

Mathematical Modeling of Blood Flow at Bifurcation of Cerebral Artery with High Degree of Stenosis*

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Abstract— The majority of numerical studies of hemodynamics in cerebral arteries are conducted using Newtonian model of blood; however, the effect of non-Newtonian blood properties in zones of recirculating flows can significantly change the flow-pattern and distribution of hemodynamic parameters in these regions. In this study, we provide an insight of blood dynamics in the bifurcation region of cerebral artery with high degree of stenosis using different non-Newtonian models of blood rheology and different flow-rate ratios between parent artery and branch. The highest differences between Newtonian and non-Newtonian fluid models are observed at zones of slow recirculating flow. For both cases Newtonian fluid model overestimates the maximum velocity magnitude compared to non-Newtonian fluid models. With increase of velocity in the branching vessel the differences between Newtonian and non-Newtonian fluid models became less significant. The study showed that the major differences between Newtonian and non-Newtonian fluids were observed in regions of low-velocity recirculation flow, where non-Newtonian blood behavior should be considered.

Keywords—cardiovascular system; cerebral hemodynamics; atherosclerosis; non-Newtonian fluids

I. INTRODUCTION

Several numerical and experimental studies have shown that hemodynamics plays an important role in genesis of various cerebral disorders [1], [2]. Therefore an assessment of hemodynamic characteristics in the cerebral arteries is important for the correct diagnostics and treatment [3]. The majority of numerical studies of hemodynamics in cerebral arteries are conducted using Newtonian model of blood e.g. [4], [5]; however, the effect of non-Newtonian blood properties in zones of recirculating flows can significantly change the flow-pattern and distribution of hemodynamic parameters in these regions [6]. For high shear rates an assumption of Newtonian fluid behavior holds true, however for low values of shear rate the relationship between shear rate and viscosity is nonlinear. It is especially important at bifurcations of cerebral arteries with high degree of stenosis where a stagnant flow and

low shear rates are presented, which leads to a high variation in viscosity values.

Typically atherosclerosis occurs at the bifurcation or at the bend of the cerebral artery, where some specific flow conditions are observed [3]. However most of the researchers assume blood as a Newtonian fluid which may lead in some cases to overestimation of hemodynamic parameters. Despite of the recent achievements in the research of blood rheology further studies should be done to clarify the role of non-Newtonian blood properties on hemodynamic parameters in cerebral arteries.

In this study, we provide an insight of blood dynamics in the bifurcation region of cerebral artery with high degree of stenosis using different non-Newtonian models of blood rheology and different flow-rate ratios between parent artery and branch.

II. MATERIALS AND METHODS

A. Idealized bifurcation model

An idealized three-dimensional model with a parent artery and a smaller branching artery was used for numerical studies. A parent vessel was considered as a straight tube with a bifurcation. The bifurcation had an angle of 90 degrees. A connection between parent vessel and branching artery had sharp corners. The diameter of inlet and outlet segments of the parent artery was 3.06 mm while the diameter of branch was 1.98 mm. The parent artery consisted of two segments, each of which has a length of 100 mm. The length of branch was 100 mm.

B. Flow conditions

Blood density was set to 1050 kg/m³. For the bifurcation model a realistic velocity curve was applied at the inlet. Average velocity over cardiac cycle was 265 mm/s. This velocity corresponds to flow rate of 7 l/h, which is physiologically relevant for cerebral artery. To model different degrees of stenosis the flow-rate ratio Q_r for parent artery and branching segment was 0.7 and 0.9. This was obtained by

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applying corresponding pressure level at the outlets of parent artery and a branch.

In order to compare the simulation results for Newtonian and non-Newtonian fluid models the dimensionless parameters - Reynolds number and Strouhal number were ensured to be the same. A representative viscosity value was used to calculate the Reynolds number for non-Newtonian fluid [1].

C. Fluid properties

The most popular models were used for the numerical studies, including: Newtonian model, Power Law model, Bird-Carreau model, Casson model and Local viscosity.

Newtonian model is the simplest assumption for modeling of blood rheology. This model considers blood as fluid with constant viscosity:

$$\eta(\dot{\gamma}) = \eta$$

where η is dynamic viscosity; $\dot{\gamma}$ is shear rate. The viscosity $\eta = 3.5 \text{ mPa} \cdot \text{s}$ was used to represent a Newtonian viscosity for numerical studies.

Power Law model:

$$\eta(\dot{\gamma}) = k\dot{\gamma}^{n-1}$$

where the k is consistency index; n is an index. The viscosity is bounded by minimum η_{\min} and maximum η_{\max} values respectively. The following parameters for Power Law model where used in this study: $k = 0.0117642$; $n = 0.8092$; $\eta_{\min} = 3.5 \text{ mPa} \cdot \text{s}$ and $\eta_{\max} = 14 \text{ mPa} \cdot \text{s}$.

