Methodology of Inspection of Absolute Stability of Pulse Distributed Control System

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***Abstract*— The article is devoted to the problem of the steady flow of thermal processes during the stabilization of the temperature field. There are considered processes in furnaces of various purposes especially in drying and roasting chambers. The system of control and stabilization of the temperature field is considered.**

***Key words*— control; stability; temperature field; Green's function**

At the present stage of the development of human civilization, automatic control systems have affected all spheres of human society. In recent years, they have deeply embedded in the systems of the agro-industrial complex. If a few decades ago, people collected and dried wheat, ground flour and baked bread with their own hands, then nowadays all these procedures do combines, drying furnaces, baking ovens and confectionery appointment. But the process of automation of manual labor leads to greater complexity of the process. So, for example, there was a problem of stabilizing the temperature field during the course of thermal processes in drying chambers (drying ovens such as SZS), heating, baking of bakery products [1–3]. However it was solved, but there is a question of stability of course of thermal processes. The stability of the process in this case is especially important, as during drying there is a large amount of excess moisture, which reduces the efficiency of the process of raising the temperature of the drying chamber.

Let us present the problem of analyzing the stability of a distributed control system for the temperature field of a drying chamber.

As an object of control, we consider an isotropic cylindrical rod. The control action is the heat flux generated by the sources, in the form of sectional heater sections, distributed along the boundary of the lateral surface of the rod. Switching on the sources is realized by means of pulse elements. At the ends of the rod zero temperature is maintained. The mathematical model of the heat propagation process will take the form of [4]:

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} + \delta(x-\xi)\delta(t-\tau)\,;\ 0 < x < l\,;\ t > 0\,;$$

$$T(0,t) = T(l,t) = 0\,;\ T(x,0) = \delta(x-\xi)\ \delta(t)\,.$$

The block diagram of the closed control system is shown in Fig. 1

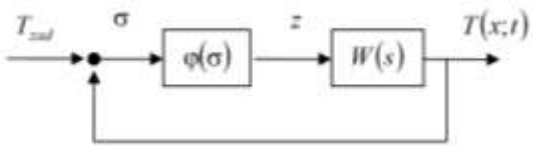

Fig. 1. Block diagram of the control system

The input signal of the non-linear link is the deviation of the system output function from the set value

$$\sigma(x,t) = T(x,t) - T_{zad}\,.$$

When the condition $T(\xi_{кр},\tau_1) - T_{zad} = 0$ is met, at the end points of the source location ξ_1 and ξ_2, at some point in time τ_1, at the observation point x_H :

$$\sigma(x_H,\tau_1) = T(x_H,\tau_1) - T_{zad}\,.$$

Where:

$$\tau_1 = \left(\frac{l}{\pi a}\right)^2 \ln\left(\frac{2\sin\frac{\pi}{l}x_{кр}\sum_{i=1}^{d}\sin\frac{\pi}{l}\xi_i}{lT_{зад}}\right)$$

Provided:

$$2\sin\frac{\pi}{l}x_{кр}\sum_{i=1}^{d}\sin\frac{\pi}{l}\xi_i \geq lT_{zad}$$

Denote the value of the signal:

$$\sigma(x_H,\tau_1) = \frac{2}{l}\exp\left[-\left(\frac{\pi a}{l}\right)^2\tau_1\right]\sin\frac{\pi}{l}x_H\sum_{i=1}^{d}\sin\frac{\pi}{l}\xi_i - T_{zad}$$

The reaction of the non-linear element $\varphi(\sigma(x,t))$ will be the total value of the pulsed effects produced at the extreme points ξ_1 and ξ_2, which can be represented as Green's function [5, 6]:

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$$G(x,t,\xi,\tau)=\frac{2}{l}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{\pi na}{l}\right)^2(t-\tau)\right]\sin\frac{\pi n}{l}x\sin\frac{\pi n}{l}\xi$$

For the observation point, which is the middle of the segment, when the sources are arranged symmetrically, you can write:

$$\phi(\sigma(x,t))=2G(x,t,\xi_{\kappa p},\tau_1)$$

The output function of a non-linear element can be represented as the sum of two values of the delta function at the observation point x. We use the formula [7, 8]:

$$\phi(\sigma(x,t))=\frac{4}{l}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{\pi na}{l}\right)^2(t-\tau_1)\right]\sin\frac{\pi n}{l}x_H\sin\frac{\pi n}{l}\xi_1$$

