Patterns of Population-based Algorithms for Continuous Global Optimization

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***Abstract*— We consider the population-based algorithms (P-algorithms) for continuous global optimization. We systematize the basic entities and, in particular, evolutionary operators of P-algorithms. We select such entities as free parameters, neighbourhood space, individual, population, union of individuals, area of individual, as well as the following evolutionary operators: population initialization, search termination, coding of individuals, randomization, selection, crossing, population control, local search. On the basis of the proposed methods of describing these entities, we present the patterns of the most well-known P-algorithms: evolutionary algorithms, behavioral algorithms inspired by animate and inanimate nature and also by human society.**

Keywords*— *global optimization; population algorithm; metaheuristic algorithm; pattern of the population algorithm

I. Introduction

In different publications P-algorithms (population-based algorithms) of continuous global search optimization are called behavioral, intellectual, metaheuristic, nature-inspired, swarms, multi-agent, etc. [1]. P-algorithms are numerous and very diverse, so more than 130 such algorithms are presented [2], and new algorithms continue to appear.

There is a small number of papers devoted to various aspects of the analysis of P-algorithms. Thus, the fundamental papers [3, 4] that use pseudocode present algorithms and operators of evolutionary algorithms. In the paper [5] general schemes of P-algorithms and their operators are also presented as a pseudocode. A high-level formalism that goes back to the works by J.Holland and is oriented to the description of adaptive P-algorithms was proposed in the paper [6] in the context of discussing the approaches that can be used to develop new P-algorithms.

II. Problem Statement

We consider the deterministic problem of continuous global unconstrained minimization

$$\min_{X \in R^{|X|}} f(X) = f(X^*) = f^*,$$

where $|X|$ is length of the vector of variable parameters X; $f(X)$ is objective function (O-function) with space values R^1; X^*, f^* are the required optimal solution and value of the O-function correspondingly. We assue that the search for the solution of the assigned problem begins in the $|X|$-dimensional parallelepiped $\Pi = \{X \mid X^- \le X \le X^+\}$, where X^-, X^+ are its lower and upper edges. We also consider the analogous problem of global conditional minimization in a convex set of admissible values of the vector X. The fitness function $\varphi(X)$ of the P-algorithm is also subject to minimization.

III. Entities of P-algorithms

We distinguish between the deterministic and stochastic entities. Also, we distinguish between the static and dynamic program-configured and dynamic adaptive entities. The definition of an entity contains the following descriptors:

$$system \in \{det, stoch\};\ control \in \{stat, prog, adapt\}.$$

Excluding evolutionary operators, we select such entities as free parameters and meta-parameters, neighborhood space, individual, population, union of individuals, neighbourhood of individual, trace of individual, etc. The less obvious descriptions of these entities are presented below. The examples of use are given in the next section.

Meta-parameters are auxiliary free parameters that determine the values of basic free parameters. The description of the free meta-parameters of the P-algorithm has the following form:

parameters $\overline{P}\,|_{ID}^{system.control}$ = <meta-parameters>:<limitations>.

Similarly, the *neighbourhood space of the individuals* is determined by the formula

space $NAME(A, P)\,|_{ID}^{system.control}$: <space metric>.

Here $NAME \in \{R_\varphi, R_X, R_T\}$; R_φ, R_X, R_T is a fitness space, a search space and a topological space, correspondingly; A, P are lists of arguments and free parameters.

Union of individuals in the space $R_\alpha, \alpha \in \{\varphi, X, T\}$ is defined by the form

$$\text{union } R_\alpha \ NAME(A,P)|_{ID}^{systemcontrol} = \text{<elements>}.$$

Area of the individual $s_i \in S$ in the same space R_α is part of this space, and the elements of the area, in terms of the space metric, are close to the position of the individual s_i:

$$\text{area } NAME(s_i, A, P)\Big|_{R_\alpha.ID}^{system\,control} : \text{<elements>}.$$

Track of the individual s_i of the population *S* in the interval $[0:t]$ is a set of pairs $(X_i(\tau), \varphi_i(\tau))$ for all $\tau \in [0:t]$:

$$\text{track } Tr(s_i, t, A, P)|^{det.stat} \{(X_i^0, \varphi_i^0),...,(X_i^t, \varphi_i^t)\}.$$

The main evolutionary operators are: population initialization, search termination, coding of individuals, randomization, selection, crossing, population control, local search. Below are the patters for describing some of these operators. The examples of use are presented in the next section.

