Mathematical Modeling of the Modulation Transfer Functions of Matrix Photodetectors in Color Interpolation

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***Abstract*— In this paper, the problems of mathematical modeling and software of methods for finding the transmission functions of modulation during the interpolation of the color of matrix photodetectors are considered. The choice of software is justified. Patternes of new color separation systems based on the colorimetric system *CIE XYZ* 1931 are presented. The introduction of new systems into digital image capture devices is justified. The influence of color interpolation on the space-frequency properties of the matrix receiver with new patterns is considered. The results of engineering calculations are presented.**

***Keywords*— mathematical modeling; software; patternes for the color separation system; matrix; interpolation; modulation transfer functions; optical transfer function**

I. Introduction

Currently, there are numerous systems of color separation of digital cameras (TFC), among which the most popular methods are with color interpolation [1, 2]. Each cell of such a matrix registers a specific base color: red, green or blue. It is basic color, all other tones are perceived as their mixing in a certain proportion. Color matching functions are the values of each primary component of light that must be present so that a person with average vision can perceive all colors of the visible spectrum. With this three primary components, the *X*, *Y*, and *Z* coordinates were aligned. Therefore, initially from the matrix, an image is obtained as a multicolored mosaic. To get a full-color image, it need to perform an interpolation algorithm. When interpolating the colors of the systems with spatial arrangement of the light filters, each element of the radiation receiver is assigned all three values of the primary colors, one of which is valid, corresponding to the color of the light filter, and two are the result of interpolation, so the image blurs, that is, the deterioration of the sharpness properties.

There are two categories of the interpolation algorithm: adaptive and non-adaptive [3-9]. Adaptive interpolators can produce distortions and errors that are unusual for the original image. Computing with adaptive color interpolation turns out to be much more complex, and for their implementation high-performance microprocessors and large memory volumes are required. For the interpolation algorithms, special volumetric programs are developed. The larger the template of the color separation system, the more complex the calculation algorithm will be and the greater the risk of blurring the images. That is why we should evaluate the color separation pattern of the matrix photodetector by the modulation transfer functions, which describes the space-frequency properties of only the color interpolation algorithm as one of the optical transmission links affecting the resolution of the digital system.

The goal of the work is to investigate on the basis of mathematical modeling new developed color separation patterns. Within the framework of the goal, the following tasks were accomplished: selection of software, analysis of new patterns, description and investigation of the optical transfer function.

It is proposed to investigate the optical transfer functions of the developed matrix photodetector with a new color separation system, shown in Fig. 1.

The spectral sensitivity of the photodetectors corresponds to the curves of addition of the colorimetric system of the *CIE* XYZ of 1931: $x(\lambda)$, $y(\lambda)$, $z(\lambda)$ shown in Fig. 2. The curve $x(\lambda)$ has a complex shape: two maxima [10].

CIE XYZ is a linear three-component color model based on the results of measuring the characteristics of the human eye and is a universal color space, in which the range of visible colors characteristic of the average person was presented. System *XYZ* inherent property – positive definiteness, i.e. any physically perceptible color is represented in the *XYZ* system only by positive values. Although not all points in *XYZ* space correspond to real colors due to the non-orthogonality of color matching functions.

It was decided to split one characteristic into two separate filters for two filters, one of which is corrected for the short-wave branch of the curve of the component *x*, where $\lambda \leq 505$ nm, and the second *X* – under the long-wave branch, where $\lambda \geq 505$ nm. Then we get a color separation system with a set of filters *XxYZ*. A small branch of the curve $x(\lambda)$ can be used as additional information to the curve $z(\lambda)$ about the blue

The reported study was funded by RFBR according to the research project № 18-37-00176.

978-1-5386-5824-6/18/$31.00 ©2018 IEEE

component of the spectrum at low illumination, as is done in the technology presented in the source [11].

