Nonlinear Control System Synthesis of Intellectual Agent's Motions Based on Optimal Control

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***Abstract*— This paper considers a nonlinear control system of an autonomous intellectual agent's motions. The aim of this work is to synthesize the control law that provides a predetermined degree of exponential stability in a closed-loop system based on Lotka–Volterra equations. The research uses the methods of optimal control theory in the synthesis of the system. Asymptotic stability is achieved by solving the Riccati equation. With the model proposed we provide the simulation and experimental results, which correspond to the quality metrics required.**

***Keywords*— nonlinear control system; random search; intellectual agent; optimal control; Riccati equation**

I. Introduction

There are many algorithms and methods for controlling autonomous intellectual agents, such as mobile robots and unmanned aerial vehicles (UAVs). Intellectual agents are capable of independent decision-making to achieve a specified purpose, for example, to search hazardous pollutants in dangerous environments. [1–4] Various biological algorithms, such as chemotaxis [5–7], flocking behavior and foraging [8–12], population development or species interaction, for example, the predator-prey model [13–19] are used in the creation of control systems. The method presented in [14–18] includes the biological algorithm of agent rotation angles control based on Lotka–Volterra equations or the model of interaction of two species. The first one is the trend to rotate to the left and the second one is the trend to rotate to the right to achieve the source of an odor or a radiation fields. The algorithm simulates the decision-making process of a living organism. According to this algorithm, the agent, depending on which of the two species prevails, rotates to the left or to the right. Thus, the agent manages its own movements to achieve a source. With such an algorithm, the stability of a control system is an important criterion of quality to be ensured. In this work, a control system has a given degree of exponential stability, therefore it is proposed to use the methods of optimal control theory [20–23] in the mathematical model of an intellectual agent's motions.

II. Task Definition

This paper considers a nonlinear continuous time-invariant positive plant with the equation of motion described by a differential equation such as:

$$\dot{x}(t)=Ax(t)+BF(x(t))\cdot u(t) \tag{1}$$

where $x(t)$ is the state vector with n state variables, the scalar $u(t)$ is the control of the system, A is the constant n x n state matrix of linear part of the nonlinear system, B is n x 1 the input matrix, the nonlinear function $F(x)$ can be written as:

$$F(x)=1+x^T F_0 x,$$

where F_0 is the constant n x n state matrix, at least a positive semidefinite matrix. Moreover, the values of the initial conditions are all positive $x_i(0)>0$, where x_i – state variables with $i=\overline{1,n}$.

We consider the task of synthesizing the control law in the form of linear-quadratic time-invariant feedback (LQTIF) such as:

$$u(t)=-Kx(t)-x^T(t)K^T Kx(t),$$

The control law provides the desired degree of exponential stability α under constraints on $Kx(t)\geq 0$, where K is 1 x n the LQTIF matrix.

The parameter α allows to estimate the time of transient responses $t_p \approx \frac{3}{\alpha}$.

III. Proposed Method

We consider a linear plant with the equation of motion written as:

$$\dot{x}(t)=Ax(t)+Bu(t), \tag{2}$$

As presented in [1], if the control law of the equation of motion (2) is described by the following equation

$$u(t)=-Kx(t)-signKx(t)\cdot x^T(t)K^T Kx(t), \tag{3}$$

the loop system is then written as:

$$\dot{x}(t)=Ax(t)-BKx(t)-BsignKx(t)\cdot x^T(t)K^T Kx(t), \tag{4}$$

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Loop system (4) provides the exponential stability with the degree of convergence α, when matrix K is obtained with the use of the Riccati equation

$$A^T P + PA - \nu K^T RK + 2\alpha P = -Q \tag{5}$$

with:

$$K = R^{-1} B^T P \tag{6}$$

where P is n x n the symmetric positive-definite matrix, Q is n x n the symmetric penalty matrix for the state vector, R is a non-zero scalar that determines the control signal constraint, v is a parameter between [0;2].

If v=2, then the Riccati equation allows to find the optimal control corresponding the principle of optimality in [2].

We consider positive systems [25] with the LQTIF matrix, such as $Kx(t)\geq 0$, where coordinates of vector $x_i(t)\geq 0$ and the matrix K has positive values. Then $signKx(t)$ in equation (4) can be reduced.

