Studio della cinematica di una gru per scarico/carico navi mediante Sintesi

Funzioni Iniziali per Analisi di Posizione

Funzione Quadrilatero

```
In[ ]:= Quadrilatero[q_, xA_, yA_, L1_, L2_, L3_, modo_, xD_, yD_] :=
  Module[{xB, yB, L5, θ5, cosα, α, θ2, xC, yC, θ3},
   xB = xA + L1 Cos[q]; (* si calcola la posizione di B *)
   yB = yA + L1 Sin[q];
   L5 = Sqrt[(xD - xB) ^2 + (yD - yB) ^2]; (* si calcola la lunghezza del lato BD *)
   θ5 = ArcTan[xD - xB, yD - yB]; (* si calcola l'angolo
    θ5: notare l'uso della funzione arcotangente a due argomenti *)
   cosα = (L5^2 + L2^2 - L3^2) / (2 L5 L2);
   (* teorema del coseno: si calcola l'argomento del coseno *)
   α = 1;
   If[Abs[cosα] ≤ 1,
    If[modo > 0, α = 1, α = -1] (* meccanismo si assembla *)
    ,
    If[modo > 0, α = 1, α = -1] (* non si assembla e α è un numero complesso *)
   ];
   θ2 = θ5 - α * ArcCos[cosα]; (* si calcola θ2 *)
   xC = xB + L2 Cos[θ2]; (* si trova il punto C *)
   yC = yB + L2 Sin[θ2];
   θ3 = ArcTan[xD - xC, yD - yC];
   (* si calcola θ3: notare l'uso della funzione arcotangente a due argomenti *)
   {θ2, θ3, {{xA, yA}, {xB, yB}, {xC, yC}, {xD, yD}}}
   (* si restituisce θ3,θ2 e il poligono ABCD *)
  ]
```

Funzione Trilatero per Estremo P e Peso K

In[ ]:=
```
Trilatero[θ1_, θ2_, LA_, LB_, xC_, yC_] :=
 {{LA Cos[θ1], LA Sin[θ1]}, {LA Cos[θ1] + LB Cos[θ2], LA Sin[θ1] + LB Sin[θ2]}, {xC, yC}}
```

Definizione Valori

In[ ]:=
```
var =
   {xA = 0,
    yA = 0,
    L1 = 10,
    L2 = 13,
    L6 = 5,
    xD = -7.5,
    yD = 13,
    L3 = 25,
    L4 = 5,
    L5 = 30,
    L1P = 18,
    L2P = 25,
    L1K = 10,
    L2K = 10,
    θ1noto = π / 6,
    θ2noto = π / 6,
    θ0 = ArcTan[xD, yD],
    L0 = √(xD^2 + yD^2)};
modo1 = -1;
modo2 = 1;
v1 = 1;
v2 = 2;
```

In[ ]:=
```
L11 = 12.2;
L22 = 13.7;
L33 = 23.8;
L44 = 5.2;
L55 = 33.9;
L66 = 6.6;
xDD = -11;
yDD = 8.7;
```

Determino le posizini dei qudrilateri

```
In[ ]:= Pol1[pol1_] := Quadrilatero[pol1, xA, yA, Lv1, Lv2, Lv6, modo2, xvD, yvD]
Pol2[pol2_] := Quadrilatero[pol2,
                                    Pol1[pol2][[3, 2, 1]],
                                    Pol1[pol2][[3, 2, 2]], Lv3, Lv4, Lv5, modo2,
                                    Pol1[pol2][[3, 3, 1]],
                                    Pol1[pol2][[3, 3, 2]]]
```

Posizione P

```
In[ ]:= Pol3[pol3_] := Trilatero[pol3, Pol2[pol3][[1]] + π + θ1noto, Lv1 + Lv3, LvP,
                               Pol2[pol3][[3, 3, 1]], Pol2[pol3][[3, 3, 2]]
                              ]
```

```
In[ ]:= Pol4[pol4_] := Trilatero[ArcTan[xvD, yvD], Pol1[pol4][[2]] - θ2noto,
   √(xvD^2 + yvD^2), L1K, Pol1[pol4][[3, 3, 1]], Pol1[pol4][[3, 3, 2]]]
```

Sintesi di Traiettoria con Correlazione

Definisco gli estremi di un range nel quale campionare gli angoli su cui valutare la funzione penalità

