Calculation of Decision-Making Damage in Managers' Qualifications Detection

Anton M. Khodunov, Maksim E. Tzaregorodtcev[1]
Voronezh State Technical University
Voronezh, Russia
[1]science2000@ya.ru

Nikolay I. Didenko
Peter the Great St. Petersburg State Polytechnic University
St. Petersburg, Russia
didenko.nikolay@mail.ru

Felix A. Desyatirikov
Dept. of Gymnasium named after Academician N.Basov
Voronezh State University
Voronezh, Russia
felixdes@ieee.org

Anna S. Nekipelova
Southwest State University
Kursk, Russia
vertakova7@ya.ru

***Abstract*— The algorithm for distinguishing hypotheses using the Neumann-Pearson criterion is implemented in the procedure of numerical quality control of decision-making decisions.**

***Keywords*— Bayesian risk; Neumann-Pearson criterion; decision theory; computer-aided learning system**

I. Introduction

The quality of the management process is largely determined by the timeliness of decision-making, the qualification of the management bodies. In this regard, the objective is to objectively assess the qualifications of the decision-maker. To do this, it is necessary to develop an automatic device that determines the optimal solution based on the state of the system and previous observations of it. The theoretical bases of calculations of this research are given in the work. The method proposed in [1, 2] is used, which establishes relationships minimizing risk, and it is also shown that decision-making is based on comparing the likelihood ratio with a threshold that depends on economic losses when making correct or false decisions and a priori probabilities of hypotheses

The aim of the work is the development of an algorithmic procedure for evaluating the quality of a management decision based on the Neumann-Pearson criterion and the methodology for calculating the economic risk in an erroneous decision.

II. Algorithm of Calculation of Risks and Losses in Decision-Making

We assume that the task is to make a decision based on the results of long-term monitoring of the state of the system. Since the result of the observation is associated with a long series of values, it is advisable to apply the probabilistic-statistical approach to determining the decision rule.

When making decisions in conditions of uncertainty, when the probabilities of possible variants of the situation are unknown, a number of criteria can be used, the choice of each of which, along with the nature of the problem being solved, the set targets and constraints, also depends on the risk appetite of decision-makers [3, 4]. Decisions are made on the basis of the accepted hypothesis about the current and predicted state of the system [5, 6].

In the general case, the a priori probabilities of hypotheses about the state of the observable system are unknown. In this case, each hypothesis is the reason for making a certain decision. Consider a system with two-point control, i.e. there are only two hypotheses H_0 and H_1, the consequences of these are opposite solutions.

The Neumann-Pearson criterion is applied when the a priori probabilities of the hypotheses $P_0 = P(H_0)$ and $P_1 = P(H_1)$ are unknown. It is known that the events of accepting hypotheses constitute a complete group, that is:

$$P_0 + P_1 = 1 \tag{1}$$

We will charge a fee for choosing a hypothesis. With the right decisions, the revenue or loss is always ensured. If wrong decisions are the economic losses of the enterprise. The value of the board is determined by the results of long-term observations. Table 1 presents options for the election of hypotheses or solutions. In the designation of the fee for choosing a hypothesis in the subscript, the first digit is the number of the chosen hypothesis, the second digit is the number of the correct hypothesis.

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TABLE I. VARIANTS FOR THE CHOICE OF HYPOTHESES

№	True hypothesis	Selected hypothesis	Damages of choice	Nature of choice
1	H_0	H_0	Π_{00}	correct
2	H_0	H_1	Π_{10}	wrong
3	H_1	H_1	Π_{11}	correct
4	H_1	H_0	Π_{01}	wrong

To calculate the Bayesian risk, it is necessary to know the loss matrix, a priori probabilities and conditional probabilities of hypotheses. Let the loss matrix be known $\overline{\Pi} = \begin{vmatrix} \Pi_{00} & \Pi_{01} \\ \Pi_{10} & \Pi_{11} \end{vmatrix}$.

The probability of deciding a hypothesis H_1 is known, provided that the hypothesis H_0 is true. This is the probability of an error of the first kind or the probability of a false alarm: $F = P(H_1|H_0)$.

Risk is the sum of risks for all possible hypothesis choices [7]. The average Bayesian risk is determined by the ratio [1, 2]:

$$R = P_0 \cdot P(H_0|H_0) \cdot \Pi_{00} + P_0 \cdot P(H_1|H_0) \cdot \Pi_{10} + \\ + P_1 \cdot P(H_1|H_1) \cdot \Pi_{11} + P_1 \cdot P(H_0|H_1) \cdot \Pi_{01}. \quad (2)$$

To calculate the Bayesian risk in the problem of evaluating the effectiveness of management decisions based on the Neumann-Pearson criterion, it is necessary to determine conditional probabilities of hypotheses.

