

Fie $\varphi: [1, \infty) \rightarrow [1, \infty)$, $\varphi(x) = \frac{1}{2} \left(x + \frac{1}{x} \right)$

* φ derivabilă pe $[1, \infty)$

* $D = [1, \infty)$, $\text{Im } \varphi = [1, \infty)$

$$\varphi'(x) = \frac{1}{2} \left(1 - \frac{1}{x^2} \right) = \frac{1}{2} \cdot \frac{x^2 - 1}{x^2}$$

$$\varphi'(x) = 0 \Leftrightarrow x = 1$$

x	1	∞
$\varphi'(x)$	0	+
$\varphi(x)$	$\varphi(1) = 1$	$\lim_{x \rightarrow \infty} \varphi(x) = \infty$

$$* |\varphi'(x)| = \frac{1}{2} \cdot \left| \frac{x^2 - 1}{x^2} \right| < \frac{1}{2}$$

$$\text{Deci } \exists g = \frac{1}{2} < 1.$$

* $\varphi(1) = 1$, $1 \in [1, \infty)$

Șirul x_n fi: $x_0 = 2$

$$x_1 = \frac{1}{2} \left(x_0 + \frac{1}{x_0} \right) = \frac{1}{2} \left(2 + \frac{1}{2} \right) = \frac{5}{4}$$

$$x_2 = \frac{1}{2} \left(x_1 + \frac{1}{x_1} \right) = \frac{1}{2} \left(\frac{5}{4} + \frac{4}{5} \right) = \frac{41}{40}$$

$$\dots$$

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{1}{x_n} \right)$$

Șirul x_n are limita 1.

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