The Control by Dynamical Quality of Automatic Systems by Method of Linear Square Optimization

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***Abstract*— The new method of regulation based on the preliminary correction of dynamic of regulation object according to the given standard model is suggested. The problem of correction is solved by method of linear-square optimization.**

Keywords*— *synthesis of regulation system; correction of object's dynamic; formalism of linear-square optimization

I. «LINEAR-SQUARE TREND» IN MODERN AUTOMATION

In spite of old history of the development the problem of quality of control processes remains the most important and still weakly evolving in the theory and practice of automatic systems. Moreover, it is necessary to state that in the researches of the last decades the succession with intuitively clear and technically profound classical ideas about the quality of control processes, worked out by native automation school [1] is lost at least. Let us note that the first conceptions about the dynamic quality of automatic regulation systems (ARS) were formed historically in the 40-th of the XX century in terms of the direct quality exponents: regulation time, over control, degree of stability and variability, etc.

In modern automation in synthesis of automation systems the requirements not desired or admissible but optimal quality of control process of synthesized system [2–5] have the most popularity. In doing so the square optimality criteria, generated class of linear-square (LS) control problems have undivided dominion [6]. They are initial for method of analytical formation of optimal regulators (AFOR) of Kalman-Letov being already classical (see, for example, the authors papers [7–11]). Here the control criterion is given in terms of integral square form from one or another exponents of real transient or from residual (disagreement) of the real and desired (standard) transients of system and it is necessary to provide their maximum proximity (see, for example, [2, c. 379-382; 3, c. 483-484; 4, c. 240-242; 5, c. 704-705]).

However, the methodology of square optimization of control processes in spite of the extraordinary popularity and apparent achievements was repeatedly subjected to sharp criticism on the side of the leading native and foreign erudites [6]. So even R.E. Bellman stated rather justified that the introduction of the integral square criterion is "the question of the mathematical convenience and often is dictated itself with desire to apply the analytical methods for solution of the problem and to get the solution in the explicit form", and concerning AFOR problem he especially emphasized that in place of the given "less important problem" they often use the initial "more realistic optimization problem".

In the present paper it is suggested the new method of synthesis of ARS based on the dynamical correction of control object which is realized by means of the application of formalism LS optimization problems. The basis for the solving problem of the correction is the idea of postulation of the desired dynamical properties of the system being synthesized in terms of the given standard model of the corrected object [12]. The algorithmization of the correction problems is based on formalism LS control problems where the optimizable integral square functionals are used for degree of deviation of formed transitional characteristics of channels of regulation from standard means.

The suggested method shows the possibility of convergence of the classical conception of the direct quality exponents of the regulation processes and AFOR methodology.

II. THE PROBLEM OF DYNAMICAL CORRECTION OF CONTROL OBJECT

Then let us suppose that input and output of control object are the scalar: $m = r = 1$. Thus, the object is described by the equations

$$\dot{\mathbf{x}} = \mathbf{A}_0 \mathbf{x} + \mathbf{B}_0 u \,, \tag{1}$$

$$y = \mathbf{C}_0 \mathbf{x} \,. \tag{2}$$

In the base of the proposed method of synthesis ARS it is assumed the idea of *decomposition* of the regulation problem into two subproblems. One of them is the subproblem of the preliminary dynamical correction of the object, and another one is the subproblem of the formation of the regulation law for the corrected object. The given idea has been embodied in block-scheme ARS, represented by Fig. 1. Here $y*$ is set point (giving the action), ε is signal of disagreement between set point and the object's output.

The regulator is composed of two blocks: *block of correction* (BCor), eliminating the dynamics of the object in agreement with the given standard dynamical model, and *block of control* (BCon), realizing the standard regulation law.

Let the desired corrected dynamics of the object be represented by standard model in terms of dynamical system of order n_{M}, described in variables of state by the equations of the form

$$\dot{\mathbf{x}}_{\mathrm{M}} = \mathbf{A}_{\mathrm{M}}\mathbf{x}_{\mathrm{M}} + \mathbf{B}_{\mathrm{M}} v\,, \qquad (3)$$

$$y_{\mathrm{M}} = \mathbf{C}_{\mathrm{M}}\mathbf{x}_{\mathrm{M}}\,, \qquad (4)$$

where $v \in \mathbf{R}$ is input, $\mathbf{x}_{\mathrm{M}} \in \mathbf{R}^{n_{\mathrm{M}}}$ is state, $y_{\mathrm{M}} \in \mathbf{R}$ is output of the model, $\mathbf{A}_{\mathrm{M}}$, $\mathbf{B}_{\mathrm{M}}$, $\mathbf{C}_{\mathrm{M}}$ are numerical matrices of the corresponding orders.