Bird-Carreau model:

$$\eta(\dot{\gamma}) = \eta_{\min} + (\eta_{\max} - \eta_{\min}) \cdot [1 + (k\dot{\gamma})^2]^{(n-1)/2}$$

The parameters of Bird-Carreau model were: $k = 0.6046$; $n = 0.3742$. Viscosities η_{\min} and η_{\max} were the same as for Power Law model.

Casson model:

$$\eta(\dot{\gamma}) = \left(\sqrt{\frac{\tau_y}{\dot{\gamma}}} + \sqrt{k} \right)^2$$

where τ_y is yield stress. The parameter values of $\tau_y = 3.6$; $k = 4.1$ were used to describe the blood viscosity with Casson model.

Local viscosity model [6]:

$$\eta(\dot{\gamma}) = a_i + b_i(\dot{\gamma} - \dot{\gamma}_i) + c_i(\dot{\gamma} - \dot{\gamma}_i)^2 + d_i(\dot{\gamma} - \dot{\gamma}_i)^3, \\ \dot{\gamma} \in [\dot{\gamma}_{i-1}, \dot{\gamma}_i], \quad i = \{1, 2, \dots, S\},$$

where a_i, b_i, c_i, d_i are coefficients of i -th spline. The local viscosity model uses a set of cubic splines to precisely interpolate a measured blood viscosity.

D. Numerical simulations

SnappyHexMesh tool was used to generate hexahedral mesh for both studies. For the bifurcation study the mesh was refined near the region of bifurcation. Mesh independence check was carried out for both studies. The 2 million cells were sufficient to capture the flow detail in the regions of interest. The same computational mesh was used both for Newtonian and non-Newtonian cases.

The flow simulations were based on the three-dimensional incompressible Navier-Stokes and continuity equations: The vessel wall was assumed to be rigid and no-slip boundary condition was applied. Open source CFD toolbox OpenFOAM was used to conduct numerical studies.

III. RESULTS

Two different Q_r flow ratios 0.7 and 0.9 were simulated by adjusting boundary conditions at outlets. The results were analyzed at the moment of systolic peak. For Newtonian model, systolic peak was observed at $t = 1.011 \text{ s}$, while for non-Newtonian models systolic peak was at $t = 1.183 \text{ s}$. This phase shift comes from the different pulse frequency for Newtonian and non-Newtonian fluid model. The velocity map for different Q_r ratios is shown in Fig. 1 for 0.7 and in Fig. 2 for 0.9. According to the results of simulation, all viscosity models predict the similar flow pattern. The main flow separates at the right boundary of the branch producing two regions of recirculation. The first one is at the left boundary of branching vessel and the second one is near the top boundary of parent vessel.

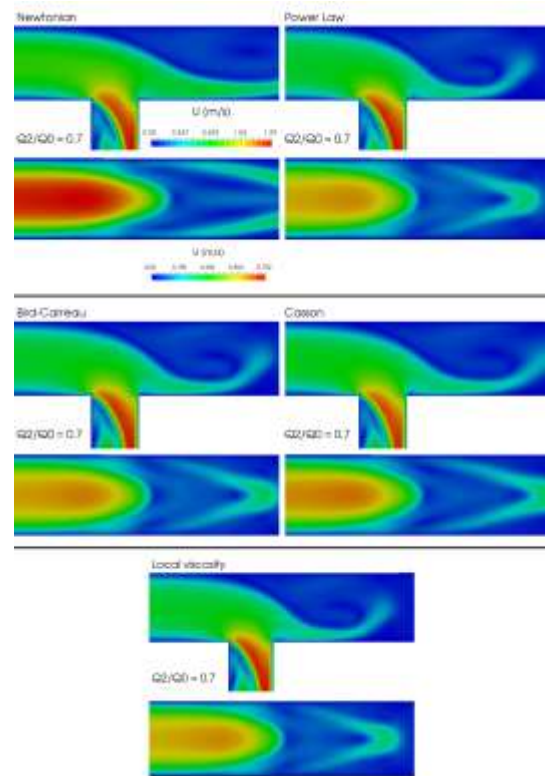


Fig. 1. Velocity distribution at the central cross-section of the bifurcation model at flow rate ratio between the vessel and branch of 0.7.

