Logică — 5.09.2023

Operații aritmetice de bază:

• Adunarea:

Ex.: b_{10}:

$$\begin{array}{r} 7304_{(10)}+ \\ 953_{(10)} \\ \hline 8257_{(10)} \end{array}$$

În caz general (în baza p):

$$\begin{array}{r} a_n a_{n-1} \dots a_1 a_0\,_{(p)}+ \\ b_m b_{m-1} \dots b_1 b_0\,_{(p)} \\ \hline c_k c_{k-1} \dots c_1 c_0\,_{(p)} \end{array}$$

$t_0 = 0$

$(a_i + b_i + t_i) : p = t_{i+1}$, rest c_i

$k = \max(m, n) + 1$

$c_k = t_k$

$$\begin{array}{r} {}^{+1\,+1+1+1\,+1} \\ 1101101_{(2)}+ \\ 1110100_{(2)} \\ \hline 11100001_{(2)} \end{array} \qquad \begin{array}{r} {}^{+1\quad +1} \\ 19AE4_{(16)}+ \\ 37247_{(16)} \\ \hline 50D2B_{(16)} \end{array}$$

$$\begin{array}{r} {}^{+1\qquad +1} \\ 12002201_{(3)}+ \\ 2200101_{(3)} \\ \hline 21210002_{(3)} \end{array} \qquad \begin{array}{r} AB97F_{(16)}+ \\ CD45_{(16)} \\ \hline \end{array}$$

• Scăderea:

• Ex.: baza 10:

$$\begin{array}{r} {}^{-1\,-1} \\ 7204_{(10)}- \\ 853_{(10)} \\ \hline 6351 \end{array}$$

În caz general (în baza p)

$A \geq B$:

$$\begin{array}{r} a_n a_{n-1} \dots a_1 a_0\,_{(p)}- \\ b_m b_{m-1} \dots b_1 b_0\,_{(p)} \\ \hline c_n c_{n-1} \dots c_1 c_0\,_{(p)} \end{array}$$

$t_0 = 0$

$$c_i \begin{cases} a_i - b_i + t_i, \text{ dacă } a_i + t_i \geq b_i, \quad t_{i+1} = 0 \\ a_i - b_i + t_i + p, \text{ dacă } t_{i+1} = -1 \end{cases}$$

$$\begin{array}{r} {}^{-1\,-1} \\ 100110_{(2)}- \\ 100011_{(2)} \\ \hline 000011_{(2)} \end{array} \qquad \begin{array}{r} {}^{-1\quad -1} \\ 16AC05_{(16)}- \\ 3B213_{(16)} \\ \hline 12F9F2_{(16)} \end{array} \qquad \begin{array}{r} {}^{-1\quad -1+3} \\ 12021_{(3)} \\ 1102_{(3)} \\ \hline 10212_{(3)} \end{array}$$

• Înmulțirea cu o cifră:

Ex.: în baza 10

$$\begin{array}{r} {}^{+1\,+1} \\ 7304_{(10)} \\ 4 \\ \hline 29216_{(10)} \end{array}$$

În caz general (în baza p):

$$\begin{array}{r} a_n a_{n-1} \dots a_1 a_0\,_{(p)} \\ b\,_{(p)} \\ \hline c_{n+1} c_n \dots c_1 c_0\,_{p} \end{array}$$

$t_0 = 0$

$(a_i \cdot b + t_i) : p = t_{i+1}$, rest c_i

$c_{n+1} = t_{n+1}$

$$\begin{array}{r} {}^{+1\ +2\ +2} \\ 3456_{(8)} \times \\ 3_{(8)} \\ \hline 12612_{(8)} \end{array}
\qquad
\begin{array}{r} {}^{+3\ +6\ +8} \\ 36BF_{(16)} \times \\ 9_{(16)} \\ \hline 1ECB7_{(16)} \end{array}$$

$15 \cdot 9 = 135 : 16 = 8, r\ 7$
128
007

$99 + 8 = 107 : 16 = 6, r\ 11$
96
$=11$

$54 + 6 = 60 : 16 = 3, r\ 12$
48
12

$27 + 3 = 30 : 16 = 1, r\ 14$
16
14

$$\begin{array}{r} {}^{+1\ +1} \\ 2431_{(5)} \times \\ 2_{(5)} \\ \hline 10412_{(5)} \end{array}$$

$6 : 5 = 1, r\ 1$
$9 : 5 = 1, r\ 4$
$5 : 5 = 1, r\ 0$

• Împărțire la o cifră

Ex.: $7304_{(10)} : 5_{(10)} = 1460, r\ 4$
5
23
20
$=30$
30
$==4$
0
4

În caz general (în baza p):

$a_n a_{n-1} \dots a_1 a_0 : b = c_n c_{n-1} \dots c_1 c_{0(p)}$, rest $r_{(p)}$

$i = \overline{n,0}$ $\qquad (t_i \cdot p + a_i) : b = c_i$, rest t_{i-1}
$t_n = 0$ $\qquad r = t_{-1}$

$1235_{(8)} : 4_{(8)} = 0247_{(8)}, r\ 1$
0
10
8
$= 2 \cdot 8 + 3$
19
16
$= 3 \cdot 8 + 5 = 29$
28
$= 1$

$42 : 3 = 14 = E$

$2A7_{(16)} : 3_{(16)} = 0E2, r\ 1$
0
$2 \cdot 16 + 10 = 42$
42
$== 7$
6
1

$1356_{(7)} : 5_{(7)} =$

Seminar 2 – Conversii

• Nr. întregi prin împărțiri consecutive:

- $198_{(10)} \rightarrow ?_{(16)}$

$198 : 16 = 12, r\,6$
$\;\;16$
$= 38$
$\;\;32$
$\;\;= 6$

$\Rightarrow 198_{(10)} \rightarrow C6_{(16)}$

$12 : 16 = 0, r\,12$

- $230_{(10)} \rightarrow ?_{(4)}$

$230 : 4 = 57, r\,2$
$\;\;20$
$= 30$
$\;\;28$
$\;\;= 2$

$\Rightarrow 230_{(10)} = 3212_{(4)}$

$57 : 4 = 14, r\,1$
$\;\;4$
$= 17$
$\;\;16$
$\;\;= 1$

$14 : 4 = 3, r\,2$
$\;\;12$
$\;\;= 2$

$3 : 4 = 0, r\,3$

• Convertiri partea fracționară prin înmulțiri succesive

$0,x_{(10)} = 0,?_{(q)} = 0,y_1 y_2 \ldots_{(q)}$

Convertiri în baza sursă

$0,x_{(p)} * q_{(p)} = y_{-1}, f_{1\,(p)}$
$0,f_{1(p)} * q_{(p)} = y_{-2}, f_{2\,(p)}$
$\ldots$

Condiția de oprire: $f_n = 0$
În mod general se obține o perioadă

Ex.: 1) $0,65 \cdot 8 = 5,20$
$\underline{0,20} \cdot 8 = 1,60$
$0,60 \cdot 8 = 4,80$
$0,80 \cdot 8 = 6,40$
$0,40 \cdot 8 = \underline{3,20}$

