



# Multinomial Processing Tree (MPT) Modeling, Part 2: Application I

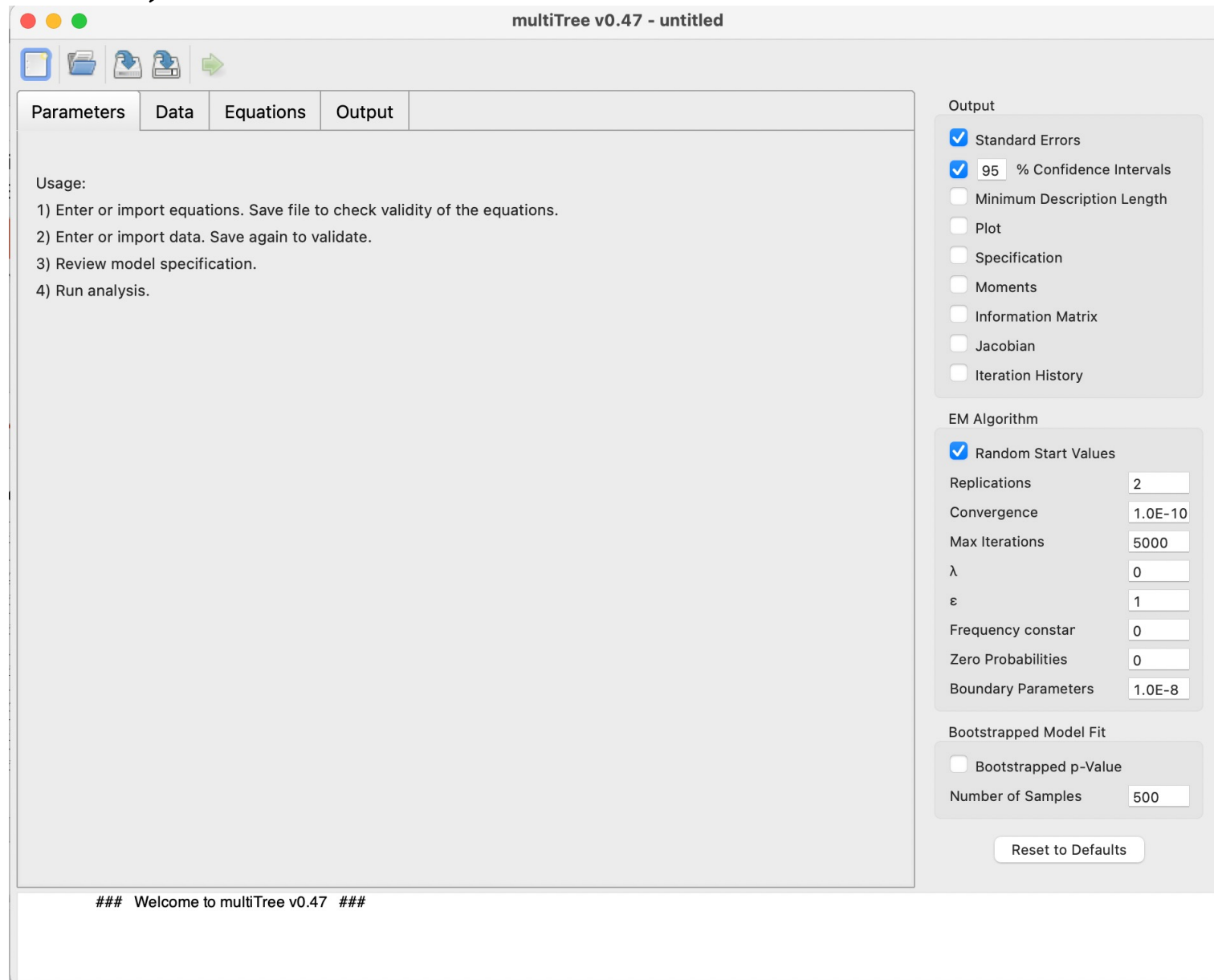
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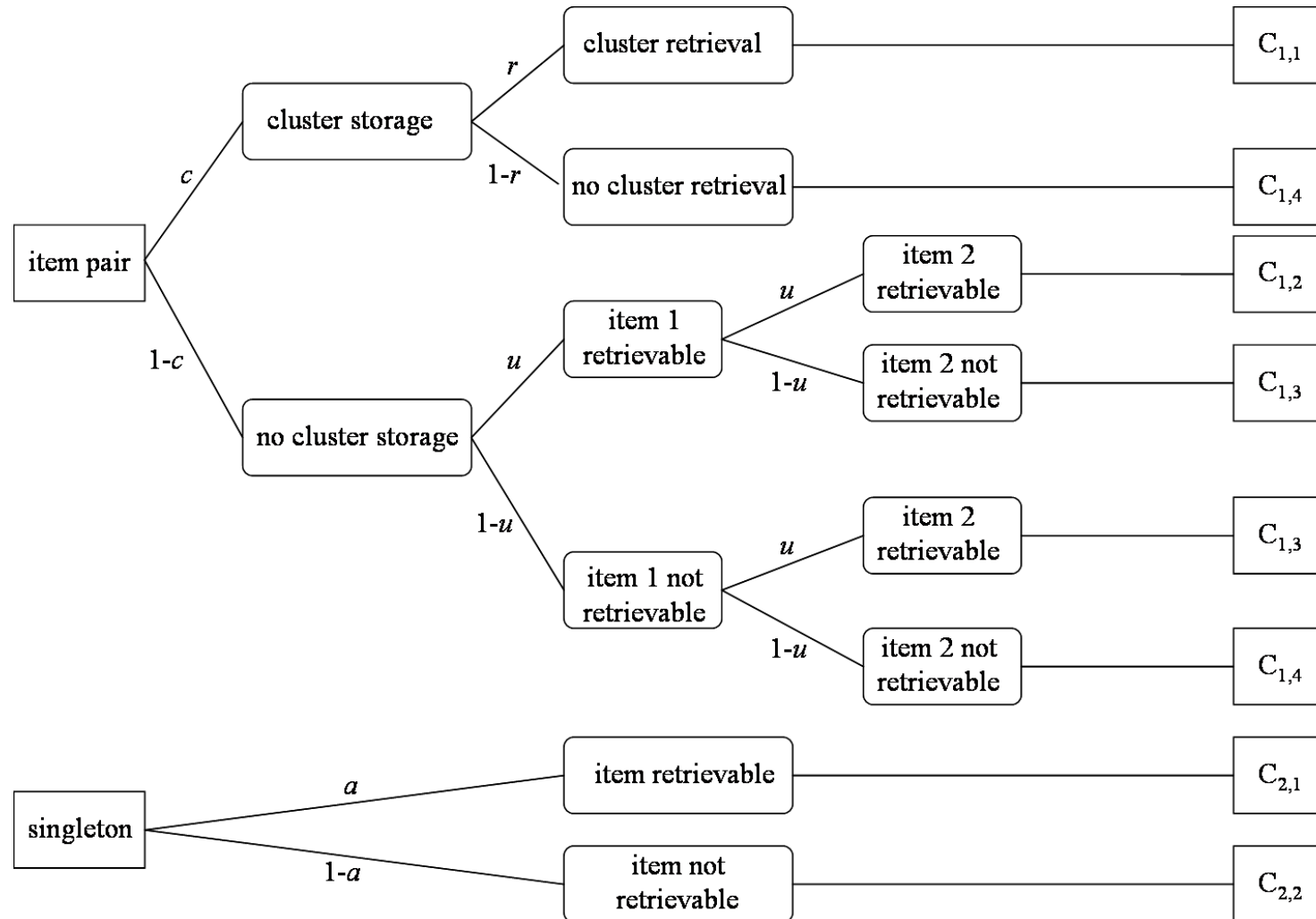
## 2) Application I

