Pipelined Frequency Transform Optimisation for Attenuation Equalisation at Channels Frequency Responses Crosspoints and Limiting Chnannel Cross-Talk Level

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***Abstract*—The paper describes modification of pipelined frequency transform digital filter bank and method of realization parameters calculation. Method utilization minimizes attenuation maximally reached at frequencies between adjacent channels. Maximal channels cross-talk estimation formula is provided to find minimal sampling rate meeting fixed constraint of both adjacent and non-adjacent channels cross-talk. Result formulas allow to calculate heterodynes frequencies and processing sampling rates required to meet stated conditions.**

***Keywords*—digital filter bank; pipelined frequency transform; frequency channels overlap; cross-talk**

I. Piplined Fourier Transform Modification

Pipelined frequency transform (PFT) architecture and details can be found in [1, 2, 3]. Application field of digital filter bank (DFB) in form of PFT is limited by low identity of resulting frequency channels. The channel identity problem can be dealt with by

- Implementing a FIR filter with a wider and more rectangular frequency response.
- Increasing processing sampling rate.

Given solutions lead to a higher adjacent channels overlap, lower adjacent channels overlap identity, higher hardware resources utilization. To deal with the problem we modify usual PFT structure: different (optimized) heterodyne frequencies are used on each signal decomposition level. The modification equalizes (minimizes maximal reached attenuation) attenuations between adjacent frequency channels and keeps FIR filter and hardware resources utilization fixed. To minimize sampling frequency a formula is obtained to estimate the higher limit of channels cross-talk level.

II. DFB Key Parameters

PFT is used to simultaneously obtain all frequency channels samples from all refining divisions (into 2; 4; 8; 16; 2N channels) of the whole frequency band. Such a usage allows to consider structure of FIR filter coefficients to reduce computational cost. Good results are shown on symmetrical coefficients.

Cross-talk is among key parameters of frequency band division systems. FFT based DFB faces the problem of adjacent channels frequency response overlapping and the problem of regularly distributed frequency response secondary maximums. PFT involves more complicated distribution of cross-talk sources. Low channel count PFT implementations are not seriously affected. Problem becomes severe in the case of decomposition into 1024 and more channels.

Most applications involve frequency band decomposition into narrow band channels to detect and locate a signal of a certain or an uncertain form. Existing PFT structure assumes critical decimation which results in highly attenuated frequency domains between resulting channels. Such an effect is caused by multiple overlapping application of FIR filter transition band at each level of filtering. That causes the effect of an attenuation mask of a complicated form. The attenuation mask is applied to the whole frequency band. The mask shadows certain frequencies and prevents corresponding signals from being detected. Frequency responses of resulting filters are shown on the Fig. 1.

Fig. 1 shows 8 channel PFT DFB. The most corrupted range on the Fig. 1 is around the 0 MHz frequency. +2.25 and −2.25 MHz ranges are less corrupted because they were attenuated by FIR filter transition band less amount of times.

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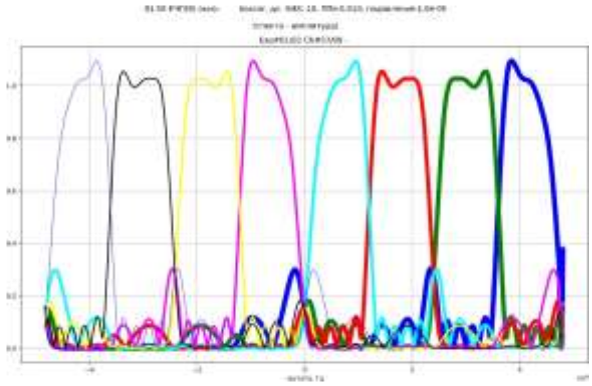

Fig. 1. PFT frequency ranges attenuation

Quality of the DFB for signal detection and location in a frequency range is defined by two key parameters: cross-talk between resulting channels, maximum reached attenuation in frequency domain between adjacent channels.

