Self-Organizing Algorithm with the Criterion of the Identifiability Degree for Correcting Autonomous Inertial Navigation Systems of Aircrafts

M. S. Selezneva[1], A. V. Proletarsky[2], K. A. Neusypin[3], V. D. Schashurin[4], T. Y. Tsibizova[5]
Bauman Moscow State Technical University
Moscow, Russia
[1]m.s.selezneva@mail.ru, [2]pav_mipk@mail.ru, [3]neysipin@mail.ru, [4]schashurin@bmstu.ru, [5]vesta952006@yandex.ru

V. M. Nikiforov
Academician Pilugin Scientific-Production Center of Automatics and Instrument-Making
Moscow, Russia
v.m.nikiforov@gmail.com

***Abstract*— Constructing high-precision non-linear error models in measuring systems of the aircraft is achieved through the use of an original numerical criterion in the algorithm to select models of the identifiability degree that allows selection of candidate models with increased identifiability characteristics of parameters in the model matrix. The methods for estimating the quality of identification about the parameters of linear stationary and nonstationary dynamical system models are investigated. The criteria for the identifiability degree about parameters of the model are investigated. A criterion for the identifiability degree about the parameters of nonlinear models in a class of dynamical systems is presented. This class includes systems that are represented using the SDC method. The criterion of the identifiability degree is used in the process of selecting models, in particular error models in navigation systems. The simulation of results demonstrated the effectiveness of the algorithm with improved properties.**

Keywords*— *aircraft; inertial navigation system; correction scheme; self-organizing algorithm; criterion for the identifiability degree

I. INTRODUCTION

Algorithmic support of control systems for various dynamic objects includes mathematical models of the study objects. Mathematical models obtained from the physical or some other laws in practical applications, as a rule, do not always accurately reflect the processes investigation. In the process of functioning a dynamic object in case of the changing conditions, some parameters of models can be changed significantly, so they need to be determined using identification algorithms [1, 2]. Identification algorithms are widely used in various technical applications: the design of instruments and control systems [3], algorithmic support of aircrafts [3], various navigation systems [4, 5], etc. For the correction of inertial navigation systems (INS) in an autonomous mode algorithms based on the Group Method of Data Handling (GMDH) are used, which is an evolutionary method and achieved through self-organization [6, 7].

The structural scheme of the INS correction, which is using the algorithm for constructing the model and prediction in the case of that the external sensor is disconnected, is presented in Fig. 1.

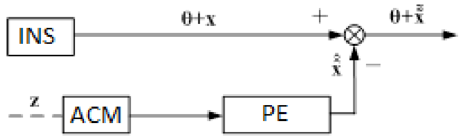

Fig. 1. Structural scheme of autonomous INS’s correction

Here ACM – algorithm for constructing a model; – error vector prediction of INS; PE – prediction errors.

Separate GMDH algorithms differ significantly in the type of support functions used, and, consequently, in the way of constructing a complete description of the object. The main ones are algorithms with a polynomial of second degree, algorithms with a linear polynomial, and probabilistic algorithms. Self-organization algorithms use criteria for selection that allow selecting models with the desired properties [6, 7].

The accuracy of determining the parameters of the model depends on the properties of the model matrix and the accuracy of the identification algorithm. The choice of the identification algorithm is determined from considerations of feasibility of implementation in the existing calculator, economic opportunities and features of the projected object. The exact characteristics of the most popular identification algorithms are known. The study of specific models for the quality of parameter identification was carried out experimentally and, in general, the application of this approach requires a series of complex experiments, a long time and significant financial costs.

Another way to solve the problem of determining the quality of identification is the analytical approach - the development of a measure criterion or the identifiability degree. Synthesis of the criterion of identifiability is expedient to carry out by analogy with the known criteria of the observability degree [8, 9, 10]. The development of a criterion for the identifiability degree is an important task, which makes it possible to calculate the quality of determining the parameters in the matrix of the model of various dynamic objects.

Known criteria for the identifiability degree [11, 12] assume an analysis of objects described by linear equations. In view of the fact that nonlinear identification algorithms are used to solve the problems of increasing the accuracy of INS, a numerical criterion for the identifiability degree of the parameters of nonlinear models is developed.

A modification of the algorithm constructed on the basis of GMDH is proposed using numerical criterion for the identifiability degree of parameters. These parameters describe nonlinear model included in criteria for selection.