Entity initialization Ξ:

$$\text{init } \Xi(A,P)\Big|_{ID}^{systemcontrol} = \Xi(0) = \Xi^0.$$

Search randomization. The operator of the evolution randomization of an individual $s_i \in S$ is determined by the expression

$$\text{rand}(s_i, A, P)|_{ID}^{systemcontrol} = X_i'.$$

The remdomized operator of individuals crossing $\{s_{i_1}, s_{i_2},..., s_{i_n}\} \in S$ in the general case has the following form:

$$\text{rand.crossing}(A,P)\Big|_{n\times m.ID}^{systemcontrol} = \{s_{j_1}', ..., s_{j_m}'\}, \quad \{s_{i_1},..., s_{i_n}\} \in A.$$

Population control. Among the operators of population control we defined operators of compression, extension, replication, splitting, dissimilation of the population, etc. The population compression operator *S*, for example, has the form

$$\text{compression } (S, A, P)\Big|_{ID}^{systemcontrol} = S'.$$

IV. Patterns of P-algorithms

As an example, we present four P-algorithms related to the classes of evolutionary algorithms, algorithms inspired by animate nature and by inanimate nature, and also related to the algorithms inspired by the processes in the human society.

A. Differential evolution DE

We consider the DE-algorithm in which the basic individual [1] is selected from the current population uniformly random.

1) Initialization

Initialization of deterministic static free parameters $|S|, |X|, X^-, X^+, a, \xi_b$ (operator init P$\Big|_{DM}^{det.stat}$) by the Decision Maker (*DM*).

We distribute individuals of the population in the area Π (operator init S$\Big|_{uniform}^{stoch.stat}$) evenly at random.

2) Population evolution

For each $i \in [1:|S|]$ we use the randomizing crossing operator rand.cross $\Big|_{3\times 1.DE}^{stoch.stat}$:

$$x_{i,j}' = \begin{cases} x_{i_1,j} + a\,(x_{i_2,j} - x_{i_3,j})\,|\,\xi_b, & j = j_1 = U_1[1:|X|], \\ x_{i,j}, & \text{else}, \end{cases}$$

$$\{s_i, s_{i_1}, s_{i_2}, s_{i_3}\} \in S;\ i = U_1[1:|S|],\ i_k = U_1[1:|S|],\ k \in [1:3],$$
$$i_1 \ne i_2 \ne i_3 \ne i,\ j = [1:|X|].$$

We select the individuals (operator select$|_{\varphi\,best}^{det.stat}$):

$$s_i(t+1) = \begin{cases} s_i', & \varphi(s_i') < \varphi(s_i), \\ s_i(t+1), & \text{иначе}. \end{cases}$$

Here $U_1[1:|S|]$ is an integer which is distributed uniformly random in the interval $[1:|S|]$.

3) Termination of the evolution process (end$|^{any}$)

Here the descriptor *any* means that the conditions of the search termination are not fixed.

DE-algorithm pattern

specifications

parameters $P|_{DM}^{det.stat} = (|S|, |X|, X^-, X^+, a, b))$:

$a \in [0;2]$, $\xi_b \in [0;1]$.

individual $S : X = (x_j, j \in [1:|X|]) \in \Pi \subset R^{|X|}$.

population $S = \{s_i, i \in [1:|S|]\}$.

initialization

init P$(P)\Big|_{DM}^{det.stat} = P$.

init S$(S,P)\Big|_{uniform}^{stoch.stat} = S(0) = S^0$.

evolution

rand.cross $(S,P)|_{3\times 1.DE}^{stoch.stat}$, $i \in [1:|S|]$.

select $(S, S')|_{\varphi.best}^{det.stat} = S(t+1)$.

termination

end $(S,P)|^{any} = (\tilde{X}^*, \tilde{f}^*)$.