- 2.1) Introduction to multiTree
- 2.2) Practical exercises
- 2.3) Order constraints
- 2.4) Testing interactions

# 2.1) Introduction to multiTree

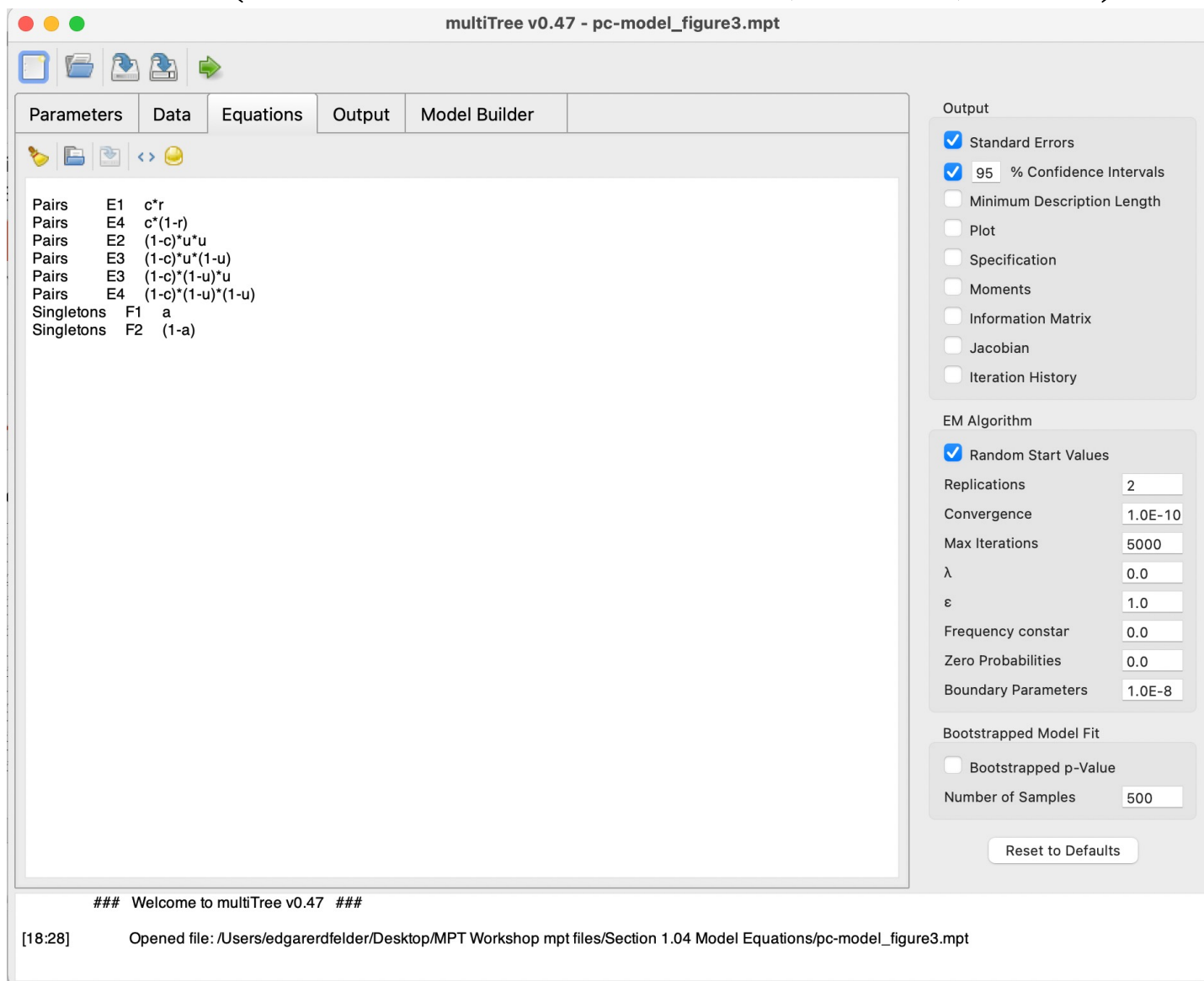