In the presented GMDH algorithm in the selection process, the models with the highest degree of identifiability are selected.

The developed algorithm allows to build models of researched processes with improved qualitative characteristics. Improvement of the quality of models is good for the selection of models for further selection, allowing for more precise parametric identification and, accordingly, more effective correction of aircraft INS in flight.

II. SELF-ORGANIZATION APPROACHES

Self-organizing approaches are used to solve complex nonlinear problems, and were presented in [6, 7]. One of the most often used self-organizing algorithms is the Group Method of Data Handling (GMDH).

GMDH is used to analyze the errors of complex navigation systems [], and is a sorting method [11]. Taking into account that the process of constructing a self-organizing model can take quite a long time, and the aging effect of the measurements inevitably arises, which has a negative effect on the accuracy of the self-organizing algorithms. To some extent, the self-organizing algorithm with redundancy of trends (Fig. 2) can prevent this negative influence, and thus, can be used for processing navigation information with less computation and the best optimal solution [12]. The functional scheme of the self-organizing algorithm with the redundancy of trends is shown in Fig. 1. The algorithm with the redundancy of the trends operates in real time and is used for the correction scheme of INS (Fig. 1) in the autonomous mode to compensate for the errors of the navigation systems on board [13].

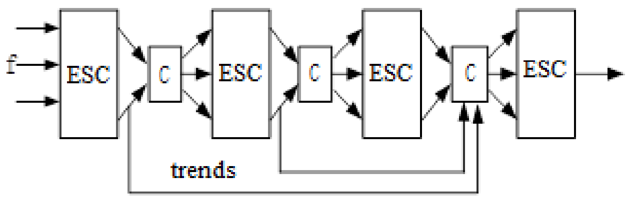

Fig. 2. Functional scheme of self-organizing algorithm with redundancy of trends

Here f – basis functions; ESC – ensemble of selection criteria; C – the way of crossing the models-applicants.

Applying various mathematical theories, the ensemble of selection criteria for the self-organization algorithm includes the following criteria [13]: the criterion for minimum of bias-consistency; the regularity criterion; the balance criterion and the simplicity criterion of the model.

Full description about the object:

$$\varphi = f_1(x_1, x_2, x_3, \dots, x_i) \tag{1}$$

It can be replaced by several specific descriptions

$$Y_1=f_1(x_1, x_2),\ y_1=f_2(x_1, x_3)\ , \dots ,\ y_m=f_1(x_{n-1}, x_n)\ ,$$

where $m=c^2{}_n$;

$$Z_1=f_1(y_1, y_2),\ z_2=f_1(y_1, y_2)\ , \dots ,\ z_p=f_1(y_{m-1}, y_m)\ ,$$

where $p=c^2{}_m$;

etc. same as above.

In the functioning process of the algorithm with the help of an ensemble of selection criteria, we can select models, which are refined on each row of selection. For constructing the model with the desired properties in an ensemble of selection criteria, it is proposed to include a criterion for the identifiability degree of nonlinear models in a class of dynamical systems. This class includes systems represented by the SDC method [14].

III. A CRITERION OF THE NONLINEAR SYSTEMS IDENTIFABILITY

Let the model of the process under investigation be described by a vector differential equation:

$$\begin{aligned}
&\frac{d}{dt}x(t) = f(t,x) + g(t,x)w(t),\ x(t_0) = x_0;\\
&y(t) = h(t,x) + v(t);\\
&f,\, g: T\times\Omega_x \to R^n,\ h: T\times\Omega_x \to R^m,\\
&(t,x) \to f(t,x),\, g(t,x),\, h(t,x),
\end{aligned} \tag{2}$$

where T − interval $[t_0, t_1]$; $x(t) \in \Omega_x$; Ω_x − area (open bound set) R^n containing the beginning; $x \in R^n$ − state of the system; $x_0 \in \Omega_x$; $w \in R^n$ − input disturbance; $y \in R^m, m \le n$ − measurement of the system; $v \in R^m$ − measuring noise; The matrices $f(t,x), g(t,x), h(t,x)$ are real and continuous.

