Mathematical Model for the Film Condensation Process

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Abstract— A mathematical model for heat transfer during film condensation is developed in a two-dimensional setting. A change in properties of the coolant as well as of the condensate film with a change in temperature is taken into account in the conservation equations. Boundary conditions for conjugation are adopted on the internal wall of the coolant flow region, on the external wall along which the condensate film flows as well as at the film-gas interface. An algorithm for solving the obtained boundary value problem is constructed. The mathematical model serves as the basis for the optimum process designing.

Keywords— mathematical model; conjugate heat and mass transfer; film condensation

I. Introduction

Heat and mass transfer processes occurring during condensation and evaporation are widely used in various fields of engineering and technology for the purpose of cooling working surfaces, as well as in various technological processes and power engineering [1, 3]. Condensation is one of the most common technological processes. Processes with condensation hold a large share in chemical engineering and many related industries [1–3]. Numerous papers have been devoted to the study of various aspects of condensation processes including review papers [1–7].

The pioneering work on heat transfer occurring during film condensation was the work by Nusselt [8], which contained a great number of simplifying assumptions [1]. As a result, the film thickness, heat transfer coefficient and amount of condensed gas (condensate flowrate) could be determined. The work produced quite satisfactory results and served as a basis for further development of the theory of film condensation calculation [1]. In that work [8], the release of heat of accumulation upon cooling the film, non-isothermicity of the cooling surface and a change in physical properties of the liquid were not taken into account [1].

Despite numerous theoretical and experimental studies of the film condensation processes in various problem formulations [1–10], conjugate heat and mass transfer during film condensation, which takes into account the nonisothermal flow of a coolant inside the condenser, the condensate film and the gas phase (vapor), has not yet been investigated sufficiently comprehensively [1].

The work is performed with financial support of the Russian Federal Property Fund, the project No. 18-43-160008

The purpose of the present paper is to develop a mathematical model for heat transfer during film condensation taking into account the thermo-hydrodynamic situation in all working areas of the heat exchanger.

II. Mathematical model

A laminar steady-state film condensation regime is considered, which can occur, for example, in heat exchangers consisting of blocks, whose heat exchange elements are made in the form of straight hollow prisms of thin metal sheets with the formation of internal slit channels for a heat carrying agent (coolant) (Fig. 1 and 2) [1].

When the condenser is in operation, the coolant flows inside the slit channel of the hollow prism. Due to cooling of the channel walls and the heat exchange through the channel walls with a vertically moving gas phase, a condensate film is formed that flows along the channel surface (Fig. 2) [1].

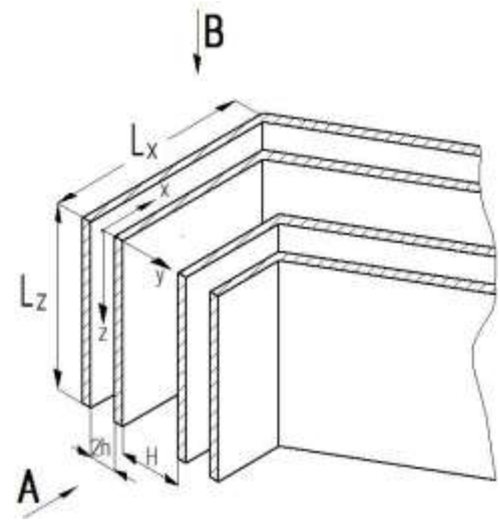

Fig. 1. Physical model of the condenser

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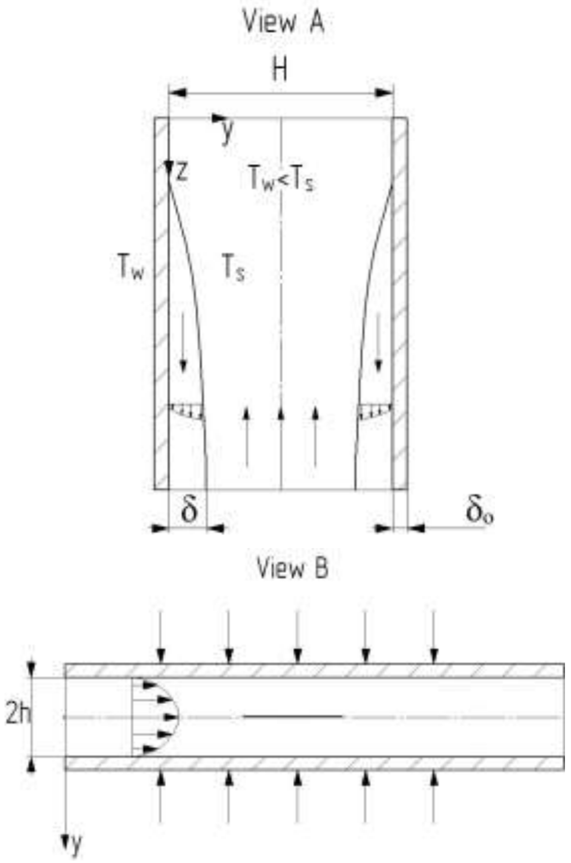

