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Factor Momentum and the Momentum Factor: a European approach

Master's thesis in Mathematics

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A study on the ability of momentum displayed by factors to enhance cross-sectional explainability in Europe, demonstrating that these factors outperform classical momentum factors.

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Abstract

This study investigates the role of factor momentum in explaining the cross-sectional variation in stock returns within European markets, extending the findings of Ehsani and Linnainmaa (2022) to a new regional context. The analysis shows that momentum among financial factors significantly outperforms traditional stock momentum, challenging conventional asset pricing models. Using two comprehensive datasets of European and global factors, this research examines whether the momentum patterns observed in U.S. markets are present in Europe and explores the interactions between factor momentum and individual stock momentum.

The findings reveal that multiple European factors exhibit notable momentum, which supports the construction of a new momentum factor (FMOM) that captures systematic factor momentum. FMOM not only surpasses the predictive power of classical stock momentum but also substantially contributes to individual stock momentum, identifying sources of return that traditional models do not fully capture. Furthermore, the use of momentum-neutral factors allows for isolating pure factor momentum from incidental stock momentum, suggesting that the profitability of momentum strategies is driven primarily by factor dynamics rather than stock-specific movements.

By offering a European perspective, this research enriches the global perspective on momentum by highlighting the importance of factor dynamics in European markets and underscores the need for asset pricing models that integrate factor momentum across different markets. All code and data used for this analysis are publicly available in a GitHub repository, ensuring transparency and reproducibility. These findings provide valuable insights for investors, emphasizing the potential of factor timing strategies over traditional stock momentum approaches to improve portfolio performance and risk management.

Keywords: Factor Momentum, Momentum Factor, Asset Pricing, European Financial Markets, Cross-sectional Stock Returns, Econometric Techniques

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Introduction

Momentum, a well-documented anomaly in financial markets, challenges the efficient market hypothesis by suggesting that past returns can predict future performance. This phenomenon is not only prevalent in individual stock returns but also extends to factor returns, as demonstrated by Ehsani and Linnainmaa (2022)[1]. Their seminal work highlights that factor momentum is a significant driver of returns, particularly in factors that explain a substantial portion of the cross-section of stock returns. This thesis seeks to extend their findings by examining factor momentum within the European context, thereby contributing to the broader understanding of momentum's role in asset pricing.

In financial markets, factors such as size, value, and momentum are commonly used to explain variations in stock returns. These factors, often constructed from portfolios of individual stocks, exhibit autocorrelations that can lead to predictable patterns in returns. Ehsani and Linnainmaa's research [1] reveals that momentum is not an isolated risk factor but rather a dynamic portfolio strategy that times other factors. Their analysis shows that momentum is concentrated in high-eigenvalue principal component factors, which capture systematic risks more effectively.

The European financial landscape, with its distinct regulatory environments and market structures, may influence the manifestation of factor momentum. This study investigates whether the factor momentum patterns found in U.S. markets are present in Europe and how they interact with individual stock momentum. By analyzing European data using econometric techniques, the research aims to evaluate the relevance of factor momentum in European markets and its implications for asset pricing, investment strategies, and risk management.

In Section 2, I provide a theoretical overview of empirical asset pricing and discuss the practical frameworks used to work with these models. In Section 3, I construct two distinct datasets that include 20 European and global factors, and I assess the feasibility of creating a new type of momentum factor by measuring the factors' autocorrelations and the profitability of a factor momentum strategy. Section 4 reviews the KNS sentiment model and tests the hypothesis that more systematic factors display greater momentum through the construction of a momentum strategy the trades the PC factors. Then, I introduce the conceptualization of factor momentum strategies as a new type of momentum factor targeting momentum in factors (FMOM). In Section 5, I investigate the sources of individual stock momentum and explore how factor momentum translates to it from a theoretical perspective. I verify the consistency of empirical data with these theories and analyze the extent to which FMOM and UMD can explain each other. This analysis is extended to include various forms of FMOM and alternative momentum factors, concluding with an examination of firm-specific returns, which are expected to be insignificant. Finally, in Section 6, I compute conditional correlations with UMD to demonstrate its relationship with all other factors and use momentum-neutral factors to assess if the profitability of momentum strategies is driven purely by factor dynamics or if it merely reflects individual stock momentum.

In conclusion, this research contributes to the ongoing discourse on momentum by providing a European perspective, thereby enriching the global understanding of factor dynamics in financial markets. To promote transparency and facilitate reproducibility, all code and data supporting this analysis are available in an open-access GitHub repository[1]. These findings have the potential to inform investment strategies and enhance the theoretical frameworks that underpin asset pricing models.

[1] `https://github.com/dani-gonzalez-muela/FinalMasterThesis`

2
Theory Review

I begin with an overview of empirical asset pricing. I start with the theoretical framework of the most basic models (CAPM, Fama-French, and Carhart) and then transition to a more practical framework of how to compute these factors.

2.1 CAPM

The Capital Asset Pricing Model (CAPM) [2] is a financial model from the 1960s used to determine the expected return on an investment ($E[r_i]$) based on its inherent risk compared to the overall market.

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f) \tag{2.1}$$

Where:
- r_f is the risk-free rate.
- $\beta_i = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)}$ is the sensitivity of the asset's return to the market return.
- $E[r_m]$ is the expected market return.

CAPM assumes efficient markets, rational investors, and single-period transactions. However, it is a simplistic model, and other models fit better the empirical data.

2.2 APT

The Arbitrage Pricing Theory (APT) [3] is a generalization of CAPM that includes multiple factors and does not specify which these factors are. The expected return on an asset i is given by:

$$E[r_i] = r_f + \sum_{j=1}^{k} \beta_{ij} F_j \tag{2.2}$$

Where:
- k is the number of factors.
- β_{ij} is the sensitivity of the asset's return to factor j.
- F_j is factor j.

2.3 Fama-French

2.3.1 Three Factor Model

The Fama-French Three-Factor Model (FF3) [4] extends CAPM by introducing two additional factors:

$$E[r_i] = r_f + \beta_{i1}(E[r_m] - r_f) + \beta_{i2}E[SMB] + \beta_{i3}E[HML] \tag{2.3}$$

Where:

- SMB (Small Minus Big): return spread between small and large-cap stocks.
- HML (High Minus Low): return spread between high and low book equity-to-market equity (BE/ME) stocks.

2.3.2 Five Factor Model

In 2015, Fama and French extended the model further to include five factors (FF5) [5]:

$$\begin{aligned} E[r_i] = r_f + \beta_{i1}(E[r_m] - r_f) + \beta_{i2}E[SMB] + \beta_{i3}E[HML] + \\ + \beta_{i4}E[RMW] + \beta_{i5}E[CMA] \end{aligned} \tag{2.4}$$

Where:

- RMW (Robust Minus Weak): spread between high and low operating profitability (OP) stocks.
- CMA (Conservative Minus Aggressive): spread between stocks that invest conservatively and aggressively.

The selection of these factors was based on empirical research that revealed patterns in stock returns not explained by CAPM. These factors have consistently demonstrated explanatory power across different markets and time periods. They reflect broad market behaviors while aiming to extend the model without adding unnecessary complexity.

2.3.3 Factor Construction

Factors are key in empirical asset pricing, and there are various ways to build them. Most methods involve sorting stocks into portfolios and computing the difference between some of the portfolio returns. The Fama-French methodology [6] is one of the most well-known approaches, and they provide factors and stock portfolios on their website [7]. I begin by explaining their original factor construction methodology, targeting US stocks, and then describe how they extended it for international equities [8].

Univariate Portfolios: For each characteristic forming the factors (BE/ME for HML, OP for RMW, and investment for CMA), three univariate portfolios are constructed based on three percentiles: bottom 30%, middle 40%, and top 30%. These portfolios are updated monthly and include all NYSE, AMEX, and NASDAQ stocks with equity data for June of that year. Breakpoints for splitting the portfolios are computed at the end of June each year using NYSE stock data. The returns for each portfolio are computed as the value-weighted returns by market cap.

Bivariate Portfolios: For each univariate portfolio, bivariate sorts are created by adding an additional split on size above or below the median market equity (ME). This approach helps avoid biases toward big or small stocks. This results in six portfolios on size and BE/ME (Table 2.1), six portfolios on size and OP, and six portfolios on size and investment.

Table 2.1: Bivariate Portfolios on Size and BE/ME

	Median ME	
	Small	**Big**
70-100th BE/ME percentile	Small Value	Big Value
30-70th BE/ME percentile	Small Neutral	Big Neutral
0-30th BE/ME percentile	Small Growth	Big Growth

Table 2.1 provides an example of bivariate portfolios formed based on size and BE/ME. The labels "Small" and "Big" refer to the split by size, while the remaining labels correspond to the split by BE/ME. Similarly, bivariate portfolios based on size and OP follow the same classification scheme, but with different labels for the OP split: "Robust," "Neutral," and "Weak." For investment, the labels would be "Aggressive", "Neutral," and "Conservative".

Factors: Finally, value-weighted returns are computed on the long minus short leg of each factor obtained by different partitions of the bivariate portfolios.

- SMB is computed as the average return on the nine small portfolios minus the nine big portfolios.

$$\begin{aligned} SMB = &\frac{1}{3}(Small\ Value + Small\ Neutral + Small\ Growth) \\ &- \frac{1}{3}(Big\ Value + Big\ Neutral + Big\ Growth) \end{aligned} \tag{2.5}$$

- HML is the average return on the two value portfolios minus average return on the two growth portfolios.

$$\begin{aligned} HML = &\frac{1}{2}(Small\ Value + Big\ Value) \\ &- \frac{1}{2}(Small\ Growth + Big\ Growth) \end{aligned} \tag{2.6}$$

- RMW is the average return on the robust operating profitability portfolios minus the average return on the weak operating profitability portfolios.

$$\begin{aligned} RMW = &\frac{1}{2}(Small\ Robust + Big\ Robust) \\ &- \frac{1}{2}(Small\ Weak + Big\ Weak) \end{aligned} \tag{2.7}$$

- CMA is the average return on the conservative portfolios minus the average return on the aggressive portfolios.

$$\begin{aligned} CMA = &\frac{1}{2}(Small\ Conservative + Big\ Conservative) - \\ &\frac{1}{2}(Small\ Aggressive + Big\ Aggressive) \end{aligned} \tag{2.8}$$

- MKT-RF is the market excess of return over the risk-free rate, usually calculated using value-weighted returns of all available stocks and the US 10-year treasury bill as the risk-free rate.

Fama and French initially focused on US stocks, but later updated the factors to cover Developed Markets [8], including countries from Europe, Asia, and Canada. The factors for these regions are computed similarly, but with a size breakpoint of 90% for big stocks and 10% for small stocks. Breakpoints are computed using stocks from the specific region.

2.4 Carhart Momentum

In 1997, Mark M. Carhart proposed momentum as its own factor, and created UMD (Up Minus Down) [9] to extend the three-factor model:

$$R_i - R_f = \alpha_i + \beta_{i1}(R_m - R_f) + \beta_{i2}SMB + \beta_{i3}HML + \beta_{i4}UMD + \epsilon_i \tag{2.9}$$

The UMD factor focuses on individual stock momentum induced by past returns. It is computed as the spread between stocks that performed well over the past 12 months and those that performed poorly. This factor significantly improves the model when explaining mutual fund returns and has been found to be significant across various asset classes. The procedure to compute the UMD is similar to that for classical factors, and the data is also available on the Fama and French website [7]. Univariate portfolios are created based on prior returns (cumulative returns from $t-12$ to $t-2$), and bivariate portfolios are created on Size and Momentum, following the same structure than Table 2.1. This results in "High", "Neutral" and "Low" as the new labels for the UMD terciles. The momentum factor is then computed as the average spread between high and low momentum portfolios.

$$\begin{aligned} UMD = &\frac{1}{2}(Small\ High + Big\ High) \\ &- \frac{1}{2}(Small\ Low + Big\ Low) \end{aligned} \tag{2.10}$$

UMD is the classical momentum factor, however, there exist different types of momentum that target momentum through different characteristics, like Industry Momentum [10], which uses prior returns to sort industries rather than individual stocks, or Intermediate Momentum [11], which targets momentum using prior returns from $t - 12$ to $t - 7$. Factors based on shorter or longer periods are usually classified differently, like short-term reversals (based on $t - 1$ returns) [12] or long-term reversals (based on $t - 13$ to $t - 60$ returns) [13].

