$$1+2+3+\ldots+(2n+1) = n(n+2)$$

1) $(2n+1) = n(n+2) \longrightarrow (2\cdot1+1) = 1(1+2)$
$$3 = 3 \ \checkmark$$

2) $(2n+1) = n(n+2) \longrightarrow \boxed{(2k+1) = k(k+2)}$

3) $(2k+1) + (2n+1) = n(n+2)$
$$\downarrow$$
$(2k+1) + (2(k+1)+1) = (k+1)(k+1+2)$
$k(k+2) + 2k+2+1 \qquad = (k+1)(k+3)$
$k^2 + 2k + 2k + 2 + 1 \qquad = k^2 + 3k + k + 3$
$\boxed{k^2 + 4k + 3} \qquad = k^2 + 4k + 3 \ \checkmark$

$$1+2+3+\ldots+(4n+1) = n(2n+3)$$

1) $(4n+1) = n(2n+3) \longrightarrow (4\cdot1+1) = 1(2\cdot1+3)$
$$5 = 5 \ \checkmark$$

2) $(4n+1) = n(2n+3) \longrightarrow (4k+1) = k(2k+3)$

3) $(4k+1) + (4n+1) = n(2n+3)$
$$\downarrow$$
$(4k+1) + (4(k+1)+1) = (k+1)(2(k+1)+3)$
$k(2k+3) + 4k+4+1 = (k+1)(2k+2+3)$
$2k^2 + 3k + 4k + 4 + 1 = 2k^2 + 2k + 3k + 2k + 2 + 3$
$\boxed{2k^2 + 7k + 5} = 2k^2 + 7k + 5 \ \checkmark$