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Integration of sales, inventory, and transportation resource planning by dynamic-demand joint replenishment problem with time-varying costs

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ABSTRACT

This paper addresses the joint replenishment problem in the context of sales and operations planning. Unlike traditional ones that assume static demands, unit costs, and transportation capacity, we consider dynamic-deterministic demands from sales plans and time-varying unit costs and transportation capacity to deal with the recent logistics conditions. We then introduce the capacitated dynamic-demand joint replenishment problem with time-varying costs. To efficiently solve this problem, a three-phase approach is proposed: (1) simplifying the problem and determining ideal inventory quantities using mixed integer linear programming, (2) estimating policy variables for each item using the covariance matrix adaptation evolution strategy, and (3) updating the ideal inventory quantities based on evaluated shortages until all demands are satisfied. Our method outperforms conventional approaches with at least 12 times faster solution runtimes in tests with up to 100 items. We also obtain the insight that the cost-efficient replenishment plan changes according to the increase rate of the unit transportation cost.

1. Introduction

Sales and operations planning (S&OP) serves as a pivotal integrated tactical planning process, ensuring a harmonious balance between demand and various supply capabilities, including production, distribution, procurement, and finance. This alignment with the sales plan empowers companies to enhance customer service, streamline inventory, reduce lead times, and provide top management with a comprehensive overview of the business (Jacobs & Chase, 2014). Notably, *distribution planning* emerges as a key S&OP process, bridging the divide between production and distribution capacity and sales objectives. Its core aim is to set precise inventory targets and maximize the utilization of warehousing and transportation resources, both internally and externally, to meet the demands outlined by the sales plan (Jacobs et al., 2011; Noroozi & Wikner, 2017).

In recent years, the rise of e-commerce has presented new challenges in distribution planning, such as handling limited logistics resource capacities, such as storage spaces, trucks, and warehouse workers. Additionally, the unit cost of transportation vehicles, especially those owned by third-party logistics (3PL) providers, surges during high-demand seasons like national holidays. In such a situation, while replenishing inventories during periods of low unit transportation costs can reduce expenses, it can also lead to increased inventory holding costs. Thus, an optimal distribution plan should strike a balance between these costs, taking into account the fluctuating unit transportation costs and capacity constraints.

Operations research technologies, including mathematical optimization, constraint programming, and simulation, play a pivotal role in bolstering S&OP decision-making (Pereira et al., 2020). Notably, distribution planning has often been approached as a *joint replenishment problem* (JRP) (Brahimi et al., 2015; Darvish et al., 2016). The JRP seeks to optimize the replenishment plan, defining both the order quantity and timing for each item, to minimize overall costs while meeting specific demands (Khouja & Goyal, 2008). This plan is typically articulated through an inventory policy, which sets guidelines on order timings and quantities, tailored by specific *policy variables* for each item. Recognized as a foundational multi-item lot-sizing problem, the JRP has garnered significant attention, especially since multi-item replenishment is a prevalent scenario in supply chain management (SCM) (Salameh et al., 2014). Various JRP models exist, differentiated by demand type: deterministic-static, stochastic-static, and deterministic-dynamic. Among these, the deterministic-dynamic demand model, also known

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as dynamic-demand JRP (DJRP), accounts for demands that are pre-known but fluctuate throughout the planning horizon (Boctor et al., 2004; Lee et al., 2001; Robinson et al., 2009).

In our research, we approach distribution planning through the lens of DJRP, given that the sales plan is characterized by deterministic-dynamic demand (Tuomikangas & Kaipia, 2014). Our primary objective is to devise a distribution plan that minimizes overall costs, factoring in time-varying unit transportation costs and capacity constraints, by optimizing inventory policy variables for each item and period. This necessitates solving the capacitated DJRP with time-varying costs (CDJRP-TC). Notably, most existing JRPs and DJRPs assume constant unit costs throughout the planning horizon. There is a noticeable gap in research addressing DJRPs that consider both time-varying unit costs and capacity constraints. Hence, this paper introduces a mathematical formulation for CDJRP-TC and presents a novel solution methodology.

This problem can be formulated as a mixed integer linear programming (MILP) model. However, it contains enormous binary variables to express the discrete behaviors of inventory policies and, therefore, only small-scale CDJRP-TC can be solved with commercial MILP solvers. Conversely, simulation-based optimization (SBO) techniques, frequently combined with metaheuristics such as genetic algorithms (GA), are often used to solve complex systems' optimization problems. However, these approaches demand significant computational resources given the numerous iterative simulation cycles involved, thus making them unsuitable for large-scale CDJRP-TC.

Therefore, to quickly solve the large-scale CDJRP-TC, we propose a three-phase approximate solution approach that combines MILP and SBO. The contributions of this paper are as follows: (1) formulation of distribution planning with outsourced transportation resources as CDJRP-TC, (2) proposal of the three-phase approach method for solving CDJRP-TC, (3) evaluation of the proposed method demonstrating its effectiveness in solving CDJRP-TC over MILP and GA, and (4) investigation of the proposed method's capability to obtain the appropriate solution according to the cost increase rate.

In Section 2, we introduce works related to this study. Section 3 explains JRPs and describes the assumptions of the CDJRP-TC. Section 4 formulates the CDJRP-TC as both an analytical and a simulation model. Section 4 presents the proposed three-phase solution approach. In Section 5, we evaluate the performance of the proposed method. We conclude in Section 6 with a summary and future work.

2. Related works

2.1. Joint replenishment problem

The joint replenishment problem (JRP) is one of the most fundamental multi-item lot-sizing models (Khouja & Goyal, 2008). The JRP determines the replenishment amounts and timings of items in the same order, generally provided by the same supplier, and corresponding warehousing and transportation resources to reduce the total cost. In JRP researches, inventory replenishment operations are often modeled with inventory policy, which is common in real-world SCM because it enables efficient operations with a few parameters. The typical inventory policies are shown in Table 1. In such cases, the goal is to optimize the policy variables of each item, site, and period to determine the appropriate inventory target.

In terms of demand type, JRPs are categorized into (1) deterministic-static, (2) stochastic-static, and (3) deterministic-dynamic demand (Bastos et al., 2017). In these classifications, JRPs with deterministic-dynamic demand, i.e., dynamic-demand JRP (DJRP), consider the demands that are known in advance but vary over the planning horizon. It is difficult to solve DJRPs analytically because the dynamic demand increases the problem's complexity. DJRPs have been studied extensively because of their importance in industry and mathematical complexity. Federgruen and Tzur (1994) proposed an efficient branch-and-bound-based heuristic approach for an uncapacitated DJRP with time-varying costs and optimized order quantities and timings for those with 20–30 items and 20–30 periods within a reasonable time. Robinson et al. (2007) studied the effective solution approaches for the DJRP and determined that a two-phase heuristic and simulated annealing (SA) outperform existing methods. (Gao et al., 2008) studied the unconstrained DJRPs and proposed two MILP formulations with tight LP relaxations that can be solved with the primal–dual algorithms.

Table 1
Classification of general inventory policies.

Policy	Policy variables	Logic (Inventory level: z)
(t, q)	Review cycle: t Reorder quantity: q	A quantity q is ordered every t days.
(t, S)	Review cycle: t Base stock position: S	A quantity $max(0, z - S)$ is ordered every t days.
(s, q)	Reorder point: s Reorder quantity: q	A quantity q is ordered if z is lower than s.
(s, S)	Reorder point: s Base stock position: S	A quantity $max(0, z - S)$ is ordered if z is lower than s.

Another important classification aspect of the JRP model is the capacity constraint; for example, production, transportation, and storage capacity. The capacitated JRP is difficult to solve within a reasonable time because even a two-item JRP with constrained capacity is NP-hard (Bushuev et al., 2015). Kiesmüller (2010) studied the stochastic-static JRP with full truckload constraints and provided approximated formulas to compute the policy variables of the (Q, S) policy, which are the variants of (s, S) policy that always satisfies the full truckload constraints. Singha et al. (2017) proposed a mathematical formulation of the JRP based on the (s, q) policy with the stochastic-static demand and the storage capacity constraint for one item and one period, which they solved using an iterative heuristic approach. (Noh et al., 2020) studied the deterministic-dynamic JRP with a can-order policy with storage capacity and carbon cap constraints. They formulated this DJRP as a deterministic and a fuzzy MILP model, respectively, and solved these models, including ten items and 20 periods, using a commercial MILP solver.

Metaheuristics such as GA, SA, and other black-box optimization methods are also used to solve constrained JRPs. Chen et al. (2019) developed a MILP model for JRPs with the shortage and partial demand substitution under budget, transportation capacity, and shipment requirement constraints. They showed that DE outperformed other GA-based heuristic approaches. Lorenzo-Espejo et al. (2022) considered the JRP of the omnichannel replenishment process under product devaluation. They solved this with the combined metaheuristic approach of particle swarm optimization (PSO) and SA. Liu et al. (2021) considered a coordinated capacitated dynamic lot-size and delivery problem and developed the six-phase heuristic (SPE) and the evolutionary algorithms (GA and DE) incorporating SPE (GA-S and DE-S, respectively). They reported that the DE-S provided more suitable solutions than the SPE and the GA-S for most problem instances.

2.2. Simulation-based optimization approaches

Recent studies dealing with complex inventory systems have widely utilized simulation models as an alternative to analytical models such as MILP. Simulation can model non-linear, discrete, and uncertain behaviors such as inventory policy and capacity constraints.

Simulation-based optimization (SBO) is one of the approaches to solving optimization problems whose objective functions are modeled by simulation. In this approach, to perceive the landscape of the objective function, solutions are evaluated by simulation, and black-box optimization methods such as metaheuristics are widely utilized as search algorithms (Jalali & Nieuwenhuyse, 2015). Baharom and Hamzah (2018) developed a simulation model mimicking the inventory system of a real-world electronics company. By Monte Carlo simulation, they determined the optimal reorder policy that results in the lowest average total inventory cost under uncertain demand and lead times. O. et al. (2017) utilized a discrete event simulation to model a supply chain subject to market demand uncertainty and proposed a new reorder point update procedure for order-up-to-level policies in continuous review systems. Nagasawa et al. (2015) built the simulation model of JRP with the can-order policy to evaluate ordering penalty, shortage penalty, and inventory penalty. They solved this JPR with 200 products and one period using a multi-objective genetic algorithm (MOGA). Chu and You (2015) used simulation to model real-world multi-echelon inventory systems under uncertainties. They propose an SBO framework for optimizing distribution inventory systems where each facility is operated with the (s, q) inventory policy by using a cutting plane algorithm. Lim et al. (2017) proposes a multi-objective simulation–optimization model for solving a S&OP problem in build-to-order industries with flexibility, uncertain demand, and impatient customers. They modeled this S&OP problem by using simulation with ϵ-constraints and solved it using SBO approaches based on random local search and SA. Grewal et al. (2015) proposed a framework to adjust reorder points and lot sizes based on a seasonal demand cycle. In this framework, a discrete-event simulation model of a capacity-constrained supply chain is developed. Then SBO experiments are performed to obtain optimal timing of adjustments, lot sizes, and reorder points.