# An example: The pair-clustering model

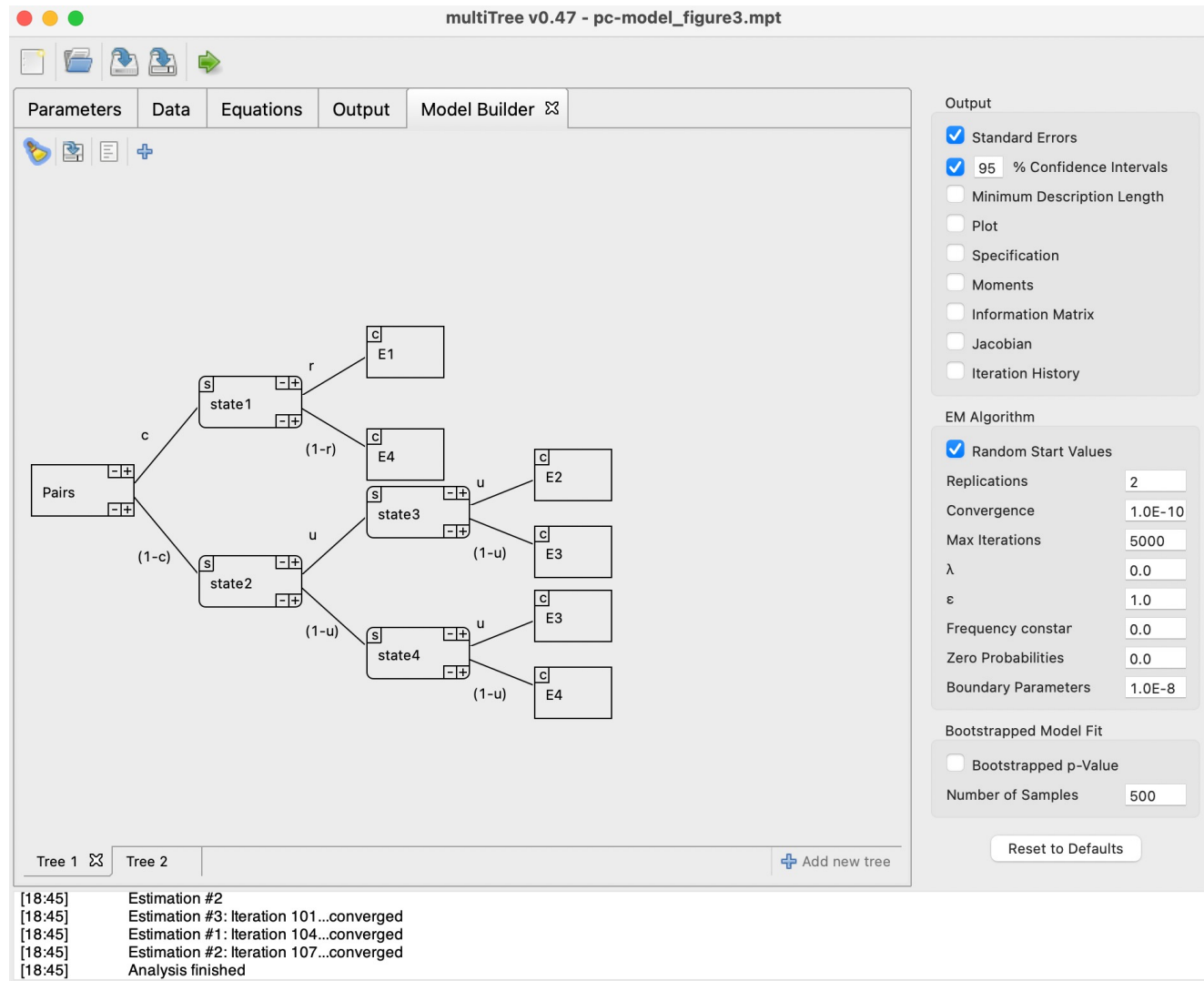


# Pair-Clustering Model

(Batchelder & Riefer, 1980,1986)