In practice, for the convenience of information processing, a discrete form of the system is often used, in which the SDC representation of the nonlinear system (2) has the form

$$\begin{aligned}
&\mathbf{x}_{k+1} = \mathbf{\Phi}(t_k, \mathbf{x}_k)\mathbf{x}_k + \mathbf{G}(t_k, \mathbf{x}_k)\mathbf{w}_k;\\
&\mathbf{y}_{k+1} = \mathbf{H}(t_{k+1}, \mathbf{x}_{k+1})\mathbf{x}_k + \mathbf{v}_{k+1}.
\end{aligned} \tag{3}$$

It is assumed that $\mathbf{w}_k$ and $\mathbf{v}_{k+1}$ are uncorrelated white Gaussian noises, and for any j and k, v_j and w_k are uncorrelated with each other (i.e. $M\left[\mathbf{v}_j\mathbf{w}_k^T\right]=0$).

Let the equation of the object in the SDC-representation and the measurement equation have the form (6). In this case, the state vector $\mathbf{x}_{k+n}$ can be expressed by its value at the initial instant of time $\mathbf{x}_k$ in the form

$$\begin{aligned}\mathbf{x}_{k+n} &= \mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_k,\mathbf{x}_k)\mathbf{x}_k\\ &+\mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_{k+1},\mathbf{x}_{k+1})\mathbf{G}(t_k,\mathbf{x}_k)\mathbf{w}_k\\ &+\cdots+\mathbf{G}(t_{k+n-1},\mathbf{x}_{k+n-1})\mathbf{w}_{k+n-1}.\end{aligned} \quad (4)$$

Substituting the expression about $\mathbf{x}_{k+n}$ in the measurement equation $\mathbf{y}_{k+n}$, and also substituting the expression about $\mathbf{x}_k$ in the resulting equation, we obtain

$$\begin{aligned}\mathbf{y}_{k+n} &= \mathbf{H}_{k+n}\mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_k,\mathbf{x}_k)\mathbf{O}_k^+\mathbf{y}_k^*\\ &-\mathbf{H}_{k+n}\mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_k,\mathbf{x}_k)\mathbf{O}_k^+\mathbf{v}_k^*\\ &+\mathbf{H}_{k+n}\mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_{k+1},\mathbf{x}_{k+1})\mathbf{G}(t_k,\mathbf{x}_k)\mathbf{w}_k\\ &+\cdots+\mathbf{H}_{k+n}\mathbf{G}(t_{k+n-1},\mathbf{x}_{k+n-1})\mathbf{w}_{k+n-1}+\mathbf{v}_{k+n},\end{aligned} \quad (5)$$

where $\mathbf{O}_k^+=\left[\mathbf{O}_k^{*T}\mathbf{O}_k^*\right]^{-1}\mathbf{O}_k^{*T}$ − pseudoinverse matrix $\mathbf{O}_k^*$.

Introduce the notation

$$\left[\lambda_{1,k}\quad \lambda_{2,k}\quad \cdots\quad \lambda_{n,k}\right]=\mathbf{H}_{k+n}\mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_k,\mathbf{x}_k)\mathbf{O}_k^+. \quad (6)$$

$$\begin{aligned}\mathbf{v}_k^0 &= \gamma_{1,k}\mathbf{w}_k+\gamma_{2,k}\mathbf{w}_{k+1}+\cdots+\gamma_{n,k}\mathbf{w}_{k+n-1}\\ &-\lambda_{1,k}\mathbf{v}_k-\lambda_{2,k}\mathbf{v}_{k+1}-\cdots-\lambda_{n,k}\mathbf{v}_{k+n-1}+\mathbf{v}_{k+n}\\ &=\mathbf{H}_{k+n}\mathbf{\Phi}(t_{k+n-1},\mathbf{x}_{k+n-1})\cdots\mathbf{\Phi}(t_k,\mathbf{x}_k)\mathbf{O}_k^+\mathbf{v}_k^*\\ &+\cdots+\mathbf{H}_{k+n}\mathbf{G}(t_{k+n-1},\mathbf{x}_{k+n-1})\mathbf{w}_{k+n-1}+\mathbf{v}_{k+n}.\end{aligned} \quad (7)$$

Then the formulation of the problem is reduced to determining the unknown non-stationary elements of the column vector $\left[\lambda_{1,k}\quad \lambda_{2,k}\quad \cdots\lambda_{n,k}\right]$ from the newly formed measurements, i.e.

