Application of the d'Alembert Method for the Pulse Shape Evaluation in Ultrasonic Problems

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***Abstract*— The pulse mode of the piezoelectric plate transducer operation in liquid was studied. The radiator was excited by the main signal at the initial time and by the compensating delayed signal. Using the d'Alembert method it was shown theoretically that it is possible to stop the transition process in the transducer by supplying the compensating pulse at the certain moment.**

Key words*— *piezoelectric transducer; d'Alembert method; compensation; acoustic pulse; electric pulse

I. Introduction

Different versions of the acoustic control method are currently used for nondestructive testing of materials and products. This is due to its simplicity, safety, low cost, the possibility of control from the one side of the product, etc. Despite the variety of types of hardware implementation of this method, they have in common the presence of a structural unit – a piezoelectric transducer (PET). This transducer must convert the electrical signal into acoustic (radiation mode) and acoustic signal into electrical one (reception mode). The main and urgent task for designers of electronic means of ultrasonic nondestructive testing is to provide conditions of short-signal radiation. A probing signal is considered to be short, if its duration does not exceed only a few half-cycles of high-frequency oscillations. The use of such type signals improves the most important characteristics of the ultrasonic testing devices (dead zone, resolution, accuracy of determination of defects coordinates).

Traditionally, short signals at the output of PET is achieved by using some hardware to expand its bandwidth (mechanical damping, matching layers, loading the output by electrical circuits, etc.). In this case, the signal shape is calculated using Laplace or Fourier transforms. These methods are often used in combination with the equivalent circuit method. Having certain advantages, these methods are rather formal, and do not allow to observe the dynamics of the transition process in the transducer when it is excited by a short electrical signal [1–3].

One can use an alternative method of determining the shape of the emitted (received) signal – the method of successive reflections (d'Alembert method). This method does not have the named disadvantage [1–5]. The essence of the method consists in summing multiple reflections of waves from the boundaries of the layer when the waves propagate inside it. It is also necessary to take into account the partial outputs of the waves from the layer when the wave falls on its boundary. The most advantageous case is the one, when a separate exciting electric pulse has a duration equal to the time of the elastic wave along the thickness (length) of the layer. In this case, pulses having different number of reflection cycles will be added up in the same phase. One must note that the sound velocity, the reflectance and transmission factors do not depend on the frequency. This means that when summing individual pulses, the shape of the output wave will remain similar to that of the original electrical pulse.

In this paper we present a theoretical study of the pulse mode of operation of the piezoelectric plate radiating into the liquid medium. By analyzing this mode, one can see that it is possible to obtain short pulses at the output of the PET.

The short duration of the probe signal emitted by the transducer can be explained as follows. The active element of the piezoelectric transducer (piezoceramic PZTNB-1 plate) is excited by an electrical signal of complex shape. It may consist, for example, of two half sine cycles of the thickness plate vibrations. The first (exciting) half-cycle causes a long transition process in the transducer. The result is an extremely long acoustic signal at the transducer output. The second half-cycle (compensating), is supplied sometime after the exciting and also causes a long transition process. If the amplitude (taking into account the sign) of the compensating half-cycle is correctly determined in advance, then two transients partially compensate each other. As a result, the PET emits a short acoustic pulse. It is possible to change the duration of the emitted acoustic signal by proper adjusting the time of the compensating half-cycle. This means that it is possible to solve the problem even using the narrow-band transducers.

II. Analysis of Piezoelectric Transducer Pulse Mode Operation

Let the of piezoceramic plate (Fig. 1) be loaded on media with specific acoustic impedances z_1 on the face side and z_2 on the back side respectively. The piezoceramic acoustic impedance of is z_c. The piezoceramic plate is excited by the electric pulse in

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the form of a half-cycle sine at the natural frequency ω_0 of plate. The voltage is U.