In[ ]:= $qmax = \frac{4589\,\pi}{12\,000};$

$qmin = \frac{8089\,\pi}{36\,000};$

Punti di Accuratezza

In[ ]:=
```
rangemedia = Range[qmin, qmax, (qmax - qmin) / 100]
```

Out[ ]=

$$\Big\{\frac{8089\pi}{36000}, \frac{135763\pi}{600000}, \frac{25633\pi}{112500}, \frac{412967\pi}{1800000}, \frac{69301\pi}{300000}, \frac{83729\pi}{360000}, \frac{105371\pi}{450000}, \frac{47147\pi}{200000}, \frac{213581\pi}{900000},$$
$$\frac{430001\pi}{1800000}, \frac{3607\pi}{15000}, \frac{435679\pi}{1800000}, \frac{219259\pi}{900000}, \frac{147119\pi}{600000}, \frac{111049\pi}{450000}, \frac{89407\pi}{360000}, \frac{24993\pi}{100000},$$
$$\frac{452713\pi}{1800000}, \frac{7118\pi}{28125}, \frac{152797\pi}{600000}, \frac{46123\pi}{180000}, \frac{464069\pi}{1800000}, \frac{38909\pi}{150000}, \frac{469747\pi}{1800000}, \frac{236293\pi}{900000}, \frac{2113\pi}{8000},$$
$$\frac{59783\pi}{225000}, \frac{481103\pi}{1800000}, \frac{80657\pi}{300000}, \frac{486781\pi}{1800000}, \frac{24481\pi}{90000}, \frac{164153\pi}{600000}, \frac{247649\pi}{900000}, \frac{498137\pi}{1800000},$$
$$\frac{3479\pi}{12500}, \frac{100763\pi}{360000}, \frac{253327\pi}{900000}, \frac{169831\pi}{600000}, \frac{128083\pi}{450000}, \frac{515171\pi}{1800000}, \frac{17267\pi}{60000}, \frac{520849\pi}{1800000},$$
$$\frac{65461\pi}{225000}, \frac{58503\pi}{200000}, \frac{264683\pi}{900000}, \frac{106441\pi}{360000}, \frac{44587\pi}{150000}, \frac{537883\pi}{1800000}, \frac{270361\pi}{900000}, \frac{181187\pi}{600000}, \frac{683\pi}{2250},$$
$$\frac{549239\pi}{1800000}, \frac{30671\pi}{100000}, \frac{554917\pi}{1800000}, \frac{139439\pi}{450000}, \frac{37373\pi}{120000}, \frac{281717\pi}{900000}, \frac{566273\pi}{1800000}, \frac{23713\pi}{75000},$$
$$\frac{571951\pi}{1800000}, \frac{57479\pi}{180000}, \frac{64181\pi}{200000}, \frac{145117\pi}{450000}, \frac{583307\pi}{1800000}, \frac{97691\pi}{300000}, \frac{117797\pi}{360000}, \frac{36989\pi}{112500},$$
$$\frac{198221\pi}{600000}, \frac{298751\pi}{900000}, \frac{600341\pi}{1800000}, \frac{3351\pi}{10000}, \frac{606019\pi}{1800000}, \frac{304429\pi}{900000}, \frac{203899\pi}{600000}, \frac{76817\pi}{225000},$$
$$\frac{4939\pi}{14400}, \frac{103369\pi}{300000}, \frac{623053\pi}{1800000}, \frac{156473\pi}{450000}, \frac{69859\pi}{200000}, \frac{63157\pi}{180000}, \frac{634409\pi}{1800000}, \frac{3319\pi}{9375}, \frac{640087\pi}{1800000},$$
$$\frac{321463\pi}{900000}, \frac{43051\pi}{120000}, \frac{162151\pi}{450000}, \frac{651443\pi}{1800000}, \frac{36349\pi}{100000}, \frac{657121\pi}{1800000}, \frac{16499\pi}{45000}, \frac{220933\pi}{600000},$$
$$\frac{332819\pi}{900000}, \frac{668477\pi}{1800000}, \frac{55943\pi}{150000}, \frac{134831\pi}{360000}, \frac{338497\pi}{900000}, \frac{75537\pi}{200000}, \frac{42667\pi}{112500}, \frac{685511\pi}{1800000}, \frac{4589\pi}{12000}\Big\}$$

Valore medio delle coordinate lungo y ottenute, Traiettoria Desiderata

In[ ]:=
```
Fdy = Block[{Lv1 = L11, Lv2 = L22, Lv3 = L33, Lv4 = L44, Lv5 = L55,
   Lv6 = L66, LvP = L1P , xvD = xDD, yvD = yDD}, Mean[Pol3[rangemedia][[2, 2]]]]
```