SBO approaches have been applied to various industrial engineering problems, such as scheduling, industrial processes, logistics, and SCM (Alrabghi & Tiwari, 2015; Junior et al., 2019). Hochmuth et al. (2010) developed a metaheuristic SBO method for a multi-location inventory model consisting of (s, S) policy with stochastic static demand. They solved this problem with PSO and GA, respectively. Suemitsu et al. (2022) developed a sequence optimization method for automated picking systems modeled by simulation. SA assisted by a pre-trained Bayesian recurrent neural network was proposed to quickly solve simulation-based sequence optimization problems. Yegul et al. (2017) tackled the production line configuration with SBO and proposed ant-colony optimization combined with a myopic search.

2.3. Hybrid simulation and optimization approaches

Optimization approaches that utilize the output of a simulation model to compute performance measures are referred to as *hybrid simulation and optimization* (Olafsson & Kim, 2002). Hybrid simulation and optimization approaches can be classified into three categories, *solution evaluation (SE)* referred to as SBO, *solution generation (SG)* and *analytical model enhancement (AME)* (Figueira & Almada-Lobo, 2014). Specifically, SG and AME use analytical and simulation models, whereas SE (SBO) approaches generally use only a simulation model. In the SG approach, the simulation model is utilized to simulate the solutions of the analytical model to compute variables as part of the overall solution. While the SG approach does not use the simulation results for feedback to the analytical model, the AME approach updates the analytical model using the simulation results.

Nikolopoulou and Ierapetritou (2012) proposed a hybrid approach combining mathematical programming and a simulation model that addresses the planning and scheduling in SCM. In this approach, a simulation is used to investigate the quality of the plans generated by the analytical model. Hrušovský et al. (2018) presented a solution framework that combines an analytical optimization model and simulation to determine intermodal transportation networks. They used the simulation model to evaluate uncertain travel times, whereas the optimization model provided the deterministic solution quickly. de Keizer et al. (2015) developed a MILP model and a hybrid approach to identify a cost-optimal network design for distributing perishable products. In this method, the MILP network design solution is checked by a discrete event simulation to account for uncertainties in supply, processing, and transport. Then, the simulation results are used to update the product quality constraints in the MILP model iteratively.

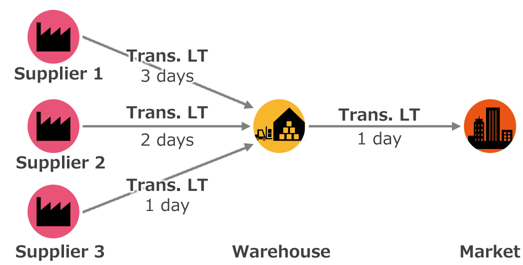

Fig. 1. Target supply chain with three suppliers, one warehouse, and one market.

Zhang et al. (2017) proposed the simulation-based Benders decomposition (BD), which is a variant of the generalized BD (Geoffrion, 1972), for efficiently solving a large-scale MILP. They proposed an algorithm to generate cuts using the simulation trajectory that utilizes simulation results as feasible solutions to the analytical model for solving joint workstation, workload, and buffer allocation problems. Forbes et al. (2021) proposed Branch-and-Simulate that combined simulation and logic-based BD where a simulation is executed at each incumbent solution to the master problem. Using the proposed method, they solved an airport check-in counter allocation problem and a nursing home shift scheduling problem within a reasonable runtime.

Moreover, Table 2 is generated to present a detailed review of the literature regarding JRP and inventory control. As mentioned above, solving CDJRP-TC is critical to distribution planning in the recent logistics environment. However, to the best of our knowledge, the existing DJRP models do not consider the time-varying unit cost and the capacity constraints simultaneously. Thus, the primary objective of this study is to formulate CDJRP-TC mathematically and propose a new solution method based on the AME approach.

3. Problem description and assumptions

This section provides an overview of the capacitated DJRP with time-varying unit costs (CDJRP-TC) based on the (s, S) policy. In this section, Section 3.1 details the inventory policies, Section 3.2 delves into the JRP, and Section 3.3 outlines the assumptions of CDJRP-TC. Our discussion references the supply chain (SC) network shown in Fig. 1, which illustrates the three-stage SC consists of suppliers, warehouses, and markets. In this structure, three suppliers, which have enough inventory to meet every shipment demand, replenish items in a warehouse based on a specific inventory policy. Subsequently, this warehouse distributes the items to markets based on a predefined sales plan. Note that these settings to simplify the SC model are adapted from the models presented by Grewal et al. (2015), Liu et al. (2021), and Nagasawa et al. (2015). For the following discussion, we categorize warehouse storage and workload as warehousing resources, while the vehicles from each supplier are considered transportation resources.

3.1. Inventory policy

As mentioned above, the inventory policy, shown in Table 1, is frequently employed for real-world inventory replenishment operations. In this study, the reorder point and base stock (order-up-to-level) (s, S) policy is used for the inventory replenishments from suppliers to a warehouse. Fig. 2 illustrates the replenishment with the (s, S) policy. The inventory level z is defined as the warehouse inventory quantity plus the quantity of orders that have been placed but have not arrived yet. In the (s, S) policy, if z has fallen below the reorder point s at the beginning of each period, the $S - z$ unit of the item is ordered to the corresponding supplier up to where the base stock is S. Note that our proposed method can be applied independently of the specific inventory policy. We adopted the (s, S) policy as a representative example in this study since this policy is one of the most extensively investigated inventory policies, as shown in the literature review.

Table 2
Detailed investigation of literature related to JRP and inventory control.

Author	Problem	Demand	Inventory policy variable	Multiple products	Time-varying unit cost	Capacity consideration: Transport	Capacity consideration: Storage	Capacity consideration: Workload	Model	Solution approach
Federgruen and Tzur (1994)	JRP	Dynamic-deterministic	Order qty. (q)	✓	✓				Analytical	Branch-and-bound-based heuristic
Robinson et al. (2007)	JRP	Dynamic-deterministic	Order qty. (q)	✓					Analytical	Two-phase heuristic, SA
Gao et al. (2008)	JRP	Dynamic-deterministic	Order qty. (q)	✓					Analytical	Primal–dual algorithms
Kiesmüller (2010)	JRP	Stochastic-Static	(s, S) variant	✓		✓			Analytical	Heuristic
Singha et al. (2017)	JRP	Stochastic-static	(s, q)	✓			✓		Analytical	Iterative heuristic
Noh et al. (2020)	JRP	Deterministic-dynamic	Can-order (s, c, S)	✓			✓		Analytical	MILP
Chen et al. (2019)	JRP	Deterministic-static	(t, q)	✓		✓			Analytical	DE, GA
Liu et al. (2021)	JRP	Deterministic-dynamic	Order qty. (q)	✓		✓			Analytical	Heuristic-DE hybrid
Lorenzo-Espejo et al. (2022)	Dynamic lot-sizing & delivery	Deterministic-static	Order qty. (q)	✓		✓			Analytical	PSO-GA hybrid
Baharom and Hamzah (2018)	Inventory control	Stochastic-static	(s, q)						Simulation	Bruteforce search with Monte Carlo simulation
O. et al. (2017)	Inventory control	Stochastic-static	(s, S)						Simulation	Heuristic
Nagasawa et al. (2015)	JRP	Deterministic-dynamic	Can-order (s, c, S)	✓					Simulation	MOGA
Chu and You (2015)	Inventory control	Stochastic-static	(s, q)						Simulation	Cutting plane algorithm
Lim et al. (2017)	Inventory control	Stochastic-static	Order qty. (q)						Simulation	Local search-SA hybrid
Grewal et al. (2015)	Inventory control	Deterministic-dynamic	(s, q)	✓			✓		Simulation	Metaheuristics (OptQuest)
Hochmuth et al. (2010)	Inventory control	Stochastic-static	(s, S)			✓	✓		Simulation	PSO, GA
Proposed method	JRP	Deterministic-dynamic	(s, S)	✓	✓	✓	✓	✓	Analytical & simulation	Hybrid simulation & optimization approach

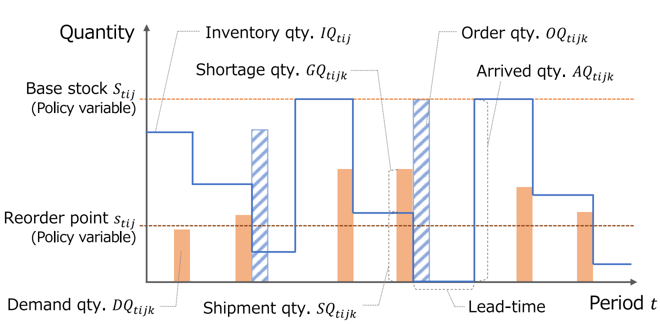

Fig. 2. Reorder point and base stock (order-up-to-level) (s, S) policy.

3.2. Description about joint replenishment problem

The joint replenishment problem (JRP) considers N items and the planning horizon with T periods. For each item i, period t, and market k, demands D_{tik} are obtained from a sales plan. To satisfy these demands, replenishment orders may be placed in each period. The concept of JRPs when $N = 3$ is shown in Fig. 3. This figure shows the inventory system based on the (s, S) policy, which replenishes inventory up to the base stock (order-up-to-level) S when the inventory level has fallen below the reorder point s. The left side of Fig. 3 shows that the joint replenishment of three items is placed at period t_1 and two items at t_3. The transportation is consolidated into two vehicles to minimize the number of vehicles because the transportation cost is determined by multiplying the number of vehicles and the unit transportation cost. The received inventories are stored in the warehouse, which consists of fixed and variable storage. All inventories are stored in the fixed storage until their total volume exceeds the fixed storage capacity. The inventory holding cost of the fixed storage is constant and incurred regardless of the inventory volume. The variable storage is used to store the inventory exceeding the fixed storage capacity. The variable inventory holding cost is proportional to the inventory volume in the variable storage.

