Robust Regulation of Technological Processes with Delay in the Class of Traditional Regulation Laws*

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Abstract— **The article considered a new method of constructing traditional regulatory laws, providing a qualitative robust stabilization of the system with delay under conditions of uncertainty of setting the value of the delay at the input and the parameters of the inertial part of the transfer function of the object, which allows us to find a compromise between the roughness and performance of the system.**

Keywords— robust control systems; PID regulation law; delay

One of the main difficulties in creating algorithmic support for solving the problems of automation of technological processes is the presence of a delay in control in the mathematical model. This is a traditional task for the theory of control, and it is devoted to a significant number of publications in a non-robust setting and using traditional regulatory laws [1–5]. Also, there are a number of works that consider non-traditional robust approaches to the management of such objects [6–10].

In the practice of automation of technological processes, most of the control systems is built using PID regulation laws, therefore, the class of traditional regulators. The construction such regulators robust tuning methods is an actual problem. The increase in robust for systems of this class is usually accompanied by a loss of performance. Therefore, the actual task is to build a methodology for the synthesis of robust regulators, which will allow for a compromise between the performance and roughness of the system with traditional regulators. The proposed method is an extension of the methods described in [1, 2].

The relevance of the robust approach is determined by the fact that the regulator must provide the quality control in the widest possible the object model coefficients range. This increases the reliability of the system in the period between two adjacent retuning of the regulator.

Consider the transfer functions of the object

$$W_0(p) = k_0 \frac{\beta_m(p)}{\alpha_n(p)} \exp(-\tau p) \quad (1)$$

where $\alpha_n(p), \beta_m(p)$ – polynomials of orders n and m, $\alpha_n(0) = \beta_m(0) = 1$, k_0 – transfer factor, $\underline{k_0} \le k_0 \le \overline{k_0}$, τ – delay time $\underline{\tau} \le \tau \le \bar{\tau}$, coefficients of polynomials $\alpha_n(p), \beta_m(p)$ can vary at given intervals.

It is necessary to notice the roots of the polynomial $\beta_m(p)$ lie strictly to the left of the imaginary axis, and the roots of the polynomial $\alpha_n(p)$ lie either to the left of the imaginary axis, or some of them are located at the zero point.

Along with the real transfer function, the nominal transfer function of the object is considered, which coincides in structure with the transfer function (1), the follow form

$$W_0^0(p) = k_0^0 \frac{\beta_m^0(p)}{\alpha_n^0(p)} \cdot \exp(-\tau_0 p), \quad (2)$$

where k_0^0, τ_0 – nominal transmission and delay values, $\beta_m^0(p), \alpha_n^0(p)$ – nominal polynomials of the numerator and denominator, $\left(m \le n, \beta_m^0(0) = \alpha_n^0(0) = 1\right)$, the roots of the polynomials $\beta_m^0(p)$ are located strictly to the left of the imaginary axis, and the roots of the polynomial $\alpha_n(p)$ either lie to the left of the imaginary axis, or some of them are located at the point zero (the number of such roots is the same as in the real model), the coefficients of the nominal model belong to the intervals indicated in (1), $\underline{k_0} \le k_0^0 \le \overline{k_0}$, $\underline{\tau} \le \tau_0 \le \bar{\tau}$.

The requirement of stability and minimum phase stability of the transfer functions (1), (2) is due to the subsequent application of the dynamic compensation method for the approximating transfer function.

In the case where the transfer function of the object has poles at the zero point, the synthesis of the regulator uses an approximation of the form

$$\frac{1}{p} \approx \frac{1}{p+\varepsilon} = \frac{\gamma}{\gamma p + 1}, \quad (3)$$

where $\varepsilon << 1$, $\gamma = \varepsilon^{-1} >> 1$.

Assumed that the inertial part of the object with a delay in control has an arbitrary complexity, and the magnitude of the delay can vary significantly. Therefore, the dynamic control method assumed to base on the concept of regulator synthesis. In

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the transfer function of the object with delay, the base part (basic transfer function), which determines the dominant component of the transition process, and for it the basic control problem is solved, providing a compromise between roughness and performance. The remaining dynamics of the object taken into account in the framework of the compensation method provided the conditions for the existence of such a solution are met.

As basic transfer functions of the object, two nominal transfer functions are selected, such as

$$W_0^0(p) = k_0^0 \exp(-\tau_0 p), \tag{4}$$

$$W_0^0(p) = \frac{k_0^0 \exp(-\tau_0 p)}{T_0 p + 1}, \tag{5}$$

where $T_0 -$ nominal value of the dominant time constant.

