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Efficient HRTF Representation Using Compact Mode HRTFs

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ABSTRACT

This paper proposes a new Head-Related Transfer Function (HRTF) representation, which utilises a rotation to reorder the energy of the HRTF in the spherical harmonic domain into a minimal number of spherical harmonics and then discards those with low energy content. This new representation is titled the compact mode HRTF. The rigid sphere and Neumann KU100 HRTFs are considered to a truncation order of $N = 30$ which requires 961 spherical harmonics. The KU100 compact mode HRTF using only 178 spherical harmonics is shown to match the full KU100 HRTF to within just a 5% error bound. Thus the compact mode scheme is proposed as an efficient method for representing, transmitting and utilising HRTFs to high orders.

1 Introduction

A range of different cues are utilised by a listener to localize an auditory event. These are broadly categorized into binaural cues that rely on differences between the signals at the listener's two ears, such as Interaural Time Differences (ITDs) and Interaural Level Differences (ILDs), as well as monoaural cues which rely on information native to the signal at one ear only [1]. Localization cues are the product of physical processes caused by the presence of the listener, such as the shadowing of sound due to the listener's head. The filtering introduced by all of these physical processes can be encoded in an acoustic transfer function, referred to as the Head-Related Transfer Function (HRTF). HRTFs by their very nature are personalized to an individual, as they depend on many factors such as the individual's own head shape, size, ear positions and pinna shape. However, there exists a number of analytical HRTF models such as the rigid sphere [2], which acts as a generic HRTF.

HRTFs are often measured in an anechoic chamber, generally requiring the measurement of a source positioned at several hundred or even thousands points around the listener (for an individualised measurement) or binaural mannequin microphone (for a generic HRTF measurement) [3]. This spatial representation of the HRTF may be transformed into the spherical harmonic domain, which has found use in many recent studies [4, 5] and provides an advantageous form of the HRTF for use in applications such as rendering [6, 7]. However, to achieve this spherical harmonic transform many measurement points are required initially. Furthermore, the spherical harmonic expansion of the HRTF must be truncated to a finite order N, requiring $(N+1)^2$ spherical harmonics. Therefore for high orders, which are required for accurate high frequency reproduction [8], the number of spherical harmonics required can become very large. This leads to increased costs with respect to acquisition, storage, transmission and rendering of the HRTF.

Reducing the number of spherical harmonics required to represent a HRTF, whilst retaining a high degree of the required information in the HRTF, would lead to a more compact HRTF representation. Such an idea has been proposed by removing the onset delay for each measurement direction resulting in an ITD equalized HRTF [9, 10, 11]. This approach refocuses some of the information of the HRTF into lower orders, however requires the ITD to be reapplied at the rendering stage and thus always requires knowledge of the position of the desired sound source. This concept was progressed further in [12] utilising a frequency-dependent delay, such that the onset delay was removed at high frequencies only whilst retaining a low frequency ITD. It has also been suggested to remove other directional and frequency-dependent information from a measured HRTF set, by equalization based on the rigid sphere (or any other desired) HRTF data set [13]. Here the remaining information also forms a lower order, more compact HRTF representation, and the removed spatial components may be restored at a later stage through filtering based on the same equalization data set.

This paper introduces a new compact representation of a HRTF, where after rotating the HRTF to reorder and focus the energy into a few select spherical harmonics, low energy spherical harmonics are discarded till the altered HRTF reproduces the original to within a given error bound. This reduced form is defined as the compact mode HRTF representation, combining the advantages of requiring less spherical harmonics however still representing the original HRTF to within a given user defined error.

The layout of the paper is as follows. First, the mathematics preliminary's are presented. Next, the spherical harmonic transform of a rigid sphere HRTF and a Neumann KU100 HRTF to a truncation order of $N = 30$ is computed to consider the distribution of the energy across the spherical harmonics. Next, a rotation of the HRTFs is exploited to reorder the energy distribution into a select few spherical harmonics. The low energy spherical harmonics for each HRTF are then progressively discarded till a set error bound is reached to create the compact mode HRTF representation. Finally, the performance of the compact mode HRTF is compared to that of the original HRTF in light of its compactness with respect to the number of spherical harmonics required.

2 Theory

2.1 The Spherical Harmonic Transform

Consider a 3D spherical coordinate system, such that a position vector $\mathbf{r} = r[\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta)]^T$ where $r \in [0,\infty]$ is the radial distance from the origin, $\phi \in [0,2\pi)$ is the azimuth and $\theta \in [0,\pi]$ is the inclination. The spatial radial quantity kr is adopted, where the wavenumber $k = \omega/c$ with ω the angular frequency and c the speed of sound. A function, $p(kr,\theta,\phi)$ may be expressed as a linear weighted summation of spherical harmonic function. This defines the Spherical Harmonic Transform (SHT) [14]

$$p_n^m = \int_\Omega p(kr,\theta,\phi) Y_n^m(\theta,\phi)^* d\Omega =: SHT\{p(kr,\theta,\phi)\} \tag{1}$$

and the Inverse Spherical Harmonic Transform ($iSHT$) [14]

$$p(kr,\theta,\phi) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} p_n^m Y_n^m(\theta,\phi) =: iSHT\{p_n^m\}. \tag{2}$$

