Spectral Method of Simulation Algorithms' Synthesis of Pseudo-Random Discrete Signals with Variable Energy Characteristics in the Walsh-Hadamard Basis

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***Abstract*— Methods of mathematical modeling can improve the quality of technological processes and real-time objects' development and research for various purposes. In the article the theoretical and applied problems are set and solved of a new method synthesis of mathematical modeling, a spectral method of pseudo - random discrete signals' simulation within the correlation theory. The method is based on the original modification of the Pugachev's simulation trigonometric model, which is been transformed into a mathematical model based on the spectral representation of signals in the basis of the functions of Walsh with the order of Hadamard. To implement the transition from the trigonometric Fourier spectrum to the Walsh–Hadamard spectrum, the synthesis and analysis of these spectra operators of mutual transformation were performed. The mathematical basis of the spectra is the specifically structured Fourier kernel, which allows to construct new effective models of stationary and non-stationary random signals. The obtained models outperform the well-known models in terms of accuracy and computational complexity.**

Keywords*— *pseudorandom signals; function of the spectral dispersion density; autocorrelation function; discrete signal's simulation; spectrum; basis functions

I. Relevance and Statement of the Research Problem

Simulation modeling which uses computer technology is one of the most effective methods of developing the complex technological processes and real-time objects for various purposes. The solution of many scientific and technical problems requires the reproduction of random signals, they make up the mathematical basis of noise and interference [1–19].

In such models, there are high requirements for the algorithm itself to simulate signals in accuracy, ease of adjustment to the specified characteristics and computational complexity, especially when simulating in hard model or real time. Among the existing classical algorithms for signal simulation within the framework of the correlation theory, which use the methods of linear transformations, canonical expansions, forming filters and expansions into trigonometric Fourier series [20-22], the algorithms based on the recursive Bykov's model [21] and the trigonometric Pugachev model [20] satisfy the greatest degree of these requirements. However, the Bykov's model has strict restrictions on the form of the given function of the spectral dispersion density (FSDD). In particular, it cannot be used to simulate signals with FSDD, containing emissions at different frequencies, typical for real signals in FSDD control systems [23]. In addition, it cannot be used in the simulation of signals such as white noise, used in research tasks in conditions of incomplete information on the statistical and energy properties of the input effects. Simulation algorithms based on Pugachev's model and using spectral series in trigonometric basis are devoid of these drawbacks, but have high computational complexity and, as a consequence, a high level of instrumental error [20]. The task of the described work was to develop a new spectral method for the synthesis of simulation algorithms with defined FSDD based on the original modification of the Pugachev's model, followed by its transformation into a model in the Walsh–Hadamard basis that does not contain multiplication operations.

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II. Modified Simulation Algorithms in the Trigonometric Basis

The modified discrete Pugachev algorithm has the following form of a Fourier series:

$$y(i)=\sum_{k=0}^{N/2}\left[\mu_k X_e(k)\cos\left(\frac{2\pi}{N}ki\right)+\alpha_k X_o(k)\sin\left(\frac{2\pi}{N}ki\right)\right], \quad i\in[0,N), \tag{1}$$

where $y(i)$ – random signal, $X_e(k)$ and $X_o(k)$ – the spectral coefficients in the cosine and sine basis functions, N – number of samples in the discrete signal detection interval, but μ_k and α_k – uncorrelated random variables with parameters (0,1), accept values ±1 equiprobably [24]. When the Fourier coefficients are equel:

$$X_e(k)=X_o(k), k\in\left[0,\frac{N}{2}\right] \tag{2}$$

The random signal $y(i)$ becomes stationary in a broad sense with an autocorrelation function (ACF):

$$R(m) = X_e^2(0) + X_o^2\left(\frac{N}{2}\right)\cos(\pi m) + \\ + 4\sum_{k=1}^{\frac{N}{2}-1} X_e^2(k)\cos\left(\frac{2\pi}{N}km\right), m \in [-N, N). \tag{3}$$

The Fourier coefficients $X_e(k)$ can be found using the FSDD $S(\omega)$ according following equations:

$$X_e^2(0) = \frac{S(0)}{N\Delta t}, X_e^2\left(\frac{N}{2}\right) = \frac{S\left(\frac{\pi}{\Delta t}\right)}{N\Delta t},$$

$$X_e^2(k) = \frac{S\left(\frac{2\pi}{N\Delta t}k\right)}{2N\Delta t}, k \in \left[1, \frac{N}{2}\right).$$

The sampling interval in time Δt is determined according to the Kotelnikov theorem.

