Planning the Pulsed Distributed Control System

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***Abstract*— The article is devoted to the question of finding the place and time of switching on section heaters with the stabilization of the temperature field based on the Green's function. The control and stabilization system of the temperature field is considered. There are presented mathematical models and exploring of temperature field for three-dimensional control object.**

Keywords — control; mathematical modeling; temperature field; Green's function

Considering the technical features of electric tunnel furnaces of conveyor type it is worth noting their low efficiency. It is directly related to the huge loss of electricity during heating of silicon carbide heating elements. Consider the possibility of replacing continuous heating elements with pulsed ones.

To solve this problem, we consider a spatially three-dimensional control object. The mathematical model of such an object has the form:

$$\frac{\partial Q(x,y,z,t)}{\partial t} - a^2\left[\frac{\partial^2 Q(x,y,z,t)}{\partial x^2} + \frac{\partial^2 Q(x,y,z,t)}{\partial y^2} + \frac{\partial^2 Q(x,y,z,t)}{\partial z^2}\right] = f(x,y,z,t);$$

$$Q(x,y,z,0) = Q_0(x,y,z);$$

$$Q(0,y,z,t) = q_1(y,z,t);\ Q(L_1,y,z,t) = q_2(y,z,t);\ Q(x,0,z,t) = q_3(y,z,t);$$

$$Q(x,L_2,z,t) = q_4(x,z,t);\ Q(x,y,0,t) = q_5(x,y,t);\ Q(x,y,L_3,t) = q_6(x,y,t);$$

$$0 \le x \le L_1;\ |0 \le y \le L_2;\ 0 \le z \le L_3;\ t \ge 0;\ a > 0;$$

We will calculate the temperature indices using the Green's function represented in the form of an infinite Fourier series [1]

$$G(x,y,z,\rho,\nu,\vartheta,t) = \frac{8}{L_1 \cdot L_2 \cdot L_3} \cdot \sum_{k,m,n=1}^{\infty} B_{k,m,n}(\cdot) \cdot \exp\left[-a^2\pi^2 \cdot t \cdot \left(\frac{k^2}{L_1^2} + \frac{m^2}{L_2^2} + \frac{n^2}{L_3^2}\right)\right]$$

$$B_{k,m,n}(\cdot) = \sin\left(\frac{k\cdot\pi\cdot x}{L_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot y}{L_2}\right)\cdot\sin\left(\frac{n\cdot\pi\cdot z}{L_3}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{L_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot\nu}{L_2}\right)\cdot\sin\left(\frac{n\cdot\pi\cdot\vartheta}{L_3}\right);$$

where d – quantity of heat sources; $p = 1,2,3,...$ – serial number of source inclusion; $z(p)$ – one of the heating sources; τ_p – moment of switching on the source number $z(p)$; l_1 ,l_2 , l_3 – spatial coordinates; p,v, ϑ – coordinates of the point source ξ; a^2 – thermal diffusivity of the material; *k,m.n* – quantity of members of the Fourier series during the expansion of the input action in width and length; *x,y,z* – coordinates of explored point; *t* – moment of time [2].

The graphically considered object will have the following form:

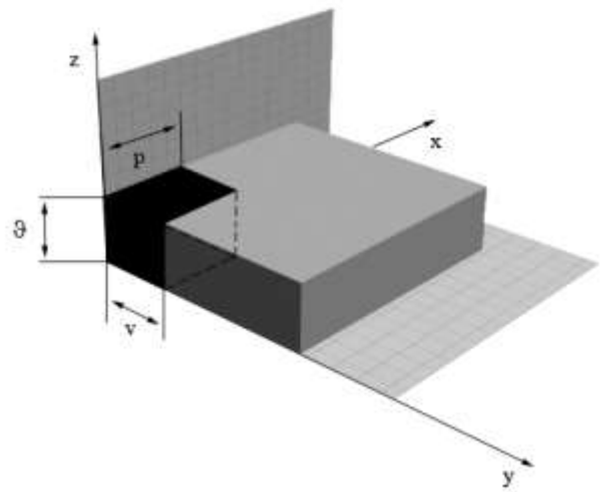

Fig. 1. Three-dimensional control object

The control system of this object will look as follows:

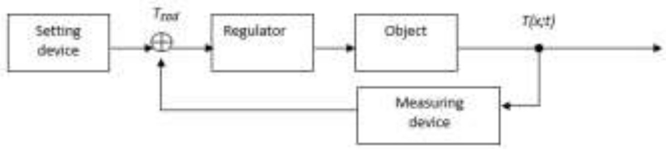

Fig. 2. Structural diagram of a closed-loop control system

The transfer function of this control object under the initial conditions and the presented mathematical model of thermal process will look like:

$$W(x,y,z,\rho,\nu,\vartheta,s) = \frac{8}{L_1 \cdot L_2 \cdot L_3} \cdot \sum_{k,m,n=1}^{\infty} \frac{B_{k,m,n}(\cdot)}{s + a^2\pi^2\left(\frac{k^2}{L_1^2} + \frac{m^2}{L_2^2} + \frac{n^2}{L_3^2}\right)}$$

Calculation of the temperature of this control object will be carried out using the following function:

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$$G(x_j, y_j, z_j, \rho, \nu, \vartheta, t) = \sum_{i=1}^{d} \frac{8}{L_1 \cdot L_2 \cdot L_3} \cdot \sum_{k,m,n=1}^{\infty} \sin\left(\frac{k\cdot\pi\cdot x_j}{L_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot y_j}{L_2}\right)\times$$
$$\times\sin\left(\frac{k\cdot\pi\cdot\rho_i}{L_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\nu_i}{L_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot z_j}{L_3}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\vartheta_i}{L_3}\right)\times$$
$$\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{L_1^2}+\frac{m^2}{L_2^2}+\frac{n^2}{L_3^2}\right)\right]\cdot\sum_{p}\sum_{k,m,n=1}^{\infty}\sin\left(\frac{k\cdot\pi\cdot x_j}{L_1}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot y_j}{L_2}\right)\times$$
$$\times\sin\left(\frac{k\cdot\pi\cdot z_j}{L_3}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho_{z(p)}}{L_1}\right)\sin\left(\frac{k\cdot\pi\cdot\nu_{z(p)}}{L_2}\right)\times\sin\left(\frac{k\cdot\pi\cdot\vartheta_{z(p)}}{L_3}\right)\times$$
$$\times\exp\left[-a^2\pi^2\cdot(t-\tau)\cdot\left(\frac{k^2}{L_1^2}+\frac{m^2}{L_2^2}+\frac{n^2}{L_3^2}\right)\right].$$

This function shows the behavior of the system at the initial time, when the system receives the first impulse. Due to the fact that the system is at rest, this pulse has a maximum amplitude. The distribution of heat through the object passes in different directions at the same speed, this is due to the homogeneity of the material. In the case of heterogeneity of the material, the temperature process will occur unevenly, which will lead to a different heating speed of the material.

Graphically, the initial heating function will have the following form:

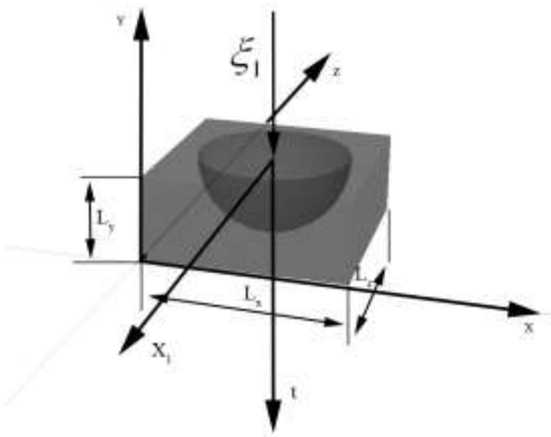

Fig. 3. Graphical representation of the initial heating function

Let's find a function that determines the time of the first control action – τ_1. The function that determines the value of the temperature field of a three-dimensional control object at a certain time *t* will be determined by a single component of the Fourier series. Expressing the value of a member of the Fourier series, we obtain the following equation [3, 4, 5, 6]:

$$T(x,y,z,t)=\frac{8}{l_1 l_2 l_3}\exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_2}y\cdot\sin\frac{\pi}{l_3}z\times\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i$$

Then if we take into account the condition $T(x,y,z,t)=T_{zad}$ that is necessary to ensure the stability of the system, we get:

$$\exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]=\frac{l_1 l_2 l_3 T_{zad}}{8\sin\frac{\pi}{l_1}x_{kr}\sin\frac{\pi}{l_2}y_{kr}\sin\frac{\pi}{l_3}z_{kr}\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}$$

Assuming that the following equations $x_{kr}=\rho_1$, $y_{kr}=\nu_1$, $z_{kr}=\vartheta_1$ are inherent in this equation, and denoting the time $t=\tau_1$, we get:

$$a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}+\frac{1}{l_3}\right)\tau_1=\ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}\nu_1\sin\frac{\pi}{l_3}\vartheta_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}{l_1 l_2 l_3 T_{zad}}\right)$$