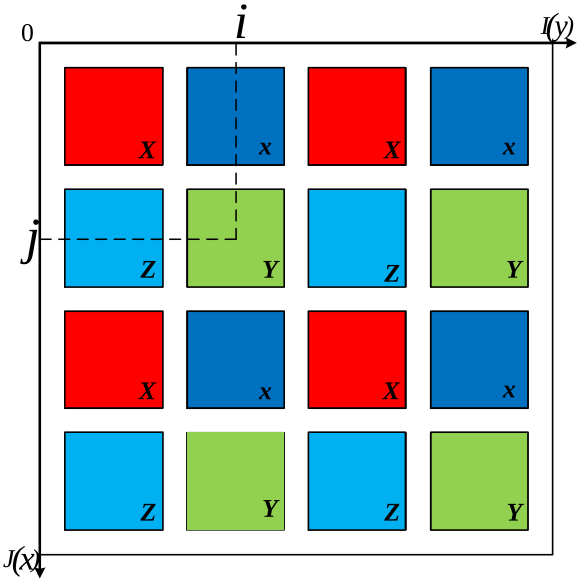

Fig. 1. Pattern of the separation system *XxYZ*

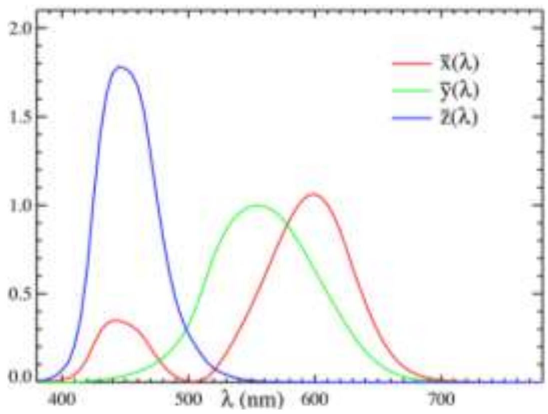

Fig. 2. Characteristics of the pattern on the basis of the colorimetric system *CIE XYZ* 1931.

In the *Super CCD* matrix, the sensor elements are grouped in pairs: the *S*-elements are saturated quickly in the illumination, receiving information about the color of the point. At the same time, *R*-elements are saturated much more slowly, why they better determine the illumination. Similarly, we can apply *x* and *Z* cells to determine the saturation in blue hues. Because of the rather large spectral characteristic, cell *Z* will fill up faster in well-illuminated scenes, while cell *x* will work well with dark little saturated scenes and objects. So these cells will complement each other when digitizing the picture. Also in the future, we can apply technology *Dual Capture Technology*.

Under this system, three patterns of color separation systems were developed. The influence of color interpolation on the spatial-frequency properties of the matrix photodetector will be considered in the framework of the first system, since the rest of the functions will be similar.

The author presents another pattern of the *XxYZ* system, created by shifting the original template on one cell down (see Figure 3). Such a configuration will get rid of defects in the image caused by diagonal lines – alliasing. In the pattern in Fig. 1, diagonal lines can be observed when using cells *x* as auxiliary cells for *Z* cells, and horizontal lines can also appear when using cell *x* as an auxiliary to cell *X*, working with the *CIE XYZ* O system of 1931. Therefore, the matrix with the proposed template can be useful in capturing images with strict straight and diagonal lines, which are often found when digital shooting industrial landscapes, cities, highways and other objects created by human.