2.5 Alternative Factors

Classical factors have demonstrated strong explanatory power for the cross-section of asset returns while also offering intuitive insights. These models were designed with simplicity in mind, but it is easy to imagine more complex models that incorporate additional factors based on different characteristics. In the following section, I will discuss some of the alternative factors utilized in the first section of this project.

Fama French Alternative Factors

The Fama-French website [7] provides univariate and bivariate portfolios constructed based on earnings-to-price (E/P), cashflow-to-price (CF/P), and dividend yield-to-price (D/P) ratios. These portfolios enable the computation of alternative factors in a manner similar to traditional Fama-French factors.

AQR Factors

This study also incorporates several factors from the AQR website [14]:

- Betting Against Beta (BAB) [15] involves constructing portfolios based on the CAPM Beta of securities. Securities are grouped into high Beta and low Beta portfolios, which are rebalanced monthly. The BAB factor is then calculated as the return difference between these portfolios, adjusted for their Beta values.
- Quality Minus Junk (QMJ) [16] measures the quality of stocks by considering profitability, growth, safety, and payout. Similar to the Fama-French methodology, portfolios are created based on size and quality. The QMJ factor is computed as the spread between high-quality and low-quality (junk) assets.

3

Autocorrelations in the returns of equity factors

In this section, I discuss the construction of a momentum factor that targets factor momentum in European markets. I begin by describing the two datasets used in the analysis. Next, I examine the autocorrelation of the factors within these datasets and evaluate the profitability of factor momentum strategies.

3.1 Data Gathering and Cleaning

The original study [1] examines 22 off-the-shelf factors for the US and global markets, covering the period from July 1963 (1990 for global factors) to December 2019. Similarly, I compiled two datasets of 20 off-the-shelf factors, as listed in Table 3.1, spanning from July 1990 (with some factors starting in 1993) to December 2023. In both datasets, returns are expressed as percentages.

3.1.1 Dataset 1

The first dataset uses the same primary sources as the original data (Fama French [7] and AQR [14] websites), but it is adapted for European factors. While most factors are directly sourced from the original data, a few required modifications, resulting in minor differences from the original factor construction methodology:

- **E/P, CE/P, and D/P:** The original study [1] computes US factors as the difference between the top and bottom deciles of the bivariate portfolios. For European portfolios, the Fama and French website [7] only contains data for univariate portfolios, which are the top 30% and bottom 30% of firms based on these ratios. I simplified the construction of these European factors as the difference between the high and low categories.
- **Global Factors:** In the original dataset, global markets are defined as developed markets excluding the US. Then, global factors returns are calculated as the value-weighted portfolio returns of these developed markets. In my study I define global factors (denoted as G_MKT-RF, G_SMB, etc.) as developed markets excluding Europe, which are Asia-Pacific, US and Canada. After computing factor returns for each region, I defined the global factors as the value-weighted returns using the lagged $t-1$ market cap of each region, following the AQR approach [14] to avoid biases. I obtained the market cap data from World Development Indicators [17].

3.1.2 Dataset 2

I utilize an off-the-shelf factors dataset developed by Theis Ingerslev Jensen, Bryan Kelly, and Lasse Heje Pedersen [18], which contains 153 factors from 93 different countries, and it is available at Kelly's website [19]. Dataset 2 spans the same time period as Dataset 1 and I construct global factors are constructed using the same methodology. To validate the consistency between datasets, Table 3.1 presents the correlations between factor monthly returns in Dataset 1 and Dataset 2.

Table 3.1: Correlations of Factor Returns Between Dataset 1 and Dataset 2

Factor	**Correlation**
MKT-RF	0.99
SMB	0.93
HML	0.80
RMW	0.78
CMA	0.73
UMD	0.95
E/P	0.60
CE/P	0.74
D/P	0.86
BAB	0.48
QMJ	0.89
G_MKT-RF	0.99
G_SMB	0.91
G_HML	0.77
G_RMW	0.73
G_CMA	0.85
G_UMD	0.96
G_E/P	0.67
G_CE/P	0.80
G_D/P	0.78
G_BAB	0.58
G_QMJ	0.67

Most factors exhibit correlations above 75%, though some, like BAB and E/P, show lower correlations than expected, 48% and 60%, respectively. These discrepancies primarily arise from differences in the methods used to compute the factors. While both datasets target the same fundamental factors, there are slight variations. For instance, the UMD factor in Dataset 1 sorts portfolios based on prior year returns $t-2$ to $t-12$, following [20]. In contrast, Dataset 2 sorts from $t-1$ to $t-11$. Also, the methodology used for constructing factor returns is not exactly the same: Dataset 1 is based on the original factor construction by Fama and French [6], which uses the bivariate portfolios described in Section 2, whereas Dataset 2 relies on the spread between high and low tercile of univariate portfolios, constructed as in [21]. Despite these differences, the overall results are consistent and align with the findings throughout the paper.

3.1.3 Summary statistics

Table 3.2: Average Statistics of Datasets 1 and 2

Factor	Start Date	Mean	Std	T-Value
SMB	July 1990	-0.2	7.5	-0.1
HML	July 1990	2.3	10.2	1.4
RMW	July 1990	3.0	6.2	2.9
CMA	July 1990	1.7	6.5	1.5
UMD	Nov 1990	8.5	13.7	3.6
E/P	July 1990	3.2	9.4	2.0
CE/P	July 1990	4.1	9.8	2.5
D/P	July 1990	2.6	9.2	1.6
BAB	July 1993	6.2	12.5	3.0
QMJ	July 1993	5.1	9.2	3.3
G_SMB	July 1990	0.3	8.9	0.2
G_HML	July 1990	2.2	10.7	1.2
G_RMW	July 1990	3.4	7.0	2.8
G_CMA	July 1990	2.3	8.5	1.6
G_UMD	Nov 1990	4.5	14.3	1.8
G_E/P	July 1990	3.4	11.4	1.8
G_CE/P	July 1990	3.4	10.5	1.8
G_D/P	July 1990	0.2	11.5	0.0
G_BAB	July 1993	4.6	13.2	2.4
G_QMJ	July 1993	4.9	8.4	3.4

Table 3.2 presents the average monthly return statistics, in percentages, for Dataset 1 and Dataset 2. There are significant differences between the observed factor returns, consistent with the findings for US factor returns [1]. For instance, SMB earns -0.2%, while UMD and BAB each earn over 5%. In contrast, the volatilities of European factors are less variable than those of US factors, ranging from 6.2% for RMW to 13.2% for global BAB, compared to 4.7% to 17.3% for US factors [1]. The t-values highlight these differences in factor returns, with approximately half of the factors showing a significant premium, as indicated by a p-value below 5% (t-value above 1.97).

3.2 Measuring Autocorrelations

As shown in Table 3.2, there is a significant premium for both European and global UMD factors. This implies that individual stocks exhibit momentum during the analyzed period, and substantial returns can be obtained with a trading strategy that buys and sells stocks based on prior year returns. As mentioned in Section 2, UMD is the classical momentum factor; however, various methods exist for computing factors that target momentum. In the original study [1], the authors propose a new momentum factor that focuses on the momentum exhibited by other factors rather than individual stocks. I now assess the feasibility of constructing a similar factor in European markets by measuring the autocorrelations displayed by the factors in Datasets 1 and 2. If the factors display momentum, there is enough evidence to support a momentum factor that targets factor momentum.

3.2.1 Regression Analysis

I perform a regression analysis between prior and future factor returns to examine autocorrelation within the factors. The target variable is the monthly factor returns, and the explanatory variable is an indicator set to 1 if the average return over the previous 12 months is positive and 0 otherwise. The regression intercept, α, measures the average factor return following a year of underperformance, while the slope, β, represents the difference between returns in up and down years. If the factors do not exhibit momentum (i.e., no autocorrelation), both the intercept and slope are expected to be 0. Table 3.3 presents these coefficients and their corresponding t-values. Additionally, Table 3.4 reports the significance levels of the t-values for 402 observations, testing the null hypothesis that the regression coefficients are equal to 0.

Table 3.3: Factor Momentum Regression

	Dataset 1				**Dataset 2**			
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
SMB	-0.21	-1.38	0.49	2.31	-0.39	-2.34	0.65	2.78
HML	-0.26	-1.23	0.92	3.37	-0.13	-0.48	0.36	0.98
RMW	0.20	1.18	0.20	0.99	0.10	0.56	0.07	0.32
CMA	-0.16	-1.21	0.54	2.92	0.02	0.16	0.21	0.98
UMD	0.81	1.53	0.02	0.04	0.82	1.68	-0.29	-0.53
E/P	0.38	1.66	-0.23	-0.76	0.06	0.25	0.40	1.35
CE/P	0.48	1.96	-0.29	-0.91	0.03	0.12	0.53	1.66
D/P	0.39	1.80	-0.26	-0.90	-0.05	-0.26	0.45	1.59
BAB	0.41	1.14	0.57	1.40	0.00	0.01	0.41	0.97
QMJ	0.52	1.99	-0.06	-0.20	0.18	0.65	0.31	0.90
G_SMB	-0.14	-0.89	0.33	1.47	-0.28	-1.34	0.60	1.98
G_HML	-0.20	-0.89	0.73	2.43	-0.41	-1.58	0.91	2.59
G_RMW	0.10	0.55	0.28	1.31	0.21	0.97	0.10	0.40
G_CMA	-0.05	-0.31	0.56	2.41	0.05	0.23	0.11	0.37
G_UMD	0.27	0.63	0.27	0.55	0.58	1.48	-0.41	-0.85
G_E/P	0.09	0.39	0.26	0.92	-0.13	-0.40	0.77	1.74
G_CE/P	0.03	0.15	0.21	0.78	0.10	0.29	0.48	1.14
G_D/P	-0.11	-0.53	0.13	0.43	-0.17	-0.57	0.44	1.08
G_BAB	-0.05	-0.15	1.02	2.76	-0.17	-0.49	0.51	1.04
G_QMJ	0.14	0.57	0.44	1.53	0.25	0.97	0.14	0.47
Pooled	0.07	1.37	0.40	6.09	0.00	-0.04	0.38	5.16

Table 3.4: Critical t-values for Different Significance Levels

Significance Level	Critical t-value
10%	1.65
5%	1.97
1%	2.59

Four of the 20 the intercepts are found to be significant at the 10% level in both datasets. Dataset 1 identifies six out of 20 factor slopes as statistically significant at the 5% level, while Dataset 2 finds seven out of 20 factor slopes significant at the 10% level. This indicates that some European factors exhibit momentum. Additionally, the pooled regression supports this conclusion: although the intercepts are not significant, the slopes in both datasets are strongly significant at the 1% level, with t-values of 6.09 and 5.16, respectively. This finding aligns with the original study [1], which also found that for US factors, the slopes in the pooled regression were significant at the 1% level (t-value of 4.22).

3.2.2 Momentum Strategy

Finally, I implement a time-series momentum strategy, which involves taking long and short positions in factors (excluding Momentum) based on whether their cumulative returns over the past year are positive or negative. Additionally, I apply a cross-sectional strategy, which involves taking long positions in factors above the median and short positions in factors below the median. The time-series strategy yields an annualized return of 3.40% (t-value = 2.76) in Dataset 1 and 4.11% (t-value = 2.41) in Dataset 2. The cross-sectional strategy yields returns of 3.59% (t-value = 3.11) in Dataset 1 and 3.07% (t-value = 1.92) in Dataset 2. The time-series strategy outperforms the cross-sectional strategy in Dataset 2 but not in Dataset 1. This finding is surprising, as [22] suggests that the time-series strategy is a pure bet on momentum, while the cross-sectional strategy implies that a high return on a factor predicts low returns on other factors, which is typically contrary to momentum. The original study [1] also supports [22]. However, despite the statistical significance of the momentum strategies at the 5% level in European factors, they are not as robust as the US strategies, which were significant at the 1% level. This disparity could be attributed to the shorter data period for European factors, starting in 1990 compared to the original study's 1960 start, where these additional 30 year period display a greater momentum. This hypothesis is supported later in Section 4.2, where I discuss how the absence of this earlier data may explain the relative weakness in my findings.