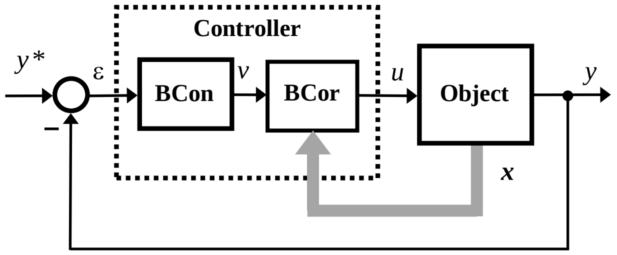

Fig. 1.

Transfer functions of the regulation object and standard model along channel "input-output" are respectively equal to

$$W(s) = \mathbf{C}(\mathbf{E}_n s - \mathbf{A})^{-1}\mathbf{B}\,,$$

$$W_{\mathrm{M}}(s) = \mathbf{C}_{\mathrm{M}}(\mathbf{E}_{n_{\mathrm{M}}} s - \mathbf{A}_{\mathrm{M}})^{-1}\mathbf{B}_{\mathrm{M}}\,,$$

where s is complex frequency, $\mathbf{E}_p$ is identity matrix p-order.

The dynamical correction of the object is dedicated to provide for it the desired dynamics in agreement with standard model (3)–(4):

$$y(t) \approx y_{\mathrm{M}}(t)\,. \qquad (5)$$

The suggested scheme solution of BCor is representative of the Fig. 2.

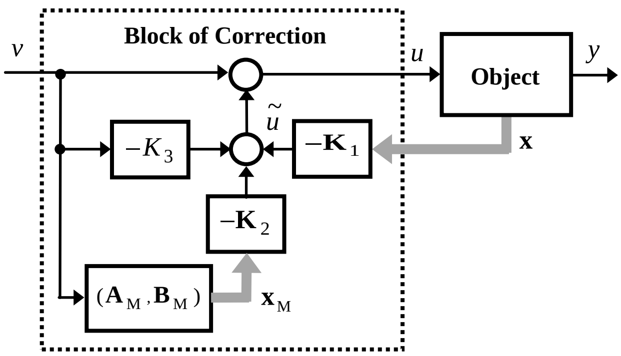

Fig. 2.

The functioning of BCor is subjected to the equations:

$$u = v + \tilde{u}\,, \qquad (6)$$

$$\tilde{u} = \tilde{u}_1 + \tilde{u}_2 + \tilde{u}_3\,, \qquad (7)$$

$$\tilde{u}_1 = -\mathbf{K}_1\mathbf{x}\,, \qquad (8)$$

$$\tilde{u}_2 = -\mathbf{K}_2\mathbf{x}_{\mathrm{M}}\,. \qquad (9)$$

$$\tilde{u}_3 = -K_3 v\,, \qquad (10)$$

Let us note that component (3) of standard model, represented by the dynamical link with mark $(\mathbf{A}_{\mathrm{M}}, \mathbf{B}_{\mathrm{M}})$ and generating vector of model's state $\mathbf{x}_{\mathrm{M}}$ is included in the structure of BCor.

For the purpose of comparison the dynamics of the corrected object and standard model we will be start from their reaction towards the applied constant action

$$v(t) = v(0) = \mathrm{const} \neq 0 \;\; (t > 0)\,, \qquad (11)$$

where the correction quality we'll be estimate at integral square functional

$$\int_0^\infty \Delta y(t)^2\, dt\,, \qquad (12)$$

where $\Delta y(t)$ is signal of disagreement of outputs of the object and model:

$$\Delta y = y - y_{\mathrm{M}}\,. \qquad (13)$$

III. BASE LINEAR-SQUARE CONTROL PROBLEM

The worked out method of structure-parametric synthesis of block's correction is based on the solution of the auxiliary LS problem for controlled system, the structure of which is represented by Fig. 3. The given system includes three components: true copy (model) of the object, dynamical standard (3), (4), and also *set point* (SP), generating test signal $v(t)$. In the considered system the signal $\tilde{u}(t)$ is the controlling one, and the signal of disagreement of outputs of the object's model $\hat{y}(t)$ and standard $y_{\mathrm{M}}(t)$ is the output:

$$\Delta\hat{y}(t) = \hat{y}(t) - y_{\mathrm{M}}(t)\,. \qquad (14)$$

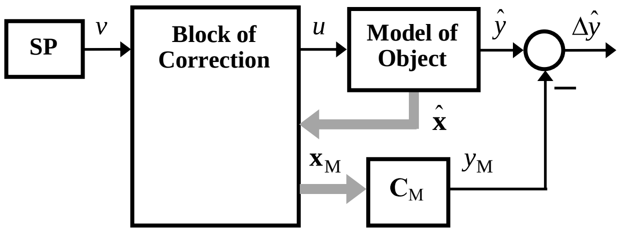

Fig. 3.