For the basic transfer function (4), suggest the basic proportional-integral (PI) regulation law is suggested

$$W_{P2}(p) = \frac{k_{P2}}{k_0^0 \tau_0} \cdot \frac{T_1 p + 1}{p}, \tag{6}$$

where $k_{P2} -$ custom parameters, $T_1 -$ custom time constant.

In the case where the transfer function using the basic PID regulation law

$$W_{P3}(p) = \frac{k_{P3}}{\tau_0 k_0^0} \cdot \frac{T_1 p + 1}{p} \cdot \frac{T_2 p + 1}{T_E p + 1}, \tag{7}$$

where $k_{P3}, T_1, T_2 -$ custom parameters, small time constant $T_E << \min(T_1, T_2)$ is introduced to ensure the condition of physical realizability of the transfer function of the regulator.

To synthesize a robust regulator, it is proposed to use the robust Nyquist criterion. As an additive measure of an open system transfer function uncertainty, inequality

$$\left| W(j\omega) - W^0(j\omega) \right| \le \gamma, \tag{8}$$

where $W(j\omega) = W_P(j\omega) W_0(j\omega)$, $W^0(j\omega) = W_P(j\omega) W_0^0(j\omega) -$ frequency transfer functions of real and nominal open systems (respectively), $W_P(j\omega) -$ frequency transfer function of the controller, $\gamma > 0 -$ positive number.

For the stability of the system it is necessary and sufficient, to have the Nyquist Plot of an open nominal system not to encompass a circle of radius γ with the center at the point $(-1, j0)$. It this article suggest to use the magnitude of the stability margin in amplitude h as a criterion for the synthesis of the system and the radius of the circle γ – to evaluate the system's roughness. Fig. 1 shows the procedure for constructing a forbidden area for Nyquist Plot, guaranteeing a given radius value γ, which is equivalent to the known procedure for constructing the forbidden area, which guarantees a given value of the oscillation index.

On the basis of the cosine theorem for the triangle OAB, we obtain

$$\cos\phi = \frac{A^2(\omega) + c^2 - \gamma^2}{2A(\omega)c} = \frac{A^2(\omega) + 1 - \gamma^2}{2A(\omega)},$$

$$1 - \gamma \le A(\omega) \le 1 + \gamma. \tag{9}$$

Based on this the forbidden area for Nyquist Plot can be constructed on the basis of relation

$$\phi(\omega) = \arccos \frac{A^2(\omega) + 1 - \gamma^2}{2A(\omega)},$$

$$1 - \gamma \le A(\omega) \le 1 + \gamma. \tag{10}$$

If the Nyquist Plot refers to the forbidden region, then the roughness of the system is estimated by the magnitude γ.

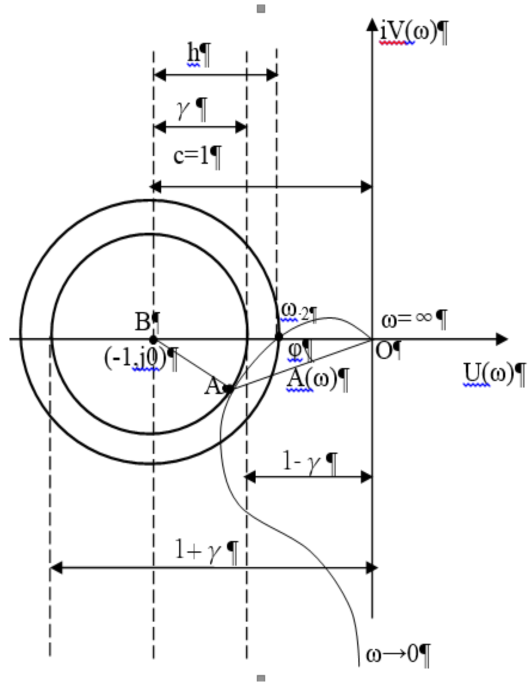

Fig. 1. Nyquist Plot of nominal open system

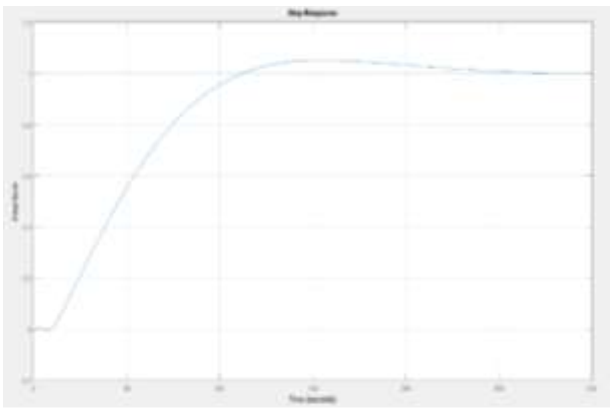

Fig. 2. Nyquist Plot of nominal open system

For the basic nominal system (4), (6), the value of the stability margin for the amplitude can be calculated from the formula

