Modeling and Optimization of the Process of Separation of Granular Materials on Sieves

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***Abstract*— A mathematical model of the process of separation of granular materials by size on multistage sieve classifiers based on the theory of Poisson processes is constructed. A system of stochastic differential equations is constructed to determine the particle size distribution function along the sieves of the classifier. The solutions obtained make it possible to find all the characteristics of the separation process of interest, including the position of receiving hoppers for the target fractions, and also determining the degree of separation efficiency. The multicriteria optimization problem for establishing constructive and regime parameters of the classifier is considered.**

***Keywords*— mathematical model; separation; granular material; probability of sieving; Poisson process; multicriteria optimization**

I. INTRODUCTION

For the separation of granular materials by size, different classifiers are used, including multistage devices. The working body of the multistage classifier is a set of oscillating sifting surfaces made in the form of a screen or sieve, which are located one above the other, forming longlines. The principle of operation of such equipment is based on the different probability of passage through the holes of the sieves of various shapes and geometric sizes. Generally the calculation of a sieve classifier is reduced to the determination of its design and regime parameters, depending on the shape and size of the material to be separated [1, 2].

Classification of granular materials by size is considered, for example, in works [1–7]. In works [1–3] the kinetics of the separation process is studied on the basis of the theory of Markov processes. It allows to determine the particle size distribution function along the sieves of the classifier. The process of separation of granular materials is considered as a diffusion process and a system of Kolmogorov–Fokker–Planck (K.–F.–P.) equations is constructed to determine the particle distribution function. Works [1, 2, 4] are devoted to the study of the probability of sifting a particle into a cell, depending on the shape and geometric dimensions of the sieve and the material to be separated. Thin layer separation of granular materials is considered in work [1]. The use of vibratory classifiers for the separation of granular materials is due to the fact that vibration converts the forces of dry friction into frictional forces of a viscous type [5-7, 10]. Conditions appear for the manifestation of differences in separation parameters. These differences do not appear in the absence of vibration (in static conditions).. Under the action of vibration, vibrational forces begin to act on the particles of the material to be separated, which lead to an increase in the intensity of separation. There are opportunities to separate particles with slightly different separation parameters.

Thus, the probabilistic nature of the process of separation and the properties of particles in the material to be separated characterizes the stochasticity of the process. Therefore, among staple approaches in the modeling of these processes, a special place is occupied by stochastic methods, in particular, the theory of Markov processes [1–3, 8]. Mathematical modeling of the separation process of granular materials can be carried out on the basis of the theory of random walk of particles on a plane, in particular, the theory of Poisson processes. [2, 9]. Modeling the separation process of granular materials in multi-stage classifiers with a large number of levels on the basis of the theory of Poisson processes allows us to overcome a number of difficulties that arise when using equations (K.-F.-P.) [2].

The purpose of work is creation of mathematical model of a separation process of granular materials on multistage classifiers with use of the theory of Poisson processes and optimization of process for determination of optimum technological and design parameters of the device on the basis of the constructed models.

II. MATHEMATICAL MODELING

Sifting of particles in the sieve cells can be regarded as Bernoulli's tests with a binomial distribution. In many applications for a binomial distribution, either a normal approximation or a Poisson distribution is constructed, so we use the theory of Poisson processes to describe the position of the particle on the sieves of the classifier [2]. We denote by ΔX_i – the distance that the particle travels through the surface of the *i*-th sieve, $i=\overline{1,m}$, where m – number of classifier sieves. Assuming that ΔX_i are random variables distributed according to the exponential law, then their sum $\Delta X_1+...+\Delta X_m$ obeys the gamma distribution with density: $g_m(x)=\lambda\dfrac{(\lambda x)^{m-1}}{(m-1)!}\exp(-\lambda x)$. Then, taking into account the

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normalization, the value of the integral $\int_{x_1}^{x_2} g_m(x)dx$ determines the probability of the particle entering the cells of the m - ith sieve on a segment [x_1,x_2]. The intensity parameter λ of the gamma distribution is related to the probability of sifting into the cell p by the ratio, $\lambda = p/2a$, where is $2a$ – the cell size in the direction of particle motion, and the average intensity value for the selected fraction with linear dimensions $(l_1, l_2) \in (l_{\min}, l_{\max})$ is calculated by the formula $\bar{\lambda} = (\int_{l_1}^{l_2} f(l)p(l)dl)/(2a \times \int_{l_1}^{l_2} f(l)dl)$, where $f(l)$ – particle size distribution in the initial material, $p(l)$ – is the probability of particle sifting into a cell as a function of linear dimensions.

