BLDC Motor in Industrial Robotic Electric Drive with Linear Optimal Speed Controller

M. P. Belov[1], D. K. Tran[2], Tran Huu Phuong[3]
Department of Robotics and Industrial Automation
Saint Petersburg Electrotechnical University “LETI”
Saint Petersburg, Russia
[1]milesa58@mail.ru, [2]tuyetnhung110807@gmail.com, [3]tranhuuphuong83@gmail.com

***Abstract*— This paper presents a new solution to synthesize speed controller for brushless motor with permanent magnets (BLDC motor) in electric drive of industrial robot. This solution based on the application of phase currents feedback and angular rotor position in Fourier transform to reduce the torque ripple. The phase inverter voltage saturation and the energy-loss process in armature steel caused by unexpected frequencies diffusion in phase current feedback are analyzed. The efficiency of torque compensation regulator with optimal control for the BLDC motor taking into account the dependence of the electromagnetic characteristics with angular rotor position is demonstrated.**

***Keywords*— BLDC motor; optimal controller; torque compensation regulator; Fourier transform**

I. MATHEMATICAL MODEL OF BLDC MOTOR

In the field of industrial robotic, BLDC motor is widely used for controlling manipulators and mobile robots electric drive. These electric drives require energy efficiency and accurate characteristic of the tracking torque in the entire speed range. Optimal torque control in BLDC motor with regard to the harmonics of the electromotive force based on Lagrange optimization [1] make reduce power dissipation and phase voltage saturation. When using method indirect optimal torque control without takes into account the corresponding dynamics of the current feedback. To ensure that not have delay of phase currents in inductive windings and motor torque, either current control with wide bandwidth or sufficiently low operating speed range are required.

This work presents an approach to increase energy-efficient torque of BLDC motor multiphase nonsinusoidal. This approach is compared the traditional motor with taking into account the dependence of inductance on the angular rotor position. We propose a method to synthesize PI speed regulator based on linear optimal control based on the application of Fourier transform of the phase currents feedback and the angular position in torque loop.

We consider general BLDC motor has p phase and n pole pairs. Stator winding phase voltage is described as

$$\mathbf{u} = \mathbf{L}\frac{d\mathbf{i}}{dt} + \mathbf{R}\mathbf{i} + \mathbf{F}(q)\dot{q}\,, \tag{1}$$

where $\mathbf{i} = [i_1, i_2, \ldots i_p]^{\mathrm{T}}$, $\mathbf{u} = [u_1, u_2, \ldots u_p]^{\mathrm{T}}$ are current vector and voltage vector of stator windings; q, $\dot{q}$ is the angular rotor position and speed; $\mathbf{F}(q)$ is the part of total magnetic flux rotor linkage; $\mathbf{L}$ is matrix of inductance stator; $\mathbf{R}$ is phase resistance stator windings matrix. Suppose that winding inductance is approximated through the truncated complex Fourier series. To simplify the analysis, winding step and distribution coefficients are chosen to negate the effect of harmonic perturbations. Denote $\mathbf{i_0} = \mathbf{e}^{\mathrm{T}}\mathbf{i}$, we equivalently rewrite differential equations following as

$$\lambda\frac{d\mathbf{i}}{dt} + \mathbf{i} - \alpha\mathbf{i_0}\mathbf{e} = \frac{1}{R}\left(\mathbf{I} - \alpha\mathbf{J}\right)\left(\mathbf{u} - \mathbf{F}(q)\dot{q}\right); \tag{2}$$

$$\lambda_0\frac{d\mathbf{i_0}}{dt} + \mathbf{i_0} = \frac{1}{R}\mathbf{e}^{\mathrm{T}}\left(\mathbf{u} - \mathbf{F}(q)\dot{q}\right), \tag{3}$$

where $\lambda = \frac{L_s - M_s}{R}$, $\lambda_0 = \frac{L_s + (p-1)M_s}{R}$ are the machine time-constants; L_s is self-inductance; M_s is mutual-inductance; $\mathbf{I}$ is the identity matrix; $\mathbf{J} = \mathbf{e}^{\mathrm{T}}\mathbf{e}$, $\mathbf{e} = [1,1,\ldots 1]^{\mathrm{T}}$, α is given by $\alpha = \frac{M_s}{(p-1)M_s + L_s}$.