$$\begin{aligned}\lambda_{1,k} &= f_{1,k}\left(y_k,\cdots,y_{k+2n-1}\right)+v_k^{00};\\ \lambda_{2,k} &= f_{2,k}\left(y_k,\cdots,y_{k+2n-1}\right)+v_{k+1}^{00};\\ &\cdots\quad\cdots\quad\cdots\\ \lambda_{n,k} &= f_{n,k}\left(y_k,\cdots,y_{k+2n-1}\right)+v_{k+n-1}^{00},\end{aligned} \quad (8)$$

where

$$\begin{bmatrix} f_{1,k}\left(y_k,\cdots,y_{k+2n-1}\right)\\ f_{2,k}\left(y_k,\cdots,y_{k+2n-1}\right)\\ \cdots\\ f_{n,k}\left(y_k,\cdots,y_{k+2n-1}\right)\end{bmatrix}=\begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+n-1}\\ y_{k+1} & y_{k+2} & \cdots & y_{k+n}\\ \cdots & \cdots & \cdots & \cdots\\ y_{k+n-1} & y_{k+n} & \cdots & y_{k+2n-2}\end{bmatrix}^{-1}\begin{bmatrix} y_{k+n}\\ y_{k+n+1}\\ \cdots\\ y_{k+2n-1}\end{bmatrix};$$

$$\begin{bmatrix} v_k^{00}\\ v_{k+1}^{00}\\ \cdots\\ v_{k+n-1}^{00}\end{bmatrix}=\begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+n-1}\\ y_{k+1} & y_{k+2} & \cdots & y_{k+n}\\ \cdots & \cdots & \cdots & \cdots\\ y_{k+n-1} & y_{k+n} & \cdots & y_{k+2n-2}\end{bmatrix}^{-1}\begin{bmatrix} v_k^0\\ v_{k+1}^0\\ \cdots\\ v_{k+n-1}^0\end{bmatrix}.$$

Therefore, the criterion for the identifiability degree of the parameters in the model of dynamic nonstationary systems has the form:

$$DI_{Nk}^i=\frac{E\left[\left(\lambda_{i,k}\right)^2\right]R_0}{E\left[\left(y_{i,k}\right)^2\right]\hat{R}_k^i}, \quad (9)$$

where $E\left[\left(\lambda_{i,k}\right)^2\right]$ variance of the arbitrary i-th parameter vector component λ; $E\left[\left(z_{i,k}\right)^2\right]$ variance of the directly measured state vector; R_0 – variance of the initial measuring noise; $\hat{R}_k^i$ variance of the resulted measuring noise.

Thus, the formalized dependence (9) is used to determine the identifiability degree of matrix parameters $\mathbf{\Phi}(t_k,\mathbf{x}_k)$.

The variance of the original measuring noise is determined from practical considerations in accordance with the operating mode of the measuring system, or the value from the passport of the measuring device is adopted. Certain difficulties arise when calculating the given measuring noise. However, when using the adaptive estimation algorithm, the variance of the given measuring noise is calculated at each step of the algorithm operation. The quality of identification or the effectiveness of identification is determined by the maximum achievable identification accuracy and the necessary time to achieve the given identification accuracy.

IV. RESULTS OF THE EXPERIMENT

In the simulation of the test model, it is assumed that only the error in determining the speed is measured. As an example, the simulation results of INS errors in a horizontal flight with a constant speed are presented. In Fig. 3 presents the modeling results about the deviation angle of the gyro-stabilized platform for the INS relative to the accompanying trihedron of the selected coordinate system and the model of this error, constructed using the GMDH algorithms.

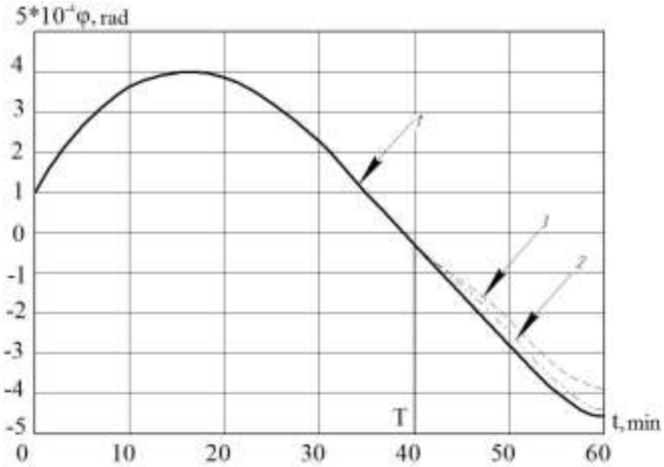

Fig. 3. Error in determining the deviation angle of the gyroscope-stabilized platform relative to the accompanying trihedron and its model, constructed by the GMDH.

