Investigation of Optimization Methods for Matching Pursuit Algorithm Based on Geoacoustic Emission Data*

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***Abstract*— The geoacoustic emission signals consist of a sequence of impulses close in shape to the modulated Berlage functions. It's possible to use the sparse approximation method for analysis such signals. This method decomposes the signal into the sum of the modulated Berlage functions describing the impulses and the modulated Gaussian functions approximating the noise. The authors developed adaptive modification of the classical matching pursuit algorithm. Its essence is the use of optimization methods for iterative refinement of the Gaussian and Berlage function parameters. The report describes numerical methods that allow to optimize the matching pursuit algorithm and to build qualitative representations of geoacoustic signals.**

Keywords—matching pursuit; adaptive matching pursuit; sparse approximation; geoacoustic emission; optimization

I. Introduction

Acoustic emission is an elastic vibration that occurs in response to deformation of the environment. The characteristics of the emerging impulse radiation depend directly on the properties of the plastic processes that generate the signals. Acoustic emission of the sound range thet occurs during rocks deformation is usually called mesoscale or geoacoustic emission. Investigations conducted in Kamchatka have shown that the highest frequency of impulse sequence occurs during strong perturbations of geoacoustic emission preceding seismic events, therefore these signals are of great interest in the research of plastic processes associated with the stability of landscapes and the formation of earthquake precursors. According to the frequency composition of the geoacoustic signal, it is possible to determine the scale of the signal generating source and to estimate the distance from it to the registration point.

For the analysis and processing of geoacoustic emission, the approach of sparse approximation was chosen, which allows us to describe signals using any set of functions (given numericaly or analytically), called a dictionary.

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II. Sparse approximation

The classical approximation problem original signal can be approximately represented as a linear combination of basis functions $g_i(t)$

$$s(t) = \sum_{i=1}^{N} \alpha_i g_i(t) + R_N(t) .$$

where α_i are weighting factors, $R_N(t)$ is residual. Basis functions should be mutually orthogonal (or be Riesz system) to ensure the convergence of the approximation to the original signal. In contrast to the classical problem of sparse approximation, in general, does not impose requirements on the orthogonal function system, which splits the signal. The problem of sparse approximation implies the representation of the signal $s(t)$ as a linear combination of the minimum possible number of functions $g_i(t)$.

$$s(t) = \sum_{i=1}^{N} \alpha_i g_i(t) + R_M(t),$$
$$\|\alpha\|_0 = M \to \min .$$

In terms of a sparse approximation, the system of functions $D = \{g_i(t)\colon i = 1 \ldots N\}$ is usually called a "dictionary". The sparseness of the constructed approximation is characterized by an l_0-norm, a pseudonorm which is equal to the number of non-zero coefficients of the decomposition, i.e.

$$\|\alpha\|_0 = \#\{i: \quad \alpha_i \neq 0, \quad i = 1 \ldots N\} .$$

An obvious advantage of the sparse approximation in comparison with the classical approaches of time-frequency analysis is the possibility of reducing the dimensionality of the signal feature space. For example, windowed Fourier transforms or continuous wavelet transforms, actively used for signal analysis in other areas of science, are extremely redundant. Another important advantage of sparse approximation can be considered the possibility of choosing physically based dictionaries, which opens up new ways of

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interpreting the results of analysis, for example, in the tasks of detecting signal sources and their characteristics.

III. DICTIONARY

The accuracy and sparseness of the constructed approximations directly depends on the chosen function dictionary. To analyze the geoacoustic signals a dictionary composed of modulated and shifted Gaussian and Berlage functions became appropriate.

The Gauss impulse (Fig. 1) is generated by the expression

$$g(t) = A \cdot \exp\left(-B\left(t_{end}\right) \cdot \Delta \cdot t^2\right) \cdot \sin(2\pi ft),$$

where A is an amplitude chosen so that $\| g(t) \|_2 = 1$; t_{end} is atom length; f is frequency from 200 to 20000 Hz; $B(t_{\text{end}})$ is a critical value of the parameter B calculated by the formula

$$B(t_{end}) = -\frac{4 \cdot \ln 0.05}{t_{end}^{\ 2}};$$

Δ is coefficient of variation of the parameter B with respect to this limit value.

