

Assignment

Case study

12) ~~given~~ large students

$$A = 4, 80, 654, 80, 654, 80, 65$$

$$B = 6, 85, 726, 85, 726, 85, 72$$

$$C = 3, 70, 603, 70, 603, 70, 60$$

Ans:- ~~Ans~~

matrix $X = \begin{bmatrix} 4 & 80 & 654 \\ 6 & 85 & 726 \\ 3 & 70 & 603 \end{bmatrix}$

13, weights $\longrightarrow 0.4, 0.3, 0.3$

Student B $\longrightarrow 6, 85, 726$

$$\text{Dot Products} = x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3$$

$$= (0.4 \times 6) + (0.3 \times 85) + (0.3 \times 726)$$

$$= 2.4 + 25.5 + 217.8$$

$$= \underline{\underline{245.7}}$$

14. ~~Ans~~

$$\begin{array}{c|ccc} 0.4 & 4 & 80 & 654 \\ 0.3 & 6 & 85 & 726 \\ 0.3 & 3 & 70 & 603 \end{array}$$

$$\text{Q14, } \begin{bmatrix} 4 & 80 & 654 \\ 6 & 85 & 726 \\ 3 & 70 & 603 \end{bmatrix} \times \begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}$$

$$\begin{aligned} 1^{\text{st}} &= 4 \times 0.4 + 80 \times 0.3 + 654 \times 0.3 \\ &= 1.6 + 24 + 196.2 \\ &= 221.8 \end{aligned}$$

$$\begin{aligned} 2^{\text{nd}} &= 6 \times 0.4 + 85 \times 0.3 + 726 \times 0.3 \\ &= 2.4 + 25.5 + 217.8 \\ &= 245.7 \end{aligned}$$

$$\begin{aligned} 3^{\text{rd}} &= 3 \times 0.4 + 70 \times 0.3 + 603 \times 0.3 \\ &= 1.2 + 21 + 180.9 \\ &= 203.1 \end{aligned}$$

$$= \begin{bmatrix} 221.8 \\ 245.7 \\ 203.1 \end{bmatrix}$$

$$\text{Q15- } \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 4 \\ 80 \\ 654 \end{bmatrix}$$

$$\begin{aligned}
 &= 2 \times 4 + 0 \times 80 + 0 \times 654 \\
 &= 0 \times 4 + 1 \times 80 + 0 \times 654 \\
 &= 0 \times 4 + 0 \times 80 + 1 \times 654
 \end{aligned}
 = \begin{bmatrix} 8 \\ 80 \\ 654 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 80 & 654 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 80 & 654 \end{bmatrix}$$

Q16, normalization

$$[4 \ 6 \ 3]$$

$$x_{\text{norm}} = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

$$\begin{aligned}
 \min &= 3 \\
 \max &= 6
 \end{aligned}$$

$$= \frac{4 - 3}{6 - 3} = 0.333$$

$$= \frac{6 - 3}{6 - 3} = 1$$

$$= \frac{3 - 3}{6 - 3} = 0 \quad = [0.333, 1, 0]$$

Q17: \rightarrow Compute the Euclidean distance b/w
Student Anna and Student B from Q12

A

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\text{Std A} = 4, 80, 654$$

$$\text{Std B} = 6, 85, 726$$

$$\begin{aligned} \text{Distance b/w A and B} &= \sqrt{(6-4)^2 + (85-80)^2 + (726-654)^2} \\ &= \sqrt{2^2 + 5^2 + 72^2} \\ &= \sqrt{4 + 25 + 5184} \\ &= \sqrt{5213} \\ &= 72.204 \end{aligned}$$

Q18 - Matrix A is $(5, 3)$
matrix B $(3, 1)$

~~If so~~ If matrix ~~weight~~ was (m, n)

(n, p)

then $m^{\frac{1}{2}}(m, p)$

$$\begin{aligned} &= (5, 3) \quad (3, 1) \\ &= (5, 1) \end{aligned}$$

Q19, = ~~Q20~~ No of Students = 100

No of Passed = 60

$$\frac{60}{100} = 0.6$$

Q20, ~~50~~ attended students = 50

Passed Students = 40

$$\frac{40}{50} = 0.8$$

Q21, A student has 70% chance of passing what is the Probability that the student fails

Percentage of Passing = 70% = 0.70

$$\begin{aligned} \text{Probability of fail} &= 1 - 0.70 \\ &= 0.30 \end{aligned}$$