$$h_1 = -20\lg k \frac{\sqrt{1+T_1^2\omega_1^2}}{\omega_1}, \tag{11}$$

where $k = k_{P2}/\tau_0$, the variable ω_1 is determined from the condition

$$\tau_0 = \frac{1}{\omega_1}\left\{\pi - arctg\left(\frac{1}{T_1\omega_1}\right)\right\}. \tag{12}$$

The specified value h_1 depends on the pair (k, T_1). Consequently, in a given range of parameter changes (k, T_1) one can find the values of these parameters, corresponding to the minimum control time for a given value of the stability margin in amplitude h_1, then the roughness γ on the basis of (10) is calculated.

In practice, this problem can be solved by a simple construction of transient characteristics with a change (k, T_1), which correspond to a predetermined value h_1, which can easily be calculated on the basis of logarithmic frequency characteristics. With an error of 5% of the determination of the control time, the fastest response in many cases can be obtained with a retuning value of 5%. The amount of retuning is mainly determined by the value k.

For the basic nominal system (5), (7)

$$h_2 = -20\lg k \frac{\sqrt{\left(1-T_1T_2\omega_2^2\right)^2 + \left(T_1+T_2\right)^2\omega_2^2}}{\sqrt{\left(T_0+T_E\right)^2\omega_2^4 + \omega_2^2\left(1-T_0T_E\omega_2^2\right)^2}}. \tag{13}$$

where $\omega_2 = \dfrac{\psi_1(\omega_1) - a_1(\omega_1, T_1, T_2)\cdot\omega_1 - a_2(\omega_1)\cdot\omega_1}{\tau_0 - a_1(\omega_1, T_1, T_2) - a_2(\omega_1)}$, $\omega_1 = \dfrac{\pi}{2\tau_0}$,

$$a_1(\omega_1, T_1, T_2) = \frac{\left(T_1+T_2\right)\left(1+T_1T_2\omega_1^2\right)}{1+\left(T_1^2+T_2^2+T_1^2T_2^2\omega_1^2\right)\omega_1^2},$$

$$a_2(\omega_1) = -\frac{\left(T_0+T_E\right)\left(1+T_0T_E\omega_1^2\right)}{1+\left(T_0^2+T_E^2+T_0^2T_E^2\omega_1^2\right)\omega_1^2}.$$

For a pair (k_{P3}, T_1), a value T_2 is determined so that condition (13) is satisfied. Next, for (k_{P3}, T_1, T_2), a transient characteristic is constructed, which determine performance and retuning are. In this case, the robust depends on the selected pair (T_1, T_2), and the retuning and performance are mainly dependent on k_{P3}, then the robust is calculated γ.

For example, consider the transfer function of a view object

$$W_0^0(p) = \frac{k_0^0\exp(-10p)}{(50p+1)^2(5p+1)} \approx \frac{k_0^0\exp(-25)}{(90p+1)(5p+1)}, \tag{14}$$

To synthesize the basic transfer function of the object (5) with $T_0 = 90c$. The second time constant is 5s. it is supposed to compensate for the additional dynamics of the regulator. Set the value of the desired margin of stability in amplitude as $h = 9\partial Б$ (0.6452). In accordance with the above procedure, set the parameters as $h = 9\partial Б$ (0.6452). In accordance with the above procedure, we set such parameters as k_{P3} and T_1. Then, the value T_2 necessary to provide a given value of the stability margin in amplitude is calculated. The solution of this problem is not unique. For example, for $k_{P3} = 0.55$, $T_1 = 60c.$, we get $T_2 = 24c.$ The transfer function of the regulator (7) at $T_E = 0.1c.$ with the application of the compensation method will have the form

$$W_{P3}(p) = \frac{0.55}{26.5\cdot 1}\cdot\frac{60p+1}{p}\cdot\frac{25.45p+1}{0.1p+1}\cdot\frac{9p+1}{0.1p+1}. \tag{15}$$

The transient response for the nominal transfer function of the object (14) with the regulator (15) is shown in Fig. 2. With the overshoot value, the control time is In this case, the radius of the circle (8) is used to estimate the roughness.

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