Studio della cinematica di una gru per scarico/carico navi

Si consideri una delle due gru della foto (matricole pari destra, dispari sinistra). Ricavare delle misure ragionevoli dalla fotografia (almeno le proporzioni).

Analisi di Posizione

Funzione Quadrilatero

In[ ]:=
```
Quadrilatero[q_, xA_, yA_, L1_, L2_, L3_, modo_, xD_, yD_] :=
 Module[{xB, yB, L5, θ5, cosα, α, θ2, xC, yC, θ3},
  xB = xA + L1 Cos[q]; (* si calcola la posizione di B *)
  yB = yA + L1 Sin[q];
  L5 = √((xD - xB)^2 + (yD - yB)^2); (* si calcola la lunghezza del lato BD *)
  θ5 = ArcTan[xD - xB, yD - yB]; (* si calcola l'angolo
   θ5: notare l'uso della funzione arcotangente a due argomenti *)
  cosα = (L5^2 + L2^2 - L3^2) / (2 L5 L2);
  (* teorema del coseno: si calcola l'argomento del coseno *)
  α = 1;
  If[Abs[cosα] ≤ 1,
   If[modo > 0, α = 1, α = -1] (* meccanismo si assembla *)
   ,
   If[modo > 0, α = 1, α = -1] (* non si assembla e α è un numero complesso *)
  ];
  θ2 = θ5 - α * ArcCos[cosα]; (* si calcola θ2 *)
  xC = xB + L2 Cos[θ2]; (* si trova il punto C *)
  yC = yB + L2 Sin[θ2];
  θ3 = ArcTan[xD - xC, yD - yC];
  (* si calcola θ3: notare l'uso della funzione arcotangente a due argomenti *)
  {θ2, θ3, {{xA, yA}, {xB, yB}, {xC, yC}, {xD, yD}}}
  (* si restituisce θ3,θ2 e il poligono ABCD *)
 ]
```

Funzione Trilatero per Estremo P e Peso K

In[ ]:=
```
Trilatero[θ1_, θ2_, LA_, LB_, xC_, yC_] :=
 {{LA Cos[θ1], LA Sin[θ1]}, {LA Cos[θ1] + LB Cos[θ2], LA Sin[θ1] + LB Sin[θ2]}, {xC, yC}}
```

Definizione Valori

```
In[ ]:= var =
          {xA = 0,
           yA = 0,
           L1 = 10,
           L2 = 13,
           L6 = 5,
           xD = -7.5,
           yD = 13,
           L3 = 25,
           L4 = 5,
           L5 = 30,
           L1P = 18,
           L2P = 25,
           L1K = 10,
           L2K = 10,
           θ1noto = π / 6,
           θ2noto = π / 6,
           θ0 = ArcTan[xD, yD],
           L0 = √(xD^2 + yD^2)};
        modo1 = -1;
        modo2 = 1;
        v1 = 1;
        v2 = 2;
```

Determino le posizini dei qudrilateri

```
In[ ]:= Pol1[pol1_] := Quadrilatero[pol1, xA, yA, Lv1, Lv2, Lv6, modo2, xvD, yvD]
        Pol2[pol2_] := Quadrilatero[pol2,
                                    Pol1[pol2][[3, 2, 1]],
                                    Pol1[pol2][[3, 2, 2]], Lv3, Lv4, Lv5, modo2,
                                    Pol1[pol2][[3, 3, 1]],
                                    Pol1[pol2][[3, 3, 2]]]
```

Posizione P

```
In[ ]:= Pol3[pol3_] := Trilatero[pol3, Pol2[pol3][[1]] + π + θ1noto, Lv1 + Lv3, L1P,
                                 Pol2[pol3][[3, 3, 1]], Pol2[pol3][[3, 3, 2]]
                                ]
```

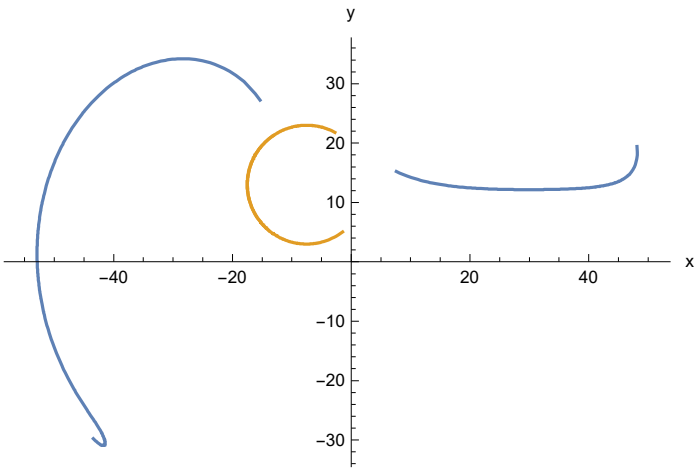
Posizione Peso k

```
In[ ]:= Pol4[pol4_] := Trilatero[ArcTan[xvD, yvD], Pol1[pol4][[2]] - θ2noto,
          Sqrt[xvD^2 + yvD^2], L1K, Pol1[pol4][[3, 3, 1]], Pol1[pol4][[3, 3, 2]]]
```