The total cost of the JRP consists of two components: the fixed cost (or major ordering cost) and the variable cost (or minor ordering cost) (Anily et al., 2009; Gao et al., 2008). The fixed cost is incurred when at least one item is replenished regardless of the order quantity, such as the setup cost, the fixed inventory holding cost, and the transportation cost. Meanwhile, the variable cost depends on the replenishment order quantity, such as the variable inventory holding

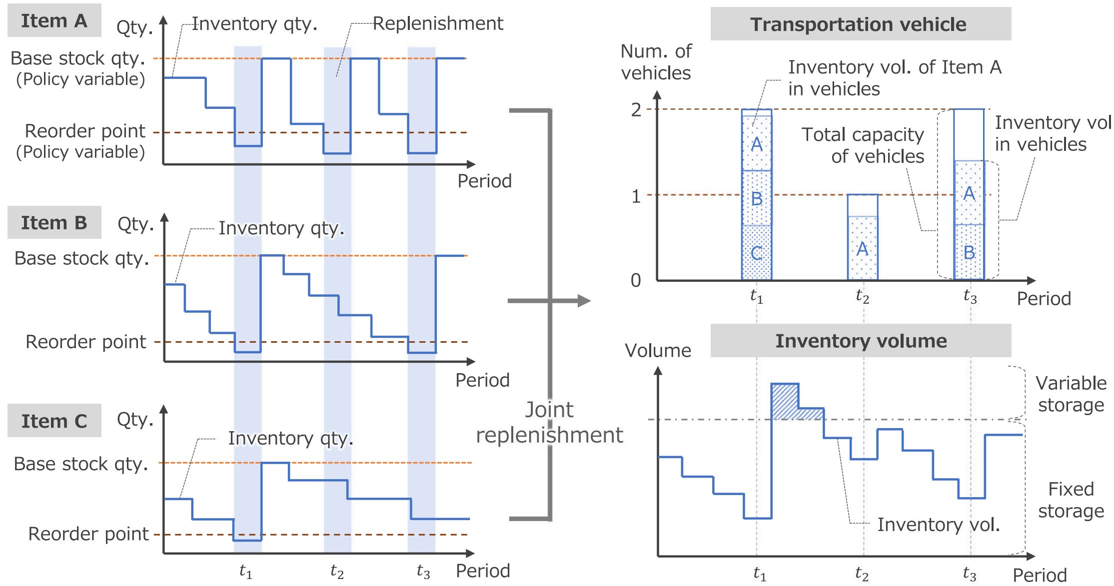

Fig. 3. Overview of the joint replenishment problem with (s, S) policy. When the inventory quantity is less than the reorder point (s), the difference between the base stock quantity (S) and the inventory quantity is ordered for replenishment. The transportation of ordered items is consolidated to minimize the number of vehicles. The replenished inventories are stored in fixed storage when the stored volume does not exceed the storage capacity. After exceeding the capacity, inventories are stored in variable storage.

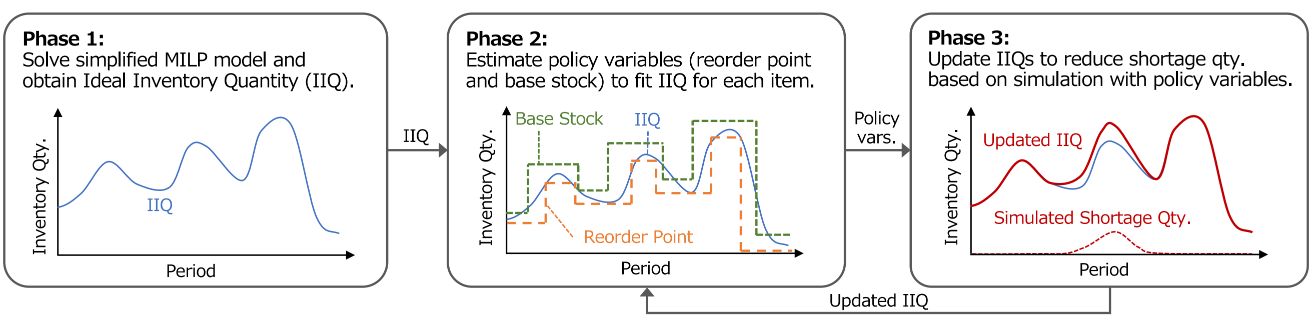

Fig. 4. Concept of the proposed three-phase solution approach.

cost and the labor cost to receive replenished items in a warehouse. This paper defines the total cost as the summation of fixed and variable inventory holding, transportation, and labor costs.

3.3. Problem assumptions

As mentioned above, this study discusses CDJRP-TCs in the supply chain network shown in Fig. 1. The target CDJRP-TCs adhere to the following problem assumptions or characteristics:

(1) The period t is defined as a day.
(2) Each item is replenished with the given path between sites, referred to as the *supply route*.
(3) The replenishment order arrives after the transportation lead time defined for each supply route.
(4) Two types of vehicles (*normal* and *extra* vehicle) are available, where the unit cost of the extra vehicle is higher than the normal vehicle.
(5) The maximum number of each type of vehicle per day is limited.
(6) The maximum workload to receive and ship items in a warehouse per day is limited.
(7) The inventory shortage is not backlogged.

4. Formulation of CDJRP-TC

This section presents a description and formulation of the CDJRP-TC based on the (s, S) policy. As mentioned above, we formulate DJRPs considering both time-varying unit costs and capacity constraints for addressing the distribution planning under recent logistics situations. To solve CDJRP-TC, we propose a three-phase solution method based on the AME approach shown in Fig. 4. The solution process of the proposed method consists of the following three phases. In Phase 1, the simplified MILP model of CDJRP-TC is solved with a MILP solver, and *ideal inventory quantities (IIQ)* of each item are obtained as the inventory quantities of the MILP solution. Phase 2 estimates the policy variables that best fit the IIQs for each item using CMA-ES. In Phase 3, the shortage of each item is evaluated with the simulation model of CDJRP-TC, whose inputs are the estimated policy variables of all items. Then the IIQs are updated according to the shortages of each item. These procedures are iterated until no shortage occurs in the simulation, and the policy variables estimated in Phase 2 are returned as a solution. The proposed method utilizes the analytical MILP model and the simulation model of CDJRP-TC. Therefore, in this section, we formulate the CDJRP-TC as both the analytical and the simulation model.

Table 3
Indexes and sets.

Notation	Description
t	index of a period (day); $t \in \mathbf{T}$.
i	index of an item; $i \in \mathbf{I}$.
j, k	index of a site; $j, k \in \mathbf{J}$.
v	index of a vehicle; $v \in \mathbf{V}$.
$\mathbf{T}$	set of periods; $\|\mathbf{T}\| = T$, $\mathbf{T} = \{t = 1, 2, 3, \dots, T\}$.
$\mathbf{I}$	set of items; $\|\mathbf{I}\| = N$.
$\mathbf{J}$	set of sites.
$\mathbf{V}$	set of vehicles.
$\mathbf{J}_S$	set of supplier sites.
$\mathbf{J}_W$	set of warehouse sites.
$\mathbf{J}_M$	set of market sites.
$\mathbf{J}_{ij}^{Pre}$	set of the predecessor sites that can replenish item i to site j.
$\mathbf{J}_{ij}^{Suc}$	set of the successor sites that can replenish item i from site j.
$\mathbf{D}$	set of given market demands; $\mathbf{D} = \{D_{tik} \mid \forall t \in \mathbf{T}, i \in \mathbf{I}, k \in \mathbf{J}_M\}$.
$\mathbf{VQ}$	set of vehicle quantities; $\mathbf{VQ} = \{VQ_{tijv} \mid \forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, v \in \mathbf{V}\}$.
$\mathbf{GQ}$	set of shortage quantities; $\mathbf{GQ} = \{GQ_{tijk} \mid \forall t \in \mathbf{T}, i \in \mathbf{I}, j, k \in \mathbf{J}\}$.
$\mathbf{IV}$	set of used variable storage volumes; $\mathbf{IV} = \{IV_{tj} \mid \forall t \in \mathbf{T}, j \in \mathbf{J}\}$.
$\mathbf{WL}$	set of used warehouse workloads; $\mathbf{WL} = \{WL_{tj} \mid \forall t \in \mathbf{T}, j \in \mathbf{J}\}$.
$\mathbf{s}$	set of reorder points; $\mathbf{s} = \{s_{tij} \mid \forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}\}$.
$\mathbf{S}$	set of base stocks; $\mathbf{S} = \{S_{tij} \mid \forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}\}$.
$\mathbf{s}_{ij}$	set of reorder points of item i and site j; $\mathbf{s}_{ij} = \{s_{tij} \mid \forall t \in \mathbf{T}\}$.
$\mathbf{S}_{ij}$	set of base stocks of item i and site j; $\mathbf{S}_{ij} = \{S_{tij} \mid \forall t \in \mathbf{T}\}$.

Table 4
Input parameters.

Notation	Description
D_{tik}	given demand of market k for item i at period t.
V_i^{Item}	unit volume of item i.
V_j^{Fix}	upper limit of the fixed storage volume of site j.
V_v^{Trans}	upper limit of transportation volume of vehicle v.
C_j^{Inv}	unit inventory holding cost of site j.
C_{tjkv}^{Trans}	unit transportation cost of vehicle v from site j to k at period t.
C_{tj}^{Labor}	unit labor cost of site j at period t.
T_{jk}	transportation lead-time from site j to k.
W_{ij}^{Labor}	unit labor workload for item i at site j.
W_{tj}^{Max}	upper limit of workload of site j at time t.
N_{tjkv}^{Trans}	upper limit of number of transportation vehicles v from site j to k at period t.
M	large positive value.
ω	positive value of shortage penalty weight.

4.1. Notations

All notations required in this paper are defined below; the indexes and sets in Table 3, the input parameters in Table 4, and the decision variables in Table 5.

4.2. Relevant costs

In this study, we consider the total cost of CDJRP-TC as the summation of the inventory holding cost, the labor cost, and the transportation cost. The inventory holding cost for a planning horizon T and all warehouses $\mathbf{J}_W$ is defined as Eq. (1). In our CDJRP-TC model, two different storage types are considered: fixed storage (i.e., an in-house warehouse) and variable storage (i.e., an external warehouse). Inventories are stored in fixed storage unless the total inventory volume exceeds the fixed storage capacity V_j^{Fix}. Then, if the total volume exceeds the capacity, the excess inventories are stored in variable storage. The inventory holding cost of the fixed storage (*fixed inventory holding cost*) is constant regardless of the inventory volume. The inventory

Table 5
Decision variables.

