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Factor Momentum and the Momentum Factor: a European approach

Master's thesis in Mathematics

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A study on the ability of momentum displayed by factors to enhance cross-sectional explainability in Europe, demonstrating that these factors outperform classical momentum factors.

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Abstract

This study investigates the role of factor momentum in explaining the cross-sectional variation in stock returns within European markets, extending the findings of Ehsani and Linnainmaa (2022) to a new regional context. The analysis shows that momentum among financial factors significantly outperforms traditional stock momentum, challenging conventional asset pricing models. Using two comprehensive datasets of European and global factors, this research examines whether the momentum patterns observed in U.S. markets are present in Europe and explores the interactions between factor momentum and individual stock momentum.

The findings reveal that multiple European factors exhibit notable momentum, which supports the construction of a new momentum factor (FMOM) that captures systematic factor momentum. FMOM not only surpasses the predictive power of classical stock momentum but also substantially contributes to individual stock momentum, identifying sources of return that traditional models do not fully capture. Furthermore, the use of momentum-neutral factors allows for isolating pure factor momentum from incidental stock momentum, suggesting that the profitability of momentum strategies is driven primarily by factor dynamics rather than stock-specific movements.

By offering a European perspective, this research enriches the global perspective on momentum by highlighting the importance of factor dynamics in European markets and underscores the need for asset pricing models that integrate factor momentum across different markets. All code and data used for this analysis are publicly available in a GitHub repository, ensuring transparency and reproducibility. These findings provide valuable insights for investors, emphasizing the potential of factor timing strategies over traditional stock momentum approaches to improve portfolio performance and risk management.

Keywords: Factor Momentum, Momentum Factor, Asset Pricing, European Financial Markets, Cross-sectional Stock Returns, Econometric Techniques

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Introduction

Momentum, a well-documented anomaly in financial markets, challenges the efficient market hypothesis by suggesting that past returns can predict future performance. This phenomenon is not only prevalent in individual stock returns but also extends to factor returns, as demonstrated by Ehsani and Linnainmaa (2022) [1]. Their seminal work challenges traditional momentum factors, which measure stock-level momentum, by proposing the construction of a momentum factor based on factor-level momentum, that is, the momentum exhibited by factors themselves. Their analysis [1] shows that factor momentum is a significant driver of returns. More specifically, it is concentrated in high-eigenvalue principal component factors, which explain a substantial portion of stock returns.

Building on this foundational work, the present study replicates and extends the analysis of factor momentum by examining its presence and implications within European markets. In financial markets, factors such as size, value, and momentum are commonly used to explain variations in stock returns. These factors, often constructed from portfolios of individual stocks, exhibit autocorrelations that can lead to predictable patterns in returns. Ehsani and Linnainmaa's research [1] reveals that momentum is not merely an isolated risk factor but rather a dynamic strategy that adjusts the weighting of other factors based on their past performance. A similar approach is taken to explore factor momentum within European contexts, contributing to the broader understanding of momentum's role in asset pricing.

The European financial landscape, with its distinct regulatory environments and market structures, may influence the manifestation of factor momentum. This study investigates whether the factor momentum patterns found in U.S. markets [1] are present in Europe and how they interact with individual stock momentum. By analyzing European data using econometric techniques, the relevance of factor momentum in European markets and its implications for asset pricing, investment strategies, and risk management are evaluated.

Section 2 provides an overview of empirical asset pricing and the practical frameworks used to apply these models. Section 3 introduces the four datasets used in this project, covering the period from July 1990 to December 2023. The first two datasets include 20 European and global factors, while the third dataset contains 154 European factors. Additionally, the third dataset includes around 400,000 monthly stock observations, which can be used to construct additional factor returns. From this, a fourth dataset is created, consisting of 154 European factor returns.

Section 4 assesses the feasibility of creating a new type of momentum factor by measuring factor autocorrelations and the profitability of a factor momentum strategy.

Section 5 reviews the KNS sentiment model and tests the hypothesis that more systematic factors exhibit greater momentum through the construction of a momentum strategy that trades PC factors. Factor momentum strategies are then introduced as a methodology for building momentum factors, leading to the definition of a momentum factor that measures factor-level momentum (FMOM).

Section 6 investigates the sources of individual stock momentum and examines how factor momentum influences it from a theoretical perspective. The consistency of empirical data with these theories is tested, and the relationship between FMOM and UMD is analyzed. This analysis is expanded to consider different forms of FMOM and alternative momentum factors, concluding with an examination of firm-specific returns, which are expected to be insignificant.

Finally, Section 7 computes conditional correlations with UMD to explore its relationship with other factors and uses momentum-neutral factors to determine whether the profitability of momentum strategies is driven by factor dynamics or reflects individual stock momentum.

In conclusion, this research contributes to the ongoing discourse on momentum by providing a European perspective, enriching the global understanding of factor dynamics in financial markets. To promote transparency and facilitate reproducibility, all code and data supporting this analysis are available in an open-access GitHub repository [1]. These findings have the potential to inform investment strategies and enhance the theoretical frameworks that underpin asset pricing models.

[1]https://github.com/dani-gonzalez-muela/Quantitative-Finance/tree/main/Master-Thesis

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Theory Review

The discussion begins with an overview of empirical asset pricing, starting with the theoretical framework of fundamental models such as CAPM, Fama-French, and Carhart. This is followed by a transition to a more practical framework for computing these factors.

2.1 CAPM

The Capital Asset Pricing Model (CAPM) [2] is a financial model from the 1960s, designed to determine the expected return on an asset i ($E[r_i]$) based on its inherent risk relative to the overall market. Understanding the CAPM requires a clear definition of the key components of return, which originate from the broader concept of return on investment (ROI). The concept of ROI, denoted by r, is a general measure used to evaluate the performance of an investment by comparing its gains or losses relative to its initial cost. ROI is typically expressed as a percentage and can be calculated as:

$$r = \frac{P_1 - P_0}{P_0}, \tag{2.1}$$

where P_0 is the initial price of the investment, and P_1 is its price at the end of the investment period. In this study, all returns are expressed as monthly returns in percentage terms. Three important return measures, all special cases of ROI, play key roles in the CAPM framework:

- $\mathbf{r_i}$: The return of an individual asset i, representing the performance of a specific financial asset, such as stocks, bonds, or other securities, over a given period.
- $\mathbf{r_m}$: The return of the market portfolio, reflecting overall market performance. Theoretically, the market portfolio includes all investable assets, weighted by their market capitalizations. Market capitalization represents the total market value of a company's publicly traded shares, calculated as: Number of Public Shares × Share Price. This weighting ensures that larger companies have a proportionately greater influence on the portfolio. Therefore, the return of the market portfolio is expressed as:

  $$r_m = \sum_{i=1}^{N} w_i \cdot r_i, \tag{2.2}$$

  where w_i is the weight of asset i in the market portfolio, determined by its market capitalization relative to the total market capitalization of all assets, and N is the total number of assets. In practice, r_m is often approximated by the return of a broad-based market index, such as the S&P 500 for the United States or the OMX Stockholm 30 Index (OMXS30) for Sweden.

- $\mathbf{r_f}$: The risk-free rate, representing the return on an investment with no risk. It is typically treated as a constant, with $E[r_f] = r_f$ and $\mathrm{Var}(r_f) = 0$. The risk-free rate serves as a baseline for evaluating riskier investments. In stable economies, this is often derived from government bonds, such as U.S. Treasury bills or Swedish government bonds.

These definitions establish the foundation for understanding the CAPM and its ability to quantify the relationship between the expected return of an asset and its risk, as captured by its sensitivity to market movements. The CAPM defines the expected return of an individual asset for a given period as:

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f) \tag{2.3}$$

Where:

- r_i is the return of the individual asset, as defined earlier.
- r_m is the return of the market portfolio, as defined earlier.
- $\beta_i = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)}$ is the sensitivity of the asset's return to the market return.

CAPM assumes efficient markets, rational investors, and single-period transactions. However, it is a simplistic model, and other models fit better the empirical data.

2.2 APT

The Arbitrage Pricing Theory (APT) [3] is a generalization of CAPM that includes multiple factors and does not specify which these factors are. The expected return on an asset i is given by:

$$E[r_i] = r_f + \sum_{j=1}^{k} \beta_{ij} F_j \tag{2.4}$$

Where:

- k is the number of factors.
- β_{ij} is the sensitivity of the asset's return to factor j.
- F_j is factor j.

It is worth noting that the final term in Equation (2.3) of the CAPM, $\beta_i(E[r_m]-r_f)$, can be interpreted as a single-factor model, where the excess market return over the risk-free rate serves as the sole factor, usually denoted as MRKT-RF. Moreover, such models commonly treat the r_f term not as a factor but as a constant baseline representing the return achieved without assuming any risk. To realize returns above this baseline, investors must assume additional risk through exposure to systematic factors.

The APT extends this framework by incorporating multiple factors, thereby accounting for a broader spectrum of systematic risks that influence asset returns. These factors aim to capture the various variables that affect stock returns, expressing their relationship in a simple and intuitive manner. Examples of factors commonly used in empirical implementations of the APT include macroeconomic variables such as GDP growth, inflation, and interest rates, as well as financial variables like market returns or company size. The APT is an unspecified model, providing researchers the freedom to choose from any number of factors. In contrast, Fama-French models, discussed next, focus exclusively on characteristic-based factors. These factors capture systematic patterns in asset returns that are directly linked to specific characteristics of a company. For example, a size factor (such as SMB) could capture the relationship between the returns of a company and its size.

2.3 Fama-French

2.3.1 Three Factor Model

The Fama-French Three-Factor Model (FF3) [4] extends CAPM by introducing two additional factors:

$$E[r_i] = r_f + \beta_{i1}(E[r_m] - r_f) + \beta_{i2}E[SMB] + \beta_{i3}E[HML] \tag{2.5}$$

Where:

- SMB (Small Minus Big):This factor captures the relationship between a company's size and its returns. It is constructed as the return difference between portfolios of small and large market equity (ME) stocks, reflecting the historical tendency of smaller firms to outperform larger ones.
- HML (High Minus Low): This factor captures the value effect by measuring the return spread between high and low book-to-market equity (BE/ME) stocks. Firms with high BE/ME ratios tend to generate higher returns compared to those with low BE/ME ratios.

2.3.2 Five Factor Model

In 2015, Fama and French extended the model further to include five factors (FF5) [5]:

$$\begin{aligned} E[r_i] = r_f + \beta_{i1}(E[r_m] - r_f) + \beta_{i2}E[SMB] + \beta_{i3}E[HML] + \\ + \beta_{i4}E[RMW] + \beta_{i5}E[CMA] \end{aligned} \tag{2.6}$$

Where:

- RMW (Robust Minus Weak): This factor captures the profitability premium by measuring the return difference between firms with high and low operating profitability (OP). More profitable firms tend to generate higher returns.
- CMA (Conservative Minus Aggressive): This factor captures the investment premium by comparing firms with conservative investment strategies to those with aggressive investment policies. Firms that invest conservatively, meaning they allocate less capital to asset growth, tend to outperform firms with high investment rates.

The selection of these factors was based on empirical research that revealed patterns in stock returns not explained by CAPM. These factors have consistently demonstrated explanatory power across different markets and time periods. They reflect broad market behaviors while aiming to extend the model without adding unnecessary complexity.

2.3.3 Factor Construction

Factors play a key role in empirical asset pricing, and there are various methods for constructing them. Most approaches involve sorting stocks into portfolios and computing return differences between selected portfolios. Since portfolios are sets of stocks, sorting stocks into portfolios means grouping them based on a defined criterion. One of the most well-known approaches is the Fama-French methodology [6], which provides ready-to-use factor returns and stock portfolios on their website [7]. This methodology constructs factor returns in three key steps: first, by forming univariate portfolios based on a specific factor; second, by extending these to bivariate portfolios; and finally, by using the bivariate portfolios to compute factor returns. This section provides a detailed explanation of these steps and describes the original approach for U.S. stocks before discussing its extension to international equities [8].

Univariate Portfolios: To compute a given factor, the first step is to construct univariate portfolios that are updated monthly based on the characteristic defining the factor (e.g., BE/ME for the HML factor). Stocks are grouped into three portfolios according to their percentile ranking for the characteristic: bottom 30%, middle 40%, and top 30%. For example, when constructing the HML factor, consider a dataset of 10,000 publicly traded stocks with monthly returns and BE/ME values from 2000 to 2024. Each month, stocks are ranked by BE/ME and assigned to one of the three portfolios based on their percentile. In the original framework, Fama and French [6] construct these portfolios each month using all NYSE, AMEX, and NASDAQ stocks with available equity data as of June each year.

Finally, the monthly returns for each of the three univariate portfolios are computed. Fama and French [6] define portfolio returns as the market capitalization-weighted average of individual stock returns, following the methodology in Equation 2.2. Applying this approach to the HML factor results in three time series of monthly returns from 2000 to 2024, corresponding to the three BE/ME portfolios: bottom 30% (0-30th percentile), middle 40% (30-70th percentile), and top 30% (70-100th percentile).

Bivariate Portfolios: Bivariate portfolios extend univariate portfolios by incorporating an additional size-based split, which helps control for size effects and ensures that the factor is not disproportionately influenced by large or small stocks. Specifically, stocks are first sorted monthly into two groups based on whether their market equity (ME) is above or below the median. Within each size group, stocks are further divided into three subgroups based on the primary characteristic (e.g., BE/ME for HML), resulting in six portfolios instead of three. This refinement provides a more robust factor construction by addressing potential biases in the univariate sorting approach.

For example, when constructing the HML factor using a dataset of 10,000 stocks with monthly returns, BE/ME, and ME data from 2000 to 2024, stocks are first sorted each month into two size groups: Small (below-median ME) and Big (above-median ME). Within each size group, stocks are further sorted into three BE/ME subgroups: bottom 30%, middle 40%, and top 30%. This process produces six portfolios that are updated monthly. Finally, for each month, the market capitalization-weighted average of stock returns is computed for each portfolio (Eq. 2.2), resulting in six time series of monthly returns from 2000 to 2024. In the case of the HML factor, the bivariate portfolios are structured as shown in Table 2.1.

Table 2.1: Bivariate Portfolios on Size and BE/ME

	Median ME	
	Small	**Big**
70-100th BE/ME percentile	Small Value	Big Value
30-70th BE/ME percentile	Small Neutral	Big Neutral
0-30th BE/ME percentile	Small Growth	Big Growth

Table 2.1 illustrates how HML's bivariate portfolios are structured. As in the univariate case, stocks are first ranked by BE/ME into percentiles. Then, they are further divided based on whether their market equity (ME) is above or below the median. The resulting groups are labeled "Small" and "Big" to indicate the size classification.

Factors: After constructing the bivariate portfolios for a given factor (e.g., Table 2.1 for HML), the final step is to compute the factor itself by combining these portfolios. A factor represents the theoretical return associated with a particular financial characteristic, capturing the difference in performance between stocks that exhibit the characteristic strongly and those that do not. For example, the HML factor measures the return difference between stocks with high BE/ME ratios and those with low BE/ME ratios.