Out[ ]=
```
15.2362
```

Valori delle coordiante lungo x ottenute

In[ ]:=
```
Fdx = Block[{Lv1 = L11, Lv2 = L22, Lv3 = L33, Lv4 = L44, Lv5 = L55,
    Lv6 = L66, LvP = L1P , xvD = xDD, yvD = yDD}, Pol3[rangemedia][[2, 1]]]
```

Out[ ]=
```
{44.5724, 44.0738, 43.7598, 43.4789, 43.2142, 42.9594, 42.7111, 42.4674, 42.2269, 41.9888,
 41.7523, 41.5171, 41.2828, 41.0489, 40.8154, 40.5818, 40.3482, 40.1143, 39.88,
 39.6451, 39.4096, 39.1734, 38.9364, 38.6984, 38.4596, 38.2196, 37.9786, 37.7364,
 37.493, 37.2484, 37.0024, 36.755, 36.5062, 36.256, 36.0042, 35.7508, 35.4959, 35.2392,
 34.9809, 34.7207, 34.4588, 34.195, 33.9293, 33.6616, 33.3919, 33.1202, 32.8463,
 32.5702, 32.2918, 32.0112, 31.7282, 31.4427, 31.1548, 30.8642, 30.5711, 30.2751,
 29.9764, 29.6748, 29.3701, 29.0624, 28.7515, 28.4372, 28.1196, 27.7984, 27.4736,
 27.1449, 26.8123, 26.4756, 26.1345, 25.7891, 25.4389, 25.0839, 24.7238, 24.3583,
 23.9872, 23.6101, 23.2268, 22.8368, 22.4399, 22.0354, 21.623, 21.202, 20.7718,
 20.3317, 19.8809, 19.4184, 18.9431, 18.4536, 17.9485, 17.4257, 16.883, 16.3173,
 15.725, 15.1012, 14.4389, 13.7284, 12.955, 12.0937, 11.0963, 9.83976, 7.29908}
```

Funzioni che definiscono gli errori rispettivamente rispetto al valore medio y e ai valori in x al variare dei parametri

In[ ]:=
```
ϵY[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Abs[Pol3[rangemedia][[2, 2]] - Fdy];
```

In[ ]:=
```
ϵX[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Abs[Pol3[rangemedia][[2, 1]] - Fdx];
```

Funzione Penalità con peso 0.01 sulle distanze in x

In[ ]:=
```
penalty[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Module[{}, Total[ϵY[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD]^2] / Length@rangemedia] +
   0.01 Module[{},
     Total[ϵX[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD]^2] / Length@rangemedia];
```

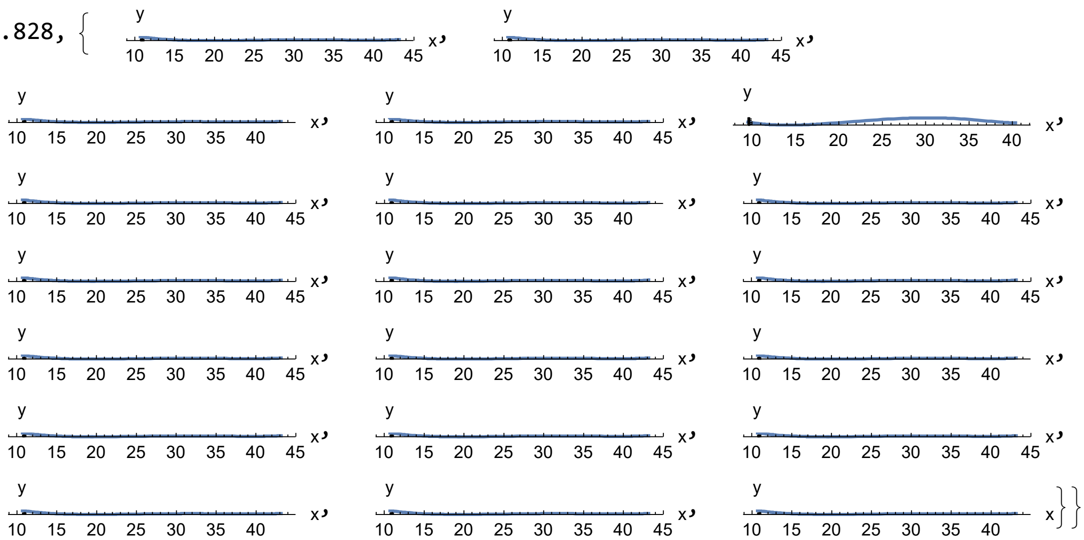
Soluzione con Vincoli

In[ ]:=
```
sol = Timing@Table[NMinimize[{penalty[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD],
    5 < Lv1, 5 < Lv2, 10 < Lv3, 2 < Lv4, 20 < Lv5, 2 < Lv6, 15 < LvP, -15 < xvD, 5 < yvD
   }, {Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD} ∈ Reals, AccuracyGoal → 5,
   PrecisionGoal → 5, Method → {"NelderMead", "RandomSeed" → i}], {i, 1, 20}]
```