In summary, I create two datasets with European factor data. Both datasets exhibit significant correlations, indicating consistency, and their summary statistics are comparable to those of US factors [1]. Factor momentum is demonstrated in two distinct ways: first, through a regression analysis of prior year returns, and second, through various factor momentum strategies. The significance of factor momentum provides enough evidence to justify the construction of a momentum factor based on these factors and to explore its implications. The following sections delve deeper into these aspects.

4

Factor Momentum and the Covariance Structure

In the following section, I construct a more robust factor momentum strategy and develop the idea proposed in Section 3 of constructing a momentum factor based on factor momentum. First, I explore the conditions under which factors exhibit momentum under the sentiment model proposed by Kozak, Nagel, and Santosh (KNS) [23]: The original study[1] hypothesizes that high eigenvalue factors display greater momentum. In the second part of the analysis, I empirically test this hypothesis and construct a more robust factor momentum strategy that trades principal component factors. Finally, I formalize the new momentum factor, Factor Momentum (FMOM), and instantiate it using the constructed factor momentum strategies.

4.1 KNS Sentiment Model

The KNS sentiment model [23] examines how sentiment-driven investors and rational arbitrageurs interact within financial markets, leading to factor momentum. Below is a conceptual explanation of the model and the agents involved involved in it:

Factor Structure of Returns

The KNS model builds on the APT [3], which posits that asset returns r_t at time t are determined by a few common factors plus some idiosyncratic randomness:

$$r_t = \beta f_t + \epsilon_t \tag{4.1}$$

Where:

- β: The matrix of factor loadings, indicating the sensitivity of each asset to each factor.
- f_t: The vector of factor returns at time t.
- ϵ_t: The vector of idiosyncratic errors at time t, with a mean of zero and unique to each asset.

Sentiment-Driven Investors

Sentiment-driven investors hold biased beliefs about asset returns, which affect their market behavior. These biases create demand shocks ξ_t, which influence asset prices:

$$\xi_t = AS_t \tag{4.2}$$

Where:

- S_t: Represents sentiment shocks at time t.
- A: The matrix that converts sentiment shocks into demand shocks.

Sentiment shocks represent sudden, unexpected changes in investor sentiment that can significantly influence market prices. These shocks can cause abrupt shifts in asset prices, impacting returns across various factors. To accurately quantify sentiment shocks, researchers often combine several metrics, each capturing different facets of investor sentiment. By integrating these diverse measures, a comprehensive understanding of sentiment shocks and their effects on the market can be achieved.

Rational Arbitrageurs

Rational arbitrageurs trade against sentiment-driven investors to exploit perceived mispricings. However, they face limitations due to risk aversion, capital constraints, and market frictions. These constraints imply that even if arbitrageurs recognize sentiment effects, they may not be able to fully exploit them due to practical limitations. As a result, sentiment-driven mispricings can persist, contributing to observed factor momentum. Their adjustment in position Δx_t is given by:

$$\Delta x_t = \alpha + \beta_x \xi_t \tag{4.3}$$

Where:

- α: Represents adjustments independent of sentiment, capturing fixed levels of risk-taking or market frictions.
- β_x: Measures the sensitivity of arbitrageurs' position adjustments to demand shocks. If $\beta_x < 1$, arbitrageurs cannot fully offset demand shocks. In a full neutralization scenario, $\Delta x_t = \xi_t$.

Market Equilibrium and Factor Returns

The interaction between sentiment-driven investors and rational arbitrageurs leads to market equilibrium, affecting factor returns r_t. The previous equations represent the general assumptions stated in the KNS model. In the paper [23], these assumptions are developed with more complexity and related to other asset pricing concepts, such as the stochastic discount factor. In one of cases, they assume that the sentiment-investor demand follows an AR(1) process, which results in the following equilibrium price:

$$R_{t+1} = D_{t+1} + a_1(\xi_{t+1} - \xi_t) - r_f a_0 - r_f a_1 \xi_t \tag{4.4}$$

Where:

- R_{t+1} is an $N \times 1$ vector of asset returns.
- D_{t+1} are the dividends.
- r_F is the risk-free rate.
- a_0 and a_1 are vectors of constants.
- ξ_t is sentiment-investor demand.

As a theoretical sentiment model, it is difficult to prove; however, it remains useful because empirical data consistently aligns with its underlying assumptions.

Persistence and Momentum

Building on the case for Equation 4.4, the original study [1] shows that the presence of factor momentum is equivalent to sentiment exhibiting a high autocorrelation. The KNS model implies that persistent sentiment leads to gradual mispricing adjustments over time, resulting in factor momentum which can by expressed by the following equation:

$$f_{t+1} = \rho f_t + \eta_t \tag{4.5}$$

Where:
- ρ: The autocorrelation coefficient.
- η_t: A noise term representing random shocks.

The study [1] also demonstrates that persistent sentiment has a more pronounced effect on high-eigenvalue factors, those that explain a significant portion of the variance in asset returns. These factors typically exhibit a higher ρ and, consequently, show more momentum. In contrast, low-eigenvalue factors, which account for a smaller portion of the variance in returns, are less influenced by sentiment persistence and therefore exhibit less momentum. Empirical evidence, such as the Baker and Wurgler sentiment index [24], supports the presence of high autocorrelation in sentiment, aligning with the KNS model's predictions. However, even with this understanding, rational arbitrageurs can not fully exploit these sentiment effects due to practical and risks associated with factor exposure. As a result, momentum persists in the market, particularly in high-eigenvalue factors where the impact of sentiment is more pronounced.

4.2 High Variance PCs and Factor Momentum

The orginal paper [1] showed that, if sentiment exhibits high autocorrelation, factors with high variance (or high eigenvalues) are expected to display stronger momentum. To empirically test this hypothesis, I introduce a large dataset of stock characteristics and conduct tests using Datasets 1 and 2, which yield statistically insignificant results. I then construct a more comprehensive factor dataset and apply a more robust test that demonstrates factors with higher variance indeed exhibit greater momentum.

4.2.1 Momentum in Systematic factors: Test 1

Stock Characteristics

For this analysis, I introduce a more large dataset that contains 12 million monthly stock observations globally and 405 stock characteristics. This dataset does not contain factor returns itself, but individual stock characteristics that can be used to compute factors based on these characteristics. It includes the original characteristics that are used to compute the off-the-shelf factors of Dataset 2, and it is accessible via Kelly's website through WRSD [25]. I filter the data to cover the period from July 1990 to December 2023 and excluded firms with a market equity below 0.01% of the aggregate European market each month. Additionally, I apply the filtering outlined in Kelly's documentation [21]. The final dataset consists of 414,837 observations and 446 columns, containing monthly stock characteristics and additional stock-specific information. However, my focus is on the 154[1] European characteristics identified by [18] as principal.

Correlation of Slope and Adjusted R^2 Scores

The first test follows the methodology outlined in the internet appendix IV of the original study [1]. I conduct a cross-sectional regression using the 10 European stock characteristics used to construct the factors listed in Table 3.2. The explanatory variables are the stock characteristics at month t, and the dependent variable is the stock's excess return at month $t+1$. For each characteristic, I calculate the adjusted R^2 score as the mean or median of all the R^2 scores, considering only those months with more than 15 observations. Finally, I examine the correlations between the adjusted R^2 scores and the slopes and t-values from the previous section. A high correlation between these metrics would indicate that more systematic factors, those explaining a larger portion of stock returns (high R^2), exhibit stronger momentum (high slope). The results of this analysis are presented in Tables 4.1 and 4.2.

Table 4.1: Factors' Slopes from Table 3.3 and Cross-Sectional Regression R^2

	Dataset 1 Slope		Dataset 2 Slope		Cross-Sectional Regression	
Anomaly	$\hat{\beta}$	$t(\hat{\beta})$	$\hat{\beta}$	$t(\hat{\beta})$	**Median R²**	**Mean R²**
SMB	0.49	2.31	0.65	2.78	0.31%	0.10%
HML	0.92	3.37	0.36	0.98	2.41%	1.05%
RMW	0.20	0.99	0.07	0.32	0.16%	-0.01%
CMA	0.54	2.92	0.21	0.98	0.22%	-0.01%
E/P	-0.23	-0.76	0.40	1.35	0.95%	0.42%
CE/P	-0.29	-0.91	0.53	1.66	0.51%	0.12%
D/P	-0.26	-0.90	0.45	1.59	1.19%	0.44%
BAB	0.57	1.40	0.41	0.97	4.03%	1.65%
QMJ	-0.06	-0.20	0.31	0.90	1.15%	0.43%

Table 4.2: Correlations between Slopes and R^2 scores

Correlations	**Dataset 1**		**Dataset 2**	
	$\hat{\rho}$	$t(\hat{\rho})$	$\hat{\rho}$	$t(\hat{\rho})$
$\rho(\hat{\beta}, R^2_{\text{median}})$	0.41	1.27	0.13	0.38
$\rho(\hat{\beta}, R^2_{\text{mean}})$	0.40	1.23	0.12	0.34
$\rho(t(\hat{\beta}), R^2_{\text{median}})$	0.17	0.49	-0.20	-0.58
$\rho(t(\hat{\beta}), R^2_{\text{mean}})$	0.16	0.47	-0.22	-0.63

Table 4.1 presents the slopes and t-values from both datasets, along with the mean and median R^2 scores obtained from the cross-sectional regressions. In Table 4.2,

[1] [18] identifies 153 main factors that can be grouped under 13 different topics. However, I combine 2 momentum characteristics (ret_{12_0} and ret_{2_0}) to create ret_{12_2}, which correspond to the original UMD factor [9].

I provide the correlations between these R^2 scores and the slopes and t-values. Contrary to the findings for US factors [1], the European data does not exhibit sufficient significance to draw definitive conclusions based on this limited number of regressions. Dataset 1 shows a weak correlation between the R^2 scores of a factor and its momentum, ranging from 0.16 to 0.41. In contrast, Dataset 2 produces opposite results, with correlations ranging from 0.13 to -0.22. Since none of these results are significant, further testing is conducted in the subsequent section to thoroughly evaluate this hypothesis.

4.2.2 Momentum in Systematic Factors: Test 2

Characteristic based Factors

The previous off-the-shelf factors are computed using the Fama and French [6] or very similar methodologies, which involve rebalancing portfolios monthly and calculating the difference between some of the top and bottom deciles. However, a more recent approach to constructing these factors was introduced in 2020 by KNS [26]. This method begins by assigning weights to each individual stock characteristic within a given month across all stock observations for that period. By combining the weighted returns of all stocks, more dynamic factors can be created that capture non-linear effects and higher-order interactions. Moreover, instead of creating a separate portfolio for each characteristic and then computing a factor based on that characteristic, this approach allows investors to build all factors simultaneously, simplifying and speeding up the process, especially when dealing with a large number of factors. The detailed process is as follows:

1. **Rank Transformation**

   Each firm characteristic $c_{i,t}$ is transformed into a cross-sectional rank $r_{c_{i,t}}$ for month t:

   $$r^s_{c_{i,t}} = \frac{\text{rank}(c^s_{i,t})}{n_t + 1} \tag{4.6}$$

   Where n_t is the number of stocks in month t.

2. **Centering and Normalizing**

   The ranks are centered around zero and normalized by the sum of absolute deviations from the mean:

   $$w_{i,t} = \frac{r^s_{c_{i,t}} - \bar{r}^s_{c_{i,t}}}{\sum_{i=1}^{n_t} \left| r^s_{c_{i,t}} - \bar{r}^s_{c_{i,t}} \right|} \tag{4.7}$$

   If a firm's characteristic $c^s_{i,t}$ is missing, the weight corresponding to this characteristic is set to zero. The normalization and ranking procedures improve stability by reducing the impact of outliers and market conditions.

3. **Calculating Factor Returns**

   The return on a factor based on characteristic j for month t is calculated as:

$$f_t^j = \sum_{i=1}^{n_t-1} w_{i,t-1}\, r_{i,t} \tag{4.8}$$

   Where $r_{i,t}$ are returns of firm i in month t.