Notation	Description
s_{tij}	positive continuous variable for reorder point of item i in site j at period t.
S_{tij}	positive continuous variable for base stock of item i in site j at period t.
z_{tij}	positive continuous variable for inventory level of item i in site j at period t.
δ_{tij}	binary variable of the ordering status, which is 1 if replenishment order of item i in site j at period t is placed, 0 otherwise.
VQ_{tijv}	positive integer variable for vehicle quantity of type v from site j to k at period t.
IQ_{tij}	positive continuous variable for inventory quantity of item i in site j at the end of period t.
AQ_{tijk}	positive continuous variable for arrived quantity of item i from site j to k at period t.
OQ_{tijk}	positive continuous variable for the positive part of the order quantity of item i at period t placed from site j to k.
OQ'_{tijk}	positive continuous variable for the negative part of the order quantity of item i at period t placed from site j to k.
DQ_{tijk}	positive continuous variable for demand quantity of item i from site j to k at period t.
SQ_{tijk}	positive continuous variable for shipment quantity of item i from site j to k at period t.
GQ_{tijk}	positive continuous variable for shortage quantity of item i from site j to k at period t.
GR_{ij}	positive continuous variable for shortage rate of item i in site j.
IV_{tj}	positive continuous variable for used variable storage volume in site j at period t.
WL_{tj}	positive continuous variable for used warehouse workload in site j at period t.
IQ_{tij}^{Ideal}	ideal inventory quantity of item i at site j in period t.
$IQ_{tij}^{Est}(\mathbf{s}_{ij}, \mathbf{S}_{ij})$	estimated inventory quantity of item i at site j in period t, which is the function of policy variables $(\mathbf{s}_{ij}, \mathbf{S}_{ij})$.

holding cost of the variable storage (*variable inventory holding cost*) is proportional to the inventory volume in the variable storage IV_{tj}. Thus, the inventory holding cost is defined as the summation of the *fixed inventory holding cost* $C_j^{Inv} V_j^{Fix}$ and *the variable inventory holding cost* $C_j^{Inv} IV_{tj}$ for each period t and warehouse $j \in \mathbf{J}_W$, as follows:

$$f_{Inv}(\mathbf{IV}) = \sum_{t \in \mathbf{T}} \sum_{j \in \mathbf{J}_W} C_j^{Inv} \left(V_j^{Fix} + IV_{tj} \right) \tag{1}$$

The labor cost for a planning horizon T and all warehouses $\mathbf{J}_W$ is defined as Eq. (2). This is the summation of the warehouse workload WL_{tj} multiplied by the unit labor cost C_{tj}^{Labor} for each period t and warehouse $j \in \mathbf{J}_W$. Here, WL_{tj} is defined as the total workload to receive the arrived quantity and to fulfill the shipment quantity.

$$f_{Labor}(\mathbf{WL}) = \sum_{t \in \mathbf{T}} \sum_{j \in \mathbf{J}_W} C_{tj}^{Labor} WL_{tj} \tag{2}$$

The transportation cost for a planning horizon T and all supply routes are defined as Eq. (3), which indicates the summation of the number of used vehicles VQ_{tjkv} multiplied by the unit transportation cost C_{tjkv}^{Trans} for each period t, supply route (j, k) and vehicle type v, as follows:

$$f_{Trans}(\mathbf{VQ}) = \sum_{t \in \mathbf{T}} \sum_{j \in \mathbf{J}_S} \sum_{k \in \mathbf{J}_W} \sum_{v \in \mathbf{V}} C_{tjkv}^{Trans} VQ_{tjkv} \tag{3}$$

4.3. Analytical model formulation: Mixed integer linear programming

This subsection provides a MILP formulation as the analytical model of the CDJRP-TC. The MILP model is formulated as follows:

$$\min \left\{ f_{Inv}(\mathbf{IV}) + f_{Labor}(\mathbf{WL}) + f_{Trans}(\mathbf{VQ}) \right\} \tag{4}$$

subject to

$$GR_{ij} = \frac{\sum_{t\in\mathbf{T}}\sum_{k\in\mathbf{J}_M} GQ_{tijk}}{\sum_{t\in\mathbf{T}}\sum_{k\in\mathbf{J}_M} D_{tik}} \leq 0 \quad (\forall i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{5}$$

$$z_{tij} = IQ_{tij} + \sum_{\tau=t+1}^{t+T_{kj}+1} \sum_{k\in\mathbf{J}_{ij}^{Pre}} AQ_{\tau ikj} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{6}$$

$$-M \times (1-\delta_{tij}) \leq s_{tij} - z_{tij} \leq M \times \delta_{tij} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{7}$$

$$0 \leq OQ_{tijk} \leq M \times \delta_{tij} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W, k \in \mathbf{J}_{ij}^{Pre}) \tag{8}$$

$$0 \leq OQ'_{tijk} \leq M \times (1-\delta_{tij}) \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W, k \in \mathbf{J}_{ij}^{Pre}) \tag{9}$$

$$OQ_{tijk} - OQ'_{tijk} = S_{tij} - z_{tij} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W, k \in \mathbf{J}_{ij}^{Pre}) \tag{10}$$

$$IQ_{tij} = IQ_{t-1\,ij} + \sum_{k\in\mathbf{J}_{ij}^{Pre}} AQ_{tikj} - \sum_{k\in\mathbf{J}_{ij}^{Suc}} SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{11}$$

$$DQ_{tijk} = \begin{cases} D_{tik} & \text{if } k \in \mathbf{J}_M, \\ OQ_{t-1\,ikj} & \text{otherwise.} \end{cases} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}_{ij}^{Suc}) \tag{12}$$

$$WL_{tj} = \sum_{i\in\mathbf{I}} W_{ij}^{Labor} \left(\sum_{k\in\mathbf{J}_{ij}^{Pre}} AQ_{tikj} + \sum_{k\in\mathbf{J}_{ij}^{Suc}} SQ_{tijk} \right) \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \tag{13}$$

$$AQ_{tijk} = SQ_{t-T_{jk}\,ijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}_{ij}^{Suc}) \tag{14}$$

$$IQ_{tij} \geq \sum_{k\in\mathbf{J}_{ij}^{Suc}} SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{15}$$

$$GQ_{tijk} = DQ_{tijk} - SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}_{ij}^{Suc}) \tag{16}$$

$$\sum_{i\in\mathbf{I}} V_i^{Item} IQ_{tij} - V_j^{Fix} \leq IV_{tj} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \tag{17}$$

$$VQ_{tjkv} \leq N_{tjkv}^{Trans} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}, k \in \mathbf{J}, v \in \mathbf{V}) \tag{18}$$

$$\sum_{i\in\mathbf{I}} V_i^{Item} SQ_{tijk} \leq \sum_{v\in\mathbf{V}} V_v^{Trans} VQ_{tjkv} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \tag{19}$$

$$WL_{tj} \leq W_{tj}^{Max} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \tag{20}$$

Eq. (4) shows the objective function; the first term denotes the inventory holding cost, the second term is the labor cost, and the third is the transportation cost. Eq. (5) shows the shortage constraint. This defines the upper limit of shortage quantity, which must be zero in this report.

Eqs. (6)–(10) are the MILP formulation of (s, S) policy defined in Table 1. Eq. (6) defines the inventory level as the warehouse inventory quantity plus the order quantities that have been placed but have not arrived yet. Eq. (7) shows that the replenishment order can be placed ($\delta_{tij} = 1$) only if the inventory level has fallen below the reorder point ($z_{tij} < s_{tij}$). Eq. (8) indicates that the positive part of the order quantity is zero ($OQ_{tijk} = 0$) if the replenishment order is not placed ($\delta_{tij} = 0$). Meanwhile, Eq. (9) defines the negative part of order quantity that is zero ($OQ'_{tijk} = 0$) if the replenishment order is placed ($\delta_{tij} = 1$). Eq. (10) means that the order quantity is equal to the difference between the base stock and inventory level ($OQ_{tijk} = S_{tij} - z_{tij}$) if the replenishment order is placed ($\delta_{tij} = 1$).

Eqs. (11)–(16) model the inventory system. Eq. (11) is the inventory-balance constraint that ensures the change in the inventory quantity between period t and $t-1$ is equal to the difference between total arrived quantities and total shipment quantity of period t for each item and site. Eq. (12) defines the demand for each item, site, and period according to the destination. Here, the first condition indicates that demands from markets are given as input values, and the second means that other demands are equal to the order quantity placed toward each site in the last period. Eq. (13) defines the warehouse workload as the summation of workloads to receive the arrived quantity and to fulfill the shipment quantity with the unit labor workload. Eq. (14) shows the relationship between the arrived quantity and shipment quantity, which indicates that the order arrives after the transportation lead time. Eq. (15) expresses the upper limit of shipping quantity and indicates that each item can be shipped no more than the inventory quantity of that period and site. Eq. (16) defines the shortage quantity as equal to the difference between demand quantity and shipment quantity.

Eqs. (17)–(20) define the capacity constraints. Eq. (17) represents the usage of the variable inventory storage, which is defined as the inventory volume that exceeds the fixed storage volume. Eq. (18) defines the upper limit of vehicle quantities for each supply route and each period. Eq. (19) shows the transportation volume constraints for each supply route and each period. Eq. (20) expresses the upper limit of the total workloads of receiving and shipping operations in a warehouse.

4.4. Discrete simulation model implementation

A simulation model of the CDJRP-TC was constructed as a time-driven simulation program to investigate the performances with respect to the policy variables. This simulation model is implemented to satisfy all assumptions defined in Section 3.3. The simulation model is expressed as Eq. (21).

$$[\mathbf{IV}, \mathbf{VQ}, \mathbf{WL}, \mathbf{GQ}] = \text{Simulator}(\mathbf{D}, \mathbf{s}, \mathbf{S}). \tag{21}$$

The input of the simulation is the set of demands $\mathbf{D}$ and the policy variables, such as reorder points $\mathbf{s}$ and base stock $\mathbf{S}$. In this simulation, each fulfillment operation, such as receiving, shipping, and replenishment order, is processed sequentially over the planning horizon ($t = 1, 2, \ldots, T$). In each period t, the fulfillment operations of each site are simulated in the order of the suppliers, the warehouses, and the markets. In each operation, target items are selected to minimize the shortage quantity sequentially while satisfying all constraints except for the upper shortage rate constraint Eq. (5). The outputs are the set of the used variable storage volumes $\mathbf{IV}$, the vehicle quantities $\mathbf{VQ}$, the warehouse workloads $\mathbf{WL}$, and the shortage quantities $\mathbf{GQ}$. These values are used to evaluate the total costs defined in Eq. (4) and the shortage rate defined in Eq. (5).

5. Solution approach

5.1. Overview of proposed method

As described in Section 4.3, CDJRP-TC can be formulated as the MILP model. However, it is still difficult to solve since it contains a vast number of binary variables δ_{tij} for modeling the (s, S) inventory policy. This problem is also challenging to solve with SBO approaches in a practical computational time. The inventory fulfillment simulation defined in Section 4.3 needs a few to hundreds of seconds for one execution when the number of items N and the planning horizon T are large. As a result, SBO approaches, which search for the optimal solution with numerous iterations of solution evaluation with simulation, require a very long calculation time. Therefore, to solve the practical scale CDJRP-TC quickly, a more efficient solution method is necessary.