The result analysis of the behavior of a positive closed-loop control system [25] with control law (4) shows that the plant (1) is exponentially stable provided that the positive semidefinite matrix is written as:

$$F_0 - K^T K \geq 0 \tag{7}$$

If conditions (7) are violated, the system is exponentially stable in area $x \in D$ limited by hyperplane (8) as:

$$Kx = 1 \tag{8}$$

with coordinates of vector $x_i\geq 0$ and $i = \overline{1,n}$.

IV. Nonlinear System Synthesis

For example, we consider the system of equation, which describes the dynamics of population change by Lotka–Volterra equations [4] as:

$$\begin{aligned} \dot{x}_1(t) &= c_1 x_1(t) x_2(t) - c_2 x_1(t), \\ \dot{x}_2(t) &= c_1 x_1(t) x_2(t) - c_3 x_2(t) \end{aligned} \tag{9}$$

where $x_1(t)\geq 0$ is the number of males in a population, $x_2(t)\geq 0$ is the number of females in a population, c_1 is the birth rate, c_2 and c_3 are death rates. If one of the coordinates becomes zero, for example, $x_1(t)=0$, then growth $x_1(t)$ stops, since the right side of the first equation (9) is zero. The other coordinate $x_2(t)$ in this case exponentially decays in $c_3>0$.

If $x_1(t)$ and $x_2(t)$ of system (11) are coordinates of the state vector $x(t)$, the system of equation (9) is written in the form (1):

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -c_2 & 0 \\ 0 & -c_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \\ + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}^T \begin{bmatrix} 0 & 0.5\cdot c_1 \\ 0.5\cdot c_1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\begin{aligned} A &= \begin{bmatrix} -c_2 & 0 \\ 0 & -c_3 \end{bmatrix}, \mathrm{B} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ F &= \begin{bmatrix} 0 & 0.5\cdot c_1 \\ 0.5\cdot c_1 & 0 \end{bmatrix} \end{aligned} \tag{10}$$

V. Simulation Experiments

We check the validation of the method proposed using equations (5), (6), (10) and the initial parameters: R=1, v=2, α=0.2, c_2=0.3, c_3=0.25, $Q = -\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$; the initial approximations $k_1=k_2=1$ and $P = \begin{bmatrix} 5 & 1 \\ 1 & 2.3 \end{bmatrix}$.

In addition to equations (5), (6) we use the Sylvester's criterion [5], and the conditions of symmetric positive definite matrix P are given by:

$$\det \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} > 0, p_{11} > 0$$

$$p_{12} = p_{21}$$

The resulting calculations are the following:

$$P = \begin{bmatrix} 3.289 & -2.13 \\ -2.13 & 2.883 \end{bmatrix}$$

$$K = R^{-1} B^T P = \begin{bmatrix} 1.159 & 0.753 \end{bmatrix}$$

Diagrams of the dynamics of population change with a non-zero initial conditions $x_1(0)=5$, $x_2(0)=3$ are illustrated in Fig. 1, 2. In Fig. 1 the system (10) with the control law is $u(t)=0$. As seen in Fig. 1, the system is unstable. In Fig. 2 the system (10) with the control law in form: $u(t) = -Kx(t) - x^T(t) K^T Kx(t)$. As seen in Fig. 2, the closed loop system is stable, and coordinates of the state vector tend to the steady-state values x_1(t)=0, x_2(t)=0. In simulation process the second coordinate of state vector x_2(t) has a negative value for some period of time, therefore, the system belongs to a wider class of systems than positive systems, which are the systems with the constraint on nonnegative linear part of control law $Kx(t)\geq 0$. The diagram of the changing function $Kx(t)$ is illustrated in Fig. 3. As seen in Fig. 3, the condition on nonnegative linear part of control law $Kx(t)\geq 0$ is done.