Let the implementation of the L results of observations be observed at one trial. For example, daily observations during the month of the exchange rate of the currency, shares or other securities. According to the H_1 hypothesis, the implementation has an m average and σ_1^2 variance, respectively, according to the H_0 hypothesis, the implementation has an m average and σ_0^2 variance.

We represent the observation by a vector $\vec{Y} = [y_1, y_2, y_3, ..., y_L]^T$. Each component of the vector is a random variable. But all components have a common. This is the mean and variance. The average value of the results of observations is determined by the formula $m = \frac{1}{L}\sum_{k=1}^{L} y_k$.

The variance of the results of observations is $d = \sigma^2 = \frac{1}{L-1}\sum_{k=1}^{L}(y_k - m)^2$.

The standard deviation is $\sigma = \sqrt{\sigma^2}$.

Let us consider a particular case of the equality of mean values of observations for both hypotheses $m_1 = m_0 = m$. The decisive rule for distinguishing between two hypotheses when the means and the variances of the results of the observations are equal is of the form [1, 2]:

$$\xi \underset{H_0}{\overset{H_1}{\gtrless}} \Pi \quad at \quad \sigma_1^2 > \sigma_0^2, \quad \text{and} \quad \xi \underset{H_0}{\overset{H_1}{\lessgtr}} \Pi \quad at \quad \sigma_1^2 < \sigma_0^2 \quad (3)$$

where $\xi = \sum_{k=1}^{L}(y_k - m)^2$ – is sufficient statistics, equal to the sum of the squared differences of observations and their mean;

$$\Pi = \frac{2 \cdot \sigma_1^2 \cdot \sigma_0^2}{\sigma_1^2 - \sigma_0^2} \cdot \left(\ln h - L \cdot \ln \frac{\sigma_0}{\sigma_1} \right) \quad (4)$$

– the threshold for comparing sufficient statistics;

$h = \frac{P_0 \cdot (\Pi_{10} - \Pi_{00})}{P_1 \cdot (\Pi_{01} - \Pi_{11})}$ – the threshold of comparison of the likelihood ratio. A priori probabilities P_0 and P_1 of the hypotheses are unknown.

The probability of accepting the H_0 hypothesis is [1, 2]:

$$P(H_0|H_0) = \begin{cases} \Phi(\Pi, L, \sigma_0), & at \quad \sigma_1^2 > \sigma_0^2, \\ 1 - \Phi(\Pi, L, \sigma_0), & at \quad \sigma_1^2 < \sigma_0^2. \end{cases}$$

In this expression, the integral probability χ^2-distribution function is applied as a function of the upper integration limit, which is equal to the comparison threshold of sufficient statistics Π, with L degrees of freedom and root-mean-square deviation σ_0

$$\Phi(\Pi, L, \sigma_0) = \int_0^{\Pi} W(\xi_0) \cdot d\xi_0 = \frac{1}{2^{\frac{L}{2}} \cdot \sigma_0^L \cdot \Gamma\left(\frac{L}{2}\right)} \cdot \int_0^{\Pi} \xi_0^{\left(\frac{L}{2}-1\right)} \cdot \exp\left\{ -\frac{\xi_0}{2 \cdot \sigma_0^2} \right\} \cdot d\xi_0$$

where $\Gamma\left(\frac{L}{2}\right) = \int_0^{\infty} t^{\frac{L}{2}-1} \cdot e^{-t} \cdot dt$ – gamma function; L – the number of degrees of freedom equal to the number of days of monitoring the state of the critical parameters of the system.

The probability of accepting a hypothesis H_1, if the hypothesis H_0 or the probability of error of the first kind is true (the probability of false alarm) is equal to.

$$P(H_1|H_0) = 1 - P(H_0|H_0) = \begin{cases} 1 - \Phi(\Pi, L, \sigma_0), & at \quad \sigma_1^2 > \sigma_0^2, \\ \Phi(\Pi, L, \sigma_0), & at \quad \sigma_1^2 < \sigma_0^2. \end{cases}$$

In accordance with the Neumann-Pearson criterion, this probability is given and is equal to F. The problem is to choose a value Π at which equality

$$F = \begin{cases} 1 - \Phi(\Pi, L, \sigma_0), & at \quad \sigma_1^2 > \sigma_0^2, \\ \Phi(\Pi, L, \sigma_0), & at \quad \sigma_1^2 < \sigma_0^2. \end{cases}$$