III. Analysis of Attenuation Reached in Frequency Domain Between Adjacent Channels

Now we will look more closely on the frequency response cross point of two neighboring channels.

We will denote as $A(f)$, the frequency response function of FIR filter used in PFT. We will consider it to be piecewise linear (Fig. 2).

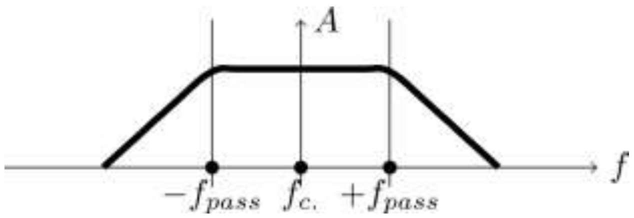

Fig. 2. Piecewise linear filter frequency response A(f)

To divide frequency band into two we move frequencies up and down by f_c from f_h and filter each with a FIR described by $A(f)$. That causes the gap in the output frequencies of the DFB. The maximum attenuation is reached at the cross point of frequency responds (Fig. 3):

$$A_{l.}(f_{h.}) = A_{r.}(-f_{h.}) = A(f_{f,r.} = f_{c.}).$$

Frequency response gap becomes deeper at each FIR filter applying level. We call — the FIR filter frequency response at the (i)th level, and — the cross point frequencies in the left and right transition bands of filter at th level (Fig. 4).

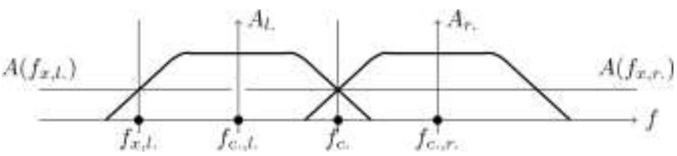

Fig. 3. Adjucent filter frequency responses

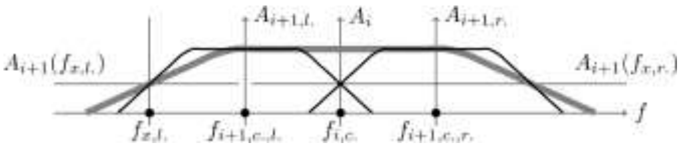

Fig. 4. Adjucent filters and higher level filter frequency responses

At the output of the (i)th level in the gap between filters $A_i^{'}$ and A_i resulting frequency response value is $A_i(f_{x,l.})$. Corresponding value at the output of $(i+1)$ th level is:

$$A_{\min,l.} = A_i(f_{x,l.}) \cdot A_{i+1}(f_{x,l.}).$$

Inside the passband of the (i)th level A_i filter the resulting maximal output attenuation at (i+1)th level is $A_{\min,r} = 1 \cdot A_{i+1}(f_{x,r.})$ times (Fig. 5).

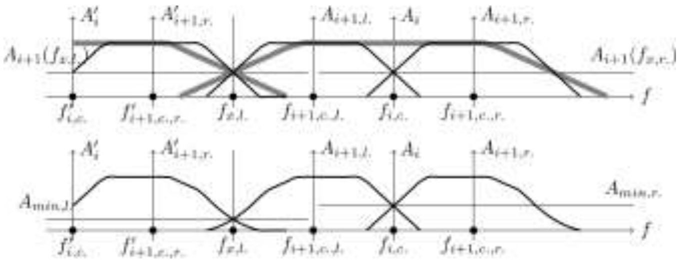

Fig. 5. Adjucent filters and higher level filter frequency responses position (at the top) and superposition (at the bottom)