Traiettoria e Distanze Max e Min

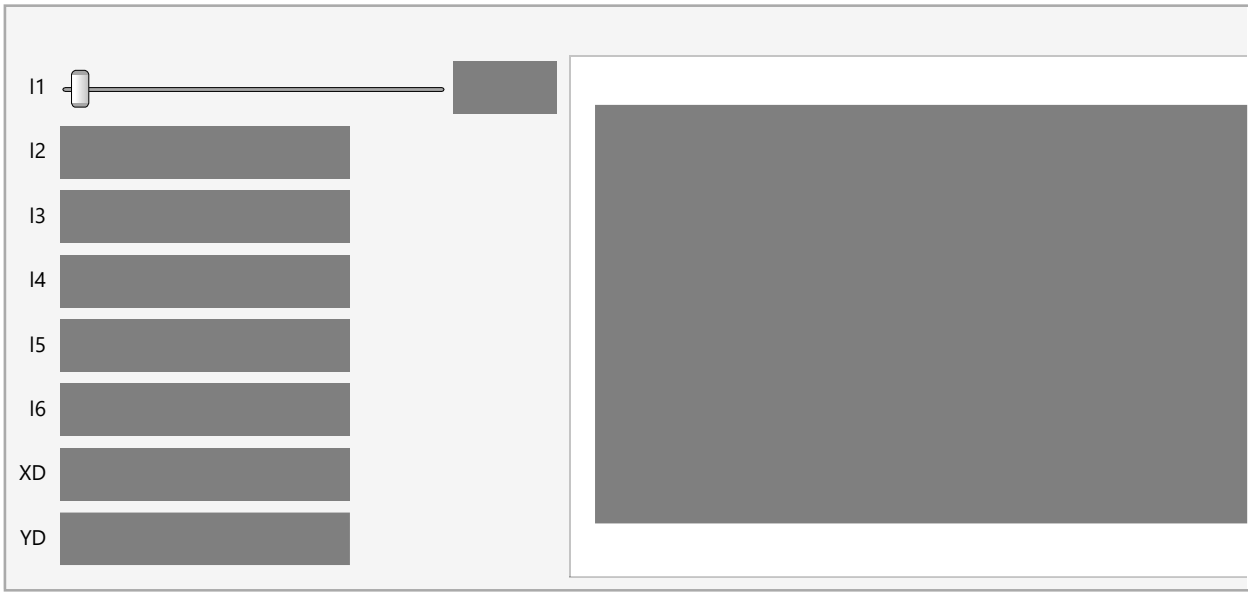
```
In[ ]:= Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
         ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]}, {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}},
          {q, 0, 2 π}, AxesLabel → {"x", "y"}]]
```

Out[ ]=



In[ ]:=
```
Manipulate[Block[{Lv1 = l1, Lv2 = l2, Lv3 = l3, Lv4 = l4, Lv5 = l5, Lv6 = l6, xvD = XD, yvD = YD},
  ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]},
    {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}}, {q, 0, 2 π}, PlotRange → {{-60, 60}, {-10, 40}}]],
 {l1, 10, 20, 0.1}, {l2, 10, 20, 0.1}, {l3, 20, 30, 0.1}, {l4, 2, 10, 0.1},
 {l5, 20, 40, 0.1}, {l6, 2, 10, 0.1}, {XD, -15, 0, 0.1}, {YD, 1, 15, 0.1}]
```

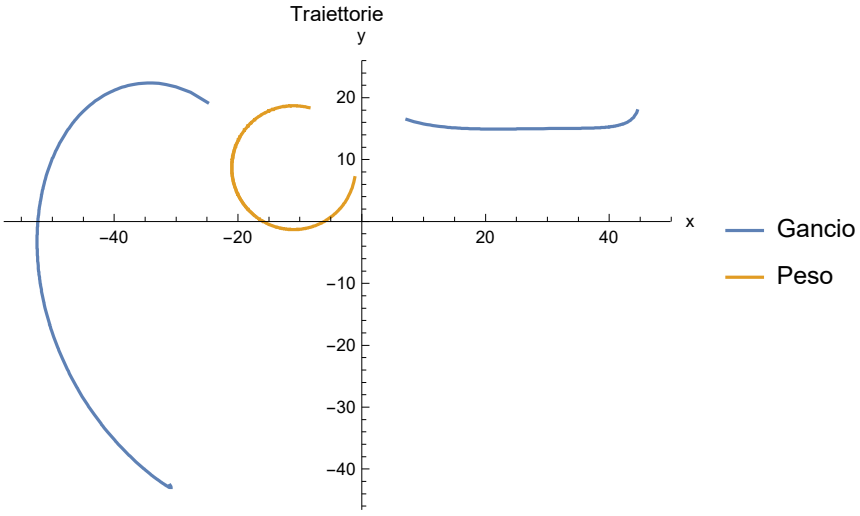
Out[ ]=



In[ ]:=
```
L11 = 12.2;
L22 = 13.7;
L33 = 23.8;
L44 = 5.2;
L55 = 33.9;
L66 = 6.6;
xDD = -11;
yDD = 8.7;
```