Using this methodology, I create Dataset 3, which consists of the characteristic-based factors derived from the stock characteristics dataset. Dataset 3 includes the 154 main factor excess returns of [18], covering the period from August 1990 to December 2023. The average correlation with the common factors between Dataset 2 and 3 is 0.72, indicating a strong consistency between these datasets. One important observation is that the stock characteristics dataset provides excess returns at $t+1$, but not raw returns. Consequently, in the third step I use excess returns instead of raw returns. This means that while Datasets 1 and 2 are expressed as percentages of raw returns, Dataset 3 is expressed as percentages of excess returns.

PC Momentum Strategy

Following the methodology of the original study [1], I use Dataset 3 to compute a momentum strategy based on the factor's principal components (PCs). The hypothesis is that if systematic factors exhibit more momentum, a momentum strategy would yield stronger returns on the first PCs. I exclude all factors related to momentum[2] , as well as those with missing values for extended periods, resulting in a final selection of 120 factors. The strategy that trades on principal factors is structured as follows:

1. **Compute Eigenvectors Using Monthly Data**

   I compute the correlation matrix of the 120 factors using monthly returns from July 1973 up to month t. Principal Component Analysis (PCA), after standardizing the data, is applied to extract the eigenvectors from this correlation matrix. These eigenvectors represent the principal components (PCs) of the monthly factor returns. The original authors required 10 years of data to start computing the strategy, but due to the shorter sample period, I reduce the requirement to 5 years. Thus, my strategy returns span from August 1995 to December 2012.

2. **Compute Monthly Returns for PC Factors**

   For each month t, I compute the returns of the new set of PC factors up to $t+1$. Given the eigenvectors extracted at month t, the strategy requires recalculating all PC returns up to month t and then adding the returns for month $t+1$. This step is necessary because the eigenvectors change each month, so the PC returns must be adjusted before executing the trade. In a real-world setup, only data up to month t is available, making the returns for

[2] I remove 14 factors related to momentum. 9 correspond to pure momentum and 5 to short term reversals.

$t+1$ the target of the trade. The return for the f-th PC factor at month t can be computed as:

$$r_{f,t}^{\mathrm{PC}} = \sum_{j=1}^{N} v_j^f \, r_{j,t} \tag{4.9}$$

where:

- v_j^f is the j-th element of the f-th eigenvector,
- $r_{j,t}$ is the return on individual factor j,
- N is the number of factors.

Intuitively, PC returns are weighted combinations of individual factor returns, where the weights are determined by the eigenvector dimensions. For example, if there are two factors, A and B, PCA would generate two principal components (PC1 and PC2) ordered by their eigenvalues. If the first principal component is expressed as $[v_A^1, v_B^1]$, the returns for PC1 at month t would be computed as:

$$r_{PC1,t} = v_A^1 \, r_{A,t} + v_B^1 \, r_{B,t} \tag{4.10}$$

3. **Demean and Adjust Variance of PC Factors**

   Using the PC returns data up to month t, I compute the variance for each individual factor. The next step is to adjust the PC factors by demeaning them (subtracting the mean) and scaling their variance to match the average variance of the individual factors. This ensures that the average returns up to month t are zero and that all factors have the same variance. Based on [27] and [28], the average factor returns tend to be positive, which could bias the strategy towards buying more than selling. Demeaning the factors neutralizes this bias, making the strategy a pure play on autocorrelations. Scaling the variance allows for the comparison of factors on an equal footing.

4. **Construct Factor Momentum Strategy**

   Using the monthly PC factor returns data, I calculate the average returns for each PC factor over the past 12 months (from $t-11$ to t). The momentum strategy is built by going long on the PC factors with positive average returns and short on those with negative average returns.

5. **Compute Return of the Momentum Strategy**

   Finally, I compute the returns of the momentum strategy at $t+1$ using the PC factor returns for month $t+1$.

I compute the PC factor momentum strategy ($FMOM_{PC}$) across various subsets of PCs, as well as a strategy that trades all PCs. It's important to note that this strategy is based on excess returns, this strategy is based on excess returns, so the reported figures in Table 4.3 represent average monthly excess returns..

Table 4.3: Performance of Momentum Strategies Across Principal Component Subsets

PC Subset	$\bar{\mathbf{r}}$	$\mathbf{t}(\bar{\mathbf{r}})$
$FMOM_{PC(1-25)}$	0.23	2.89
$FMOM_{PC(26-50)}$	0.10	4.47
$FMOM_{PC(51-75)}$	0.06	3.32
$FMOM_{PC(76-100)}$	0.05	3.22
$FMOM_{PC(101-120)}$	0.01	0.57
$FMOM_{PC(1-120)}$	0.10	3.96

Trading all the PC factors yields positive monthly average returns of 0.10% with a t-value of 3.96, indicating that factors display significant momentum, as already found in Section 3. Notably, the first PC subsets show higher returns and t-values compared to later subsets, reinforcing the idea that "more systematic factors display more momentum". While these findings support the initial hypothesis, it is not uncommon for some subsets, such as the second or third, to show stronger significance despite the first subset's higher average returns.

These results align with the observed patterns in U.S. factors during the time period I analyzed. In the original study [1], the PC is divided into two halves: the first covering 1970 to 1996, and the second from 1996 to 2019, which overlaps with my own period of study. In the first half, the study's results are more significant, whereas in the second half, the results are less significant and similar my findings. This strengthens the argument I made in Section 3 that factor momentum exhibits similar importance across both the U.S. and Europe but varies across different time periods, being stronger in earlier years. This time-period difference—where results tend to be weaker in later years—emerges as a recurring pattern in my analysis. To maintain clarity and consistency, I will briefly revisit this finding in later sections, noting how time-period effects influence the significance of factor momentum across various datasets and models, without going into detail each time.

Comparing $\mathbf{FMOM_{PC}}$ Strategies

Finally, I examine the relationships among different subsets of PC factors. Regressions are conducted where the highest eigenvalue strategy, $FMOM_{PC(1-25)}$, serves as the explanatory variable, and the target variables are the lower eigenvalue strategies, augmented with the FF5 model[3], as shown in Equation (4.11). I also perform the inverse regression, where $FMOM_{PC(1-25)}$ is the target variable, and the explanatory variables are the lower eigenvalue strategies combined with the FF5 model, as shown in Equation (4.12).

$$FMOM_{PC(\text{26-50}),t} = \alpha + \beta FMOM_{PC(\text{1-25}),t} + FF5_t + \epsilon_t^{(\text{1-25})} \tag{4.11}$$

$$FMOM_{PC(\text{1-25}),t} = \alpha + \beta FMOM_{PC(\text{26-50}),t} + FF5_t + \epsilon_t^{(\text{26-50})} \tag{4.12}$$

[3]Note that writing FF5 refers to computing a regression for the five factor model with 5 different coefficients for each factor (Eq. 2.4)

In this context, a significant alpha indicates that the momentum strategy being targeted cannot be fully explained by the explanatory variables, suggesting the presence of additional sources of returns not captured by the explanatory variables. Conversely, a non significant alpha suggests that the target variable is redundant, as it can be explained by the explanatory variables, in the literature it is common to say "the explanatory variable spans the target variable". On the other hand, a significant slope coefficient demonstrates a strong relationship between the explanatory and target variables, reflecting how changes in one are systematically associated with changes in the other. The results are summarized in Tables 4.4 and 4.5. I report the intercepts α and the coefficients β with their corresponding t-values in parenthesis. For all subsequent tables, values presented in parentheses will refer to the t-values of the corresponding coefficients.

Table 4.4: Explaining low eigenvalue strategies with $FMOM_{PC(1-25)}$ (Eq. 4.11).

Coefficients	**Target Variable**			
	$\mathbf{FMOM_{PC(26-50)}}$	$\mathbf{FMOM_{PC(51-75)}}$	$\mathbf{FMOM_{PC(76-100)}}$	$\mathbf{FMOM_{PC(101-120)}}$
$\hat{\alpha}$	0.7 (3.43)	0.03 (1.58)	0.02 (1.35)	-0.02 (-1.80)
$\hat{\beta}$	0.13 (7.51)	0.13 (8.72)	0.11 (10.75)	0.12 (12.68)
Adj. R^2	27.65%	28.61%	45.51%	51.48%

Table 4.4 shows that PC factor momentum strategies are significantly correlated, with slope t-values between 7.51 and 12.68. This is noteworthy, considering that the individual PC factors are nearly orthogonal. According to the original study [1], these positive correlations suggest synchronization among the factors, meaning they tend to be either profitable or unprofitable simultaneously. Similarly, the slope coefficients from other regressions in the project are consistently significant, which is expected; therefore, I will present the results in future tables without further explicit analysis of these coefficients. The alphas indicate that the strategy trading the first PC factors spans the three lower-order PC strategies but is not able to span the 26-50 strategy due to the significant coefficient (3.43). Additionally, there is an observed alpha decay that contrasts with [26], which suggested that a model with a few number of low-order PC factors could effectively expected returns of anomaly portfolios.

Table 4.5: Explaining $FMOM_{PC(\text{1-25})}$ with low eigenvalue strategies (Eq. 4.12)

Coefficients	**Explanatory Variable**			
	$\mathbf{FMOM_{PC(26\text{-}50)}}$	$\mathbf{FMOM_{PC(51\text{-}75)}}$	$\mathbf{FMOM_{PC(76\text{-}100)}}$	$\mathbf{FMOM_{PC(101\text{-}120)}}$
$\hat{\alpha}$	0.16 (2.68)	0.18 (3.26)	0.17 (3.08)	0.24 (4.71)
$\hat{\beta}$	1.09 (7.51)	1.45 (8.72)	2.38 (10.75)	2.70 (12.68)
Adj. R^2	54.41%	56.60%	60.40%	64.03%

Table 4.5 shows that the conclusions from Table 4.4 do not hold the other way around. The correlations hold, but the alphas are always significant, ranging from 2.68 to 4.71, and do not decay with lower eigenvalue strategies, meaning that a FMOM strategy based on the first PC factors contains information that the lower eigenvalue strategies are not able to capture.

To summarize, more systematic factors are consistently more profitable, in line with the KNS sentiment model [26]. When measuring momentum, high eigenvalue factors significantly enhance the explanatory power of the cross-section, as they can explain low eigenvalue factors, but not vice versa. This also suggests that high eigenvalue factors may be more predictable, offering superior explanatory power and better capturing systematic risks [29]. These findings are consistent with observations in U.S. markets [1], and suggest that the variations in significance are attributed to time differences rather than geographical disparities

4.3 Factor Momentum: Conceptualizing Momentum Strategies as Factors

In financial analysis, momentum strategies are often employed to measure the persistence of asset returns over time. In an asset pricing context, this persistence can be used as evidence to construct a factor that targets this persistence, as I already discussed in Section 3. Now my aim is to conceptualize the different factor momentum strategies that I constructed, as a momentum factor that targets factor momentum.

1. Factor Momentum as a Factor

Let us define this new factor as Factor Momentum (FMOM). It represents a specialized form of factor analysis where the momentum of factors is scrutinized as an independent concept. As we have seen in Section 1, traditional momentum factors target individual stock momentum. The classical momentum factor is UMD but there can be other types of momentum factors that measure momentum in different ways. The proposed FMOM shifts the focus to the momentum of factors themselves. Whereas the UMD methodology involves building different portfolios and then computing the difference between long and short legs, one can directly compute an FMOM factor by going long/short on the top/bottom performers. This conceptualization is crucial because it allows us to view momentum strategies not merely as trading tactics but as methodology to construct momentum factors. This shift on the focus from individual stocks to factors can allow to understand better and exploit systematic patterns that influence broader market dynamics.

2. $\mathbf{FMOM_{PC}}$: Principal Components Momentum as a Factor

The $FMOM_{PC}$ strategy exemplifies this concept effectively. In this strategy, PC factors are employed to capture and exploit momentum effects. By trading these PCs, which are weighted averages of the original factors, $FMOM_{PC}$ essentially becomes a specific instance of FMOM. This approach leverages the systematic components of factors (those with high eigenvalues) to maximize momentum opportunities.