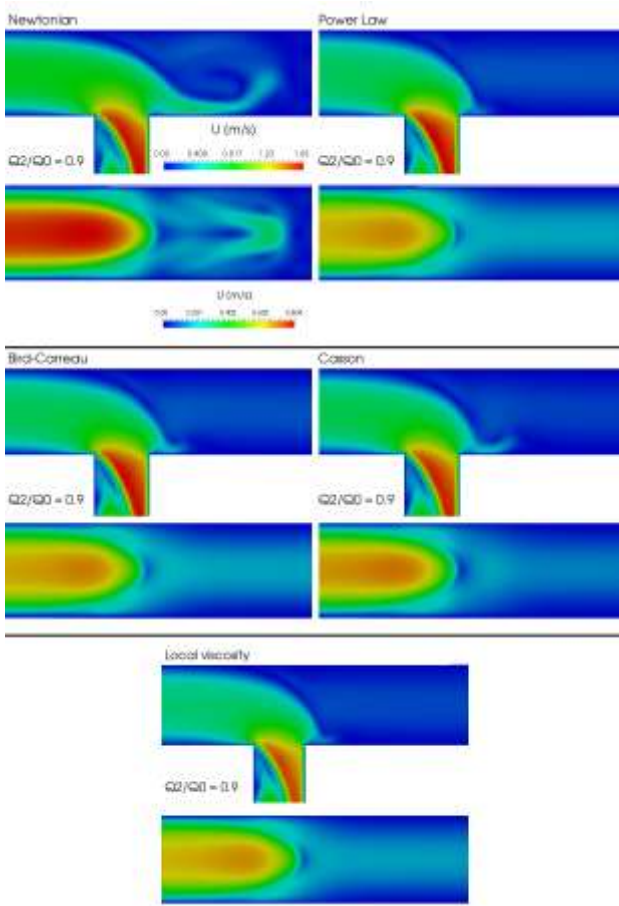


Fig. 2. Velocity distribution at the central cross-section of the bifurcation model at flow rate ratio between the vessel and branch of 0.9

A region of high velocity values is located near the right boundary of branch vessel after the separation point. The maximum velocity values in this region for different flow ratios Q_r are presented in Table 1. As it can be seen, Newtonian model produces the highest velocity magnitude for every Q_r flow-rate ratio.

It should be noted that with increase of flow-rate ratio Q_r the recirculation zone near the left boundary of branch vessel is shrinking, while another recirculation zone near the top boundary of parent vessel is growing. Also for Newtonian fluid the recirculating zone at the top boundary is about 1.5 times larger than for non-Newtonian fluid models.

The computed velocity fields for different viscosity models were post-processed to calculate streamlines. The presented in Fig. 3 and 4 streamlines visualize two flow recirculation zones near the bifurcation region. These zones are characterized by low velocity values. The velocity magnitude in these zones is

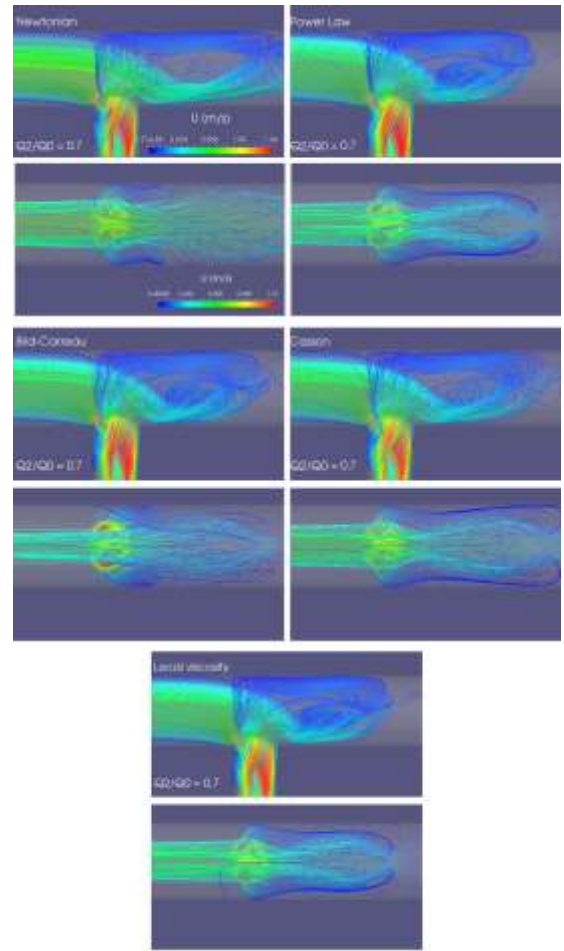


Fig. 3. Streamlines at the central cross-section of the bifurcation model at flow rate ratio between the vessel and branch of 0.7

about $8.12 \cdot 10^{-5}$ m/s. The flow recirculation zone near the left boundary of branch decreases with increase of flow-rate ratio. On the other side, the flow recirculation zone near top boundary of parent vessel increases.

The results show that non-Newtonian properties have a significant impact on blood flow characteristics in the studied cases, especially in the zones of recirculation, and have to be considered in the future investigations.

IV. DISCUSSION

Hemodynamic factors such as velocity and pressure gradients, regions of high and low shear stresses play an important role in forming atherosclerotic plaques. This is especially important at bends and bifurcations, where the flow is disturbed and secondary flows are created.