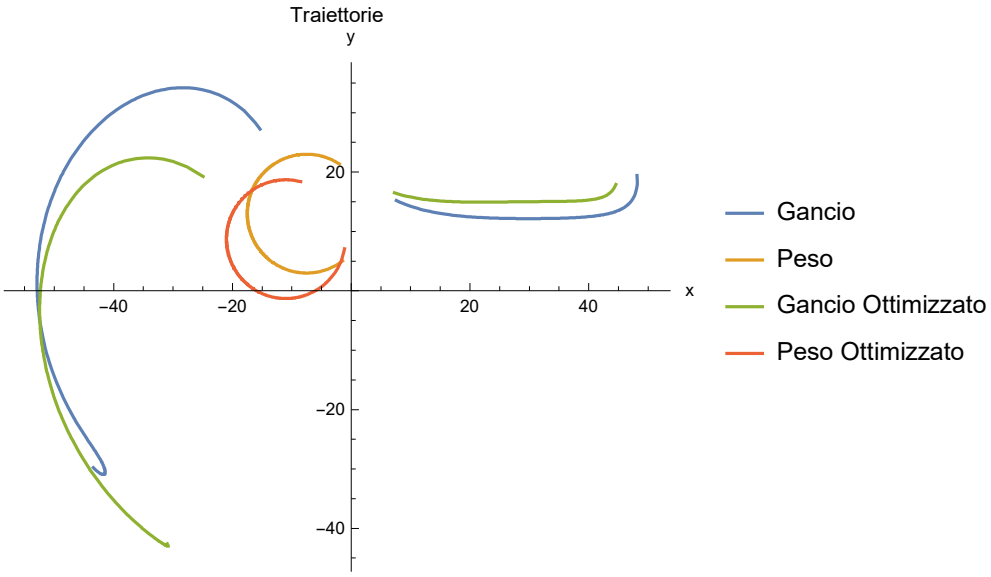
In[ ]:=
```
Block[{Lv1 = L11, Lv2 = L22, Lv3 = L33, Lv4 = L44, Lv5 = L55, Lv6 = L66, xvD = xDD, yvD = yDD},
 ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]}, {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}},
  {q, 0, 2 π}, AxesLabel → {"x", "y"},
  PlotLabel → "Traiettorie", PlotLegends → {"Gancio", "Peso"}]]
```

Out[ ]=



In[ ]:=
```
ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]} /.
    {Lv1 → L1, Lv2 → L2, Lv3 → L3, Lv4 → L4, Lv5 → L5, Lv6 → L6, xvD → xD, yvD → yD},
   {Pol4[q][[2, 1]], Pol4[q][[2, 2]]} /. {Lv1 → L1, Lv2 → L2, Lv3 → L3, Lv4 → L4,
     Lv5 → L5, Lv6 → L6, xvD → xD, yvD → yD}, {Pol3[q][[2, 1]], Pol3[q][[2, 2]]} /.
    {Lv1 → L11, Lv2 → L22, Lv3 → L33, Lv4 → L44, Lv5 → L55, Lv6 → L66, xvD → xDD, yvD → yDD},
   {Pol4[q][[2, 1]], Pol4[q][[2, 2]]} /. {Lv1 → L11, Lv2 → L22, Lv3 → L33,
     Lv4 → L44, Lv5 → L55, Lv6 → L66, xvD → xDD, yvD → yDD}},
  {q, 0, 2 π}, AxesLabel → {"x", "y"}, PlotLabel → "Traiettorie",
  PlotLegends → {"Gancio", "Peso", "Gancio Ottimizzato", "Peso Ottimizzato"}]
```

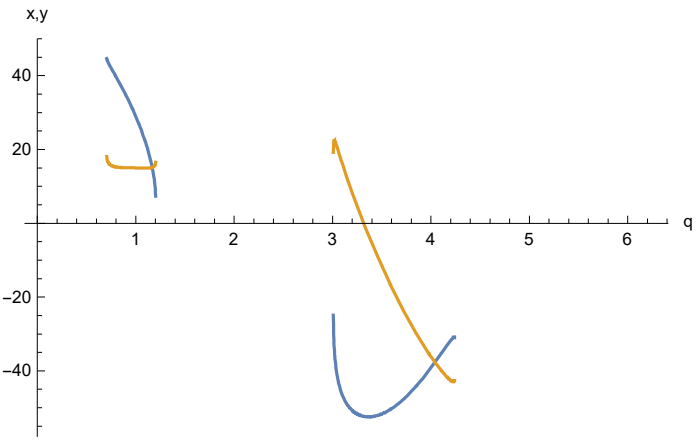
Out[ ]=



In[ ]:=
```
L1 = L11;
L2 = L22;
L3 = L33;
L4 = L44;
L5 = L55;
L6 = L66;
xD = xDD;
yD = yDD;
```

In[ ]:=
```
Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
 Plot[{Pol3[q][[2, 1]], Pol3[q][[2, 2]]}, {q, 0, 2 π}, AxesLabel → {"q", "x,y"}]]
```