$0,65_{(10)} \longrightarrow 0,5(1463)_{(8)}$

2) $0,46_{(10)} \rightarrow ?_{(4)}$

$0,46 \cdot 4 = 1,84$
$\underline{0,84} \cdot 4 = 3,36$
$0,36 \cdot 4 = 1,44$
$0,44 \cdot 4 = 1,76$
$0,76 \cdot 4 = 3,04$
$0,04 \cdot 4 = 0,16$
$0,16 \cdot 4 = 0,64$
$0,64 \cdot 4 = 2,56$
$0,56 \cdot 4 = 2,24$
$0,24 \cdot 4 = 0,96$
$0,96 \cdot 4 = \underline{3,84}$

$0,46_{(10)} \rightarrow 0,1(3113002203)_{(4)}$

$16 \cdot$
$\;\;5$
$\overline{60}$

3) $0,50 \cdot 16 = 6,00$

$0,500$

• Convertire prin substituție:

$$x_n x_{n-1} \dots x_1 x_0, x_{-1} x_{-2} \dots x_{-m\,(p)} = ?_{(q)}$$

$$x_{n(q)} \cdot p_{(q)}^{n} + x_{n-1(q)} \cdot p_{(q)}^{n-1} + \dots + x_{1(q)} \cdot p_{(q)} + x_{0(q)} + x_{-1(q)} \cdot p_{(q)}^{-1} + \dots + x_{-m(q)} \cdot p_{(q)}^{-m}$$

Ex.: 1) $124,24_{(8)} \to ?_{(10)}$

$$1 \cdot 8^2 + 2 \cdot 8^1 + 4 \cdot 8^0 + 2 \cdot \frac{1}{8} + 4 \cdot \frac{1}{64} =$$

$$= 64 + 16 + 32 + {}^{4)}\frac{1}{4} + \frac{1}{16}$$

$$= 84 + \frac{5}{16}$$

$$= 84,3125_{(10)}$$

$5 : 16 = 0,3125$
0
50
48
=20
16
=40
32
=80
80
==

2) $12,31_{(4)} \to ?_{(10)}$

$$12,31_{(4)} = 1 \cdot 4 + 2 + 3 \cdot \frac{1}{4} + 1 \cdot \frac{1}{16}$$

$$= 4 + 2 + {}^{4)}\frac{3}{4} + \frac{1}{16}$$

$$= 6 + \frac{13}{16}$$

$$= 6,8125_{(10)}$$

$13 : 16 = 0,8125$
0
130
128
==20
16
=40
32
=80
80
==

3) $1AB,4C_{(16)} \to ?_{(10)}$

$$1AB,4C_{(16)} = 1 \cdot 16^2 + 10 \cdot 16 + 11 + {}^{16)}4 \cdot \frac{1}{16} + 12 \cdot \frac{1}{256}$$

$$= 256 + 160 + 11 + \frac{64+12}{256}$$

$$= 427 + \frac{72}{256}$$

$$= 427,2812\dots$$

256+
171
427

$$\frac{72}{256}^{(2} = \frac{36}{128}^{(2} = \frac{18}{64}^{(2} = \frac{9}{32}$$

$9 : 32 = 0,2812$
0
90
64
260
256
==40
32
=80
64
160

• Conversii rapide:

$$?_{(p)} \to ?_{(p^k)}$$

$010.100.011.001,011.100.110.101.100_{(2)}$
$\downarrow$
$2\ 4\ 3\ 1,\ 3\ 4\ 6\ 5\ 4_{(8)}$

$1011.1101.1001,1001.1110.1010_{(2)}$
$\downarrow$
$B\ D\ 9,\ 9\ E\ A_{(16)}$

$437102,76613_{(8)}$
$\downarrow$
$100.011.111.001.000.010,111.110.110.001.011_{(2)}$

$BF27,D35C_{(16)}$
$\downarrow$
$1011.1111.0010.0111,1101.0011.0101.1100_{(2)}$

$27641,11273_{(8)}$
$\downarrow$
$0010.1111.1010.0001,0010.0101.0111.0110_{(2)}$
$\downarrow$
$2\ F\ A\ 1,\ 2\ 5\ 7\ 6_{(16)}$

$A37,B01_{(16)}$
$\downarrow$
$101.000.110.111,101.100.000.001_{(2)}$
$\downarrow$
$5\ 0\ 6\ 7,\ 5\ 4\ 0\ 1_{(8)}$

2	16
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

2	8
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

• Conversia printr-o bază intermediară:

Din $p \xrightarrow{} 10 \xrightarrow{} q$ (subst, :+x) Din $10 \to 8/16 \to 2$ (:+x, rapid)

Din $2 \to 8/16 \to 10$ (rapid, subst.)

$64 \cdot 8 = 512$ $512 \cdot 8 = 4096$ $64 \cdot 6 = 384$

$001.011.001.010.111,\ 110.011.101_{(2)} = ?_{(10)}$

$15127,635_{(8)} = 1\cdot 8^4 + 5\cdot 8^3 + 1\cdot 8^2 + 2\cdot 8 + 7 + 6\cdot \frac{1}{8} + 3\cdot\frac{1}{64} + 5\cdot\frac{1}{512}$
$= 4096 + 5\cdot 512 + 1\cdot 64 + 16 + 7 + \frac{384 + 24 + 5}{512}$
$= 4096 + 2560 + 87 + \frac{413}{512}$
$= 6743,$

$384 + 24 = 408$, $408 + 5 = 413$

$4096 + 2560 = 6656$, $6656 + 87 = 6743$

$413 : 512 = 0,80662$
4130
4096
== 340
3400
3072
= 3280
3072
= 1080
1024
== 560

$368 : 16 = 23, r0$
32
= 48
48
==

$23 : 16 = 1, r7$
16
= 7

$1 : 16 = 0, r1$

$\Rightarrow 368_{(10)} \to 170_{(16)}$

$2112,01_{(3)} = ?_{(7)}$

$2112,01_{(3)} = 2\cdot 3^3 + 1\cdot 3^2 + 1\cdot 3 + 2 + 0\cdot\frac{1}{3} + 1\cdot\frac{1}{9}$
$= 2\cdot 27 + 9 + 3 + 2 + 0 + \frac{1}{9}$
$= 54 + 14 + \frac{1}{9}$
$= 68,111_{(10)}$

$1 : 9 = 0,(1)$
0
10

$68 : 7 = 9, r5$
63
= 5

$9 : 7 = 1, r2$

$1 : 7 = 0, r1$

$0,111 \cdot 7 = 0,777$

$0,777 \cdot 7 = 5,439$

$125,05$

Seminar 3 – 13.10.2023 – Reprezentare numere: Temă 918 Sub3 !!!