Here, the operator * denotes complex conjugation and $\int_\Omega d\Omega = \int_0^{2\pi}\int_0^{\pi} \sin(\theta) d\theta d\phi$ is integration over the unit 2-sphere, S^2. This is a form of Fourier transform to the spherical harmonic domain, where the coefficient p_n^m gives the weighting for the order n and degree m spherical harmonic, given by $Y_n^m(\theta,\phi)$. The complex spherical harmonics are defined by [15]

$$Y_n^m(\theta,\phi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}} P_n^m(\cos\theta) e^{-m\phi} \tag{3}$$

where $P_n^m(\cos\theta)$ is the associated Legendre polynomial. The real part of the first few spherical harmonic functions for positive m only is shown in Fig. 1. The spherical harmonics form an orthonormal basis over S^2. The orthogonality condition for the spherical harmonics is given by [15]

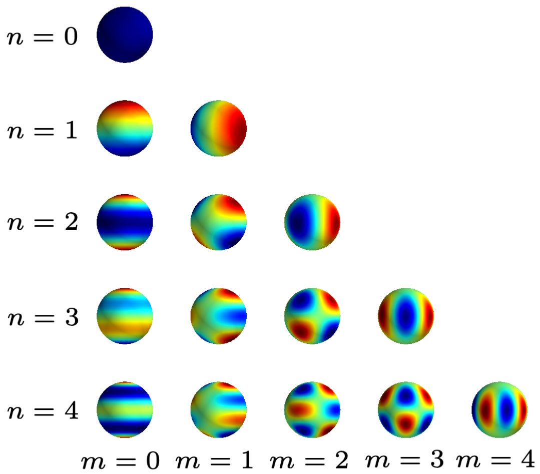


Fig. 1: Real part of a set of spherical harmonics mapped to the surface of a sphere. The colour map represents positive/negative amplitude.

$$\int_{\Omega} Y_n^m(\theta,\phi) Y_{n'}^{m'}(\theta,\phi)^* d\Omega = \delta_{nn'}\delta_{mm'} \tag{4}$$

where δ_{pq} is the Kronecker delta function defined as

$$\delta_{pq} = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}. \tag{5}$$

The expansion of a function in spherical harmonics requires an infinite number of terms. In practise this must be truncated to a finite order N, where the error between the function and truncated representation is the combined contribution of all spherical harmonic terms greater than N.

2.2 Rotations In Spherical Harmonics

Once represented using spherical harmonics, the soundfield may be altered and manipulated. One key manipulation is rotation of a soundfield. A rotation may be described by a set of Euler angles (α,β,γ) [16]. A rotation in spherical harmonics is given by [17]

$$Y_n^m(\theta,\phi) = \sum_{m'=-n}^{n} D^n_{m'm}(\alpha,\beta,\gamma) Y_n^{m'}(\theta,\phi). \tag{6}$$

Here $D^n_{m'm}(\alpha,\beta,\gamma)$ is the Wigner-D function, defined using the Wigner-d function $d^n_{m'm}(\beta)$ [17]

$$D^n_{m'm}(\alpha,\beta,\gamma) = e^{-jm'\alpha} d^n_{m'm}(\beta) e^{-jm\gamma} \tag{7}$$

The rotation of a spherical harmonic is order constant, that is a spherical harmonic of order n and degree m may be written as a weighted summation of all the spherical harmonics of the same order n. This means a rotation may be applied even if the order is truncated to a finite number. The rotation also preserves energy within a given order. It may be shown that the rotation can be applied directly to the spherical harmonic coefficients of a function, such that the coefficients of the rotated field, $\tilde{p}_m^n$ are given by [17]

$$\tilde{p}_n^{m'} = \sum_{m=-n}^{n} p_n^m D^n_{m'm}(\alpha,\beta,\gamma) \tag{8}$$

hence a rotation in spherical harmonics is order constant and may be applied directly to the spherical harmonic coefficients.

2.3 HRTFs In Spherical Harmonics

Any general HRTF, $HRTF(ka,\theta,\phi)$ with $r=a$ the radius of the head, may be expressed in spherical harmonics

$$HRTF(ka,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} H_n^m(\omega) Y_n^m(\theta,\phi) \tag{9}$$

however if there is no analytical expression for the HRTF the $H_n^m(\omega)$ frequency coefficients must be found numerically.