B. Particle swarm optimization algorithm – PSO.

1) Initialization

Initialization of free parameters by the *DM* (operator init P$\Big|_{DM}^{det.stat}$).

Initialization of the population by uniformly random distribution of individuals in the area Π, and the vectors of their initial velocities $\Delta X_i, i \in [1:|S|]$ in the area Π_Δ (оператор init S$\Big|_{uniform}^{stoch.stat}$).

2) Population evolution

To each of the individuals $s_i \in S$ we apply the randomizing crossing operator rand.crossing$|_{|S|\times 1.PSO}^{stoch.stat}$:

$$X_i' = X_i + b_I \Delta X_i^- + U_{|X|}(0;b_C) \otimes (X_i^* - X_i) +$$
$$+ U_{|X|}(0;b_S) \otimes (X_i^{**} - X_i),$$

$$\varphi(X_i^*) = \min_{\tau \in [0:t]} \varphi(X_i(\tau)), \quad \varphi(X_i^{**}) = \min_{j \in N_i} \varphi(X_j^*).$$

Here $U_{|X|}(0;b)$ is a $|X|$-dimensional vector of random numbers uniformly distributed in the interval $[0;b]$; N_i is a set of neighbours of the individual s_i [1].

3) Termination of the evolution process (end$|^{any}$)

Pattern of the PSO algorithm

specifications

parameters $P|_{DM}^{det.stat} = (|S|, |X|, X^-, X^+, b_I, b_C, b_S)$:

$b_I = 0{,}7298$, $b_C = b_S = 1{,}49618$.

individual S: $X = (x_j, j \in [1:|X|]) \in \Pi \subset R^{|X|}$.

population $S = \{s_i, i \in [1:|S|]\}$.

space R_T $|^{det.stat}$: $\mu_{R_T}(s_i, s_j) = r(s_i, s_j)$, $i, j \in [1:|S|]$.

track $Tr(s_i, t, P)|^{det.stat} = \{X_i^0, ..., X_i^t\}$, $i \in [1:|S|]$.

initialization

init P$(P)\Big|_{DM}^{det.stat} = P$.

init S$(P)\Big|_{uniform}^{stochstat} = S(0) = S^0$.

evolution

select$(Tr_i, P)|_{best}^{det.stat} = X_i^*$, $i \in [1:|S|]$.

select$(G_i, P)|_{best}^{det.stat} = X_i^{**}$, $i \in [1:|S|]$.

rand.crossing $(s_i, X_i^*, X_i^{**}, P)\Big|_{|S|\times 1.PSO}^{stoch.stat} = s_i(t+1)$; $i \in [1:|S|]$.

termination

end $(P)|^{any} = (\tilde{X}^*, \tilde{f}^*)$.

Here we use the following definitions: μ_{R_T} is a proximity metric for individuals in the space R_T; $r(s_i, s_j)$ is the distance between the vertexes s_i, s_j in the neighborhood graph $G = G_T(S)$.

C. Electromagnetic algorithm EM

1) Initialization

Initialization of free parameters by the *DM* (operator init P$\Big|_{DM}^{det.stat}$).

Programmatic initialization of the population by uniformly random distribution of individuals in the area Π (operator init $S\Big|_{uniform}^{stoch.stat}$).