In practice, the Fama-French methodology [6] constructs factors by computing the return difference every month between the top and bottom groups of the bivariate portfolios. This approach is equivalent to an investment strategy that goes long on the top-ranked stocks (70-100th percentile) and short on the bottom-ranked stocks (0-30th percentile)[1], rebalancing monthly based on the characteristic being measured.

[1] A long position refers to buying an asset with the expectation that its value will rise, allowing the investor to sell it later at a higher price for a profit. A short position, on the other hand, involves borrowing and selling an asset with the expectation that its price will decline, so it can be repurchased later at a lower price, generating a profit from the difference. In practice, the total return of a long-short strategy consists of adding the returns of the long positions and subtracting the returns of the short ones.

For instance, HML captures the value effect by computing returns each month as the average return on the value portfolios minus the average return on the growth portfolios. These portfolios correspond to the respective bivariate portfolios from Table 2.1:

$$HML = \frac{1}{2}(Small\ Value + Big\ Value) - \frac{1}{2}(Small\ Growth + Big\ Growth) \tag{2.7}$$

Since the bivariate portfolios from the previous HML example consited of six monthly time-series returns from 2000 to 2024, this monthly averaging would result in a single time-series of monthly factor returns over the same period, typically expressed as a percentage.

Other factors, such as SMB, RMW, or CMA, follow a similar procedure, using their corresponding bivariate portfolios sorted by their defining characteristic: ME for SMB, OP for RMW, and investment for CMA. The MKT-RF factor, which represents market excess return ($E[r_m] - r_f$) in the Fama-French model (Eq. 2.6), is the only factor constructed differently. Instead of sorting stocks into portfolios monthly, it computes $E[r_m]$ as the monthly weighted returns by market capitalization of all available stocks, and r_f as the US 10-year treasury bill. While Fama and French initially developed their model for U.S. stocks, they later expanded it to cover Developed Markets [8], including Europe, Asia, and Canada.

2.4 Carhart Momentum

In 1997, Mark M. Carhart introduced momentum as an independent factor and developed the UMD (Up Minus Down) factor [9] to extend the three-factor model:

$$E[r_i] = r_f + \beta_{i1}(E[R_m] - R_f) + \beta_{i2}SMB + \beta_{i3}HML + \beta_{i4}UMD + \epsilon_i \tag{2.8}$$

UMD captures the momentum effect, which refers to the tendency of stocks with high past returns to continue performing well, while those with low past returns tend to underperform. UMD is constructed as the return difference between stocks with a strong momentum characteristic and those with a weak momentum characteristic, where this characteristic is measured as cumulative returns from month $t - 12$ to $t - 2$).

Following the Fama-French methodology [6], UMD is constructed using bivariate portfolios based on momentum characteristics, maintaining the same structure as Table 2.1 but replacing the "Value," "Neutral," and "Growth" labels with "High," "Neutral," and "Low" momentum. The UMD returns are then computed monthly analogously to Equation 2.7, as the average return spread between high and low momentum portfolios:

$$UMD = \frac{1}{2}(Small\ High + Big\ High) \\ - \frac{1}{2}(Small\ Low + Big\ Low) \tag{2.9}$$

Factor returns and portfolio data for UMD are publicly available on the Fama and French website [7], making it a key reference in momentum research. While UMD is the standard momentum factor, alternative definitions of momentum exist. For example, Intermediate Momentum [11] captures momentum using returns from $t-12$ to $t-7$, excluding the most recent months. These measures focus on individual stock momentum (also known as stock-level momentum), meaning they track the persistence of returns at the stock level. However, momentum can also be analyzed within specific stock groups. For instance, Industry Momentum [10] examines the return persistence of stocks within particular industries, reflecting the tendency for entire sectors to sustain trends over time.

2.5 Alternative Factors

Classical factors have demonstrated strong explanatory power of asset returns while also offering intuitive insights. These models were designed with simplicity in mind and track well-known stock characteristics. However, it is possible to construct more complex models that incorporate additional factors based on different financial characteristics. Any measurable stock characteristic could serve as the basis for a factor, provided it offers sufficient explanatory power. This section introduces some alternative factors that will be utilized in the first part of the project.

Fama French Alternative Factors

In addition to the classical factors, the Fama-French website [7] provides univariate and bivariate portfolios constructed based on alternative characteristics such as earnings-to-price (E/P), cashflow-to-price (CF/P), and dividend yield-to-price (D/P) ratios. These portfolios enable the computation of alternative factors following the Fama-French methodology [6].

AQR Factors

AQR Capital Management is a well-known asset management firm that provides a wide range of alternative factors on its website [12]. This section briefly describes the factors used in this project:

- Betting Against Beta (BAB) [13] constructs portfolios based on the CAPM Beta of different stocks (β_i from Equation 2.3). Following the Fama-French methodology [6], stocks are grouped into high-beta and low-beta bivariate portfolios, which are rebalanced monthly. The BAB factor return is then computed as the spread between these portfolios.

- Quality Minus Junk (QMJ) [14] follows a similar methodology [6], but constructs portfolios based on stock quality, which is measured using a combination of different characteristics. The QMJ factor is then calculated as the return spread between high-quality and low-quality stocks.

3
Data Gathering and Cleaning

The original study [1] begins by analyzing a dataset containing 22 factors for the US and global markets, covering the period from July 1963 (1990 for global factors) to December 2019. The analysis is then extended to a more comprehensive dataset, enabling the construction of factor returns for 54 US factors.

Similarly, this project begins with two datasets containing monthly returns for 22 European and global factors, spanning from July 1990 (with some factors starting in 1993) to December 2023. These factors correspond to the 11 factors explained in the previous section, once for each market. They are listed in Table 3.2 and express monthly returns as percentages.

Subsequently, a more comprehensive dataset is introduced, covering 412,013 monthly stock observations from July 1990 to December 2023. This dataset contains stock characteristics and is then used to construct a dataset of 154 European monthly factor returns (expressed as percentages).

Detailed files and procedures for these datasets can be found below and on [GitHub].

3.1 Dataset 1: Monthly Factor Returns (Fama-French & AQR)

The first dataset uses the same primary sources as the original study (Fama-French [7] and AQR [12] websites), with a focus on European factors. While most factors are directly sourced from the original data, a few required modifications, resulting in minor differences from the original factor construction methodology [6].

- **SMB, HML, RMW, CMA, MKT-RF:** The monthly factor returns for the European Fama-French factors [8] are available on the Fama-French website [7], specifically in the "Fama/French European 5 Factors" section. The dataset was downloaded directly and used without alterations. The data is provided in CSV files, often including additional information, such as yearly returns or company data. For this project, only the standard factor data from the first table in the CSV file was used, and the extra information was excluded.
- **UMD:** Similarly, the UMD factor for Europe can be obtained from the Fama-French website [7] under the "European Momentum Factor (Mom)" section. As with the previous factors, only the first dataset of the file should be extracted.

- **E/P, CE/P, and D/P:** To obtain the data, visit the Fama-French website [7] and access the file "Index Portfolios formed on B/M, E/P, CE/P, and D/P." (Ind_Eur_With_UK.Dat). Unlike the previous datasets, which contain ready-to-use factor returns, this file includes the monthly returns of univariate portfolios based on the E/P, CE/P, and D/P characteristics. These portfolios correspond to the top 30% and bottom 30% of firms based on these ratios. While the original study [1] computes US factors as the difference between the top and bottom deciles of bivariate portfolios, the European factors in this project were simplified by calculating the difference between the high and low categories of the univariate portfolios every month.
- **BAB and QMJ:** The monthly datasets for these two factors can be found in the AQR website [12]. Specifically, the files can be obtained from the "Betting Against Beta: Equity Factors Data, Monthly" and "Quality Minus Junk Factors, Monthly" sections of their website. They come as CSV files, with the first sheet in each file, labeled "BAB Factors" and "QMJ Factors", containing all the factor return data.
- **Global Factors:** In the original study [1], global markets are defined as developed markets excluding the US, specifically Europe and Asia-Pacific. Factor returns for these global markets are calculated as the monthly weighted returns of the developed regions, weighted by the lagged 1-year market capitalization of each region, as shown in Equation 2.2, but with a 1-year delay. This delay, a standard practice described by AQR [12], helps avoid biases. In this study, global factors (denoted as G_MKT-RF, G_SMB, etc.) are defined as developed markets excluding Europe, which include Asia-Pacific and North America. The construction process involves two steps:
  1. Compute monthly factor returns for the Asia-Pacific and North America regions.
  2. Construct global factors as the weighted average of the monthly returns for both regions, with weights based on the lagged 1-year market capitalization of each region. Market capitalization data, sourced from World Development Indicators [15] and covering all countries from 1975 to 2022, is used to compute the weights. The market capitalization for a region (e.g., North America) is the sum of the market capitalizations of all countries in that region (i.e., Canada and the United States).

  Below is described how to access the files used to build each of the global factors:

  G_SMB, G_HML, G_RMW, G_CMA, G_MKT-RF: These classical Fama-French factors are available for North America, Asia-Pacific (excluding Japan), and Japan. They can be downloaded from the Fama-French website [7] under the following names: "Fama/French Japanese 5 Factors", "Fama/French North American 5 Factors", and "Fama/French Asia Pacific ex Japan 5 Factors".

  G_UMD: The UMD factor is also available on the Fama-French website [7] for the same regions under: "Japanese Momentum Factor (Mom)," "Asia Pacific ex Japan Momentum Factor (Mom)," and "North American Momentum Factor (Mom)."

G_E/P, G_CE/P, G_D/P: These factors are not directly available and must be constructed using the top 30% and bottom 30% of univariate portfolios, following the same approach used for their European counterparts. The data is available on the Fama-French website [7] for the US, Canada, and Asia-Pacific. For Canada, the relevant information is found in the "Country Portfolios formed on B/M, E/P, CE/P, and D/P" file, while the Asia-Pacific data is provided in the "Index Portfolios formed on B/M, E/P, CE/P, and D/P" file. In the case of the US, the required portfolios are contained in separate files: "Portfolios Formed on Earnings/Price," "Portfolios Formed on Cashflow/Price," and "Portfolios Formed on Dividend Yield."

G_BAB, G_QMJ: These monthly factors are available for North America and Asia-Pacific on the AQR website [12], within the same files as the European versions but located in a different column of the corresponding CSV files.

After obtaining the 22 different monthly factor returns for European and global markets, they were merged into a single dataset. Dataset 1 consists of 22 columns, each representing a factor, and 402 rows, corresponding to each month from July 1990 to December 2023. Each cell in Dataset 1 contains the monthly return (in percentage) of one of the 22 factors.

3.2 Dataset 2: Monthly Factor Returns (Kelly)

The second dataset contains the same 22 factors over the same time period as the first dataset but was obtained from a different source that used slightly different procedures [16]. Theis Ingerslev Jensen, Bryan Kelly, and Lasse Heje Pedersen [16] developed a comprehensive factor dataset covering 153 monthly factor returns across 93 countries, which is available on Kelly's website [17] under the "Factors Returns" section. The full dataset was downloaded using the filters: "Region/Country: All Countries," "Theme/Factor: All 153 Factors," "Data Frequency: Monthly," and "Weighting: Capped Value Weighting."

From this dataset, only the factors and time periods corresponding to those in Dataset 1 were selected, yielding monthly factor returns for 93 countries from July 1990 to December 2023. To compute broader returns for the European and global markets, the country-level factor returns must be aggregated. Specifically, each month, the return for a given region is calculated as the weighted average of the factor returns of all countries within that region, using each country's one-year lagged market capitalization as a weight [12]. For example, the European market return is computed as the weighted average of the returns of all European countries. Similarly, the global market return is obtained by aggregating the returns of countries from the North American and Asia-Pacific regions.

The resulting Dataset 2 consists of 22 columns and 402 rows, representing monthly factor returns from July 1990 to December 2023 (expressed as percentages). Since this dataset covers the same factors and time period as Dataset 1, a high degree of similarity is expected. To evaluate the consistency between the datasets, Table 3.1 reports the correlations between factor returns across both datasets.

Table 3.1: Correlations of Factor Returns Between Dataset 1 and Dataset 2

Factor	**Correlation**
MKT-RF	0.99
SMB	0.93
HML	0.80
RMW	0.78
CMA	0.73
UMD	0.95
E/P	0.60
CE/P	0.74
D/P	0.86
BAB	0.48
QMJ	0.89
G_MKT-RF	0.99
G_SMB	0.91
G_HML	0.77
G_RMW	0.73
G_CMA	0.85
G_UMD	0.96
G_E/P	0.67
G_CE/P	0.80
G_D/P	0.78
G_BAB	0.58
G_QMJ	0.67

Table 3.1 reports the correlations between Dataset 1 and Dataset 2, which were explained in detail above. Most factors exhibit correlations above 75%, though some, like BAB and E/P, show lower correlations than expected, 48% and 60%, respectively. These discrepancies primarily arise from differences in the methods used to compute the factors. While both datasets target the same fundamental factors, there are slight variations. For instance, the UMD factor in Dataset 1 sorts portfolios based on prior year returns $t-2$ to $t-12$, following [18]. In contrast, Dataset 2 sorts from $t-1$ to $t-11$ [19]. Also, the methodology used for constructing factor returns is not exactly the same: Dataset 1 is based on the original factor construction by Fama and French [6], which uses the bivariate portfolios described in Section 2, whereas Dataset 2 relies on the spread between high and low tercile of univariate portfolios, constructed as in [19]. Despite these differences, the overall results are consistent and align with the findings throughout the paper.

3.2.1 Summary statistics

This subsection presents the summary statistics for the 22 monthly factor returns from Datasets 1 and 2. The statistics are computed separately for each dataset and then averaged, as reported in Table 3.2. This approach facilitates a comparison of factor profitability across Europe and provides insights into how these factors differ from the US factors analyzed in the original study [1]. By examining these statistics, key differences in factor performance and volatility can be identified, forming the basis for further analysis.

Table 3.2: Average Statistics of Datasets 1 and 2

Factor	Start Date	Mean	Std	T-Value
SMB	July 1990	-0.2	7.5	-0.1
HML	July 1990	2.3	10.2	1.4
RMW	July 1990	3.0	6.2	2.9
CMA	July 1990	1.7	6.5	1.5
UMD	Nov 1990	8.5	13.7	3.6
E/P	July 1990	3.2	9.4	2.0
CE/P	July 1990	4.1	9.8	2.5
D/P	July 1990	2.6	9.2	1.6
BAB	July 1993	6.2	12.5	3.0
QMJ	July 1993	5.1	9.2	3.3
G_SMB	July 1990	0.3	8.9	0.2
G_HML	July 1990	2.2	10.7	1.2
G_RMW	July 1990	3.4	7.0	2.8
G_CMA	July 1990	2.3	8.5	1.6
G_UMD	Nov 1990	4.5	14.3	1.8
G_E/P	July 1990	3.4	11.4	1.8
G_CE/P	July 1990	3.4	10.5	1.8
G_D/P	July 1990	0.2	11.5	0.0
G_BAB	July 1993	4.6	13.2	2.4
G_QMJ	July 1993	4.9	8.4	3.4

Table 3.2 reports the summary statistics (in percentage) for the monthly factor returns in Datasets 1 and 2, covering the period from July 1990 (or 1993 for certain factors) to December 2023. Dataset 1 follows the Fama-French methodology, while Dataset 2 is sourced from [16]. For each factor, the table reports its start date, mean return, standard deviation, and t-value. The t-value, calculated as the mean return divided by its standard error, provides a measure of statistical significance. A t-value above 1.97 corresponds to a p-value below 5%, indicating statistical significance at the conventional level.