In this study, the influence of rheological model of blood was investigated by numerical simulations of blood flow in an idealized model of bifurcation with high degree stenosis. For that purpose, three most widely used non-Newtonian viscosity models were employed for a description of blood behavior.

TABLE I. MAXIMUM VELOCITY (M/S) FOR DIFFERENT FLOW RATIOS

Q_r	Newtonian	Power Law	Bird-Carreau	Casson	Local viscosity
0.7	1.374	1.349	1.342	1.344	1.351
0.9	1.741	1.733	1.728	1.73	1.737

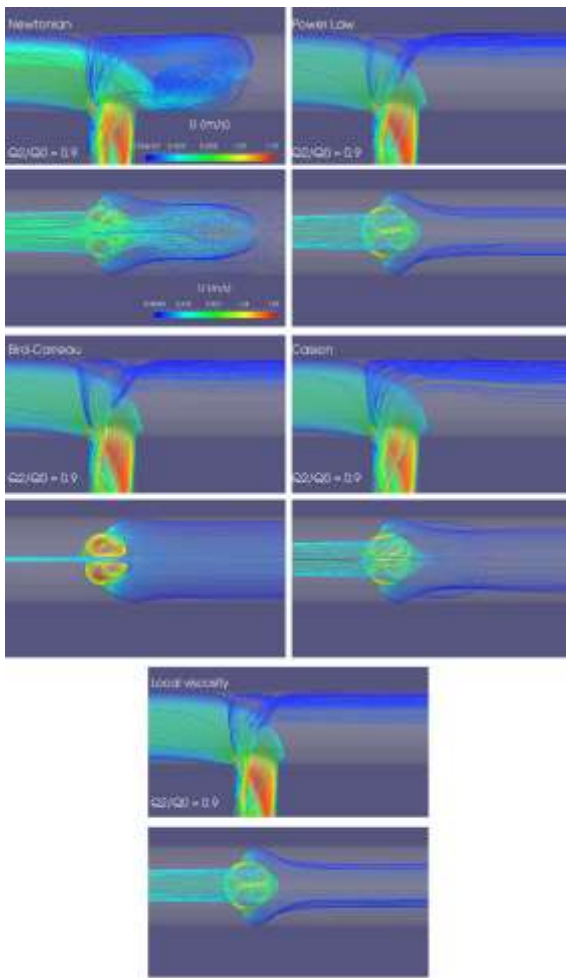


Fig. 4. Streamlines the central cross-section of the bifurcation model at flow rate ratio between the vessel and branch of 0.9

The results were compared with the data obtained from simulations utilizing a Newtonian assumption.

Numerical simulation is a fast method of computing hemodynamic parameters whereas it is difficult to be measured, e.g. viscosity distribution. Newtonian model of fluid is a common assumption for simulation of blood flow in large arteries, however for small arteries, especially for cerebral arteries with atherosclerosis; this assumption may not be precise in all cases. In regions with slow flow and recirculating flow, blood exhibits non-Newtonian behavior.

Fisher and Rossmann used four constitutive models of blood to investigate the influence of non-Newtonian behavior on flow patterns and fluid mechanical forces. They found that the choice of constitutive model has measurable influence on the numerical prediction of progress of cerebral disorders due to fluid stresses, though less influence than aneurysm morphology [7]. To eliminate an influence of patient-specific geometry of the artery and evaluate only difference between different viscosity models, we used only the idealized bifurcation model.

In the recent research Suzuki et al. [8] demonstrated that irrespective to the artery size, numerical simulations with either Newtonian or non-Newtonian viscosity assumption could lead to values different from those of the patient-specific viscosity model for hemodynamic parameters such as normalized WSS. To eliminate this effect we've used a Local viscosity model which perfectly fit the measured sample viscosity.

In our study the highest differences between Newtonian and non-Newtonian fluid models are observed especially at zones prone to atherogenesis. Using idealized geometry we've modeled the high degree of stenosis and assessed influence of different rheological models excluding the impact of individual features of patient's geometry. This study extends the recent studies that evaluate effect of non-Newtonian rheological properties of blood on distribution of hemodynamic parameters in regions of bifurcation of cerebral arteries

The presented study has some limitations. The number of numerical studies should be extended to derive the statistical implications from the results. Also the assumption of rigid walls was used, which could potentially affect the viscosity distribution. Future studies will address the listed limitations.

V. CONCLUSIONS

For both degrees of stenosis Newtonian fluid model overestimates the maximum velocity magnitude compared to non-Newtonian fluid models. With increase of velocity in the branching vessel the differences between Newtonian and non-Newtonian fluid models became less significant. The study showed that the major differences between Newtonian and non-Newtonian fluids were observed in regions of low-velocity recirculation flow, where non-Newtonian blood behavior should be considered.

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