2) Population evolution

To each of the individuals $s_i \in S$, $i \neq i_b$ we apply the randomizing crossing operator rand.crossing$\Big|_{|S|\times 1.EM}^{stoch.stat}$:

$$X_i' = X_i + \lambda_1 U_1(1) \frac{F_i}{\|F_i\|_E} \otimes V_i, \quad X_{i_b}' = X_{i_b};$$

$$F_i = \sum_{j=1,\, j\neq i}^{|S|} F_{i,j} = \sum_{j=1,\, j\neq i}^{|S|} \begin{cases} (X_j - X_i)\dfrac{q_i\, q_j}{\|X_j - X_i\|_E^2}, & \varphi_j < \varphi_i, \\ (X_i - X_j)\dfrac{q_i\, q_j}{\|X_j - X_i\|_E^2}, & \varphi_i \le \varphi_j; \end{cases}$$

$$q_i = \exp\left(-|X| \frac{\varphi_i - \varphi_{i_b}}{\sum_{j\in[1:|S|],\, j\neq i} (\varphi_j - \varphi_{i_b})}\right);$$

$$v_{i,j} = \begin{cases} (x^+ - x_{i,j}), & F_{i,j} > 0, \\ (x_{i,j} - x^-), & F_{i,j} \le 0, \end{cases} \quad j \in [1:|X|], \ j \neq i.$$

For each of the individuals $s_i \in S$ we use a linear stochastic local search by means of the algorithm RLS (operator local$|_{RLS}^{stoch.stat}$):

$$X_i'' = X_i' + \lambda_2 V_i, \quad i \in [1:|S|],$$

$$v_{i,j} = u_{sign}^{\pm 1} U_1(0;1), \ j \in [1:|X|], \ \lambda_2 = \alpha \max_{j\in[1:|X|]} (x^+ - x^-).$$

3) Termination of the evolution process (end$|_{\hat{t}}^{det.stat}$)

Condition of the search termination: $t = \hat{t}$.

Pattern of the EM-algorithm

specifications

parameters $P|_{DM}^{det.stat} = (|S|, |X|, X^-, X^+, \lambda_1, \alpha, \hat{\tau}_l, \hat{t})$:

$\alpha \in (0;1)$, $\hat{\tau}_l >> |X|$, $\hat{t} = 25\,|X|$.

space R_X $|^{det.stat}$: $\mu(s_i, s_j) = \|X_i - X_j\|_E$.

individuals S: $X(s) = (x_j, j \in [1:|X|]) \in \Pi \subset R^{|X|}$.

population $S = \{s_i, i \in [1:|S|]\}$.

initialization

init P$\Big|_{DM}^{det.stat} = P$.

init S$(P)\Big|_{uniform}^{stoch.stat} = S(0)$.

evolution

rand.crossing $(s_i, P)\Big|^{stoch.stat}_{|S|\times 1.EM} = s'_i$; $i \in [1:|S|]$.

local $(s'_i, P)\,|^{stoch.stat}_{RLS} = s''_i = s_i(t+1)$; $i \in [1:|S|]$.

termination

end $(P)\,|^{det.stat}_{\tilde{t}} = (\tilde{X}^*, \tilde{f}^*)$.

D. Simple mind evolutionary algorithm (SMEC)

1) Initialization

Initialization of free parameters by the *DM* (operator init P $|^{det.stat}_{DM}$).

Programmatic uniformly random distribution of individuals $s^b_i, i \in [1:|S^b|]$ of the leading unit S^b in the area Π (operator init $\mathbf{S}\Big|^{stoch.stat}_{uniform}$). Analogous initialization of the individuals $s^w_i, i \in [1:|S^w|]$ of the lagging unit S^w (operator init S $\Big|^{stoch.stat}_{uniform}$).

2) Population evolution

Local search in the neighbourhood of the individual s^b_i (operator local $|^{stoch.stat}_{Normal}$):

$$X^b_{i,j} = X^b_{i,1} + N_{|X|}(0, \sigma), \quad X^b_{i,1} = X^b_i, \quad j \in [2:n_g];$$

$$X'^b_i = X^b_{i,j_b}, \quad \min_{j\in[1:n_g]} \varphi(X^b_{i,j}) = \varphi(X^b_{i,j_b}); \ i \in [1:|S^b|].$$

Analogous search in the neighbourhood of the individual s^w_i, $i \in [1:|S^w|]$ (operator local $|^{stoch.stat}_{Normal}$).