The rigid sphere is a useful approximation of a real HRTF, where the head is modelled as a symmetric rigid sphere with the ears situated at two diametrically opposed positions on either side of the head. As the pinna are not modelled, the rigid sphere is mostly valid below $4,000$ Hz however remains useful as a simple analytical HRTF model. For an incident plane wave arriving from direction $(\theta_{inc},\phi_{inc})$, the pressure at the surface of the sphere ($kr = ka$) is [14]

$$p(ka,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} 4\pi j^n R_n(ka) Y_n^m(\theta,\phi) Y_n^m(\theta_{inc},\phi_{inc})^* \tag{10}$$

with j the imaginary unit. $R_n(ka)$ is the n-th order radial filter defined as

$$R_n(ka) = \frac{j}{(ka)^2} \frac{1}{h_n^{(2)\prime}(ka)} \tag{11}$$

where $h_n^{(2)\prime}(x)$ is the derivative of the Hankel function of the second kind. Hence the rigid sphere HRTF, $HRTF_{RS}$, may be expressed through the use of an inverse spherical harmonic transform such that

$$HRTF_{RS}(ka,\theta,\phi) = \sum_{n=0}^{\infty}\sum_{m=-n}^{n} H_n^m(\omega)Y_n^m(\theta,\phi) \tag{12}$$

and as there is an analytical expression for the HRTF, then $H_n^m(\omega) = 4\pi j^n R_n(ka)Y_n^m(\theta_{inc},\phi_{inc})$.

3 Spherical Harmonic Energy Reordering

As defined in Eqn. 8, a rotation in spherical harmonics may be applied directly to the spherical harmonic coefficients of the function. This operation is order constant and preserves the energy of the function. A rotation may be considered as re-expressing the function in a different basis of spherical harmonics, defined as before in Eqn. 3, however just in a rotated frame of reference. Hence, it is possible to reorder the energy of the spherical harmonic coefficients of a signal by applying a rotation. In the case of a HRTF, if it is possible to reorder the majority of the energy of the HRTF into a few selected spherical harmonics, then this can lead to a more efficient representation of the HRTF using less spherical harmonics - the compact mode representation.

To consider the energy in each spherical harmonic the rigid sphere HRTF and a Neumann KU100 HRTF were considered. The rigid sphere was calculated as per Eqn. 12 with a head radius of $a = 0.10$ m as this gives a similar fit to the characteristics of a Neumann KU100. For the KU100 the far-field 2702 point spherical Lebedev grid measurements of the Neumann KU100 by TH Köln was used [18]. To read the KU100 SOFA files and perform the spherical harmonic transforms, the python SOFIA Soundfield-Analysis Toolbox was used [19]. For both HRTFs, the truncation order was set to $N = 30$ corresponding to an aliasing frequency as per the standard rule $N = kr$ [8] to $\approx 18,200$ Hz. This truncation order was chosen as this spatial aliasing frequency is high enough to not create issues in the recreation of full-range audio signals, however resulted in a considerable number of spherical harmonics under consideration, totalling $(30+1)^2 = 961$ coefficients.

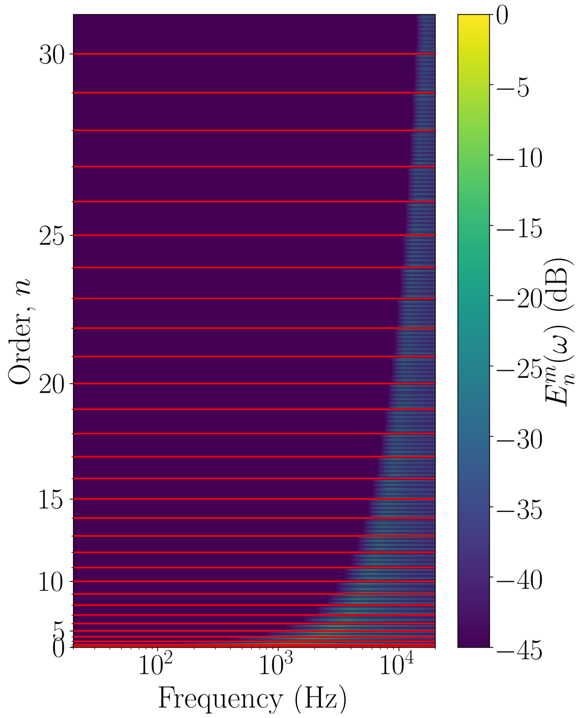
The spherical harmonic transform of the KU100 HRTF was performed to find the coefficients of its spherical harmonic decomposition. For the rigid sphere HRTF, Eqn. 12 was used to calculate its spherical harmonic coefficients up to the truncation order. Next, the normalised modal energy of each spherical harmonic coefficient as a function of frequency, $E_n^m(\omega)$, was calculated as

$$E_n^m(\omega) = \frac{|H_n^m(\omega)|^2}{\sum_{n'=0}^{\infty}\sum_{m'=-n'}^{n'} |H_{n'}^{m'}(\omega)|^2}. \tag{13}$$