The Berlage impulse (Fig. 2) is generated by the expression

$$g(t) = A \cdot t^{n(p_{\max}) \cdot \Delta} \cdot \exp\left(-\frac{n(p_{\max}) \cdot \Delta}{p_{\max} \cdot t_{end}} \cdot t\right) \cdot \cos\left(2\pi ft + \frac{\pi}{2}\right),$$

where A is an amplitude chosen so that $\| g(t) \|_2 = 1$; $p_{\max}$ is a maximum position with respect to the length atom, $p_{\max} \in [0.01, 0.4]$; f is frequency from 200 to 20000 Hz; $n(p_{\max})$ is a critical value of the parameter n calculated by the formula

$$n(p_{\max}) = \frac{\ln 0.05}{\ln \frac{1}{p_{\max}} - \frac{1}{p_{\max}} + 1};$$

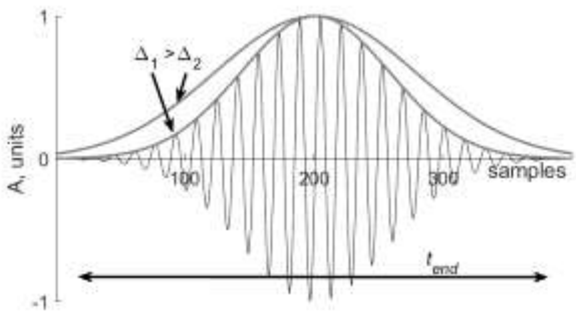

Fig. 1. Gauss impulse

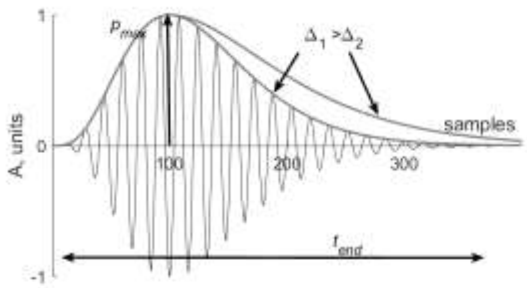

Fig. 2. Berlage impulse

Δ is coefficient of variation of the parameter n with respect to this limit value.

IV. MATCHUNG PURSUIT

The problem of sparse approximation is not solvable in polynomial time. The exact solution algorithm requires a complete search of all possible combinations of functions from the dictionary, i.e. has the factorial complexity O(N!).

One of the most commonly used algorithms for the approximate solution of the problem of sparse approximation is the Matching Pursuit (MP) algorithm. This algorithm was proposed by Mallat and Shang in 1993 [1].

$$\left\| s(t) - \sum_{i=1}^{N} \alpha_i g_i(t) \right\|_2 \to \min,$$
$$\|\alpha\|_0 \le N.$$

The matching pursuit algorithm implements greedy strategy. At each stage of the algorithm locally optimal solution is taken, i.e. a function having the largest scalar product with the signal being studied is selected. This algorithm has the cubic computational complexity $O(n^3)$ if the matrix of scalar products is computed by definition and quadratic-logarithmic $O(n^2 \log n)$, if the calculations are performed using a Fast Fourier Transform.

V. ADAPTIVE MATCHING PURSUIT

Unfortunately, the matching pursuit algorithm has a few significant disadvantages. At first to ensure sufficient accuracy of decomposition it is required to use large dictionaries which causes a gradual increase in the speed of the algorithm execution. Secondly, because the functions are selected from an unchanging dictionary, the resulting decompositions are distinguished by a "coarse" discretization in the parameter space. Fig. 3 shows decomposition example of a signal with a fundamental frequency of 8.9 kHz by a dictionary containing Berlage impulses with frequencies of 1, 5, 10 and 15 kHz. As a result, decomposition consisting of 25 functions (Fig. 3 shows the first 10 of them) with a frequency of 10 kHz was constructed with an error of 7%.

To solve the listed above problems the authors proposed to improve the classical algorithm so that on dictionaries of limited size it is possible to build decompositions of the required accuracy [2,3,4]. Because the parameters of the function having the largest value of the scalar product with the signal are determined at each iteration of the matching pursuit algorithm, the iteration of the matching pursuit can be conditionally described as the problem of finding the functional maximum

$$F(g) = \langle s(t), g(t-\tau, \mathbf{p}) \rangle \to \max_{\mathbf{p}} .$$

The main idea of the proposed improvements is to apply optimization methods to find parameters **p** of the function having the maximum scalar product with the signal. The developed algorithm was called the Adaptive Matching Pursuit (AMP).