The statistics reveal substantial variation across factors, with significant differences in factor returns, consistent with findings for US factor returns [1]. For instance, SMB earns -0.2%, while UMD and BAB each earn over 5%. In contrast, the volatilities of European factors exhibit less variation than their US counterparts, ranging from 6.2% for RMW to 13.2% for global BAB, compared to 4.7% to 17.3% for US factors [1]. The t-values highlight these differences, with approximately half of the factors exhibiting a significant premium, as indicated by a p-value below 5% (t-value above 1.97). These findings provide a preliminary view of factor profitability and risk in European markets, indicating that European and US factors present similar patters, setting the stage for further analysis in subsequent sections.

3.3 Dataset 3: European Stock Characteristics

Dataset 2 is obtained from Kelly's website [17], which provides 153 monthly factors covering 93 countries. These factors are constructed from a broader dataset comprising 25 million stock observations from 1925 to 2024. Each observation includes 443 variables, detailing monthly stock returns alongside firm-specific characteristics. These characteristics include the financial attributes used for constructing classical factor returns, such as SMB and HML [6], as well as a variety of non-traditional characteristics that facilitate the construction of factors based on broader attributes. A comprehensive description of this broader dataset is available in [16] and can be accessed through WRDS [20], which requires a valid subscription. For this project, access was provided through the University of Gothenburg.

Dataset 3 is a filtered subset of this main dataset, obtained by downloading the original data from WRDS and applying a few filters. The dataset can be retrieved by navigating to the WRDS main page, selecting "Get Data," then "Contributed Data Forms," and finally "Global Stock Returns and Characteristics" [20]. This section allows users to submit queries to the main dataset by specifying filters, after which WRDS processes and delivers the data to the user's account. For this study, a date filter was applied, restricting the dataset to the period from July 1990 to December 2023. The specific query used for this extraction (Query 8679286) can be replicated with the appropriate WRDS license [2]. The resulting dataset contains 12 million monthly stock observations from 93 countries, each with 443 variables. Due to its size (25GB), it could not be uploaded directly to Github and one must follow the steps described above to obtain the data.

Additional filtering was performed locally to create the final version of Dataset 3. Only European stocks were retained, and following the recommendations in Kelly's documentation [19], firms with market equity below 0.01% of the total European market capitalization in a given month were excluded. Additionally, only stocks listed on major exchanges, as identified by CRSP, were kept.

After applying these filters, Dataset 3 consists of 412,031 monthly stock observations from July 1990 to December 2023, with 443 variables each. While it does not contain factor returns directly, it provides stock-level characteristics that allow for the construction of factor returns.

[2] `https://wrds-www.wharton.upenn.edu/query-manager/query/8679286/#payload_formatted_collapsed_section`

3.4 Dataset 4: Monthly Factor Returns (KNS methodology)

Factors in Dataset 1 and Dataset 2 are precomputed factor returns, meaning that they have already been calculated using well-established methodologies, such as the Fama and French [6] framework or similar methods introduced by Kelly and his co-authors [16]. These methodologies follow a structured process, as explained in Section 2.3.3:

1. Start with a universe of stocks: Obtain a large dataset of stock characteristics, which will be used as the basis for constructing factors. For instance, Dataset 2 is built from a much larger dataset, Dataset 3, which contains raw stock characteristics for millions of stocks.
2. Portfolio Sorting: For each factor, stocks are grouped monthly into different portfolios based on their characteristics' percentiles. For example, the HML factor creates monthly portfolios as shown in Table 2.1. Then, compute the monthly returns of each portfolio as the weighted average by market capitalization of the stocks in each portfolio.
3. Factor Computation: Monthly factor returns are obtained by taking the difference in monthly returns between portfolios corresponding to the top and bottom percentiles of each characteristic.

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While this approach has been effective, it is computationally intensive and slow, especially when dealing with multiple factors, making it hard for researchers or investors who need to calculate a large number of factors.

A more recent, efficient approach to factor construction was introduced in 2020 by Kozak, Nagel, and Santosh (KNS) [21]. Their method allows for the simultaneous calculation of all factors, making the process faster and more scalable, particularly when many factors need to be created. The key innovation in this approach is that, instead of calculating each factor separately, it focuses on assigning weights to each stock characteristic for every stock in a given month. The weighted average of the returns of all available stocks in that month is then calculated, using a different set of weights for each characteristic. This method captures non-linear relationships and higher-order interactions among stock characteristics that might be missed using the traditional approach. Here's how the process works, step by step:

1. **Rank Transformation**

   First, each stock's characteristic i at time t is ranked among all other stocks's characteristic i for that month. For example, if data for 100 stocks with 10 characteristics each is available, then at every month t, each characteristic is ranked separately from 1 to 100, where a rank of 100 corresponds to the stock with the highest value for that characteristic and a rank of 1 corresponds to the lowest. The rank is then scaled to a range between 0 and 1 by dividing by the total number of stocks in that month.

$$r^s_{c_{i,t}} = \frac{\text{rank}(c_{i,t})^s}{n_t + 1} \tag{3.1}$$

Where:

- n_t is the number of stocks in month t
- $c^s_{i,t}$ corresponds to the characteristic i of stock s at month t.
- rank$c^s_{i,t}$ is the rank operator that, given a characteristic i and a month t, ranks the available stocks at month t from 1,2... to n_t based on the value of the characteristic i at month t. The suscript s returns the ranking value for a given stock s.

These rankings are recalculated every month to adjust for any changes in the data, resulting in one different ranking for very month and characteristic.

2. **Centering and Normalizing**

Once the rankings are assigned, the next step is centering and normalizing them. The ranks for each month and characteristic are centered around zero and then normalized by the sum of absolute deviations from the mean. This process helps eliminate extreme values and outliers that might distort the results. The formula for centering and normalizing the ranks is:

$$w^s_{i,t} = \frac{r^s_{c_{i,t}} - \bar{r}_{c_{i,t}}}{\sum_{i=1}^{n_t} \left| r^s_{c_{i,t}} - \bar{r}_{c_{i,t}} \right|} \tag{3.2}$$

Where:

- $\bar{r}_{c_{i,t}}$ is the average rank of characteristic i at month t.

If any stock has missing data for a characteristic, the corresponding weight is set to zero.

3. **Calculating Factor Returns**

The final step is calculating the returns for a factor i at month t, based on the underlying characteristic i.The return on a factor is computed as the weighted average of the returns for all stocks in a given month, where the weights $w^s_{i,t}$ are recalculated each month for each characteristic:

$$f_{i,t} = \sum_{s=1}^{n_t - 1} w^s_{i,t-1}\, r_{s,t} \tag{3.3}$$

Where:

- $r_{i,t}$ are the returns of stock i in month t
- $w_{i,t-1}$ is the weight assigned to stock i based on its characteristic rank in the previous month.

Dataset 4 is constructed by applying the KNS methodology to the stock characteristics from Dataset 3. Dataset 3 contains 414,837 stock observations, each with 443 variables, including characteristics and other relevant information. While this setup allows for the computation of approximately 443 factors (excluding variables like date and returns), only the 154 main European factors are computed. These factors correspond to the 154 principal factors identified by [16][3]. These factors are the most influential and tend to drive asset pricing and investment strategies.

The resulting Dataset 4 consists of 154 characteristic-based factors derived from the stock characteristics in Dataset 3. It covers the period from August 1990 to December 2023, resulting in a dataset of 401 rows and 154 columns. It follows the same format as Dataset 1 and Dataset 2 but with one fewer observation. Each cell contains the monthly excess return (in percentage) of one of the 154 factors, where the rows represent the time period, and the columns correspond to the factors. It is also important to note that excess returns are used instead of raw returns. The excess return of an asset refers to its return above the risk-free rate, expressed as $R_i = r_i - r_f$. Adding the risk-free rate to the excess return yields the standard raw return. While Datasets 1 and 2 contain raw return percentages, Dataset 3 expresses returns as percentages of excess returns.

Factors from Dataset 2 are also present in Dataset 3, and they should be similar since they measure the same characteristic but use different factor construction methodologies. This expectation holds, as the average correlation between the common factors in Dataset 2 and Dataset 3 is 0.72, indicating strong consistency across datasets.

[3] [18] identifies 153 main factors that can be grouped under 13 different topics. However, I combine 2 momentum characteristics (ret_{12_0} and ret_{2_0}) to create ret_{12_2}, which correspond to the original UMD factor [9].

4

Autocorrelations in the returns of equity factors

Traditional momentum factors, such as UMD [9], measure momentum in individual stocks. However, momentum can also be assessed within specific groups of stocks, such as Industry Momentum [10], as discussed in Section 2.4. The original study replicated in this project [1] challenges this traditional approach by proposing the construction of a momentum factor that measures momentum at the factor level (i.e., momentum exhibited by other factors), also known as factor momentum. Factors capture the performance of stock characteristics, so factor momentum would suggest that stocks grouped by a particular characteristic exhibit momentum.

The first step in constructing a factor that measures factor momentum is to determine whether factors themselves display momentum. This section evaluates the feasibility of such a factor in European markets, using Dataset 1 and Dataset 2. Momentum in these datasets is assessed using two tests. The first involves examining the autocorrelation of the factors, while the second assesses the profitability of factor momentum strategies. If these two tests demonstrate that European factors exhibit momentum, then it would be justified to develop a momentum factor based on factors rather than individual stocks.

4.1 Regression Analysis

To examine autocorrelation within the factors, regression analyses are performed between past and future factor returns. Each factor of Dataset 1 and Dataset 2 is analyzed individually using a one-dimensional regression, where the dependent variable is the monthly factor return, and the explanatory variable is an indicator variable set to 1 if the factor's average return over the previous 12 months is positive, and 0 otherwise. The regression model for each factor is specified as:

$$r_t = \alpha + \beta I_{t-12} + \epsilon_t \tag{4.1}$$

Where:

- r_t represents the return of the factor at time t.
- I_{t-12} is the indicator variable set to 1 if the average return over the previous 12 months is positive, and 0 otherwise.
- α and β are the regression coefficients.
- ϵ_t is the error term.

Additionally, a pooled regression is performed, where the data for all factors is aggregated to assess the overall momentum exhibited across both European and global factors. This pooled regression merges the data of all the factors and provides an estimate of the average momentum displayed by the factors. In this case, the intercept and slope represent the combined effects of all factors.

In this particular regression (4.1), the regression coefficients α and β have an economic interpretation that allows us to determine if the monthly factor returns display momentum based on their coefficient values:

- **Intercept (α)**: The intercept represents the average factor return following a year of underperformance [1]. If there is no momentum (i.e., no autocorrelation), the intercept is expected to be zero. A negative α suggests that underperforming factors tend to continue underperforming, while a positive α suggests the opposite.

- **Slope (β)**: The slope represents the difference in factor returns between years of positive performance (when $I_{t-12} = 1$) and years of negative performance (when $I_{t-12} = 0$) [1]. Under a no momentum scenario, the slope is expected to be 0. A positive β indicates better performance following positive returns, while a negative β suggests the opposite, showing a relationship between past and future returns.

Table 4.1 presents the coefficients from the individual regressions for each factor, as well as the pooled regression, along with their corresponding t-values. Recall that t-values, introduced earlier in Table 3.2, measure statistical significance, indicating how different the coefficients are from zero. Table 4.2 reports the critical t-values for different significance levels (10%, 5%, and 1%), based on 402 observations. If the absolute value of a calculated t-value exceeds the corresponding critical t-value at a given significance level, the coefficient is statistically significant, suggesting momentum in the factor returns. The 5% level is typically considered the benchmark for significance. However, a 10% level, while not significant enough, can still provide useful insights into the direction of the results and may warrant discussion. For example, if the absolute t-value of a coefficient exceeds 1.97, as indicated in Table 4.2, it is statistically significant at the 5% level, implying strong evidence of momentum in the factor returns.

Table 4.1: Factor Momentum Regression

	Dataset 1				Dataset 2			
Anomaly	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$
SMB	-0.21	-1.38	0.49	2.31	-0.39	-2.34	0.65	2.78
HML	-0.26	-1.23	0.92	3.37	-0.13	-0.48	0.36	0.98
RMW	0.20	1.18	0.20	0.99	0.10	0.56	0.07	0.32
CMA	-0.16	-1.21	0.54	2.92	0.02	0.16	0.21	0.98
UMD	0.81	1.53	0.02	0.04	0.82	1.68	-0.29	-0.53
E/P	0.38	1.66	-0.23	-0.76	0.06	0.25	0.40	1.35
CE/P	0.48	1.96	-0.29	-0.91	0.03	0.12	0.53	1.66
D/P	0.39	1.80	-0.26	-0.90	-0.05	-0.26	0.45	1.59
BAB	0.41	1.14	0.57	1.40	0.00	0.01	0.41	0.97
QMJ	0.52	1.99	-0.06	-0.20	0.18	0.65	0.31	0.90
G_SMB	-0.14	-0.89	0.33	1.47	-0.28	-1.34	0.60	1.98
G_HML	-0.20	-0.89	0.73	2.43	-0.41	-1.58	0.91	2.59
G_RMW	0.10	0.55	0.28	1.31	0.21	0.97	0.10	0.40
G_CMA	-0.05	-0.31	0.56	2.41	0.05	0.23	0.11	0.37
G_UMD	0.27	0.63	0.27	0.55	0.58	1.48	-0.41	-0.85
G_E/P	0.09	0.39	0.26	0.92	-0.13	-0.40	0.77	1.74
G_CE/P	0.03	0.15	0.21	0.78	0.10	0.29	0.48	1.14
G_D/P	-0.11	-0.53	0.13	0.43	-0.17	-0.57	0.44	1.08
G_BAB	-0.05	-0.15	1.02	2.76	-0.17	-0.49	0.51	1.04
G_QMJ	0.14	0.57	0.44	1.53	0.25	0.97	0.14	0.47
Pooled	0.07	1.37	0.40	6.09	0.00	-0.04	0.38	5.16

Table 4.2: Critical t-values for Different Significance Levels

Significance Level	**Critical t-value**
10%	1.65
5%	1.97
1%	2.59

Four of the 20 the intercepts are found to be significant at the 10% level in both datasets. Dataset 1 identifies six out of 20 factor slopes as statistically significant at the 5% level, while Dataset 2 finds seven out of 20 factor slopes significant at the 10% level. This indicates that some European factors exhibit momentum. Additionally, the pooled regression supports this conclusion: although the intercepts are not significant, the slopes in both datasets are strongly significant at the 1% level, with t-values of 6.09 and 5.16, respectively. This finding aligns with the original study [1], which also found that for US factors, the slopes in the pooled regression were significant at the 1% level (t-value of 4.22).