Fig. 2. Operation scheme of the heat exchanger during film condensation

In the region of the flow of a coolant and of a gas (Fig. 2), the following relationships hold true: $L_z \approx L_x \gg 2h(H>2h)$. Then the original system of equations governing the steady plane laminar flow of the coolant inside the prism as well as of the condensate film and the gas (vapor) phase in the Cartesian coordinate system x, y, z is written in the form [1]:

$$\frac{\partial V_{1x}}{\partial x} + \frac{\partial V_{1y}}{\partial y} = 0, \tag{1.1}$$

$$\rho_1 \left(V_{1x} \frac{\partial V_{1x}}{\partial x} + V_{1y} \frac{\partial V_{1x}}{\partial y} \right) = -\frac{\partial p_1}{\partial x} + F_{1x} + \frac{\partial}{\partial y}(\mu_1 \frac{\partial V_{1x}}{\partial y}), \tag{1.2}$$

$$-\frac{\partial p_1}{\partial y} + F_{1y} = 0, \tag{1.3}$$

$$-\frac{\partial p_1}{\partial z} + F_{1z} = 0, \tag{1.4}$$

$$\rho_1 c_{1p_1} (V_{1x} \frac{\partial T_1}{\partial x} + V_{1y} \frac{\partial T_1}{\partial y}) = \frac{\partial}{\partial y}(\lambda_1 \frac{\partial T_1}{\partial y}), \tag{1.5}$$

$$\frac{\partial V_{2y}}{\partial y} + \frac{\partial V_{2z}}{\partial z} = 0, \tag{2.1}$$

$$\rho_2 \left(V_{2y} \frac{\partial V_{2z}}{\partial y} + V_{2z} \frac{\partial V_{2z}}{\partial z} \right) = -\frac{\partial p_2}{\partial z} + F_{2z} + \frac{\partial}{\partial y}(\mu_2 \frac{\partial V_{2z}}{\partial y}), \tag{2.2}$$

$$-\frac{\partial p_2}{\partial x} + F_{2x} = 0, \tag{2.3}$$

$$-\frac{\partial p_2}{\partial y} + F_{2y} = 0, \tag{2.4}$$

$$\rho_2 c_{2p_2} (V_{2y} \frac{\partial T_2}{\partial y} + V_{2z} \frac{\partial T_2}{\partial z}) = \frac{\partial}{\partial y}(\lambda_2 \frac{\partial T_2}{\partial y}), \tag{2.5}$$

$$\frac{\partial V_{3y}}{\partial y} + \frac{\partial V_{3z}}{\partial z} = 0, \tag{3.1}$$

$$\rho_3 \left(V_{3y} \frac{\partial V_{3z}}{\partial y} + V_{3z} \frac{\partial V_{3z}}{\partial z} \right) = -\frac{\partial p_3}{\partial z} + F_{3z} + \frac{\partial}{\partial y}(\mu_3 \frac{\partial V_{3z}}{\partial y}), \tag{3.2}$$

$$-\frac{\partial p_3}{\partial x} + F_{3x} = 0, \tag{3.3}$$

$$-\frac{\partial p_3}{\partial y} + F_{3y} = 0, \tag{3.4}$$

$$\rho_3 c_{3p_3} (V_{3y} \frac{\partial T_3}{\partial y} + V_{3z} \frac{\partial T_3}{\partial z}) = \frac{\partial}{\partial y}(\lambda_3 \frac{\partial T_3}{\partial y}), \tag{3.5}$$