Coduri de reprezentare a nr. întregi:

n biți = k octeți

	nr. supraunitar	nr. subunitar
$[x]_d$	$\begin{cases} 0\,\lvert x\rvert, & x\ge 0 \\ 1\,\lvert x\rvert, & x<0 \end{cases}$	$\begin{cases} 0,\lvert x\rvert, & x\ge 0 \\ 1,\lvert x\rvert, & x<0 \end{cases}$
$[x]_i$	$\begin{cases} 0\,\lvert x\rvert, & x\ge 0 \\ 1\,\overline{\lvert x\rvert}, & x<0 \end{cases}$	$\begin{cases} 0,\lvert x\rvert, & x\ge 0 \\ 1,\overline{\lvert x\rvert}, & x<0 \end{cases}$
$[x]_c$	$\begin{cases} 0\,\lvert x\rvert, & x\ge 0 \\ 1\,\ldots, & x<0 \end{cases}$	$\begin{cases} 0,\lvert x\rvert, & x\ge 0 \\ 1,\ldots, & x<0 \end{cases}$
	Ca la Varcea	Ca la Varcea

n = 8

$x, y \ge 1,$ (x: +, y: −) $\qquad s, t < 1$ (s: +, t: −)

1) $x = 17_{(10)} = 10001_{(2)}$

$[x]_d = 00010001_{(2)}$

$[x]_i = 00010001_{(2)}$

$[x]_c = 00010001_{(2)}$

2) $y = -981$

$|-981| = 981 : 16 = 61,\!...5$
96
$=21$
16
$=5$

$61 : 16 = 3,\!..13$
48
13

$3 : 16 = 0,\!..3$

$981 = 3D5_{(16)}$
$= 0011\,|\,11010101_{(2)}$

$[y]_d = 11010101\ b$

$[y]_i = 10101010\ b$

$[y]_c = 10101011\ b$

3) $s = 0,23_{(10)} = 0,0011101_{(2)}$

$0,23 \cdot 2 = 0,46$
$0,46 \cdot 2 = 0,92$
$0,92 \cdot 2 = 1,84$
$0,84 \cdot 2 = 1,68$
$0,68 \cdot 2 = 1,36$
$0,36 \cdot 2 = 0,72$
$0,72 \cdot 2 = 1,44$
$0,44 \cdot 2 = 0,88$

$[s]_d = 00011101$

$[s]_i = 00011101$

$[s]_c = 00011101$

4) $t = -0,56_{(10)} = 0,8F_{(16)} = 0,10001111_{(2)}$

$0,56 \cdot 16$: 336, 56, $8,96$

$0,96 \cdot 16$: 576, 96, $15,36$ → F

$[t]_d = 11000111_{(2)}$

$[t]_i = 10111000_{(2)}$

$[t]_c = 10111001_{(2)}$

Operații în cod complementar:

$[x]_c \oplus [y]_c = \underbrace{[x+y]_c}_{n \text{ biți}}$

$[x]_c = 00010001$
$[y]_c = 10101011$
$[5]_c = 00011101$
$[-7]_c = 10111001$

$[x]_c \ominus [y]_c = [x]_c + [-y]_c$

Obs.: „+” ⊕ „+” = „−”; „−” ⊕ „−” = „+” ⇒ depășire

$[x]_c \oplus [y]_c = \begin{array}{r} 00010001 \\ 10101011 \\ \hline 11111100 \end{array}$ Nu avem depășire

$[x]_c \ominus [y]_c = [x]_c \oplus [-y]_c = \begin{array}{r} 00010001 \\ 01010101 \\ \hline 01100110 \end{array}$

$[y]_c \oplus [y]_c = \begin{array}{r} 10101011 \\ 10101011 \\ \hline 1\ 01010110 \end{array}$ Avem depășire

$[5]_c \oplus [5]_c = \begin{array}{r} 00011101 \\ 00011101 \\ \hline 00111010 \end{array}$ Nu avem depășire

Reprezentare nr. în virgulă fixă:

$[x]_{10} = \_\_\_,\_\_\__{(2)}$

S	ℤ	{x}

17 16 15 8 7 0

$n = 16$

$x = 47,23_{(10)} = 2F,3A_{(16)} = 0010\ 1111,0011\ 1010_{(2)}$

$47 : 16 = 2$, rest 15; 32, 15

$2 : 16 = 0$, rest 2

$0,23 \cdot 16 = 3,68$

$0,68 \cdot 16 = 10,88$

$00101111,00111010$

$y = -27,33$

$27,33 = 1B,54_{(16)} = 0001\ 1011,0101\ 0100_{(2)}$

$27 : 16 = 1$, rest 11; 16, 11

$1 : 16 = 0$, rest 1

$0,33 \cdot 16 = 5,28$

$0,28 \cdot 16 = 4,48$

$10011011,01010101$

Virgula mobilă:

$[x]_{10} = \_\_\_,\_\_\__{(2)}$ → mantisă subunitară: $0,\ldots$; mantisă supraunitară: $1,\ldots$

1) $x = 47,23 = 2F,3AE14$

$0,88 \cdot 16$
528
88
14,08

$0,08 \cdot 16$
048
8
1,28

$0,28 \cdot 16$
168
28
4,48

$= 0010\,1111,0011\,1010\,1110\,0001\,0010_{(2)}$
$= 1,0111100\,111010\,1110000\,10010 \cdot 2^5$

127+
5
132

$132 : 16 = 8, r\,4$
128
= = 4
$8 : 16 = 0, r\,8$

$0,10000010,01111001110101110000101$

$132 = 84_{(16)} = 10000010$

2) $y = -27,33_{(10)} = 1B,547AE_{(16)} =$

$27 : 16 = 1, r\,11$
16
11
$1 : 16 = 0, r\,1$

$0,33 \cdot 16$
198
33
5,28

$0,28 \cdot 16$
168
28
4,48

$0,48 \cdot 16$
288
48
7,68

$0,68 \cdot 16$
408
68
10,88

$0,88 \cdot 16$
528
88
14,08

$= 0001\,1011,0101\,0100\,0111\,1010_{(2)}$
$= 0,110\,11010\,101000\,1111010 \cdot 2^5$

$1,10000100,11011010101000111101011$

3) $z = \frac{-15}{16^2}$

mantisă între 1 și 2

$= -15 \cdot 16^{-2}$
$= -0,0F_{(16)}$
$= -0,0000\,1111$
$= -1,111 \cdot 2^{-5}$

127
−5
$122 = 7A_{(16)} = 0111\,1010$

$122 : 16 = 7, r\,10$
112
= 10
$7 : 16 = 0, r\,7$

$1,0111\,1010,11100000000000000000000$

9.1.8. 3) $(p \wedge q \to r) \to (p \to r) \wedge q$

$((p \wedge q) \to r) \to ((p \to r) \wedge q)$

$((\neg p \vee \neg q) \vee r) \to ((\neg p \vee r) \wedge q)$

$(\neg p \vee \neg q \vee r) \to ((\neg p \wedge q) \vee (r \wedge q))$

$\boxed{(p \wedge q \wedge \neg r) \vee (\neg p \wedge q) \vee (r \wedge q)} \longrightarrow$ FND

p	q	r	$p \wedge q \wedge \neg r$	$\neg p \wedge q$	$r \wedge q$		
0	0	0	0	0	0	0	✗
0	0	1	0	0	0	0	✗
0	1	0	0	1	0	1	✓
0	1	1	0	1	1	1	✓
1	0	0	0	0	0	0	✗
1	0	1	0	0	0	0	✗
1	1	0	1	0	0	1	✓
1	1	1	1	0	1	1	✓

$p \wedge q \wedge \neg r$ $\quad$ $\neg p \wedge q$

$\neg p$: F $\quad$ q: T

T $\quad$ T $\quad$ F

Seminar 4 – 26.10.2023:

9.1.2. 1) $A = \neg p \vee (q \wedge \neg r) \to p \wedge \neg q \wedge r$

Use tabelul de adevăr și dacă e contingentă scrie modelele sau anti-modelele, care sunt mai puține

$(\neg p \vee (q \wedge \neg r)) \to (p \wedge \neg q \wedge r)$

$(p \wedge (\neg q \vee r)) \vee (p \wedge \neg q \wedge r)$

$\underbrace{(p \wedge \neg q)}_{A} \vee \underbrace{(p \wedge r)}_{B} \vee \underbrace{(p \wedge \neg q \wedge r)}_{C}$

p	q	r	A	B	C	
F	F	F	F	F	F	F
F	F	A	F	F	F	F
F	A	F	F	F	F	F
F	A	A	F	F	F	F
A	F	F	A	F	F	A
A	F	A	A	A	F	A
A	A	F	F	F	F	F
A	A	A	A	A	A	A