The maximum value of these effects at the point x_H will be observed at the time $t_{\max}$, where $t_{\max}$ – the time of arrival of the maximum signal from the source to the observation point, is determined by the formula.

$$\begin{cases}\dfrac{\left(\frac{l}{2}-\xi_1\right)^2}{2a^2}+\tau_1; & npu \quad \dfrac{3l}{10}\le\xi_1\le\dfrac{l}{2}\\ -\dfrac{\left(\frac{l}{10}-\xi_1\right)^2}{2a^2}+\dfrac{l^2}{25a^2}+\tau_1; & npu \quad \dfrac{l}{10}\le\xi_1\le\dfrac{3l}{10}\\ \dfrac{\left(\frac{l}{10}-\xi_1\right)^2}{4a^2}+\dfrac{l^2}{25a^2}+\tau_1; & npu \quad 0\le\xi_1\le\dfrac{l}{10}\end{cases}$$

Thus, the expression of the output function of a non-linear element at the observation point is the value of the function

$$\phi(\sigma(x_H,t_{\max}))=\frac{4}{l}\sum_{n=1}^{\infty}\exp\left[-\left(\frac{\pi na}{l}\right)^2(t_{\max}-\tau_1)\right]\sin\frac{\pi n}{l}x_H\sin\frac{\pi n}{l}\xi_1$$

These expressions define an implicit relationship between the input and output signals of a non-linear element.

The angular coefficient of the straight line limiting the sector in which the non-linear characteristic is located is defined as the ratio of the value $\varphi(\sigma(x_H,t_{\max}))$ to the value $\sigma(x_H,\tau_1)$, so

$$\phi\left(\sigma\left(t_{\max};x_H\right)\right)\quad k=\frac{\phi(\sigma(x_H,t_{\max}))}{\sigma(x_H,\tau_1)},$$

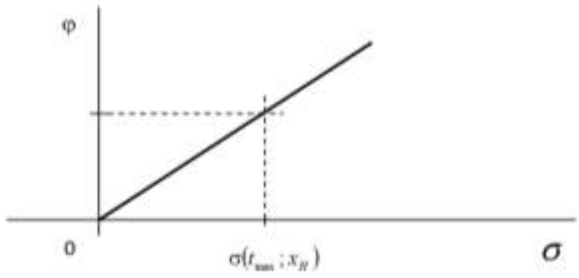

Fig. 2. The angle limiting sector of the non-linear characteristic

In systems with distributed parameters, the amplifying link can be represented as:

$$K(G)=E_1\left[\frac{n_1-1}{n_1}+\frac{1}{n_1}G\right],\qquad 0\le G\le\infty$$

Having accepted $E_1=k$ and $n_1=1$, we define values of angular coefficients for each spatial mode.

$$K_n=E_1G_n\,.$$

Then, having chosen a real number q, it is possible to construct for each spatial mode a Popov line passing through the point $\left(-\frac{1}{K_n},0\right)$ of the real axis and the point (0, q) of the imaginary axis of the complex plane.

The transfer function of the object in the n mode of input action can be represented as [9–11]:

$$W_n(s)=\frac{\exp[\beta_n x_H]+\exp[-\beta_n x_H]}{\exp[\beta_n l]+\exp[-\beta_n l]},\ (n=\overline{1,\infty}),$$

where $\beta_n=\left(\frac{s}{a}+\varphi_n^2\right)^{\frac{1}{2}}$, – observation point. For frequency analysis, let's say $s=j\omega$. When the frequency ω changes from 0 to ∞, the function $W_n(j\omega)$ will describe the hodograph for each spatial mode. The modified frequency characteristic $W^*(j\omega)$ is used for the analysis of absolute stability. It is known that when the frequency changes from zero to infinity, the vector $W^*(j\omega)=\mathrm{Re}(W(j\omega))+\omega\,\mathrm{Im}(W(j\omega))$ will also describe the hodograph for each spatial mode. Exploring several spatial modes allows to analyze the absolute stability of the considered class of non-linear systems.