3. $\mathbf{FMOM_{ind}}$: Time-Series Momentum as a Factor

Similarly, the time-series momentum strategy computed earlier in Section 2 embodies the same principle. It represents an alternative method for calculating FMOM, targeting persistence in individual factors rather than PC factors, though, as shown in this section, it is less significant than $FMOM_{PC}$. From now on, I denote $FMOM_{ind}$ as the time-series momentum strategy applied to the 134 individual factors from Dataset 3, excluding momentum factors.

4. Implications and Broader Context

By framing momentum strategies as factors, we gain a deeper insight into their role within the factor structure of financial markets. This perspective enables a comprehensive analysis of how these strategies interact with factors and influence market dynamics. It also emphasizes that FMOM is not limited to a particular strategy but is a broad concept that spans various approaches to trading and analysis.

In this section, I reviewed the KNS sentiment model [23] and how persistent sentiment demand can lead to persistence in factor returns. Additionally, the model suggests that more systematic factors exhibit stronger momentum, a finding that aligns with my results and the original study [1]. Notably, this alignment indicates that momentum strengths are comparable for both European and U.S. factors. I then introduced FMOM as a factor measuring momentum through factor momentum and constructed it as an instantiation of the strategies analyzed in previous sections. A natural question that arises now is how FMOM compares to factors based on individual stock momentum, such as UMD, which will be explored in the next section.

5

Factor Momentum and Individual Stock Momentum

Exploring the relationship between FMOM and individual stock momentum reveals insights into their interactions and significance in factor analysis. I begin by analyzing the various sources that can generate momentum in individual stocks. Then, I demonstrate that FMOM not only significantly contributes to individual stock momentum but also fully captures it, as it successfully explains UMD. Next, I extend the analysis to compare FMOM with other forms of individual stock momentum: it also explains most of the momentum factors. Moreover, none of the factors are able to explain it, which implies that FMOM captures additional sources of momentum. This explainability depends on the instance of FMOM, so I subsequently explore its strength across different subsets of factors, where $FMOM_{PC(120)}$ stands as the most robust method. Finally, I delve deeper into one of the potential sources of individual stock momentum, firm-specific momentum, showing that it is insignificant.

5.1 Sources of Individual Stock Momentum

If stock returns follow a factor structure, momentum in these factors can be transmitted to individual stock returns, leading to cross-sectional momentum [30]. In multifactor models like CAPM and APT, the excess return of stock i at time t ($R_{i,t} = r_{i,t} - r_{f,t}$) is given by F factors:

$$R_{i,t} = \sum_{f=1}^{F} \left(B_i^f r_t^f + \epsilon_{i,t} \right) \tag{5.1}$$

Where:

- B_i^f is the loading of stock i on factor j.
- r_t^f is the return of factor j at time t.
- $r_{f,t}$ is the risk-free rate at time t.
- $\epsilon_{i,t}$ is the idiosyncratic error term for stock i.

Under a multifactor model, the original study [1] examines the payoffs of a cross-sectional momentum strategy on individual stocks and concludes that the expected value of a momentum strategy portfolio π_t^{mom} can be expressed as:

$$\mathbb{E}[\pi_t^{mom}] = \sum_{f=1}^{F} \left[\text{cov}(r_{t-1}^f, r_t^f) \sigma_{\beta^f}^2 \right] + \sum_{f=1}^{F} \sum_{\substack{g=1 \\ g \neq f}}^{F} \left[\text{cov}(r_{t-1}^f, r_t^g) \text{cov}(\beta^f, \beta^g) \right] + \frac{1}{N} \sum_{i=1}^{N} [\text{cov}(\epsilon_{i,t-1}, \epsilon_{i,t})] + \sigma_\eta^2 \tag{5.2}$$

Where:

- N is the number of stocks.
- $\sigma_{\beta^f}^2$ is the cross-sectional variance of the portfolio loadings.
- σ_η^2 is the cross-sectional variance of the stocks' unconditional expected returns.

Equation 5.2 contains four terms that explain the potential sources of profit in a cross-sectional momentum strategy.

1. **Autocorrelation in Factor Returns:** When factor returns exhibit positive autocorrelation, it contributes to momentum profits. This effect is further amplified by variations in the factor loadings across different stocks. Even if individual factors show only weak autocorrelation, a larger number of factors can lead to noticeable momentum in stock returns.
2. **Lead-Lag Relationships Between Factors:** Momentum profits can also arise from timing differences in factor returns. This is influenced by the cross-serial covariance between factor returns and the relationships between factor loadings. For example, if the return of one factor today predicts the return of another factor tomorrow, and if the loadings on these factors are positively correlated, this can enhance momentum profits.
3. **Autocorrelation in Firm-Specific Returns:** Momentum profits may stem from the autocorrelation in the returns of individual stocks. This implies that if a stock's returns exhibit patterns of persistence or reversal, it can contribute to the overall success of momentum strategies.
4. **Variation in Mean Returns Across Stocks:** Differences in average returns among stocks contribute to momentum profits through mechanisms like those proposed by [31]. This means that variations in the expected returns of individual securities play a role in the effectiveness of momentum strategies.

Understanding the implications of these sources is crutial for the remaining sections. Equation 5.2 defines the potential sources, but does not define the magnitude of each source. By definition, FMOM directly captures sources 1 and 2, so if these are significant contributors to momentum, FMOM should be able to explain a significant of individual stock momentum. Classical models like the FF5 implicitly capture source 4, suggesting that a combination of FMOM and FF5 would be sufficient to account for sources 1, 2, and 4. However, none of the factor models address source 3, meaning that in the presence of firm-specific return autocorrelation, FMOM should not be able to explain all of the individual stock momentum. These implications are the focus of the following sections. Lastly, note that source 4 implies a relationship between individual stock momentum and factors, as these aim to capture differences in mean returns. This connection is further examined in Section 6.

5.2 Does FMOM contribute to individual stock momentum?

Now, I examine the extent to which FMOM contributes to individual stock momentum, finding that sources 1 and 2 are significant. I then explore the relationship between FMOM and UMD, and show that FMOM combined with FF5 directly spans UMD. Together, they capture all four sources of individual stock momentum.

5.2.1 Pricing UMD-sorted portfolios with UMD and FMOM

If FMOM contributes to individual stock momentum, the fit of a model combining FMOM with FF5 (Eqs. 5.5 and 5.6) should significantly improve over a model that includes only FF5 (Eq. 5.3). Such an improvement would suggest that sources 1 and 2 are key drivers of momentum, as FMOM predominantly captures these two sources. To test this hypothesis, I use various asset pricing models to price portfolios sorted by individual stock momentum. To create these portfolios I use UMD-based sorting: stocks are sorted monthly into deciles according to their prior year returns, skipping a month. I then calculate the excess return for each decile portfolio as the excess returns from the previous month. For each decile, I regress the portfolio's excess return at month t on different sets of explanatory variables:

$$R_{i,t} = \alpha + FF5_t + \epsilon_t \tag{5.3}$$

$$R_{i,t} = \alpha + FF5_t + \beta UMD_t + \epsilon_t \tag{5.4}$$

$$R_{i,t} = \alpha + FF5_t + \beta FMOM_{ind} + \epsilon_t \tag{5.5}$$

$$R_{i,t} = \alpha + FF5_t + \beta FMOM_{PC(1-120)} + \epsilon_t \tag{5.6}$$

These regressions assess how well different factor models explain individual stock momentum portfolios, indicating the contribution of each set of factors. Since the portfolios are constructed using the same sorting method as UMD, the combination of UMD and the FF5 model (Eq. 5.4) serves as a natural benchmark. In assessing the fit of each model, I focus on two key metrics:

Alpha: Alphas serve as a proxy for understanding how well a model captures portfolio returns. Low alphas suggest that the factors in the model explain the returns effectively.

GRS F-value: While alphas provide an intuitive sense of fit, the Gibbons, Ross, and Shanken (GRS) F-value [32] offers a more comprehensive measure. It accounts not only for the alphas but also for the residuals of the model, making it a stricter test of overall model performance. The GRS test sets the null hypothesis that the model is valid, meaning all alphas should equal zero. Thus, a lower GRS F-value indicates a better fit. The GRS F-value is expressed as:

$$GRS = \frac{(T - N - K)}{N} \cdot \frac{\alpha' \Sigma^{-1} \alpha}{1 + \bar{f}' \Omega^{-1} \bar{f}} \tag{5.7}$$

Where:

- T: Number of periods.
- N: Number of portfolios.
- K: The number of factors in your asset pricing model.
- α: An $N \times 1$ vector representing the intercepts (alphas) from the regression on the portfolios' excess returns.
- Σ: The $N \times N$ covariance matrix of the residuals.
- Ω: A $K \times K$ covariance matrix of the factor returns.
- $\bar{f}$: The vector of factor means.

A significant reduction in the GRS F-value when incorporating FMOM into the FF5 model would indicate that it meaningfully contributes to explain individual stock momentum, confirming that sources 1 and 2 are significant.

To ensure consistency, I perform each regression three times, using three different sets of portfolios and factors:

1. **First Set**
   The target variables are the excess returns of quintile momentum portfolios, constructed from the 5x5 portfolios on size and momentum available on the Fama-French website [7]. The FF5 and UMD factors are sourced from Dataset 1. The results are summarized in Table 5.1.

2. **Second and Third Sets**
   The target variables in these regressions are decile momentum portfolios, formed using the UMD sorting method based on the stock characteristics dataset. Since this dataset is used to construct Datasets 2 and 3, I compute the regressions twice, once with the FF5 and UMD factors from Dataset 2 and again from Dataset 3. The average results are reported in Table 5.2.

In regards to $FMOM_{PC}$ and $FMOM_{ind}$, I utilize the two instances defined in Section 4.3. These factors are computed using Dataset 3, as there is insufficient data to compute them using Datasets 1 or 2. Consequently, one would expect that the regressions on the first set yield worse results.

Table 5.1: Portfolio regressions for quintile portfolios

Quintile	FF5	FF5 + UMD		FF5 + $\mathbf{FMOM_{ind}}$		FF5 + $\mathbf{FMOM_{PC(1-120)}}$	
	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Losers	-0.41 (-3.26)	0.07 (1.62)	-0.64 (-54.44)	-0.18 (-1.98)	-1.79 (-19.47)	-0.15 (-1.51)	-4.05 (-18.09)
2	-0.13 (-2.18)	0.05 (1.26)	-0.23 (-22.15)	-0.03 (-0.59)	-0.66 (-13.12)	0.01 (0.19)	-1.54 (-13.07)
3	0.01 (0.30)	0.01 (0.25)	0.00 (0.20)	0.02 (0.59)	-0.03 (-0.73)	0.05 (1.04)	-0.13 (-1.32)
4	0.19 (3.57)	0.02 (0.67)	0.22 (23.93)	0.11 (2.53)	0.61 (13.10)	0.11 (2.28)	1.34 (11.95)
Winners	0.54 (5.58)	0.18 (4.42)	0.48 (42.89)	0.38 (4.86)	1.24 (15.75)	0.32 (3.83)	2.91 (15.64)
Winners - Losers	0.75 (3.58)	-0.08 (-1.85)	1.12 (91.68)	0.36 (2.41)	3.03 (19.67)	0.30 (1.78)	6.94 (18.70)
Avg $\lvert\hat{\alpha}\rvert$	0.34	0.07		0.18		0.15	
GRS F-value	13.31	5.07		7.86		4.67	
GRS p-values	0.00%	0.01%		0.00%		0.01%	

Examining the GRS F-values of the quintile portfolios priced using Dataset 1 factors allows for meaningful comparisons, yielding similar results to those observed with U.S. portfolios [1]. The FF5 model performs relatively poorly when pricing these portfolios, yielding an F-value of 13.31. However, performance improves significantly when augmenting FF5 with UMD, resulting in an F-value of 5.07, which captures the four sources of individual stock momentum. Furthermore, FMOM significantly enhances the FF5 model, indicating that FMOM contributes to individual stock momentum through sources 1 and 2. Specifically, $FMOM_{ind}$ yields an F-value of 7.86, while $FMOM_{PC(1-120)}$ surprisingly outperforms UMD, achieving a GRS F-value of 4.67. As noted by the original study [1], this is unexpected, as UMD is specifically designed to target these momentum portfolios.

