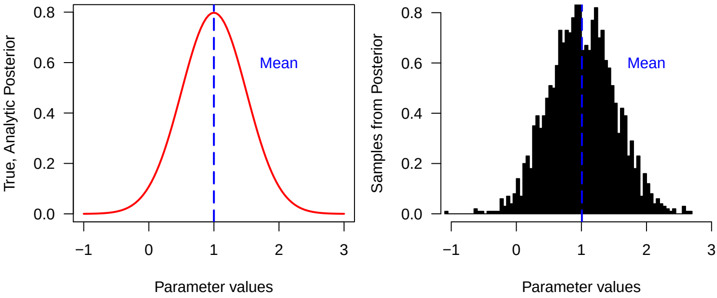
$$p(\theta \mid x) = \frac{p(x \mid \theta)p(\theta)}{p(x)}$$

- Interpretation: "What have we learned about the parameters θ given the data x?"



Bayesian Estimation

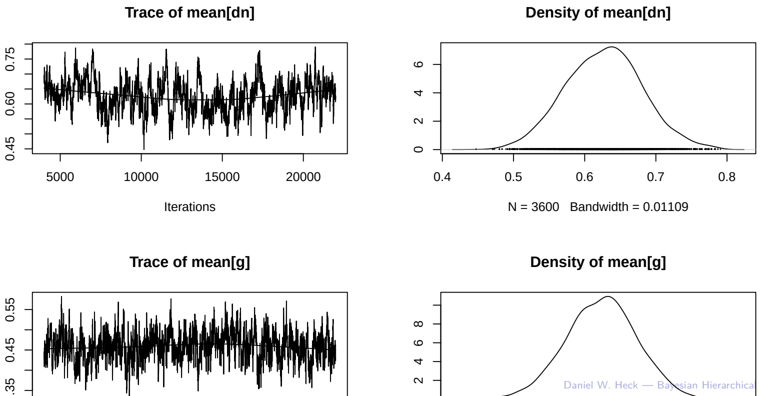
- Problem: We need to work with the posterior function $p(\theta \mid x)$
  - What is the mean/mode/95% credibility interval of θ?
  - Often, this is analytically not tractable
- Solution: We draw random samples from the posterior distribution
  - Logic: It is easier to draw conclusions from these random samples than deriving solutions for the analytical posterior (which is a function!)
  - Example: Computing the mean of a normal distribution requires to solve:



Bayesian Estimation

Markov Chain Monte Carlo (MCMC) Sampling

1. Draw random samples of the posterior distribution for *all* parameters (individual and group level)
2. Summarize parameter samples (e.g., mean, SD, density, . . .)



Bayesian Estimation

Markov chain Monte Carlo (MCMC)

- General method to draw posterior samples
- In a hierarchical model, there are many (!) parameters
  - Group-level means and covariances, person parameters, ...
  - Intuitively, this method moves around and searches for parameter values with high posterior density
- There are software packages that draw random samples for many models of interest
  - JAGS, WinBUGS, OpenBUGS, Stan, ...

Summary of Bayesian estimation

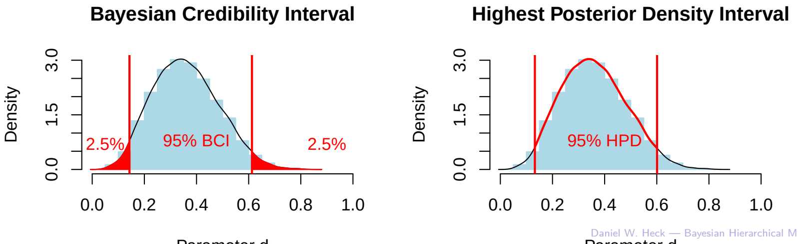
1) Develop a model (=> psychological theory, multiTree)
2) Get posterior (MCMC) samples (JAGS, TreeBUGS)
3) Summarize these samples (e.g., mean of group-level parameters μ_D, μ_g,...)

Advantages of MCMC

Advantages of MCMC: Uncertainty

Advantages of MCMC sampling

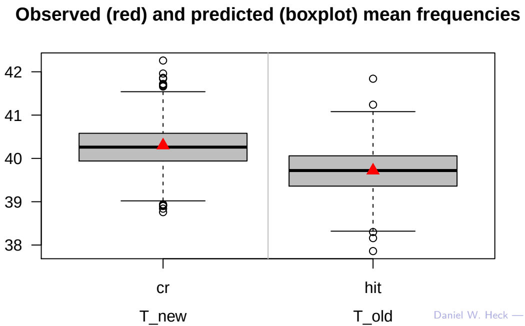
- Theoretical: No asymptotic assumptions about minimal sample size
- Practical: It is easy to quantify uncertainty
  - Bayesian credibility interval (BCI): What are the 2.5%- and 97.5%-quantiles of the parameter values?
  - Highest posterior density interval (HPD or HDI): What are the 95% most plausible parameter values?
  - For probability parameters, these intervals will always be in the interval $[0, 1]$



Advantages of MCMC: Model Fit

Does the model fit the data?

- Graphical comparison: observed vs. predicted frequencies
- Use posterior samples of the MPT parameters to sample new data (= posterior predictive)
- Compare whether these predicted data (boxplot) are in line with the observations (red points)



Summary

Hierarchical MPT Models

- Individual level: Assume separate MPT parameters for each person
- Group level
  - Beta-MPT: Beta distribution of person parameters
  - Latent-trait MPT: Normal distribution of probit-transformed parameters
- Bayesian model fitting: Draw posterior samples via MCMC

Appendix

Appendix: Standard vs. Hierarchical MPT Modeling

Currently open questions:

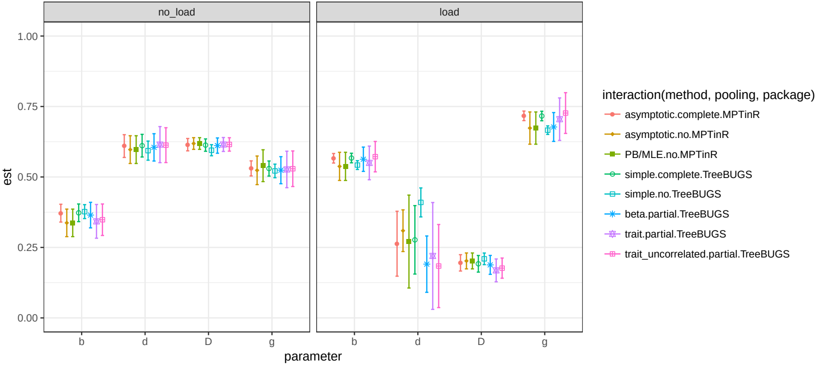
- How much do results actually differ when using different MPT model versions (standard, hierarchical, beta, latent-trait, ...)?
- Which MPT model version should be used in practice?

Large-scale reanalysis project

- Network of MPT researchers (organized by Beatrice Kuhlmann & Julia Groß)
- Reanalysis of existing data sets to compare:
  - Fixed-effects vs. hierarchical
  - Maximum-likelihood vs. Bayes
  - Different hierarchical level-2 structures (beta, multiv. normal, independent univ. normal)
- Software: "A multiverse pipeline for MPT models"
  - Maximum likelihood: `MPTinR` (Henrik Singmann)
  - Bayes: `TreeBUGS`
  - Available at: https://github.com/mpt-network/MPTmultiverse

Appendix: Standard vs. Hierarchical MPT Modeling: Reanalysis

- Source-monitoring model (data by Bayen & Kuhlmann, 2011)
- Plot: Difference in parameters across two groups



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