Out[ ]=



In[ ]:=
```
qmax = 0;
qmin = π / 2;
Timing@Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
  Do[If[Pol3[i][[2, 1]] ∈ Reals,
                                                    If[i > qmax, qmax = i, 0], 0],
    {i, 0, π / 2, π / 36 000}]]
Timing@Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
  Do[If[Pol3[π / 2 - i][[2, 1]] ∈ Reals,
                                                    If[π / 2 - i < qmin, qmin = π / 2 - i, 0], 0],
    {i, 0, π / 2, π / 36 000}]]
```

Out[ ]= {42.4531, Null}

Out[ ]= {42.5, Null}

In[ ]:=
```
qmax
```

Out[ ]=
$$\frac{4589\,\pi}{12\,000}$$

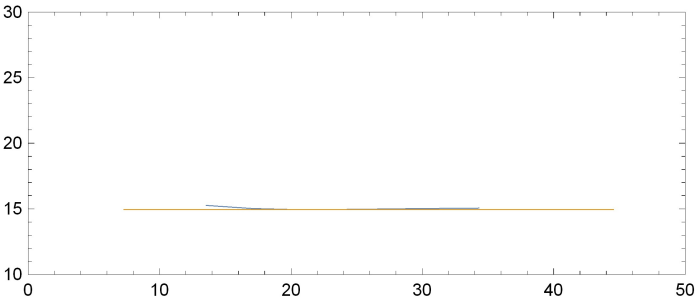
In[ ]:=
```
qmin
```

Out[ ]=
$$\frac{8089\,\pi}{36\,000}$$

In[ ]:=
```
domQ = FunctionDomain[Pol3[q][[2, 2]], q];
```

```
In[ ]:= miny = Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
          FindMinimum[{Pol3[q][[2, 2]], domQ}, {q, π / 3}]]
Out[ ]= {14.9397, {q → 1.10163}}

In[ ]:= Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
         ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]}, {r, miny[[1]]}},
          {q, 0, π / 2}, {r, Pol3[qmin][[2, 1]], Pol3[qmax][[2, 1]]},
          PlotRange → {{0, 50}, {10, 30}}, AxesLabel → {"x", "y"}]]
```

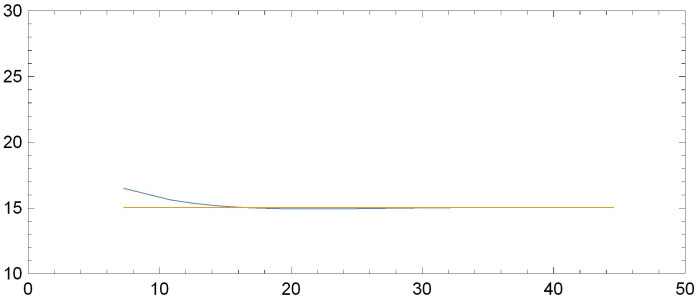
Out[ ]=



La traiettoria va da dx a sx al variare di q

```
In[ ]:= diffsx = Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
          Pol3[qmax][[2, 2]] - miny[[1]]]
Out[ ]= 1.55785

In[ ]:= diffdx = Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
          Pol3[qmin][[2, 2]] - miny[[1]]]
Out[ ]= 3.02837

In[ ]:= point = Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
           Array[Pol3[#][[2, 2]] &, 500, {qmin, qmax}]];

In[ ]:= mediana = Median[point]
Out[ ]= 15.0512

In[ ]:= Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
         ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]}, {r, mediana}},
          {q, qmin, qmax}, {r, Pol3[qmin][[2, 1]], Pol3[qmax][[2, 1]]},
          PlotRange → {{0, 50}, {10, 30}}, AxesLabel → {"x", "y"}]]
```

Out[ ]=



In[ ]:=
```
deviazione = StandardDeviation[point]
```

Out[ ]=
```
0.426381
```

Retta piu piana possibile

In[ ]:=
```
pointpiano = Array[Pol3[#][[2, 2]] &, 500, {0, π / 2}];
```

In[ ]:=
```
(*Timing@Solve[StandardDeviation[pointpiano]≤5,{Lv1,Lv2,Lv3,Lv4,Lv5,Lv6}]*)
```

Analisi di Velocità

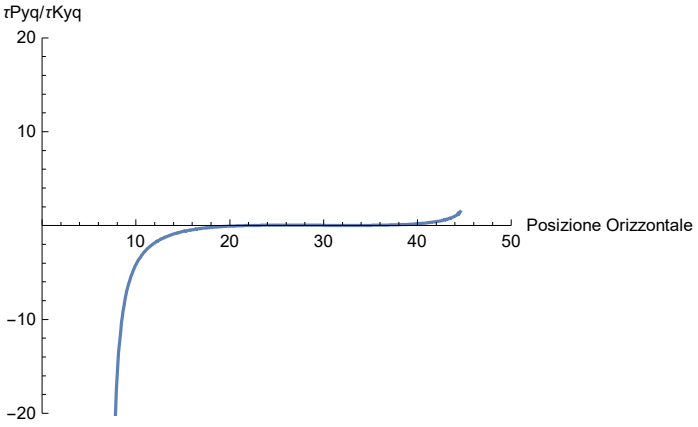
Rapporto Velocità Gancio/Velocità Peso

In[ ]:=
```
τPyq = ∂q Pol3[q][[2, 2]];
```

In[ ]:=
```
τKyq = ∂q Pol4[q][[2, 2]];
```

In[ ]:=
```
Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
 ParametricPlot[{Pol3[q][[2, 1]], τPyq / τKyq}, {q, qmin, qmax},
  PlotRange → {{0, 50}, {-20, 20}}, AxesLabel → {"Posizione Orizzontale", "τPyq/τKyq"}]]
```