4.2 Momentum Strategy

The last part of this section implements a time-series momentum strategy, which involves taking long and short monthly positions in factors (excluding Momentum) based on whether their cumulative returns over the past year are positive or negative. Additionally, a cross-sectional strategy is applied, where long positions are taken in factors above the median, and short positions are taken in factors below the median. These strategies directly capture factor momentum. Similar to the definition of UMD in Section 2.4, they exploit the tendency of factors with strong past performance to continue outperforming, while weaker assets tend to lag. If these momentum strategies generate positive returns, it indicates that factors themselves exhibit momentum.

The time-series strategy yields an average yearly return of 3.40% (t-value = 2.76) in Dataset 1 and 4.11% (t-value = 2.41) in Dataset 2. The cross-sectional strategy yields returns of 3.59% (t-value = 3.11) in Dataset 1 and 3.07% (t-value = 1.92) in Dataset 2. Despite being statistically significant at the 5% level in European factors, these momentum effects are weaker than those observed in U.S. data, where significance is at the 1% level. One possible explanation is the shorter dataset in Europe, which begins in 1990, compared to the U.S. study, which starts in 1960. The additional 30 years in the U.S. data exhibit stronger momentum effects. This idea is further explored in Section 5.2, where the impact of missing earlier data on these results is discussed.

Furthermore, the time-series strategy outperforms the cross-sectional strategy in Dataset 2 but not in Dataset 1. This finding is unexpected, as [22] suggests that the time-series strategy is a pure bet on momentum, while the cross-sectional strategy implies that a high return on a factor predicts low returns on other factors, which is typically contrary to momentum. The original study by [1] also supports [22].

In summary, factor momentum is demonstrated through two distinct methods: first, via a regression analysis of prior year returns, and second, through the implementation of various factor momentum strategies. The significance of factor momentum provides substantial evidence to support the creation of a momentum factor based on these factors, rather than on individual stocks. The rest of this project will examine these aspects in greater detail.

5

Factor Momentum and the Covariance Structure

In this section, a momentum factor is constructed based on factor-level momentum, as introduced in Section 4. The analysis begins by examining the conditions under which factors exhibit momentum within the sentiment model proposed by Kozak, Nagel, and Santosh (KNS) [23]. The original study by [1] suggests that factors with high eigenvalues display stronger momentum. This hypothesis is then tested empirically, leading to the development of a more robust factor momentum strategy using principal component factors. Finally, the connection between factor momentum strategies and the construction of factors that capture factor-level momentum is established, demonstrating their equivalence. These momentum-based factors, referred to as Factor Momentum (FMOM), capture factor-level momentum and can be directly derived from the constructed factor momentum strategies.

5.1 KNS Sentiment Model

The KNS sentiment model [23] is an asset pricing model that incorporates investor sentiment as a factor influencing market prices and returns. It examines how the interaction between sentiment-driven investors and rational arbitrageurs shapes factor returns, ultimately leading to factor momentum. The model distinguishes between two types of investors:

- **Sentiment Investors** – Traders whose demand for assets is influenced by psychological factors, causing prices to deviate from their fundamental values. Past sentiment influences future market conditions, allowing sentiment-driven price fluctuations to persist over time rather than being immediately corrected by arbitrageurs.
- **Arbitrageurs** – Rational investors who optimize their portfolios based on expected returns and risks. These investors seek to exploit mispricings caused by sentiment-driven investors by trading against them. However, their ability to correct these mispricings is limited by risk aversion, capital constraints, and market frictions. As a result, sentiment-driven price distortions are not fully corrected, and sentiment-driven price movements persist.

The interaction between these two investor types leads to a market equilibrium where factor returns are influenced by different variables. This relationship is formalized in an explicit return equation (Equation C5 in [23]), which captures how sentiment-driven mispricing persists despite the presence of rational investors. For a detailed mathematical derivation, refer to the KNS study [23].

Building on this results, the original study [1] demonstrates that persistent sentiment yields factor-level momentum, with a stronger impact on high-eigenvalue factors—those that explain a large portion of the variance in asset returns. In contrast, low-eigenvalue factors, which explain a smaller portion of the variance, are less affected by sentiment persistence and exhibit weaker momentum. Empirical evidence, such as the Baker and Wurgler sentiment index [24], supports the presence of high autocorrelation in sentiment. Therefore, under the KNS sentiment model, high-eigenvalue factors, also known as systematic factors, should exhibit more momentum.

5.2 High Variance PCs and Factor Momentum

Under the KNS sentiment model [23], if sentiment exhibits high autocorrelation, systematic factors are expected to display stronger momentum. To empirically test this hypothesis, two tests are conducted. The first test measures the correlations between the factors from Dataset 1 and Dataset 2, yielding statistically insignificant results. A second, more robust test is conducted using Dataset 3, which shows that factors with higher variance indeed exhibit greater momentum.

5.2.1 Momentum in Systematic factors: Test 1

Correlation of Slope and Adjusted R^2 Scores

The first test follows the methodology outlined in the internet appendix IV of the original study [1]. The goal is to measure the relationship between systematic factors (those explaining more variation in returns) and momentum. Systematic factors are identified by having a high R^2, which reflects how well the factor explains the variation in returns. Momentum, on the other hand, can be measured by the slopes β computed in Table 5.1. By calculating the correlation between the adjusted R^2 values and β for different factors, we can test if the empirical data aligns with the hypothesis of higher eigenvalue factors explain more momentum. Adjusted R^2 is used instead of normal R^2 because it corrects for the number of factors in the regression. While normal R^2 always increases when adding more factors (even if they don't actually explain returns), adjusted R^2 only increases if the new factors provide real explanatory power. This avoids overestimating how well the model explains returns [1].

This tests explores the idea and computes the correlation between the β values of the 9 European factors (excluding MKT-RF and UMD) from Datasets 1 and 2, and their adjusted R^2 value. The β values for these 9 factors were computed earlier in Table 5.1. The adjusted R^2 values are computed using Dataset 3, which is the broader stock characteristic dataset that contains 412,031 monthly stock observations with 443 variables each.

For each of the 9 factors, the characteristic forming each factor (e.g. ME for SMB, BE/ME for HML, etc.) is selected, and a cross-sectional regression is performed between the characteristic and the observed returns at month t. Dataset 3 is filtered for 412,031 observations with 4 columns corresponding to (Characteristic, Return, Stock ID, Date). The cross-sectional regression groups these observations by date, such that for each of the 402 months from July 1990 to December 2023, a regression is applied using all the observations for that month:

$$r_{i,t} = \alpha_t + \beta_t X_{i,t} + \epsilon_{i,t} \tag{5.1}$$

Where:

- $r_{i,t}$ is the return of stock i at time t,
- $X_{i,t}$ is the characteristic of stock i at time t,
- α_t and β_t are the regression coefficients estimated at each time t,
- $\epsilon_{i,t}$ is the error term.

For example in the case of HML, the regression 5.1 would be applied 402 times, grouping the stocks from Dataset 3 by month. $r_{i,t}$ would correspond to the returns of each of the available stocks i at month t and $X_{i,t}$ would correspond to the value of the BE/ME characteristic of each available stock i at month t. As a result, 402 different adjusted R^2 values would be obtained from the monthly regressions, indicating how systematic the HML factor is over time.

After performing the cross-sectional regression for each of the 9 factor‘s characteristic, 9 different time series would be obtained, one per factor, containing the R^2 values for each month. This information could be effectively summarized by computing the mean and median R^2 values for each factor. For each characteristic, the adjusted R^2 score is calculated as the mean or median of all the R^2 scores.

Finally, the correlations between the adjusted R^2 scores and the slopes and t-values from the previous section are examined. A high correlation between these metrics would suggest that more systematic factors, those explaining a larger portion of stock returns (high R^2), exhibit stronger momentum (high slope). The results of this analysis are presented in Tables 5.1 and 5.2.

Table 5.1: Factors’ Slopes from Table 4.1 and Cross-Sectional Regression Adj. R^2

	Dataset 1 Slope		**Dataset 2 Slope**		**Cross-Sectional Regression**	
Anomaly	$\hat{\beta}$	$t(\hat{\beta})$	$\hat{\beta}$	$t(\hat{\beta})$	**Median R²**	**Mean R²**
SMB	0.49	2.31	0.65	2.78	0.31%	0.10%
HML	0.92	3.37	0.36	0.98	2.41%	1.05%
RMW	0.20	0.99	0.07	0.32	0.16%	-0.01%
CMA	0.54	2.92	0.21	0.98	0.22%	-0.01%
E/P	-0.23	-0.76	0.40	1.35	0.95%	0.42%
CE/P	-0.29	-0.91	0.53	1.66	0.51%	0.12%
D/P	-0.26	-0.90	0.45	1.59	1.19%	0.44%
BAB	0.57	1.40	0.41	0.97	4.03%	1.65%
QMJ	-0.06	-0.20	0.31	0.90	1.15%	0.43%

Table 5.2: Correlations between Slopes and R^2 scores

Correlations	**Dataset 1**		**Dataset 2**	
	$\hat{\rho}$	$t(\hat{\rho})$	$\hat{\rho}$	$t(\hat{\rho})$
$\rho(\hat{\beta}, R^2_{\text{median}})$	0.41	1.27	0.13	0.38
$\rho(\hat{\beta}, R^2_{\text{mean}})$	0.40	1.23	0.12	0.34

Table 5.1 presents the slopes and t-values from both datasets, along with the mean and median R^2 scores obtained from the cross-sectional regressions. In Table 5.2, I provide the correlations between these R^2 scores and the slopes. Contrary to the findings for US factors [1], the European data does not exhibit sufficient significance to draw definitive conclusions based on this limited number of regressions. Dataset 1 shows a weak correlation between the R^2 scores of a factor and its momentum, at around 0.16. In contrast, Dataset 2 produces opposite results, with correlations close to 0.12. Since none of these results are significant, further testing is conducted in the subsequent section to thoroughly evaluate this hypothesis.

5.2.2 Momentum in Systematic Factors: Test 2

PC Momentum Strategy

Following the methodology of the original study [1], Dataset 4 is used to compute a momentum strategy based on the factor's principal components (PC). This is because, if systematic factors exhibit more momentum, a momentum strategy would yield stronger returns on the first PCs, as these explain a higher variability of the returns. All factors related to momentum are excluded[4], as well as those with missing values for extended periods, resulting in a final selection of 120 factors. For every month t, the strategy that trades PC factors is computed as:

1. **Compute Eigenvectors Using Monthly Data**

   For a given month t, compute the covariance matrix of the standarized factor returns, from July 1993 up to month t. Apply Principal Component Analysis (PCA) to extract the eigenvectors from its covariance matrix. As there are 120 factors, this results in 120 PCs, expressed as 120 eigenvectors, ordered by their variance. Each of these eigenvectors points in an orthogonal direction, corresponding to a 120-dimensional vector in the space of the 120 monthly factor returns. By definition, the first eigenvector points in the direction of the highest variance, making this eigenvector the most systematic.

   The original authors required 10 years of data to start computing the strategy, but due to the shorter sample period, the requirement is reduced to 5 years. Thus, the strategy returns span from July 1998 to December 2023.

2. **Compute Monthly Returns for PC Factors**

[4] 14 factors related to momentum are removed. 9 correspond to pure momentum and 5 to short-term reversals.

Using the computed eigenvectors at moth t, PC factors returns are defined as the weighted combination of the 120 factors, using as weights the eigenvector dimension pointing to each factor (each eigenvector is a 120-dimensional vector where each dimension corresponds to a factor). For instance, the first eigenvector provides the linear combination of weights that exhibit the most variance at month t.

The strategy requires recalculating all PC factor returns from July 1993 to month $t+1$. This is necessary because the eigenvectors extracted in step 1 change every month, meaning that the PC component factors point in a different direction with each additional month of data. The return for the f-th PC factor at month t can be computed as:

$$r_{f,t}^{\mathrm{PC}} = \sum_{j=1}^{N} v_j^f \, r_{j,t} \tag{5.2}$$

where:
- v_j^f is the j-th element of the f-th eigenvector,
- $r_{j,t}$ is the return on individual factor j,
- N is the number of factors.

Imagine a simple scenario where there are two factors, A and B, PCA would generate two principal components (PC1 and PC2) ordered by their eigenvalues. If the first principal eigenvector is expressed as $[v_A^1, v_B^1]$, the returns for PC1 at month t would be computed as the weighted combination of the factors A and B at month t, using the weights of the principal eigenvector:

$$r_{PC1,t} = v_A^1 \, r_{A,t} + v_B^1 \, r_{B,t} \tag{5.3}$$

This step results in a set of 120 PC monthly factor returns that extend from July 1993 to month $t+1$, where each factor return is a monthly percentage computed as in Equation 5.2. In step 4, using the PC components computed up to month t, a PC momentum strategy is determined, so $t+1$ data is also needed in to asses the result of the strategy. In a real-world setup, only data up to month t is available, making the returns for $t+1$ unknown until the returns of the strategy are realized.

3. Demean and Adjust Variance of PC Factors

Using the PC returns data up to month t and factor data up to month t, PC factor returns are adjusted to have a mean of 0 and a variance equal to the average variance of the individual factors. This ensures that the average returns up to month t are zero and that all factors have the same variance. Based on [25] and [26], the average factor returns tend to be positive, which could bias the strategy towards buying more than selling. Demeaning the factors neutralizes this bias, making the strategy a pure play on autocorrelations. Scaling the variance allows for the comparison of factors on an equal footing.

4. **Construct Factor Momentum Strategy**

   At month t, he adjusted monthly factor returns for the 120 PC component factors, created using data from July 1993 to month t, are available. The momentum strategy on the PC component factors uses this data to make an investment decision on the PC factors at month t, and the plain returns of the same PC factors are observed at month $t+1$. The PC momentum strategy applies the same methodology that the strategy from Section 4.2: at the current month t, the average returns for each PC factor over the past 12 months (from $t-11$ to t) are calculated. Then, PC factors with positive average returns are longed, and those with negative average returns are shorted.

5. **Compute Return of the Momentum Strategy**

   The returns of the momentum strategy at $t+1$ are then computed using the PC factor returns for month $t+1$: of the PC factors that were longed in Step 4, and subtract the returns of the PC factors that were shorted. This gives the returns for the PC momentum strategy at month $t+1$, based on information up to month t. To compute the returns for other months, all steps must be repeated starting from the appropriate month.