With machine connected without neutral line, the phase current constraint must be $i_0 = 0$. From Eq. (3), $\mathbf{e}^{\mathrm{T}}\left(\mathbf{u} - \mathbf{F}(q)\dot{q}\right) = 0$, with stability state current $i_0(t) = i_0(0)e^{-\lambda_0 t}$. In this case, the value i_0 in eq. (2) is vanished, so the dynamic BLDC motor equation without a neutral line is simplified and represented in the form as

$$\lambda\frac{d\mathbf{i}}{dt} + \mathbf{i} = \frac{1}{R}\left(\mathbf{B}\mathbf{u} - \mathbf{F}'(q)\dot{q}\right), \tag{4}$$

where $\mathbf{B} = \frac{1}{p}\begin{bmatrix} p-1 & -1 & \ldots & -1 \\ -1 & p-1 & \ldots & -1 \\ \ldots & \ldots & \ldots & \ldots \\ -1 & -1 & \ldots & p-1 \end{bmatrix}$; $\mathbf{F}'(q) = \mathbf{B}\mathbf{F}(q)$.

On the other hand, the electromagnetic torque τ is the result electrical energy to mechanical energy converting [2].

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$$\boldsymbol{\tau} = \mathbf{F}^{\mathrm{T}}(q)\mathbf{i} = \mathbf{F}'^{\mathrm{T}}(q)\mathbf{i}\ . \qquad (5)$$

Eq. (4) and (5) completely describe parametric general model of multiphase nonsinusoidal synchronous machines with permanent magnets.

The time derivative of the torque expression (5) has the following form

$$\dot{\boldsymbol{\tau}} = \mathbf{F}'^{\mathrm{T}}(q)\frac{d\mathbf{i}}{dt} + \mathbf{i}^{T}\frac{\partial \mathbf{F}'(q)}{\partial q}\dot{q}\ . \qquad (6)$$

From (4), (5), (6), we have:

$$\boldsymbol{\tau} + \lambda\dot{\boldsymbol{\tau}} = \frac{1}{R}\mathbf{F}'^{\mathrm{T}}(q)\mathbf{u}' - \frac{1}{R}\left\|\mathbf{F}'(q)\right\|^{2}\dot{q} + \mathbf{F}'_{q}(q)\lambda\mathbf{i}^{\mathrm{T}}\dot{q}\ , \qquad (7)$$

where $\mathbf{u}' = \mathbf{B}u$, $\mathbf{F}'_{q}(q) = \dfrac{\partial \mathbf{F}'(q)}{\partial q}$.

From (7), the torque motor consists of two parts:

$$\boldsymbol{\tau} + \lambda\dot{\boldsymbol{\tau}} = \frac{1}{R}\mathbf{F}'^{\mathrm{T}}(q)\mathbf{u}' - \frac{1}{R}\left\|\mathbf{F}'(q)\right\|^{2}\dot{q} + \mathbf{F}'_{\mathbf{q}}(q)\lambda\mathbf{i}^{\mathrm{T}}\dot{q} = \mathbf{u}_{p} + \mathbf{u}_{n}\ , \qquad (8)$$

where $\mathbf{u}_p$ is the main control signal of electromagnetic torque, $\mathbf{u}_n$ could be used to minimize power dissipation to achieve maximum efficiency machine and reduce the inverter phase voltages saturation.

In this case, we need solving two problems:

- Optimize the current loop in (4) to prevent electric drive voltage saturation and reduce the loss in armature steel, which will lead to increased machine productivity and smooth torque.
- Determine the optimal torque regulator. The solution of this problem is based on solving linear optimization in (8).

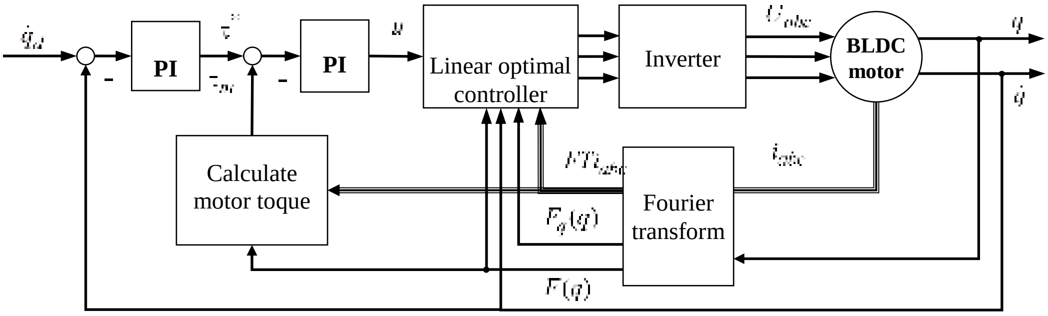

Fig. 1. Block diagram of the BLDC motor with linear optimal speed controller

Fig. 1 shows block diagram of the BLDC motor with linear optimal speed controller. In which, the torque loop is based on the application of a linear current feedback and angular rotor position through Fourier transform. Analysis of the Fourier transform is given in [1].