To facilitate further account let us agree to call the considered auxiliary system as the *extended model* of the object.

Then let us assume that the point adjuster generates exponential *weakly damped* signal:

$$v(t) = v(0)\exp(-\gamma t)\,, \qquad (15)$$

$$\gamma << 1\,. \qquad (16)$$

Thus, the dynamics of state and the output of the extended object's model are described by the equations:

$$\dot{\hat{\mathbf{x}}}=\mathbf{A}\hat{\mathbf{x}}+\mathbf{B}(\tilde{u}+v), \tag{17}$$

$$\dot{\mathbf{x}}_{\mathrm{M}}=\mathbf{A}_{\mathrm{M}}\mathbf{x}_{\mathrm{M}}+\mathbf{B}_{\mathrm{M}}v, \tag{18}$$

$$\dot{v}=-\gamma v, \tag{19}$$

$$\Delta\hat{y}=\mathbf{C}\hat{\mathbf{x}}-\mathbf{C}_{\mathrm{M}}\mathbf{x}_{\mathrm{M}}, \tag{20}$$

Moreover, its dynamical order is equal to

$$\tilde{n}=n+n_{\mathrm{M}}+1.$$

Let us emphasize that the variable v enters into the composition of endogenous variables of the controlled system.

Let us form state vector of the system (17)–(20)

$$\tilde{\mathbf{x}}=\begin{bmatrix}\hat{\mathbf{x}}\\ \mathbf{x}_{\mathrm{M}}\\ v\end{bmatrix}. \tag{21}$$

Then the equations of state and output of the given system may be written as the following vector-matrix form:

$$\dot{\tilde{\mathbf{x}}}=\tilde{\mathbf{A}}\tilde{\mathbf{x}}+\tilde{\mathbf{B}}\tilde{u}, \tag{22}$$

$$\Delta\hat{y}=\tilde{\mathbf{C}}\tilde{\mathbf{x}}, \tag{23}$$

where matrices $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}$, $\tilde{\mathbf{C}}$ have the following block structure (zero blocks stay empty)

$$\tilde{\mathbf{A}}=\left[\begin{array}{c:c:c}\mathbf{A} & & \mathbf{B}\\ \hdashline & \mathbf{A}_{\mathrm{M}} & \mathbf{B}_{\mathrm{M}}\\ \hdashline & & -\gamma\end{array}\right],\quad \tilde{\mathbf{B}}=\left[\begin{array}{c}\mathbf{B}\\ \hdashline \\ \hdashline \\ \end{array}\right],\quad \tilde{\mathbf{C}}=[\ \mathbf{C}\ \vdots\ -\mathbf{C}_{\mathrm{M}}\ \vdots\quad\ ]. \tag{24}$$

Now let us direct our attention to the auxiliary LS stabilization problem of output $\Delta\hat{y}(t)$ of the system {22), (23):

$$\int_0^\infty [g(\Delta\hat{y}(t))^2+(\tilde{u}(t))^2]dt\to\min, \tag{25}$$

where $g>0$.

Rightly to wait that by means of large values choice of weight coefficient criterion

$$g>>1, \tag{26}$$

it's possible to make signal $\Delta\hat{y}(t)$ arbitrarily near zero in sense of metric of functional space $L_2[0,\infty)$, what according to (14) means the nearness in this metric of signals $\hat{y}(t)$ and $y_{\mathrm{M}}(t)$:

$$\hat{y}(t)\approx y_{\mathrm{M}}(t). \tag{27}$$

Really, it can be shown that this result is guaranteed in case of complete controllability and observability of the object (1), (2) and follows from known solvability conditions of the considered optimization problem (25).

Let us explain logic of the introducing into setting of solved LS control problem of damped test action (15) instead of constant one (11). It was inspired by the requirement for integrability of the square of stabilized action $\tilde{u}(t)$ what conversely provides the convergence of the functional (25) for considered process of stabilization. Besides, such practice one can justify having the following concept: selecting according to (16) parameter of damping γ small enough, we get variant with *quasistationary* signal $v(t)$ anyhow nearly constant signal (11).