# multiTree Model Builder: Pairs



# multiTree Model Builder: Singletons

The screenshot displays the multiTree v0.47 software interface. The main window is titled "multiTree v0.47 - pc-model\_figure3.mpt". It features a tabbed interface with "Parameters", "Data", "Equations", "Output", and "Model Builder" tabs. The "Model Builder" tab is active, showing a hierarchical model structure. The model consists of a root node labeled "Singletons" with two sub-nodes: "F1" and "F2". The branch to "F1" is labeled "a", and the branch to "F2" is labeled "(1-a)". Each node has a small icon with a minus and plus sign. The right sidebar contains several sections: "Output" with checkboxes for "Standard Errors", ".95 % Confidence Intervals", "Minimum Description Length", "Plot", "Specification", "Moments", "Information Matrix", "Jacobian", and "Iteration History"; "EM Algorithm" with a checked "Random Start Values" checkbox and input fields for "Replications" (2), "Convergence" (1.0E-10), "Max Iterations" (5000),  $\lambda$  (0.0),  $\epsilon$  (1.0), "Frequency constar" (0.0), "Zero Probabilities" (0.0), and "Boundary Parameters" (1.0E-8); and "Bootstrapped Model Fit" with a "Bootstrapped p-Value" checkbox and "Number of Samples" (500). A "Reset to Defaults" button is at the bottom right. The bottom status bar shows a log of messages: "[18:45] Estimation #2", "[18:45] Estimation #3: Iteration 101...converged", "[18:45] Estimation #1: Iteration 104...converged", "[18:45] Estimation #2: Iteration 107...converged", and "[18:45] Analysis finished".

# Data

The screenshot displays the multiTree v0.47 software interface. The main window is titled "multiTree v0.47 - pc-model\_figure3.mpt". The "Data" tab is selected, showing a list of data points: E1 90, E2 14, E3 84, E4 212, F1 102, and F2 298. The "Data Type" is set to "Category Frequencies" and the "Analyze" button is visible. The "Output" panel on the right includes checkboxes for "Standard Errors", "95 % Confidence Intervals", "Minimum Description Length", "Plot", "Specification", "Moments", "Information Matrix", "Jacobian", and "Iteration History". The "EM Algorithm" panel includes checkboxes for "Random Start Values" and input fields for "Replications" (2), "Convergence" (1.0E-10), "Max Iterations" (5000),  $\lambda$  (0.0),  $\epsilon$  (1.0), "Frequency constar" (0.0), "Zero Probabilities" (0.0), and "Boundary Parameters" (1.0E-8). The "Bootstrapped Model Fit" panel includes a checkbox for "Bootstrapped p-Value" and an input field for "Number of Samples" (500). A "Reset to Defaults" button is located at the bottom right. The bottom status bar shows "Set 1" and "Set 2" tabs, and a "Add new data set" button. The bottom log window displays the following text: [18:45] Estimation #2, [18:45] Estimation #3: Iteration 101...converged, [18:45] Estimation #1: Iteration 104...converged, [18:45] Estimation #2: Iteration 107...converged, and [18:45] Analysis finished.



# Model definition and analysis

multiTree v0.47 - pc-model\_figure3.mpt

Parameters Data Equations Output Model Builder

Hierarchical Model Families

☐ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).

☐ Compare current model against baseline model

a = u 0.15793

c free 0.41563

r free 0.25263

u free 0.15793

Specification

Number of trees	2
Number of categories	6
Number of free categories	4
Number of parameters	4
Number of constrained parameters	1
Degrees of freedom	1

Output

☒ Standard Errors

☒ 95 % Confidence Intervals

☐ Minimum Description Length

☐ Plot

☐ Specification

☐ Moments

☐ Information Matrix

☐ Jacobian

☐ Iteration History

EM Algorithm

☒ Random Start Values

Replications 2

Convergence 1.0E-10

Max Iterations 5000

$\lambda$  0.0

$\epsilon$  1.0

Frequency constar 0.0

Zero Probabilities 0.0

Boundary Parameters 1.0E-8

Bootstrapped Model Fit

☐ Bootstrapped p-Value

Number of Samples 500

Reset to Defaults

### Welcome to multiTree v0.47 ###

[18:41] Opened file: /Users/edgarerdfelder/Desktop/MPT Workshop mpt files/Section 1.04 Model Equations/pc-model\_figure3.mpt

[18:42] File /Users/edgarerdfelder/Desktop/MPT Workshop mpt files/Section 1.04 Model Equations/pc-model\_figure3.mpt saved.

# Output

The screenshot displays the multiTree v0.47 software interface. The main window is titled "multiTree v0.47 - pc-model\_figure3.mpt". It features a menu bar with "Parameters", "Data", "Equations", "Output", and "Model Builder". The "Output" tab is selected, showing the following content:

File: /Users/edgarerndfelder/Desktop/MPT Workshop mpt files/Section 1.04 Model Equations/pc-Data Set 2: Title: bayen1990aggregated\_young\_lag0

Estimation proceeded normally.