Under the action of electric voltage on the right side of the plate arise waves of elastic displacement $\xi_1 = \xi_{m1} e^{-jk_1 x}$ and $\xi_2 = \xi_{m2} e^{-jk_c x}$ where k_1 and k_c – are wavenumbers in media 1 and in piezoceramics respectively, ξ_{m1} and ξ_{m2} – are magnitude values of displacements. The x-axis is directed along the plate thickness. The electric field strength: $E = U/d$, where d – the thickness of piezoelectric plates. In this case we neglect the direct piezoelectric effect. On the boundary with medium 1, the conditions of continuity of displacements and elastic stresses are fulfilled. From here at $x = 0$ we obtain that $\xi_{m1} = \xi_{m2}$. For elastic stresses we have:

$$\sigma_c = c_{33}^D \frac{\partial \xi_2}{\partial x} - e_{33} \frac{U}{d} = j\omega_0 z_c \xi_{m2} - e_{33} \frac{U}{d} ;$$

$$\sigma_1 = \rho_1 c_1^2 \frac{\partial \xi_1}{\partial x} = -j\omega z_1 \xi_{m1} .$$

Here c_{33}^D – the elastic modulus of piezoelectric ceramic at constant electric induction; e_{33} – piezoelectric constant; ρ_1 and c_1 – density of medium 1 and sound velocity. From conditions $\xi_{m1} = \xi_{m2}$ and $\sigma_c = \sigma_1$ (σ_c and σ_1 – normal stress along the x-axis) we obtain: $\xi_{m1} = AD_1$, where $A = e_{33}U/(2j\omega d z_c)$, $D_1 = 2z_c/(z_1 + z_c)$ – the displacement transmittance from the piezoelectric to medium 1. Similarly, for the left face we obtain $\xi_{m3} = -AD_2$, where $D_2 = 2z_c/(z_2 + z_c)$.

Oscillations of the plate can be analyzed independently from each other. The resulting signal is obtained by adding the components emitted by each surface separately. Waves propagating from the right surface to the left one will be reflected from it with a reflectance factor $R_2 = (z_c - z_2)/(z_c + z_2)$, and waves propagating from the left face to the right will be reflected from it with a reflectance factor $R_1 = (z_c - z_1)/(z_c + z_1)$. Waves coming from inside the transducer to the right surface will partially come out into the medium with specific acoustic impedance z_1 with transmittance factor D_1. Similarly, in a medium with a specific acoustic impedance z_2, the waves will come out with a transmittance factor D_2.

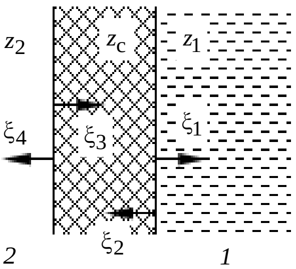

Рис. 1. Waves in media

Numbers of half-cycles of transient process are numbered: $i = 0, 1, 2, \dots$. Normalize the transient magnitudes to the amplitude of the initial half-cycle AD_1. Then the amplitudes of the normalized process in each of the half-periods will be:

$a_0 = 1$;

$a_i = D_1 R_2 (R_1 R_2)^{(i/2)-1} = D_1 R_2 R^{(i/2)-1}$ – for even $i \neq 0$;

$a_i = -D_2 R^{(i-1)/2}$ – for odd numbers $\underline{i}$; $R = R_1 R_2$.

Consider a specific example. As the active material we choose piezoceramics PZTNB-1 ($z_c = 30 \cdot 10^6$ Pa·s/m). We assume that the plate is loaded on water ($z_1 = 1.5 \cdot 10^6$ Pa·s/m), and the back surface is bordered by air ($z_2 = 0$). Offset amplitudes normalized to the amplitude of the initial half-period ξ/ξ_0 (taking into account the sign), calculated by the previous formulas are given in Tab. 1. One can see from tab.1, that the transient process caused by an exciting electrical pulse of one half-cycle is strongly tightened. In fact, it exceeds 20 half-cycles.