$$\lambda_4 \frac{\partial^2 T_4}{\partial y^2} = 0. \tag{3.6}$$

Here the equations (1.1)-(1.5) govern a motion of the coolant inside the hollow straight prism; equations (2.1)-(2.5) govern a flow of the falling film; equations (3.1)-(3.5) govern a motion of the gas (vapor); equation (3.6) governs heat transfer through the wall. $F_{1y} = F_{1x} = 0$, $F_{1z} = \rho_1 g$; $F_{2y} = F_{2x} = 0$, $F_{2z} = \rho_2 g$; $F_{3x} = F_{3z} = 0$, $F_{3z} = \rho_3 g$; $\mu_i = \mu_i(T_i), i = \overline{1,3}$ (only a change in viscosity with temperature is taken into account); indices 1, 2 and 3 refer to the coolant, liquid film and gas phase, respectively [1].

The system of equations (1.1)–(3.6) is solved with appropriate boundary conditions of the form [1]:

$$\frac{\partial V_{1x}}{\partial y} = 0,\ \frac{\partial T_1}{\partial y} = 0,\ V_{1y} = 0 \text{ at } y = 0; \tag{4.1}$$

$$V_{1x} = V_{1y} = 0,\ \lambda_1 \frac{\partial T_1}{\partial y} = \lambda_4 \frac{\partial T_4}{\partial y} \text{ at } y = \pm h; \tag{4.2}$$

$$T_4 = T_2,\ \lambda_4 \frac{\partial T_4}{\partial y} = \lambda_2 \frac{\partial T_2}{\partial y},\ V_{2y} = V_{2z} = 0,$$

$$-\lambda_2 \frac{\partial T_2}{\partial y} = \alpha(T_{10} - T_{cm}) \approx \alpha(T_{1cp} - T_{4cp})$$

$$\text{at } y = h + \delta_0; \tag{4.3}$$

$$\mu_2 \frac{\partial V_{2z}}{\partial y} = \mu_3 \frac{\partial V_{3z}}{\partial y} + \frac{d\sigma_2}{\partial z},\ V_{2z} = V_{3z},\ T_2 = T_3 = T_{2S},$$

$$p_2 = p_3 + \sigma_2 \frac{d^2\delta}{dz^2} \text{ at } y = h + \delta_0 + \delta; \tag{4.4}$$

$$\frac{\partial T_3}{\partial y}=0\,,\ \frac{\partial V_{3z}}{\partial y}=0\,,\ V_{3y}=0$$

at $y=\delta_1=h+\delta_0+\dfrac{H}{2}$; (4.5)

$V_{1x}=V_{1вх}$, $T_1=T_{1вх}$, $p_1=p_{1вх}$ at $x=x_{вх}$; (5.1)

$V_{2z}=V_{2zвх}$, $T_2=T_{2вх}$ at $z=z_{вх}=z_0$; (5.2)

$V_{3z}=V_{3zвх}$, $T_3=T_{3вх}$, $V_{3y}=0$, $p_3=p_{3вх}$

at $z=L_z$, (5.3)

where α is heat transfer coefficient (taking into account thermal resistance of the wall); σ_2 is surface tension coefficient of the film; T_{10} is temperature of the cooling medium (coolant); T_c, T_{2S} are wall temperature and saturation temperature, respectively,

$$T_{10}\approx T_{1cp}=\frac{1}{h}\int_0^h T_1 dy=B_1\frac{h^2}{6}+C_1\,,$$

$$T_{cm}=T_{4cp}=\frac{1}{\delta_0}\int_h^{h+\delta_0} T_4 dy=C_4\left(h+\frac{\delta_0}{2}\right)+C_5\,. \tag{5.4}$$

Solution of the boundary value problem (1.1) – (1.5), (2.1) – (2.5), (3.1) – (3.6), (4.1) – (4.5) and (5.1) – (5.3) represents a complex problem. In this regard, the solution to the problem is considered for a particular case only.

III. Heat transfer under slow flow conditions

In this case, the problem is substantially simplified and, in the momentum conservation equations, one can neglect the left-hand sides (inertia terms). Such flow regimes occur in many cases of film condensation and the conservation equations (1.1)–(1.5), (2.1)–(2.5), (3.1)–(3.6) can be integrated (solved) using the boundary conditions (4.1)-(4.5) and (5.1)–(5.3) [1].