Formation of a new population (operator dissimilation$\Big|^{det.stat}_{SMEC}$):

$$\text{if } \varphi(s^b_i) > \varphi(s^w_k), k \in [1:|S^w|], \text{ then } s^w_k \to S^b, s^b_i \to S^w;$$
$$i \in [1:|S^b|];$$

$$\text{if } \varphi(s^w_k) > \varphi(s^b_i), \ i \in [1:|S^b|], \text{ then delete } s^w_k, \text{ init } s^w_k\,|^{stoch.stat}_{uniform};$$
$$k \in [1:|S^w|].$$

3) Termination of the evolutionary process (end $|^{det.stat}_{\varphi\ stagnation}$).

End condition for iterations is stagnation of the computation process (free parameters: $\delta t, \delta\varphi$).

Pattern of the SMEC algorithm

specifications

parameters

$P\,|^{det.stat}_{DM} = (|S|, |X|, X^-, X^+, |S^b|, |S^w|, \sigma, \delta t, \delta\varphi$):

$|S^b| + |S^w| = |S|, \ \sigma > 0$.

individuals S: $X(s) = (x_j, \ j \in [1:|X|]) \in \Pi \subset R^{|X|}$.

union $S^b\,|^{stoch.dynamic} = \{s^b_i, i \in [1:|S^b|]\}$.

union $S^w\,|^{stoch.dynamic} = \{s^w_i, i \in [1:|S^w|]\}$.

population $S = \{s_i, i \in [1:|S|]\} = S^b \cup S^w$.

initialization

init P$\Big|^{det.stat}_{DM} = P$.

init union $S^b(P)\Big|^{stoch.stat}_{uniform} = S^b(0)$.

init union $S^w(P)\Big|^{stoch.stat}_{uniform} = S^w(0)$.

evolution

local $(s_i, P)\,|^{stoch.stat}_{Normal} = s'_i, \quad i \in [1:|S|]$.

dissimilation $(S^b, S^w, P)\Big|^{det.stat}_{SMEC} = S' = S(t+1)$.

termination

end $(S, P)\,|^{det.stat}_{\varphi\ stagnation} = (\tilde{X}^*, \tilde{f}^*)$.

V. Conclusion

With the help of the proposed notation we recorded the patterns of the basic P-algorithms presented in [1]. This experience showed convenience and laconism of the proposed means. During the work the author plans to develop a software system designed for automated synthesis of P-algorithms.

References

[1] Karpenko A.P. Modern Algorithms of Search Ooptimization. Algorithms Inspired by Nature. Moskow. BMSTU publishing, 2014. p. 446. (In Russian).

[2] Bo Xing, Wen-Jing Gao. Innovative Computational Intelligence: A Rough Guide to 134 Clever Algorithms. Springer International Publishing Switzerland, 2014. 451 p.

[3] Evolutionary Computation. 1. Basic Algorithms and Operators. Edited by Thomas Back, David B. Fogel, Zbigniew Michalevich. Institute of Physics Publishing, Bristol and Philadelphia. 2004. 339 p.

[4] Evolutionary Computation. 2. Advanced Algorithms and Operators. Edited by Thomas Back, David B. Fogel, Zbigniew Michalevich. Institute of Physics Publishing, Bristol and Philadelphia, 2004. 339 p.

[5] Luke S. Essentials of Metaheuristics. Access Mode: http://cs.gmu.edu/~sean/book/metaheuristics/ (access date 06.01.2018).

[6] Brownlee J. Clever Algorithms: Nature-Inspired Programming Recipes. Access Mode: http://www.cleveralgorithms.com/nature-inspired/index.html (date of circulation 06.01.2018).

[7] Skobtsov Yu.A. Speranskiy D.V. Evolutionary computation: Teaching aid. Moskow. The National Open University " INTUIT", 2012. p. 331. (In Russian).

[8] De Jong K.A. Evolutionary Computation: a Unified Approach. Massachusetts Institute of Technology. 2006. 256 p.