TABLE I. VALUES OF DISPLACEMENT AMPLITUDES NORMALIZED TO THE AMPLITUDE OF THE INITIAL HALF-LIFE

Number of half-period	0	1	2	3	4	5	6	7	8	9
ξ/ξ_0	1	–2	1.91	–1.81	1.72	–1.64	1.56	–1.48	1.41	–1.34

The calculation of the transient process, produced for the case of excitation of the plate by one half-cycle of the electric voltage, can serve as a basis for the calculation in the case of its excitation by a system of single-half-wave pulses of the same frequency. The resulting process can be obtained as a superposition of individual processes with the given amplitudes and time shift.

When calculating, it is convenient to consider the dimensionless time $T = t/(T_0/2)$, where t – physical time, and T_0 – duration of the period of natural oscillations.

If the process, generated by one exciting pulse, denote by $s_0(T)$, and the number of subsequent exciting pulses is j, then the resulting process can be written as $s(T) = s_0(T) + \sum_{i=1}^{j} a_i s_0(T - T_i)$, where a_i – amplitudes of exciting pulses, $T_1 < T_2 < \dots \dots < T_j \leq T - 1$ and T_i – integer numbers.

We shall show that the problem of obtaining a short acoustic pulse can be solved by supplying at some time a compensating electric pulse also in the form of a half-period of the sine wave of the natural frequency, with a certain amplitude and polarity opposite to the polarity of the initial process in the half-cycle of the compensating pulse. Let in some half-period with an even number $i \neq 0$ a compensating pulse U_{comp} is applied. Denote by a the ratio of amplitudes U_{comp}/U, where U – the amplitude of the initial excitation pulse. In this half-period, the amplitude ξ_i of the initial process is determined by the expression $\xi_i = D_1 R_2 R^{(i/2)-1}$, and the amplitude of the compensating process $\xi_{comp} = -a$. The resulting signal in this half-period is equal to $\xi_{res} = D_1 R_2 R^{(i/2)-1} - a$. In the next half-period (odd number) we have $\begin{cases} \xi_{i+1} = -D_2 R^{\frac{i+1-1}{2}} \\ \xi_{comp} = aD_2 \end{cases}$.

The resulting process must be zero in the half-period with the number $i+1$, i. e. $\xi_{res} = D_2\left(a - R^{i/2}\right) = 0$, where does it follow that $a = R^{i/2}$.

Let us consider what will happen in the next (even) half-period with the number ($i+2$). The basic process $\xi_{i+2} = D_1 R_2 R^{\frac{i+2}{2}-1} = D_1 R_2 R^{i/2}$. The offsetting process $\xi_{comp} = -D_1 R_2 R^0 a = -D_1 R_2 R^{i/2}$. The offsetting process $\xi_{res} = \xi_{i+2} + \xi_{comp} = 0$. It follows that in all subsequent half-cycles the transition process is completely compensated.

Now we shall consider the case when the compensating pulse is applied into the half-cycle with an odd number. Then the main process: $\xi_i = -D_2 R^{(i-1)/2}$. The compensating process in this half-period: $\xi_{comp} = a$. The resulting process is: $\left(\xi_i\right)_{res} = -D_2 R^{\frac{i-1}{2}} + a$. At the next half-period (even) with the number $(i+1)$ $\xi_{i+1} = D_1 R_2 R^{\frac{i-1}{2}}$; $\xi_{comp} = -aD_2$. The resulting process in this half-period $\xi_{res} = D_1 R_2 R^{\frac{i-1}{2}} - D_2 a = 0$, from where $a = \left(\frac{D_1 R_2}{D_2}\right) R^{\frac{i-1}{2}}$. Calculations have shown, that in the half-periods with even numbers compensation remains full, and in the half-periods with odd numbers is not complete. In this case the amplitudes of the resulting process are small, not exceeding 0.4% of the initial amplitude of the process.

