Computational Analysis of Surface Acoustic Wave Propagation in Crystals under Rotation

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***Abstract*—The results of computational analysis of rotation effect on surface acoustic wave propagation in crystals are given; wave properties that can be used as an informative signal are revealed; new conception for gyro sensitive element based on surface acoustic waves is provided.**

Keywords—surface acoustic waves; rotational velocity; computational analysis; gyro sensitive element

I. Introduction

The simulation of physical processes in anisotropic medium is often reduced to complex mathematical models. One of the practical realizations of continuous media mechanics, considered as a field of mathematical analysis, is the propagation of surface acoustic waves (SAWs) in crystals.

The actuality of research made in this field is associated with the rapid development of SAW acoustoelectronic devices and its wide application in a variety of technical areas.

One of the directions of such researches is the investigation of rotation influence on SAW parameters. Solid-state angular velocity sensors based on acoustic waves represent a potential alternative to existing MEMS sensors that have operational limitations in high dynamic systems due to the presence of torsion suspension elements in its design.

The first work devoted to the study of SAW propagation under rotation is [1], where the basic equations are given. Some general theoretical relations were also obtained by the authors [2]. The attempt to create a SAW gyro sensor design was also made [3], but it's operability was not confirmed either theoretically nor experimentally.

In this connection, the study of this scientific problem remains relevant and opens the high potential for innovative approach to solve the problem of creating SAW navigation sensors.

II. The theory of SAW propagation under rotation

The SAW propagation in a solid piezoelectric medium in the presence of rotation can be described by a system of differential equations (1) obtained from the joint solution of the motion equation and the piezoelectric effect equation:

$$\begin{cases} \rho\cdot\left(\dfrac{\partial^2\xi_i}{\partial t^2}+2(\epsilon_{ink}\ \Omega_n)\dfrac{\partial\xi_k}{\partial t}+(\Omega_i\Omega_k\xi_k-\Omega_k\Omega_k\xi_i)\right)= \\ =C_{iklm}\dfrac{\partial^2\xi_m}{\partial x_k\partial x_l}+e_{jik}\dfrac{\partial^2\varphi}{\partial x_j\partial x_k} \\ e_{prs}\dfrac{\partial^2\xi_s}{\partial x_p\partial x_r}-\varepsilon_{pq}\dfrac{\partial^2\varphi}{\partial x_p\partial x_q}=0 \end{cases} \qquad (1)$$

where ρ is a medium density, ξ_{i0} – displacement vector components, C_{iklm} – elasticity tensor, ϵ_{ink} – Levi-Civita symbol, Ω_i– angular velocity components, x_k, t – space and time coordinates, e_{jik}, ε_{pq} - tensors of piezoelectric and dielectric constants, respectively, φ – electrical potential.

The solution of system (1) is found as a linear combination of plane harmonic waves described by expressions below

$$\xi_i=\xi_{i0}\cdot\cos(\omega t-k_m x_m)$$
$$\varphi=\varphi_0\cdot\cos(\omega t-k_m x_m)$$

where ξ_{i0} is a wave amplitude, k_m – wave vector components, ω - radial frequency.

Let $Wi = \Omega i/\omega$ be the relative rotational velocity. Then, taking into account its smallness for all practically realizable cases, for the joint solution of system (1) will have the equation below:

$$\begin{bmatrix} \Gamma_{11}-\rho\cdot v^2 & \Gamma_{12}+2j\rho\cdot v^2W_3 & \Gamma_{13}-2j\rho\cdot v^2W_2 & \gamma_1 \\ \Gamma_{12}-2j\rho\cdot v^2W_3 & \Gamma_{22}-\rho\cdot v^2 & \Gamma_{23}+2j\rho\cdot v^2W_1 & \gamma_2 \\ \Gamma_{13}+2j\rho\cdot v^2W_2 & \Gamma_{23}-2j\rho\cdot v^2W_1 & \Gamma_{33}-\rho\cdot v^2 & \gamma_3 \\ \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \end{bmatrix}\times$$
$$\times\begin{bmatrix}\xi_{01}\\ \xi_{02}\\ \xi_{03}\\ \varphi_0\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix},$$

where $\Gamma_{im}=C_{iklm}l_k l_l$, $\gamma_i=e_{jik}l_j l_k$, $\gamma_4=-\varepsilon_{pq}l_p l_q$, l_i – directing cosine, v – phase velocity.

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The expression obtained is a system of Green-Christoffel type equations for rotating solid anisotropic medium. Thus, the problem of finding the velocities and polarizations of partial waves reduces to a generalized task of finding eigenvectors and eigennumbers.

III. COMPUTATIONAL ANALYSIS OF SAW PARAMETERS UNDER ROTATION

For problem under consideration, there is no linear analytical solution so it is necessary to use numerical method algorithms. In this case, iterative search for SAW velocities satisfies a combination of boundary conditions and Green-Christoffel type equations.

Using the software developed by the authors, mathematical modeling and calculation analysis of SAW parameters variation and dependence on angular velocity were carried out.