In the first approximation, the probability of sifting p can be represented as a product of the probabilities of two independent events [2,4]: $p = p_g \times p_v$, where p_g – geometric probability, depending on the size and shape of the hole of the screen and the particles of the material to be separated, and p_v – is the probability that depends on the velocity of the particle, which is determined by formula

$$p_v = 2 - (\Phi(z) + \Phi(z_0)), \quad (1)$$

where $z = (V_a - V_k)/\sigma$, $z_0 = V_k/\sigma$, V_k, σ – the parameters of the normal law are determined from the experimental data in the process of identification of the constructed models, V_a – the amplitude of the particle velocity relative to the screen, $\Phi(x)$ – is the standard normal distribution function. These issues are considered in works [1,2].

For determination the amplitude of the relative velocity V_a it is necessary to study vibrational motion of granular materials. Movement of granular materials under vibration is considered in works [1,4-7,10]. Depending on on the physical and mechanical properties of the of the material to be separated, the particle size distribution, etc. the motion of dispersed materials over a vibrating surface can be modeled both in the single-particle approximation [1,4] and on the basis of methods of mechanics of heterogeneous media [10]. Simulation of the motion of particles as a material point can be carried out with thin layer motion of the materials being separated. For the purpose of the most complete use of the residence time of the material to be separated on the surface of the sieve of the classifiers the regular forward-backward motion mode with instant stops without flipping is used [1,4].

Sifting of particles along sieves of the multistage classifier generally can be considered as nonstationary Poisson process with a distribution density [2]:

$$f(x, \Delta x) = \Lambda(x, \Delta x) \frac{\bar{\Lambda}^{(m-1)}(x, \Delta x)}{(m-1)!} \exp(-\bar{\Lambda}(x, \Delta x)),$$

where $\Lambda(x, \Delta x)$ – the intensity of sifting of particles along the sieves on a segment [x, x+Δx], $\bar{\Lambda}(x, \Delta x)$ – its average value, m – is the number of sieves. The distribution function $F_i(x)$ is constructed for the random quantities ΔX_i, $i = \overline{1, m}$. As the sieves work together, these probabilities for $i \geq 2$ will be conditional. The mathematical expectation of the number of particles passing through the sieve depends not only on the length of the section, but also on its position along the length of the sieve. If we determine the distribution law for the first sieve as $F_1(x) = 1 - F_0(x)$, where $F_0(x) = P\{\Delta X_1 \geq x\}$ – the probability that at the segment of the sieve $(0; x)$ in the direction of movement of the particles and width $2b$ (cell size in the transverse direction), no events will appear. Taking into account the independence of the coordinate of the process changes at disjoint intervals (no aftereffect for the Poisson process) $F_0(x)$ must satisfy the equation: [2]: $F_0(x + \Delta x) = F_0(x)F_0(\Delta x)$, where the interval (0, x) is long, Δx - short. In the first approximation, we can assume that the intensity of events at the point x is determined as a linear function: $\mu(x) = \lambda_0 x/\Delta x_{av}$, where Δx_{av} – the average distance that the particle travels along the surface of the sieve, defined in the form $\Delta x_{av} = 1/\bar{\lambda}$, λ_0 – the intensity of arrival of the selected particles on the site in question. Thus, the instantaneous density can be determined from expression $\mu(x) = Cx$, where parameter $C = \lambda_0 \bar{\lambda}$, which, like the intensity $\bar{\lambda}$, depends on the probability of sifting into a cell p. Taking into account the non-stationarity and ordinary nature of the event flow, the probability of the absence of an event in the interval Δx in the first approximation can be estimated from expression $F_0(\Delta x) = 1 - C x \Delta x$, and probability of one event as $Cx\Delta x$.