Out[ ]=

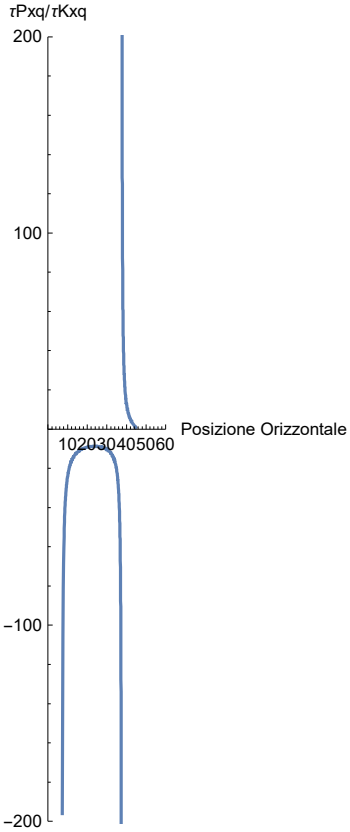


In[ ]:=
```
τPxq = ∂q Pol3[q][[2, 1]];
```

In[ ]:=
```
τKxq = ∂q Pol4[q][[2, 1]];
```

```
In[ ]:= Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
         ParametricPlot[{Pol3[q][[2, 1]], τPxq / τKxq}, {q, qmin, qmax},
          PlotRange → {{0, 60}, {-200, 200}}, AxesLabel → {"Posizione Orizzontale", "τPxq/τKxq"}]]
```

Out[ ]=



Analisi di Accelerazione

Velocità Orizzontale Costante

```
In[ ]:= xP' == τPxq q';
        q' == cost / τPxq;
        q'' == ∂t q;
        q'' == cost q' ∂q cost / τPxq ;
```

```
In[ ]:= cost = 1;
```

```
In[ ]:= dq1 = cost / τPxq;
```

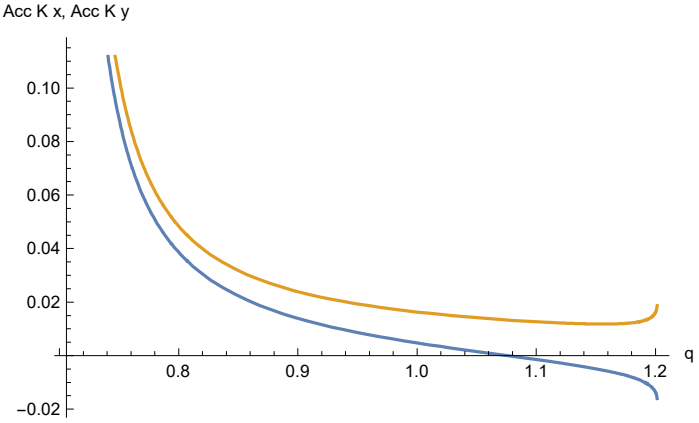
```
In[ ]:= dq2 = cost dq1 ∂q (cost / τPxq);
```

Accelerazione Orizzontale Contrappeso

quello fatto da me

```
In[•]:= pXK'' == τKxq q'' + (∂q τKxq) q'^2;

In[•]:= dpXK2 = τKxq dq2 + (∂q τKxq) (dq1)^2;

In[•]:= pYK'' == τKyq q'' + (∂q τKyq) q';

In[•]:= dpYK2 = τKyq dq2 + (∂q τKyq) (dq1)^2;

In[•]:= Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
         Plot[{dpXK2, dpYK2}, {q, qmin, qmax}, AxesLabel → {"q", "Acc K x, Acc K y"}]]
```

Out[•]=

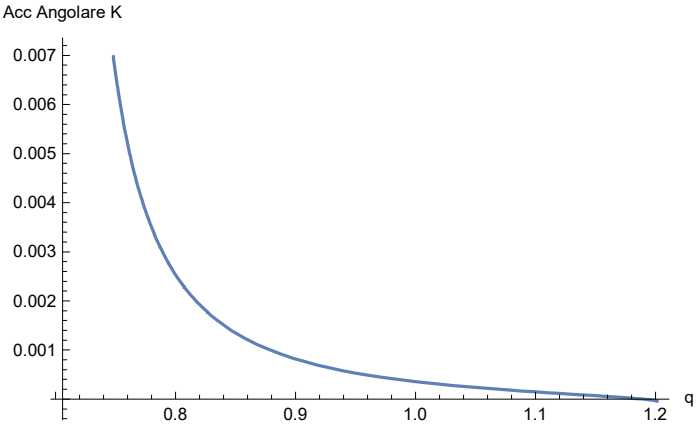


Il Grafico potrebbe essere in effetti corretto perche il peso esegue una traiettoria a circonferenza

