# Optimization Assignment - Genetic Algorithm

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## Problem:

Tumors cells are supposed following the Gompertz model:

$$N'(t) = N(t)(\lambda ln \frac{\Theta}{N(t)} - \sum_{j=1}^{d} k_j \sum_{i=0}^{n-1} C_{ij} X_{(\tau_i, \tau_{i+1}]}(t))$$

$$= N(t)(\lambda ln(\Theta) - \lambda ln(N(t)) - drift_i) \forall t \in (\tau_i, \tau_{i+1}]$$

Where: - $drift_i = \sum_{i=1}^{d} k_j C_{ij}$
- $\lambda = 0.336$ is the "evolution speed" of the model
- $\Theta = 10^1 2$ is a kind of carrying capacity of the tumor
- N(t) is the number of tumor cells at time t, N(0) = 20000 is the initial number of tumor cells, $N_m ax = 0.95\Theta$ is the maximum number of tumor cells an alive body can contain
- n = 10 is the number of treatment sessions, given at times $\tau_0 = 0 < \tau_1 = 3 < ... < \tau_9 = 30 < \tau_{10} = 33$. The treatment given at time $\eta_i$ is supposed to act until time $\eta_{i+1}$
- $C_{ij} \in 0, 1, 2, ...C_{max,j} = 15$ denotes the concentration of drug j at dose i
- $\overline{k} = [k_1, ..., k_d] = [0.12, 0.0502, 0.0637, 0.1347, 0.0902, 0.0546, 0.0767, 0.1121, 0.0971, 0.0403]$ represent the effectivenesses of the corresponding drugs
- The cumulative concentration $\sum_{i=0}^{n-1} C_{ij}$ of each drug must be under $C_{cum,j} = 127$ for every j = 1,..,d. - We have parameters $\eta_{kj}$ to find curative solutions

$$\begin{bmatrix} 0.0036 & 0.0098 & 0.0061 & 0.0009 & 0.0003 & 0.0108 & 0.0045 & 0.0021 & 0.0096 & 0.0125 \\ 0.0063 & 0.0082 & 0.0062 & 0.0062 & 0.0083 & 0.013 & 0.0039 & 0.0019 & 0.0015 & 0.005 \\ 0.0129 & 0.0018 & 0.0116 & 0.0021 & 0.009 & 0.0129 & 0.0054 & 0.0049 & 0.0093 & 0.0066 \\ 0.0053 & 0.0086 & 0.0067 & 0.0029 & 0.0089 & 0.0054 & 0.0042 & 0.0095 & 0.0112 & 0.0092 \end{bmatrix}$$

- We have parameters $\eta_{kj}$ to find palliative solutions

$$\begin{bmatrix} 0.00612 & 0.01666 & 0.01037 & 0.00153 & 0.00051 & 0.01836 & 0.00765 & 0.00357 & 0.01632 & 0.02125 \\ 0.01071 & 0.01394 & 0.01054 & 0.01054 & 0.01411 & 0.0221 & 0.00663 & 0.00323 & 0.00255 & 0.0085 \\ 0.02193 & 0.00306 & 0.01972 & 0.00357 & 0.0153 & 0.02193 & 0.00918 & 0.00833 & 0.01581 & 0.01122 \\ 0.00901 & 0.01462 & 0.01139 & 0.00493 & 0.01513 & 0.00918 & 0.00714 & 0.01615 & 0.01904 & 0.01564 \end{bmatrix}$$

Finding a treatment is curative if $N(\tau_i) \leq 1000$ for 3 consecutive values $\tau_{\ell-2}, \tau_{\ell-1}$ and $\tau$ with $\ell \in 2, 4, ..., n$.

Finding a treatment is palliative if it has no dose or treatment. There, when dealing with palliative treatments the term $\sum_{j=1}^{d} k_j C_{ij}$ in Gompertz model must be taken equal to zero for $t > \tau_n$.

To find a curative treatment which, among the curative options, it is the most effective in the sense that it minimises the following fitness function:

$$I = \int_{\tau_0}^{\tau_\ell} N(t)dt$$

If the curative treatment does not exist, try to find the palliative treatment such that maximises the lifespan of the patient.

1. Ideas

   I will consider individuals with the treatments in order to solve the problem with genetic algorithm. I look for a curative treatment with minimum I. In doing this, I consider each generation, there can be treatments which does not satisfy the constraints of the model. These individuals are chose to reproduce the small fitness value. The stopping criterion is that I is minimized if the best individual was found does not change in four consecutive generative steps. The curative treatment will not exist if I found ten consecutive infeasible generations. In this case, I need to find a palliative solution which is maximized the lifespan in order to keep the patient alive as long as possible.

   - The choices in order to generate new population:
   **Crossover**: uniform crossover with probability `probability_cross` $= 0.2$
   **Mutation**: The single mutation bit $C_{ij}$ is performed by changing it with $C_{max,j} - Cij$ with `probability_cross` $= 0.3$

2. Results

   I run the program with the parameters $\eta_{k,j}$, a curative treatment was found after 18 generative steps, the fitness value is 3989.589067. Its doses are:

$$\begin{bmatrix}
15 & 14 & 14 & 14 & 15 & 6 & 12 & 15 & 13 & 14 \\
8 & 11 & 13 & 14 & 2 & 7 & 10 & 13 & 14 & 1 \\
12 & 3 & 14 & 3 & 13 & 10 & 14 & 13 & 10 & 6 \\
14 & 15 & 11 & 2 & 7 & 14 & 12 & 6 & 5 & 14 \\
11 & 3 & 4 & 7 & 3 & 13 & 14 & 13 & 5 & 2 \\
2 & 3 & 8 & 13 & 1 & 13 & 9 & 9 & 11 & 11 \\
1 & 7 & 8 & 11 & 12 & 8 & 3 & 5 & 8 & 9 \\
1 & 4 & 14 & 4 & 12 & 7 & 7 & 5 & 4 & 10 \\
11 & 1 & 3 & 11 & 9 & 5 & 1 & 12 & 7 & 5 \\
6 & 9 & 15 & 11 & 6 & 6 & 7 & 1 & 2 & 5
\end{bmatrix}$$

I run the program with the parameters $\eta_{k,j}$, a palliative treatment was found after 27 generative steps, the lifespan of the patient is 52.098909. Its doses are:

$$\begin{bmatrix}
2 & 8 & 2 & 11 & 2 & 13 & 2 & 2 & 14 & 6 \\
3 & 11 & 0 & 1 & 1 & 14 & 13 & 9 & 1 & 3 \\
8 & 6 & 6 & 3 & 2 & 12 & 12 & 6 & 1 & 1 \\
5 & 5 & 2 & 10 & 14 & 3 & 3 & 7 & 13 & 6 \\
7 & 9 & 5 & 5 & 1 & 4 & 15 & 2 & 4 & 9 \\
5 & 11 & 4 & 12 & 9 & 0 & 3 & 1 & 8 & 8 \\
2 & 8 & 12 & 15 & 6 & 4 & 6 & 15 & 1 & 12 \\
0 & 9 & 10 & 5 & 1 & 8 & 11 & 12 & 6 & 1 \\
3 & 4 & 15 & 11 & 1 & 2 & 5 & 14 & 4 & 11 \\
14 & 14 & 2 & 15 & 9 & 6 & 10 & 9 & 4 & 2
\end{bmatrix}$$