Q22, $P(A) = 2, 4, 6$

$P(B) = 4, 5, 6$

$$a) P(A) = \frac{3}{6} = \frac{1}{2}$$

$$b) P(B) = \frac{3}{6} = \frac{1}{2}$$

$$c) P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

$$d) P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

23) Find x and y if possible such that

$$A) \begin{bmatrix} 2x+y & 3 & 10 \\ y+1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 10 \\ 3 & -2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & -4 & -6 & x-y \end{bmatrix} = -2 \begin{bmatrix} -3 & 2 & 2x+2y & 13 \end{bmatrix}$$

$$A) \quad 2x + y = 2 \quad \text{--- (1)}$$

$$y + 1 = 3 \quad \text{--- (2)}$$

$$y = 2$$

$$2x + 2 = 2$$

$$2x = 0$$

$$x = 0$$

$$B) \quad \begin{bmatrix} 6 & -4 & -6 & x-y \end{bmatrix} = -2 \begin{bmatrix} -3 & 2 & 2x+2y & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -4 & -6 & x-y \end{bmatrix} = \begin{bmatrix} 6 & -4 & -4x-4y & -26 \end{bmatrix}$$

$$x - y = -26 \quad \text{--- (1)}$$

$$-4x - 4y = -26 \quad \text{--- (2)}$$

$$x = -26 + y \quad (3)$$

$$3 - 2 \Rightarrow -4(-26 + y) - 4y = -6$$

$$= 104 - 4y - 4y = -6$$

$$-8y = 104 - 6$$

$$-8y = -6 + 104$$

$$-8y = 98$$

$$y = \frac{98}{-8} \quad 8y = 100$$

$$y = \frac{100}{8} \quad y = 12.5$$

$$x - y = -26$$

$$x - 12.5 = -26$$

$$x = -26 + 12.5$$

$$x = -13.5$$

24) A, B, C, D and E are matrices given by

$$A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 2 & 7 & -5 \end{bmatrix}$$

$$b = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix} \quad c = \begin{bmatrix} 3 \\ 5 \\ -11 \end{bmatrix} \quad f = \begin{bmatrix} -1 & 0 & 2 \\ -2 & -3 & 4 \\ 1 & 4 & -3 \end{bmatrix}$$

$A B$ 2×3 3×3 Possible 1st col 2nd row

$B C$ 3×3 1×5 not

$A D$ 2×3 2×2 not

$E F$ 3×1 3×3 not

$F E$ 3×3 3×1 Possible

$$AB = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}_{2 \times 3} \times \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}_{3 \times 3}$$

$$\begin{aligned} & -1 \times 1 + 1 \times 0 + -2 \times -1 \\ & = -1 + 0 + 2 \\ & = 1 \end{aligned}$$

$$-1 \times 2 + 1 \times -3 + -2 \times -2$$

$$\begin{aligned} & -2 + -3 + 4 \\ & = -1 \end{aligned}$$

$$-2 \times 0 + -1 \times 4 + -2 \times 3$$

$$= 0 + -4 + -6 = -10$$

$$FE = \begin{bmatrix} -1 & 0 & 2 \\ -2 & -3 & 4 \\ 1 & 4 & -3 \end{bmatrix} \quad E = \begin{bmatrix} 3 \\ 5 \\ -11 \end{bmatrix}$$

$$\begin{aligned} -1 \times 3 + -22 &= -28 \\ -6 + -15 + 33 &= 12 \\ 3 + 20 + 33 &= 56 \end{aligned} \quad = \begin{bmatrix} -28 \\ 12 \\ 56 \end{bmatrix}$$

253 Given the matrix

find the eigenvalues and eigenvectors

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \lambda \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\begin{bmatrix} 2-\lambda & 0 \\ 0 & 3-\lambda \end{bmatrix}$$

$$\boxed{\lambda = 2 \quad \text{or} \quad \lambda = 3}$$

Eigen vectors

for $\lambda = 2$

$$(A - \lambda I) x = 0$$

$$\begin{bmatrix} 2-2 & 0 \\ 0 & 3-2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_2 = 0$$

$$x_1 = k$$

$$\cancel{x_2} \quad \cancel{0} \quad x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} 2-3 & 0 \\ 0 & 3-3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$u_1 = 0 \quad u = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$u_2 = k$$