Whence,

$$\tau_1=\frac{1}{a^2\pi^2\left(\frac{1}{l_1}+\frac{1}{l_2}+\frac{1}{l_3}\right)}\ln\left(\frac{8\sin\frac{\pi}{l_1}\rho_1\sin\frac{\pi}{l_2}\nu_1\sin\frac{\pi}{l_3}\vartheta_1\sum_{i=1}^{N}\sin\frac{\pi}{l_1}\rho_i\sin\frac{\pi}{l_2}\nu_i\sin\frac{\pi}{l_3}\vartheta_i}{l_1 l_2 l_3 T_{zad}}\right)$$

In a similar way, we find the place for the inclusion of a control action, which, according to the tolerances indicated above, will have the form:

$$\sin\left(\frac{k\cdot\pi\cdot x}{l_1}\right)=\frac{\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{m\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot\nu}{l_2}\right)\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,\nu,t)}$$
$$\left(\frac{k\cdot\pi\cdot x}{l_1}\right)=\arcsin\frac{\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{m\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot\nu}{l_2}\right)\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,\nu,t)}$$
$$x=\arcsin\frac{\frac{4}{l_1\cdot l_2}\cdot\sum_{k,m=1}^{\infty}\sin\left(\frac{m\cdot\pi\cdot y}{l_2}\right)\cdot\sin\left(\frac{k\cdot\pi\cdot\rho}{l_1}\right)\cdot\sin\left(\frac{m\cdot\pi\cdot\nu}{l_2}\right)\times\exp\left[-a^2\pi^2\cdot t\cdot\left(\frac{k^2}{l_1^2}+\frac{m^2}{l_2^2}\right)\right]}{G(x,y,\rho,\nu,t)}\cdot\left(\frac{l_1}{k\cdot\pi}\right)$$

Analogically,

$$y=\frac{l_2}{\pi}\cdot\arcsin\frac{\frac{8}{l_1 l_2 l_3}\exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_3}z\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i}{T(x,y,z,t)}$$
$$z=\frac{l_3}{\pi}.\arcsin\frac{\frac{8}{l_1 l_2 l_3}\exp\left[-a^2\pi^2 t\left(\frac{1}{l_1^2}+\frac{1}{l_2^2}+\frac{1}{l_3^2}\right)\right]\cdot\sin\frac{\pi}{l_1}x\cdot\sin\frac{\pi}{l_2}y\cdot\sum_{i=1}^{d}\sin\frac{\pi}{l_1}\rho_i\cdot\sin\frac{\pi}{l_2}\nu_i\cdot\sin\frac{\pi}{l_3}\vartheta_i}{T(x,y,z,t)}.$$

The obtained equations can be used to calculate the time and place of inclusion of temperature sources [7, 8].

Let's explore the thermal diffusivity of a three-dimensional control object. We will use the mathematical apparatus of the environment of mathematical operations Mathcad 14. For the mathematical modeling of the process, we take the following values of the variables parameters of the system: $l_1=l_2=l3=10$, $k=10$, $d=10$, $T_{зад}$ = 1...500, $a^2=0{,}01$, $x_1=y_1=z_1=v_1=p_1=Q_1=1$, $y,x,v,p_i\in\{1,2,3,4,5,6,7,8,9\}$, $\tau=3$. Entering such values in Mathcad 14, we get the values shown in the Table 1.

TABLE I. RESULTS OF EXPLORING THERMAL DIFFUSIVITY

№ source	d=5	d=6	d=8	d=9	d=10
1.	0,08	0,004	6,32	7,44	6,05
2.	0,06	0,003	4,70	5,53	4,50
3.	0,045	0,002	3,49	4,11	3,34
4.	0,034	0,001	2,60	3,06	2,49
5.	0,025	0,001	1,93	2,27	1,85
6.	0,018	0,0009	1,43	1,69	1,37
7.	0,014	0,0007	1,06	1,26	1,02
8.	0,010	0,0005	7,95	9,37	7,61
9.	0,007	0,0004	5,91	6,97	5,66
10.	0,005	0,0002	4,40	5,18	6,05

Analysis of the results of the work of the developed software package for stabilizing temperature fields showed:

- The ability to stabilize the temperature field within the permissible values through the use of pulsed heating elements.
- Formulas for calculating the location and time of inclusion of temperature sources were obtained.
- The developed software package can be used for any heating elements regardless of the technological process.

The place and time of switching on the heating elements of the sectional heater is searched in this work. This work shows that it is possible to reach a predetermined temperature regime replacing solid heating elements with pulsed ones.

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