Model Fit

PD<sup>lambda</sup>=0.0 (df=1) = 0.00731 p = 0.93188

ln(likelihood) = -673.97963  
AIC = 1353.95926  
BIC = 1368.01310  
Delta AIC = -1.99269  
Delta BIC = -6.67731

Parameter Estimates, Standard Errors, and Confidence Intervals

a	=	u		
c	=	0.44813	(0.06124)	[0.32811 - 0.56816]
r	=	0.50209	(0.07298)	[0.35905 - 0.64512]
u	=	0.25431	(0.02021)	[0.21470 - 0.29392]

On the right side, there are two panels. The "Output" panel has checkboxes for "Standard Errors" (checked), "95 % Confidence Intervals" (checked), "Minimum Description Length", "Plot", "Specification", "Moments", "Information Matrix", "Jacobian", and "Iteration History". The "EM Algorithm" panel has a checkbox for "Random Start Values" (checked) and input fields for "Replications" (2), "Convergence" (1.0E-10), "Max Iterations" (5000), "λ" (0.0), "ε" (1.0), "Frequency constar" (0.0), "Zero Probabilities" (0.0), and "Boundary Parameters" (1.0E-8). The "Bootstrapped Model Fit" panel has a checkbox for "Bootstrapped p-Value" and an input field for "Number of Samples" (500). A "Reset to Defaults" button is located at the bottom right.

At the bottom of the main window, a log shows the following messages:

```
[18:45] Estimation #2  
[18:45] Estimation #3: Iteration 101...converged  
[18:45] Estimation #1: Iteration 104...converged  
[18:45] Estimation #2: Iteration 107...converged  
[18:45] Analysis finished
```

## 2.2) Practical exercises

- Estimate the pair-clustering model for young and old participants jointly (lag 0 word pairs only)
- Does  $c$  differ significantly between age groups?
- Does  $r$  differ significantly between age groups?

## 2.3 Order constraints

Simple idea:

To impose the order constraint

$$c(\text{old}) \leq c(\text{young}),$$

introduce a new MPT parameter  $s_c$  (called shrinkage parameter) and set

$$c(\text{old}) = s_c \cdot c(\text{young})$$

# Model without order constraints

a_old	= u_old0	0.15793
a_young	= u_young0	0.25431
c_old0	free	0.41563
c_young0	free	0.44813
r_old0	free	0.25263
r_young0	free	0.50209
u_old0	free	0.15793
u_young0	free	0.25431

## Model Fit

$PD^{\lambda=0.0}$  (df=2) = 0.15539      p = 0.92525

ln(likelihood) = -1176.19517  
 AIC = 2364.39034  
 BIC = 2396.65690  
 Delta AIC = -3.84461  
 Delta BIC = -14.60013

## Parameter Estimates, Standard Errors, and Confidence Intervals

a_old	= u_old0		
a_young	= u_young0		
c_old0	= 0.41563	(0.08733)	[0.24446 - 0.58679]
c_young0	= 0.44813	(0.06124)	[0.32810 - 0.56816]
r_old0	= 0.25263	(0.06107)	[0.13294 - 0.37232]
r_young0	= 0.50209	(0.07298)	[0.35905 - 0.64513]
u_old0	= 0.15793	(0.01738)	[0.12385 - 0.19200]
u_young0	= 0.25431	(0.02021)	[0.21470 - 0.29392]

# Imposing order constraint on $c$

**Add parametric order constraints**

Parametric order constraints of the form  $a < b$  may be implemented by applying a reparameterization scheme such that  $a = b * \alpha_A$  (Method A sensu Knapp & Batchelder, 2004). Note that this method does not reduce the number of parameters in a model and, therefore, does not permit using standard procedures of nested model testing. Although you may set multiple order constraints at once (such as  $a < b < c < d$ ), it is recommended to add one order constraint at a time.

☒ Replace current equations

Please select parametric order constraints:

a_old	free
a_young	free
c_old0	< c_young0
c_young0	free

OK Cancel

# Model with order constraint on $c$

a_old	= u_old0	0.5
a_young	= u_young0	0.5
c_young0	free	0.5
r_old0	free	0.5
r_young0	free	0.5
s_c_old0	free	0.5
u_old0	free	0.5
u_young0	free	0.5

## Model Fit

$PD^{\lambda=0.0}$  (df=2) = 0.15539       $p = 0.92525$

ln(likelihood) = -1176.19517

AIC = 2364.39034

BIC = 2396.65690

Delta AIC = -3.84461

Delta BIC = -14.60013

## Parameter Estimates, Standard Errors, and Confidence Intervals

a_old	= u_old0		
a_young	= u_young0		
c_young0	= 0.44813	(0.06124)	[0.32811 - 0.56816]
r_old0	= 0.25263	(0.06107)	[0.13294 - 0.37233]
r_young0	= 0.50208	(0.07298)	[0.35905 - 0.64512]
s_c_old0	= 0.92745	(0.23247)	[0.47183 - 1.38308]
u_old0	= 0.15793	(0.01738)	[0.12385 - 0.19200]
u_young0	= 0.25431	(0.02021)	[0.21470 - 0.29392]

## 2.4 Testing interactions

Simple and straightforward idea:

To test the  $H_0$  that the relative age decline in cluster *storage*  $c$  and cluster *retrieval*  $r$  is the same (i.e., there is no interaction between memory process and age for lag 0 pairs), set

$$c(\text{old}) = s_c \cdot c(\text{young})$$

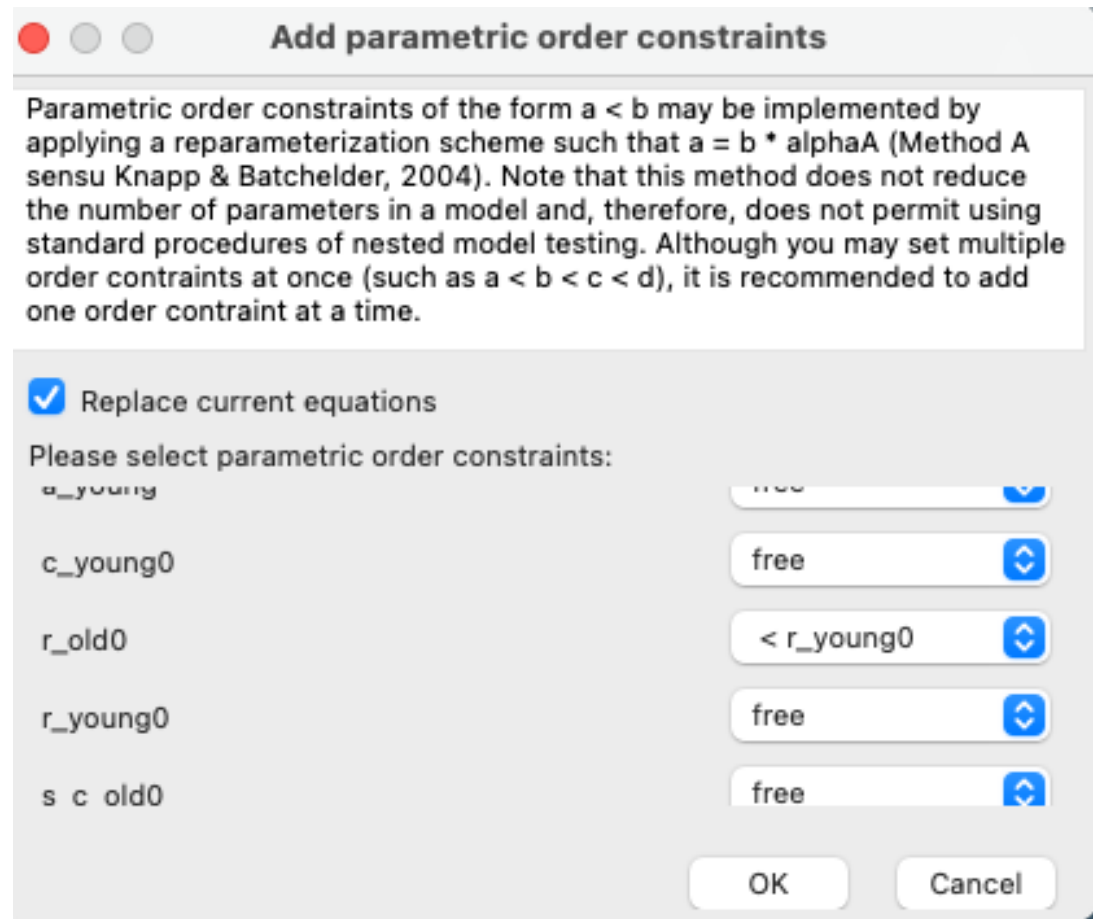
$$r(\text{old}) = s_r \cdot r(\text{young})$$

and test the equality constraint

$$H_0: s_c = s_r$$



# Additional order constraint on $r$



# No constraint on $s$ parameters

Hierarchical Model Families

☒ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).  
☐ Compare current model against baseline model

a_old	= u_old0	0.5
a_young	= u_young0	0.5
c_young0	free	0.5
r_young0	free	0.5
s_c_old0	free	0.5
s_r_old0	free	0.5
u_old0	free	0.5
u_young0	free	0.5

Specification

- Number of trees
- Number of categories
- Number of free categories
- Number of parameters
- Number of constrained parameters
- Degrees of freedom

## Model Fit

$PD^{\lambda=0.0}$  (df=2) = 0.15539 p = 0.92525

ln(likelihood) = -1176.19517

AIC = 2364.39034

BIC = 2396.65690

Delta AIC = -3.84461

Delta BIC = -14.60013

## Parameter Estimates, Standard Errors, and Confidence Intervals

a_old	= u_old0		
a_young	= u_young0		
c_young0	= 0.44813	(0.06124)	[0.32811 - 0.56816]
r_young0	= 0.50208	(0.07298)	[0.35905 - 0.64512]
s_c_old0	= 0.92745	(0.23247)	[0.47183 - 1.38308]
s_r_old0	= 0.50317	(0.14193)	[0.22500 - 0.78134]
u_old0	= 0.15793	(0.01738)	[0.12385 - 0.19200]
u_young0	= 0.25431	(0.02021)	[0.21470 - 0.29392]

# Equality constraint on $s$ parameters

**Hierarchical Model Families**

☐ Define current model as new baseline model (needs to be compared against a baseline model)