9.1.1.3.: verifică $p \uparrow (q \uparrow r) \equiv (p \uparrow q) \uparrow r$

p	q	r	$q \uparrow r$	$p \uparrow (q \uparrow r)$	$p \uparrow q$	$(p \uparrow q) \uparrow r$
F	F	F	T	T	T	T
F	F	T	T	T	T	F
F	T	F	T	T	T	T
F	T	T	F	T	T	F
T	F	F	T	F	T	T
T	F	T	T	F	T	F
T	T	F	T	F	F	T
T	T	T	F	T	F	T

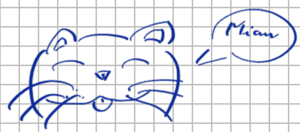
S. 1. 3 3)

$U \models V$: $p \to q = U$; $(p \to q) \to (p \to q \wedge r) = V$

? → Consecință logică

$(p \to q) \models ((p \to r) \to (p \to q \wedge r))$

p	q	r	$p \to q$	$q \wedge r$	$p \to q \wedge r$	$p \to r$	V
F	F	F	T	F	T	T	T
F	F	T	T	F	T	T	T
F	T	F	T	F	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	F	F	F	T
T	F	T	F	F	F	T	F
T	T	F	T	F	F	F	T
T	T	T	T	T	T	T	T



$(\overbrace{p \to (q \vee r)}^{U}) \to (\overbrace{(p \to q) \vee (p \to r)}^{V})$

p	q	r	$q \vee r$	U	$p \to q$	$p \to r$	V	
F	F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T	T
T	F	F	F	F	F	F	F	T
T	F	T	T	T	F	T	T	T
T	T	F	T	T	T	F	T	T
T	T	T	T	T	T	T	T	T

Seminar 5 – 2. 11. 2023:

Seminar 6 – 9.11.2023:

9.1.6. 3) $(U\to(V\to Z))\to(V\to(U\to Z))\equiv$

$\equiv\neg(\neg U\vee(\neg V\vee Z))\vee(\neg V\vee(\neg U\vee Z))$

$\equiv(U\wedge V\wedge\neg Z)\vee\neg U\vee\neg V\vee Z$ (FND cu 4 cuburi)

$\equiv(\underbrace{U\vee\neg U}_{T}\vee\neg V\vee Z)\wedge(\underbrace{V\vee\neg U\vee\neg V}_{T}\vee Z)\wedge(\underbrace{\neg Z\vee\neg U\vee\neg V\vee Z}_{T})$ (FNC cu 3 clauze tautologice)

9.1.9. $(U\to V)\wedge(U\wedge V\to Z)\wedge(U\wedge\neg Z)\equiv$

$\equiv(\neg U\vee V)\wedge(\neg(U\wedge V)\vee Z)\wedge(U\wedge\neg Z)$

$\equiv(\neg U\vee V)\wedge(\neg U\vee\neg V\vee Z)\wedge(U\wedge\neg Z)$

$\equiv(\neg U\vee V)\wedge(\neg U\vee\neg V\vee Z)\wedge U\wedge\neg Z$ (FND cu 4 cuburi)

$\equiv(\neg U\wedge\neg U\wedge U\wedge\neg Z)\vee(\neg U\wedge\neg V\wedge U\wedge\neg Z)\vee(\neg U\wedge Z\wedge U\wedge\neg Z)\vee$
$(V\wedge\neg U\wedge U\wedge\neg Z)\vee(V\wedge\neg V\wedge U\wedge\neg Z)\vee(V\wedge Z\wedge U\wedge\neg Z)$

$\equiv F\vee F\vee F\vee F\vee F\vee F$

$\equiv F$

9.1.11. Folosind tablele semantice deduceți tipul formulei A. Dacă A este consistentă, scrieți toate modelele sale

$A=(p\wedge q\to r)\to(p\to r)\wedge q$ (1) ✓

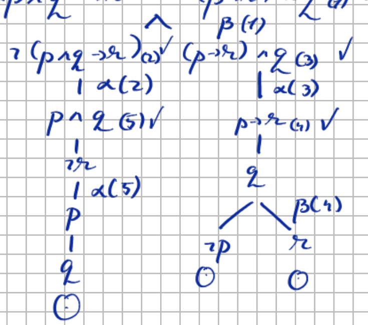


Tabla completă și deschisă ⇒ A- consistentă

$FND(A)=(\neg r\wedge p\wedge q)\vee(\neg p\wedge q)\vee(r\wedge q)$

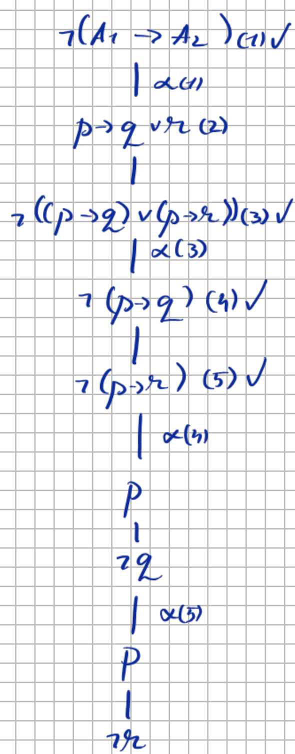
Seminar 7 – 16.11.2023:

1. Dem. că e A este tautologie

3) $A=(p\to q\vee r)\leftrightarrow(p\to q)\vee(p\to r)$

$A\equiv A_1\leftrightarrow A_2\equiv(A_1\to A_2)\wedge(A_2\to A_1)$

A tautologie ⟺ $A_1\to A_2$ tautologie
$A_2\to A_1$ tautologie



$\neg(A_1 \to A_2)$ (1) ✓

$\mid \alpha(1)$

$p \to q \vee r$ (2) ✓

$\neg((p \to q) \vee (p \to r))$ (3) ✓

$\mid \alpha(3)$

$\neg(p \to q)$ (4) ✓

$\neg(p \to r)$ (5) ✓

$\mid \alpha(4)$

p

$\boxed{\neg q}$

$\mid \alpha(5)$

p

$\neg r$

$\beta(2)$

$\neg p$ $\qquad$ $q \vee r$ (6) ✓

$\otimes$ $\qquad$ $\beta(6)$

$\boxed{q}$ $\qquad$ r

$\otimes$ $\qquad$ $\otimes$

Tabelul este închis și complet $\xRightarrow{T.C.C.}$ $A_1 \to A_2$ tautologie (1)

$\neg(A_2 \to A_1) \equiv \neg((p \to q) \vee (p \to r) \to (p \to q \vee r))$ (1) ✓

$\mid \alpha(1)$

$(p \to q) \vee (p \to r)$ (2) ✓

$\neg(p \to q \vee r)$ (3) ✓

$\mid \alpha(3)$

p

$\neg(q \vee r)$ (4) ✓

$\mid \alpha(4)$

$\neg q$

$\neg r$

$\beta(2)$

$p \to q$ (5) ✓ $\qquad$ $p \to r$ (6) ✓

$\beta(5)$ $\qquad$ $\beta(6)$

$\neg p$ $\quad$ q $\qquad$ $\neg p$ $\quad$ r

$\otimes$ $\quad$ $\otimes$ $\qquad$ $\otimes$ $\quad$ $\otimes$

9.1.16.