Rapporto di velocità di $\theta 6$

```
In[•]:= τθ6 = ∂q Pol1[q][[1]];

In[•]:= d2θ6 = τθ6 dq2 + dq1^2 ∂q τθ6;

In[•]:= Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
         Plot[{d2θ6}, {q, qmin, qmax}, AxesLabel → {"q", "Acc Angolare K"}]]
```

Out[•]=



Animazione

In[ ]:=
```
Manipulate[
 Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
      Show[
           ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]},
     {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}}, {q, qmin, qmax},
    PlotStyle → Directive[Black, Dashed], PlotRange → {{-30, 50}, {0, 50}}],
           ListPlot[{Pol1[q][[3]], Pol2[q][[3]], Pol3[q], Pol4[q]},
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-30, 50}, {0, 50}}]]], {q, 0, π / 2}]
```

Out[ ]=

Configurazioni Singolari

Equazione di Chiusura

```
In[ ]:= eq1x = (Lv1 + Lv3) Cos[θ1] + Lv4 Cos[θ4] + Lv5 Cos[θ5] + Lv6 Cos[θ6] + Lv0 Cos[θ0];
        eq1y = (Lv1 + Lv3) Sin[θ1] + Lv4 Sin[θ4] + Lv5 Sin[θ5] + Lv6 Sin[θ6] + Lv0 Sin[θ0];
        eq2x = Lv1 Cos[θ1] + Lv2 Cos[θ2] + Lv6 Cos[θ6] - Lv0 Cos[θ0];
        eq2y = Lv1 Sin[θ1] + Lv2 Sin[θ2] + Lv6 Sin[θ6] - Lv0 Sin[θ0];

In[ ]:= jak = {{∂θ2 eq1x, ∂θ4 eq1x, ∂θ5 eq1x, ∂θ6 eq1x}, {∂θ2 eq1y, ∂θ4 eq1y, ∂θ5 eq1y, ∂θ6 eq1y},
          {∂θ2 eq2x, ∂θ4 eq2x, ∂θ5 eq2x, ∂θ6 eq2x}, {∂θ2 eq2y, ∂θ4 eq2y, ∂θ5 eq2y, ∂θ6 eq2y}};

In[ ]:= jak

Out[ ]= {{0, -Lv4 Sin[θ4], -Lv5 Sin[θ5], -Lv6 Sin[θ6]}, {0, Lv4 Cos[θ4], Lv5 Cos[θ5], Lv6 Cos[θ6]},
         {-Lv2 Sin[θ2], 0, 0, -Lv6 Sin[θ6]}, {Lv2 Cos[θ2], 0, 0, Lv6 Cos[θ6]}}

In[ ]:= FullSimplify[Det[jak]]

Out[ ]= Lv2 Lv4 Lv5 Lv6 Sin[θ4 - θ5] Sin[θ2 - θ6]

In[ ]:= sol = Timing@Solve[Det[jak] == 0, {θ2, θ4, θ5, θ6}, Reals]

Out[ ]= {0.21875, {{θ5 → ConditionalExpression[θ4 - 2 π c1, c1 ∈ ℤ]},
           {θ5 → ConditionalExpression[-π + θ4 - 2 π c1, c1 ∈ ℤ]},
           {θ6 → ConditionalExpression[θ2 - 2 π c1, c1 ∈ ℤ]},
           {θ6 → ConditionalExpression[-π + θ2 - 2 π c1, c1 ∈ ℤ]}}}

In[ ]:= soldef = sol /. c1 → 0

Out[ ]= {0.21875, {{θ5 → θ4}, {θ5 → -π + θ4}, {θ6 → θ2}, {θ6 → -π + θ2}}}

In[ ]:= tablesol1 =
         Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD, Lv0 = L0},
          Table[{eq1x, eq1y, eq2x, eq2y} /. soldef[[2, 1]] /. soldef[[2, i]], {i, 3, 4}]]

Out[ ]= {{-7.5 + 36. Cos[θ1] + 6.6 Cos[θ2] + 39.1 Cos[θ4],
           13. + 36. Sin[θ1] + 6.6 Sin[θ2] + 39.1 Sin[θ4], 7.5 + 12.2 Cos[θ1] + 20.3 Cos[θ2],
           -13. + 12.2 Sin[θ1] + 20.3 Sin[θ2]}, {-7.5 + 36. Cos[θ1] - 6.6 Cos[θ2] + 39.1 Cos[θ4],
           13. + 36. Sin[θ1] - 6.6 Sin[θ2] + 39.1 Sin[θ4],
           7.5 + 12.2 Cos[θ1] + 7.1 Cos[θ2], -13. + 12.2 Sin[θ1] + 7.1 Sin[θ2]}}

In[ ]:= tablesol2 =
         Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD, Lv0 = L0},
          Table[{eq1x, eq1y, eq2x, eq2y} /. soldef[[2, 2]] /. soldef[[2, i]], {i, 3, 4}]]

Out[ ]= {{-7.5 + 36. Cos[θ1] + 6.6 Cos[θ2] - 28.7 Cos[θ4],
           13. + 36. Sin[θ1] + 6.6 Sin[θ2] - 28.7 Sin[θ4], 7.5 + 12.2 Cos[θ1] + 20.3 Cos[θ2],
           -13. + 12.2 Sin[θ1] + 20.3 Sin[θ2]}, {-7.5 + 36. Cos[θ1] - 6.6 Cos[θ2] - 28.7 Cos[θ4],
           13. + 36. Sin[θ1] - 6.6 Sin[θ2] - 28.7 Sin[θ4],
           7.5 + 12.2 Cos[θ1] + 7.1 Cos[θ2], -13. + 12.2 Sin[θ1] + 7.1 Sin[θ2]}}
```

```
In[ ]:= sol1min =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol1[[1, i]] == 0}, {x, qmin}]], {i, 1, 4}];

In[ ]:= sol2min =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol1[[2, i]] == 0}, {x, qmin}]], {i, 1, 4}];

In[ ]:= sol3min =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol2[[1, i]] == 0}, {x, qmin}]], {i, 1, 4}];

In[ ]:= sol4min =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol2[[2, i]] == 0}, {x, qmin}]], {i, 1, 4}];

In[ ]:= sol1max =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol1[[1, i]] == 0}, {x, qmax}]], {i, 1, 4}];

In[ ]:= sol2max =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol1[[2, i]] == 0}, {x, qmax}]], {i, 1, 4}];

In[ ]:= sol3max =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol2[[1, i]] == 0}, {x, qmax}]], {i, 1, 4}];

In[ ]:= sol4max =
  Timing@Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD,
      yvD = yD, Lv0 = L0, θ1 = x, θ2 = Pol1[x][[1]], θ4 = Pol2[x][[1]]},
        FindRoot[{tablesol2[[2, i]] == 0}, {x, qmax}]], {i, 1, 4}];
```