A NUMERICAL EXAMPLE

Consider the control object with the following parameters: l=0,45 m length of the rod; $x_H = l/2$ – observation point; a= 0,0044 – the coefficient of thermal conductivity of the material. Let the number of heater sections r=20, so $\xi_1=l/20$=0,0225 – middle of the left extreme section, $\xi_{20}=l-\xi_1$=0,4275 – middle of the right extreme section [12].

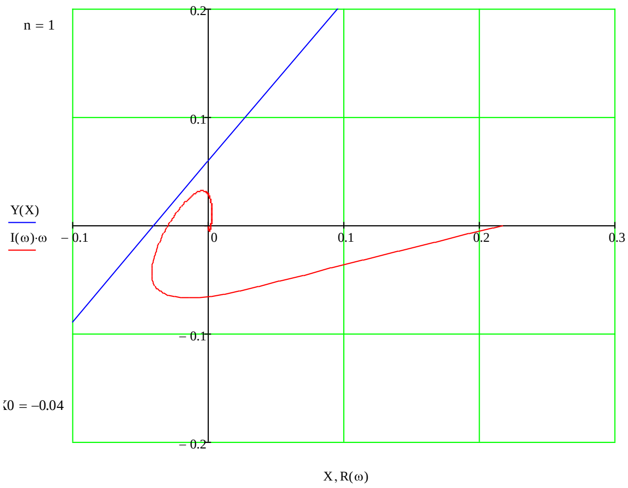

Fig. 3. The relative position of a hodograph and direct line when *r*=20; *n*=1

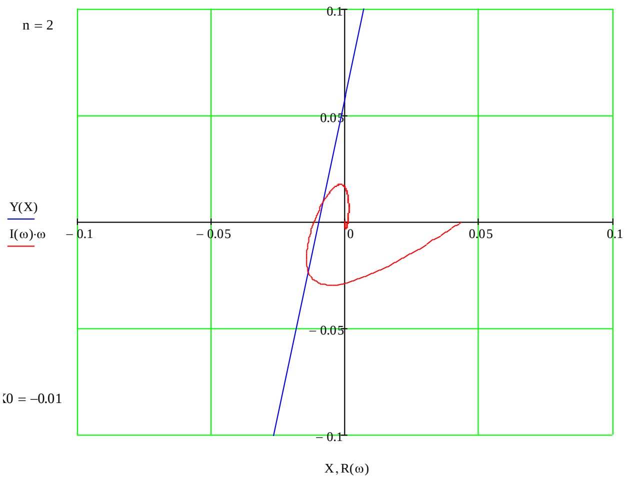

Fig. 4. The relative position of a hodograph and direct line when r=20; n=2

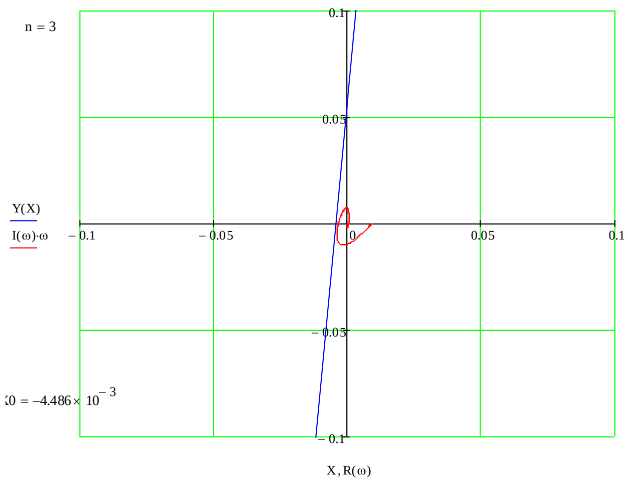

Fig. 5. The relative position of a hodograph and direct line when r=20; n=3

For the second mode, the hodograph crosses the Popov line, therefore, with the number of sections equal to 20, the system is not stable. Let the number of partitions be 23, then the analysis of the 4 spatial modes shows that the system is stable.