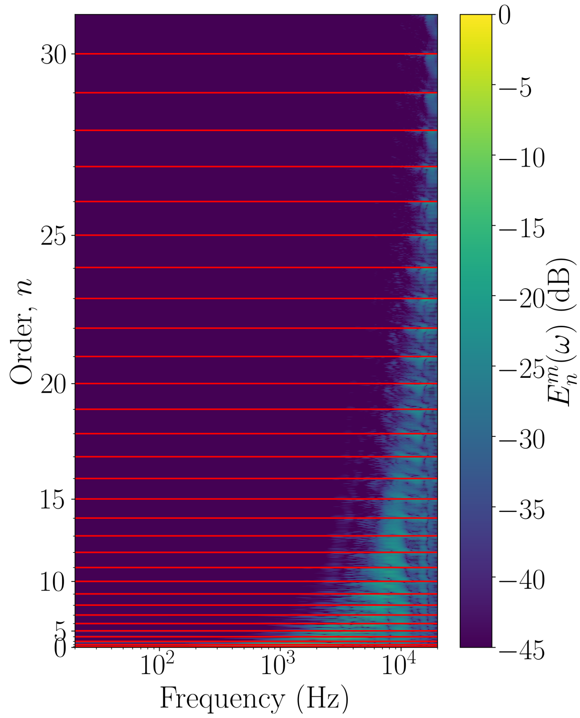
For this and future calculations, the infinite limit in the denominator was approximated by a truncation order of 40, which gives an aliasing frequency as per $N = kr$ of $\approx 24,000$ Hz.

The results are shown in Fig. 2a and 2b for the left ear of each HRTF. Here the normalised energy across all frequencies is shown, where the spherical harmonic coefficients are ordered in increasing order n and m, such that for each given order $m = -n$ is shown first, and $m = n$ shown last. As $m \in [-n,n]$ there are $(2n+1)$ spherical harmonic coefficients for each order. For this first head orientation, the ears are aligned along the y axis. It is clear that for each order and for both HRTFs the energy is spread across all degrees of that given order. Furthermore, as per the $N = kr$ rule for low frequencies only low orders are required and contain any significant energy, whilst as the frequency increase higher orders are significant. Due to the high truncation order chosen, the effects of spatial aliasing above $18,200$ Hz are just slightly noticeable.

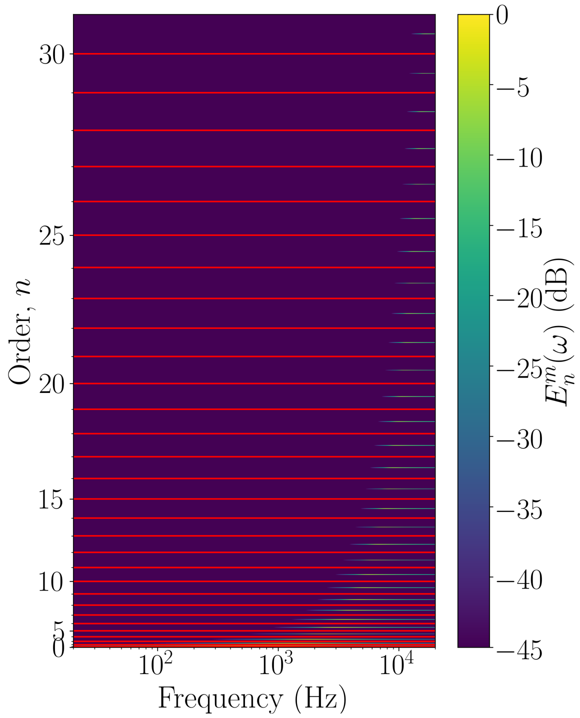
Next, a rotation will be applied that takes advantage of the axisymmetric nature of the rigid sphere. Because of this property, if the rigid sphere HRTF is rotated such that the ears are aligned along the z axis, only the $m = 0$ degree spherical harmonics are required to fully reproduce the HRTF. In doing so, the rotation is effectively reordering the energy into the $m = 0$ spherical harmonics only. Hence, a rotation is applied to the spherical harmonic coefficients as in Eqn. 8 where to rotate the



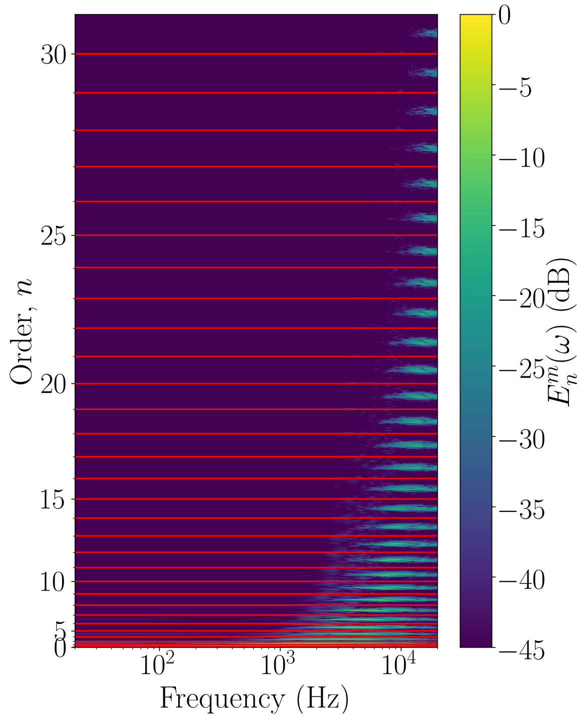
(a) Rigid sphere HRTF.