3) $\neg(p\to r)\to\neg p \models (p\to q)\to(p\to r)$

Tabela închisă $\Longrightarrow$ are loc relația de consecință logică

$((\neg(p\to r)\to\neg p)\wedge\neg((p\to q)\to(p\to r)))$ (1) √

$\alpha(1)$

$\neg(p\to r)\to\neg p$ (2) √

$\neg((p\to q)\to(p\to r))$ (3) √

$\alpha(3)$

$p\to q$ (4) √

$\neg(p\to r)$ (5) √

$\alpha(5)$

p

$\neg r$

$\beta(4)$

$\neg p$ ⊗ | q

$\beta(2)$

$p\to r$ (6) √ | $\neg p$ ⊗

$\beta(6)$

$\neg p$ ⊗ | r ⊗

9.1.22

Seminar 8 — 23.11.2023:

9.1.19*

③ $\underbrace{p \wedge (p \to r)}_{U_1}, \underbrace{q \vee r}_{U_2} \models \underbrace{p \to (q \to r)}_{V}$

$\neg V \equiv \neg(p \to (q \to r)) \equiv \neg(\neg p \vee (\neg q \vee r)) \equiv p \wedge q \wedge \neg r$

$U_1 \equiv p \wedge (p \to r) \equiv p \wedge (\neg p \vee r)$

$U_2 = q \vee r$

$S^0 = \{\underbrace{p}_{C_1}, \underbrace{q}_{C_2}, \underbrace{\neg r}_{C_3}, \underbrace{\neg p \vee r}_{C_4}, \underbrace{q \vee r}_{C_5}\}$

$C_6 = res_p(C_1, C_4) = r$

$C_7 = res_r(C_3, C_4) = \neg p$

$C_2 = res_q(C_3, C_5)$

$S^1 = \{\underbrace{r}_{C_6}, \underbrace{\neg p}_{C_7}\}$

$C_8 = res_r(C_6, C_3) = \square \Rightarrow$ S inconsistentă $\Rightarrow$ Are loc relația de consecință logică

9.1.25:

3) $\underbrace{q \wedge r \to p}_{U_1}, \underbrace{p \vee q}_{U_2}, \underbrace{q \to r}_{U_3} \vdash \underbrace{p}_{V}$

$\neg V \equiv \neg p \overset{not}{=} C_1$

$U_1 \equiv (q \wedge r) \to p \equiv \neg(q \wedge r) \vee p \equiv \neg q \vee \neg r \vee p \overset{not}{=} C_2$

$U_2 \equiv p \vee q \overset{not}{=} C_3$

$U_3 \equiv q \to r \equiv \neg q \vee r \overset{not}{=} C_4$

$S = \{\underbrace{\neg p}_{C_1}, \underbrace{\neg q \vee \neg r \vee p}_{C_2}, \underbrace{p \vee q}_{C_3}, \underbrace{\neg q \vee r}_{C_4}\}$

$S \setminus Y$ - consistentă $\Rightarrow S \setminus Y = \{C_2, C_3, C_4\}$

$Y = \{C_1\}$

$C_5 = res_p\{C_1, C_2\} = \neg q \vee \neg r$

$C_6 = res_p\{C_1, C_3\} = q$

$C_7 = res_p\{C_5, C_6\} = \neg r$

$C_8 = res_q(C_4, C_6) = r$

$C_9 = res_p(C_7, C_8) = \square$ - inconsistentă

$\overset{TCC}{\Longrightarrow}$ S inconsistentă $\Rightarrow$ are loc deducția logică

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1. 24.*

3) $S_1 = \{q \vee r, \cancel{\neg p}, \neg q \vee r, p \vee \neg r\}$

Pas1: Eliminare clauze tautologice: $\nexists$

Pas2: Elim. clauze subsumate: $\nexists$

Pas 3: Elim. clauze care contin literar pur: $\nexists$

Pas 4: Clauza unitate: $\neg p$

$S_2 = \{q \vee r, \neg q \vee r, \neg r\}$: $\neg r$ - clauză unitate

$S_3 = \{q, \neg q\}$: q - clauză unitate

$S_4 = \{\square\}$ - inconsistentă $\xRightarrow{TCC}$ S_1 - inconsistentă

Seminar 10 - 7.12.2023:

5. 1.24. Construiți o respingere liniară din mulțimea de clauze S. Există respingeri input și unit din S?

3) $S = \{\underset{C_1}{q \vee r}, \underset{C_2}{\neg p}, \underset{C_3}{\neg q \vee r}, \underset{C_4}{p \vee \neg r}\}$

$C_1 = q \vee r$, $C_3 = \neg q \vee r$

$C_5 = r$, $C_4 = p \vee \neg r$

$C_6 = p$, $C_2 = \neg p$

$C_7 = \square$ $\xRightarrow{TCC}$ S inconsistentă

Aceasta este o respingere input $\xRightarrow{T. Echiv.}$ $\exists$ respingere unit

$C_2 = \neg p$, $C_4 = p \vee \neg r$

$C_5 = \neg r$, $C_1 = q \vee r$

$C_6 = q$, $C_3 = \neg q \vee r$

$C_7 = r$, $C_4 = p \vee \neg r$

$C_8 = p$, $C_2 = \neg p$

$C_9 = \square$ $\xRightarrow{TCC}$ S este inconsistentă

Este o rezoluție unit și input

9.1.26. 3) Dem. legea silogismului $(p\to q)\to((q\to r)\to(p\to r))$ utilizând o metodă directă

$A: (p\to q)\to((q\to r)\to(p\to r))$

p	q	r	$p\to q$	a) $q\to r$	b) $p\to r$	a) $\to$ b)	A
F	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	F	F	T
T	F	T	F	T	T	T	T
T	T	F	T	F	F	T	T
T	T	T	T	T	T	T	T

$\Rightarrow A$ este Tautologie

Predicate:

9.2.2 Suma a două nr. pare este un număr par

$D=\mathbb{N}$

const –

$f:\mathbb{N}^2\to\mathbb{N},\ f(x,y)=x+y$

$P:\mathbb{N}\to\{T,F\}$, $P(x)=$ „x este par"

$(\forall x)(\forall y)\ (P(x)\wedge P(y)\to P(f(x,y)))$

9.2.4* Dați o interpretare cu dom. finit și cu dom. ∞ pt. form. de mai jos a.î. una să fie model și cealaltă anti-model

$U=((\forall x)P(x)\to(\forall x)Q(x))\to\forall(x)\ (P(x)\to Q(x))$

$I_1=\langle D_1, m_1\rangle$

$D_1=\mathbb{N}\ (\infty)$

$m_1(P):\mathbb{N}\to\{T,F\}$, $m(P)(x)=$ „x impar"

$m_1(Q):\mathbb{N}\to\{T,F\}$, $m(Q)(x)=$ „$x\ \vdots\ 1$"

$$\begin{aligned}
v^{I_1}(U) &= v^{I_1}((\forall x)P(x)\to(\forall x)Q(x))\to v^{I_1}((\forall x)(P(x)\to Q(x)))\\
&= v^{I_1}((\forall x)(P(x)))\to v^{I_1}((\forall x)Q(x))\to v^{I_1}((\forall x)(P(x)\to Q(x)))\\
&= (F\to T)\to T\\
&= T\to T\\
&= I_1 \text{ este model cu dom. } \infty
\end{aligned}$$