One way to solve the problem is to use the inverse function of the integral χ^2 distribution of probability

$$\Pi = \begin{cases} \Phi^{-1}((1-F), L, \sigma_0), & at \quad \sigma_1^2 > \sigma_0^2, \\ \Phi^{-1}(F, L, \sigma_0), & at \quad \sigma_1^2 < \sigma_0^2. \end{cases} \quad (5)$$

When the value of the comparison threshold of sufficient statistics Π is found, the threshold for comparing the likelihood ratio from equation (4) is determined:

$$\ln h = -L \cdot \ln \frac{\sigma_0}{\sigma_1} - \Pi \cdot \frac{\sigma_1^2 - \sigma_0^2}{2 \cdot \sigma_1^2 \cdot \sigma_0^2}.$$

The likelihood ratio comparison threshold is

$$h = \exp\left\{ - L \cdot \ln \frac{\sigma_0}{\sigma_1} - \Pi \cdot \frac{\sigma_1^2 - \sigma_0^2}{2 \cdot \sigma_1^2 \cdot \sigma_0^2} \right\}. \quad (6)$$

In the methodology [1, 2], the a priori probabilities of the hypotheses corresponding to a given probability of a false alarm are determined as follows. From the expression for the likelihood ratio comparison threshold, we express one of the a priori probabilities of the hypotheses, P_1 for example, from this relation

$$P_1 = \frac{P_0 \cdot (\Pi_{10} - \Pi_{00})}{h \cdot (\Pi_{01} - \Pi_{11})}.$$

We will take into account (1), then we will find P_1 (and hence, P_0 also):

$$P_1 = \frac{1}{1 + h \cdot \dfrac{\Pi_{01} - \Pi_{11}}{\Pi_{10} - \Pi_{00}}} \quad (7)$$

Thus, the threshold for comparing sufficient statistics Π, the likelihood ratio comparison threshold h, and the a priori probabilities of the hypotheses P_1 and P_0 - are determined from the given false alarm probability F.

The probability of accepting a hypothesis H_1, if it is true, or the probability of a correct decision about the hypothesis H_1 is

$$P(H_1|H_1) = \begin{cases} 1 - \Phi(\Pi, L, \sigma_1), & at \quad \sigma_1^2 > \sigma_0^2, \\ \Phi(\Pi, L, \sigma_1), & at \quad \sigma_1^2 < \sigma_0^2. \end{cases} \quad (8)$$

In this expression, the integral probability χ^2-distribution function is applied as a function of the upper integration limit, which is equal to the comparison threshold of sufficient statistics Π, with L degrees of freedom and root-mean-square deviation σ_1

$$\Phi(\Pi, L, \sigma_1) = \int_0^{\Pi} W(\xi_1) \cdot d\xi_1 = \frac{1}{2^{\frac{L}{2}} \cdot \sigma_1^L \cdot \Gamma\left(\frac{L}{2}\right)} \cdot \int_0^{\Pi} \xi_1^{\left(\frac{L}{2} - 1\right)} \cdot \exp\left\{ -\frac{\xi_1}{2 \cdot \sigma_1^2} \right\} \cdot d\xi_1$$

The probability of accepting a hypothesis H_0, if the hypothesis H_1 is true, or the probability of a second kind error (the probability of missing a message)

$$P(H_0|H_1) = 1 - P(H_1|H_1) = \begin{cases} \Phi(\Pi, L, \sigma_1), & at \quad \sigma_1^2 > \sigma_0^2, \\ 1 - \Phi(\Pi, L, \sigma_1), & at \quad \sigma_1^2 < \sigma_0^2. \end{cases}$$

Thus, [1, 2], the calculation relationships are determined to determine the conditional probabilities included in the expression for the average Bayesian risk and the a priori probabilities of the hypotheses corresponding to the known probability of false alarm by the Neumann-Pearson criterion.

III. Example of Implementation of Algorithm

The model task represents the calculation of risks and losses of the enterprise from operations in the securities market. The initial data for the calculation are as follows.

1. The standard deviation of the share price is:
– on the hypothesis H_0 of "buy shares" σ_0 = 28.6900 \$.
– on the hypothesis H_1 of "sell shares" σ_1 = 16.5300 \$.

2. The average value of the share price of an enterprise m is determined by the results of observations in the past month.