The algorithm for solving the equations (1.1)–(1.5), (2.1)–(2.5), (3.1)–(3.6) is as follows:

1. It follows from the equations (1.3)-(1.4), (2.3)–(2.4) and (3.3)–(3.4), respectively, that

$$p_1(x,z)=\rho_1 gz+p_1(x)\,,\ p_2=p_2(z)\,,\ p_3=p_3(z)\,, \tag{6}$$

where $p_1(x), p_2(z), p_3(z)$ are yet unknown functions, which will be defined below.

2. For integrating the energy conservation equations (1.5), (2.5) and (3.5), the approximate Slezkin method is used, according to which the following functions are introduced [1, 11]:

$$B_1(x)=\frac{\rho_1 c_{1p_1}}{\lambda_1\cdot h}\int_0^h (V_{1x}\frac{\partial T_1}{\partial x}+V_{1y}\frac{\partial T_1}{\partial y})dy\,, \tag{7.1}$$

$$B_2(z)=\frac{\rho_2 c_{2p_2}}{\lambda_2\cdot\delta}\int_{h+\delta_0}^{h+\delta_0+\delta} (V_{2y}\frac{\partial T_2}{\partial y}+V_{2z}\frac{\partial T_2}{\partial z})dy\,, \tag{7.2}$$

$$B_3(z)=\frac{\rho_3 c_{3p_3}}{\lambda_3\left(\dfrac{H}{2}-\delta\right)}\int_{h+\delta_0+\delta}^{h+\delta_0+\frac{H}{2}} (V_{3y}\frac{\partial T_3}{\partial y}+V_{3z}\frac{\partial T_3}{\partial z})dy\,, \tag{7.3}$$

where $B_1(x), B_2(z), B_3(z)$ and $\delta(z)$ are yet unknown functions.

Then after integrating the equations:

$B_1(x)=\dfrac{\partial^2 T_1^2}{\partial y^2}$, $B_2(z)=\dfrac{\partial^2 T_2}{\partial y^2}$, $B_3(z)=\dfrac{\partial^2 T_3}{\partial y^2}$ and $\lambda_4\dfrac{\partial^2 T_4}{\partial y^2}=0$ with the appropriate boundary conditions, one obtains the following expressions for the temperatures T_1, T_2 and T_3, T_4:

$$T_1=B_1(x)\frac{y^2}{2}+C_1\,,\ T_2=B_2(z)\frac{y^2}{2}+C_2\cdot y+\tilde{C}_2\,,$$

$$T_3=B_3(z)(\frac{y^2}{2}-\delta_1(z)\cdot y)+C_3\,,\ T_4=C_4\cdot y+C_5\,,$$

$$\delta_1(z)=h+\delta_0+\frac{H}{2}\,, \tag{8}$$

where $C_1, C_2, \tilde{C}_2, C_3, C_4, C_5$ are integration constants, which are determined via the boundary conditions (4.1)–(4.5).

3. From the equations (1.1)–(1.2), (2.1)–(2.2) and (3.1)–(3.2), fields of the velocities $V_{1x}, V_{1y}, V_{2y}, V_{2z}, V_{3y}, V_{3z}$ are determined for known dependencies of the temperatures T_1, T_2, T_3 (8). Here polynomial dependences of viscosities $\mu_i(T_i)$ on temperature are used as follows:

$$\frac{\mu_i(T_{i0})}{\mu_i(T_i)}=1+\sum_{k=1}^{n_i}\alpha_{ik}T_{ik}^{\,k}\,,\ (T_i=T_i-T_{i0}) \tag{9.1}$$

3.1. For example, from the equation (1.2) one can determine V_{1x} with the account of the dependence (9.1):

$$V_{1x}(x,y)=\int_h^y \frac{p_1'(x)\cdot y}{\rho_1\cdot\mu_1(T_1)}dy=\tilde{p}_1'(x)\left\{\frac{\left(y^2-h^2\right)}{2}+\right.$$

$$+a_{11}\left[B_1\frac{\left(y^4-h^4\right)}{8}+\left(C_1-T_{10}\right)\cdot\frac{\left(y^2-h^2\right)}{2}\right]+$$

$$+a_{12}\left[B_1^2\frac{\left(y^6-h^6\right)}{24}+2B_1\left(C_1-T_{10}\right)\cdot\frac{\left(y^4-h^4\right)}{8}+\right.$$

$$+\left(C_1-T_{10}\right)^2\cdot\frac{\left(y^2-h^2\right)}{2}\Bigg]\Bigg\},\qquad(9.2)$$

where $\tilde{p}_1'(x)=\dfrac{p_1'(x)}{\rho_1\mu_1(T_{10})}$, $n=2$ in (9.1).

