The Task of Assessment of the System Functioning Efficiency under Conditions of Uncertainty

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***Abstract*— We investigate the features of assessment of the system functioning efficiency under conditions of uncertainty related to environment influence. We describe the model of structure and functional connections of system in form of a multi-commodity network. In this model we propose the algorithm for a task of evaluating the system functioning efficiency based on the estimations of difficulty of achieving the goal.**

***Keywords*— efficiency; quality; difficulty of achieving the goal; risk of not achieving the goal**

I. About System Efficiency

In accordance with the efficiency assessment theory [1], it is possible to assess the quality of any object or system only during its use for its intended purpose. Thus the most objective assessment is the one considering the efficiency of application. Yet the design, organizing and application are in fact connected with unpredictable events in the future and thus have a degree of uncertainty. The objective characteristic of a system's quality (the level of its adaptability to achieving a requested result in the conditions of real interference with random factors) can only be the possibility of the system's task fulfillment that characterizes the given system's abilities at given conditions [2].

Thus the estimation of the system functioning efficiency considering possible external influences must conform to certain requirements and bear a ternary structuring logic:

First, while assessing the efficiency it is necessary to consider the system's ability to achieve its goal. This characteristic directly depends on the efficiency of the elements comprising the system, because for a systematic approach the characteristics of the elements that define their interactions among each other and influence the achievement of the goal are of foremost importance [2].

Second, it is necessary to integrally assess the quality of the system's functioning. This is done by estimating to what extent the elements meet requirements, because in equal conditions it is the more difficult to get a result of a certain quality, the lower is the quality of the resource and the higher the requirements to the result [3].

Third, it is necessary to estimate of the possibility of the system achieving its task by finding the risk of not achieving the task on condition that all tasks of the separate elements are achieved.

In our survey we propose to use the estimations of the type of difficulty of the goal achievement introduced by I. B. Russman as these dimensionless and universal indexes meet all given requirements. From the one hand, the difficulty is a generalized characteristic of the resource's quality or lack of quality that is considered not only their qualities but also the requirements posed by the system that arise from the requirement of the system's general functionality [3]. From the other hand the probability-based interpretation of difficulty can be understood as a risk of not achieving the goal while all requirements are met.

Thus we will interpret the system functioning efficiency as the degree of its achieving its goal (also the risk of not achieving it) that is found by estimating the fulfillment of requirements to the elements of the system (them achieving their goals).

II. The Model of System in Form of a Multi-Commodity Network

To visually represent the structure and character of the connection between the elements of the system it can be shown as a graph with the system elements as vertices. The links of this graph comprise the interaction of the elements among themselves and the flows that pass through these links are the numerical estimation of these interactions. To show the bilateral character of the elements' interaction the schematic of the system may be described by a mathematical model called the multi-commodity flow network (MC-net). It is comprised of two graphs built on a single array of vertices - the nodes of the system [4]. The first, physical graph determines the simulated physical structure of the network by links connecting nodes. The same nodes are the vertices of another, logical, network graph that determines the connection structure of the elements, i.e. the structure of requirements to the flows throughout the network. The links of the logical graph connect the pairs of elements in the system that require to be connected, the so-called attracted pairs. The inclusion of the two described graphs into one MC-net is caused by the fact that the logical connections can only be achieved via links of the physical graph. The "multi-commodity" term is explained by the noninterchangeabilty of the flows of different attracted pairs. It

978-1-5386-5824-6/18/$31.00 ©2018 IEEE

is considered that these flows correspond to different types of products [4].

If the measure of the requirements of the attracted pairs is known then a corresponding flow measured in arbitrary units is attributed to the links of the logical network. The same arbitrary units are used to measure the bandwidth of the physical graph links. These numbers limit the flow through a given link between any of the attracted pairs. The task of distributing the flows in the network is achieved by laying a route for any of the logical graph links between any pair of nodes through the physical graph links [4]. It is also necessary to respect the physical bandwidth limitations and it is preferable to respect the logical limitations that are defined by the requirements of the elements' functioning. It is clear that it is necessary to achieve the highest possible flow with the given bandwidths of the attracted pairs. A network is called allowable if all such flows satisfy the logical requirements of the elements. Such a network can effectively function as intended. Otherwise the net requires developments or reconsiderations of conditions for those pairs of elements, the requirements of which cannot be met.

The system allows uncertainty arising due to the bandwidth vector of the links in the MC-net physical graph [4]. We consider the cause of this to be the drop of bandwidth or malfunction of separate links of the physical graph of the MC-net resulting from random or intended damages of the network. Further we will name as an incident any unexpected or undesired event (or their combination) capable of interfering with the system's functionality. The degree of the incident's influence we will call the weight of the incident. The specific localization of the influence that arises from the incident (the link that is damaged) and the distribution of this influence throughout the links is considered unknown. Thus, the warranted estimation of the MC-net functional capabilities proposes the search of the worst incident influence for the network, i.e. finding the case of bandwidth drop throughout the links that leads to the maximum harm to the MC-net functionality. At the same time the functioning of the network is still understood as the fulfillment of extreme bandwidth requirements for the attracted pairs.