From the above it can be seen that when the compensating pulse is applied to the half-period i with an even number, not only for the unilaterally loaded piezoplate ($z_2 = 0$), the full compensation is achieved, but also for arbitrary acoustic loads z_1 and z_2 in the half-period with the number ($i+2$), the resulting process will be zero. In fact, this means that generally for a damped transducer, when $z_2 \neq 0$ full compensation is possible.

Interesting is the case when $z_1 = z_2$, i. e. the case of symmetric two-side load. Herewith $R_1 = R_2$, $R = R_1^2$, $D_2 = D_1$, and the amplitude of the compensating pulse $a = R_1^i$. Taking all these conditions into account in the formula for the resulting process, it can be shown that $\left(\xi_{i+2}\right)_{res} = 0$. Thus, we have a case of complete compensation.

In general case of a damped transducer ($z_2 \neq 0$ and $z_2 \neq 1.5 \cdot 10^6$ Pa·s/m) we have the formula $\left(\xi_{i+2}\right)_{res} = R^{\frac{i+1}{2}} D_2 \left[-1 + \frac{D_1^2 R_2}{D_2^2 R_1}\right]$, which can be used to calculate the amplitudes of the process different from zero at different i and different z_2. Since the process decreases with the growth of the number i, it is advisable to make calculations for $i = 1$ to estimate the maximum amplitude after compensation.

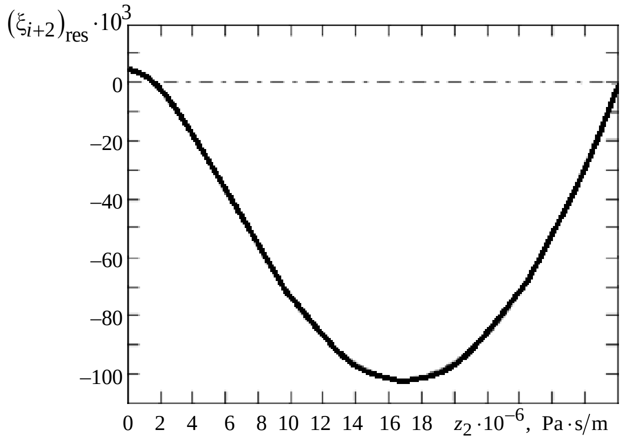

Рис. 2. Dependence $\left(\xi_{i+2}\right)_{res}$ on the specific impedance of the damper z_2

Fig. 2 shows the dependence of $\left(\xi_{i+2}\right)_{res}$ at $i = 1$ from the specific impedance z_2 of the damper. With growth of z_2 from 0 up to $15.6 \cdot 10^6$ Pa·s/m we can see a monotonous decrease of ξ_{res} from $4.53 \cdot 10^{-3}$ to –0.1, going through 0 at $z_2 = 1.5 \cdot 10^6$ Pa·s/m and then again increases. It is interesting to note that in the symmetric mode of radiation ($z_1 = z_2 = 1.5 \cdot 10^6$ Pa·s/m) incomplete compensation becomes complete. With further growth of z_2, the function changes the sign and increases in absolute value until z_2 exceeds the value $17 \cdot 10^6$ Pa·s/m. Further growth of z_2 leads to a decrease in the absolute value of the function $\left(\xi_{i+2}\right)_{res}$, which becomes zero at $z_2 = z_c$ (the case of the “ideal” damper).

III. Conclusion

Thus, using the d'Alembert method, the pulse mode of operation of the piezoelectric plate emitter was studied. The possibility of termination of the transient process was shown. The method allowing to determine the amplitudes of the electric compensating pulse is proposed. Mathematical expressions for the amplitude of the compensating pulse for different moments of time and different degrees of damping of the transducer are obtained. It is noted that the compensation may be complete and incomplete. In the case of incomplete compensation, the maximum amplitudes of acoustic pulses after the impact of the compensating pulse were estimated. It was shown as well, that incomplete compensation goes to full compensation at $z_2 = z_1$, i. e. if the piezoelectric plate is loaded symmetrically.

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