All GRS p-values are significant, indicating that none of these models can be considered fully valid. One potential explanation for this poor fit is that I could only obtain quintile portfolios based on Dataset 1 factors, rather than decile portfolios, resulting in broader portfolios that are harder to price accurately. Additionally, while the FMOM variables are expected to be highly correlated, they originate from different datasets.

Another notable observation is that my results for FMOM in European markets are relatively weaker than those found for U.S. markets. In the original study [1], both $FMOM_{ind}$ and $FMOM_{PC}$, which traded only 20 factors each, outperformed Carhart's UMD. However, in my analysis, $FMOM_{ind}$ is constructed using 134 factors from Dataset 3 (excluding momentum) and fails to surpass UMD. When constructed with the 18 non-momentum factors from Dataset 1, the GRS F-value rises to 9.55, further worsening the fit. Similarly, $FMOM_{PC}$ only outperforms UMD when using its most comprehensive form, $FMOM_{PC(1-120)}$. Using $FMOM_{PC(1-25)}$ results in a GRS F-value of 7.83. As discussed in Section 4.2, the main reason for these discrepancies seems to be the missing data from 1960 to 1990 in my analysis.

Table 5.2: Mean regression results for decile portfolios using Dataset 2 and Dataset 3

Decile	FF5	FF5 + UMD		FMOM$_{ind}$		FF5 + FMOM$_{PC(1-120)}$	
	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Losers	-0.72 (-4.16)	-0.07 (-0.56)	-1.28 (-19.24)	-0.31 (-1.97)	-2.09 (-10.72)	-0.18 (-1.06)	-4.86 (-10.70)
2	-0.37 (-3.15)	0.11 (1.36)	-0.92 (-23.08)	-0.08 (-0.73)	-1.51 (-11.73)	-0.04 (-0.37)	-3.21 (-10.48)
3	-0.07 (-0.72)	0.21 (2.44)	-0.52 (-12.53)	0.11 (1.12)	-0.88 (-7.35)	0.17 (1.55)	-1.99 (-6.96)
4	-0.15 (-1.94)	-0.05 (-0.62)	-0.18 (-5.03)	-0.12 (-1.46)	-0.19 (-1.99)	-0.11 (-1.22)	-0.37 (-1.59)
5	0.10 (1.19)	0.13 (1.49)	-0.04 (-1.29)	0.10 (1.15)	-0.05 (-0.45)	0.12 (1.27)	-0.06 (-0.24)
6	0.16 (2.18)	0.09 (1.23)	0.14 (3.45)	0.08 (1.05)	0.27 (2.91)	0.04 (0.51)	0.76 (3.44)
7	0.25 (3.41)	0.06 (0.92)	0.37 (10.80)	0.14 (1.89)	0.58 (6.58)	0.13 (1.62)	1.38 (6.61)
8	0.27 (3.23)	0.01 (0.08)	0.50 (14.50)	0.09 (1.20)	0.89 (9.33)	0.08 (0.89)	1.86 (8.21)
9	0.12 (1.16)	-0.27 (-3.46)	0.74 (19.24)	-0.09 (-0.92)	1.04 (8.64)	-0.11 (-0.96)	2.21 (7.52)
Winners	0.61 (4.24)	0.09 (0.81)	0.99 (18.09)	0.25 (1.91)	1.77 (11.02)	0.18 (1.19)	3.83 (9.84)
Winners − Losers	1.33 (5.06)	0.16 (1.06)	2.27 (29.26)	0.57 (2.53)	3.86 (14.16)	0.36 (1.46)	8.69 (13.43)
Avg $\lvert\hat{\alpha}\rvert$	0.38	0.11		0.18		0.13	
GRS F-value	15.76	2.43		3.42		1.81	
GRS p-values	0.00%	0.83%		0.02%		5.59%	

The decile portfolios priced using Dataset 2 and Dataset 3 exhibit similar relative performance to those in Dataset 1. The FF5 model shows the highest GRS F-value at 15.76, which significantly decreases to 2.43 when UMD is introduced.

Additionally, $FMOM_{ind}$ improves the fit, yielding a GRS F-value of 3.42. Notably, $FMOM_{PC(1-120)}$ surpasses UMD, achieving a GRS value of 1.81.

In this case, the absolute performance of the models is better. While the FF5 model still yields a p-value close to 0, combining it with momentum factors results in substantially increased GRS p-values compared to Table 5.1. The model combining FF5 with FMOM_{PC} yields a p-value of 5.59%, indicating that none of the alphas are significantly different from zero; therefore, the model fits well in explaining all sources of individual stock momentum.

As seen in Table 5.1, to outperform UMD, it is necessary to use the strongest form of FMOM. $FMOM_{ind}$ is dominated by UMD, and constructing a less demanding version of $FMOM_{ind}$ that trades the 20 factors from Dataset 2 significantly worsens the GRS F-value to 8.08. When utilizing $FMOM_{PC(1-25)}$, the GRS value increases to 2.84, which is still a respectable F-value but falls short of UMD.

Overall, FMOM demonstrates a strong ability to explain individual stock momentum through sources 1 and 2. Furthermore, $FMOM_{PC(1-120)}$ not only outperforms UMD but also provides a valid empirical model, suggesting that it might be a better factor for explaining individual stock momentum than UMD.

5.2.2 Explaining UMD with FMOM

Tables 5.1 and 5.2 are consistent showing that FMOM to individual stock momentum through sources 1 and 2. Moreover, FMOM might capture individual stock momemntum better than UMD. To further investigate this hypothesis, I conduct a regression where UMD is the target variable, and the explanatory variables comprise various $FMOM_{PC}$ strategies augmented by the FF5 model (Eq. 5.8). The results of this analysis are reported in Table 5.3

$$UMD_t = \alpha + FF5_t + \beta \text{FMOM}_{\text{PC(Subset)}} + \epsilon_t \tag{5.8}$$

Table 5.3: Explaining UMD with FMOM

	FF5		$\mathbf{FMOM_{PC(Subset)}}$		$\mathbf{R^2}$
Subset	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$	
None	0.40	5.42			48.74%
1-25	0.18	2.55	0.78	12.68	66.14%
26-50	0.27	3.25	1.17	5.70	54.28%
51-75	0.29	3.64	1.70	7.30	56.73%
76-100	0.25	3.37	3.21	10.53	62.33%
101-120	0.35	4.76	3.02	9.61	60.69%
1-120	0.13	1.81	2.50	13.77	68.00%

The regressions indicate that FMOM can explain UMD and capture the four sources of individual stock momentum, particularly when utilizing the most comprehensive version that trades all the PC factors (t-value of 0.13). In contrast, the strategies that trade lower eigenvalue factors obtain significant alphas that do not span UMD.

Notably, models incorporating higher eigenvalue factors significantly reduce alpha compared to those using lower eigenvalue factors. For instance, I obtained a t-value of 2.55 for the first 25 PC factors, which increases to 4.76 in the lowest eigenvalue subset. This observation aligns with findings from Section 4 and the original study [1], which suggest that higher eigenvalue factors are more explanatory.

To summarize, I validate the hypothesis that factor momentum contributes to individual stock momentum by using various asset pricing models to explain portfolios sorted on UMD. Furthermore, the results reveal that FMOM outperforms UMD when using $FMOM_{PC(1-120)}$. This raises the question of whether FMOM can can explain momentum better than UMD. Time-series regressions confirm that this is indeed the case, demonstrating that FMOM can also explain UMD and capture at least its four sources. These findings prompt two critical questions: First, does this relationship hold when comparing FMOM to other types of momentum factors? Second, is FMOM capable of capturing additional sources of profits that individual momentum factors do not?

5.3 Alternative Momentum Factors: Spanning Tests

In addition to the standard UMD factor, I extend the analysis to show how FMOM also explains other forms of individual stock momentum. Morevoer, I run spanning tests to show that FMOM captures additional sources of profits not accounted for by any other momentum factors. Using the original UMD methodology [9] on the stock characteristics dataset, I construct three alternative momentum factors and recompute UMD. This recomputation is necessary because the UMD factor in Dataset 3 follows the methodology outlined in Section 4, rather than the original UMD approach, which sorts stocks based on their prior year returns, skipping a month. Similarly, these additional factors create sortings based on other variables: Intermediate Momentum [11], which uses returns from $t-7$ to $t-12$ months; Year 1-lagged return Momentum (Lagged Momentum) [33], which considers returns lagged by one year; and Current price to high price over the last year Momentum (CP/HP Momentum) [34], which sorts based on the current price divided by the highest price over the past year. Table 5.4 provides summary statistics on the monthly excess returns of these factors, expressed as percentages.

Table 5.4: Performance of Different Types of Momentum

Momentum Definition	Monthly Returns			FF5 Model	
	$\bar{\mathbf{r}}$	**Std. Dev.**	$\mathbf{t}(\bar{\mathbf{r}})$	$\hat{\boldsymbol{\alpha}}$	$\mathbf{t}(\hat{\boldsymbol{\alpha}})$
Individual Stock Momentum					
Standard Momentum	0.55	4.06	2.75	0.75	4.80
Intermediate Momentum	0.50	3.30	3.04	0.64	4.39
Lagged Momentum	0.40	4.20	1.91	0.60	3.92
CP/HP Momentum	0.43	4.68	1.87	0.57	3.42
Factor Momentum					
$FMOM_{ind}$	0.18	1.00	3.56	0.20	5.01
$FMOM_{PC}$	0.10	0.46	3.96	0.11	5.31

Standard and Intermediate Momentum show statistical significance, with t-values of 2.75 and 3.04, respectively. In contrast, Lagged and CP/HP Momentum are near the 5% significance threshold of 1.97. The FF5 model alphas are also significant, ranging from 3.42 to 4.80, indicating that the strategies based on these types of momentum offer a premium unexplained by the classical five-factor model. This suggests they could serve as potential additions to a traditional pricing model. Additionally, the FMOM factors demonstrate the strongest significance, with t-values of 3.56 and 3.96. The FF5 model alpha t-values are also the highest (5.01 and 5.31), suggesting that augmenting the FF5 with FMOM provides a better fit than using any of the individual stock momentum measures. To further validate these findings, I compute regressions where the target variable are the different types of individual stock momentum (UMD_t^*) and the explanatory variables are the two main FMOM strategies augmented with FF5. The results of these regressions are summarized in Table 5.5.

$$UMD_t^* = \alpha + FF5_t + \beta FMOM_t + \epsilon_t \tag{5.9}$$

Table 5.5: Explaining Individual Stock Momentum with FMOM

Explanatory Variable	$\mathbf{FMOM_{ind}}$		$\mathbf{FMOM_{PC(1-120)}}$	
	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$
Standard Momentum	0.28 (2.18)	2.41 (15.23)	0.17 (1.17)	5.40 (13.96)
Intermediate Momentum	0.40 (2.77)	1.29 (7.36)	0.35 (2.18)	2.94 (6.96)
Lagged Momentum	0.08 (0.71)	2.62 (18.31)	0.00 (-0.02)	5.85 (16.89)
CP/HP Momentum	0.12 (0.86)	2.30 (13.06)	0.13 (0.77)	4.91 (11.52)

The first row shows consistency with previous findings: regressing the recomputed Standard Momentum on $FMOM_{PC(1-120)}$, yields intercept and slope t-values (1.17 and 13.96) that closely match those in Table 5.3 (1.81 and 13.77). The most comprehensive version of FMOM continues to effectively span UMD. In contrast, $FMOM_{ind}$

remains less robust than $FMOM_{PC}$ and fails to explain UMD, as evidenced by the significant alpha (2.18).

For the alternative momentum factors, both FMOM variants successfully span all except Intermediate Momentum, which exhibits t-values for alpha of 2.77 and 2.18, respectively. These results are consistent with the original study [1], although the significance levels are lower due to the time-period difference discussed in Section 4.2. Additionally, $FMOM_{ind}$ consistently yields higher alphas, reinforcing the view that $FMOM_{PC}$ is the stronger variant of FMOM.