For LS problem (22), (23), (25) the equation of Rikkaty has the following form:

$$\mathbf{P}\tilde{\mathbf{B}}\tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{P}-\mathbf{P}\tilde{\mathbf{A}}-\tilde{\mathbf{A}}^{\mathrm{T}}\mathbf{P}-g\tilde{\mathbf{C}}^{\mathrm{T}}\tilde{\mathbf{C}}=0. \tag{28}$$

As a result from the solution of the optimization problem (22)–(25) is the linear regulation law

$$\tilde{u}=-\tilde{\mathbf{K}}\tilde{\mathbf{x}}, \tag{29}$$

where

$$\tilde{\mathbf{K}}=-\tilde{\mathbf{B}}^{\mathrm{T}}\mathbf{P}. \tag{30}$$

Breaking down this matrix (according to (21)) into blocks for orders $1\times n$, $1\times n_{\mathrm{M}}$ and 1×1:

$$\tilde{\mathbf{K}}=[\ \mathbf{K}_1\ \vdots\ \mathbf{K}_2\ \vdots\ K_3\ ], \tag{31}$$

we'll get unknown parameters $\mathbf{K}_1$, $\mathbf{K}_2$ and K_3 of BCor (6)–(10).

Let us note that the required result of the correction (5) follows from (27) due to identically with chains of signal's transformation $v(t)$ into signals $y(t)$ and $\hat{y}(t)$ in schemes in Fig. 2 and 3, so that with coinciding initial states of the object and its model the equality is obeyed by

$$y(t)=\hat{y}(t).$$

Even from here and from (13), (14) it follows that value of functional being optimized (25) in the conditions of its parametrization (26) will be define the value of control criterion of the correction (12).

IV. Synthesis of Automatic Regulation System

As noted above, the regulation problem according to the suggested block structure ARS (Fig. 1) falls into two *independent* subproblems:

1) the problem of dynamical correction of object according to the given standard;

2) the problem of formation of the regulation law starting from standard dynamics of the corrected object.

As a result from the solution of the indicated subproblems is the construction of block correction and block regulation in the suggested structure of regulator.

An example. Let us consider the control object of the third order described by the equations (1)–(2) with matrices of state, control and output:

$$\mathbf{A}_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.5 & 1 \\ 0 & 0 & -1 \end{bmatrix}, \quad \mathbf{B}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C}_0 = [1.5 \ \ 0 \ \ 0].$$

The transfer function of the given object is equal to

$$W_0(s) = \frac{3}{s(s+1)(2s+1)}.$$

Let us take the following standard model of the dynamics of the corrected object:

$$W_{\mathrm{M}}(s) = \frac{1}{s+1}. \tag{32}$$

The result for the solution of base LS control problem for $g = 10^4$ is the following:

$$\mathbf{K}_1 = [150.00 \ \ 51.57 \ \ 9.20], \quad K_2 = -68.69, \quad K_3 = -30.30.$$

Let us note the interesting fact: matrix amplification factors $\mathbf{K}_1$ of BCor in the given and preceding examples are in close agreement!

Fig. 4 illustrates the result of the realized dynamical correction of the object: one can see that the transitional characteristic of the corrected object is near standard.

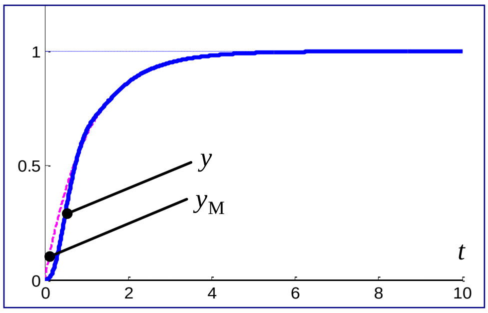

Fig. 4.

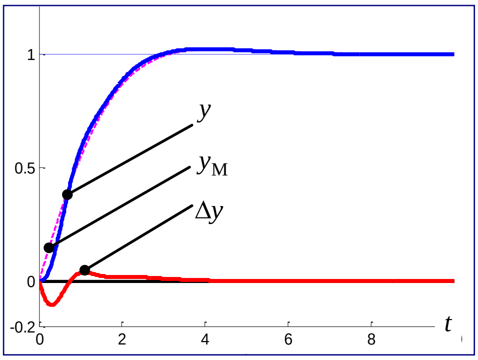

Fig. 5.

In BCon we'll realize PI-regulation law with the transfer function in the form:

$$W_R(s) = k_P + \frac{k_I}{s}.$$

Let us choose the following turning of the regulator: $k_P = 0.6$, $k_I = 1$. In this case for standard transfer function of the corrected object (38) means the following transfer function of closed system:

$$\hat{W}(s) = \frac{0.6s+1}{s^2+1.6s+1}. \tag{33}$$

The poles of the last one are equal to $-0.8 \pm 0.6i$.

In Fig. 5 the factual (entire line) and standard (dotted line) transfer characteristics of ARS and too the difference between them are represented.

It is seen, that the given transfer characteristics are practically the same, moreover, the spectrum of synthesized ARS is

$$\Lambda = \{-0.8243 \pm 0.6124i, -2.1746 \pm 4.4937i, -5.7070\}.$$

Thus, the dominating poles of the closed system are near the poles of desired transfer function (33).

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