$I_2=\langle D_2, m_2\rangle$

$D_2=\{2,4\}$

$m_2(P):D_2\to\{T,F\}$, $m_2(P)(x)=$ „x par"

$m_2(Q):D_2\to\{T,F\}$, $m_2(Q)(x)=$ „

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Seminar 10 – 14. 12. 2023:

5.2.7

$U = (\forall x)(\exists y)((\exists z) P(z) \wedge (\exists v)(Q(x,v) \to (\exists z)\, Q(y,z)))$

Pasul 3:

$\equiv (\forall x)(\exists y)((\exists z) P(z) \wedge (\exists u)(\neg Q(x,u) \vee (\exists x)\, Q(y,x)))$

Pasul 4:

$\equiv (\forall x)(\exists y)(\exists z)(\exists u)(\exists t)(P(z) \wedge (\neg Q(x,u) \vee Q(y,t)))$

Pasul 5:

$y \leftarrow f(x);\quad z \leftarrow g(x);\quad u \leftarrow h(x);\quad t \leftarrow j(x)$

$(\forall x)(P(g(x)) \wedge (\neg Q(x,h(x)) \vee Q(f(x),j(x))))$

Pasul 6:

$U^{Sg1} = P(g(x)) \wedge (\neg Q(x,h(x)) \vee Q(f(x),j(x)))$

Pasul 7 FNC:

$P(g(x)) \wedge (\neg Q(x,h(x)) \vee Q(f(x),j(x)))$

5.2.13.

$\neg((\forall x)(P(x) \to Q(x)) \to ((\forall x)(P(x) \to (\forall x)(Q(x)))) \; \cancel{(1)}$

$\downarrow \alpha(1)$

$(\forall x)(P(x) \to Q(x)) \; \cancel{(2)}$

$\downarrow$

$\neg((\forall x) P(x) \to (\forall x) Q(x)) \; \cancel{(3)}$

$\downarrow \alpha(3)$

$(\forall x) P(x) \; \cancel{(4)}$

$\downarrow$

$\neg((\forall x) Q(x)) \; \cancel{(5)}$

$\downarrow \delta(5)$ – a const. nouă

$\boxed{\neg Q(a)}$

$\downarrow \gamma(4)$

$\boxed{P(a)}$

$\downarrow$

$(\forall x)(P(x))(4')$

$\downarrow \gamma(2)$

$P(a) \to Q(a) \; \cancel{(6)}$

$\downarrow$

$(\forall x)(P(x) \to Q(x))\, (2')$

$\swarrow \quad \searrow \beta(6)$

$\boxed{\neg P(a)}$ $\qquad$ $\boxed{Q(a)}$

$\otimes$ $\qquad$ $\otimes$

$\neg(((\forall x)\ P(x) \to (\forall x)\ Q(x)) \to \forall(x)(P(x) \to Q(x)))\ \cancel{(1)}$

$\downarrow \alpha(1)$

$(\forall x)\ P(x) \to (\forall x)\ Q(x)\ \cancel{(2)}$

$\downarrow$

$\neg((\forall x)(P(x) \to Q(x)))\ \cancel{(3)}$

$\downarrow \delta(3)$ — a const. nouă

$\neg(P(a) \to Q(a))\ \cancel{(4)}$

$\downarrow \alpha(4)$

$P(a)$

$\downarrow$

$\neg Q(a)$

$\swarrow \quad \searrow \beta(2)$

$\neg((\forall x)\ P(x))\ \cancel{(5)}$ | $(\forall x)\ Q(x)\ \cancel{(6)}$

b - const. nouă $\downarrow \delta(5)$ | $\downarrow \gamma(6)$

$\neg P(b)$ | $Q(a)$

$\odot$ | $\otimes$

9.2.14.

$U = (\forall y)(\exists x)\ P(x,y) \Leftrightarrow (\exists y)(\exists x)\ P(x,y)$

$\neg((\exists y)(\exists x)\ P(x,y) \to (\forall y)(\exists x)\ P(x,y))\ \cancel{(1)}$

$\downarrow \alpha(1)$

$(\exists y)(\exists x)\ P(x,y)\ \cancel{(2)}$

$\downarrow$

$\neg((\forall y)(\exists x)\ P(x,y))\ \cancel{(3)}$

$\downarrow \delta(2)$ — a const. noi

$(\exists x)\ P(x,a)\ \cancel{(4)}$

$\downarrow \delta(3)$ — b const. nouă

$\neg((\exists x)\ P(x,b))\ (5)$

$\downarrow \delta(4)$ — c const. nouă

$P(c,a)$

$\downarrow \gamma(5)$ — a, b, c

$\neg P(a,b)$

$\downarrow$

$\neg P(b,b)$

$\downarrow$

$\neg P(c,b)$

$\downarrow$

$\neg((\exists x)\ P(x,b))\ (5')$

Seminar 12 - 15.12.2023:

9.2.8.
$\begin{cases} A_1 = P(a, f(x), g(h(a))) \\ A_2 = P(y, f(z), g(z)) \end{cases}$

$\begin{cases} A_3 = P(x, g(f(a)), h(x, y)) \\ A_4 = P(f(y), z, g(y)) \end{cases}$

$\begin{cases} A_5 = P(g(y), x, f(g(y))) \\ A_6 = P(z, f(z, u), f(u)) \end{cases}$

Pas 1: Același sim pred?

Pas 2: Aceeași aritate?

Pas 3: Dacă $\neq$ primul caracter $\neq$ $\exists$ subst.

Pentru A_1, A_2:

Pas 1: DA (P)

Pas 2: DA (3)

Pas 3: $\theta_1 = [y \leftarrow a]$

$\theta_1(A_1) = P(a, f(x), g(h(a)))$

$\theta_1(A_2) = P(a, f(z), g(z))$

$\theta_2 = [x \leftarrow z]$

$\theta_2(\theta_1(A_1)) = P(a, f(z), g(h(a)))$

$\theta_2(\theta_1(A_2)) = P(a, f(z), g(z))$

$\theta_3 = [z \leftarrow h(a)]$

$\theta_3(\theta_2(\theta_1(A_1))) = P(a, f(h(a)), g(a))$

$\theta_3(\theta_2(\theta_1(A_2))) = P(a, f(h(a)), g(a))$

$\Rightarrow A_1, A_2$ - unificabil, $MGU(A_1, A_2) = [y \leftarrow a, x \leftarrow h(a), z \leftarrow h(a)]$

Pentru A_3, A_4

Pas 1: DA (P)

Pas 2: DA (3)

Pas 3:

$\theta_1 = [x \leftarrow f(y)]$

$\theta_1(A_3) = P(f(y), g(f(a)), h(f(y), y))$

$\theta_1(A_4) = P(f(y), z, g(y))$

$\theta_2 = [z \leftarrow g(f(a))]$

$\theta_2(\theta_1(A_3)) = P(f(y), g(f(a)), h(f(y), y))$

$\theta_2(\theta_1(A_4)) = P(f(y), g(f(a)), g(y))$

Nu se pot sub 2 funcții $\Rightarrow A_3, A_4$ NU sunt unificabile

Pentru A_5, A_6:

Pas 1: DA

Pas 2: DA

Pas 3:

$\theta_1 = [z \leftarrow g(y)]$

$\theta_1(A_5) = P(g(y), x, f(g(y)))$

$\theta_1(A_6) = P(g(y), f(g(y), u), f(u))$

$\theta_2 = [x \leftarrow f(g(y), u)]$

$\theta_2(\theta_1(A_5)) = P(g(y), f(g(y), u), f(g(y)))$

$\theta_2(\theta_1(A_6)) = P(g(y), f(g(y), u), f(u))$

$\theta_3 = [u \leftarrow g(y)]$

$\theta_3(\theta_2(\theta_1(A_5))) = P(g(y), f(g(y), g(y)), f(g(y)))$

$\theta_3(\theta_2(\theta_1(A_6))) = P(g(y), f(g(y), g(y)), f(g(y)))$

$\Rightarrow A_5, A_6$ sunt unificabile

$MGU = [z \leftarrow g(y), x \leftarrow f(g(y), g(y)), u \leftarrow g(y)]$

9.2.17.3.