II. Optimal Control in Torque Loop

We define:

$$\mathbf{u}_{p} = \mathbf{F}(q)\dot{q} + \mathbf{R}\left(\mathbf{u} - \lambda\mathbf{i}^{\mathrm{T}}\mathbf{F}_{q}(q)\right)\boldsymbol{\eta}(q)\ , \qquad (9)$$

where $\boldsymbol{\eta}(q) = [\eta_1(q), \eta_2(q), ..., \eta_p(q)]^{\mathrm{T}}$; u- auxiliary input control.

From (8) and (9), we obtain the differential equation of torque closed-loop:

$$\boldsymbol{\tau} + \lambda\dot{\boldsymbol{\tau}} = \dot{q}\lambda\mathbf{i}^{\mathrm{T}}\mathbf{F}_{q}(q) + \left(\mathbf{u} - \dot{q}\lambda\mathbf{i}^{\mathrm{T}}\mathbf{F}_{q}(q)\right)\mathbf{F}'^{\mathrm{T}}(q)\boldsymbol{\eta}(q). \qquad (10)$$

Eq. (10) will become first order linear differential equation if the following conditions are satisfied: $\mathbf{F}'^{\mathrm{T}}(q)\boldsymbol{\eta}(q) = 1, \forall q \in R$. and the norm $\boldsymbol{\eta}(q) = \dfrac{\mathbf{F}'(q)}{\left\|\mathbf{F}'(q)\right\|^{2}} \to \min$.

The main control input signal in (10) has the following form:

$$\mathbf{u}_{p} = \mathbf{F}(q)\dot{q} + \mathbf{R}\frac{\left(\mathbf{u} - \lambda\mathbf{i}^{\mathrm{T}}\mathbf{F}_{q}(q)\right)}{\left\|\mathbf{F}'(q)\right\|^{2}}\mathbf{F}'(q). \qquad (11)$$

The control linearization in (11) not only ensures minimization of power loss, but also reduces windings phase voltage saturation. On the other hand, it increases the efficiency of the machine and the possibility of smooth torque.

The voltage u must satisfy the following conditions:

$$\begin{cases} -U_{\max}\mathbf{e} \le \mathbf{u} \le U_{\max}\mathbf{e} \\ \mathbf{e}^{\mathrm{T}}\mathbf{u}_{q} = 0 \text{ или } \mathbf{B}\mathbf{u}_{q} = \mathbf{u}_{q} \end{cases}, \qquad (12)$$

where $U_{\max}$ is the maximum voltage of the inverter.

From (4) and (11), the linear equation of current loop has the form:

$$\lambda\frac{d\mathbf{i}}{dt} + \left(\mathbf{e} + \lambda\dot{q}\boldsymbol{\Lambda}\right)\mathbf{i} = \frac{\mathbf{u}(t)}{\left\|\mathbf{F}'(q)\right\|^{2}}\mathbf{F}'(q) + \frac{1}{R}\mathbf{u}_{q}, \qquad (13)$$

where $\Lambda = \dfrac{\mathbf{F}'(q)\mathbf{F}_{q}^{\mathrm{T}}(q)}{\left\|\mathbf{F}'(q)\right\|^{2}}$.

Suppose that the main of power dissipation source is copper losses. To optimize current loop, we can use u_p in (11), lower and upper limits of the optimal control input signal u_q are defined as:

$$-\mathbf{u}_p - \mathbf{e}U_{max} \le \mathbf{u}_q \le -\mathbf{u}_p + \mathbf{e}U_{max}$$

With the feedback linearization u_p, the torque loop transfer function of linear system in (11) is obtained by Laplace transform:

$$\frac{\tau(s)}{u(s)} = \frac{1}{\lambda s + 1}, \tag{14}$$

where λ is motor torque constant. The optimal PI controller has transfer function as following:

$$u = \left(K_{\mathrm{Pc}} + \frac{K_{\mathrm{Ic}}}{s}\right)\left(\tau^* - \tau\right) = \left(K_{\mathrm{Pc}} + \frac{K_{\mathrm{Ic}}}{s}\right)\left(\tau^* - F'^{\mathrm{T}}(q)i\right), \tag{15}$$

where τ^* is desired torque; $K_{\mathrm{Pc}}, K_{\mathrm{Ic}}$ are the proportional and integral coefficient of the current regulator.