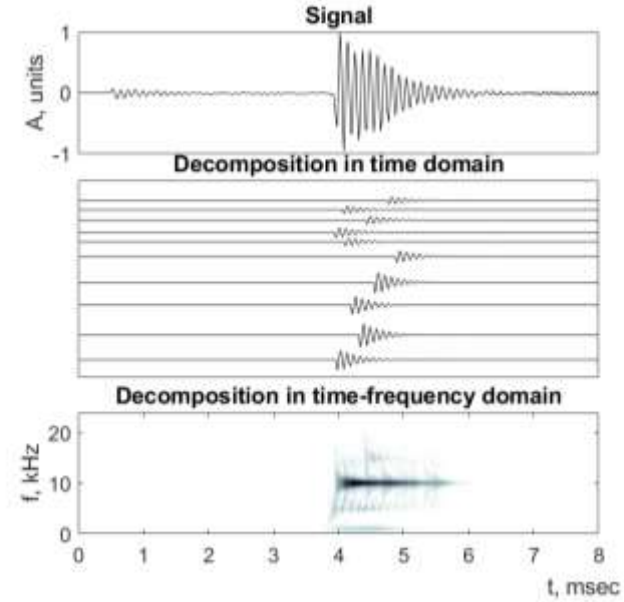

Fig. 3. An example of a "coarse" discretization in the dictionary parameter space

A. Gradient descent method

The maximum is determined by the direction along the gradient vector. For the Gaussian function the gradient is defined as follow

$$\nabla F_G = \left(\frac{\partial F}{\partial \Delta}, \frac{\partial F}{\partial t_{end}}, \frac{\partial F}{\partial f} \right),$$

for the Berlage function as

$$\nabla F_B = \left(\frac{\partial F}{\partial p_{\max}}, \frac{\partial F}{\partial \Delta}, \frac{\partial F}{\partial t_{end}}, \frac{\partial F}{\partial f} \right).$$

Because the functions are given analytically the calculation of their partial derivatives was carried out by an analytical method and according to the properties of the scalar product, the gradient vector can be calculated as follows

$$\nabla F_G = \left(\left\langle s(t), \frac{\partial g}{\partial \Delta} \right\rangle, \left\langle s(t), \frac{\partial g}{\partial t_{\text{end}}} \right\rangle, \left\langle s(t), \frac{\partial g}{\partial f} \right\rangle \right),$$

$$\nabla F_B = \left(\left\langle s(t), \frac{\partial g}{\partial p_{\max}} \right\rangle, \left\langle s(t), \frac{\partial g}{\partial \Delta} \right\rangle, \left\langle s(t), \frac{\partial g}{\partial t_{\text{end}}} \right\rangle, \left\langle s(t), \frac{\partial g}{\partial f} \right\rangle \right)$$

for the functions of Gauss and Berlage respectively.

The refined parameters are determined according to the following scheme

$$\mathbf{p}^{[i+1]} = \mathbf{p}^{[i]} + \lambda \cdot \nabla F$$

where λ is gradient step size, is may be constant or variable.

The initial approximation $\mathbf{p}^{[0]}$ is determined by the matching pursuit method on the initial dictionary of functions.

Fig. 4 shows a refinement example of the frequency of first function included in the signal decomposition from Fig. 3. The graph shows the dependence of the scalar products values on the shift τ and frequency f and the refinement path f. With the gradient descent method, the initial frequency value of the 9.7 kHz is refined to 9 kHz.

B. Grid search method

The simplest method of maximum search consisting in the sequential calculation of the functional for the given functions (determined by the parameters **p**) and the search for the maximum value. The specified parameters can be selected randomly or with a uniform pitch. With minor changes this method can be used to optimize $F(g)$:

- initial grid is a uniform grid whose nodes correspond to $g_i(t)$, the half of the sampling step of the parameter values $\boldsymbol{\lambda} = (0.5\ \lambda_p;\ 0.5\ \lambda_\Delta;\ 0.5\ \lambda_t;\ 0.5\ \lambda_f)$ is selected as the grid step;
- using MP to determine the initial approximation for the parameters $\mathbf{p}^{[0]}$, $i = 0$;
- in the neighborhood of $\mathbf{p}^{[i]}$ to construct a new grid containing three points for each parameter: $\mathbf{p}^{[i]} - \boldsymbol{\lambda}/2$, $\mathbf{p}^{[i]}$, $\mathbf{p}^{[i]} + \boldsymbol{\lambda}/2$; define a new approximation $\mathbf{p}^{[i+1]}$, and if it matches with $\mathbf{p}^{[i]}$ then $\boldsymbol{\lambda} = \boldsymbol{\lambda}/2$;
- $i = i + 1$ and repeat from the third step.