We will utilize a standard mathematical structural model of a multi-commodity flow network, $S=(V,P)$, that is specified with arrays of: $V=\{v_1,...,v_n\}$ - network nodes and $P=\{p_1,...,p_m\}\in V\times V$ - attracted pairs and links of the logical network graph. The corresponding index arrays we will determine as: $N=\{1,...,n\}$ and $M=\{1,...,m\}$, so $V=\{v_i\}_{i\in N}$ and $P=\{p_k\}_{k\in M}$.

For any vertex $v\in V$ we will designate through $S(v)$ an array of indexes of the outgoing links and through $T(v)$ the array of incoming links [4].

For each k of the attracted pairs we will designate a $p_k=(v_{sk},v_{tk})$, where $s_k<t_k$, the vertex v_{sk} is called the source and v_{tk} - the drain of the *k* product. The value of g_k is the flow between the source and the drain for each of the attracted pairs $p_k\in P$.

To estimate the maximum possible flow for each attracted pairs we will consider the extreme case when the flow of each product occupies the entire physical net. This will be showing the extreme level of requirements for this attracted pair.

The network has quantitative limitations that are defined by the bandwidth capacity of the physical graph [5].

We will formally attach to each link (v_i,v_j) a number $c_{ij}\geq 0$ called the bandwidth capacity of the link (v_i,v_j) that is counted in the flow units that the network is created for. The matrix $c=\{c_{ij}\}$ sets the limitations through inequalities for the flow distributions in the net, i.e. the sum of all flows attracted through the link should not exceed the link bandwidth. Moreover, to all links $p_k\in P$ of the logical graph numerical values $y_k\geq 0$ are attached. They are calculated in arbitrary units of flow that are required to pass through the current logical link of the MC-net [4].

If the vector of requirements is known, the problem of the acceptability of the net to the given requirements vector is posed, i.e. checking the condition of the existence of a distributions of flows where $g_k\geq y_k$, $k=1,...,m$.

It is obvious that to solve the problem of acceptability it is not obligatory to build every possible flow distribution in the physical network. It is enough to find distributions allowing maximum flows between all attracted pairs. We will denote as z_k the biggest of all possible g_k.

The array of the maximum flows between the attracted pairs we will denote as:

$$Z(c)=\{z_k\} \qquad (1)$$

This flow matrix provides the maximum functioning efficiency to the network.

As in the case of external disturbances (incidents) the bandwidth matrix can change, it is sensible to introduce several additional parameters. Let us assume that SC is an array of *c* matrices within the set uncertainty. It can be regarded as $SC=\{C\mid C'\leq C\leq C^0\}$ where C^0 is the matrix of primary bandwidths of the net (in the best case scenario when no incidents occur), and $C'=0$ (in the worst case scenario when all links are completely destroyed). Not to reduce all calculations of vulnerability to the trivial 0 value we will solve this problem with an array $C^\gamma=\{c^\gamma\mid\gamma\in[0,1]\}$, where the parameter γ has the meaning of the incident weight and shows the expected bandwidth drop for any link of the net (i.e. what part of the primary bandwidth was destroyed). Because any link of the physical graph could be influenced by the incident for a more reliable estimation we will assume a drop of all bandwidths in accordance with the parameter.

The variable c_{ij}^γ here and elsewhere will designate the remnant bandwidth of the physical graph links of the MC-net after an incident of the weight:

$$c_{ij}^{\gamma} = \{c_{ij}^{\gamma} \le c_{ij}^{0} \mid c_{ij}^{\gamma} = (1-\gamma)c_{ij}^{0}\} \quad (2)$$

This matrix will be used in further calculations for assessing the network efficiency.

III. Algorithm of MC-net efficiency analysis

Step 1. Building an MC-net. Definition of the physical and logical graphs of the network, the primary bandwidth matrix C^0 and the requirements vector y, basing on the information about the elements of the system and their interconnections.

Step 2. Building an array of maximum flows between each of the attracted pairs. An algorithm of searching the maximum flow for each of the attracted pairs should be used in the case of a relatively small number of vertices of the logical graph. Currently multiple methods are known and any can be used to solve this task. We use the algorithm of Ford-Fulkerson, described in [5]. If the number of attracted pairs is close to the number of all pairs of vertices of the physical graph, it is sensible to use the Gomory-Hu algorithm, described in [6].

In any case we get an array of maximum flows between all vertex pairs $Z(c)$, which can be used to estimate the ability of the net to conform to the requirements of the system components, i.e. to efficiently function.

Step 3. Building a matrix of difficulties of meeting the flow requirements of the attracted pairs. Let us estimate the quality of the calculated streams z_k using the formula:

$$\mu_k = (z_k - \underline{Z}) / (\bar{Z} - \underline{Z}) \quad (3)$$

where $\bar{Z} = \max_{1 \le k \le m} z_k$, $\underline{Z} = \min_{1 \le k \le m} z_k$.