Practical implementation of the algorithm (1) at equality (2) requires costs of $\left(3N^2\right)/2$ total multiplication and addition operations. Its further simplification can be achieved by transformation in the basis of the Walsh–Hadamard functions that take values ± 1 only [25-30].

III. The Simulation Algorithms in the Basis of Walsh– Hadamard

The transformation from the trigonometric model to the Walsh–Hadamard model can be done by converting the trigonometric spectrum $\left\{\mu_k X_e(k), \alpha_k X_e(k)\right\}$ into the Walsh–Hadamard spectrum $\left\{Y_A(k)\right\}$ using the linear operator:

$$Y_A(k) = \sum_{m=0}^{\frac{N}{2}} \mu_m X_e(m)\Phi_e(k,m) + \\ + \sum_{m=1}^{\frac{N}{2}-1} \alpha_m X_e(m)\Phi_o(k,m), k \in [0, N),$$

where

$$\left.\begin{aligned} &\Phi_e(k,m) = \frac{1}{N}\sum_{i=0}^{N-1}\cos\left(\frac{2\pi}{N}mi\right) had(k,i), \\ &\qquad m \in \left[0, \frac{N}{2}\right], k \in [0, N); \\ &\Phi_o(k,m) = \frac{1}{N}\sum_{i=0}^{N-1}\sin\left(\frac{2\pi}{N}ki\right) had(k,i), \\ &\qquad m \in [1, N), k \in [0, N); \end{aligned}\right\} \tag{4}$$

are the components of the transform kernel (Fourier kernel [29]):

$$\Phi(k,m) = \left\{\Phi_e(k,m), \Phi_o(k,m)\right\}.$$

Here

$$had(k,i) = \cos\left(\pi\sum_{\nu=1}^{n} k_\nu i_\nu\right), \; n = \log_2 N$$

are the Walsh functions, where k_ν and i_ν mean ν-st bits of the n- bits binary codes of the values k and i.

The analysis have shown, that for any $N = 2^n, n = 1, 2, \ldots$ the Fourier kernel matrix contains two single values, with $2\left(N^3 - 1\right)/3$ zero elements and $\left(N^2 + 2\right)/3$ nonzero elements. Moreover, non-zero elements and corresponding spectral coefficients can be formed into $n+1$ independent groups, allowing to present the operator of the Fourier and Walsh spectra transformation in the following form:

$$\left.\begin{aligned} &Y_A(0) = \mu_0 X_e(0); Y_A(1) = \mu_{\frac{N}{2}} X_e\left(\frac{N}{2}\right); \\ &Y_A\left(2^{\gamma-2} + j\right) = \\ &= \sum_{m=0}^{2^{\gamma-3}-1}\left\{\mu_{\frac{2N}{2^\gamma}(1+2m)} Z_e\left(\frac{2N}{2^\gamma}(1+2m), 2^{\gamma-2} + j\right) + \right. \\ &\left. + \alpha_{\frac{2N}{2^\gamma}(1+2m)} Z_o\left(\frac{2N}{2^\gamma}(1+2m), 2^{\gamma-2} + j\right)\right\}; \\ &\gamma = 3, 4, \ldots, n+1; \; j = 0, 1, \ldots, 2^{\gamma-2} - 1; \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} &Z_e\left(\frac{2N}{2^\gamma}(1+2m), 2^{\gamma-2} + j\right) = \\ &= X_e\left(\frac{2N}{2^\gamma}(1+2m)\right)\Phi_e\left(2^{\gamma-2} + j, \frac{2N}{2^\gamma}(1+2m)\right), \\ &Z_o\left(\frac{2N}{2^\gamma}(1+2m), 2^{\gamma-2} + j\right) = \\ &= X_o\left(\frac{2N}{2^\gamma}(1+2m)\right)\Phi_o\left(2^{\gamma-2} + j, \frac{2N}{2^\gamma}(1+2m)\right), \\ &\gamma = 3, 4, \ldots, n+1; m = 0, 1, \ldots, 2^{\gamma-3} - 1; \\ &j = 0, 1, \ldots, 2^{\gamma-2} - 1; \end{aligned}\right\} \tag{6}$$

and the imitating signal is restored with the help of such a series of Walsh–Hadamard:

$$y(i) = Y_A(0) + Y_A(1) had(1,i) + \\ + \sum_{\gamma=3}^{n+1}\sum_{j=0}^{2^{\gamma-2}-1} Y_A\left(2^{\gamma-2} + j\right) had\left(2^{\gamma-2} + j, i\right), i \in [0, N). \tag{7}$$