☒ Compare current model against baseline model

a_old	= u_old0	0.14604
a_young	= u_young0	0.2592
c_young0	free	0.47009
r_young0	free	0.47633
s_c_old0	= s_r_old0	0.6906
s_r_old0	free	0.6906
u_old0	free	0.14604
u_young0	free	0.2592

## Model Fit

PD $\lambda$ =0.0 (df=3) = 1.29956 p = 0.72924

ln(likelihood) = -1176.76726  
 AIC = 2363.53452  
 BIC = 2390.42331  
 Delta AIC = -4.70044  
 Delta BIC = -20.83372

## Difference to Baseline Model (Difference = Current - Baseline)

PD $\lambda$ =0.0 (df=1) = 1.14417 p = 0.28477

AIC difference = -0.85583  
 BIC difference = -6.23359

## Parameter Estimates, Standard Errors, and Confidence Intervals

a_old	= u_old0		
a_young	= u_young0		
c_young0	= 0.47009	(0.05473)	[0.36283 - 0.57735]
r_young0	= 0.47633	(0.06177)	[0.35527 - 0.59739]
s_c_old0	= s_r_old0		
s_r_old0	= 0.69060	(0.05922)	[0.57454 - 0.80667]
u_old0	= 0.14604	(0.01271)	[0.12112 - 0.17095]
u_young0	= 0.25920	(0.01965)	[0.22068 - 0.29771]

# Conclusion

- There is no significant interaction effect between age (young vs. old) and memory process (cluster storage vs. retrieval) for lag 0 word pairs at least:
- $G^2(1) = 1.14, p = .28$
- This is better in line with the idea of a general cognitive age decline rather than a specific decline limited to retrieval.
- Note, however, that this insignificant outcome might be due to lack of statistical power.
- (We return to the power issue later.)

# Further exercise

- The article by Schmidt, Erdfelder & Heck (2023) extends this analysis to lag 15 presentation conditions for word pairs (with similar results).
- The corresponding mpt files are also available in Part 2 of the workshop zip file.

# Model including lag 0 and lag 15 pairs (without order constraints)

multiTree v0.47 - pc-model\_2g\_2lags.mpt

Parameters Data Equations Output Model Builder

Hierarchical Model Families

☒ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).

☐ Compare current model against baseline model

a_old	free	0.17034
a_young	free	0.24627
c_old0	free	0.44142
c_old15	free	0.1653
c_young0	free	0.44142
c_young15	free	0.1653
r_old0	free	0.2367
r_old15	free	0.4598E
r_young0	free	0.5122E
r_young15	free	1.0
u_old0	= a_old	0.17034
u_old15	= a_old	0.17034
u_young0	= a_young	0.24627
u_young15	= a_young	0.24627

Specification

Number of trees	6
Number of categories	20
Number of free categories	14
Number of parameters	14
Number of constrained parameters	4
Degrees of freedom	4