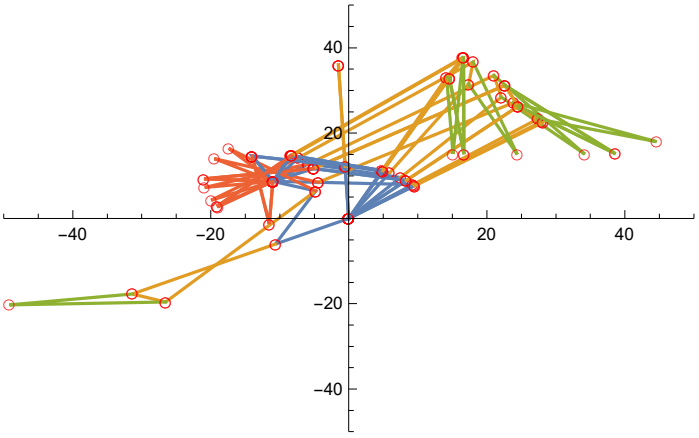
```
In[ ]:= TableForm[{sol1min, sol2min, sol3min, sol4min, sol1max, sol2max, sol3max, sol4max}]
```

Out[ ]//TableForm=

	$x \to 0.90929$
0.15625	$x \to 0.68276 - 3.19482\,10^{-15}\,i$
	$x \to 0.367049 + 0.367326\,i$
	$x \to 0.821198$
	$x \to 0.705888$
0.109375	$x \to 0.674436 - 4.62853\,10^{-15}\,i$
	$x \to 1.61252 + 1.02294\,10^{-16}\,i$
	$x \to 1.15365$
	$x \to 1.06688$
0.109375	$x \to 0.279669 + 0.230121\,i$
	$x \to 0.367049 + 0.367326\,i$
	$x \to 0.821198$
	$x \to 1.16674$
0.09375	$x \to 3.65522$
	$x \to 1.61252 + 1.02294\,10^{-16}\,i$
	$x \to 1.15365$
	$x \to 0.90929$
0.09375	$x \to 1.20368 + 4.79267\,10^{-15}\,i$
	$x \to 0.367049 + 0.367326\,i$
	$x \to 0.821198$
	$x \to 0.705888$
0.125	$x \to 1.20357 + 4.45094\,10^{-15}\,i$
	$x \to 1.61252$
	$x \to 1.15365$
	$x \to 1.06688$
0.109375	$x \to 0.279669 + 0.230121\,i$
	$x \to 0.367049 + 0.367326\,i$
	$x \to 0.821198$
	$x \to 1.16674$
0.0625	$x \to 0.422426 + 0.317199\,i$
	$x \to 1.61252$
	$x \to 1.15365$

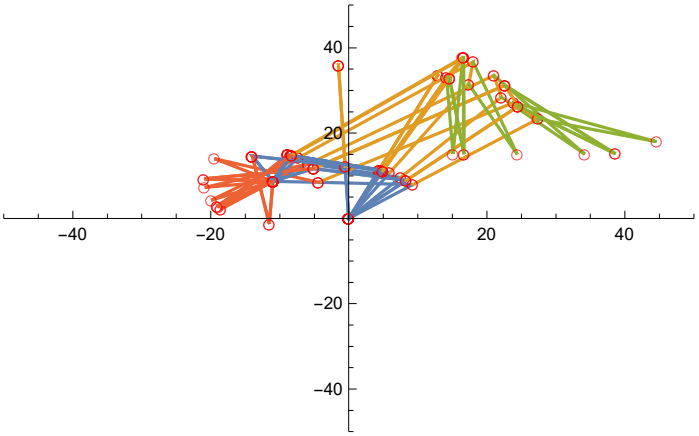
In[ ]:=
```
Show[
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol1min[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}],
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol2min[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}],
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol3min[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}],
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol4min[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}]
]
```