Fig. 5 shows our proposed three-phase solution method. The solution process of the proposed method consists of the following three phases. In Phase 1, we formulate the simplified MILP model of CDJRP-TC by ignoring the (s, S) inventory policy from the MILP formulation defined in Section 4.3. This simplified MILP model can be solved much shorter than the original MILP problem because the simplified

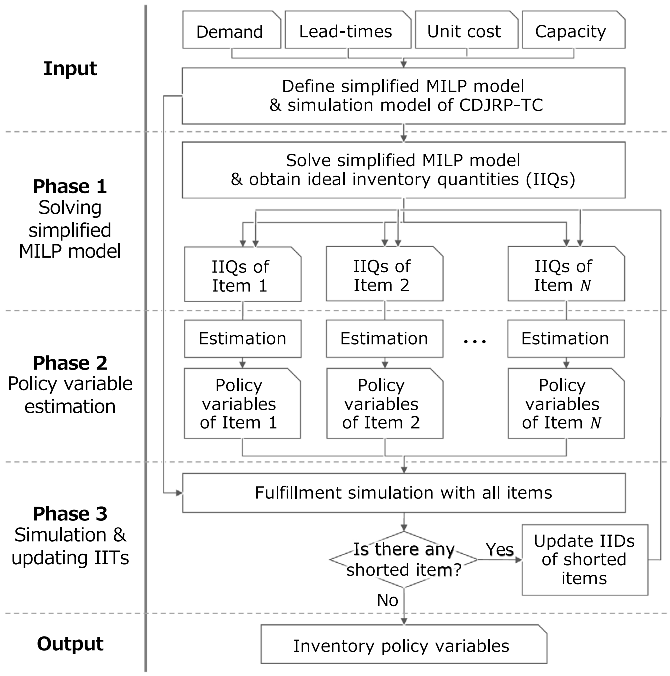

Fig. 5. Flowchart of the proposed method. In Phase 1, the simplified MILP model that ignores inventory policies is solved, and ideal inventory quantities (IIQs) are obtained. In Phase 2, the policy variables that best fit the IIQs are estimated for each item in parallel. In Phase 3, the fulfillment simulation with estimated policy variables is executed, and the shorted items' IIQs are updated to reduce the shortage quantities.

model contains few binary variables δ_{tij}. Then the ideal inventory quantities (IIQs) of each item, which is the solution to the simplified MILP problem, are obtained. The IIQs may not satisfy the constraints of the inventory policy. However, the replenishment order quantities and timings of the IIQs are still valuable information as these two factors are essential for minimizing the total cost while satisfying constraints under the time-varying costs.

In Phase 2, therefore, we estimate the policy variables (s_{tij}, S_{tij}) that mimic the IIQs' replenishment order quantities and timings. In this policy variable estimation (PVE) process, the policy variables of each item are estimated by CMA-ES to minimize the difference between the IIQs and the estimated inventory quantities (EIQs), which are the function of policy variables. Here, we ignored the capacity constraints, such as warehouse inventory, transportation, and workload capacities, because the PVE is executed for each item separately. Therefore, the policy variables obtained by PVE can violate the capacity constraints when used in operations simultaneously.

In Phase 3, the inventory fulfillment simulation, whose inputs are the policy variables of all items estimated in PVE, is executed to account for the capacity constraints. This simulation satisfies all capacity constraints automatically, as mentioned in Section 4.4. When these policy variables violate capacity constraints, the simulation outputs the positive shortage quantities. This means that these policy variables are a feasible solution to the CDJRP-TC if no shortage has occurred. Otherwise, the policy variables must be modified to reduce the shortage quantity to zero. Here, the policy variables are revised by executing PVE with the IIQs updated by the shortage quantities.

Phase 2 and 3 are iterated until no shortage occurs in the simulation, and the estimated policy variables of the final iteration are returned as a feasible solution of the target CDJRP-TC. Additionally, Warehousing and transportation resources are obtained as the used variable storage, warehouse workloads, and vehicle quantities in the final solution. We note that our proposed method is a heuristic approach to provide a practical and efficient solution, especially for large-scale CDJRP-TCs where obtaining an exact solution might be computationally infeasible.

5.2. Phase 1: Simplified MILP optimization

In this phase, the CDJRP-TC model is simplified by removing Eqs. (6)–(10), which models (s, S) inventory policies, as follows:

$$\min \left\{ f_{Inv}(\mathbf{IV}) + f_{Labor}(\mathbf{WL}) + f_{Trans}(\mathbf{VQ}) \right\} \quad \text{(Eq. (4))}$$

subject to

$$GR_{ij} = \frac{\sum_{t\in\mathbf{T}} \sum_{k\in\mathbf{J}_M} GQ_{tijk}}{\sum_{t\in\mathbf{T}} \sum_{k\in\mathbf{J}_M} D_{tik}} \leq 0 \quad (\forall i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (5))}$$

$$IQ_{tij} = IQ_{t-1\,ij} + \sum_{k\in\mathbf{J}^{Pre}_{ij}} AQ_{tikj} - \sum_{k\in\mathbf{J}^{Suc}_{ij}} SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (11))}$$

$$DQ_{tijk} = \begin{cases} D_{tik} & \text{if } k \in \mathbf{J}_M, \\ OQ_{t-1\,ikj} & \text{otherwise.} \end{cases} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}^{Suc}_{ij}) \quad \text{(Eq. (12))}$$

$$WL_{tj} = \sum_{i\in\mathbf{I}} W^{Labor}_{ij} \left(\sum_{k\in\mathbf{J}^{Pre}_{ij}} AQ_{tikj} + \sum_{k\in\mathbf{J}^{Suc}_{ij}} SQ_{tijk} \right) \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \quad \text{(Eq. (13))}$$

$$AQ_{tijk} = SQ_{t-T_{jk}\,ijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}^{Suc}_{ij}) \quad \text{(Eq. (14))}$$

$$IQ_{tij} \geq \sum_{k\in\mathbf{J}^{Suc}_{ij}} SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (15))}$$

$$GQ_{tijk} = DQ_{tijk} - SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}^{Suc}_{ij}) \quad \text{(Eq. (16))}$$

$$\sum_{i\in\mathbf{I}} V^{Item}_{i} IQ_{tij} - V^{Fix}_{j} \leq IV_{tj} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \quad \text{(Eq. (17))}$$

$$VQ_{tjkv} \leq N^{Trans}_{tjkv} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}, k \in \mathbf{J}, v \in \mathbf{V}) \quad \text{(Eq. (18))}$$

$$\sum_{i\in\mathbf{I}} V^{Item}_{i} SQ_{tijk} \leq \sum_{v\in\mathbf{V}} V^{Trans}_{v} VQ_{tjkv} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \quad \text{(Eq. (19))}$$

$$WL_{tj} \leq W^{Max}_{tj} \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \quad \text{(Eq. (20))}$$

In this simplified model, any item can be replenished at any time with arbitrary quantities while other constraints (Eqs. (5) and (11)–(20)) are satisfied. By solving this simplified model as MILP, the IIQs (IQ^{Ideal}_{tij}) are obtained as the solution's inventory quantities IQ_{tij}.

5.3. Phase 2: Policy variable estimation

In Phase 2, the policy variables (s_{tij}, S_{tij}) of each item and site are estimated to mimic the IIQs' replenishment order quantities and timings. This process is referred to as policy variable estimation (PVE), whose concept is shown in Fig. 6. The reorder point is shown as the red line and the base stock as the orange line, which are the policy variables to be estimated. IIQs (IQ^{Ideal}_{tij}) obtained in Phase 1 are illustrated as the dotted cyan line. Here, we define *estimated inventory quantities (EIQs)*, shown in the blue line. EIQs $(IQ^{Est}_{tij}(\mathbf{s}_{ij}, \mathbf{S}_{ij}))$ are the inventory quantities obtained by replenishment based on the given policy variables $(\mathbf{s}_{ij}, \mathbf{S}_{ij})$.

To estimate the policy variables that mimic the IIQs' replenishment order quantities and timings, we formulate the PVE as an item-wise optimization problem that minimizes the difference between IIQs and EIQs. The PVE of item i and site j is formulated as follows.

$$\min \left\{ \sum_{t\in\mathbf{T}} \sum_{j\in\mathbf{J}_W} \left(IQ^{Est}_{tij}(\mathbf{s}_{ij}, \mathbf{S}_{ij}) - IQ^{Ideal}_{tij} \right)^2 + \omega \sum_{t\in\mathbf{T}} \sum_{j\in\mathbf{J}_W} \sum_{k\in\mathbf{J}_M} GQ^2_{tijk} \right\} \tag{22}$$

subject to

$$z_{tij} = IQ_{tij} + \sum_{\tau=t+1}^{t+T_{kj}+1} \sum_{k\in\mathbf{J}^{Pre}_{ij}} AQ_{\tau ikj} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (6))}$$

$$-M \times (1 - \delta_{tij}) \leq s_{tij} - z_{tij} \leq M \times \delta_{tij} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (7))}$$

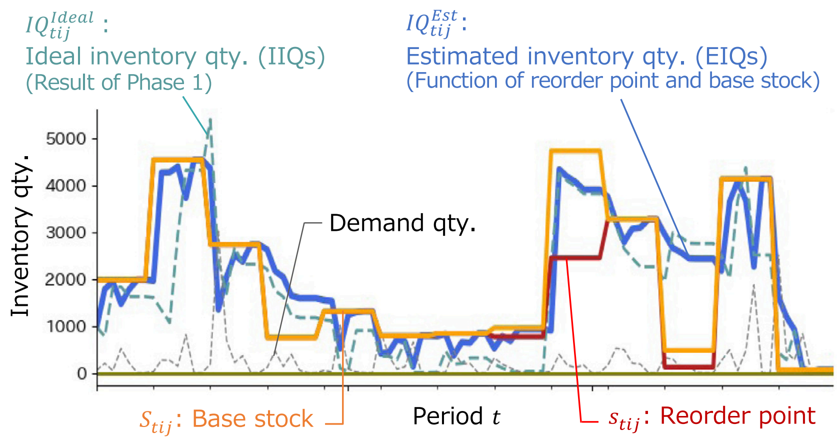

Fig. 6. The concept of the policy variable estimation. The policy variables are estimated for each item to minimize the difference between EIQs and IIQs.

$$0 \le OQ_{tijk} \le M \times \delta_{tij} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W, k \in \mathbf{J}_{ij}^{Pre}) \quad \text{(Eq. (8))}$$

$$0 \le OQ'_{tijk} \le M \times (1 - \delta_{tij}) \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W, k \in \mathbf{J}_{ij}^{Pre}) \quad \text{(Eq. (9))}$$

$$OQ_{tijk} - OQ'_{tijk} = S_{tij} - z_{tij} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W, k \in \mathbf{J}_{ij}^{Pre}) \quad \text{(Eq. (10))}$$

$$IQ_{tij} = IQ_{t-1\,ij} + \sum_{k \in \mathbf{J}_{ij}^{Pre}} AQ_{tikj} - \sum_{k \in \mathbf{J}_{ij}^{Suc}} SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (11))}$$

$$DQ_{tijk} = \begin{cases} D_{tik} & \text{if } k \in \mathbf{J}_M, \\ OQ_{t-1\,ikj} & \text{otherwise.} \end{cases} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}_{ij}^{Suc}) \quad \text{(Eq. (12))}$$

$$WL_{tj} = \sum_{i \in \mathbf{I}} W_{ij}^{Labor} \left(\sum_{k \in \mathbf{J}_{ij}^{Pre}} AQ_{tikj} + \sum_{k \in \mathbf{J}_{ij}^{Suc}} SQ_{tijk} \right) \quad (\forall t \in \mathbf{T}, j \in \mathbf{J}_W) \quad \text{(Eq. (13))}$$

$$AQ_{tijk} = SQ_{t-T_{jk}\,ijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}_{ij}^{Suc}) \quad \text{(Eq. (14))}$$

$$IQ_{tij} \ge \sum_{k \in \mathbf{J}_{ij}^{Suc}} SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \quad \text{(Eq. (15))}$$

$$GQ_{tijk} = DQ_{tijk} - SQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}, k \in \mathbf{J}_{ij}^{Suc}) \quad \text{(Eq. (16))}$$

Eq. (22) shows that the policy variables are optimized to minimize the squared errors between IQ_{tij}^{Est} and IQ_{tij}^{Ideal} with the shortage penalty, which is a relaxed formulation of Eq. (5). Additionally, Eqs. (6)–(16) describe the inventory system of item i. In item-wise PVE, we ignore the capacity constraints (Eqs. (17)–(20)) since these evaluations require all items' information.