Then the difference equation for the probability $F_0(x)$ has the form [2]: $F_0(x + \Delta x) = (1 - C x \Delta x)F_0(x)$. Passing to the limit as $\Delta x \to 0$, we can write the following stochastic differential equation $F_0'(x) = -C xF_0(x)$. Its solution, taking into account the initial condition $F_0(0) = 1$ has the form $F_0(x) = \exp(-C x^2/2)$. Thus, the distribution function for the first sieve has the form $F_1(x) = 1 - \exp(-C x^2/2)$. The law of distribution of the quantity ΔX_i, $i = \overline{2, m}$, is determined from the system of stochastic differential equations [2]:

$$F_j'(x) = -C xF_j(x) + C xF_{j-1}(x), \; j = \overline{1, m}. \quad (2)$$

First of all, we are interested in the law of distribution of quantities ΔX_m on the lower tier. Taking into account the initial conditions $F_0(0) = 1$ and $F_j(0) = 0$ for all $j = \overline{1, m}$, from the system of stochastic differential equations we obtain the distribution function and the distribution density of ΔX_m:

$$F_m(x) = 1 - \exp(-Cx^2/2) \times \sum_{i=0}^{m-1} (Cx^2/2)^i / i!,$$

$$f_m(x) = Cx(Cx^2/2)^{m-1} \exp(-Cx^2/2)/(m-1)!. \quad (3)$$

The coordinates of the position of the receiving hopper x_1 and x_2 for the target fraction are determined on the basis of the distribution law of ΔX_m at given value for extraction of the target fraction η_*: $F_m(x_1) = (1-\eta_*)/2$ and $F_m(x_2) = (1+\eta_*)/2$, (4)

The parameter C value is determined from the second equation (4) with $x_2 = L$, where L – the length of the sieve, then, the coordinate x_1 is determined from the first equation (4). As a result, the recovery factor and separation efficiency are determined by the formulas [2]:

$$\eta = \int_{x_1}^{x_2} f_m(x)dx \text{ and } E = \eta(1-\delta) \times 100\%, \quad (5)$$

where δ – relative content of non-target products in the target hopper (purity of separation).

Calculations show (fig. 1) that mathematical expectations ΔX_m for the considered fractions disperse with increase in quantity of tiers of the qualifier *m*. It positively influences on a separation process. In fig. 2 distribution ΔX_m for the chosen fractions along the lower sieve is given. It is shown in which section of the lower sieve the particles of the selected fractions are sieved into its cells. Calculations show that if the values of the design parameters are set (pre-calculated), namely, the shape and dimensions of the holes, the length and number of sieves, and the requirement for extraction of the target fraction, it is possible to pick up best value of amplitude of speed V_a. Operating parameters of the device: amplitude and frequency of oscillations A and ω, angles of inclination and vibration α и β should be selected in such a way that the efficiency or the extraction coefficient take the greatest values.

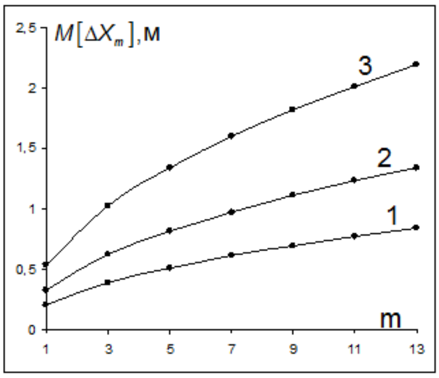

Fig. 1. Dependence of the mathematical expectation ΔX_m from the number of tiers m with the mean square deviation for shallow (1) - 0,117 m; average (2) - 0.185 m; large (3) - 0.269 m fractions

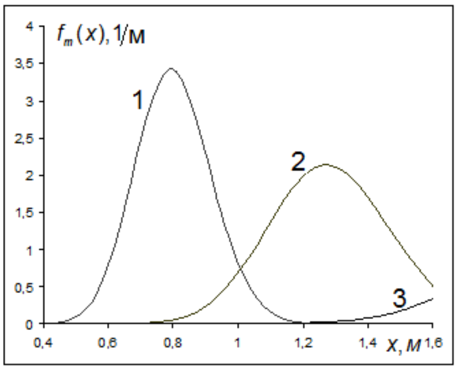

Fig. 2. The distribution of the number of particles of fine (1), medium (2) and large (3) fractions along the lower sieve at m = 12 and the mean square deviation ΔX_m for small (1) - 0,117 m; average (2) - 0.185 m; large (3) - 0.269 m fractions

Velocity parameters V_k and σ in dependence (1) are determined during the identification of the constructed models, the calculated values of the the extraction coefficient for the target fraction or separation efficiency can be compared with the experimental values. The experimental values are determined according to the selected model of motion by the regime parameters in the course of the classifier operation: amplitude and frequency of oscillations A and ω, and angles of inclination and vibration α and β.