Consider all FIR filters are identical and there are a factor two decimation between levels and frequency shift by $f_{i+1,h.}$. Coordinate system transformation is

$$f_{i+1,c.,l.} = f_{i,c.} + f_{i+1,h.}, \quad f_{i+1,c.,r.} = f_{i,c.} - f_{i+1,h.},$$

gives

$$A_{i+1,l.}(f) = A_i\big(2(f + f_{i+1,h.})\big), \tag{1}$$

$$A_{i+1,r.}(f) = A_i\big(2(f - f_{i+1,h.})\big).$$

It is easy seen that increasing the distance between filters centers causes maximal reached attenuation to raise. Raising at least one of maximal reached attenuation causes the raise of the maximum reached result attenuation of the whole DFB. Considering the form of $A_i(f)$ and fixed frequency range let us evaluate the minimal maximal attenuation reached

$$A_{\min,r.} = A_{\min,l.},$$

$$1 \cdot A_{i+1,l.}(f_{x,r.}) = A_i(f_{x,l.}) \cdot A_{i+1,l.}(f_{x,l.}). \tag{2}$$

Fig. 6 shows the result graphically. Maximal reached attenuations in one transition band case and in two transition bands case are equal.

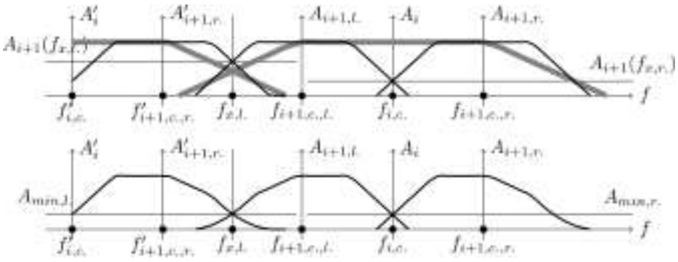

Fig. 6. Adjucent filters and higher level filter frequency responses position (at the top) and superposition (at the bottom)

Applying (2):

$$A_i\left(2\left(f_{x,r.}+f_{i+1,h.}\right)\right)=A_i\left(f_{x,l.}\right)\cdot A_i\left(2\left(f_{x,l.}+f_{i+1,h}\right)\right). \quad (3)$$

Letting

$$f_{i,c}=0, \quad (4)$$

we obtain $f_{x,r.}=0$, a $f_{x,l.}=-f_{i,h.}$. Combining these we can rewrite (3) as:

$$A_i\left(2f_{i+1,h.}\right)=A_i\left(-f_{i,h.}\right)\cdot A_i\left(2\left(-f_{i,h.}+f_{i+1,h.}\right)\right), \quad (5)$$

$$f_{i,h.}>f_{i+1,h.}>0,\ f_{1,h.}=f_{h.}.$$

IV. Linear approximation

Assuming (2) we will find formula for $f_{i+1,h.}$. It makes maximal reached rejection to be minimal. Consider piecewise linear function $A_i(f)$ to be:

$$A_i(f)=\begin{cases} a_i\cdot\left(f+b_i\right)+c_i,\ f\in\left[-f_{cut},-f_{pass}\right) \\ A_{\max},\ f\in\left[-f_{pass},f_{pass}\right] \\ -a_i\cdot\left(f-b_i\right)+c_i,\ f\in\left(f_{pass},f_{cut}\right] \end{cases}, \quad (6)$$

$$f_{cut}>f_{pass}>0, \quad (7)$$

$$A_1=A,a_1=a,b_1=b,c_1=c.$$

Combining (5), (6) and (7) gives us equation with solutions:

$$f_{i+1,h.}=\frac{-a_i^2b_i^2+3a_i^2b_i f_{i,h.}-2a_i^2 f_{i,h.}^2}{2a_i\left(a_ib_i-a_i f_{i,h.}+c_i+1\right)}+$$

$$+\frac{-2a_ib_ic_i+a_ib_i+3a_ic_i f_{i,h.}-c_i^2+c_i}{2a_i\left(a_ib_i-a_i f_{i,h.}+c_i+1\right)}.$$