When a pair of the policy variables $(\mathbf{s}_{ij}, \mathbf{S}_{ij})$ are given, its objective value (Eq. (22)) can be easily evaluated because IQ_{tij}^{Est} and GQ_{tijk} are obtained by calculating the above one-item inventory model (Eqs. (6)–(16)) from $t = 1$ to T sequentially. We, therefore, solved PVE by using the metaheuristics approach. In this study, the covariance matrix adaptation evolution strategy (CMA-ES), which is one of the metaheuristics for continuous optimization problems, is utilized (Beyer & Schwefel, 2002; Nomura et al., 2020). CMA-ES facilitates optimization by updating the parameters of the multivariate Gaussian distribution and searches for the optimal policy variables by sampling candidate solutions from the distribution. Additionally, PVE can be separately solved for each item because of ignoring the capacity constraints to joint multi-item inventory models. This separation makes it possible to solve PVE quickly with reduced solution space and parallel computation.

5.4. Phase 3: Update of ideal inventory quantities

The policy variables estimated in Phase 2 can violate the capacity constraints (Eqs. (17)–(20)) because these are ignored in PVE. Thus, in Phase 3, we validate the shortage quantities (GQ_{tijk}) under the satisfaction of all capacity constraints by using the inventory simulation with all items defined in Section 3.3. If all items satisfy the shortage constraints (Eq. (5)), the current policy variables are the solution to the target CDJRP-TC.

Otherwise, the current policy variables $(\mathbf{s}_{ij}, \mathbf{S}_{ij})$ must be updated to satisfy the shortage constraint. However, it is difficult to modify the policy variables directly to reduce shortage because of their discrete and complex behaviors. Therefore, we consider updating the policy variables through PVE with the IIQs (IQ_{tij}^{Ideal}) revised with the simulation result. Here, Eqs. (15) and (16) lead to the following inequation between the shortage quantity and inventory quantity:

$$IQ_{tij} + \sum_{k \in \mathbf{J}_{ij}^{Suc}} GQ_{tijk} \le \sum_{k \in \mathbf{J}_{ij}^{Suc}} DQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{23}$$

In this inequation, the left-hand side is the inventory quantity and the total shortage quantity of item i in site j at period t, and the right-hand side is the total demand quantity at the same period. Note that the demand quantity DQ_{tijk} cannot be controlled directly since the simulation calculates it. A simple way to reduce the total shortage quantity is to increase the inventory quantity of the shorted item, site, and period as follows:

$$IQ_{tij} \leftarrow IQ_{tij} + \sum_{k \in \mathbf{J}_{ij}^{Suc}} GQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{24}$$

Now, we can assume that IQ_{tij} takes similar values to EIQ (IQ_{tij}^{Est}) since both are calculated based on the same policy variables. In addition, Eq. (22) indicates that EIQs become close to IIQs. Therefore, Eq. (24) can be rewritten with this approximation ($IQ_{tij} \approx IQ_{tij}^{Est} \approx IQ_{tij}^{Ideal}$) as follows:

$$IQ_{tij}^{Ideal} \leftarrow IQ_{tij}^{Ideal} + \sum_{k \in \mathbf{J}_{ij}^{Suc}} GQ_{tijk} \quad (\forall t \in \mathbf{T}, i \in \mathbf{I}, j \in \mathbf{J}_W) \tag{25}$$

In our proposed method, the IIQs are updated as Eq. (25), and PVE is executed again for shorted items. Phase 2–3 are continued until all items satisfy the shortage constraint, and the policy variables are returned as the solution to the target CDJRP-TC when all items satisfy the shortage constraints.

6. Computational experiments

We conducted numerical experiments to investigate the effectiveness of the proposed method and compared it with conventional optimization approaches MILP and SBO.

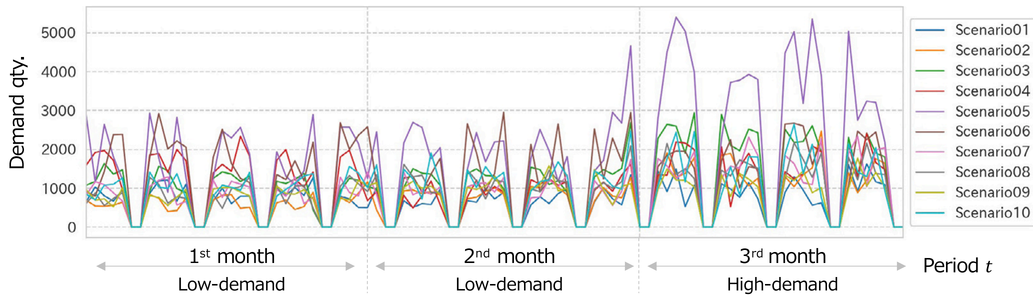

Fig. 7. Ten random demand patterns, in which the first and second months are low-demand seasons and the third month high-demand.

Table 6
Transportation conditions of optimization runtimes comparison.

From	To	Vehicle	Month	Upper num. of vehicles	Unit trans. cost
Supplier 1	Warehouse	Normal	1st–3rd	1	200
		Extra	1st–2nd	4	240
			3rd	4	360
Supplier 2	Warehouse	Normal	1st–3rd	3	150
		Extra	1st–2nd	7	180
			3rd	7	270
Supplier 3	Warehouse	Normal	1st–3rd	3	100
		Extra	1st–2nd	9	120
			3rd	9	180
Warehouse	Market	Normal	1st–3rd	Unlimited	0

6.1. Conditions

In the following experiments, we consider the CDJRP-TC of the supply chain with three suppliers, one warehouse, and one market shown in Fig. 1. In these problems, the policy variables of the (s, S) inventory policy for each item and period in a warehouse $(\{s_{tij}, S_{tij}\} \mid \forall t, i, j \in \mathbf{J}_W)$ are optimized to minimize the total cost while satisfying all market demands. As described in Section 3.1, the policy variables can be updated for each item and each week. In a week, the policy variables of each item (s, S) take the same value, and the planning horizon T is 91 days (13 weeks or three months). Therefore, the number of policy variables to be optimized is $26 \times T \times N$, where N is the number of items.

Deterministic-dynamic demands and item properties were randomly generated with the following settings. For the sake of simplicity, we use $U[*, *]$ to denote the corresponding uniform distribution. Fig. 7 shows the generated ten patterns of the time-series total demands.

- Unit volume of item i: $V_i^{Item} \sim U[0.5, 4.0]$,
- Supplier of each item to the warehouse is randomly selected in the following ratio, [Supplier 1, Supplier 2, Supplier 3] = [15%, 25%, 60%],
- Demand quantity of item i in period t: D_{tik} - Demand basis of item i: $\bar{D}_i \sim U[0, 10000]$, - D_{tik} if period t is in the 1st or 2nd month: $D_{tik} \sim U[0, \bar{D}_i]$, - D_{tik} if period t is in the 3rd month: $D_{tik} \sim U[0, 1.3 \times \bar{D}_i]$.

The other settings of CDJRP-TC are set as follows; $W_{ij}^{Labor} = 0.003$, $W_{tj}^{Max} = 150$, $C_{tj}^{Labor} = 5$, $C_j^{Inv} = 2$, $V_j^{Fix} = 10^6$, $V_v^{Trans} = 1{,}700$. Additionally, the proposed method's hyper-parameters were set as follows; The time limit of Phase 1 was 60 seconds. The CMA-ES population was 48, the upper limit of iteration was 200, and $\omega = 20$. The computing environment was a Ryzen 3950X CPU (3.70 GHz, 32threads) and 128 GB RAM. All MILP models were solved by CPLEX 12.8.

6.2. Optimization result comparison

In this experiment, we compared the optimization results of the CDJRP-TCs using the proposed and conventional methods. We evaluated the three problem sizes of CDJRP-TCs with 10, 50, and 100 items. Ten problem instances with different randomly generated demands were solved for each problem size, and the averages and standard deviations of the results were compared. Their time limits were set as follows: 100 seconds for 10-item problems, 1800 seconds for 50-item problems, and 3600 seconds for 100-item problems.

Table 6 shows the transportation conditions; unit transportation costs C_{tjkv}^{Trans} and the upper limit number of the vehicles N_{tjkv}^{Trans}. All problems must satisfy the shortage constraint defined in Eq. (5). Therefore, there is no feasible solution when the shortage rate exceeds zero. As conventional methods, we used the SBO approach with GA and MILP, respectively. In the SBO approach, the problems were formulated with the simulator described in Section 4.4 and solved by GA with the following settings: the population size is set to 100, and uniform crossover and Gaussian mutation are set as genetic operators as proposed by Hochmuth et al. (2010). In the MILP approach, the problems formulated as the MILP model in Section 4.3 were directly solved with the commercial MILP solver. In addition, we investigated the optimality gap by comparing each solution's total cost with the analytical optimal solution obtained by MILP run for 3600 seconds. Due to the problem size, we could obtain the analytically optimal solution and report the optimality gap successfully only for Experiment 1.

The computational results are shown in Table 7. First, we discuss the result of Experiment 1, which contains ten items. Table 7 shows that the proposed method solved all problem instances, whereas GA solved seven instances, and MILP only solved two instances. The total cost of the MILP solutions was the lowest of all methods, but its shortage rate was the worst.