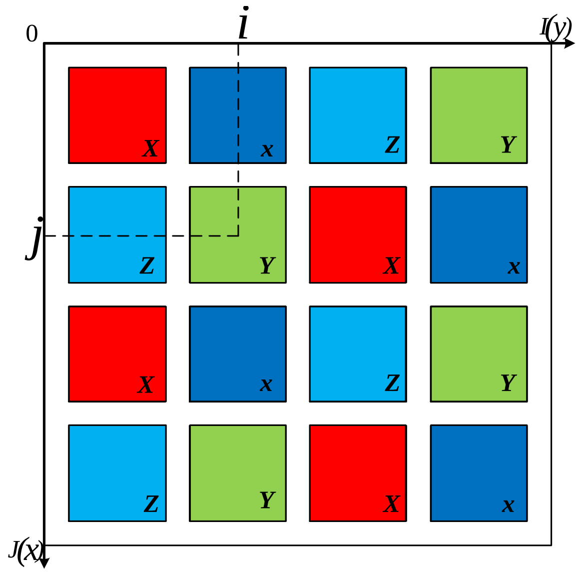

Fig. 3. Pattern of the separating system *XxYZ* with shift

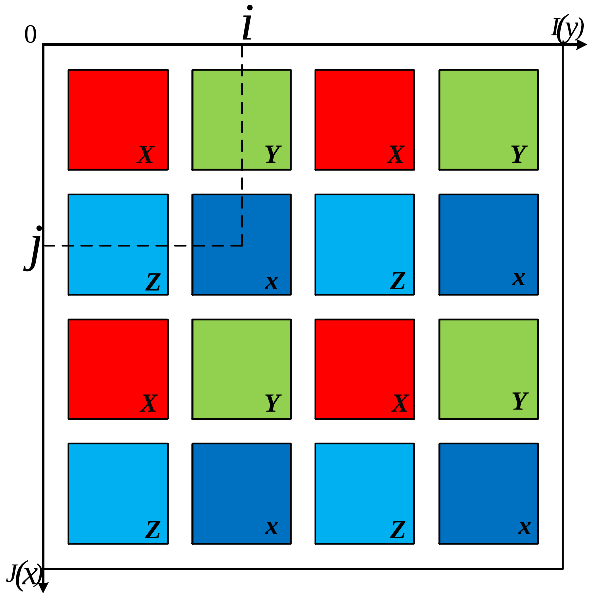

Fig. 4. Pattern of the separation system *XYxZ*

A pattern with a combination of *XYxZ* cells is also proposed. The only difference from the first pattern is that diagonal lines can be observed when using cells *x* as auxiliary cells for cells *X*, and horizontal lines when applying cell *x* as an auxiliary to cell *Z*, working with the *CIE XYZ* system of 1931. This t pattern is well used for photography of wildlife, where strict straight lines are extremely rare (except for the horizon).

II. Research of optical transmission functions

Based on the analysis of various programming languages and their advantages and disadvantages, the *Mathcad* program was chosen as the most convenient, understandable and easy-to-learn software package. The main advantage of the package is the natural mathematical language, on which the solved problems are formed. The program has a wide graphics capabilities, extensible from version to version. The practical application of the package significantly increases the effectiveness of intellectual work. Making changes, the user immediately sees the results, at any time and can make changes. Also the program has a friendly and intuitive interface.

To research the effect of color interpolation on the spatial-frequency properties of a matrix photodetector, a linear interpolation algorithm was chosen. When performing color interpolation, each element of the receiver is assigned the value of all primary colors. One basic color is considered "true", which the cell itself registers, the remaining colors are "calculated", because are found from the average value from neighboring cells. In the *XxYZ* system, the color interpolation algorithm for each element will be similar to the calculations of the receivers with coordinates (*i*, *j*), (*i* + 1, *j*), (*i*, *j* + 1) and (*i* + *1*, *j* + 1). Calculation and construction of graphs will be carried out in the program *Mathcad*.

For the element of the radiation receiver with coordinates (*i*, *j*), the output signal will be:

$$X'(i;j)=\frac{1}{4}\left[X(i-1;j-1)+X(i+1;j-1)+X(i+1;j-1)+X(i-1;j+1)\right];$$

$$x`(i;j)=\frac{1}{2}\left[x(i;j-1)+x(i;j+1)\right];$$

$$Y`(i;j)=Y(i;j);$$

$$Z`(i;j)=\frac{1}{2}\left[Z(i-1;j)+Z(i+1;j\right].$$

For the element of the radiation receiver with coordinates (*i*+1,*j*), the output signal will be:

$$X`(i+1;j)=\frac{1}{2}\left[X(i+1;j-1)+X(i+1;j+1)\right];$$

$$x`(i+1;j)=\frac{1}{4}\left[x(i;j-1)+x(i+2;j-1)+x(i;j+1)+x(i+2;j+1)\right];$$

$$Y`(i+1;j)=\frac{1}{2}\left[Y(i;j)+Y(i+2;j)\right];$$

$$Z`(i+1;j)=Z(i+1;j).$$

For the element of the radiation receiver with coordinates (*i*, *j*+1), the output signal will be:

$$X1`(i;j+1)=\frac{1}{2}\left[X(i-1;j+1)+X(i+1;j+1)\right];$$

$$x`(i;j+1)=x(i;j+1);$$

$$Y`(i;j+1)=\frac{1}{2}\left[Y(i;j)+Yi;j+2)\right];$$

$$Z`(i;j+1)=\frac{1}{4}\left[Z(i-1;j)+Z(i+1;j)+Z(i-1;j+2)+Z(i+1;j+2)\right].$$

For the element of the radiation receiver with coordinates (*i*+1, *j*+1), the output signal will be:

$$X`(i+1;j+1)=X(i+1;j+1);$$

$$x`(i+1;j+1)=\frac{1}{2}\left[x(i;j+1)+x(i+2;j+1)\right];$$

$$Y`(i+1;j+1)=\frac{1}{4}\left[Y(i;j)+Y(i+2;j)+Y(i;j+2)+Y(i+2;j+2)\right];$$

$$Z`(i+1;j+1)=\frac{1}{2}\left[Z(i+1;j)+Z(i+1;j+2)\right].$$

In the space-frequency domain, an *optical transfer function* (OTF) is described, which in this case is due to signal shifts along the elements of the radiation receiver. When finding optical transfer functions, we take for each color, that the images are infinite, of the same color, and during the interpolation an image is created, each element of which is the result of interpolation.