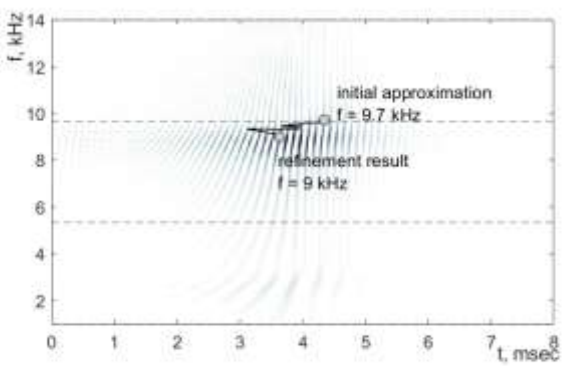

Fig. 4. Parameter refinement using gradient descent method

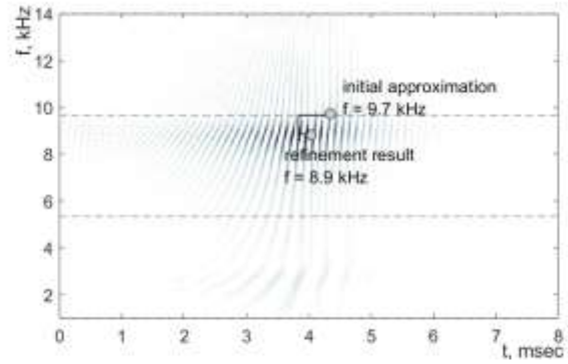

Fig. 5. Parameter refinement using grid search method

VI. CONSLUSIONS

The proposed numerical schemes were tested on a sample of 100 real geoacoustic signals. At each iteration of the algorithm after refining the parameters of the atom by the formula

$$ERR_N = \frac{\left\| s(t) - \sum_{i=1}^{N} \alpha_i \cdot g_i(t) \right\|}{\left\| s(t) \right\|} \cdot 100\%,$$

the ratio of the residual to the original signal was calculated. In Fig. 6 are shown the dependence of *ERR* on the algorithm iteration number. Both schemes demonstrate a good result regarding the correspondence of the decompositions to the real signal. Table I shows the execution time of the algorithms for a different number of refinement iterations. Because with the appropriate setting of parameters both algorithms build qualitative approximations on small size dictionary with approximately the same accuracy, then in further studies any of them can be used. Fig. 7 shows the decomposition of the signal from Fig. 3 on the same dictionary using AMP algorithm based on the grid search method. As result decomposition consisting of 3 functions with a frequency of 8.8-9 kHz was constructed with an error of 6%. The problem of "coarse" discretization is solved.

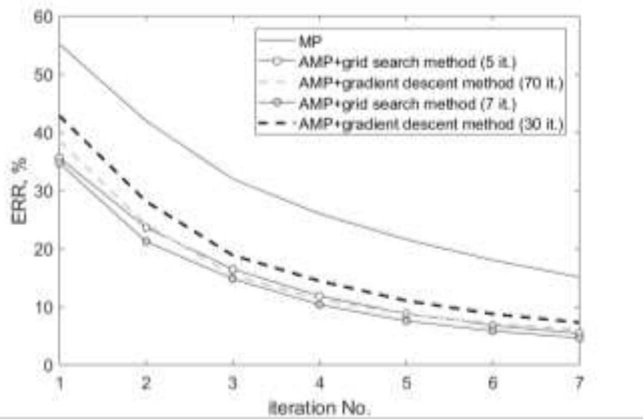

Fig. 6. Errors for different refinment methods

TABLE I EXECUTION TIME OF ALGORITHMS

Algorithm	*Execution time (sec)*
MP	15.54
AMP + gradient descent method (30 it.)	49.78
AMP + gradient descent method (70 it.)	84.3
AMP + grid search method (5 it.)	91.73
AMP + grid search method (7 it.)	111.31

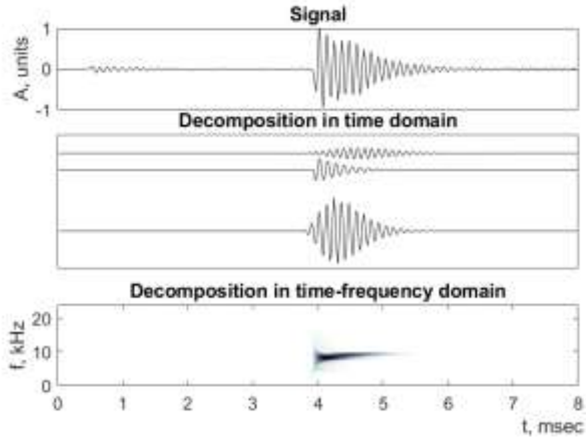

Fig. 7. The solution of the problem of "coarse" discretization in the dictionary parameter space

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