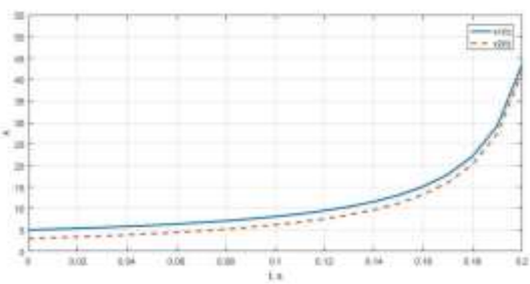

Fig. 1. The diagram of the dynamics of population change with $u(t)=0$

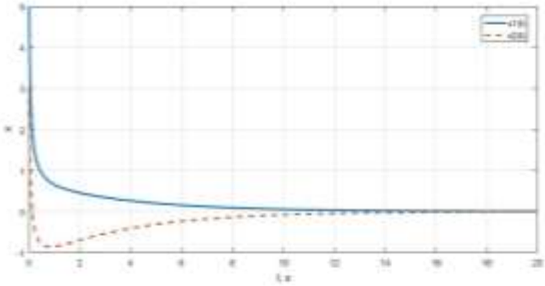

Fig. 2. The diagram of the dynamics of population change with LQTIF

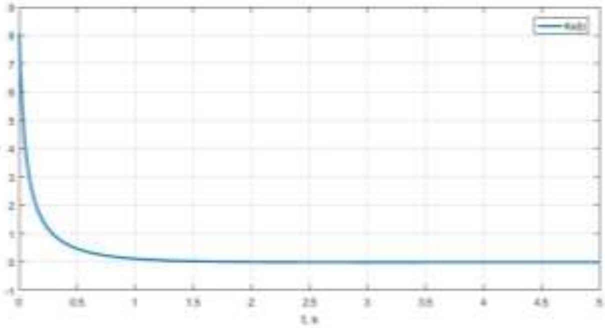

Fig. 3. The diagram of the changing function $Kx(t)$

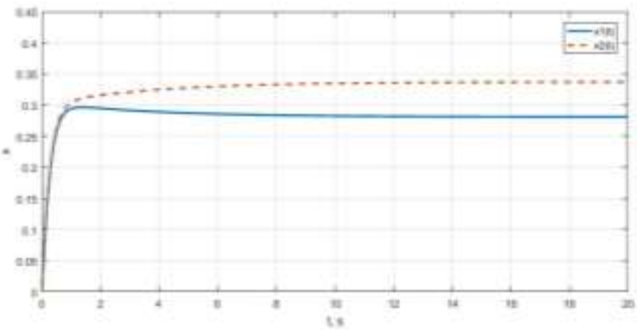

Fig. 4. The diagram of the transient responses with $u(t)$=1

The diagram of the transient responses with a unit step signal $u(t)$=1 under zero initial conditions is illustrated in Fig. 4. The transient response is less than 15 seconds, which corresponds to the predetermined one in the synthesis of the feedback of stability degree α=0.2.

The diagram of the dynamics of population change with constraint $x(t)\geq 0$ is illustrated in Fig. 5. As seen in figure 5, after x_2(t)=0, the coordinate x_1(t) exponentially decays. The population tends to zero.

Equations of population are used in control systems of mobile robots and quadcopters [6], [7], in which the coordinates are the rotation angles of the robot to the left and to the right, and for a quadcopter the rotation upwards and the rotation downwards are added.

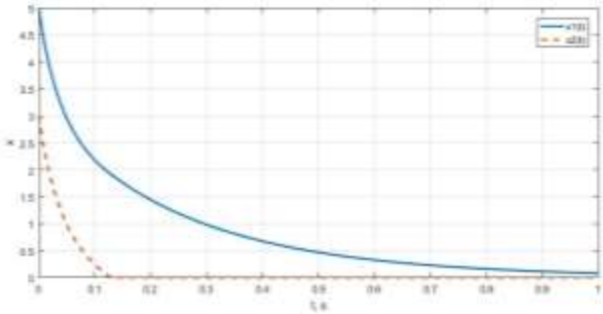

Fig. 5. The diagram of the dynamics of population change with x(t)≥0

VI. CONCLUSION

This work has considered the use of optimal control theory methods in the synthesis of the control system of robotic intellectual agent. The coefficients of the system are calculated with the use of the Riccati equation. The simulation results of the system have proven that the system synthesized is exponentially stable and can be used in terrestrial mobile robots and quadcopters.

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