Four-Dimensional Constellations for Improving Energy Efficiency of Communication Systems

Lyubov A. Antyufrieva
Laboratory of multimedia systems and technologies
Moscow Institute of Physics and Technology (State University)
Moscow, Russia
antyufieva@phystech.edu

***Abstract*— In the article the problem of energy efficiency of communication systems is considered. Methods for constructing optimal multidimensional constellations are analyzed using four-dimensional example.**

Keywords— digital signal processing; energy efficiency of communication systems; Shannon limit; constellation modulation; communication systems design

I. Introduction

An important aspect in communication systems design is to minimize energy potential of the line to limit the power of radiofrequency radiation and to reduce interference with other systems. Improving the energy efficiency of the system allows increasing the speed and reliability of data transmission, and hence the bandwidth of the channel, while maintaining the transmitter power.

II. Multidimensional constellations

In the fundamental work [1] Shannon defined the limiting channel capacity

$$R_f = \log_2(1 + SNR) \tag{1}$$

where R_f – channel capacity per unit frequency, SNR – signal-to-noise ratio. In deriving formula (1), the idea of representing a complex signal as a point in a multidimensional space is used. The foundations of the concept of multidimensional surface and volume-spherical constellations are laid in his later work [2] and then developed in a number of articles and a monograph [3, 4, 5].

We define $\left\{\overrightarrow{c_1}, \overrightarrow{c_2}, ..., \overrightarrow{c_M}\right\}$ – a set of N-dimensional vectors characterizing the signal points of the constellation, $d_{jk} = \| \overrightarrow{c_j} - \overrightarrow{c_k} \|$ – the distance between the signal points. $d_{\min} = \min\limits_{j \neq k}\left(d_{jk}\right),\ \ 1 \le j,k \le M$ – minimum distance between the signal points. The average energy of a symbol E_S is determined by the expression $E_S = \dfrac{1}{M}\sum\limits_{k=1}^{M}\left\|\overrightarrow{c_k}\right\|^2$.

In [6] the problem of constructing an energy-optimal constellation is reduced to the problem of decreasing the average energy of a symbol while maintaining the minimum distance between signal points, which in turn reduces to the task of finding the densest packing of *N*-dimensional spheres. To demonstrate the effectiveness of this method, the dependence of the symbol error rate (SER) on the signal-to-noise ratio per bit of transmitted information is used. Yet this does not reflect the possibility of reducing the number of bit error rate provided by the Gray code.

Constellations with *N>2* are used in optical communication channels, where it is possible to decompose the signal into 4 components due to polarization [6, 7]. There is no fundamental prohibition of the use of multidimensional constellations in standard radio channels. To construct such constellation, it is possible to combine several I-Q constellations orthogonal in time or frequency. The disadvantage of using such multidimensional constellations is the need for an exponential increase in the number of signal points, in order to maintain the channel capacity per unit frequency (2). Therefore, we restrict our considerations to the cases of *N=2* and *N=4*.

$$R_f = \frac{2}{N}\log_2 M \tag{2}$$

III. Two-dimensional and four-dimensional constellations

For comparison of constellations a simulation was performed in which the bit error rate (BER) from the signal-to-noise ratio per one bit of information in a channel with additive white Gaussian noise was calculated.

A. Hypercubic constellation

Any *N*-dimensional hypercubic constellation is a linear combination of two-dimensional quadrature phase shift keying (QPSK) and is equal to the two-dimensional constellation in the terms of energy efficiency [6]. The same applies to linear combinations of quadrature amplitude modulation. A distinctive feature of such ensembles is the possibility of using the Gray code to reduce the number of bit errors. For comparison with four-dimensional ensembles, two-dimensional QPSK and 16-QAM using the Gray code are selected.

978-1-5386-5824-6/18/$31.00 ©2018 IEEE

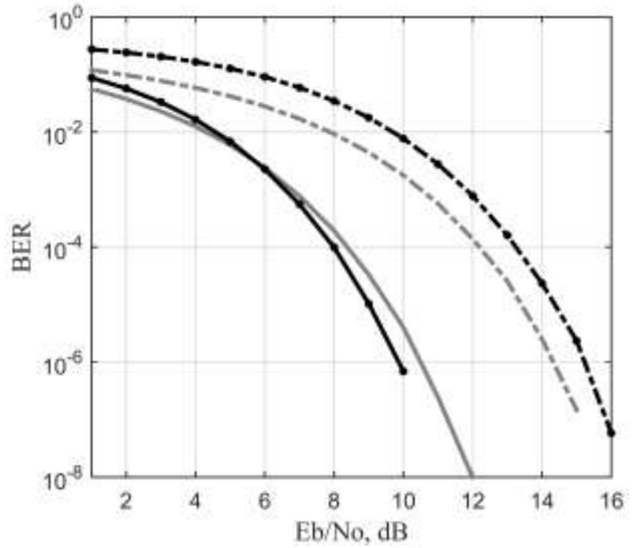

Fig. 1. Dependence of the bit error rate (BER) from the ratio of the bit energy to the noise energy. A black solid line is a dense packing of four-dimensional spheres $C_{4,16}$. Gray solid - QPSK. The gray dot-dash is 16-QAM. Black dot-dash - 4-dimensional surface-spherical constellation *M = 256.*