3.2. After $V_{1x}(x,y)$ is determined from equation (1.1) using the boundary condition $V_{1y}(x,h)=0$, one can obtain the transverse velocity $V_{1y}(x,y)$ in the form:

$$V_{1y}(x,y)=\int_h^y[\int_y^h[\frac{p_1'(x)\cdot y}{\rho_1\mu_1(T_1)}]_x' dy]dy=$$
$$=\tilde{p}_1''(x)\left[W_{11}(x,y,z)-W_{11}(x,h,z)\right]+\qquad(10.1)$$
$$+\tilde{p}_1'(x)\left[W_{12}(x,h,z)-W_{12}(x,y,z)\right],$$

where

$$W_{11}(x,y,z)=\left(\frac{h^2y}{2}-\frac{y^3}{6}\right)+a_{11}\left[B_1\left(\frac{h^4y}{8}-\frac{y^5}{40}\right)+\right.$$
$$\left.+\left(C_1-T_{10}\right)\cdot\left(\frac{y^3}{6}-\frac{h^2y}{2}\right)\right]+a_{12}\left[B_1^2\left(\frac{y^7}{168}-\frac{h^6\cdot y}{24}\right)+\right.\qquad,\quad(10.2)$$
$$\left.+2B_1\left(C_1-T_{10}\right)\cdot\left(\frac{y^5}{40}-\frac{h^4\cdot y}{8}\right)+\left(C_1-T_{10}\right)^2\cdot\left(\frac{y^3}{6}-\frac{h^2\cdot y}{2}\right)\right]$$

$$W_{12}(x,y,z)=a_{11}\left[B_1'\left(\frac{h^4y}{8}-\frac{y^5}{40}\right)+C_1'\cdot\left(\frac{y^3}{6}-\frac{h^2y}{2}\right)\right]+$$
$$+a_{12}\left[2B_1B_1'\left(\frac{y^7}{168}-\frac{h^6\cdot y}{24}\right)+2B_1'\left(C_1-T_{10}\right)\left(\frac{y^5}{40}-\frac{h^4\cdot y}{8}\right)+\right.\quad(10.3)$$
$$\left.+2B_1C_1'\left(\frac{y^5}{40}-\frac{h^4\cdot y}{8}\right)+2C_1'\left(C_1-T_{10}\right)\left(\frac{y^3}{6}-\frac{h^2\cdot y}{2}\right)\right],$$

3.3. Similarly, the velocity fields $V_{2y}, V_{2z}, V_{3y}, V_{3z}$ are determined from equations (2.1)–(2.2) and (3.1)–(3.2) using the appropriate boundary conditions.

4. In all of the obtained solutions for the velocity and temperature fields, there are still unknown parameters $B_1(x), B_2(z), B_3(z)$ and $\delta(z)$. For their determination, the dependencies (7.1)–(7.3) and a condition for a change in the condensate flowrate Q along the length of the condenser z are used, taking into account the mass flux during condensation $Q_г$ and a condition of energy balance in the condensate film:

$$\frac{dQ}{dz}=Q_г,\ Q=\rho_2\int_{h+\delta_0}^{h+\delta_0+\delta}V_{2z}dy,\qquad(11.1)$$

where $Q_г$ is determined from the relationship:

$$q=-\lambda_2\left.\frac{\partial T_2}{\partial y}\right|_{h+\delta_0}=Q_г c_{p_2}\left(T_3-T_{2s}\right)+h_2Q_г,\qquad(11.2)$$

$T_{2s}, T_w=T_c$ are saturation temperature and wall temperature, respectively.

This system of four differential equations is solved by using numerical methods at each step of the algorithm.

5. Pressure differentials in these expressions are determined from the conditions of mass conservation for the coolant, condensate and gas phase.

IV. Conclusion

The mathematical model in concrete particular agrees with the previously known solutions given in [1–3, 8]. For example, if one does not consider the flow of the cooling medium inside the hollow prism and takes its temperature $T_1=T_0$ and the wall temperature $T_4=T_c$ and neglects the convective heat transfer in equation (2.5) ($B_2(z)=0$) as well as friction at the gas-liquid interphase under the boundary conditions (4.3)–(4.4), then one can see that our solution completely coincides with the solution presented in [8].

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