$(\forall x)(\neg P(x) \wedge \neg Q(x) \rightarrow R(x)), (\forall y)(R(y) \rightarrow W(y)), (\forall x)(W(x) \rightarrow P(x)), \neg P(a), \neg P(b), \neg W(c) \vdash (\exists z)\ Q(z)$

$S_1 = \{ \underset{C_1}{P(x) \vee Q(x) \vee R(x)}, \underset{C_2}{\neg R(y) \vee W(y)}, \underset{C_3}{\neg W(x) \vee P(x)}, \underset{C_4}{\neg P(a)}, \underset{C_5}{\neg P(b)}, \underset{C_6}{\neg W(c)}, \underset{C_7}{\neg Q(z)})$

$$S_1 = \{ \underset{C_1}{P(x) \vee Q(x) \vee R(x)},\ \underset{C_2}{\neg R(y) \vee W(y)},\ \underset{C_3}{\neg W(x) \vee P(x)},\ \underset{C_4}{\neg P(a)},\ \underset{C_5}{\neg R(b)},\ \underset{C_6}{\neg W(c)},\ \underset{C_7}{\neg Q(z)})$$

$C_7 = \neg Q(z)$ $\qquad$ $C_1 = P(x) \vee Q(x) \vee R(x)$

$[z \leftarrow x]$

$C_8 = P(x) \vee R(x)$ $\qquad$ $C_2 = \neg R(y) \vee W(y)$

$[x \leftarrow y]$

$C_9 = P(y) \vee W(y)$ $\qquad$ $C_3 = \neg W(x) \vee P(x)$

$[x \leftarrow y]$

$C_{10} = P(y)$ $\qquad$ $C_4 = \neg P(a)$

$[y \leftarrow a]$

$C_{11} = \square \xrightarrow{T.C.C.}$ S este inconsistentă $\Rightarrow$ are loc reducția

3.2.21.5.

$$(\forall x)(\exists y)\neg(P(y,y) \leftrightarrow \neg P(x,y)) \overset{not}{=\!=} U$$

$$\neg U \equiv (\exists x)(\forall y)((P(y,y) \to \neg P(x,y)) \wedge (\neg P(x,y) \to P(y,y)))$$

$$\neg U \equiv (\exists x)(\forall y)(\neg P(y,y) \vee \neg P(x,y)) \wedge (P(x,y) \vee P(y,y))$$

$$\neg U \equiv \cancel{(\forall y)}(\underset{C_1}{\neg P(y,y) \vee \neg P(a,y)}) \wedge (\underset{C_2}{P(a,y) \vee P(y,y)})$$

$$S = \{ \neg P(y,y) \vee \neg P(a,y),\ P(a,y) \vee P(y,y) \}$$

$C_1 \mid \xrightarrow[{[y \leftarrow a]}]{fact} \neg P(a,a)$

$C_2 \mid \xrightarrow[{[y \leftarrow a]}]{fact} P(a,a)$

$C_1 \qquad C_2$

$C_3 = \square \xrightarrow{T.C.C.}$ S – inconsistentă $\Rightarrow$ U tautologie

Seminar 13 − 11.01.2024:

9.3.1. Pt. următoarele funcții booleene de 3 variabile, date prin intermediul tabelelor de valori, scrieți cele 2 forme canonice: conjunctivă și disjunctivă. Simplificați funcțiile utilizând diagramele Veitch.

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

$m_0 = \bar{x}\,\bar{y}\,\bar{z}$
$m_1 = \bar{x}\,\bar{y}\,z$
$m_2 = \bar{x}\,y\,\bar{z}$
$M_3 = x \vee \bar{y} \vee \bar{z}$
$m_4 = x\,\bar{y}\,\bar{z}$
$M_5 = \bar{x} \vee y \vee \bar{z}$
$M_6 = \bar{x} \vee \bar{y} \vee z$
$M_7 = \bar{x} \vee \bar{y} \vee \bar{z}$

m_0	M_3
1	1
0	1
0	1
0	0
0	1
0	1
0	1
0	1

$$FCC(f) = M_3 \wedge M_5 \wedge M_6 \wedge M_7 = (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{y} \vee z) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$$

$$FCD(f) = m_0 \vee m_1 \vee m_2 \vee m_4 = \bar{x}\,\bar{y}\,\bar{z} \vee \bar{x}\,\bar{y}\,z \vee \bar{x}\,y\,\bar{z} \vee x\,\bar{y}\,\bar{z}$$

	x	x	$\bar{x}$	$\bar{x}$
y			m_2	
$\bar{y}$		m_4	m_0	m_1
	z	$\bar{z}$	$\bar{z}$	z

$max_1 = m_4 \vee m_0 = \bar{y}\,\bar{z}$
$max_2 = m_0 \vee m_1 = \bar{x}\,\bar{y}$
$max_3 = m_2 \vee m_0 = \bar{x}\,\bar{z}$

$M(f) = \{max_1, max_2, max_3\}$
$C(f) = M(f) \Rightarrow$
$\Rightarrow$ caz 1 de simplificare
$f'(x,y,z) = \bar{y}\,\bar{z} \vee \bar{x}\,\bar{y} \vee \bar{x}\,\bar{z}$

9.3.2. Simplificați următoarele funcții booleene de 4 variabile, date prin formele disjunctive, utilizând diagrama Veitch

$$f_3(x_1,x_2,x_3,x_4) = \underbrace{\bar{x}_1 x_2 x_3 x_4}_{m_7} \vee \underbrace{\bar{x}_1 x_2 \bar{x}_3 \bar{x}_4}_{m_4} \vee \underbrace{\bar{x}_1 x_2 x_3 \bar{x}_4}_{m_6} \vee \underbrace{x_1 \bar{x}_2 x_3 \bar{x}_4}_{m_{10}} \vee \underbrace{x_1 \bar{x}_2 \bar{x}_3 \bar{x}_4}_{m_8} \vee \underbrace{\bar{x}_1 \bar{x}_2 \bar{x}_3 \bar{x}_4}_{m_0} \vee \underbrace{x_1 \bar{x}_2 x_3 x_4}_{m_{11}} \vee \underbrace{x_1 \bar{x}_2 \bar{x}_3 x_4}_{m_9} \vee \underbrace{\bar{x}_1 \bar{x}_2 \bar{x}_3 x_4}_{m_1}$$

	x_1	x_1	$\bar{x}_1$	$\bar{x}_1$	
x_2				m_7	x_4
x_2			m_4	m_6	$\bar{x}_4$
$\bar{x}_2$	m_{10}	m_8	m_0		$\bar{x}_4$
$\bar{x}_2$	m_{11}	m_9	m_1		x_4
	x_3	$\bar{x}_3$	$\bar{x}_3$	x_3	