Let us rewrite:

$$k_{i+i,h.}=\frac{f_{i+1,h.}}{f_{i,h.}},$$

$$k_{i+1,h.}=\frac{-a_i^2b_i^2+3a_i^2b_i f_{i,h.}-2a_i^2 f_{i,h.}^2}{2a_i f_{i,h.}\left(a_ib_i-a_i f_{i,h.}+c_i+1\right)} + \frac{-2a_ib_ic_i+a_ib_i+3a_ic_i f_{i,h.}-c_i^2+c_i}{2a_i f_{i,h.}\left(a_ib_i-a_i f_{i,h.}+c_i+1\right)}. \quad (8)$$

Let us introduce recursive formulas for coefficients used in (8). Combining (1) and (4) with lower domain of (6) yields

$$A_{i+1,l.}(f)=A_i\left(2\left(f+f_{i+1,h.}\right)\right)=A_i\left(2(f)\right)=$$

$$=a_i\cdot\left(2f+b_i\right)+c_i=2a_i\cdot\left(f+\frac{b_i}{2}\right)+c_i,$$

we obtain:

$$a_{i+1}=2a_i,\ b_{i+1}=\frac{b_i}{2},\ c_{i+1}=c_i. \quad (9)\ (10)\ (11)$$

V. Quadratic approximation

Let us examine quadratic approximation. Assume $B_i(f)$ model of FIR filter frequency response to be quadratic:

$$B_i(f)=\begin{cases} a_{l.,i}f^2+b_{l.,i}f+b_{l.,i},\ f\in\left[-f_{cut},-f_{pass}\right) \\ A_{\max},\ f\in\left[-f_{pass},f_{pass}\right] \\ a_{r.,i}f^2+b_{r.,i}f+c_{r.,i},\ f\in\left(f_{pass},f_{cut}\right] \end{cases}, \quad (12)$$

$$f_{cut}>f_{pass}>0,\ B_i=B. \quad (13)$$

Combining (5) with (12), (13) we can rewrite (5) as

$$a_{r.,i}\left(2f_{i+1,h.}\right)^2+b_{r.,i}\left(2f_{i+1,h.}\right)+c_{r.,i}=$$

$$=\left(a_{l.,i}\left(-f_{i,h.}\right)^2+b_{i.,i}\left(-f_{i,h.}\right)+c_{l.,i}\right)\times$$

$$\times\begin{pmatrix} a_{l.,i}\left(2\left(-f_{i,h.}+f_{i+1,h.}\right)\right)^2+ \\ +b_{i.,i}\left(2\left(-f_{i,h.}+f_{i+1,h.}\right)\right)+c_{l.,i} \end{pmatrix},$$

$$f_{cut}>f_{i,h.}>f_{pass}.$$

Clearly,

$$a_i=a_{r.,i}=a_{l.,i},\ b_i=b_{r.,i}=-b_{l.,i},\ c_i=c_{r.,i}=c_{l.,i},$$

yields quadratic equation with solutions:

$$k_{i+1,h.} = \frac{4a^2 f_{i,h.}^3 + 5abf_{i,h.}^2 + 4acf_{i,h.}}{4af_{i,h}\left(af_{i,h.}^2 + bf_{i,h} + c - 1\right)} + \\ + \frac{b^2 f_{i,h} + bc + b \pm \sqrt{D}}{4af_{i,h}\left(af_{i,h.}^2 + bf_{i,h} + c - 1\right)}, \quad (14)$$

where we have set

$$D = -4a^3cf_{i,h.}^4 + 16a^3 f_{i,h.}^4 + a^2b^2 f_{i,h.}^4 - 8a^2bcf_{i,h.}^3 + \\ +32a^2bf_{i,h.}^3 - 8a^2c^2 f_{i,h.}^2 + 24a^2cf_{i,h.}^2 + 2ab^3 f_{i,h.}^3 - \\ -2ab^2 f_{i,h.}^2 + 18ab^2 f_{i,h.}^2 - 8abc^2 f_{i,h.} + 24abcf_{i,h.} - \\ -4ac^3 + 8ac^2 - 4ac + b^4 f_{i,h.}^2 + 2b^3cf_{i,h.} + 2b^3 f_{i,h.} + \\ +b^2c^2 + 2b^2c + b^2.$$