(b) KU100 HRTF.



(c) Rigid sphere HRTF after rotation.



(d) KU100 HRTF after rotation.

Fig. 2: Normalised energy per spherical harmonic coefficient for the left ear of each HRTF. The coefficients are ordered in increasing order n and degree $m \in [-n, n]$ and the red lines indicate the start of each new order.

ears from being aligned along the y axis to the z axis, the Euler angles are set to $(\alpha = 0, \beta = \pi/2, \gamma = \pi/2)$. The result of the normalised energy per coefficient after the rotation is shown in Fig. 2c. It is clear that now only the $m = 0$ spherical harmonics contain any energy, where on the figure the $m = 0$ are positioned exactly between each red line indicating a change in order. Therefore, a rigid sphere HRTF can be represented in an efficient formulation if the correct rotation is applied to align the ears with the z axis, such that all the energy is formulated in the $m = 0$ spherical harmonics and therefore these are all that is required to represent the HRTF.

The rigid sphere is only an approximation of a real HRTF, and not valid above 4000 Hz where the pinna effects are important [3]. However, across the low to mid frequencies it can simulate a real HRTF satisfactorily. Therefore, it is expected that with the KU100 HRTF, a similar effect might be observed however not at all frequencies and not to the same extent - this is due to the fact that the phenomenon occurs in such a specific manner with the rigid sphere due to its perfect symmetry. Consequently, the energy in the KU100 HRTF spherical coefficients after the same rotation applied as to the rigid sphere, such that the ears align along the z axis, is shown in Fig. 2d. Interestingly, there is also a strong reordering of the energy in the HRTF to focus around the $m = 0$ modes. Whilst modes outside of $m = 0$ are still required as they contribute significantly, it appears that for each given order the extreme degree modes where $|m| \to n$ are less important as they contain considerably less energy. Furthermore, this effect is observed across all orders. Thus, by applying this rotation the energy of the KU100 HRTF has been reordered to focus around spherical harmonic coefficients with m close to 0. In this sense, a more efficient representation of the KU100 HRTF has been developed, such that less spherical harmonic coefficients are required to represent the HRTF.

The normalised total energy across all frequencies for each given spherical harmonic coefficient, $E^m_{n,tot}$, is defined as

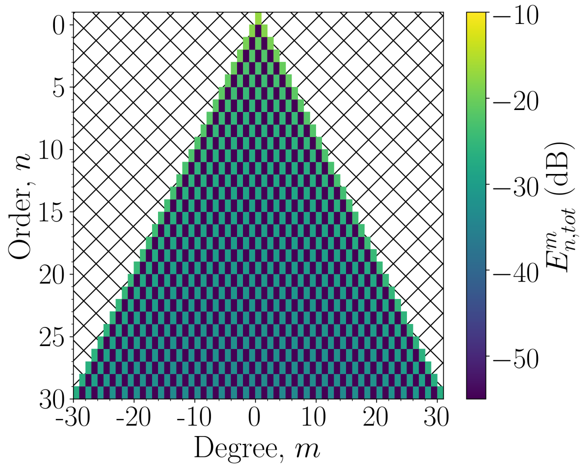
$$E^m_{n,tot} = \frac{\int |H^m_n(\omega)|^2 d\omega}{\int \sum_{n'=0}^{\infty} \sum_{m'=-n'}^{n'} |H^{m'}_{n'}(\omega)|^2 d\omega}. \qquad (14)$$

Fig. 3 shows the normalised total energy for the left ear of the rigid sphere and KU100 before and after the rotation. It is clear that after the rotation the energy for both representations is focused about the $m = 0$ modes, with the rigid sphere fully defined using just $m = 0$ modes. For the KU100 it is clearly seen that the outer orders as $|m| \to n$ are in some instances $30 - 40$ dB down from the main peaks centred around $m = 0$. Finally, it may be observed that in general lower orders contain more energy than the higher orders. For the KU100 HRTF the focusing is stronger at low orders, relating to how the rigid sphere is a good HRTF approximation at low to mid frequencies. However with the KU100 as the order increases the energy is spread considerably more, for example past $n = 7$ corresponding to $\approx 4,250$ Hz as per $N = kr$ further enforcing the accepted validity region of the rigid sphere. However, even here there is still a significant reordering of the energy to modes with m close to 0.