In Fig. 3 the following notations are used: 1 – deviation angle of the gyroscope-stabilized platform relative to the accompanying trihedron, obtained with the help of the test model; 2 – deviation angle of the gyroscope-stabilized platform, obtained with the help of the GMDH model with the criterion of the identifiability degree; 3 – deviation angle of the gyroscope-stabilized platform, obtained with the help of the classical GMDH model.

In the time interval 0-T, a measurement sample for the GMDH is formed. Based on the measurement sample, a model is constructed. The results of calculating deviation angle of the gyroscope-stabilized platform for INS using the obtained model of the classical GMDH and GMDH with a criterion of the identifiability degree have demonstrated the advantage of the latter.

V. CONCLUSIONS

A numerical criterion for the identifiability degree of the parameters in a class of non-linear dynamic process models is developed, based on the SDC-representation. An algorithm for identification based on the GMDH method is developed, and the criterion for the identifiability degree of the parameters in the process model studied is included in the selection criteria ensemble.

REFERENCES

[1] Balonin N.A. Teoremy identifitsiruemosti. Saint-Petersburg. Publ. Politekhnika. 2010. 48 c. (In Russian).

[2] Tsibizova T.Y., P'o S., Selezneva M.S. Matematicheskoe modelirovanie dinamicheskikh sistem s ispol'zovaniem parametricheskoy identifitsiruemosti. Sovremennye naukoemkie tekhnologii. 2018. No. 1. Pp. 54-60. (In Russian).

[3] Ley W., Wittmann K., Hallmann W. Handbook of space technology. John Wiley & Sons. 2009. 908 p. 97. Noureldin A., Karamat T.B., Georgy J. Fundamentals of inertial navigation, satellite-based positioning and their integration. Springer-Verlag Berlin Heidelberg, 2013. 314 p.

[4] Groves P.D. Principles of GNSS, inertial, and multisensor integrated navigation systems. Artech House. 2013. 800 p.

[5] Ben-Israel A., Greville Thomas N.E. Generalized inverses: Theory and applications. Springer. 2003. 420 p.

[6] Shen K., Selezneva M.S., Neusypin K.A., Proletarsky A.V. Novel variable structure measurement system with intelligent components for flight vehicles. Metrology and Measurement Systems. 2017. V. 24. No. 2. Pp. 347-356. (In Russian).

[7] Neusypin K.A. [et al.] Aircraft self-organization algorithm with redundant trend. Journal of Nanjing University of Science and Technology. 2014. No. 5. Pp. 602-607.

[8] Ablin H.L. Criteria for degree of observability in a control system // Retrospective Theses and Dissertations. Paper 3188. Iowa State University. 1967. 74 p.

[9] Brown R.G. Not just observable, but how observable? National Electronics Conference Proceedings. No. 22. 1966. Pp. 409-714.

[10] Gauthier J.P., Kupka I. Deterministic observation theory and applications. Cambridge University Press, 2001. 240 p.

[11] Kai Shen', Neusypin K.A., Selezneva M.S., P'o Si Tkhu. Razrabotka chislennogo kriteriya stepeni identifitsiruemosti parametrov nelineynoy modeli atmosfernykh letatel'nnykh apparatov. (In Russian).

[12] [12] Shen Kai, Neusypin K.A., Proletarskiy A.V. Razrabotka kriteriya stepeni identifitsiruemosti parametrov modeli dinamicheskikh nestatsionarnykh sistem. Avtomatizatsiya. Sovremennye tekhnologii. 2017. V 71, No. 10. Pp. 415- 420. (In Russian).

[13] Neusypin K.A. Sovremennye sistemy i metody navedeniya, navigatsii i upravleniya letatel'nymi apparatami. Moskow. Publ. MGOU, 2009. (In Russian).

[14] Afanas'yev V.N. Upravlenie nelineynymi neopredelennymi dinamicheskimi ob"ektami. Moskow. Librokom. URSS. 2015. 224 p. (In Russian).