The PC factor momentum strategy ($FMOM_{PC}$) is computed across various subsets of the 120 PCs to evaluate whether momentum performs better in PC factors with the highest eigenvalues compared to those with the lowest. This strategy is based on excess returns, as Dataset 4 consists of excess returns. Each strategy generates a time series of monthly excess returns, expressed as a percentage, from July 1998 to December 2023. Therefore, the statistics reported in Table 5.3 represent average monthly excess returns for each PC factor momentum strategy, denoted as $\bar{r}$.

Table 5.3: Performance of Momentum Strategies Across Principal Component Subsets

PC Subset	$\bar{\mathbf{r}}$	$\mathbf{t}(\bar{\mathbf{r}})$
$FMOM_{PC(1-25)}$	0.23	2.89
$FMOM_{PC(26-50)}$	0.10	4.47
$FMOM_{PC(51-75)}$	0.06	3.32
$FMOM_{PC(76-100)}$	0.05	3.22
$FMOM_{PC(101-120)}$	0.01	0.57
$FMOM_{PC(1-120)}$	0.10	3.96

Trading all the PC factors yields positive monthly average returns of 0.10% with a t-value of 3.96, indicating that factors display significant momentum, as already found in Section 4. Notably, the first PC subsets show higher returns and t-values compared to later subsets, reinforcing the idea that "more systematic factors display more momentum". While these findings support the initial hypothesis, it is not uncommon for some subsets, such as the second or third, to show stronger significance despite the first subset's higher average returns.

These results align with the observed patterns in U.S. factors during the analyzed time period. In the original study [1], the PC is divided into two halves: the first covering 1970 to 1996, and the second from 1996 to 2019, which overlaps with the period of study. In the first half, the study's results are more significant, whereas in the second half, the results are less significant and similar to these findings. This strengthens the argument made in Section 4 that factor momentum exhibits similar importance across both the U.S. and Europe but varies across different time periods, being stronger in earlier years. This time-period difference—where results tend to be weaker in later years—emerges as a recurring pattern in this analysis. To maintain clarity and consistency, this finding will be briefly revisited in later sections, noting how time-period effects influence the significance of factor momentum across various datasets and models, without going into detail each time.

Comparing $\mathbf{FMOM_{PC}}$ Strategies

Finally, the relationships among different subsets of PC factors are examined. Different multiple regressions are conducted where the highest eigenvalue strategy, $FMOM_{PC(1-25)}$, is augmented with the FF5 model[5] as the explanatory variables, and the target variables are the lower eigenvalue strategies ($FMOM_{PC(26-50)}$, $FMOM_{PC(51-75)}$, etc.), as shown in Equation (5.4). The inverse regression is also applied, where $FMOM_{PC(1-25)}$ is the target variable, and the explanatory variables are the lower eigenvalue strategies combined with the FF5 model, as shown in Equation (5.5).

$$FMOM_{PC(\text{26-50}),t} = \alpha + \beta FMOM_{PC(\text{1-25}),t} + FF5_t + \epsilon_t^{(\text{1-25})} \tag{5.4}$$

$$FMOM_{PC(\text{1-25}),t} = \alpha + \beta FMOM_{PC(\text{26-50}),t} + FF5_t + \epsilon_t^{(\text{26-50})} \tag{5.5}$$

In this context [1], a significant alpha indicates that the momentum strategy being targeted cannot be fully explained by the explanatory variables, suggesting the presence of additional sources of returns not captured by the explanatory variables. Conversely, a non significant alpha suggests that the target variable is redundant, as it can be explained by the explanatory variables. In the literature, it is common to describe a non-significant alpha by stating that "the explanatory variables span the target variable," meaning a linear combination of the explanatory variables is sufficient to capture the target variable.

On the other hand, a significant slope demonstrates a strong relationship between the explanatory and target variables, reflecting how changes in one are systematically associated with changes in the other. The results are summarized in Tables 5.4 and 5.5. The intercepts α and the slope coefficients β are reported with their corresponding t-values in parenthesis. For all subsequent tables, values presented in parentheses will refer to the t-values of the corresponding coefficients.

[5] Note that writing $FF5_t$ refers to performing a multiple regression including the five factors of the five-factor model (Eq. 2.6). Each of these terms has a β coefficient and acts as a regressor. Thus, Equation 5.4 is a multiple regression with six β coefficients. However, since only the first one is of interest, it is the only one explicitly included in the equation. This will hold in the rest of the project.

Table 5.4: Explaining low eigenvalue strategies with $FMOM_{PC(1-25)}$ (Eq. 5.4).

Coefficients	**Target Variable**			
	$\mathbf{FMOM_{PC(26-50)}}$	$\mathbf{FMOM_{PC(51-75)}}$	$\mathbf{FMOM_{PC(76-100)}}$	$\mathbf{FMOM_{PC(101-120)}}$
$\hat{\alpha}$	0.7 (3.43)	0.03 (1.58)	0.02 (1.35)	-0.02 (-1.80)
$\hat{\beta}$	0.13 (7.51)	0.13 (8.72)	0.11 (10.75)	0.12 (12.68)
Adj. R^2	27.65%	28.61%	45.51%	51.48%

Table 5.4 shows that PC factor momentum strategies are significantly correlated, with slope t-values between 7.51 and 12.68. This is noteworthy, considering that the individual PC factors are nearly orthogonal. According to the original study [1], these positive correlations suggest synchronization among the factors, meaning they tend to be either profitable or unprofitable simultaneously. Similarly, the slope coefficients from other regressions in the project are consistently significant, which is expected; therefore, future tables will present these results without further explicit analysis of these coefficients. The alphas indicate that the strategy trading the first PC factors spans the three lower-order PC strategies but is not able to span the 26-50 strategy due to the significant coefficient (3.43). Additionally, there is an observed alpha decay that contrasts with [21], which suggested that a model with a few number of low-order PC factors could effectively expected returns of anomaly portfolios.

Table 5.5: Explaining $FMOM_{PC(1\text{-}25)}$ with low eigenvalue strategies (Eq. 5.5)

Coefficients	**Explanatory Variable**			
	$\mathbf{FMOM_{PC(26\text{-}50)}}$	$\mathbf{FMOM_{PC(51\text{-}75)}}$	$\mathbf{FMOM_{PC(76\text{-}100)}}$	$\mathbf{FMOM_{PC(101\text{-}120)}}$
$\hat{\alpha}$	0.16 (2.68)	0.18 (3.26)	0.17 (3.08)	0.24 (4.71)
$\hat{\beta}$	1.09 (7.51)	1.45 (8.72)	2.38 (10.75)	2.70 (12.68)
Adj. R^2	54.41%	56.60%	60.40%	64.03%

Table 5.5 shows that the conclusions from Table 5.4 do not hold the other way around. The correlations hold, but the alphas are always significant, ranging from 2.68 to 4.71, and do not decay with lower eigenvalue strategies, meaning that a momentum strategy based on the first PC factors contains information that the lower eigenvalue strategies are not able to capture.

To summarize, more systematic factors are consistently more profitable, in line with the KNS sentiment model [21]. When measuring momentum, high eigenvalue factors significantly enhance the explanatory power of the cross-section, as they can explain low eigenvalue factors, but not vice versa. This also suggests that high eigenvalue factors may be more predictable, offering superior explanatory power and better capturing systematic risks [27]. These findings are consistent with observations in U.S. markets [1], and suggest that the variations in significance are attributed to time differences rather than geographical disparities

5.3 Factor Momentum: Conceptualizing Momentum Strategies as Factors

Momentum strategies are often employed to measure the persistence of asset returns over time. In an asset pricing context, these strategies can also be applied to construct a factor that targets this persistence. This subsection describes how trading strategies and factors are connected, showing that momentum strategies can be used to create momentum factors. More specifically, the factor momentum strategies described in the previous sections are now introduced as new momentum factors that measure factor-level momentum.

1. Factor Momentum as a Factor

Recalling from Section 1, traditional momentum factors like UMD target individual stock momentum (also known as stock-level momentum). However, the factor proposed in the original study [1] shifts the focus to factor-level momentum. This type of factor, which captures factor-level momentum, is denoted as Factor Momentum (FMOM). It represents a specialized form of factor analysis where the momentum exhibited by other factors is directly measured.

UMD follows the Fama-French methodology [6], described in Section 2.4, which involves constructing bivariate portfolios and computing the return difference between the top and bottom portfolios. As stated in Section 2.3.3, "This approach is equivalent to an investment strategy that goes long on the top-ranked stocks and short on the bottom-ranked stocks". This equivalence is crucial, as it allows trading strategies to be viewed not merely as investment tactics but as a methodology for constructing factors.

Similarly, the momentum strategies computed in Sections 2.4 and 5.2.2 can be interpreted as an alternative methodology for building factors. Since these strategies trade based on factor-level momentum, they serve as specific instances of FMOM, corresponding to $FMOM_{PC}$ and $FMOM_{ind}$.

2. $\mathbf{FMOM_{PC}}$: Principal Components Momentum as a Factor

The $FMOM_{PC}$ strategy from Section 5.2.2 exemplifies this concept effectively. In this strategy, PC factors are employed to capture and exploit momentum effects. By trading these PC factors, which are weighted averages of the original factors, $FMOM_{PC}$ becomes a specific instance of FMOM. This approach leverages the systematic components of factors (those with high eigenvalues) to maximize momentum opportunities.

3. $\mathbf{FMOM_{ind}}$: Time-Series Momentum as a Factor

Similarly, the time-series momentum strategy computed earlier in Section 4.2 embodies the same principle. It represents an alternative method for calculating FMOM, targeting persistence in individual factors rather than PC factors. However, as shown in this section, it is less significant than $FMOM_{PC}$.

This strategy was originally applied to the 22 factors of Dataset 1 and 2. From now on, $FMOM_{ind}$ refers to the time-series momentum strategy applied to the 120 individual factors from Dataset 4 that were used to compute PC factor momentum strategy.

In this section, the KNS sentiment model [23] was reviewed, illustrating how persistent sentiment-driven demand can lead to persistence in factor returns. Additionally, the model suggests that more systematic factors exhibit stronger momentum, a finding that aligns with both my results and those of the original study [1]. Notably, this alignment indicates that momentum strengths are comparable for both European and U.S. factors.

FMOM was then introduced as a new type of momentum factor that captures factor-level momentum, constructed as an instantiation of the strategies analyzed in previous sections. A natural question now arises: How does the FMOM factor compare to the momentum factors based on individual stock momentum, such as UMD? This question will be explored in the next section.

6

Factor Momentum and Individual Stock Momentum

Exploring the relationship between FMOM and individual stock momentum reveals insights into their interactions and significance in factor analysis. The analysis begins by examining the various sources that can generate momentum in individual stocks. Then, it is demonstrated that FMOM not only significantly contributes to individual stock momentum but also fully captures it, as it successfully explains UMD. Next, the analysis is extended to compare FMOM with other forms of individual stock momentum: it also explains most of the momentum factors. Moreover, none of the factors are able to explain it, which implies that FMOM captures additional sources of momentum. This explainability depends on the instance of FMOM, and its strength is therefore examined across different subsets of factors, where $FMOM_{PC(120)}$ stands as the most robust method. Finally, one of the potential sources of individual stock momentum, firm-specific momentum, is examined, showing that it is insignificant.

6.1 Sources of Individual Stock Momentum

If stock returns follow a factor structure, momentum in these factors can be transmitted to individual stock returns, leading to cross-sectional momentum [28]. In multifactor models like CAPM and APT, the excess return of stock i at time t ($R_{i,t} = r_{i,t} - r_{f,t}$) is given by F factors:

$$R_{i,t} = \sum_{f=1}^{F} B_i^f r_t^f + \epsilon_{i,t} \tag{6.1}$$

Where:

- B_i^f is the loading of stock i on factor j.
- r_t^f is the return of factor j at time t.
- $r_{f,t}$ is the risk-free rate at time t.
- $\epsilon_{i,t}$ is the idiosyncratic error term for stock i.

Under a multifactor model, the original study [1] examines the payoffs of a cross-sectional momentum strategy on individual stocks and concludes that the expected value of a momentum strategy portfolio π_t^{mom} can be expressed as:

$$
\begin{aligned}
\mathbb{E}[\pi_t^{mom}] = \sum_{f=1}^{F} &\left[\text{cov}(r_{t-1}^f, r_t^f)\sigma_{\beta^f}^2\right] + \\
&\sum_{f=1}^{F} \sum_{\substack{g=1 \\ g \neq f}}^{F} \left[\text{cov}(r_{t-1}^f, r_t^g)\text{cov}(\beta^f, \beta^g)\right] + \\
&\frac{1}{N} \sum_{i=1}^{N} [\text{cov}(\epsilon_{i,t-1}, \epsilon_{i,t})] + \sigma_\eta^2
\end{aligned} \tag{6.2}
$$

Where:

- N is the number of stocks.
- $\sigma_{\beta^f}^2$ is the cross-sectional variance of the portfolio loadings.
- σ_η^2 is the cross-sectional variance of the stocks' unconditional expected returns.

Equation 6.2 contains four terms that explain the potential sources of profit in a cross-sectional momentum strategy.

1. **Autocorrelation in Factor Returns:** When factor returns exhibit positive autocorrelation, it contributes to momentum profits. This effect is further amplified by variations in the factor loadings across different stocks. Even if individual factors show only weak autocorrelation, a larger number of factors can lead to noticeable momentum in stock returns.
2. **Lead-Lag Relationships Between Factors:** Momentum profits can also arise from timing differences in factor returns. This is influenced by the cross-serial covariance between factor returns and the relationships between factor loadings. For example, if the return of one factor today predicts the return of another factor tomorrow, and if the loadings on these factors are positively correlated, this can enhance momentum profits.
3. **Autocorrelation in Firm-Specific Returns:** Momentum profits may stem from the autocorrelation in the returns of individual stocks. This implies that if a stock's returns exhibit patterns of persistence or reversal, it can contribute to the overall success of momentum strategies.
4. **Variation in Mean Returns Across Stocks:** Differences in average returns among stocks contribute to momentum profits through mechanisms like those proposed by [29]. This means that variations in the expected returns of individual securities play a role in the effectiveness of momentum strategies.

Understanding the implications of these sources is crutial for the remaining sections. Equation 6.2 defines the potential sources, but does not define the magnitude of each source. By definition, FMOM directly captures sources 1 and 2 [1], so if these are significant contributors to momentum, FMOM should be able to explain a significant of individual stock momentum. Classical models like the FF5 implicitly capture source 4 [1], suggesting that a combination of FMOM and FF5 would be sufficient to account for sources 1, 2, and 4. However, none of the factor models address source 3[1], meaning that in the presence of firm-specific return autocorrelation, FMOM should not be able to explain all of the individual stock momentum. These implications are the focus of the following sections. Lastly, note that source 4 implies a relationship between individual stock momentum and factors, as these aim to capture differences in mean returns. This connection is further examined in Section 7.

6.2 Does FMOM contribute to individual stock momentum?

The extent to which FMOM contributes to individual stock momentum is now examined, finding that sources 1 and 2 are significant. The relationship between FMOM and UMD is then explored, demonstrating that FMOM combined with FF5 directly spans UMD. Together, they capture all four sources of individual stock momentum.