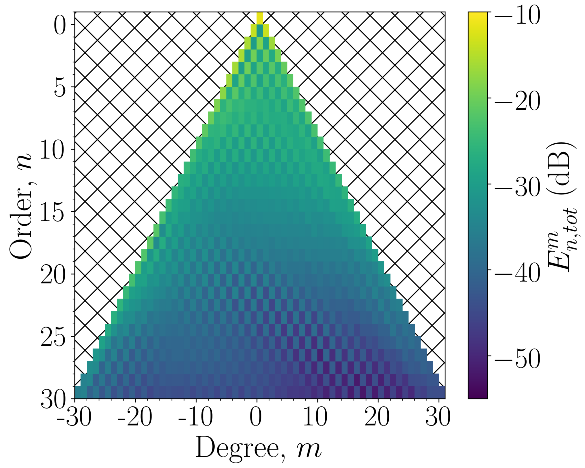
Following this reordering of the energy of the HRTF, it is suggested that by selectively removing the low energy modes an accurate, but more compact, representation of the HRTF can be achieved. This is defined as the compact mode HRTF. Whilst for the special case of the rigid sphere its compact mode representation only requires the $m = 0$ modes to fully recreate it, for any real (or simply not axisymmetric) HRTF this is not the case. However, it may still be represented by a reduced number of spherical harmonic coefficients leading to its compact mode form.

4 The Compact Mode HRTF Representation

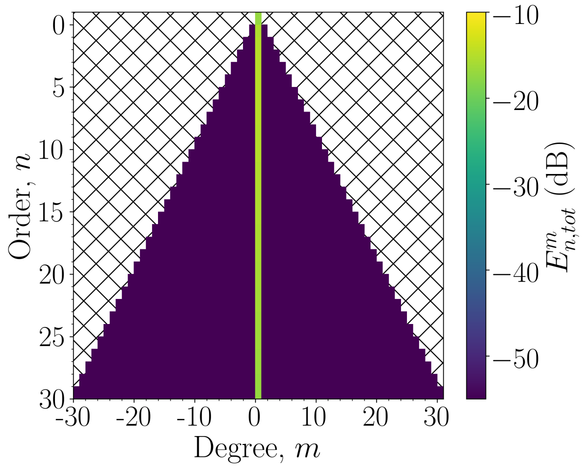
Having ascertained that the correct rotation reorders the energy into a select few spherical harmonics, the question remains as to exactly which and how many spherical harmonic should be discarded to create the compact mode HRTF. For a real HRTF such as the KU100, increasing the number of discarded spherical harmonics will increase the compactness and efficiency of the compact mode representation. However, it will come at the cost of reducing the accuracy of the reproduced binaural signals of the soundfield, as unlike the rigid sphere HRTF there is still energy outside of the $m = 0$ modes. Thus, an error bound is set for the compact mode HRTF, such that the lowest energy spherical harmonic coefficients are progressively discarded till



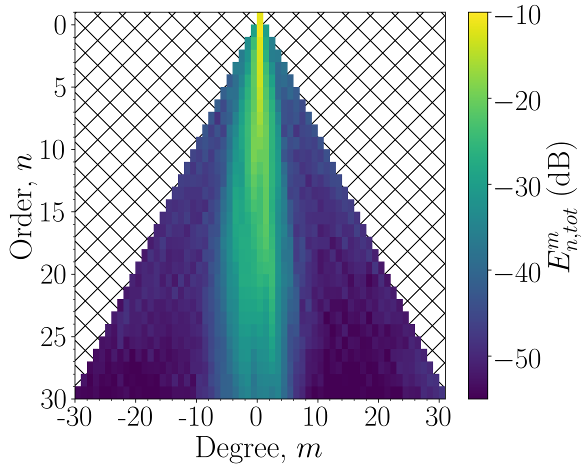
(a) Rigid sphere HRTF.



(b) KU100 HRTF.



(c) Rigid sphere HRTF after rotation.



(d) KU100 HRTF after rotation.

Fig. 3: Total normalised energy per spherical harmonic coefficient across all frequencies for the left ear HRTFs. The coefficients are ordered in sets of order n with degree $m \in [-n, n]$.

the minimum number required to reproduce the HRTF within the given error bound are left.

The error is defined as the difference between the original HRTF and the compact mode HRTF in the spatial domain, which is equivalent to all the spherical harmonic coefficients that are removed. The set α is defined as the set of spherical harmonics which are not discarded. Thus the error, ε, is

$$\begin{aligned}\varepsilon &= \frac{\int\int_{\Omega} |H(ka,\theta,\phi) - \tilde{H}(ka,\theta,\phi)|^2 d\Omega\, d\omega}{\int\int_{\Omega} |H(ka,\theta,\phi)|^2 d\Omega\, d\omega} \\ &= \frac{\displaystyle\int \sum_{n=0}^{\infty} \sum_{\substack{m=-n \\ n,m\notin\alpha}}^{n} |H_n^m(\omega)|^2 d\omega}{\displaystyle\int \sum_{n'=0}^{\infty} \sum_{m'=-n'}^{n'} |H_{n'}^{m'}(\omega)|^2 d\omega}\end{aligned} \tag{15}$$