3. Damages with the right decisions are equal:

– when selecting H_0, when the correct H_0: Π_{00}=41400.0 \$

– when selecting H_1, when the correct H_1: Π_{11}=28200.0 \$

Damages with the wrong decisions are equal::

– when selecting H_0, when the correct H_1:

Π_{01}= .411000E+07 \$

– when selecting H_1, when the correct H_0:

Π_{10}= .286000E+07 \$

4. The probability of choosing a hypothesis H_1 when the correct hypothesis is H_0 (probability of false alarm) is: $F = P(H_1|H_0)$= .100

5. The results of observations of the share price y_k for the past month are:

k	1	2	3	4	5
y_k	971.000	985.900	1010.60	1017.10	999.500
k	6	7	8	9	10
y_k	1001.70	981.200	994.200	974.800	1000.50
k	11	12	13	14	15
y_k	999.100	993.800	1004.10	1003.50	992.100
k	16	17	18	19	20
y_k	970.900	993.000	959.000	1010.90	1009.10
k	21	22	23	24	25
y_k	1002.30	958.300	997.800	1001.60	998.500
k	26	27	28	29	30
y_k	996.300	977.100	987.800	986.300	993.600

The number of observations is $L = 30$.

Note that $m_1 = m_0 = m$, $\sigma_1^2 < \sigma_0^2$.

Lets calculate the risks and damage

1. Average value of the stock price of the enterprise

$m = \frac{1}{L}\sum_{k=1}^{L} y_k$ =992.387.

2. Dispersion of the results of observations

$\sigma^2 = \frac{1}{L-1}\sum_{k=1}^{L}(y_k - m)^2$.

Standard deviation of stock price $\sigma = \sqrt{\sigma^2}$ = 14.7505.

3. The threshold for comparing sufficient statistics from (5)

$\Pi = \Phi^{-1}(F, L, \sigma_0)$= 16955.6.

4. Sufficient statistics is $\xi = \sum_{k=1}^{L}(y_k - m)^2$ =6309.77.

5. The rules for making decisions when the mean values are equal $m_1 = m_0$, depending on the variance ratio $\sigma_1^2 > \sigma_0^2$, or $\sigma_1^2 < \sigma_0^2$ have the form (3). This means at $m_1 = m_0$, and $\sigma_1^2 < \sigma_0^2$ a decision is made in favor of the hypothesis H_1 if sufficient statistics ξ are less than or equal to the threshold of comparison Π. In the course of the decision it was established that sufficient statistics are less than the threshold $\xi < \Pi$, so when $m_1 = m_0$ and $\sigma_1^2 < \sigma_0^2$ makes the decision: 1 – the hypothesis H_1 of "sell shares".

The signs of the decision: the variance according to the hypothesis H_1 is less than the variance according to the hypothesis H_0 , $\sigma_1^2 < \sigma_0^2$; the average of the hypothesis H_1 is equal to the average according to the hypothesis H_0, $m_1 = m_0$; sufficient statistics are less than or equal to the threshold, $\xi \le \Pi$.

6. The comparison threshold of the likelihood ratio from (6):

h =65.7585.

7. The a priori probability of the hypothesis H_1 from (7):

P_1=0.0103919.

8. The a priori probability of the hypothesis H_0 from (1):

$P_0 = 1 - P_1$=0.989608.

9. Probability of the correct decision on the hypothesis H_0 equal to

$P(H_0|H_0) = 1 - F$ =0.90.

10. The probability of an incorrect decision about a hypothesis H_1 is in principle given - this is the probability of a false alarm

$P(H_1|H_0) = F$ =0.10.

11. Probability of the correct decision on the hypothesis H_1 from (8) (for the calculation of the integrated χ^2-distribution function with threshold Π= 16955.6, with the number of degrees of freedom L= 30, with a standard deviation of σ_1= 16.5300 and with $\sigma_1^2 < \sigma_0^2$ the desired probability equal to)

$P(H_1|H_1) = \Phi(\Pi, L, \sigma_1)$=0.999485.

12. The probability of a wrong decision about the hypothesis H_0 at $m_1 > m_0$

$P(H_0|H_1) = 1 - P(H_1|H_1)$=0.514984E–03.

13. Average losses or the average Bayesian risk from (2):

R = 320216.0 $.

Conclusion

It is shown that the minimum risk corresponds to the decision making on the basis of comparing the likelihood ratio to a threshold that depends on economic losses when making correct or false decisions and a priori probabilities of hypotheses. The calculation of the losses in the decision-making by the Neumann-Pearson criterion for equal mean values and unequal variances of the observation results was carried out. The received decisions are realized in system of computer definition of qualification of the administrative personnel.

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