OTF for the component *X* takes the form:

$$W^{X}_{i;j}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x)\cdot\cos(p_y\Omega_y);$$

$$W^{X}_{i;j+1}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x);$$

$$W^{X}_{i+1;j}(\Omega_x;\Omega_y)=\cos(p_y\Omega_y);$$

$$W^{X}_{i+1;j+1}(\Omega_x;\Omega_y)=1.$$

OTF for the component *x* takes the form:

$$W^{x}_{i;j}(\Omega_x;\Omega_y)=\cos(p_y\Omega_y);$$

$$W^{x}_{i;j+1}(\Omega_x;\Omega_y)=1;$$

$$W^{x}_{i+1;j}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x)\cdot\cos(p_y\Omega_y);$$

$$W^{x}_{i+1;j+1}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x).$$

OTF for the component *Y* takes the form:

$$W^{Y}_{i;j}(\Omega_x;\Omega_y)=1;$$

$$W^{Y}_{i;j+1}(\Omega_x;\Omega_y)=\cos(p_y\Omega_y);$$

$$W^{Y}_{i+1;j}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x);$$

$$W^{Y}_{i+1;j+1}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x)\cdot\cos(p_y\Omega_y).$$

OTF for the component *Z* takes the form:

$$W^{Z}_{i;j}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x);$$

$$W^{Z}_{i;j+1}(\Omega_x;\Omega_y)=\cos(p_x\Omega_x)\cdot\cos(p_y\Omega_y);$$

$$W^{Z}_{i+1;j}(\Omega_x;\Omega_y)=1;$$

$$W^{Z}_{i+1;j+1}(\Omega_x;\Omega_y)=\cos(p_y\Omega_y).$$

Take as the optical transfer functions of each primary color the mean value of the optical transfer functions described above. Thus, one-dimensional modulation transfer functions (MTF) color interpolations for each primary color, taking into account the influence of the color interpolation algorithm, of the *XxYZ* system, can be described by the following expression:

$$T^X_{XxYZ}(N) = T^x_{XxYZ}(N) = T^Y_{XxYZ}(N) = T^Z_{XxYZ}(N) = 0{,}5 + 0{,}5\cos(2\pi pN)$$

where $T^X_{XxYZ}(N)$ is the MTF interpolation of the X component; $T^x_{XxYZ}(N)$ – MTF interpolation of the component x; $T^Y_{XxYZ}(N)$ – MTF interpolation of the Y component; $T^Z_{XxYZ}(N)$ – MTF interpolation of the Z component; N is the spatial frequency, mm^{-1}; p – the size of the radiation receiver element (6.424 μm).

In Fig. 5 shows the graphs of the MTF of the color interpolation of the proposed *XxYZ* system calculated in the *Mathcad* program.

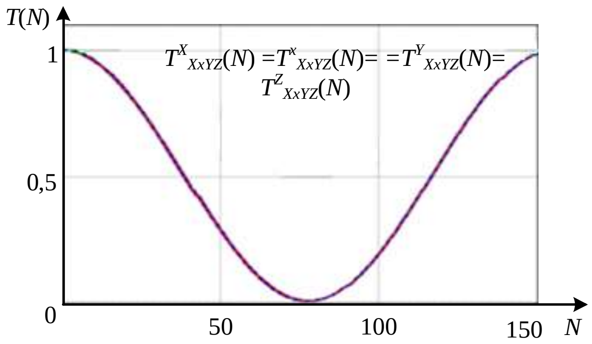

Fig. 5. Curves of the modulation transfer function of *XxYZ* pattern

III. OBJECT OF RESEARCH

In general, the calculated OPF systems for multi-element matrix photodetectors are satisfactory. The modulation transfer functions of the *XxYZ* system are oscillatory in nature and a slight decrease in the MTF of the channels of all three primary colors is observed, as is also observed in the standard *Bayer* pattern [12]. To obtain ideal characteristics of MTF, new multilayer systems [13] should be used, which are expensive in production and have a high level of noise. For such systems, high-performance microprocessors and large amounts of memory are required.

The main advantage of the developed *XxYZ* pattern is the spectral characteristics of the color separation filters, which are based on the curves of the addition of the color system *CIE XYZ* 1931. Application of the characteristics of this system allows to extend the color coverage of digital systems, in comparison with the standard RGB color spaces. Thus, in the proposed color separation systems *XxYZ*, the spatial frequency characteristics show a fairly acceptable result.

In the future, it is possible to study the transmission functions of the color interpolation modulation of the color separation system *XxYZ* with upward shift by one pixel, and the *XYxZ* template with the alternation of light filters with the characteristics of the curve $x(\lambda)$. It is proposed to apply bicubic-interpolation and compare the results of the research. The work may be interest to specialists in the field of optoelectronic devices and digital devices, in particular.

ACKNOWLEDGMENT

Parvulyusov Yuri Borisovich, candidate of technical sciences, professor, thanks for the help and support.

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