B. *Dense packing of N-dimensional spheres*

As an algorithm of the densest packing of *N*-dimensional spheres modeling the attraction of a random arrangement of *M N*-dimensional spheres of the same radius [6] is used. The best known four-dimensional ensemble of signals with *M = 16* (3) [6] corresponds to the channel capacity per unit frequency QPSK. In this ensemble of signals, the Gray code is not applicable.

$$c_{4,16} = \{\left(a+\sqrt{2},0,0,0\right),\left(a,\pm\sqrt{2},0,0\right), \left(a,0,\pm\sqrt{2},0\right),\left(a,0,0,\pm\sqrt{2}\right), \left(a-c,\pm 1,\pm 1,\pm 1\right),\left(a-c-1,0,0,0\right)\} \tag{3}$$

where $a=\left(1+\sqrt{2}+9c\right)/16$, $c=\sqrt{2\sqrt{2}-1}$.

C. *Surface-spherical constellations*

As a surface-spherical ensemble of signals, a 4-dimensional constellation *M = 256* corresponding to the channel capacity per unit frequency 16-QAM, calculated in the monograph [5], is taken. The disadvantage of surface-spherical constellations is that they become energy-efficient only at large *N*, when the bulk of the volume of the multidimensional sphere is concentrated at the surface, and also the impossibility of the Gray code being applicable.

D. *Simulation results*

The results of modeling the dependence of the bit error rate on the ratio of the energy of the bit to the noise energy are shown in Fig. 1. With a large E_b/N_0, the ensemble produces fewer errors than QPSK. The gain of QPSK at low E_b/N_0 is due to the Gray code. It follows from the data given in [8] that the using of this constellation can give an energy gain at high code rates and a high signal-to-noise ratio.

IV. Conclusion

The results of the simulation demonstrate the possibility of using constellations built on the basis of tight packing of 4-dimensional spheres with a high signal-to-noise ratio. In the future, it is planned to add forward error correction coding and use of non-uniform constellations.

Acknowledgment

I express my sincere gratitude to Mark Aronovich Bykhovsky for the materials given.

References

[1] Shannon C. Communication in the presence of noise. Proc. IRE. Vol. 37, I. 1. 1949. Pp 10-21. DOI: 10.1109/JRPROC.1949.232969

[2] Shannon C. Probability of error for optimal codes in a Gaussian channel. The Bell System Technical Journal. Vol. 38. I. 3. 1959. Pp. 611 – 656. DOI: 10.1002/j.1538-7305.1959.tb03905.x

[3] Bykhovsky M.A. *Pomehoustojchivost' priema optimal'nyh signalov, raspolozhennyh na poverhnosti N-mernogo shara* [Noise immunity of the optimal signals' reception located on a surface of N-dimensional sphere]. *Jelektrosvjaz'* [Telecommunications]. No. 3. 2016. Pp. 40-46 (in Russian).

[4] Bykhovsky M.A. *Teoreticheskie osnovy proektirovanija sistem svjazi s vysokoj jenergeticheskoj jeffektivnost'ju* [Theoretical foundations of designing communication systems with high energy efficiency]. *Cifrovaja obrabotka signalov* [digital signal processing]. No. 2. 2017. Pp 3-8 (in Russian).

[5] Bykhovsky M.A. Giperfazovaja moduljacija – optimal'nyj metod peredachi soobshhenij v gaussovskih kanalah svjazi. [Hyperphase Modulation - the optimal method of message transmission in the Gaussian communication channels]. Technosphera. 2018. 310 p. (in Russian).

[6] Karlsson M., Agrell E. Four-dimensional optimized constellations for coherent optical transmission systems. 36th European Conference and Exhibition on Optical Communication (ECOC). 2010, DOI: 10.1109/ECOC.2010.5621574.

[7] Karlsson M., Agrell E. Power-efficient modulation formats in coherent transmission systems. Journal of Lightwave Technology. Vol. 27. I. 22. 2009. Pp. 5115-5126. DOI: 10.1109/JLT.2009.2029064.

Tahir B., Schwarz S. BER comparison between Convolutional, Turbo, LDPC, and Polar codes. 24th International Conference on Telecommunications (ICT). 2017. DOI: 10.1109/ICT.2017.7998249.