Out[ ]=



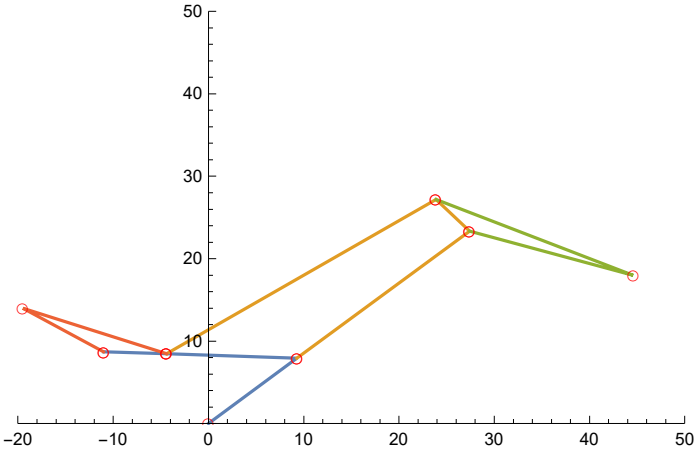
In[ ]:=
```
Show[
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol1max[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}],
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol2max[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}],
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol3max[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}],
 Table[Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
   ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol4max[[2, i]],
    Joined → True, PlotMarkers → {Style["○", Red], Medium},
    PlotRange → {{-50, 50}, {-50, 50}}]], {i, 4}]
]
```

Out[ ]=



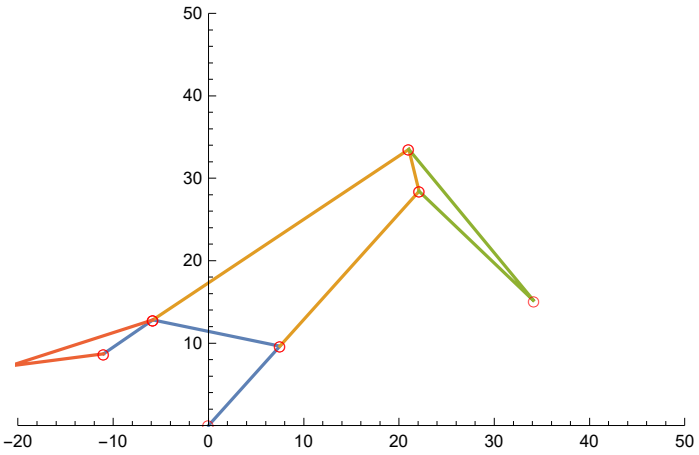
In[ ]:=
```
Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
    ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol2min[[2, 1]],
  Joined → True, PlotMarkers → {Style["○", Red], Medium}, PlotRange → {{-20, 50}, {0, 50}}]]
```

Out[ ]=



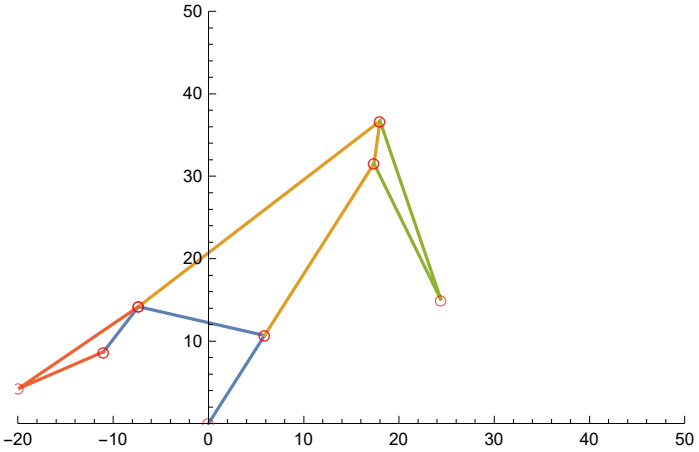
In[ ]:=
```
Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
    ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol1min[[2, 1]],
  Joined → True, PlotMarkers → {Style["○", Red], Medium}, PlotRange → {{-20, 50}, {0, 50}}]]
```

Out[ ]=



In[ ]:=
```
Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
    ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol3max[[2, 1]],
  Joined → True, PlotMarkers → {Style["○", Red], Medium}, PlotRange → {{-20, 50}, {0, 50}}]]
```

Out[ ]=



In[ ]:=
```
Block[{Lv1 = L1, Lv2 = L2, Lv3 = L3, Lv4 = L4, Lv5 = L5, Lv6 = L6, xvD = xD, yvD = yD},
    ListPlot[{Pol1[x][[3]], Pol2[x][[3]], Pol3[x], Pol4[x]} /. sol4max[[2, 1]],
  Joined → True, PlotMarkers → {Style["○", Red], Medium}, PlotRange → {{-20, 50}, {0, 50}}]]
```

Out[ ]=