Out[ ]=
```
{2636.19,
 {{0.00914697, {Lv1 → 12.5062, Lv2 → 10.1896, Lv3 → 23.2255, Lv4 → 4.05343, Lv5 → 21.8689,
    Lv6 → 9.12117, LvP → 17.8031, xvD → -2.34685, yvD → 12.0746}},
  {0.00915246, {Lv1 → 11.3307, Lv2 → 10.1958, Lv3 → 24.3988, Lv4 → 2.86035,
    Lv5 → 22.5513, Lv6 → 11.6428, LvP → 17.8022, xvD → -2.60487, yvD → 7.84073}},
  {0.00913699, {Lv1 → 14.9634, Lv2 → 8.09417, Lv3 → 20.7625, Lv4 → 3.0468,
    Lv5 → 22.1696, Lv6 → 10.9904, LvP → 17.8, xvD → -2.93015, yvD → 8.87083}},
  {0.00915145, {Lv1 → 11.5408, Lv2 → 10.2507, Lv3 → 24.1915, Lv4 → 3.64707,
    Lv5 → 22.7654, Lv6 → 9.27676, LvP → 17.8034, xvD → -2.57977, yvD → 10.7285}},
  {0.0978522, {Lv1 → 11.7422, Lv2 → 7.05716, Lv3 → 21.7398, Lv4 → 4.57348,
    Lv5 → 26.8536, Lv6 → 11.1151, LvP → 16.3567, xvD → -8.63022, yvD → 5.11236}},
  {0.00914815, {Lv1 → 13.0039, Lv2 → 9.17266, Lv3 → 22.7261, Lv4 → 3.86796,
    Lv5 → 23.3965, Lv6 → 8.63016, LvP → 17.8024, xvD → -3.05266, yvD → 11.0772}},
  {0.00917608, {Lv1 → 13.4106, Lv2 → 9.60282, Lv3 → 22.3176, Lv4 → 2.92422,
    Lv5 → 27.8802, Lv6 → 7.95263, LvP → 17.8012, xvD → -5.2116, yvD → 7.35891}},
  {0.00916702, {Lv1 → 9.5186, Lv2 → 10.7249, Lv3 → 26.2113, Lv4 → 3.14696,
    Lv5 → 24.5067, Lv6 → 9.39213, LvP → 17.8019, xvD → -2.93353, yvD → 8.71532}},
  {0.00913076, {Lv1 → 14.8851, Lv2 → 8.76538, Lv3 → 20.8454, Lv4 → 3.49704,
    Lv5 → 20.3847, Lv6 → 11.3707, LvP → 17.8027, xvD → -2.26392, yvD → 10.4849}},
  {0.00911892, {Lv1 → 11.6565, Lv2 → 16.2313, Lv3 → 24.0805, Lv4 → 9.32013,
    Lv5 → 20., Lv6 → 17.454, LvP → 17.8075, xvD → -13.5061, yvD → 25.9757}},
  {0.0091014, {Lv1 → 10.8989, Lv2 → 16.8344, Lv3 → 24.8401, Lv4 → 9.14382,
    Lv5 → 20., Lv6 → 18.7984, LvP → 17.8084, xvD → -14.9998, yvD → 25.9702}},
  {0.00916411, {Lv1 → 9.15681, Lv2 → 11.0935, Lv3 → 26.5732, Lv4 → 2.91567,
    Lv5 → 24.3361, Lv6 → 10.6167, LvP → 17.802, xvD → -2.88829, yvD → 7.54432}},
  {0.00915125, {Lv1 → 11.5757, Lv2 → 10.3846, Lv3 → 24.1554, Lv4 → 4.02701,
    Lv5 → 22.8561, Lv6 → 8.39966, LvP → 17.8027, xvD → -2.54034, yvD → 11.9072}},
  {0.00915932, {Lv1 → 8.05412, Lv2 → 10.8537, Lv3 → 27.6766, Lv4 → 2.23833,
    Lv5 → 27.721, Lv6 → 10.1572, LvP → 17.8018, xvD → -3.46948, yvD → 5.}},
  {0.00915313, {Lv1 → 11.6052, Lv2 → 10.4164, Lv3 → 24.1271, Lv4 → 3.41788,
    Lv5 → 22.1363, Lv6 → 10.4464, LvP → 17.8034, xvD → -2.47647, yvD → 9.90516}},
  {0.00916035, {Lv1 → 10.6669, Lv2 → 10.9478, Lv3 → 25.0635, Lv4 → 3.68052,
    Lv5 → 22.7204, Lv6 → 9.36036, LvP → 17.8027, xvD → -2.47033, yvD → 10.7331}},
  {0.00915807, {Lv1 → 10.3709, Lv2 → 10.5496, Lv3 → 25.3597, Lv4 → 3.07073,
    Lv5 → 23.52, Lv6 → 10.374, LvP → 17.8024, xvD → -2.76752, yvD → 8.50309}},
  {0.00914987, {Lv1 → 9.03594, Lv2 → 10.4694, Lv3 → 26.6917, Lv4 → 2.16053,
    Lv5 → 26.532, Lv6 → 10.8878, LvP → 17.8013, xvD → -3.2075, yvD → 5.00133}},
  {0.0091288, {Lv1 → 15.3241, Lv2 → 8.42666, Lv3 → 20.4075, Lv4 → 3.48364,
    Lv5 → 20.4915, Lv6 → 11.3716, LvP → 17.8039, xvD → -2.37588, yvD → 10.3825}},
  {0.00917801, {Lv1 → 15.7413, Lv2 → 11.8156, Lv3 → 19.9819, Lv4 → 2.,
    Lv5 → 29.5669, Lv6 → 8.2748, LvP → 17.7987, xvD → -5.92998, yvD → 5.00018}}}}
```

