Stochastic Modelling and Numerical Methods in Finance: SABR and Heston

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Thesis Focus

The thesis centers on option pricing in financial mathematics, specifically studying two stochastic volatility models—SABR and Heston—and applying advanced numerical techniques to price vanilla options efficiently and accurately.

Key Components

1. **Background and Motivation**
    - The Black-Scholes model, while fundamental, assumes constant volatility, which does not match observed market behaviors.
    - Stochastic volatility models allow volatility to vary randomly, better capturing market dynamics but are computationally complex.
2. **Models Studied**
    - **SABR Model:** Captures volatility smile, relatively simpler with fewer parameters and easier calibration; widely used in interest rate derivatives.
    - **Heston Model:** Includes mean reversion in volatility, has an explicit option pricing formula but more challenging calibration due to extra parameters.
3. **Numerical Methods**
    - Monte Carlo simulations are essential for pricing since closed-form solutions are often unavailable.
    - Variance reduction techniques (Control Variates, Antithetic Variates, Conditional Monte Carlo, and combinations thereof) significantly improve the precision and computational speed of option price estimators.
4. **Results – Variance Reduction**
    - Applied variance reduction methods to SABR and Heston models, showing dramatic improvements in estimator precision and reduced computational cost.
    - The combination of Control Variate + Antithetic + Conditional Monte Carlo yielded up to 460x improvement in SABR and 30x in Heston compared to naive Monte Carlo.
5. **Implied Volatility Surfaces**
    - Simulated surfaces reflect market-observed volatility smiles and skews, unlike the constant volatility assumption of Black-Scholes.

    - SABR surfaces matched well with Hagan approximations; Heston surfaces demonstrated complex shapes consistent with empirical phenomena.

6. **Model Calibration**
    - Calibrated SABR parameters using real market data, minimizing the error between observed and model-implied volatilities per maturity.
    - Achieved calibration results very close to industry benchmarks, producing accurate implied volatility surfaces.

7. **Conclusions and Future Directions**
    - Combining advanced variance reduction techniques is key to practical and efficient stochastic volatility model pricing.
    - Stochastic volatility models better reflect market realities and offer practical tools for traders and risk managers.
    - Future work could focus on improving computational efficiency, applying to local volatility or exotic options, and extending calibration methods.

Here is a list of potential interview questions related to your bachelor thesis on stochastic volatility models (SABR and Heston) along with suggested answers to help you prepare:

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1. What motivated you to study the SABR and Heston models for option pricing?

Suggested Answer:
The Black-Scholes model, while foundational, assumes constant volatility, which contradicts observed market phenomena such as volatility smiles and skews. The SABR and Heston models are popular stochastic volatility models that address these limitations by allowing volatility to be random and dynamic. Studying these models enables more realistic option pricing that aligns better with real market data.

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2. Can you explain the main differences between the SABR and Heston models?

Suggested Answer:
The SABR model focuses on a single forward price with volatility following a correlated stochastic process, and it has fewer parameters, making calibration simpler. The Heston model captures the instantaneous variance evolving with mean reversion and volatility of volatility but has more parameters and is more complex to calibrate. Both model stochastic volatility but differ in formulation and complexity.

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3. Why are variance reduction techniques important in Monte Carlo simulations for option pricing?

Suggested Answer:
Monte Carlo simulations require large numbers of samples for accurate estimates,

which can be computationally expensive. Variance reduction techniques like control variates, antithetic variates, and conditional Monte Carlo reduce the variance of the estimator, improving precision without needing more samples, thus speeding up computation and making simulations more efficient.

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4. What variance reduction methods did you apply, and which one performed best?

Suggested Answer:
I applied control variates, antithetic variates, conditional Monte Carlo, and their combinations. The combination of control variate, antithetic, and conditional Monte Carlo yielded the most significant improvement—up to 460 times faster computational performance for the SABR model and 30 times for the Heston model compared to naive simulations.

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5. How did you simulate the implied volatility surface?

Suggested Answer:
I used Monte Carlo methods enhanced by variance reduction techniques to compute option prices over a grid of maturities and strikes. By numerically inverting the Black-Scholes formula on these prices, I obtained implied volatilities, which were then plotted to visualize the implied volatility surface. This approach captures the market-observed volatility smile and skew, unlike the flat surface from the Black-Scholes model.

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6. How did you calibrate the SABR model parameters?

Suggested Answer:
I used real market data of vanilla options with various maturities and strikes from CaixaBank. Calibration was done maturity-by-maturity by minimizing the squared error between observed implied volatilities and those predicted by the SABR model using the Hagan approximation. Optimization was performed using scipy.minimize. Calibration results closely matched industry benchmarks, with a mean absolute error of 0.15.

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7. Why did you choose to calibrate only the SABR model and not Heston?

Suggested Answer:
The SABR model has fewer parameters and a closed-form approximation for implied volatility, making calibration more straightforward and faster. The Heston model, while more realistic, is more complex and computationally expensive to calibrate, especially as it lacks closed-form implied volatility solutions, requiring more involved numerical techniques.

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8. What challenges did you face while implementing the models and methods?

Suggested Answer:
Challenges included handling the computational cost of large Monte Carlo simulations, ensuring numerical stability during simulations (especially avoiding negative variance in

Heston’s CIR process), and tuning parameters for variance reduction methods. Efficient implementation of combined variance reduction required careful coding to maintain unbiasedness and reduce variance effectively.

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9. How do your findings contribute to the field of financial mathematics?

Suggested Answer:
This work demonstrates that advanced variance reduction techniques can dramatically improve the computational efficiency of stochastic volatility models, making them more practical for real-world option pricing and calibration. It also provides validated algorithms for simulating implied volatility surfaces and calibrating models based on market data, helping bridge theory and practice.

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10. What future work or extensions do you suggest based on your thesis?

Suggested Answer:
Future work could focus on adapting these computational methods to local volatility models or exotic option pricing, improving algorithmic efficiency further, and exploring deep learning or advanced machine learning techniques for model calibration and option pricing. Additionally, integrating these models into real-time trading platforms presents an exciting application