The total costs and the shortage rates of each instance are shown in Fig. 8. In these figures, the x-axis means the problem instance ID and the y-axis of the upper chart represents the total cost, and that of the lower chart is the shortage rate in the log scale. In these figures, when no feasible solution was obtained in the problem instance, the result is illustrated as a hatched bar, which indicates the result of the best infeasible solution. Fig. 8 shows that GA obtained six feasible solutions. MILP obtained two feasible solutions with the lowest objective values, while it returned infeasible solutions for the other eight instances. As shown in Fig. 8, MILP infeasible solutions have larger shortage rates than those of the other two methods. These results indicate that the MILP is an unstable approach to solving CDJRP-TCs. On the other hand, the proposed method obtained feasible solutions for all problem instances, as well as feasible solutions with the lowest objective values of all methods in the seven instances.

Next, we analyzed problem instance #1 in detail. The time-series total inventory quantities obtained by each method are shown in Fig. 9. In this figure, the x–axis indicates period t, and the y–axis represents each date's total inventory and demand quantities. The dotted line is the demand quantities. The blue line is the total inventory quantities

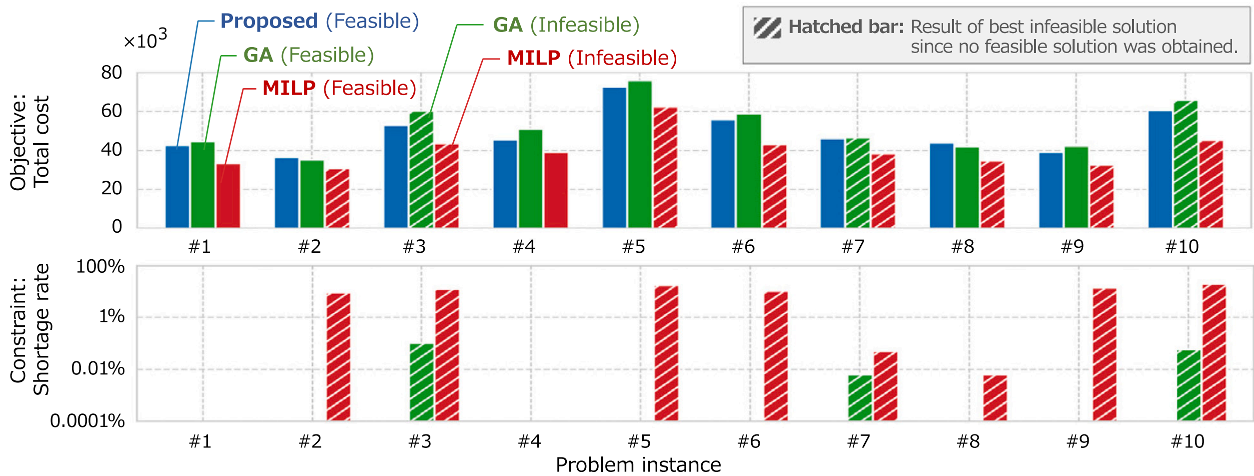

Fig. 8. Objective and constraint value comparison of Experiment 1, 10-item CDJRP-TC.

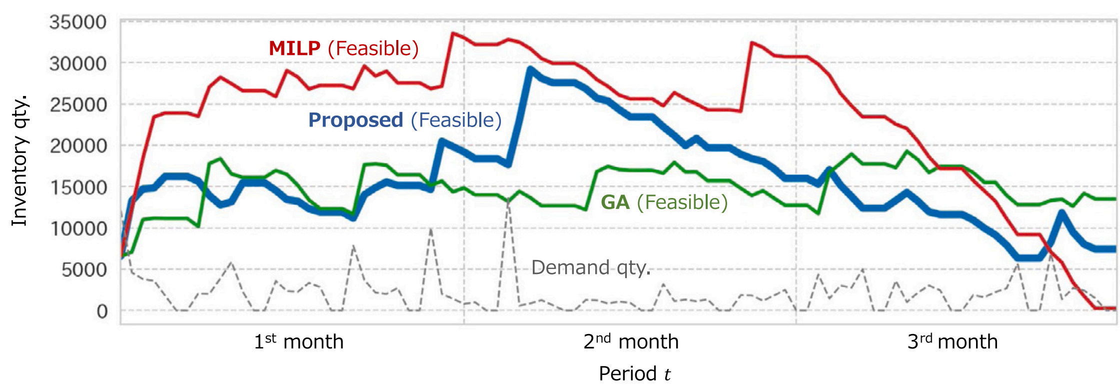

Fig. 9. Time-series total inventory quantities for the instance #1 of Experiment 1, 10-item CDJRP-TC.

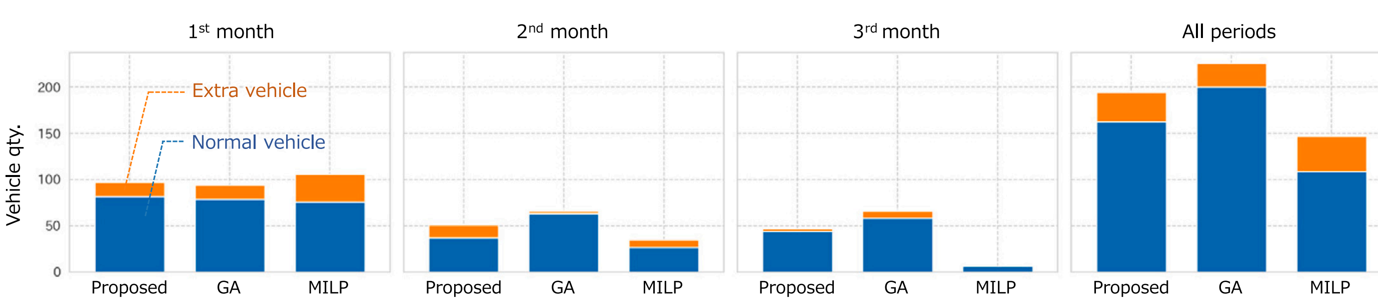

Fig. 10. Monthly used vehicle quantities for the instance #1 of Experiment 1, 10-item CDJRP-TC.

obtained by the proposed method, the green line is that of GA, and the red line is that of MILP. This figure shows that the inventory quantities take the largest values for MILP, the proposed method, and GA in that order, which is the same order as their objective values shown in Table 7. This indicates that the more inventories are replenished in the first and second months (when unit transportation costs are low), the

Table 7
Optimization results of CDJRP-TC with 10, 50, and 100 items.

Setting	Method	Solved instances	Shortage rate [%]	Total cost [×10³]	Optimality gap [%]	Runtime [sec.]
Experiment 1	Proposed	10/10	0.0 ± 0.0	49.7 ± 11.0	11.9 ± 12.9	72 ± 26
10 items	GA	7/10	0.02 ± 0.04	52.4 ± 12.7	16.2 ± 12.6	100
≤100 sec.	MILP	2/10	8.3 ± 7.8	40.4 ± 9.3	24.8 ± 29.0	100
Experiment 2	Proposed	10/10	0.0 ± 0.0	145.3 ± 20.2	–	75 ± 11
50 items	GA	0/10	0.09 ± 0.04	169.1 ± 28.0	–	1800
≤1800 sec.	MILP	0/10	56.7 ± 9.2	63.7 ± 13.4	–	1800
Experiment 3	Proposed	10/10	0.0 ± 0.0	298.9 ± 18.2	–	295 ± 88
100 items	GA	0/10	0.41 ± 0.01	342.2 ± 20.0	–	3600
≤3600 sec.	MILP	0/10	58.8 ± 3.2	109.7 ± 8.6	–	3600

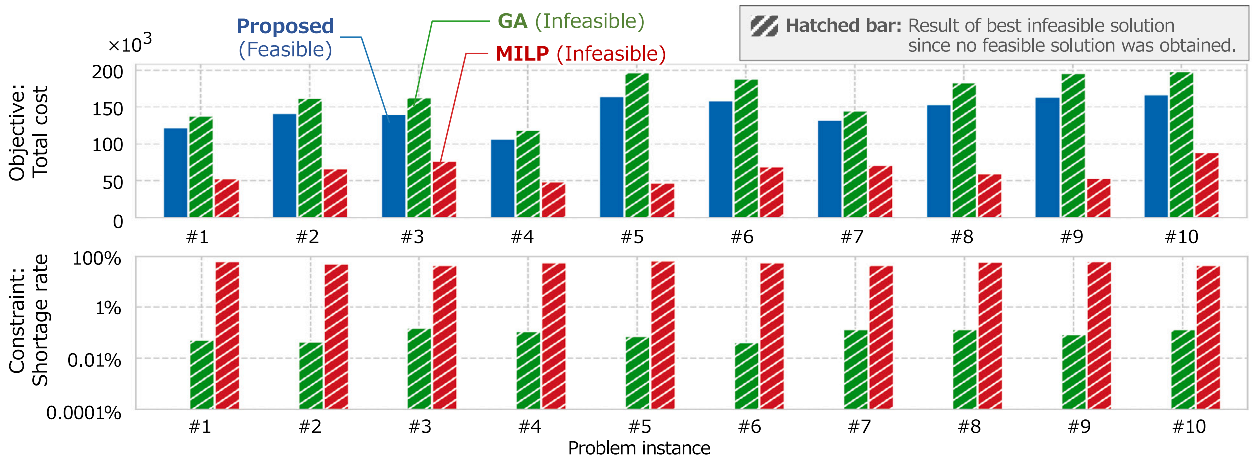

Fig. 11. Objective and constraint value comparison of Experiment 2, 50-item CDJRP-TC.

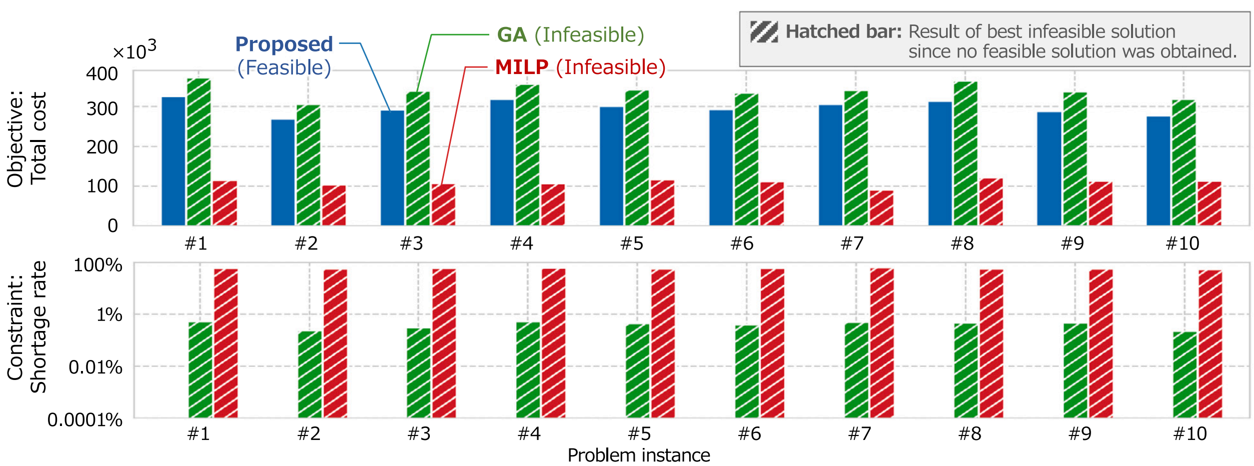

Fig. 12. Objective and constraint value comparison of Experiment 3, 100-item CDJRP-TC.

fewer inventories will need to be replenished in the third month (when unit transportation costs are high), leading to a lower total cost. This relationship can also be seen in Fig. 10, which shows the total number of monthly used vehicles of each method's solution. This figure shows that only the GA solution used the extra vehicle in the third month, and the fourth chart, which illustrates the total vehicle quantities for all periods, shows that GA used more vehicles. These results suggest that MILP and the proposed method can obtain solutions to replenish inventory according to the change in unit transportation costs.