Since FMOM can explain most individual momentum factors, I also performed inverse regressions to assess whether the alternative momentum factors can span FMOM, and whether $FMOM_{PC}$ and $FMOM_{ind}$ can expand each other. The results of these inverse regressions are presented in Table 5.6

$$FMOM_t = \alpha + FF5_t + \beta UMD_t^* + \epsilon_t \tag{5.10}$$

Table 5.6: Explaining FMOM with UMD

Target Variable	$\mathbf{FMOM_{ind}}$		$\mathbf{FMOM_{PC(1\text{-}120)}}$	
	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$
Standard Momentum	0.08 (2.49)	0.16 (15.23)	0.06 (3.38)	0.07 (13.96)
Intermediate Momentum	0.14 (3.58)	0.10 (7.36)	0.08 (4.05)	0.04 (6.96)
Lagged Momentum	0.09 (3.09)	0.18 (18.31)	0.06 (3.83)	0.08 (16.89)
CP/HP Momentum	0.12 (3.61)	0.13 (13.06)	0.07 (3.99)	0.06 (11.52)
All Forms Momentum	0.09 (2.78)	—	0.06 (3.62)	—
$FMOM_{ind}$	—	—	0.03 (3.31)	0.39 (34.86)
$FMOM_{PC(1-120)}$	-0.02 (-1.01)	2.01 (34.86)	—	—

None of the alternative forms of momentum can explain FMOM on their own, even when all are included in the regression. The alphas remain significant across all cases, with the lowest t-value at 2.78 when regressed against all alternative momentum factors. This indicates that, in addition to explaining individual momentum, FMOM captures additional sources of profit. Consistent with previous findings, the alpha t-values for $FMOM_{PC}$ tend to be higher than those for $FMOM_{ind}$. Finally, the last two rows demonstrate that $FMOM_{PC}$ spans $FMOM_{ind}$, as indicated by an alpha t-value of -1.01, while the reverse does not hold, yielding a t-value of 3.31. This reinforces the conclusion that $FMOM_{PC}$ is the dominant version of $FMOM_{ind}$.

In conclusion, the findings in European markets suggest that FMOM effectively explains all returns in individual momentum factors and contains information not captured by the other factors. This reinforces the conclusions drawn for US markets [1]: “individual stock momentum is, at least in large part, a manifestation of factor momentum. An investor who trades individual stock momentum indirectly times factors; she would do better by timing the factors directly.” Furthermore, the construction method of FMOM is crucial: $FMOM_{PC}$ not only proves to be a significantly stronger factor but also spans $FMOM_{ind}$, highlighting the superiority of the principal components approach in capturing momentum effects.

5.4 Alternative Sets of Factors: Spanning Tests

Building on the previous findings, this section investigates how the number of factors used to compute FMOM affects its explanatory power. For each potential subset length, ranging from 1 to 120, I generate 50 random combinations of factors of that length. This process allows for the construction of 50 distinct instances of both $FMOM_{ind}$ and $FMOM_{PC}$ factors for each length, with each instance based on a different random subset of factors. Subsequently, I perform two regressions for each constructed FMOM factor to assess their relative strength.

- **Explaining FMOM with FF5**
  Measures how the constructed FMOM improves FF5. A significant alpha suggests that FMOM provides additional information beyond FF5 and it could be considered as valid additional factor.

$$FMOM_t = \alpha + \beta FF5_t + \epsilon_t \tag{5.11}$$

- **Explaining UMD with FF5 and FMOM**
  Mesaures the ability of FMOM to explain UMD, as in Table 5.3. In this case, an unsignificant alpha indicates that the combination of FF5 and FMOM capture UMD's returns.

$$UMD_t = \alpha + FF5_t + \beta FMOM_t + \epsilon_t \tag{5.12}$$

The results are illustrated in Figure 5.1, For each subset of length "Number of Factors", in the x-axis, I plot the average t($\hat{\alpha}$) from the 50 regressions performed on that lenght.

(a) $FMOM_{PC}$ (b) $FMOM_{ind}$

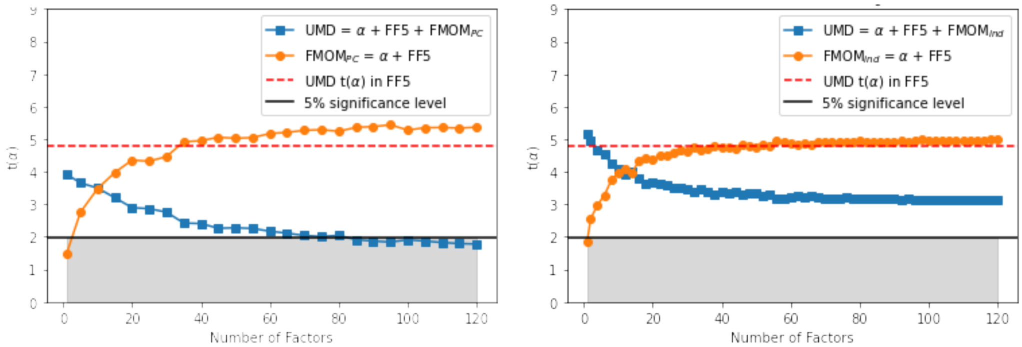

Figure 5.1: Measuring FMOM strength for different subsets

The figure illustrates consistent patterns regarding $FMOM_{PC}$ and $FMOM_{ind}$. Focusing on the first regression (orange), the t-values increase with the number of factors, and with just 2 factors, both instances of FMOM are significant enough to be considered as additional factors. In the case of $FMOM_{PC}$ (left subplot), at around 40 factors, the significance level surpasses that of UMD against FF5 (red line), indicating a greater explanatory power. For $FMOM_{ind}$ (right subplot), one needs to consider 60 factors to consistently surpass UMD.

In contrast, in the second regression (blue), the t-values decrease as the number of factors increases. $FMOM_{PC}$ is not able to span UMD until one includes 70 factors, and $FMOM_{ind}$ never achieves this. These patterns aligns with the original study, and support the idea that "individual stock momentum is an aggregation of the autocorrelations found in factor returns – the more factors, the better they capture UMD" [1].

5.5 Do Firm-Specific Returns Display Momentum?

In this final subsection, I explore a consequence that is implicit in my findings. In Section 5.1 I examine the different sources of individual stock momentum and state that FMOM combined with FF5 should capture sources 1,2 and 4 of individual momentum, but not 3 (momentum in firm-specific returns). Therefore, if FMOM captures UMD, and therefore, the four sources, as demonstrated in Section 5.2, it follows that firm-specific returns should not exhibit momentum. To test the consistency of this implication, I employ residual momentum (RMOM) [35] to conclude that firm-specific momentum is not a significant source of momentum.

In a perfectly specified asset pricing model, residuals should ideally represent randomness or firm-specific returns. For instance, if assets follow an FF5 model and are cross-sectionally regressed using FF5, the residuals correspond to firm-specific returns. However, if they are regressed using an FF3 model, the omitted factors will remain in the residuals, distorting their interpretation. As noted, "in practice, firm-specific returns are difficult to isolate" due to incomplete knowledge of all influencing factors, reliance on estimated rather than true factor returns, and the inherent noise in estimating factor loadings [1].

The concept behind RMOM is to compute these residuals given a pricing model and construct a momentum factor based on them, reflecting the momentum effects of factors not included in the model. Under a perfect asset pricing model that includes all relevant factors, strong residual momentum would indicate momentum in firm-specific returns. However, it is challenging to asses weather the specified asset pricing model is complete. Consequently, significant RMOM suggests in practice that there are additional sources of momentum not captured by the model, which may arise from either firm-specific returns or omitted factors.

I now investigate the profitability of residual momentum strategies. Using the stock characteristics dataset, I compute residuals for the CAPM, FF3, and FF5 models. For each month t, I perform a time-series regression to estimate the factor loadings (slope coefficients) from month $t-72$ to $t-13$, requiring a minimum of three years of data. Residuals for each stock at time t are computed as the average residuals from month $t-12$ to $t-2$. Following the original UMD methodology [9], I create an RMOM factor based on these residuals and reconstruct UMD using the same stocks as those used for RMOM. Table 5.7 reports the average monthly returns of each momentum factor and presents a regression where RMOM is explained by FMOM.

$$\text{RMOM}_t = \alpha + \beta\text{FMOM}_t + \text{FF5}_t + \epsilon_t \tag{5.13}$$

Table 5.7: Explaining RMOM with FMOM

Momentum Variable	Average Return	Explanatory Variable			
		$\mathbf{FMOM_{ind}}$		$\mathbf{FMOM_{PC}}$	
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Raw Returns (UMD)	0.43 (2.06)	0.17 (1.20)	2.24 (12.97)	0.02 (0.16)	4.90 (12.07)
CAPM Residual (RMOM)	0.40 (2.76)	0.23 (1.99)	1.66 (12.19)	0.16 (1.30)	3.34 (10.29)
FF3 Residual (RMOM)	0.23 (2.30)	0.10 (1.01)	0.73 (6.18)	0.07 (0.70)	1.66 (6.15)
FF5 Residual (RMOM)	0.14 (1.69)	0.03 (0.31)	0.49 (4.93)	0.01 (0.14)	0.98 (4.23)

The recomputed UMD shows an average return of 0.43% with a t-value of 2.46, which is lower compared to the one computed in Table 5.4. The universe of stocks used to recompute UMD appears to weaken its significance, as now both FMOM factors span it, yielding alphas of 1.20 and 0.16.

Regarding RMOM strategies, their profitability decreases as more factors are added to the pricing model: 0.40 for CAPM residuals, 0.23 for FF3 residuals, and 0.14 for FF5 residuals. This decline is expected, as the addition of factors displaying momentum should reduce RMOM performance. Similarly, the alphas from FMOM regressions decrease as more factors are added, with higher values observed when regressing on $FMOM_{ind}$, reflecting its weaker performance compared to $FMOM_{PC}$. The alphas range from 0.31 to 1.99, indicating that FMOM explains all the RMOM factors except for the RMOM variant computed from CAPM residuals with $FMOM_{ind}$. Therefore, FMOM captures all sources of individual stock momentum, indicating no momentum in firm-specific returns. By extension, FMOM also captures the momentum displayed by any omitted factors: “whatever these additional factors beyond the five-factor model are, they are still largely the same as those found in the FMOM factor” [1].

In conclusion, Section 5 provides compelling evidence supporting the hypothesis that FMOM not only significantly contributes to individual stock momentum in European marketsbut also explains all individual momentum sources while capturing additional sources that none of the other momentum factors can explain. In particular, the explanatory power of FMOM highly depends on the number of factors employed. $FMOM_{PC(1-120)}$, provides the greater explanatory power, even explaining the other FMOM instances. Finally, the analysis of RMOM shows that FMOM spans RMOM, consistent with the implication that there is no significant firm-specific momentum. Overall, these results underscore the importance of FMOM in asset pricing models and suggest that investors might achieve better results by directly timing factors rather than relying on individual stock momentum strategies.

6

Momentum vis-à-vis with other factors

In this final section, I investigate the dynamics of momentum strategies and their interactions with classical factors. As explained in Section 5.1, the fourth source of individual stock momentum—variation in mean returns across stocks—is linked to classical factors, suggesting a relationship between momentum and these factors. The analysis begins by examining both the unconditional and conditional correlations between the UMD factor and the classical factors. The findings reveal that momentum tends to load positively on factors that have recently performed well and negatively on those that have underperformed, indicating that it shares systematic risk with them. Rather than being an independent factor, momentum appears to reflect broader dynamics among these factors. This raises an important question: does factor momentum arise purely from timing other factors, or is it simply a reflection of stock-level momentum? To address this, I isolate pure factor momentum using momentum-neutral factors and find that the factor momentum observed is indeed a true characteristic of the factors themselves, not merely an effect of individual stock momentum.

6.1 Unconditional and Conditional Correlations with UMD

A noteworthy observation regarding individual stock momentum is its low correlation with the five classical factors. Table 5.3 shows an R^2 value of 48.74% when regressing UMD on the FF5. According to the original study, "this might imply that that factors unrelated to the market, size, value, profitability, and investment factor must explain the remaining 52% of the variation. Alternatively, these estimates might suggest that momentum is a distinct risk factor" [1]. However, unconditional correlations are not the best proxy for measuring this relationship. By definition, when constructing UMD, if a factor has performed well, stocks within that factor will have performed well, and UMD will tend to pick those stocks. Therefore, it is more insightful to study the conditional correlations based on the prior year's return, selecting observations only when the factor return over the prior year has been positive or negative. I compute correlations between UMD and the factors in Table 6.1. The European factors are computed with respect to UMD and the Global factors with respect to G_UMD. Additionally, I include the z-value, which is the significance value of a test where the null hypothesis is that both conditional correlations are equal. These findings are reported in Table 6.1 as the average results obtained using Dataset 1 and Dataset 2.