$max_1 = m_8 \vee m_9 \vee m_{10} \vee m_{11} = x_1 \bar{x}_2$
$max_2 = m_8 \vee m_0 \vee m_1 \vee m_9 = \bar{x}_2\,\bar{x}_3$
$max_3 = m_4 \vee m_6 = \bar{x}_1\,x_2\,\bar{x}_4$
$max_4 = m_6 \vee m_7 = \bar{x}_1\,x_2\,x_3$
$max_5 = m_0 \vee m_4 = \bar{x}_1\,\bar{x}_3\,\bar{x}_4$

$M(f) = \{max_1, max_2, max_3, max_4, max_5\}$
$C(f) = \{max_1, max_2, max_4\}$

$M(f) \neq C(f) \neq \emptyset \rightarrow$ caz II

$g(x_1,x_2,x_3,x_4) = x_1\bar{x}_2 \vee \bar{x}_2\,\bar{x}_3 \vee \bar{x}_1 x_2 x_3$
$h_1(x_1,x_2,x_3,x_4) = \bar{x}_1\,x_2\,\bar{x}_4$
$h_2(x_1,x_2,x_3,x_4) = \bar{x}_1\,\bar{x}_3\,\bar{x}_4$

$$f_1'(x_1,x_2,x_3,x_4) = g(x_1,x_2,x_3,x_4) \vee h_1(x_1,x_2,x_3,x_4) = x_1\bar{x}_2 \vee \bar{x}_2\bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee \bar{x}_1 x_2 \bar{x}_4$$

$$f_2'(x_1,x_2,x_3,x_4) = g(x_1,x_2,x_3,x_4) \vee h_2(x_1,x_2,x_3,x_4) = x_1\bar{x}_2 \vee \bar{x}_2\bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee \bar{x}_1\bar{x}_3\bar{x}_4$$

9.3.3. Simplificați următoarele funcții booleene de 3 variabile, date prin mintermenii expresiilor, utilizând diagrama Karnaugh.

3) $f(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_7$

x_1 \ x_2x_3	00	01	10	11
0		m_1	m_3	m_2
1	m_4	m_5	m_7	

$max_1 = m_1 \vee m_3 \vee m_5 \vee m_7 = x_3$

$max_2 = m_3 \vee m_2 = \overline{x_1}\, x_2$

$max_3 = m_4 \vee m_5 = x_1 \overline{x_2}$

$$M(f) = \{max_1,\ max_2,\ max_3\}$$

$$C(f) = M(f) \Rightarrow caz\ \underline{I} \Rightarrow f'(x_1,x_2,x_3) = x_3 \vee \overline{x_1}x_2 \vee x_1\overline{x_2}$$

9.3.7. Simplificați următoarele funcții booleene de n var. utilizând diagrama Karnaugh:

$f(x_1,x_2,x_3,x_4) = x_1x_4 \vee x_1\overline{x_2}\,\overline{x_3}\,\overline{x_4} \vee \overline{x_1}\,\overline{x_2}\,\overline{x_4} \vee \overline{x_1}\,\overline{x_3} \vee x_3x_4$

x_1	x_2	x_3	x_4	x_1x_4	$x_1\overline{x_2}\,\overline{x_3}\,\overline{x_4}$	$\overline{x_1}\,\overline{x_2}\,\overline{x_4}$	$\overline{x_1}\,\overline{x_3}$	x_3x_4	f	
0	0	0	0	0	0	1	1	0	1	m_0
0	0	0	1	0	0	0	1	0	1	m_1
0	0	1	0	0	0	1	0	0	1	m_2
0	0	1	1	0	0	0	0	1	1	m_3
0	1	0	0	0	0	0	1	0	1	m_4
0	1	0	1	0	0	0	1	0	1	m_5
0	1	1	0	0	0	0	0	0	0	
0	1	1	1	0	0	0	0	1	1	m_7
1	0	0	0	0	1	0	0	0	1	m_8
1	0	0	1	1	0	0	0	0	1	m_9
1	0	1	0	0	0	0	0	0	0	
1	0	1	1	1	0	0	0	1	1	m_{11}
1	1	0	0	0	0	0	0	0	0	
1	1	0	1	1	0	0	0	0	1	m_{13}
1	1	1	0	0	0	0	0	0	0	
1	1	1	1	1	0	0	0	1	1	m_{15}

x_3x_4 \ x_1x_2	00	01	11	10
00	m_0	m_4		m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2			

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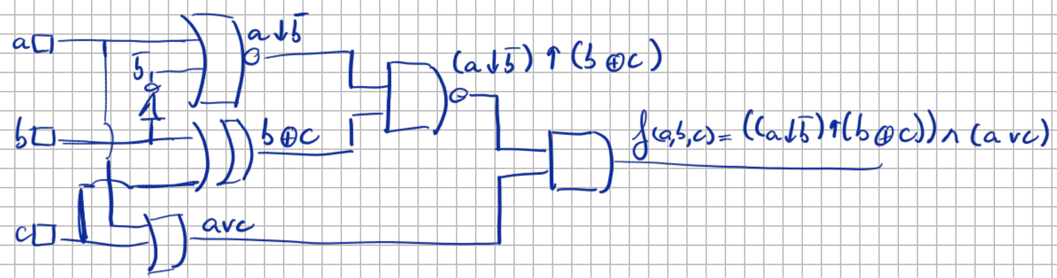
S.3.9. Să se deseneze un circuit cu 3 variabile, să se simplifice și să se deseneze circuitul simplificat



a	b	c	$\bar{b}$	A: $a \downarrow \bar{b}$	B: $b \oplus c$	$A \uparrow B$	$a \vee c$	f	
0	0	0	1	0	0	1	0	0	
0	0	1	1	0	1	1	1	1	m_1
0	1	0	0	1	1	0	0	0	
0	1	1	0	1	0	1	1	1	m_3
1	0	0	1	0	0	1	1	1	m_4
1	0	1	1	0	1	1	1	1	m_5
1	1	0	0	0	1	1	1	1	m_6
1	1	1	0	0	0	1	1	1	m_7

$$S(f) = \{(0,0,1),(0,1,1),(1,0,0),(1,0,1),(1,1,0),(1,1,1)\}$$

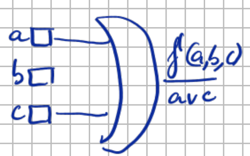
min \ max	max_1	max_2
m_1	*	
m_3	*	
m_4		*
m_5	*	*
m_6		*
m_7	*	*

$C(f) = \{max_1, max_2\}$

$M(f) = C(f) \Rightarrow$ Caz I $\Rightarrow$

$\Rightarrow$ o formă simplificată

$f(a,b,c) = c \vee a$



S.3.4. Utilizând met. lui Quine simplificați următoarele funcții booleene de 3 var

3) $f(x_1,x_2,x_3) = x_2(x_1 \uparrow x_3) \vee \overline{x_2} x_3$

$= x_2(\overline{x_1 \wedge x_3}) \vee \overline{x_2} x_3$

$= x_2(\overline{x_1} \vee \overline{x_3}) \vee \overline{x_2} x_3$

$= \overline{x_1} x_2 \vee x_2 \overline{x_3} \vee \overline{x_2} x_3$

$= \overline{x_1} x_2 \vee x_3$

	max_1	max_2
m_1		*
m_2	*	
m_3	*	*
m_5		*
m_7		*

$M(f) = C(f) \Rightarrow$ Cazul I $\Rightarrow$

$\Rightarrow$