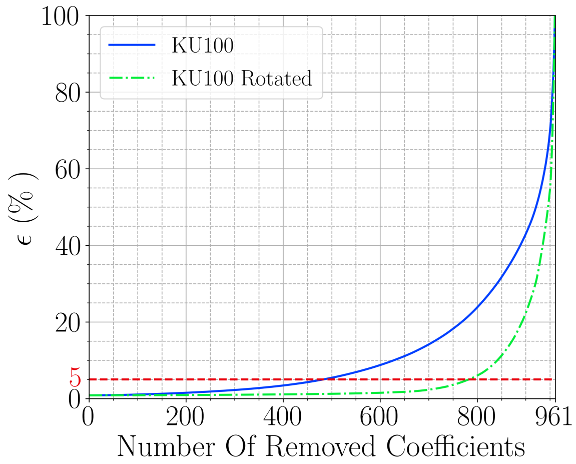


Fig. 4: Cumulative error for the left ear of the KU100 as spherical harmonic coefficients are removed.

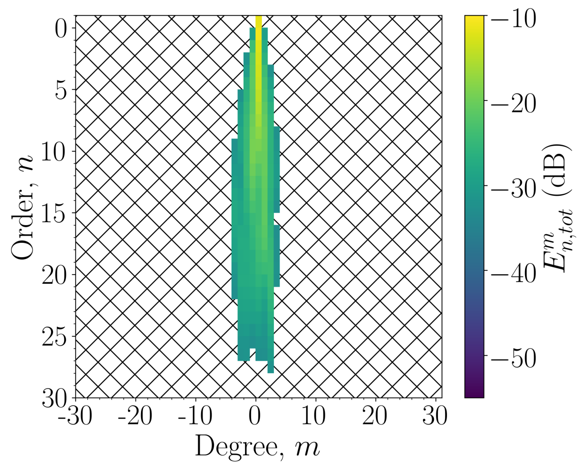


Fig. 5: Total normalised energy per spherical harmonic coefficient across all frequencies, for the left ear KU100 compact mode HRTF.

with $\tilde{H}(ka, \theta, \phi)$ the compact mode HRTF. A small error bound of 5% was set, such that the compact mode HRTF should still have 95% of the energy of the original HRTF.

The cumulative error, calculated by reordering the spherical harmonic coefficients in increasing order of energy and progressively discarding till all were removed, is shown in Fig. 4 for the left ear of the KU100. Working to the 5% error bound, it was found that a total of 783 coefficients could be discarded following the rotation. The same procedure performed for the KU100 without applying the rotation resulted in 513 coefficients which could be discarded. This means that for this error bound, only $961 - 783 = 178$ spherical harmonic coefficients are actually required, whereas for the unrotated HRTF $961 - 483 = 478$ are needed. This is a substantially large decrease, and is closer to the number of spherical harmonics for a truncation order of just $N = 12$ when processed with traditional methods. The effect of this discarding is shown in Fig. 5 where it may be observed that the remaining orders focus around the $m = 0$ modes as these have the highest energy content.

Interestingly, for very high orders all spherical harmonics have been removed. This reveals the nature of reducing the modes based purely on energy criterion - it does not necessarily remove spherical harmonics in increasing degree m or order n but purely in order of increasing energy. Thus, if it is desired that certain spherical harmonics should be retained no matter what the energy content, a different criteria should be considered to create the compact mode representation. Finally, the fact that all modes are removed for high orders above $n = 28$ suggests that for an initial truncation order of $N = 30$, the maximum order of the compact mode HRTF is just $N = 28$ working to a 5% error bound.

5 HRTF Representation Comparison

Due to the way the modes have been removed, the HRTF in the spatial and frequency domain should retain the vast majority of the information of the original HRTF. Hence, the compact mode KU100 HRTF was transformed back into the spatial domain using an *iSHT* and the HRTF entry for an incident source directly in front of the head such that such that $\theta, \phi = 0$ was chosen for analysis. The original measurement from the database is compared to not only the compact mode representation, but also to the HRTF truncated to order $N = 30$ in the spherical harmonic domain to demonstrate the effect of truncation, which will naturally be included within the compact mode representation. Hence, Fig. 6 shows the left ear KU100 measurement for these three HRTF representations.