In case if the kernel elements $\Phi(k,m)$ (4) and intermediate values $Z_e(k,m)$ and $Z_o(k,m)$ (6) have been calculated in advance and keep in computer memory, than the simulation algorithm (7) will not have non-trivial multiplications, while it’s quick realization using fast-Walsh transformations [29, 31, 32] will require $[N^2+3N(n-1)+2]/3$ addition operations only, it is more than 4 times less compared to the modified Pugachev's algorithm (1). So the Hadamard model (7) is an effective computational tool for simulation of stationary random signals. The ACF of the process (7) coincides with the ACF (3) of the process (1) and (2), which was synthesized according to the modified model.

It is possible to simplify the simulation algorithm in the Walsh–Hadamard basis if within each group of coefficients in the system (5) take:

$$\mu_{\frac{2N}{2^\gamma}(1+2m)} = \alpha_{\frac{2N}{2^\gamma}(1+2m)} = \beta_{\gamma-1}.$$

Then when calculating the Hadamard spectrum from equations (5) the random parameters μ_k and α_k could be taken out of a sign of the sum. As a result the spectral Hadamard coefficients will become deterministic and they can be calculated in advance at the stage of the simulation algorithm's setup. The Hadamard simulation series will be in this case:

$$y(i) = \beta_0 X_e(0) + \beta_1 X_e\left(\frac{N}{2}\right) had(1,i) + \sum_{\gamma=3}^{n+1} \beta_{\gamma-1} \sum_{j=0}^{2^{\gamma-2}-1} Y_A\left(2^{\gamma-2}+j\right) had\left(2^{\gamma-2}+j,i\right); \quad (8)$$
$$i \in [0,N),$$

where

$$Y_A\left(2^{\gamma-2}+j\right) = \sum_{m=0}^{2^{\gamma-3}-1} \left\{ Z_e\left(\frac{2N}{2^\gamma}(1+2m), 2^{\gamma-2}+j\right) + Z_o\left(\frac{2N}{2^\gamma}(1+2m), 2^{\gamma-2}+j\right) \right\};$$
$$\gamma = 3,4,\ldots,n+1;\ j = 0,1,\ldots,2^{\gamma-2}-1.$$

Direct implementation of the algorithm (8) will lead to costs in N^2 additions, while it's quick implementation will require nN additions only. There is no multiplications in the algorithm (8). Win operations' rate compared to algorithm (1) will be $1{,}5N/n$.

It should be borne in mind, however, that the obtained random process will be non-stationary, it will have same FSDD as signal (7) has. The exclusion of multiplication operations from algorithms (7) and (8) as well as the reduction of the addition operations' number increase their overall instrumental accuracy.

IV. Conclusion

Thus, in this work there are presented the theoretical and applied fundamentals of the synthesis of the spectral method for simulating stationary and nonstationary random signals in the framework of the correlation theory. A modification of the classical Pugachev's model has been performed with the aim of reducing its computational complexity. Its further transformation to the basic system of Walsh functions in Hadamard ordering is carried out using a linear operator containing elements of the Fourier kernel of a specific structure. The isolation of independent groups of the Fourier kernel's non-zero elements allowed to avoid performing insignificant arithmetic operations.

Taking into account the fact that the kernel elements and intermediate coefficients can be calculated in advance at the stage of tuning the simulation algorithm, the algorithm itself will not contain cumbersome multiplication operations. The use of fast Walsh transforms provides an advantage to the proposed simulation algorithm for addition operations more than 4 times in comparison with the modified Pugachev's algorithm. The ACF of the random signal obtained by the new algorithm coincides with the corresponding ACF calculated from the modified Pugachev's model.

Further simplification of the algorithm in the Walsh–Hadamard basis is achieved by balancing uncorrelated random variables μ_k and α_k. This allowed us to proceed to the calculation of the deterministic Hadamard coefficients also at the stage of tuning the simulation algorithm, and to realize the algorithm itself without multiplication operations. As a result, the total gain in the number of computational operations compared with the modified Pugachev's model was a multiple of the ratio of the number of samples to the number of bits representing the arguments of the Walsh functions. An additional effect of this simplification is the nonstationarity of the generated signals, which along with the stationary ones are widely used in the analysis and synthesis of real-time systems for various purposes.

The absence in the resulting algorithm of simulation of multiplication operations and the reduction of the number of addition operations gives an improvement in its instrumental accuracy in comparison with the known algorithms implemented on spectral series in the basis of trigonometric functions.

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