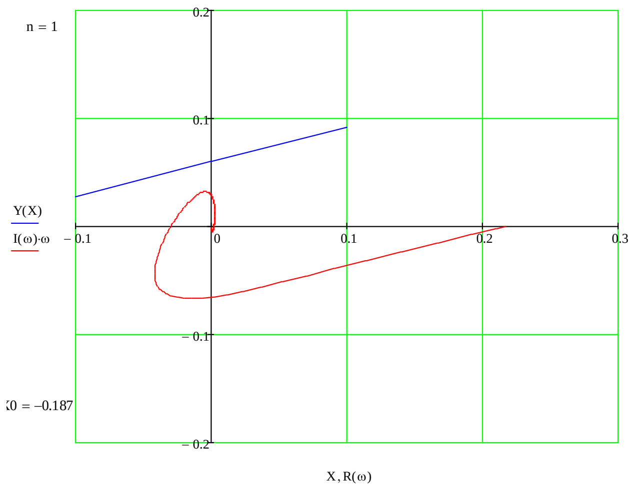

Fig. 6. The relative position of a hodograph and Popov line when r=23; n=1

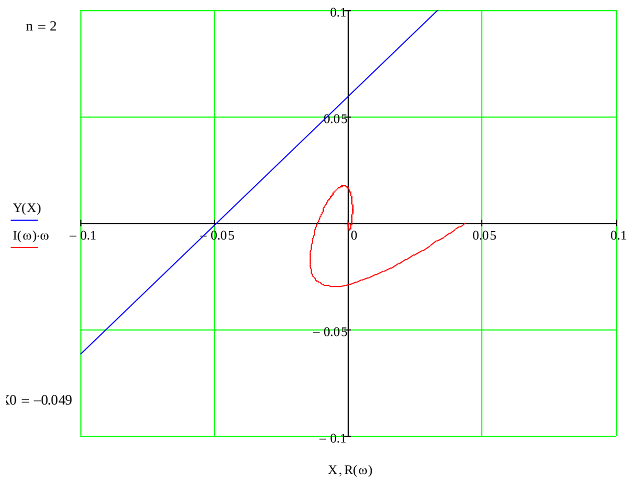

Fig. 7. The relative position of a hodograph and Popov line when r=23; n=2

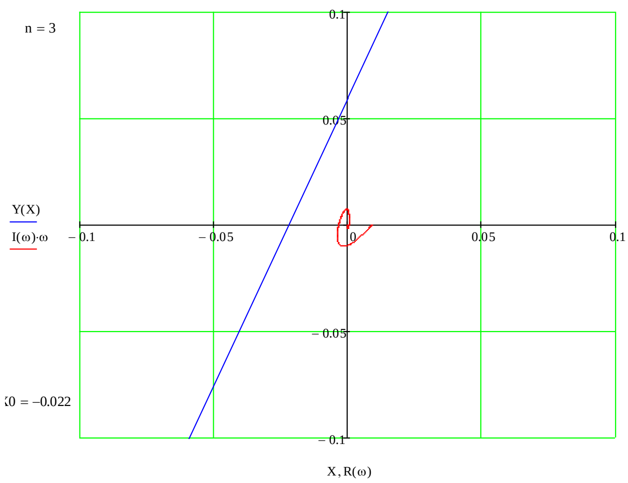

Fig. 8. The relative position of a hodograph and Popov line when r=23; n=3

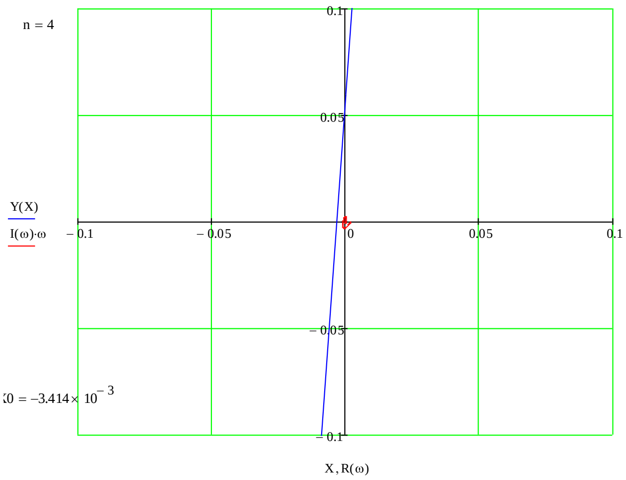

Fig. 9. The relative position of a hodograph and Popov line when r=23; n=4

Conclusion

The dependence of the stability of a non-linear distributed system on the value of the discretization step of the control actions is established. From a practical point of view, the quantity of sampling points for the considered object can be interpreted as the quantity of sections of the sectional heater. Increasing the sampling step, starting at a certain value, causes the stable system to become unstable.

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