6.2.1 Pricing UMD-sorted portfolios with UMD and FMOM

If FMOM contributes to individual stock momentum, the fit of a model combining FMOM with FF5 (Eqs. 5.5 and 5.6) should significantly improve over a model that includes only FF5 (Eq. 5.3). Such an improvement would suggest that sources 1 and 2 are key drivers of momentum, as FMOM predominantly captures these two sources. To test this hypothesis, various asset pricing models are used to price portfolios sorted by individual stock momentum. These portfolios, explained below, are created using UMD-based sorting: stocks are sorted monthly into deciles according to their prior year returns, skipping a month. The excess return for each decile portfolio i is then calculated monthly as $R_{i,t} = r_{i,t} - r_f$. Then, for each decile, the portfolio's excess return at month t is regressed on four different sets of explanatory variables:

$$R_{i,t} = \alpha + FF5_t + \epsilon_t \tag{6.3}$$

$$R_{i,t} = \alpha + FF5_t + \beta UMD_t + \epsilon_t \tag{6.4}$$

$$R_{i,t} = \alpha + FF5_t + \beta FMOM_{ind} + \epsilon_t \tag{6.5}$$

$$R_{i,t} = \alpha + FF5_t + \beta FMOM_{PC(1-120)} + \epsilon_t \tag{6.6}$$

These multiple regressions asses how well different factor models explain individual stock momentum portfolios, indicating the contribution of each set of factors. Since the portfolios are constructed using the same sorting method as UMD, the combination of UMD and the FF5 model (Eq. 5.4) serves as a natural benchmark. In assessing the fit of each model, two metrics are used:

Alpha: Alphas serve as a proxy for understanding how well a model captures portfolio returns. Low alphas suggest that the factors in the model explain the returns effectively.

GRS F-value: While alphas provide an intuitive sense of fit, the Gibbons, Ross, and Shanken (GRS) F-value [30] offers a more comprehensive measure. It accounts not only for the alphas but also for the residuals of the model, making it a stricter test of overall model performance. The GRS test sets the null hypothesis that the model is valid, meaning all alphas should equal zero. Thus, a lower GRS F-value indicates a better fit. The GRS F-value is expressed as:

$$GRS = \frac{(T-N-K)}{N} \cdot \frac{\alpha' \Sigma^{-1} \alpha}{1 + \bar{f}' \Omega^{-1} \bar{f}} \tag{6.7}$$

Where:
- T: Number of periods.
- N: Number of portfolios.
- K: The number of factors in your asset pricing model.
- α: An $N \times 1$ vector representing the intercepts (alphas) from the regression on the portfolios' excess returns.
- Σ: The $N \times N$ covariance matrix of the residuals.
- Ω: A $K \times K$ covariance matrix of the factor returns.
- $\bar{f}$: The vector of factor means.

A significant reduction in the GRS F-value when incorporating FMOM into the FF5 model would indicate that it meaningfully contributes to explain individual stock momentum, confirming that sources 1 and 2 are significant.

To ensure consistency, each of the regressions are performed three times, using different sets of portfolios and factors:

1. **First Set**
   The target variables are the excess returns of quintile momentum portfolios, constructed from the 5x5 portfolios on size and momentum available on the Fama-French website[7] [6]. The FF5 and UMD factors are sourced from Dataset 1. The results are summarized in Table 6.1.

2. **Second and Third Sets**
   The target variables in these regressions are decile momentum portfolios, created using the UMD sorting method on Dataset 3, which contains monthly stock characteristics. Since this dataset is used to construct Datasets 2 and 4, the regressions are run twice: once with the FF5 and UMD factors from Dataset 2, and again with those from Dataset 4. The average results are reported in Table 6.2.

In regard to $FMOM_{PC}$ and $FMOM_{ind}$, the two instances defined in Section 5.3 are utilized. These factors are computed using Dataset 4, as there is insufficient data to compute them using Datasets 1 or 2. Therefore, it is expected that the regressions on the first set will yield worse results.

[6] Note that quintile portfolios are used in this set instead of deciles due to data availability for European portfolios. These portfolios are available [here]. This data contains portfolios for different quintiles of size and momentum, which were merged to obtain 5 quintiles based solely on momentum

Table 6.1: Portfolio regressions for quintile portfolios

Quintile	**FF5**	**FF5 + UMD**		**FF5 + FMOM$_{\mathbf{ind}}$**		**FF5 + FMOM$_{\mathbf{PC(1-120)}}$**	
	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Losers	-0.41 (-3.26)	0.07 (1.62)	-0.64 (-54.44)	-0.18 (-1.98)	-1.79 (-19.47)	-0.15 (-1.51)	-4.05 (-18.09)
2	-0.13 (-2.18)	0.05 (1.26)	-0.23 (-22.15)	-0.03 (-0.59)	-0.66 (-13.12)	0.01 (0.19)	-1.54 (-13.07)
3	0.01 (0.30)	0.01 (0.25)	0.00 (0.20)	0.02 (0.59)	-0.03 (-0.73)	0.05 (1.04)	-0.13 (-1.32)
4	0.19 (3.57)	0.02 (0.67)	0.22 (23.93)	0.11 (2.53)	0.61 (13.10)	0.11 (2.28)	1.34 (11.95)
Winners	0.54 (5.58)	0.18 (4.42)	0.48 (42.89)	0.38 (4.86)	1.24 (15.75)	0.32 (3.83)	2.91 (15.64)
Winners - Losers	0.75 (3.58)	-0.08 (-1.85)	1.12 (91.68)	0.36 (2.41)	3.03 (19.67)	0.30 (1.78)	6.94 (18.70)
Avg $\lvert\hat{\alpha}\rvert$	0.34	0.07		0.18		0.15	
GRS F-value	13.31	5.07		7.86		4.67	
GRS p-values	0.00%	0.01%		0.00%		0.01%	

Examining the GRS F-values of the quintile portfolios priced using Dataset 1 factors allows for meaningful comparisons, yielding similar results to those observed with U.S. portfolios [1]. The FF5 model performs relatively poorly when pricing these portfolios, yielding an F-value of 13.31. However, performance improves significantly when augmenting FF5 with UMD, resulting in an F-value of 5.07, which captures the four sources of individual stock momentum. Furthermore, FMOM significantly enhances the FF5 model, indicating that FMOM contributes to individual stock momentum through sources 1 and 2. Specifically, $FMOM_{ind}$ yields an F-value of 7.86, while $FMOM_{PC(1-120)}$ surprisingly outperforms UMD, achieving a GRS F-value of 4.67. As noted by the original study [1], this is unexpected, as UMD is specifically designed to target these momentum portfolios.

All GRS p-values are significant, indicating that none of these models can be considered fully valid. One potential explanation for this poor fit is that I could only obtain quintile portfolios based on Dataset 1 factors, rather than decile portfolios, resulting in broader portfolios that are harder to price accurately. Additionally, while the FMOM variables are expected to be highly correlated, they originate from different datasets.

Another notable observation is that my results for FMOM in European markets are relatively weaker than those found for U.S. markets. In the original study [1], both $FMOM_{ind}$ and $FMOM_{PC}$, which traded only 20 factors each, outperformed Carhart's UMD. However, in my analysis, $FMOM_{ind}$ is constructed using 134 factors from Dataset 3 (excluding momentum) and fails to surpass UMD. When constructed with the 18 non-momentum factors from Dataset 1, the GRS F-value rises to 9.55, further worsening the fit. Similarly, $FMOM_{PC}$ only outperforms UMD when using its most comprehensive form, $FMOM_{PC(1-120)}$. Using $FMOM_{PC(1-25)}$ results in a GRS F-value of 7.83. As discussed in Section 5.2, the main reason for these discrepancies seems to be the missing data from 1960 to 1990 in my analysis.

Table 6.2: Mean regression results for decile portfolios using Dataset 2 and Dataset 4

Decile	FF5	FF5 + UMD		FMOM$_{\mathbf{ind}}$		FF5 + FMOM$_{\mathbf{PC(1-120)}}$	
	$\hat{\alpha}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Losers	-0.72 (-4.16)	-0.07 (-0.56)	-1.28 (-19.24)	-0.31 (-1.97)	-2.09 (-10.72)	-0.18 (-1.06)	-4.86 (-10.70)
2	-0.37 (-3.15)	0.11 (1.36)	-0.92 (-23.08)	-0.08 (-0.73)	-1.51 (-11.73)	-0.04 (-0.37)	-3.21 (-10.48)
3	-0.07 (-0.72)	0.21 (2.44)	-0.52 (-12.53)	0.11 (1.12)	-0.88 (-7.35)	0.17 (1.55)	-1.99 (-6.96)
4	-0.15 (-1.94)	-0.05 (-0.62)	-0.18 (-5.03)	-0.12 (-1.46)	-0.19 (-1.99)	-0.11 (-1.22)	-0.37 (-1.59)
5	0.10 (1.19)	0.13 (1.49)	-0.04 (-1.29)	0.10 (1.15)	-0.05 (-0.45)	0.12 (1.27)	-0.06 (-0.24)
6	0.16 (2.18)	0.09 (1.23)	0.14 (3.45)	0.08 (1.05)	0.27 (2.91)	0.04 (0.51)	0.76 (3.44)
7	0.25 (3.41)	0.06 (0.92)	0.37 (10.80)	0.14 (1.89)	0.58 (6.58)	0.13 (1.62)	1.38 (6.61)
8	0.27 (3.23)	0.01 (0.08)	0.50 (14.50)	0.09 (1.20)	0.89 (9.33)	0.08 (0.89)	1.86 (8.21)
9	0.12 (1.16)	-0.27 (-3.46)	0.74 (19.24)	-0.09 (-0.92)	1.04 (8.64)	-0.11 (-0.96)	2.21 (7.52)
Winners	0.61 (4.24)	0.09 (0.81)	0.99 (18.09)	0.25 (1.91)	1.77 (11.02)	0.18 (1.19)	3.83 (9.84)
Winners − Losers	1.33 (5.06)	0.16 (1.06)	2.27 (29.26)	0.57 (2.53)	3.86 (14.16)	0.36 (1.46)	8.69 (13.43)
Avg $\lvert\hat{\alpha}\rvert$	0.38	0.11		0.18		0.13	
GRS F-value	15.76	2.43		3.42		1.81	
GRS p-values	0.00%	0.83%		0.02%		5.59%	

The decile portfolios priced using Dataset 2 and Dataset 4 exhibit similar relative performance to those in Dataset 1. The FF5 model shows the highest GRS F-value at 15.76, which significantly decreases to 2.43 when UMD is introduced. Additionally, $FMOM_{ind}$ improves the fit, yielding a GRS F-value of 3.42. Notably, $FMOM_{PC(1-120)}$ surpasses UMD, achieving a GRS value of 1.81.

In this case, the absolute performance of the models is better. While the FF5 model still yields a p-value close to 0, combining it with momentum factors results in substantially increased GRS p-values compared to Table 6.1. The model combining FF5 with FMOM$_{PC}$ yields a p-value of 5.59%, indicating that none of the alphas are significantly different from zero; therefore, the model fits well in explaining all sources of individual stock momentum.

As seen in Table 6.1, to outperform UMD, it is necessary to use the strongest form of FMOM. $FMOM_{ind}$ is dominated by UMD, and constructing a less demanding version of $FMOM_{ind}$ that trades the 20 factors from Dataset 2 significantly worsens the GRS F-value to 8.08. When utilizing $FMOM_{PC(1-25)}$, the GRS value increases to 2.84, which is still a respectable F-value but falls short of UMD.

Overall, FMOM demonstrates a strong ability to explain individual stock momentum through sources 1 and 2. Furthermore, $FMOM_{PC(1-120)}$ not only outperforms UMD but also provides a valid empirical model, suggesting that it might be a better factor for explaining individual stock momentum than UMD.

6.2.2 Explaining UMD with FMOM

Tables 6.1 and 6.2 are consistent, showing that FMOM contributes to individual stock momentum through sources 1 and 2. Moreover, FMOM might capture individual stock momentum better than UMD. To further investigate this hypothesis, an additional multiple regression is conducted, where UMD is the target variable, and the explanatory variables comprise various $FMOM_{PC}$ strategies augmented by the FF5 model (Eq. 6.8). The results of this analysis are reported in Table 6.3[7].

[7]The alpha values reported under FF5 in Table 6.3 simply refer to the alpha coefficient from Equation 6.8. In the case where the Subset is None, the same regression is applied but does not

$$UMD_t = \alpha + FF5_t + \beta \text{FMOM}_{\text{PC(Subset)}} + \epsilon_t \tag{6.8}$$

Table 6.3: Explaining UMD with FMOM

	FF5		**$\text{FMOM}_{\text{PC(Subset)}}$**		**R^2**
Subset	$\hat{\alpha}$	$t(\hat{\alpha})$	$\hat{\beta}$	$t(\hat{\beta})$	
None	0.40	5.42			48.74%
1-25	0.18	2.55	0.78	12.68	66.14%
26-50	0.27	3.25	1.17	5.70	54.28%
51-75	0.29	3.64	1.70	7.30	56.73%
76-100	0.25	3.37	3.21	10.53	62.33%
101-120	0.35	4.76	3.02	9.61	60.69%
1-120	0.13	1.81	2.50	13.77	68.00%

The regressions indicate that FMOM can explain UMD and capture the four sources of individual stock momentum, particularly when utilizing the most comprehensive version that trades all the PC factors (t-value of 0.13). In contrast, the strategies that trade lower eigenvalue factors obtain significant alphas that do not span UMD. Notably, models incorporating higher eigenvalue factors significantly reduce alpha compared to those using lower eigenvalue factors. For instance, a t-value of 2.55 was obtained for the first 25 PC factors, increasing to 4.76 in the lowest eigenvalue subset. This observation aligns with findings from Section 5 and the original study [1], which suggest that higher eigenvalue factors are more explanatory.

To summarize, the hypothesis that factor momentum contributes to individual stock momentum is validated by using various asset pricing models to explain portfolios sorted on UMD. Furthermore, the results reveal that FMOM outperforms UMD when using $FMOM_{PC(1-120)}$. This raises the question of whether FMOM can can explain momentum better than UMD. Time-series regressions confirm that this is indeed the case, demonstrating that FMOM can also explain UMD and capture at least its four sources. These findings prompt two critical questions: First, does this relationship hold when comparing FMOM to other types of momentum factors? Second, is FMOM capable of capturing additional sources of profits that individual momentum factors do not?

6.3 Alternative Momentum Factors: Spanning Tests

In addition to the standard UMD factor, the analysis is extended to examine how FMOM explains other forms of individual stock momentum. Moreover, spanning tests[8] are conducted to determine whether FMOM captures additional sources of profits not accounted for by other momentum factors.

include the $FMOM_{PC(Subset)}$ term; therefore, there is no Beta coefficient reported.