The theoretical studies and numerical calculations have shown that in a number of piezoelectric crystals there are so-called "non-piezoelectric" ("non-piezoactive") directions, where SAW propagation is not followed by the electric field.

Figure 1 shows the dependence of the normalized electrical potential for the *Y*-cut of LGS (La3Ga5SiO14) on the angle between the X_1 axis and the SAW propagation direction - the zero value corresponds to the "non-piezoactive" direction.

A critically important feature of the "non-piezoelectric" directions, revealed as a result of analysis, is that under rotation the acoustic wave becomes accompanied by an electric potential wave φ whose amplitude is proportional to the angular velocity Ω, which can be the basis for creation of new SAW angular velocity sensors. As for the practical application, the constancy of the SAW velocity is important, as well as the linearity of the electric potential change dependence on the relative angular velocity for all workable values.

Figure 2 shows the dependence of the normalized electrical potential for the *Y*-cut of LGS (La3Ga5SiO14) on the relative angular velocity *W* in different directions.

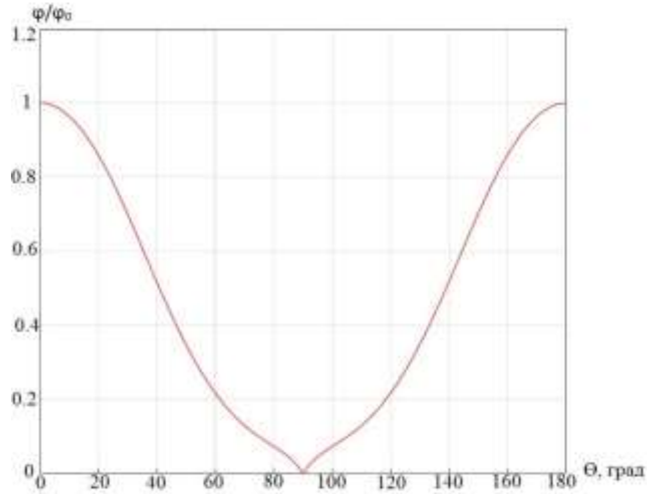

Fig. 1. «Non-piezoelectric» direction in $La_3Ga_5SiO_{14}$ crystal

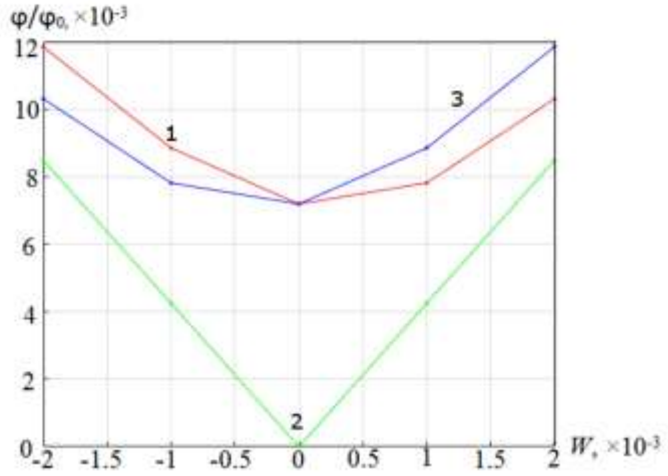

Fig. 2. The dependense of normalized electrical potential on relative angular velocity in $La_3Ga_5SiO_{14}$ crystal (1 - 89.8°; 2 – 90°; 3 – 90.2°)

IV. PRACTICAL APPLICATION OF COMPUTATIONAL ANALYSIS RESULTS

The revealed dependence of the electrical potential, following the SAW propagation, on the angular velocity allowed the authors to propose a fundamentally new concept of the constructive realization of the SAW solid-state gyro sensitive element [6], [7]. The appearance of the electric potential and its proportionality to the rotational velocity in the "non-piezoactive" directions, which had been found using numerical analysis, are the base of the proposed constructive realizations.

The design of the SAW solid-state gyro sensor element is shown in Figure 3, where 1 is the waveguide substrate, 2 - "non-piezoelectric" direction, 3 - radiating transducer, 4 - receiving transducer, 5 - the absorber.

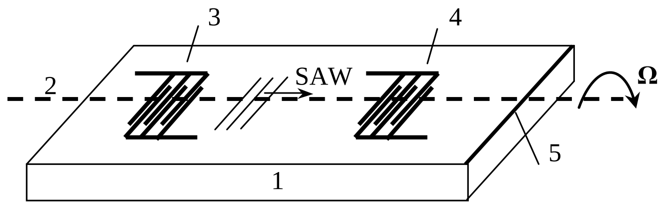

Fig. 3. Sensitive element of SAW solid-state gyro

V. SUMMARY

The paper presents the results of mathematical modeling and calculation analysis of SAW parameter dependence on angular velocity. Based on the results obtained, the SAW parameters, whose change from the rotational velocity can be used as an informative signal, are revealed. This allowed the authors to propose a fundamentally new concept of constructive implementation of SAW solid-state angular velocity sensor.

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