Therefore $f_{i+1,h.}$ is

$$f_{i+1,h.} = \frac{4a^2 f_{i,h.}^3 + 5abf_{i,h.}^2 + 4acf_{i,h.}}{4a\left(af_{i,h.}^2 + bf_{i,h.} + c - 1\right)} + \\ + \frac{b^2 fi;h: + bc + b \pm p\sqrt{D}}{4a\left(af_{i,h.}^2 + bf_{i,h.} + c - 1\right)}.$$

Let us find recursive formula for (14) coefficients. Combining (1), (4) and left range (12) yields:

$$B_{i+1,j.}(f) = B_i(2(f + f_{i+1,h.})) = \\ = B_i(2(f)) = a_i(2f)^2 + b_i(2f) + c_i,$$

therefore

$$a_{i+1} = 4a_i,\ b_{i+1} = 2b_i,\ c_{i+1} = c_i.$$

VI. Calculation of Maximum Reached Attenuation in Gaps Between Channels

After N levels of filtering the rejection between channels is identical. Therefore it is identical to rejection between newly divided channels:

$$A_{min,N} = A_N(f_{N,h.}) = A_1(2^{N-1} f_{N,h.}) = \\ = A(2^{N-1} f_{1,h.} \prod_{i=2}^{N} k_{i,h.}). \quad (15)$$

Given equation for $f_{i,h.}$ allows to choose filter frequency response $A(f)$ and heterodyne frequency f_h, which guarantee maximal minimal $A_{\min,N}$ on the fixed number of levels N.

VII. Cross-Talk Level

Scaling up $f_{i,h.}$ while keeping critical decimation ratio raises level of cross-talk. Define $A_i(f)$ at Ω. On each processing level signal is shifted by $f_{i,h.}$ and decimated. Therefore channel cross-talk power $P_{i,out}$ on (i)th level will not exceed $P_{i,out,\max}$:

$$P_{i+1,out,\max} = \int_{\Omega/\left[-\frac{f_{s.}}{2};\frac{f_{s.}}{2}\right]} A_{i+1}^2(f)df = \\ = \int_{\Omega/\left[-f_{s.}-2f_{i+1,h.};f_{s.}-2f_{i+1,h.}\right]} A_i^2(f)df.$$

Considering decimation on each processing level we can assume $A_i(f) = A(f)$. The symmetry of $A(f)$, gives:

$$P_{i+1,out,\max} = \int_{\Omega/\left[-\frac{f_{s.}}{2}-f_{i,h.};\frac{f_{s.}}{2}-f_{i,h.}\right]} A^2(f)df.$$

Total cross-talk in channel

$$P_{\Sigma,out} = \sum_{i=1}^{N} P_{i,out,\max} = \\ = \sum_{i=1}^{N} \int_{\Omega/\left[-\frac{f_{s.}}{2}-f_{i,h.};\frac{f_{s.}}{2}-f_{i,h.}\right]} A^2(f)df$$

will not exceed sum of maximal cross-talks:

$$P_{\Sigma,out} \le P_{\Sigma,out,\max} = \\ = N \int_{\Omega/\left[-\frac{f_{s.}}{2}-\min_i f_{i,h.};\frac{f_{s.}}{2}-\max_i f_{i,h.}\right]} A^2(f)df. \quad (16)$$

(16) is our cross-talk level estimation.

VIII. Conclusions

Equations (15) and (16) with (9), (10), (11), (8) can be used to find parameters providing required maximal reached attenuation between channels and required level of cross-talk.

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