Table 6.1: UMD Correlations

Factor	$\hat{\rho}$	$\hat{\rho}^{+}$	$\hat{\rho}^{-}$	**z-value**
		Europe		
Pooled	-0.08	0.17	-0.43	8.80
SMB	-0.02	0.17	-0.19	3.62
HML	-0.51	-0.26	-0.77	11.52
RMW	0.41	0.54	-0.04	5.03
CMA	-0.10	0.21	-0.53	9.40
E/P	-0.46	-0.25	-0.71	9.74
CE/P	-0.50	-0.34	-0.75	10.34
D/P	-0.54	-0.44	-0.69	6.38
BAB	0.44	0.57	0.12	3.41
QMJ	0.60	0.69	0.30	2.66
		Global		
G_SMB	-0.08	0.10	-0.29	4.31
G_HML	-0.44	-0.05	-0.78	16.37
G_RMW	0.09	0.35	-0.42	9.04
G_CMA	-0.13	0.25	-0.62	12.16
G_E/P	-0.31	0.09	-0.70	13.54
G_CE/P	-0.33	0.03	-0.74	15.17
G_D/P	-0.24	0.28	-0.69	14.61
G_BAB	0.24	0.61	-0.24	7.38
G_QMJ	0.34	0.44	-0.00	3.63

On average, the unconditional correlations tend to be low. While some factors, such as RMW or G_QMJ, exhibit high positive correlations (0.41 and 0.34), others show significant negative correlations, like HML (-0.51) and D/P (-0.54). These opposing correlations balance out in the pooled regression, resulting in an overall correlation of -0.08. In contrast, the conditional correlations display a different pattern, with most factors showing positive conditional correlations when the prior year's return has been positive and negative otherwise. The differences between these two types of correlations, as indicated by the z-values, are all significant at the 5% level. This demonstrates that momentum is significantly related to all factors, capturing the autocorrelations among these factors and thus sharing systematic risk. While unconditional correlations average out to zero—leading many factor models to treat UMD as a distinct factor—the conditional correlation analysis suggests this distinction may arise from momentum's switching behavior between the long and short legs of the factors [1].

6.2 Momentum in Momentum-Neutral Factors

Finding that momentum captures the dynamics between factors, rather than being an independent factor is consistent with the argument from Section 5, that individual stock momentum can be better explained by targeting the factors, rather than specific companies. However, since factors are constructed from individual stocks, momentum ultimately arises from individual stock returns. This raises the question: is momentum truly inherent to the factors (pure factor momentum), or is it merely a reflection of individual stock momentum (incidental momentum)?

"A source of ambiguity in demonstrating causality is that individual stock momentum may induce incidental momentum in factor returns" [1]. When a factor has performed well over the prior year, stocks in its long leg will tend to outperform those in its short leg. If there is no pure factor momentum, but individual stock momentum favors, for example, the size factor (such as small technology companies gaining popularity), it may appear as if the factor itself exhibits momentum—even if it is actually driven by individual stock momentum. This effect is known as incidental momentum. To control for this effect and isolate pure factor momentum, I construct momentum-neutral factors, which aim to capture the pure momentum of the factor. A factor exhibits incidental momentum if:

$$\sum_{i=1}^{N} w_{i,t} r_{i,t-12,t-2} \neq 0, \tag{6.1}$$

This means that a factor's past returns, computed using current weights, are nonzero. Note the resemblance to Equation 4.8, which was used to build Dataset 3 factors from stock characteristics. To remove the incidental momentum effect, I define a new set of weights $x_{i,t}$ that adjust the original weights minimally to achieve orthogonality. The objective function, defined and solved in the original study [1], is:

$$\min_{x_i} \sum_{i} (w_i - x_i)^2 \quad \text{subject to} \quad \sum_{i=1}^{N} x_i = 0 \quad \text{and} \quad \sum_{i=1}^{N} x_i r_{i,t-12,t-2} = 0. \tag{6.2}$$

The new weights are equivalent to the residuals from running a cross-sectional regression of the original factor weights on past returns:

$$w_{i,t} = \alpha + \beta r_{i,t-12,t-2} + x_{i,t}. \tag{6.3}$$

The key idea is to make the long and short legs of a portfolio indistinguishable from each other based on past returns. In the case of pure factor momentum, constructing these legs using past returns should result in similar returns for both legs, as they would be equally influenced by the factor's momentum effects. If the returns differ, it could indicate the presence of incidental stock momentum, where individual stocks' momentum is affecting the factor's performance. By applying this approach, both legs should exhibit similar performance over the prior year, whether positive or negative, thereby isolating and capturing pure factor momentum.

In Table 6.2, I compare two versions of the PC factor momentum strategy from Section 4: one computed using the Dataset 3 factors and the other using momentum-neutral factors.

Table 6.2: Factor Momentum Performance for Different Sets of PCs

Subset of PCs	$\mathbf{FMOM_{PC}}$		$\mathbf{FMOM_{Neutral}}$	
	$\bar{r}$	$\mathbf{t}(\bar{r})$	$\bar{r}$	$\mathbf{t}(\bar{r})$
1-25	0.23	(2.89)	0.29	(5.33)
26-50	0.10	(4.47)	0.09	(4.77)
51-75	0.06	(3.32)	0.04	(3.03)
76-100	0.05	(3.22)	0.02	(2.24)
101-120	0.01	(0.57)	0.01	(1.56)
1-120	0.10	(3.96)	0.09	(5.60)

Momentum-neutral factors strategies tend to be more profitable than the original factors strategies. The absolute differences reflect the amount of incidental momentum, which does not constitute a significant portion of the original factor momentum strategy. This suggests that FMOM stems from pure factor momentum. Finally, I confirm this hypothesis by measuring how much individual stock momentum explains factor momentum. I run regressions where $FF5$ targets the different forms of $FMOM_{PC}$ strategies (1: eq 6.4), and where $FMOM_{PC(1-120)}$ and $FMOM_{Neutral(1-120)}$ target each other (2: eq. 6.5 and 6.6). The results are reported in Table 6.3.

$$FMOM_t = \text{FF5}_t + \epsilon_t \tag{6.4}$$

$$FMOM_{PC(1-120),t} = \alpha + \text{FF5}_t + FMOM_{Neutral(1-120),t} + \epsilon_t \tag{6.5}$$

$$FMOM_{Neutral(1-120),t} = \alpha + \text{FF5}_t + FMOM_{PC(1-120),t} + \epsilon_t \tag{6.6}$$

Table 6.3: Regressions Targeting different Forms of $FMOM$ (eq. 6.4, 6.5 and 6.6)

Explanatory Variable	$\mathbf{FMOM_{PC(1\text{-}120),t}}$		$\mathbf{FMOM_{Neutral(1\text{-}120),t}}$	
	(1)	**(2)**	**(1)**	**(2)**
$\hat{\alpha}$	0.11 (5.31)	0.01 (0.70)	0.09 (6.08)	0.03 (2.92)
$FMOM_{PC(1\text{-}120),t}$	-	-	-	0.59 (24.43)
$FMOM_{Neutral(1\text{-}120),t}$	-	1.08 (24.44)	-	-
R^2	44.00%	79.92%	26.82%	73.75%

The results indicate that momentum-neutral factors effectively dominate the original momentum factors. When regressing them against FF5 (Eq. 6.4), both alphas are significant, with t-values of 5.31 and 6.08, demonstrating that they can be considered new, statistically significant factors in addition to the classical ones. However, when running regression (Eq. 6.5), $FMOM_{PC}$ is not able to explain $FMOM_{Neutral}$ (t-value of 0.70). The reverse regression shows the opposite; $FMOM_{Neutral}$ significantly improves upon the original factors (t-value of 2.92), meaning that $FMOM$ is not merely incidental but is genuinely distinct: returns in factor momentum are derived purely from the momentum inherent in individual factors.

In conclusion, momentum can be viewed as "the sum of autocorrelations found in other factors" [1] rather than a completely independent factor, which is consistent with the argument that investors can achieve a more robust momentum by timing these factors rather than relying solely on stock-level momentum. As a result of these autocorrelations, momentum stocks share similar factor exposures, meaning that investors trading momentum bear systematic risk because all winners and all losers have similar factor exposures. These risks arise from pure factor momentum rather than incidental stock momentum, demonstrating that factor momentum is an inherent characteristic of the factors themselves.

7

Conclusion

This thesis investigates the role of factor momentum within European markets, establishing it as a significant predictor of cross-sectional stock returns. Unlike traditional momentum factors that rely on patterns within individual stocks, factor momentum operates as a broader mechanism, driven by autocorrelation in factor returns. The findings reveal that European factors with recent strong performance continue to yield substantial premiums, challenging conventional asset pricing models and contributing to the landscape of investable anomalies. This allows for the construction of a new factor targeting momentum in factors (FMOM), which is even more effective when based on principal component (PC) factors rather than individual factors. This is consistent with the KNS sentiment model, where PC component factors exhibit more momentum. Evidence supports the hypothesis that FMOM not only significantly contributes to individual stock momentum (both UMD and alternative momentum) but also explains all individual momentum sources, capturing additional components that other momentum factors cannot. Specifically, the explanatory power of FMOM increases with the number of factors included. Finally, it is shown that UMD is not independent but rather linked to all the factors, suggesting that momentum arises from factor dynamics rather than stock-level momentum. This supports findings that a momentum investor would benefit more from timing factors than stocks, as systematic risk emerges from common factor exposures. Furthermore, this systematic risk appears to arise from pure factor momentum rather than incidental momentum effects.

This thesis highlights the strategic value of factor momentum, particularly in the European context, where data limitations and market structures differ from those in the U.S. My results are relatively weaker due to the absence of data from 1960 to 1990, a period in which the original study shows stronger effects. Additionally, in some cases, the European factors had to be constructed differently than in the U.S. study (e.g., using top and bottom 30% portfolios instead of deciles) due to data limitations. Although access to more comprehensive data could enhance the robustness and comparability of my findings, they remain consistent with those of the original study [1].

The original authors [1] raised two key questions for future research. First, while factor momentum aligns with the KNS sentiment model, which suggests that momentum may arise from the mispricing behaviors of sentiment-driven investors, they observed that in earlier periods, momentum also appears in lower-eigenvalue factors, potentially due to arbitrage. They suggest developing a test to determine whether these premiums arise from risk or mispricing, and a rational model with varying risk premiums might clarify the sentiment-driven demand within the KNS framework. Secondly, the data implies that momentum is driven entirely by factors, suggesting an absence of firm-specific momentum. However, unseen factors or estimation errors may affect these findings, indicating a need for a more rigorous approach to fully isolate firm-specific returns.

In addition to these questions, there are several next steps that could advance this research. First, my findings indicate that the strength of factor momentum appears to depend more on the time period than on geographical location. Further investigation could examine these periods in more detail and explore the relationship between factor momentum and macroeconomic variables like interest rates, inflation, or GDP growth. This would also enable an analysis of whether factor momentum is driven by broader economic conditions or if pure factor data suffices. Secondly, extending this analysis to other geographical regions (e.g., Asia-Pacific or emerging markets) could validate whether factor momentum behaves consistently across various market structures and regulatory environments. Third, I demonstrate how momentum appears to time classical factors rather than individual stocks; this analysis could be expanded to a broader set of factors to uncover any additional relationships. Fourth, I used a traditional method for defining momentum eigenvalue factors, based on PCs of prior-year factor returns. While this approach is straightforward and explainable, machine learning techniques could identify optimal methods for constructing factor returns and generating highly profitable factor components without relying on abstract theoretical models. Finally, this research suggests that momentum investors would benefit more by timing factors, and that systematic risk exists purely from factor momentum. It would be worthwhile to further examine the pricing of this risk and whether momentum strategies offer returns that justify this risk in comparison to the factors themselves.

In summary, while this study provides a solid foundation on factor momentum in European markets, addressing these areas and broadening future research will deepen the understanding of factor dynamics and their implications for both academic finance and investment practice.

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