III. Optimal Control in Speed Loop

This article presents the model of an electric drive based on a three-phase brushless permanent magnet motor with six pairs of poles. By Kirchhoff's law, we get the equations:

$$\begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_a(q_e) - M & 0 & 0 \\ 0 & L_b(q_e) - M & 0 \\ 0 & 0 & L_c(q_e) - M \end{bmatrix}\frac{d}{dt}\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix},$$

where u_a, u_b, u_c is phase voltages on the stator windings; R is resistance of stator windings; M is mutual inductance of stator windings; $L_a(q_e), L_b(q_e), L_c(q_e)$ are functions of stator winding inductance; e_a, e_b, e_c are phase counter electromotive forces on the stator windings; i_a, i_b, i_c are phase currents through the stator windings. Equations of the electromagnetic and the mechanical torque are:

$$\tau_e = \frac{1}{\dot{q}}\left(e_a i_a + e_b i_b + e_c i_c\right);$$

$$J\frac{d\dot{q}}{dt} + B\dot{q} = \tau_e - \tau_{\mathrm{L}},$$

where J is motor inertia; B is viscous coefficient of friction; $\dot{q}$ is angular velocity and τ_L is loading torque. The inductance of stator winding is determined by following expression [3–6]:

$$L(q_e) = L_0(1 + K_L)\cos q_e,$$

where L_0 is the nominal inductance of stator winding; K_L is coefficient of dependence; the ratio between the electrical and mechanical rotational angles of the rotor is $q_e = \frac{n}{2}q$; n is the number of poles.

When using the optimal PI control in (15), electromagnetic torque τ_e equal to the required torque τ^*. In this case, the optimal speed controller has the form

$$\tau^*(s) = \left(K_{\mathrm{Ps}} + \frac{K_{\mathrm{Is}}}{s}\right)(\dot{q}_{\mathrm{d.}} - \dot{q}),$$

where $\dot{q}_{\mathrm{d.}}$ is the desired speed; K_{Ps}, K_{Is} are proportional and integral coefficients of the speed controller.

IV. Experimental Results

Simulation of BLDC motor with the parameters are presented in Table 1.

TABLE I. Parameter of motor (RBE 03011C series)

Parameters	Value
Inductance of the stator phase, mH	14
Stator phase resistance, Ohm	5.33
Coefficient back EMF, Nm/amp	0.159
Dependent coefficient of the magnetic flux (K_L)	0.1
Number of pole pairs	06
Coefficient friction, Nm·s.	$1.5 \cdot 10^{-3}$
Torque of inertia, kg·m^2	$6.41 \cdot 10^{-4}$
Continuous stall torque, Nm	5.06
Max. Torque, Nm	15.09

The simulation results are shown in Fig. 2, 3, 4, 5. In this way, desired speed equals 20 rad/s. Load amplitude sinusoidal torque of the is 5 Nm with frequency is 55 Hz. The coefficients of PI regulator are chosen as follows: $K_{\mathrm{Pc}} = 300$, $K_{\mathrm{Ic}} = 100$; $K_{\mathrm{Ps}} = 300$, $K_{\mathrm{Is}} = 100$.

V. Conclusion

- The application of the Fourier transforms in phase currents feedback leads to significant increase phase currents amplitude, minimization of losses (Fig. 2 and Fig. 3). In practice, we can choose the optimal frequency of the Fourier transforms to reduce the influence of the unexpected frequency of the feedback phase currents in the current loop.
- When applying of the Fourier transform for the feedback signals in the current loop, the motor torque ripple is smoother than the traditional model (Fig. 4).
- The speed ripples are removed and the transition time of motor speed is significantly reduced (Fig. 5).