[8]Spanning tests measure the t-value of the alpha coefficient in a multiple linear regression. As stated in Section 5.2.2: "In the literature, it is common to describe a non-significant alpha by

Using the original UMD methodology [9] on the stock characteristics dataset (Dataset 3), three alternative momentum factors are constructed, and UMD is recomputed. This recomputation is necessary because the UMD factor in Dataset 4 follows the methodology outlined in Section 5, rather than the original UMD approach, which sorts stocks based on their prior year returns, skipping a month.

These other forms of individual stock momentum are factors created using sortings based on different variables: Intermediate Momentum [11], which uses returns from $t-7$ to $t-12$ months; Year 1-lagged return Momentum (Lagged Momentum) [31], which considers returns lagged by one year; and Current Price to High Price over the Last Year Momentum (CP/HP Momentum) [32], which sorts stocks based on the current price divided by the highest price over the past year.

Table 6.4 provides summary statistics on the monthly excess returns of these factors, expressed as percentages, as well as the alpha coefficient from a linear regression where these momentum factors are explained by the FF5 model, as shown in Equation 6.9:

$$MomentumFactor_t = \alpha + FF5_t + \epsilon_t \tag{6.9}$$

Following the logic of spanning tests, a more significant alpha, measured by its t-value, indicates that the momentum factor adds more information to the classical five-factor model, improving the explanatory power when modeling asset returns with a combination of the FF5 model and this momentum factor.

Table 6.4: Performance of Different Types of Momentum

Momentum Definition	Monthly Returns			FF5 Model	
	$\bar{\mathbf{r}}$	**Std. Dev.**	$\mathbf{t(\bar{r})}$	$\hat{\boldsymbol{\alpha}}$	$\mathbf{t(\hat{\boldsymbol{\alpha}})}$
Individual Stock Momentum					
Standard Momentum	0.55	4.06	2.75	0.75	4.80
Intermediate Momentum	0.50	3.30	3.04	0.64	4.39
Lagged Momentum	0.40	4.20	1.91	0.60	3.92
CP/HP Momentum	0.43	4.68	1.87	0.57	3.42
Factor Momentum					
$FMOM_{ind}$	0.18	1.00	3.56	0.20	5.01
$FMOM_{PC}$	0.10	0.46	3.96	0.11	5.31

Standard and Intermediate Momentum show statistical significance, with t-values of 2.75 and 3.04, respectively. In contrast, Lagged and CP/HP Momentum are near the 5% significance threshold of 1.97. The FF5 model alphas are also significant, ranging from 3.42 to 4.80, indicating that the strategies based on these types of momentum offer a premium unexplained by the classical five-factor model. This

stating that the explanatory variables span the target variable," indicating that the target variable does not add significant new information beyond the explanatory variables.

suggests they could serve as potential additions to a traditional pricing model. Additionally, the FMOM factors demonstrate the strongest significance, with t-values of 3.56 and 3.96. The FF5 model alpha t-values are also the highest (5.01 and 5.31), suggesting that augmenting the FF5 with FMOM provides a better fit than using any of the individual stock momentum measures. To further validate these findings, regressions are computed where the target variables are the different types of individual stock momentum (UMD_t^*) and the explanatory variables are the two main FMOM strategies augmented with FF5. The results of these regressions are summarized in Table 6.5.

$$UMD_t^* = \alpha + FF5_t + \beta FMOM_t + \epsilon_t \tag{6.10}$$

Table 6.5: Explaining Individual Stock Momentum with FMOM

Explanatory Variable	**$\mathbf{FMOM_{ind}}$**		**$\mathbf{FMOM_{PC(1-120)}}$**	
	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$	$\hat{\boldsymbol{\alpha}}$	$\hat{\boldsymbol{\beta}}$
Standard Momentum	0.28 (2.18)	2.41 (15.23)	0.17 (1.17)	5.40 (13.96)
Intermediate Momentum	0.40 (2.77)	1.29 (7.36)	0.35 (2.18)	2.94 (6.96)
Lagged Momentum	0.08 (0.71)	2.62 (18.31)	0.00 (-0.02)	5.85 (16.89)
CP/HP Momentum	0.12 (0.86)	2.30 (13.06)	0.13 (0.77)	4.91 (11.52)

The first row shows consistency with previous findings: regressing the recomputed Standard Momentum on $FMOM_{PC(1-120)}$, yields intercept and slope t-values (1.17 and 13.96) that closely match those in Table 6.3 (1.81 and 13.77). The most comprehensive version of FMOM continues to effectively span UMD. In contrast, $FMOM_{ind}$ remains less robust than $FMOM_{PC}$ and fails to explain UMD, as evidenced by the significant alpha (2.18).

For the alternative momentum factors, both FMOM variants successfully span all except Intermediate Momentum, which exhibits t-values for alpha of 2.77 and 2.18, respectively. These results are consistent with the original study [1], although the significance levels are lower due to the time-period difference discussed in Section 5.2. Additionally, $FMOM_{ind}$ consistently yields higher alphas, reinforcing the view that $FMOM_{PC}$ is the stronger variant of FMOM.

Since FMOM can explain most individual momentum factors, the inverse regressions are also performed to assess whether the alternative momentum factors can span FMOM, and whether $FMOM_{PC}$ and $FMOM_{ind}$ can expand each other. The results of these inverse regressions are presented in Table 6.6

$$FMOM_t = \alpha + FF5_t + \beta UMD_t^* + \epsilon_t \tag{6.11}$$

Table 6.6: Explaining FMOM with UMD

Target Variable	$\mathbf{FMOM_{ind}}$		$\mathbf{FMOM_{PC(1\text{-}120)}}$	
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Standard Momentum	0.08 (2.49)	0.16 (15.23)	0.06 (3.38)	0.07 (13.96)
Intermediate Momentum	0.14 (3.58)	0.10 (7.36)	0.08 (4.05)	0.04 (6.96)
Lagged Momentum	0.09 (3.09)	0.18 (18.31)	0.06 (3.83)	0.08 (16.89)
CP/HP Momentum	0.12 (3.61)	0.13 (13.06)	0.07 (3.99)	0.06 (11.52)
All Forms Momentum	0.09 (2.78)	—	0.06 (3.62)	—
$FMOM_{ind}$	—	—	0.03 (3.31)	0.39 (34.86)
$FMOM_{PC(1-120)}$	-0.02 (-1.01)	2.01 (34.86)	—	—

None of the alternative forms of momentum can explain FMOM on their own, even when all are included in the regression. The alphas remain significant across all cases, with the lowest t-value at 2.78 when regressed against all alternative momentum factors. This indicates that, in addition to explaining individual momentum, FMOM captures additional sources of profit. Consistent with previous findings, the alpha t-values for $FMOM_{PC}$ tend to be higher than those for $FMOM_{ind}$. Finally, the last two rows demonstrate that $FMOM_{PC}$ spans $FMOM_{ind}$, as indicated by an alpha t-value of -1.01, while the reverse does not hold, yielding a t-value of 3.31. This reinforces the conclusion that $FMOM_{PC}$ is the dominant version of $FMOM_{ind}$.

In conclusion, the findings in European markets suggest that FMOM effectively explains all returns in individual momentum factors and contains information not captured by the other factors. This reinforces the conclusions drawn for US markets [1]: "individual stock momentum is, at least in large part, a manifestation of factor momentum. An investor who trades individual stock momentum indirectly times factors; she would do better by timing the factors directly." Furthermore, the construction method of FMOM is crucial: $FMOM_{PC}$ not only proves to be a significantly stronger factor but also spans $FMOM_{ind}$, highlighting the superiority of the principal components approach in capturing momentum effects.

6.4 Alternative Sets of Factors: Spanning Tests

Building on the previous findings, this section examines how the number of factors used to compute FMOM influences its explanatory power. For each subset length, ranging from 1 to 120, 50 random combinations of factors of that length are generated. This process produces 50 distinct instances of both $FMOM_{ind}$ and $FMOM_{PC}$ for each length, with each instance derived from a different random subset of factors. Subsequently, two regressions are performed for each constructed FMOM factor to evaluate their relative strength:

- **Explaining FMOM with FF5**
  Measures how the constructed FMOM improves FF5. A significant alpha suggests that FMOM provides additional information beyond FF5 and it could be considered as valid additional factor.

$$FMOM_t = \alpha + \beta FF5_t + \epsilon_t \tag{6.12}$$

- **Explaining UMD with FF5 and FMOM**
  Mesaures the ability of FMOM to explain UMD, as in Table 6.3. In this case, an unsignificant alpha indicates that the combination of FF5 and FMOM capture UMD's returns.

$$UMD_t = \alpha + FF5_t + \beta FMOM_t + \epsilon_t \tag{6.13}$$

The results are illustrated in Figure 6.1. For each subset length ("Number of Factors") on the x-axis, the average t($\hat{\alpha}$) from the 50 regressions performed at that length is plotted.

(a) $FMOM_{PC}$ (b) $FMOM_{ind}$

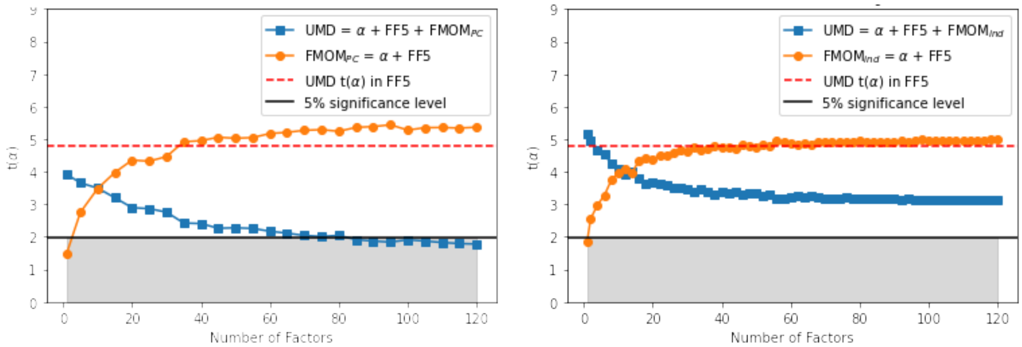

Figure 6.1: Measuring FMOM strength for different subsets

The figure illustrates consistent patterns regarding $FMOM_{PC}$ and $FMOM_{ind}$. Focusing on the first regression (orange), the t-values increase with the number of factors, and with just 2 factors, both instances of FMOM are significant enough to be considered as additional factors. In the case of $FMOM_{PC}$ (left subplot), at around 40 factors, the significance level surpasses that of UMD against FF5 (red line), indicating a greater explanatory power. For $FMOM_{ind}$ (right subplot), one needs to consider 60 factors to consistently surpass UMD.

In contrast, in the second regression (blue), the t-values decrease as the number of factors increases. $FMOM_{PC}$ is not able to span UMD until one includes 70 factors, and $FMOM_{ind}$ never achieves this. These patterns aligns with the original study, and support the idea that "individual stock momentum is an aggregation of the autocorrelations found in factor returns – the more factors, the better they capture UMD" [1].

6.5 Do Firm-Specific Returns Display Momentum?

In this final subsection, a consequence that is implicit in the findings is explored. In Section 6.1, the different sources of individual stock momentum are examined, stating that FMOM combined with FF5 should capture sources 1, 2, and 4 of individual momentum, but not 3 (momentum in firm-specific returns). Therefore, if FMOM captures UMD, and thus all four sources, as demonstrated in Section 6.2, it follows that firm-specific returns should not exhibit momentum. To test the consistency of this implication, residual momentum (RMOM) [33] is employed to conclude that firm-specific momentum is not a significant source of momentum.

In a perfectly specified asset pricing model, residuals should ideally represent randomness or firm-specific returns. For instance, if assets follow an FF5 model and are cross-sectionally regressed using FF5, the residuals correspond to firm-specific returns. However, if they are regressed using an FF3 model, the omitted factors will remain in the residuals, distorting their interpretation. As noted, "in practice, firm-specific returns are difficult to isolate" due to incomplete knowledge of all influencing factors, reliance on estimated rather than true factor returns, and the inherent noise in estimating factor loadings [1].

The concept behind RMOM is to compute these residuals given a pricing model and construct a momentum factor based on them, reflecting the momentum effects of factors not included in the model. Under a perfect asset pricing model that includes all relevant factors, strong residual momentum would indicate momentum in firm-specific returns. However, assessing whether the specified asset pricing model is complete is challenging. Consequently, significant RMOM suggests in practice that there are additional sources of momentum not captured by the model, which may arise from either firm-specific returns or omitted factors.

The profitability of residual momentum strategies is now investigated. Using the stock characteristics dataset, residuals for the CAPM, FF3, and FF5 models are computed. For each month t, a time-series regression is performed to estimate the factor loadings (slope coefficients) from month $t-72$ to $t-13$, requiring a minimum of three years of data. Residuals for each stock at time t are computed as the average residuals from month $t-12$ to $t-2$. Following the original UMD methodology [9], an RMOM factor is created based on these residuals, and UMD is reconstructed using the same stocks as those used for RMOM. Table 6.7 reports the average monthly returns of each momentum factor and presents a regression where RMOM is explained by FMOM.

$$\text{RMOM}_t = \alpha + \beta \text{FMOM}_t + \text{FF5}_t + \epsilon_t \tag{6.14}$$

Table 6.7: Explaining RMOM with FMOM

Momentum Variable	**Average Return**	**Explanatory Variable**			
		FMOM_{ind}		**FMOM_{PC}**	
		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\alpha}$	$\hat{\beta}$
Raw Returns (UMD)	0.43 (2.06)	0.17 (1.20)	2.24 (12.97)	0.02 (0.16)	4.90 (12.07)
CAPM Residual (RMOM)	0.40 (2.76)	0.23 (1.99)	1.66 (12.19)	0.16 (1.30)	3.34 (10.29)
FF3 Residual (RMOM)	0.23 (2.30)	0.10 (1.01)	0.73 (6.18)	0.07 (0.70)	1.66 (6.15)
FF5 Residual (RMOM)	0.14 (1.69)	0.03 (0.31)	0.49 (4.93)	0.01 (0.14)	0.98 (4.23)

The recomputed UMD shows a monthly average return of 0.43% with a t-value of 2.06, which is lower compared to the one computed in Table 6.4. The universe of stocks used to recompute UMD appears to weaken its significance, as now both FMOM factors span it, yielding alphas of 1.20 and 0.16.

Regarding RMOM strategies, their profitability decreases as more factors are added to the pricing model: 0.40 for CAPM residuals, 0.23 for FF3 residuals, and 0.14 for FF5 residuals. This decline is expected, as the addition of factors displaying momentum should reduce RMOM performance. Similarly, the alphas from FMOM regressions decrease as more factors are added, with higher values observed when regressing on $FMOM_{ind}$, reflecting its weaker performance compared to $FMOM_{PC}$. The alphas range from 0.31 to 1.99, indicating that FMOM explains all RMOM factors except for the RMOM variant computed from CAPM residuals with $FMOM_{ind}$. Therefore, FMOM captures all sources of individual stock momentum, indicating no momentum in firm-specific returns. By extension, FMOM also captures the momentum displayed by any omitted factors: "whatever these additional factors beyond the five-factor model are, they are still largely the same as those found in the FMOM factor" [1].