Then it is necessary to estimate the requirements to the quality of these flows:

$$\varepsilon_k = (y_k - \underline{Z}) / (\bar{Z} - \underline{Z}) \quad (4)$$

It is necessary to note that both types of parameters μ and ε change in the interval [0,1] and it is required that $\varepsilon_k \le \mu_k \forall k$ for any attracted pair [3]. Combinations do not meet the minimum quality requirements if this condition is not met. In the other cases the difficulty factor [3] will be equal to:

$$d_k = \varepsilon_k (1-\mu_k) / \mu_k (1-\varepsilon_k) \quad (5)$$

in addition $d_k = 0$ when $\varepsilon_k = \mu_k = 0$ and $d_k = 1$ when $\varepsilon_k = \mu_k = 1$.

Let us additionally introduce weight coefficients α_k that vary in the range $0 < \alpha_k \le 0{,}1$.

Then the final array of difficulties of meeting the requirements of the attracted pairs at the current array of maximum flows can be set as:

$$D(Z) = \{d_k^{\alpha_k} \mid d_k^{\alpha_k} = 1-(1-d_k)^{\alpha_k}\} \quad (6)$$

The integral difficulty can be found with the formula:

$$D = \sum_{k=1}^{m} d_k^{\alpha_k} \quad (7)$$

This parameter designates the integral difficulty of meeting the requirements of the attracted pairs of the network and serves as a criterion of the efficiency of the system. The higher the difficulty, the more difficult it is to meet the bilateral requirements of the components of the system with the given bandwidths within the network. If $D = 1$ the difficulty is extreme and the system is in a very vulnerable state. If at least one of the d_k factors exceed 1 (in the case $\varepsilon_k > \mu_k$), then the integral factor $D > 1$ also, and this means that the flow between this pair of vertices does not conform to the requirements and the system is not functioning efficiently. In this case an increase of bandwidth capacity or changing the requirements for the elements is needed.

Step 4. Estimating the weight of the incidents' influence. It is possible to estimate the system efficiency after an incident's influence. First it is required to calculate the weight of the incident, the parameter $\gamma \in [0,1]$. If it is unknown we propose to analyze it by solving the problem of estimating the efficiency of risk elimination [7].

The final calculated parameter will carry the meaning of an integral estimation of the efficiency of resilience to risks of system malfunction and will show the probability of the expected damage to the bandwidths in the network.

Step 5. Analysis of system efficiency during the influence of an incident. Let us calculate the matrix C^{γ} of the expected post-incident bandwidths. The weight of the incident was calculated in step 4 with the formula:

$$c_{ij}^{\gamma} = (1-\gamma)c_{ij}^{0} \quad (8)$$

Now it is necessary to again solve the problem of finding the array of maximum flows (repeat step 2 with the matrix C^{γ}) and find the arrays of difficulties of meeting the requirements of attracted pairs (repeat step 3 with the matrix $Z(C^{\gamma})$). Then the calculated integral difficulty factor $D(\gamma)$ will characterize the efficiency of the system's functioning in the conditions of the incident with the weight γ. If the network remains allowable ($D(\gamma) < 1$) it will be considered that the system is functioning efficiently enough to sustain the influence of an incident of the expected weight and meet the bilateral requirements of all system components.

Step 6. The search of the functioning efficiency limit for a system. It is sensible to search for a maximum incident weight that a system can sustain, irrespective of whether the network is allowable after the influence of the incident (which is taken into consideration in step 4). Step 5 is repeated with and array $SC^{\gamma} = \{C^{\gamma}\}$ in which the incident weight γ changes with an increment of the desired amount. If the system is allowable after step 5, then the incident weight increases otherwise decreases.

The system's reliability limit will be found when the integral difficulty factor $D(\gamma)$ will exceed 1. This weight of the incident γ is the maximum that the current system is capable of sustaining without failing to comply to the bilateral requirements of the elements. Let us assign the limit of the system's effective functioning in the conditions of external influences as $\gamma_{\lim}$.

Step 7. Benchmarking assessment of the efficiency. The difficulty factor of meeting the requirements of attracted pairs can be used to compare the efficiency of 2 or several systems. Steps 1-6 are carried out for all systems then.

The comparison of the parameters D from step 3 allows to define which system best meets the requirements of attracted pairs with no external influences. The parameters $D(\gamma)$ from step 5 allow comparing the systems according to their vulnerability to influences of the same weight. Finally, the comparison of the $\gamma_{\lim}$ parameters from step 6 allow to define which system has the highest limit of functioning efficiency.

The system with the maximum $\gamma_{\lim}$ should be considered the most efficient in the sense of functioning in the conditions of external influences.

ACKNOWLEDGMENT

We would like to express our appreciation to professor of the Department "Computer Aided Design and Management" of Saint-Petersburg State Institute of Technology Alexander Bolshakov for advices and valuable comments, also to Svjatoslav Averin for his assistance with translation.

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