Next, we discuss the results of Experiments 2 and 3, which contain 50 and 100 items, respectively. Table 7 shows that the only proposed method obtained feasible solutions for all problem instances, whereas GA and MILP could not find any feasible solutions within the time limits. The total costs and shortage rates of each instance in 50-item and 100-item CDJRP-TCs are shown in Figs. 11 and 12, respectively. These figures show that GA could not obtain feasible solutions in any instance; moreover, the objective values of GA's infeasible solutions were larger than those of the feasible solutions obtained by the proposed method. Additionally, the infeasible solutions obtained by MILP have much higher shortage rates than those of the proposed method and GA. It indicates that the MILP approach can only be used for small-scale problems. In terms of the runtime to obtain feasible solutions, the proposed method could find feasible solutions within 75 seconds for 50-item cases and 295 seconds for 100-item cases on average, while GA and MILP could not find any feasible solutions within 1800 and 3600 seconds, respectively. From this result, it can be seen that the proposed method obtains feasible solutions in at least 12 times shorter runtimes than GA and MILP for large-scale CDJRP-TCs. It indicates that the heuristic approach in the proposed method is effective for finding feasible solutions for large-scale CDJRP-TCs. These results suggest that the proposed method is the most scalable approach to solving CDJRP-TCs of the conventional methods.

6.3. Planning capability for different unit transportation costs

In this section, we investigate the proposed method's capability to obtain the appropriate solution according to the increase in unit transportation costs. We conducted the 100-item CDJRP-TCs with different patterns of the cost increase rate.

Table 8 shows the three patterns of transportation conditions. Each pattern has a different cost increase rate of the unit transportation costs. *+0%* means that unit costs do not increase in the third month. *+50%* means that unit costs increase 50% in the third month from the first and second month. In the same way, *+100%* means that unit costs increase 100% in the third month. The other problem settings are the same as the 100-item CDJRP-TCs described in Section 6.2. Using the proposed method, we randomly generated ten demands and solved the 100-item

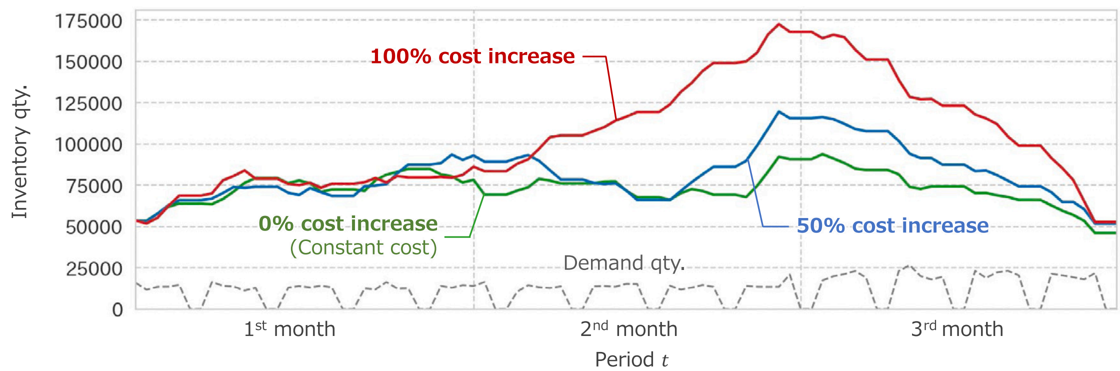

Fig. 13. Time-series total inventory quantities for each cost increase rate.

Table 8
Transportation conditions of different cost increase patterns.

From	To	Vehicle	Month	Upper num. of vehicles	Unit trans. cost		
					+0%	+50%	+100%
Supplier 1	Warehouse	Normal	1st–3rd	1	200	200	200
		Extra	1st–2nd	4	240	240	240
			3rd	4	240	360	480
Supplier 2	Warehouse	Normal	1st–3rd	3	150	150	150
		Extra	1st–2nd	7	180	180	180
			3rd	7	180	270	360
Supplier 3	Warehouse	Normal	1st–3rd	3	100	100	100
		Extra	1st–2nd	9	120	120	120
			3rd	9	120	180	240

Table 9
Optimization result comparison with different unit cost increase rates.

Result	Month	Unit trans. cost increase		
		+0%	+50%	+100%
Normalized avg. inventory qty.	1st	1.00	1.10	1.10
	2nd	1.06	1.24	1.56
	3rd	1.06	1.24	1.72
Normalized total vehicle qty.	1st	1.00	1.10	1.04
	2nd	0.93	0.91	1.19
	3rd	0.99	0.94	0.69

CDJRP-TCs with these demands and the transportation conditions with each cost increase rate.

Table 9 shows the normalized results of the 100-item CDJRP-TCs with each cost increase rate. This table shows the average inventory quantity (AIQ) and the total vehicle quantity (TVQ), normalized by each result of the first month with *+0%* cost increase rate. This table shows that when the cost increase rate is higher, the AIQs of the second and third months become larger, and the TVQs of those months become smaller. For example, the third month's AIQs with *+50%* and *+100%* cost increase rates are 17.0% and 62.3% larger than that with *+0%* cost increase rate, respectively. In the same period, the TVQs are reduced by 5.1% in *+50%* case and by 30.3% in *+100%* compared to *+0%* case. These results indicate that the unit transportation cost has a significant impact on distribution planning.

Next, we focused on one problem instance and analyzed it in detail. The time-series total inventory quantities obtained by each cost increase rate are shown in Fig. 13. In this figure, the x–axis indicates period t, and the y–axis indicates each date's total inventory and demand quantities. The dotted line is the demand quantities. The green, blue, and red lines are the inventory quantities of *+0%*, *+50%*, *+100%* cost increase cases. This figure shows that when the cost increase rate is higher, more inventories are stored at the end of the second month to reduce replenishment in the third month. Fig. 14 also illustrates that the more extra vehicles are used in the first and second months, the fewer extra vehicles are used in the third month when the cost increase rate is higher.

Fig. 15 illustrates the monthly vehicle quantities used to replenish from Supplier 1, 2, and 3 with each cost increase rate. From this figure, the above relationship can be seen more clearly in the increased unit transportation cost from each supplier. Replenishment from Supplier 1, whose unit transportation cost is the highest of all suppliers, changes most drastically with the cost increase rate. This is because the transportation cost from such a supplier has a larger impact on the total cost than other suppliers. These results demonstrate that the proposed method can obtain the appropriate solution according to increases in unit transportation costs.

7. Conclusion

In this paper, we investigated the formulation and solution approach to support distribution planning in the sales and operations planning (S&OP) process. To consider dynamic-deterministic demands obtained as the sales plan and time-varying unit costs of vehicles owned by third-party logistics providers, we formulate this problem as the capacitated dynamic-demand JRP with time-varying costs (CDJRP-TC), and proposed a three-phase solution approach based on the hybrid simulation & optimization. Numerical experiments with up to 100 items demonstrate that the proposed method could obtain feasible solutions within at least 12 times shorter runtimes than the conventional methods (GA, MILP) through computational experiments. Additionally, we investigated the capability of the proposed method to obtain the appropriate solutions according to the cost increase rate of unit transportation cost.

From these computational results, we can obtain the following managerial insights for the distribution planning under the unit cost increase:

1. For the distribution planning to minimize the total cost, inventory replenishment in the period with the low unit transportation cost should be increased to reduce the number of transportation vehicles in the period with the high unit cost and suppress the transportation cost.
2. The appropriate amount of inventory replenishment in the low unit cost periods depends on the cost increase rate, and the higher the rate, the larger amount of replenishment is recommended while satisfying the capacity constraints and increase of other costs.

In future work, we can consider improving the policy variable estimation in Phase 2 and updating IIQs in Phase 3 to solve larger CDJRP-TCs, for example, by using machine learning techniques.

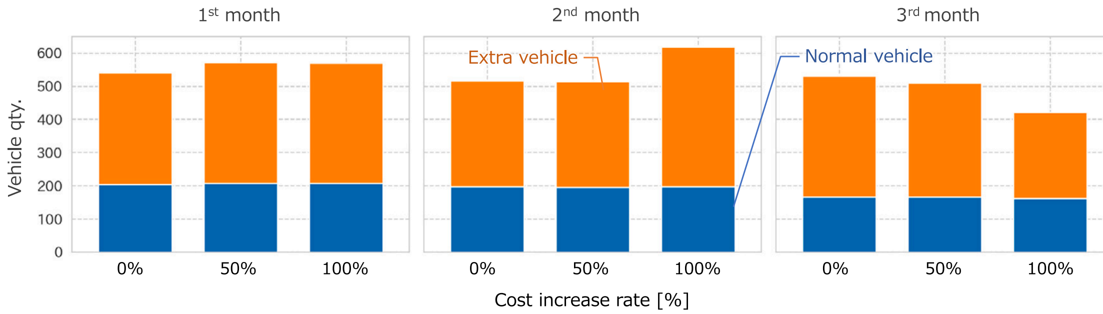

Fig. 14. Monthly used vehicle quantities from all suppliers for each cost increase rate.

Fig. 15. Monthly used vehicle quantities from supplier 1–3 for each cost increase rate.

CRediT authorship contribution statement

Issei Suemitsu: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing. **Naoko Miyashita:** Conceptualization, Software. **Junko Hosoda:** Conceptualization, Funding acquisition, Methodology, Project administration, Software, Supervision, Validation, Writing – review & editing. **Yoshihito Shimazu:** Conceptualization, Data curation, Funding acquisition, Project administration, Resources. **Takahiro Nishikawa:** Conceptualization, Data curation, Funding acquisition, Project administration, Resources. **Kazuhiro Izui:** Supervision, Writing – review & editing.

Data availability

The data that has been used is confidential.

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