In conclusion, Section 6 provides compelling evidence supporting the hypothesis that FMOM not only significantly contributes to individual stock momentum in European markets but also explains all individual momentum sources while capturing additional sources that none of the other momentum factors can explain. In particular, the explanatory power of FMOM highly depends on the number of factors employed. $FMOM_{PC(1-120)}$ provides the greatest explanatory power, even explaining the other FMOM instances. Finally, the analysis of RMOM shows that FMOM spans RMOM, consistent with the implication that there is no significant firm-specific momentum. Overall, these results underscore the importance of FMOM in asset pricing models and suggest that investors might achieve better results by directly timing factors rather than relying on individual stock momentum strategies.

7

Momentum vis-à-vis with other factors

In this final section, the dynamics of momentum strategies and their interactions with classical factors are investigated. As explained in Section 6.1, the fourth source of individual stock momentum—variation in mean returns across stocks—is linked to classical factors, suggesting a relationship between momentum and these factors. The analysis begins by examining both the unconditional and conditional correlations between the UMD factor and the classical factors. The findings reveal that momentum tends to load positively on factors that have recently performed well and negatively on those that have underperformed, indicating that it shares systematic risk with them. Rather than being an independent factor, momentum appears to reflect broader dynamics among these factors. This raises an important question: does factor momentum arise purely from timing other factors, or is it simply a reflection of stock-level momentum? To address this, pure factor momentum is isolated using momentum-neutral factors, revealing that the observed factor momentum is indeed a true characteristic of the factors themselves, not merely an effect of individual stock momentum.

7.1 Unconditional and Conditional Correlations with UMD

A noteworthy observation regarding individual stock momentum is its low correlation with the five classical factors. Table 6.3 reports an R^2 value of 48.74% when regressing UMD on the FF5. According to the original study, "this might imply that factors unrelated to the market, size, value, profitability, and investment factor must explain the remaining 52% of the variation. Alternatively, these estimates might suggest that momentum is a distinct risk factor" [1]. However, unconditional correlations are not the best proxy for measuring this relationship. By definition, when constructing UMD, if a factor has performed well, stocks within that factor will have performed well, and UMD will tend to select those stocks. A more insightful approach involves studying the conditional correlations based on the prior year's return, selecting observations only when the factor return over the prior year has been positive or negative. Correlations between UMD and the factors are computed in Table 7.1. The European factors are computed with respect to UMD, and the Global factors with respect to G_UMD. Additionally, the z-value is included, representing the significance value of a test where the null hypothesis states that both conditional correlations are equal. These findings are reported in Table 7.1 as the average results obtained using Dataset 1 and Dataset 2.

Table 7.1: UMD Correlations

Factor	$\hat{\rho}$	$\hat{\rho}^{+}$	$\hat{\rho}^{-}$	**z-value**
		Europe		
Pooled	-0.08	0.17	-0.43	8.80
SMB	-0.02	0.17	-0.19	3.62
HML	-0.51	-0.26	-0.77	11.52
RMW	0.41	0.54	-0.04	5.03
CMA	-0.10	0.21	-0.53	9.40
E/P	-0.46	-0.25	-0.71	9.74
CE/P	-0.50	-0.34	-0.75	10.34
D/P	-0.54	-0.44	-0.69	6.38
BAB	0.44	0.57	0.12	3.41
QMJ	0.60	0.69	0.30	2.66
		Global		
G_SMB	-0.08	0.10	-0.29	4.31
G_HML	-0.44	-0.05	-0.78	16.37
G_RMW	0.09	0.35	-0.42	9.04
G_CMA	-0.13	0.25	-0.62	12.16
G_E/P	-0.31	0.09	-0.70	13.54
G_CE/P	-0.33	0.03	-0.74	15.17
G_D/P	-0.24	0.28	-0.69	14.61
G_BAB	0.24	0.61	-0.24	7.38
G_QMJ	0.34	0.44	-0.00	3.63

On average, the unconditional correlations tend to be low. While some factors, such as RMW or G_QMJ, exhibit high positive correlations (0.41 and 0.34), others show significant negative correlations, like HML (-0.51) and D/P (-0.54). These opposing correlations balance out in the pooled regression, resulting in an overall correlation of -0.08. In contrast, the conditional correlations display a different pattern, with most factors showing positive conditional correlations when the prior year's return has been positive and negative otherwise. The differences between these two types of correlations, as indicated by the z-values, are all significant at the 5% level. This demonstrates that momentum is significantly related to all factors, capturing the autocorrelations among these factors and thus sharing systematic risk. While unconditional correlations average out to zero—leading many factor models to treat UMD as a distinct factor—the conditional correlation analysis suggests this distinction may arise from momentum's switching behavior between the long and short legs of the factors [1].

7.2 Momentum in Momentum-Neutral Factors

Finding that momentum captures the dynamics between factors, rather than being an independent factor is consistent with the argument from Section 6, that individual stock momentum can be better explained by targeting the factors, rather than specific companies. However, since factors are constructed from individual stocks, momentum ultimately arises from individual stock returns. This raises the question: is momentum truly inherent to the factors (pure factor momentum), or is it merely a reflection of individual stock momentum (incidental momentum)?

"A source of ambiguity in demonstrating causality is that individual stock momentum may induce incidental momentum in factor returns" [1]. When a factor has performed well over the prior year, stocks with a high value on the factor characteristic tend to outperform those with a lower value. This idea can also be expressed as "the long leg tends to outperform the short leg." If no pure factor momentum exists, but individual stock momentum favors, for example, the size factor (such as small technology companies gaining popularity), it may appear as if the factor itself exhibits momentum—even if it is actually driven by individual stock momentum. This effect is known as incidental momentum. To control for this effect and isolate pure factor momentum, momentum-neutral factors are constructed to capture the pure momentum of the factor. A factor exhibits incidental momentum if:

$$\sum_{i=1}^{N} w_{i,t} r_{i,t-12,t-2} \neq 0, \tag{7.1}$$

This implies that a factor's past returns, computed using current weights, are non-zero. Notably, this resembles Equation 3.3, which was used to construct Dataset 4 factors from stock characteristics (Dataset 3). To eliminate the incidental momentum effect, a new set of weights, $x_{i,t}$, is defined to minimally adjust the original weights while achieving orthogonality. The objective function, as defined and solved in the original study [1], is:

$$\min_{x_i} \sum_i (w_i - x_i)^2 \quad \text{subject to} \quad \sum_{i=1}^{N} x_i = 0 \quad \text{and} \quad \sum_{i=1}^{N} x_i r_{i,t-12,t-2} = 0. \tag{7.2}$$

The new weights correspond to the residuals obtained from a cross-sectional regression of the original factor weights on past returns:

$$w_{i,t} = \alpha + \beta r_{i,t-12,t-2} + x_{i,t}. \tag{7.3}$$

The key idea is to make the long and short legs of a portfolio indistinguishable from each other based on past returns. In the case of pure factor momentum, constructing these legs using past returns should result in similar returns for both legs, as they would be equally influenced by the factor's momentum effects. If the returns differ, it could indicate the presence of incidental stock momentum, where individual stocks' momentum is affecting the factor's performance. By applying this approach, both legs should exhibit similar performance over the prior year, whether positive or negative, thereby isolating and capturing pure factor momentum.

In Table 7.2, two versions of the PC factor momentum strategy from Section 5 are compared: one computed using the Dataset 4 factors and the other using momentum-neutral factors.

Table 7.2: Factor Momentum Performance for Different Sets of PCs

Subset of PCs	$\mathbf{FMOM_{PC}}$		$\mathbf{FMOM_{Neutral}}$	
	$\bar{r}$	$\mathbf{t}(\bar{r})$	$\bar{r}$	$\mathbf{t}(\bar{r})$
1-25	0.23	(2.89)	0.29	(5.33)
26-50	0.10	(4.47)	0.09	(4.77)
51-75	0.06	(3.32)	0.04	(3.03)
76-100	0.05	(3.22)	0.02	(2.24)
101-120	0.01	(0.57)	0.01	(1.56)
1-120	0.10	(3.96)	0.09	(5.60)

Momentum-neutral factor strategies tend to be more profitable than the original factor strategies. The absolute differences reflect the amount of incidental momentum, which does not constitute a significant portion of the original factor momentum strategy. This suggests that FMOM stems from pure factor momentum. Finally, this hypothesis is confirmed by measuring how much individual stock momentum explains factor momentum. Regressions are run where $FF5$ targets the different forms of $FMOM_{PC}$ strategies (Eq. 7.4), and where $FMOM_{PC(1-120)}$ and $FMOM_{Neutral(1-120)}$ target each other (Eq. 7.5 and 7.6). The results are reported in Table 7.3, where column (1) refers to Equation 7.4, column (2) on the left side refers to Equation 7.5, and column (2) on the right side refers to Equation 7.6.

$$FMOM_t = \mathrm{FF5}_t + \epsilon_t \tag{7.4}$$

$$FMOM_{PC(1-120),t} = \alpha + \mathrm{FF5}_t + FMOM_{Neutral(1-120),t} + \epsilon_t \tag{7.5}$$

$$FMOM_{Neutral(1-120),t} = \alpha + \mathrm{FF5}_t + FMOM_{PC(1-120),t} + \epsilon_t \tag{7.6}$$

Table 7.3: Regressions Targeting different Forms of $FMOM$ (Eq. 7.4, 7.5 and 7.6)

Explanatory Variable	$\mathbf{FMOM_{PC(1\text{-}120),t}}$		$\mathbf{FMOM_{Neutral(1\text{-}120),t}}$	
	(1)	**(2)**	**(1)**	**(2)**
$\hat{\alpha}$	0.11 (5.31)	0.01 (0.70)	0.09 (6.08)	0.03 (2.92)
$FMOM_{PC(1\text{-}120),t}$	-	-	-	0.59 (24.43)
$FMOM_{Neutral(1\text{-}120),t}$	-	1.08 (24.44)	-	-
R^2	44.00%	79.92%	26.82%	73.75%

The results indicate that momentum-neutral factors effectively dominate the original momentum factors. When regressing them against FF5 (Eq. 7.4), both alphas are significant, with t-values of 5.31 and 6.08, demonstrating that they can be considered new, statistically significant factors in addition to the classical ones. However, when running regression (Eq. 7.5), $FMOM_{PC}$ is not able to explain $FMOM_{Neutral}$ (t-value of 0.70). The reverse regression (Eq. 7.6) shows the opposite; $FMOM_{Neutral}$ significantly improves upon the original factors (t-value of 2.92), meaning that $FMOM$ is not merely incidental but is genuinely distinct: returns in factor momentum are derived purely from the momentum inherent in individual factors.

In conclusion, momentum can be viewed as "the sum of autocorrelations found in other factors" [1] rather than a completely independent factor, which is consistent with the argument that investors can achieve a more robust momentum by timing these factors rather than relying solely on stock-level momentum. As a result of these autocorrelations, momentum stocks share similar factor exposures, meaning that investors trading momentum bear systematic risk because all winners and all losers have similar factor exposures. These risks arise from pure factor momentum rather than incidental stock momentum, demonstrating that factor momentum is an inherent characteristic of the factors themselves.

8
Conclusion

This thesis investigates the role of factor momentum in European markets, establishing it as a significant predictor of cross-sectional stock returns. Unlike traditional momentum, which arises from patterns within individual stocks, factor momentum reflects broader dynamics and is driven by factor-level autocorrelations. The findings reveal that European factors with recent strong performance continue to yield substantial premiums, challenging conventional asset pricing models and contributing to the landscape of investable anomalies. This allows for the construction of a new factor targeting momentum in factors (FMOM), which is even more effective when based on principal component (PC) factors rather than individual factors. This is consistent with the KNS sentiment model, where PC component factors exhibit more momentum. Evidence supports the hypothesis that FMOM not only significantly contributes to individual stock momentum (both UMD and alternative momentum) but also explains all individual momentum sources, capturing additional components that other momentum factors cannot. Specifically, the explanatory power of FMOM increases with the number of factors included. Finally, it is shown that UMD is not independent but rather linked to all the factors, suggesting that momentum arises from factor dynamics rather than stock-level momentum. This supports findings that a momentum investor would benefit more from timing factors than stocks, as systematic risk emerges from common factor exposures. Furthermore, this systematic risk appears to arise from pure factor momentum rather than incidental momentum effects.

This thesis highlights the strategic value of factor momentum, particularly in the European context, where data limitations and market structures differ from those in the U.S. My results are relatively weaker due to the absence of data from 1960 to 1990, a period in which the original study shows stronger effects. Additionally, in some cases, the European factors had to be constructed differently than in the U.S. study (e.g., using top and bottom 30% portfolios instead of deciles) due to data limitations. Although access to more comprehensive data could enhance the robustness and comparability of my findings, they remain consistent with those of the original study [1].

The original authors [1] raised two key questions for future research. First, while factor momentum aligns with the KNS sentiment model, which suggests that momentum may arise from the mispricing behaviors of sentiment-driven investors, they observed that in earlier periods, momentum also appears in lower-eigenvalue factors, potentially due to arbitrage. They suggest developing a test to determine whether these premiums arise from risk or mispricing, and a rational model with varying risk premiums might clarify the sentiment-driven demand within the KNS framework. Secondly, the data implies that momentum is driven entirely by factors, suggesting an absence of firm-specific momentum. However, unseen factors or estimation errors may affect these findings, indicating a need for a more rigorous approach to fully isolate firm-specific returns.

In addition to these questions, there are several next steps that could advance this research. First, my findings indicate that the strength of factor momentum appears to depend more on the time period than on geographical location. Further investigation could examine these periods in more detail and explore the relationship between factor momentum and macroeconomic variables like interest rates, inflation, or GDP growth. This would also enable an analysis of whether factor momentum is driven by broader economic conditions or if pure factor data suffices. Secondly, extending this analysis to other geographical regions (e.g., Asia-Pacific or emerging markets) could validate whether factor momentum behaves consistently across various market structures and regulatory environments. Third, I demonstrate how momentum appears to time classical factors rather than individual stocks; this analysis could be expanded to a broader set of factors to uncover any additional relationships. Fourth, I used a traditional method for defining momentum eigenvalue factors, based on PCs of prior-year factor returns. While this approach is straightforward and explainable, machine learning techniques could identify optimal methods for constructing factor returns and generating highly profitable factor components without relying on abstract theoretical models. Finally, this research suggests that momentum investors would benefit more by timing factors, and that systematic risk exists purely from factor momentum. It would be worthwhile to further examine the pricing of this risk and whether momentum strategies offer returns that justify this risk in comparison to the factors themselves.

In summary, while this study provides a solid foundation on factor momentum in European markets, addressing these areas and broadening future research will deepen the understanding of factor dynamics and their implications for both academic finance and investment practice.

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