Output

☒ Standard Errors

☒ 95 % Confidence Intervals

☐ Minimum Description Length

☐ Plot

☐ Specification

☐ Moments

☐ Information Matrix

☐ Jacobian

☐ Iteration History

EM Algorithm

☒ Random Start Values

Replications 2

Convergence 1.0E-10

Max Iterations 20000

$\lambda$  0.0

$\epsilon$  1.0

Frequency constar 0.0

Zero Probabilities 0.0

Boundary Parameters 1.0E-8

Bootstrapped Model Fit

☐ Bootstrapped p-Value

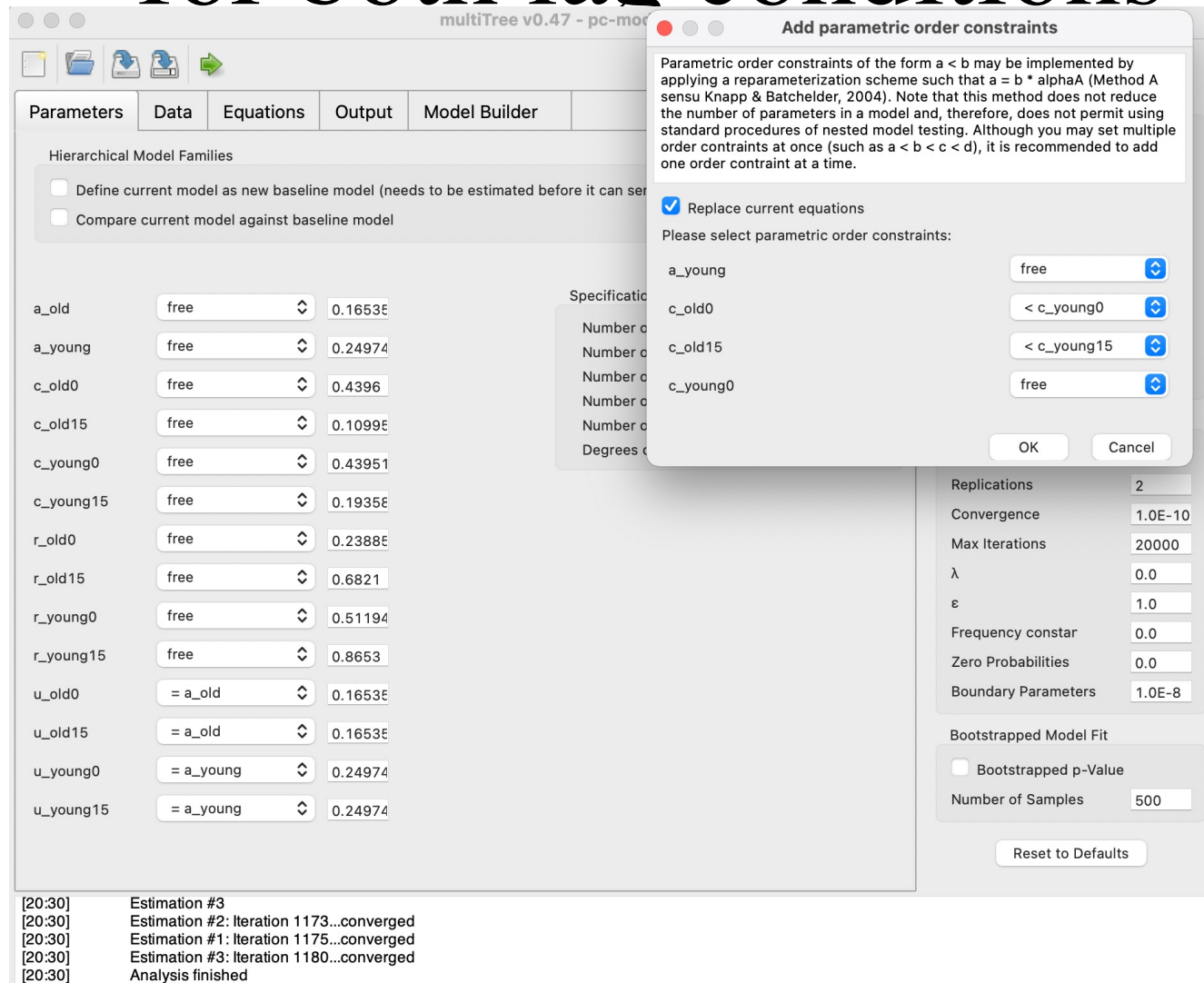
Number of Samples 500

Reset to Defaults

### Welcome to multiTree v0.47 ###

[20:30] Opened file: /Users/edgarerdfelder/Desktop/pc-model\_2g\_2lags.mpt

# Imposing order constraints on $c$ for both lag conditions



# Model with order constraints on $c_0$ and $c_{15}$

The screenshot displays the multiTree v0.47 software interface. The main window is titled "multiTree v0.47 - pc-model\_2g\_2lags.mpt". It features a tabbed interface with "Parameters", "Data", "Equations", "Output", and "Model Builder" tabs. The "Parameters" tab is active, showing a list of parameters with their status (free) and values (0.5). A "Specification" box on the right lists model statistics: Number of trees (6), Number of categories (20), Number of free categories (14), Number of parameters (14), Number of constrained parameters (0), and Degrees of freedom (0). The "Output" panel on the right shows options for Standard Errors, 95% Confidence Intervals, Minimum Description Length, Plot, Specification, Moments, Information Matrix, Jacobian, and Iteration History. The "EM Algorithm" panel shows options for Random Start Values, Replications (2), Convergence (1.0E-10), Max Iterations (20000),  $\lambda$  (0.0),  $\epsilon$  (1.0), Frequency constar (0.0), Zero Probabilities (0.0), and Boundary Parameters (1.0E-8). The "Bootstrapped Model Fit" panel shows options for Bootstrapped p-Value and Number of Samples (500). A "Reset to Defaults" button is located at the bottom right. The bottom status bar shows a log of reparameterization steps.

multiTree v0.47 - pc-model\_2g\_2lags.mpt

Parameters Data Equations Output Model Builder

Hierarchical Model Families

☐ Define current model as new baseline model (needs to be estimated before it can serve as a baseline).

☐ Compare current model against baseline model

a\_old free 0.5

a\_young free 0.5

c\_young0 free 0.5

c\_young15 free 0.5

r\_old0 free 0.5

r\_old15 free 0.5

r\_young0 free 0.5

r\_young15 free 0.5

s\_c\_old0 free 0.5

s\_c\_old15 free 0.5

u\_old0 free 0.5

u\_old15 free 0.5

u\_young0 free 0.5

u\_young15 free 0.5

Specification

Number of trees 6

Number of categories 20

Number of free categories 14

Number of parameters 14

Number of constrained parameters 0

Degrees of freedom 0

Output

☒ Standard Errors

☒ 95 % Confidence Intervals

☐ Minimum Description Length

☐ Plot

☐ Specification

☐ Moments

☐ Information Matrix

☐ Jacobian

☐ Iteration History

EM Algorithm

☒ Random Start Values

Replications 2

Convergence 1.0E-10

Max Iterations 20000

$\lambda$  0.0

$\epsilon$  1.0

Frequency constar 0.0

Zero Probabilities 0.0

Boundary Parameters 1.0E-8

Bootstrapped Model Fit

☐ Bootstrapped p-Value

Number of Samples 500

Reset to Defaults

[20:32] Reparameterizing... c\_old0 < c\_young0

[20:32] Reparameterizing... Current size of tree old\_lag0\_Pairs: 5

[20:32] Reparameterizing... c\_old15 < c\_young15

[20:32] Reparameterizing... Current size of tree old\_lag15\_Pairs: 5

[20:32] Reparameterization completed.