15, Given matrix =

$$B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \quad [A - \lambda I] = 0$$

$$\begin{bmatrix} 4 - \lambda & 1 \\ 2 & 3 - \lambda \end{bmatrix} = 0$$

$$= (4 - \lambda)(3 - \lambda) - (1)(2)$$

$$= 12 - 4\lambda - 3\lambda + \lambda^2 - 2$$

$$= \lambda^2 - 7\lambda + 10$$

$$\therefore \lambda = 5 \quad \text{or } \lambda = 2$$

16)

$$A = \begin{bmatrix} -2 & 1 \\ 12 & -3 \end{bmatrix}$$

$$\text{eigenvalue} = \det(A - \lambda I) = 0$$

$$\begin{bmatrix} -2 - \lambda & 1 \\ 12 & -3 - \lambda \end{bmatrix} = 0$$

$$(-2-\lambda)(-3-\lambda) - (1)(12) = 0$$

$$6 + 2\lambda + 3\lambda + \lambda^2 - 12 = 0$$

$$\lambda^2 - 5\lambda - 6 = 0$$

$$\lambda = 1 \quad \text{or} \quad \lambda_2 = -6$$

for $\lambda_1 = 1$

$$(A - \lambda I) x = 0$$

$$\begin{bmatrix} -2-1 & 1 \\ 12 & -3-1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 1 \\ 12 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + x_2 = 0$$

$$x_2 = 3x_1$$

$$\text{let } x_1 = 1$$

$$x_2 = 3$$

$$\cancel{x_1 = 1} \quad x = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

for $\lambda_2 = -6$

$$\begin{bmatrix} -2-(-6) & 1 \\ 12 & -3-(-6) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$2x_1 + x_2 = 0 \quad x_2 = -2x_1$$

$$\text{let } x_1 = 1 \Rightarrow x_2 = -2$$

$$x = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

17 e-commerce site

$$i) (4 \times 3 + 2 \times 1 + 1 \times 0) \\ = 12 + 2 = 14$$

ii) is of 3rd

$$(4 \times 1 + 4 \times 3 + 1 \times 2) \\ = 4 + 6 + 2 \\ = 12$$

$$\text{match} = \text{Product A} = 14$$

18, A logistic company models

$$\text{weight vector} \quad \text{weights (N)} = \begin{bmatrix} 0.5 \\ 1.2 \\ -0.8 \end{bmatrix}$$

Delivery location matrix

$$X = \begin{bmatrix} 10 & 0.5 & 0.6 \\ 15 & 0.8 & 0.3 \\ 8 & 0.3 & 0.9 \end{bmatrix}$$

$$\text{wt} = 10 \times 0.5 + (0.5)(1.2) + (0.6)(-0.8)$$

$$= 15 + 0.6 - 0.48$$

$$= \underline{5.12}$$

$$= (15 \times 0.5) + (0.8 \times 1.2) + (0.3) (-0.8)$$

$$= 7.5 + 0.96 - 0.24$$

$$= \underline{8.22}$$

$$(8) (0.5) + (0.3) (1.2) + (0.9) (-0.8)$$

$$= 4 + 0.36 - 0.72$$

$$= 3.64$$

$$\text{Product delivery time} \begin{bmatrix} 5.12 \\ 8.22 \\ 3.64 \end{bmatrix}$$

14, Climat

$$P(\text{Cloudy} / \text{Rain}) = \frac{25}{30}$$

$$P(\text{Cloudy}) = \frac{40}{100}$$

$$\text{P(Rain)} \quad P(R/C) = \frac{P(C/R) P(R)}{P(C)}$$

$$= \frac{\left(\frac{25}{30} \right) \left(\frac{30}{100} \right)}{\frac{40}{100}} = \frac{25}{40} = 0.625$$

20, ~~for~~ telephone

customer = 3, 1, 123

weight for Prediction model = $[1.5, 2, 0, -0.3]$

$$\begin{aligned}\text{Churn score} &= 1.5 \times 3 + 2.0 \times 1 + (-0.3) \times 12 \\ &= 4.5 + 2 - 3.6 \\ &= \underline{\underline{2.9}}\end{aligned}$$

21, a) $P(\text{Study} > 5 \text{ hours})$

$$\frac{50}{100} = 0.5$$

b, $P(\text{Pass})$

$$= \frac{70}{100} = 0.7$$

c, $P(\text{Pass} / \text{study} > 5)$

$$\begin{aligned}\text{P} &= \frac{B(40/100)}{50/100} \\ &= 0.8\end{aligned}$$