III. OPTIMIZATION OF THE SEPARATION PROCESS

To determine the optimal design and operational parameters of the classifier, the optimization problem is considered in a multi-criteria formulation, the efficiency of separation and the performance of the multi-stage classifier are chosen as criteria [1, 2]:

$$\begin{aligned} &\max Q(A,\omega,\alpha,\beta,h,B) = \rho_c hBV_{av}, \\ &\max E(A,\omega,\alpha,\beta,D,L,\eta,\ m) = \eta(1-\delta)\times 100\%, \\ &\text{under conditions}: x_j^{\min} \le x_j \le x_j^{\max}, \\ &\varphi_k^{\min} \le \varphi_k(A,\omega,\alpha,\beta) \le \varphi_k^{\max}, \end{aligned} \quad (6)$$

where $x_j^{\min}$, $x_j^{\max}$ – the smallest and largest value of the components of the vector $\bar{x} = (A,\omega,\alpha,\beta,D,L,h,\eta,m)$, φ_k - functional limitations associated with the selected high-speed mode, ρ_c – bulk density, B – sieve width, L – sieve length, m – number of sieves, A and ω – amplitude and frequency of oscillation, α and β – angles of inclination and vibration, V_{av} – the average speed of movement of the granular material on the sieve, h – The thickness of a layer of granular material fed from the hopper, η – extraction of target fraction.

For the solution of a multicriteria problem (6) it is possible to use various approaches [11]. For example, at the first stage the optimal values of the constructor parameters of the classifier are determined: number of tiers *m*, length and width of sieves L and B, form and cell size D. Knowing the requirement of extraction η, parameter L is calculated, and the number of tiers of m is determined by purity of separation. Further, the values of the tuning parameters A, α can be selected before operation, and the optimum values of the control parameters ω, β are chosen by the decision maker from the solution of the optimization problem (6), for example, from some set of Pareto optimal solutions [2]. To construct the Pareto set, a linear convolution of the normalized objective functions (6) is made and its maximum is determined for different values of the weight coefficients. In the space of criteria an image of the Pareto set is constructed, from which the optimal values of the control parameters ω, β are selected. The Pareto area contains, as a rule, many elements, in this connection there is a need for further narrowing of this area. To do this, you can use different procedures [11].

To determine the optimum values of the regime and constructive parameters of the classifier, a computational experiment is performed based on the constructed mathematical model. A granular material on a polymer base was used, the particles of which have a cylindrical shape ranging from 0.2 mm to 1.4 mm and identical diameters of 0.5 mm. The bulk density of the material is 1150 kg/m^3, the content of the target fraction 0.5-0.9 mm in the material is 60–75%.

Calculations were carried out at values: L = 1.6 m; B = 1.0 m; m = 12 pieces; $A =$ 5 mm; $\alpha = -5^0$; $h = 3$ mm; $\eta_* =$ 0.9; cell sizes 2a × 2b = 3.5 × 3.5 mm with round hole diameter D = 1.5 mm. As a result of solving the multicriteria optimization problem (6), a compromise solution was obtained: E =81.5 %, $Q = 1490$ kg/h at optimal values of the parameters: $\omega =$ 54.6 s^{-1} и $\beta =$ 10.5°. At the same time, the average speed was V_{av} = 0.118 m/s, amplitude of relative speed V_a = 0.36 m/s, the position coordinates of the receiving hopper for the target fraction are equal to: x_1 = 0.98 m, x_2 = 1.6 m, and the purity of the separation (the relative content of non-target products in the target hopper) $\delta =$ 10.4%.

IV. CONCLUSION

The theory of random processes, in particular Poisson processes, supplemented by experimental studies to determine the parameters of the model, makes it possible to build mathematical models and, on this basis, to optimize the process of separation granular materials in size on multi-stage sieve classifiers.

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