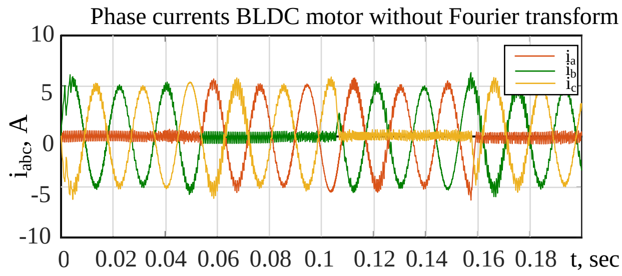

Fig. 2. Output phase currents of BLDC motor without Fourier transforms

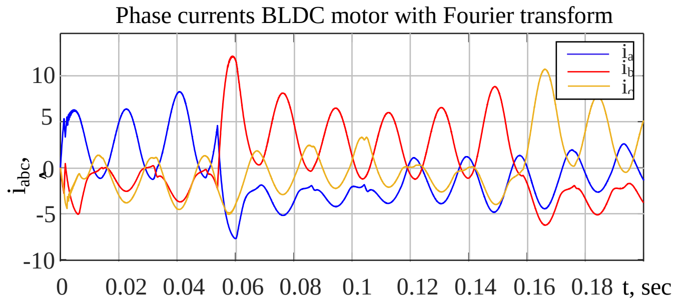

Fig. 3. Output phase currents of BLDC motor with Fourier transforms

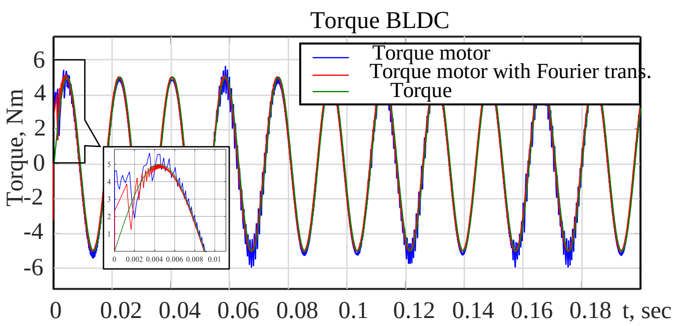

Fig. 4. Torque of the BLDC motor

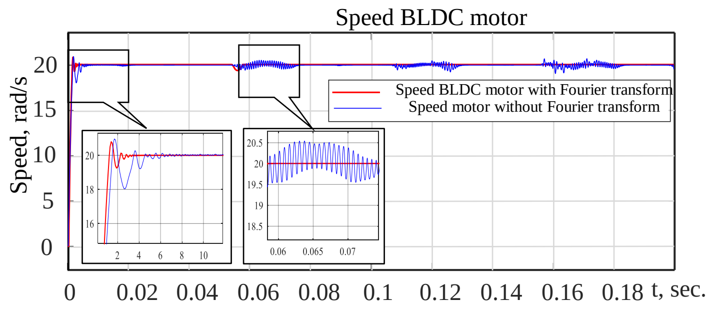

Fig. 5. Speed of the BLDC motor

REFERENCES

[1] F. Aghili, "Optimal and fault-tolerant torque control of servo motors subject to voltage and current limits," IEEE Transactions on Control Systems Technology 21. vol. 4. pp. 1440-1448. 2013.

[2] P. Krause, O. Wasynczuk, S. D. Sudhoff, S. Pekarek, Analysis of electric machinery and drive systems, John Wiley & Sons. vol. 75. pp. 557-600. 2013.

[3] A.Y. L'vovich, Elektromekhanicheskiye sistemy. ucheb. Posobiye [Electromechanical systems]. Leningrad. un-ta. 296 p. 1989. (In Russian)

[4] D. Uayst, G. Vudson, Elektromekhanicheskoye preobrazovaniye energii [Electromechanical Energy Transformation]. Moscow–Leningrad: Energiya. 528 p. 1964. (In Russian)

[5] Chen Yong, Jun Tang, Dong-sheng Cai, and Xia Liu, "Torque Ripple Reduction of Brushless dc motor on current prediction and overlapping commutation," PRZEGLĄD ELEKTROTECHNICZNY (Electrical Review). ISSN. pp. 0033-2097. 2012.

[6] Gavrilov S. V., Zang D. T., Tkhan N. D. Upravleniye elektroprivodom na osnove beskollektornogo dvigatelya s postoyannymi magnitami [Control of an electric drive based on a brushless motor with permanent magnets]// Izv. SPbETU «LETI». 2016. № 8. pp. 53-62. (In Russian)