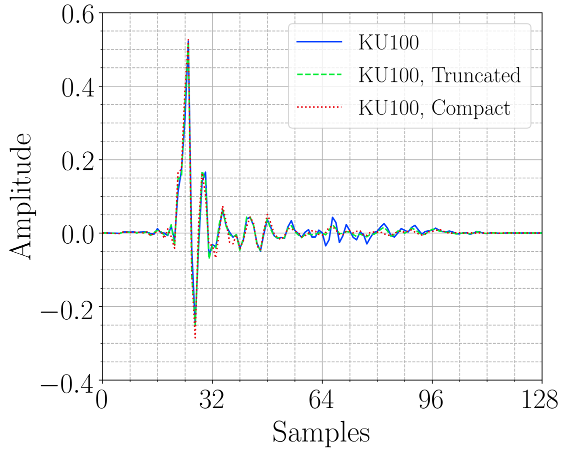
From Fig. 6 the truncation and compact mode representation both result in a smoothing of the information following the main peak of the impulse response. In general, the truncated and compact mode HRTFs either underestimate or overestimate the peaks and ripples in the impulse responses but also in the magnitude of the transfer function. This deviation remains in the order of $1-2$ dB from the original, except for a few extreme deviations closer to $3-4$ dB at $3,700$ Hz and $14,000$ Hz. These deviations may be observed as a smoothing of the original transfer function. Beyond $18,000$ Hz the effect of spatial aliasing is seen with a boost in the very high frequencies. This is most prominent in the compact mode HRTF likely due to the complete removal of very high order modes, which have greatest influence at high frequencies.

Finally, considering the difference in the phase response between the original HRTF and either the truncated or the compact mode representation respectively reveals that there is little error introduced by both representations. The compact mode HRTF introduces slightly larger differences in the phase response at high frequencies, where it is noticeable that the differences become more erratic and larger. Again, this is likely due to the complete removal of the highest order modes.

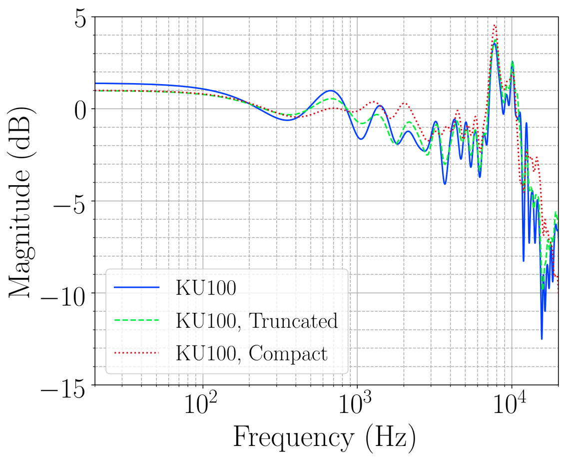
6 Conclusions

This paper has introduced the compact mode HRTF representation. This HRTF representation utilises a rotation to reorder the energy distribution across the spherical harmonic coefficients to focus around the $m = 0$ spherical harmonics. The low energy modes are then progressively discarded till a user defined error bound is reached, to create a more compact HRTF requiring less spherical harmonics at the cost of an introduced error.

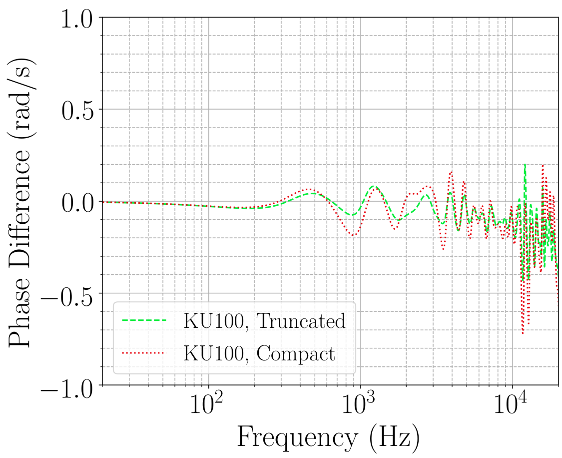
The scheme is considered for both a rigid sphere and a KU100 HRTF calculated up to truncation order $N = 30$ in the spherical harmonic domain, which results in 961 spherical harmonic coefficients. The rotation is demonstrated to focus the energy of the HRTF to around the modes with m close to 0. Removing the modes such that low energy modes are discarded first, it is shown that the KU100 following this rotation can be fully represented to just a 5% error bound using only 178 spherical harmonics. This is a considerably large reduction in the size of the data set for a small error bound. Considering the HRTF for a given source position the compact mode KU100 HRTF recreates that of the full 30th order representation with only slight deviations of $1-2$ dB in the magnitude response and small errors in the phase response.



(a) Left ear impulse responses.



(b) Magnitude of the left ear HRTF.



(c) Phase difference of the left ear HRTF between the original HRTF and the truncated or compact mode representation.

Fig. 6: KU100 comparison between the original HRTF, an order $N = 30$ truncation and the compact mode representation.

For future work, other schemes for deciding which spherical harmonics may be discarded are to be considered. Furthermore, whilst the rotation presented is motivated by the results of the axisymmetric rigid sphere energy reordering, this is just an approximation of a HRTF. Therefore, the optimal alignment for the efficient representation of a real HRTF may be a different rotation and is to be investigated. This may be framed as a search for a sparse representation of the HRTF working to an error bound. Finally, application of the compact mode HRTF to efficient real-time rendering is also to be investigated.

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