In[ ]:=
```
Timing@Table[ParametricPlot[
   {{Pol3[q][[2, 1]] /. Last@sol[[2, i]], Pol3[q][[2, 2]] /. Last@sol[[2, i]]}},
   {q, qmin, qmax}, AxesLabel → {"x", "y"}, PlotLegends → Automatic], {i, 1, 20}]
```

Out[ ]= {192.828, {



}}

Dal Table prendo la soluzione che più si avvicina ad una retta parallela all’asse x

In[ ]:=
```
paramsol = 11;
```

```
Manipulate[
     Show[
          ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]} /. Last@sol[[2, paramsol]],
      {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}} /. Last@sol[[2, paramsol]], {q, qmin, qmax},
   PlotStyle → Directive[Black, Dashed], PlotRange → {{-30, 50}, {0, 50}}],
          ListPlot[{Pol1[q][[3]] /. Last@sol[[2, paramsol]],
    Pol2[q][[3]] /. Last@sol[[2, paramsol]],
    Pol3[q] /. Last@sol[[2, paramsol]], Pol4[q] /. Last@sol[[2, paramsol]]},
   Joined → True, PlotMarkers → {Style["○", Red], Medium},
   PlotRange → {{-30, 50}, {0, 50}}]], {q, qmin, qmax}]
```

Sintesi di Traiettoria senza Correlazione

```
data = Array[0, Length[Fdx]];
```

In un vettore inserisco rispettivamente i dati delle coordinate x e i rispettivi valori del movente.

```
For[i = 1, i <= Length[Fdx], i++, data[[i]] = {Fdx[[i]], rangemedia[[i]]}];
data;
```

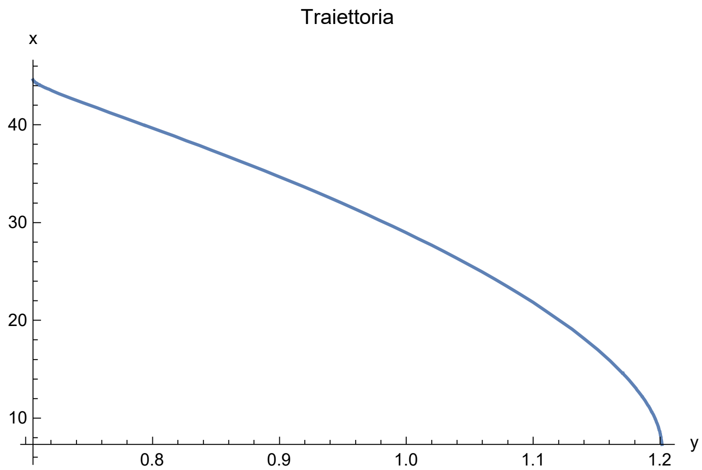
Ora svolgo un fitting dei dati per ottenere il moto della gru in funzione della posizione x.
Applico sia un modello lineare che polinomiale.

In[ ]:=
```
lm = LinearModelFit[data, x, x]
poli = Fit[data, {1, x, x^2, x^3}, x]
```

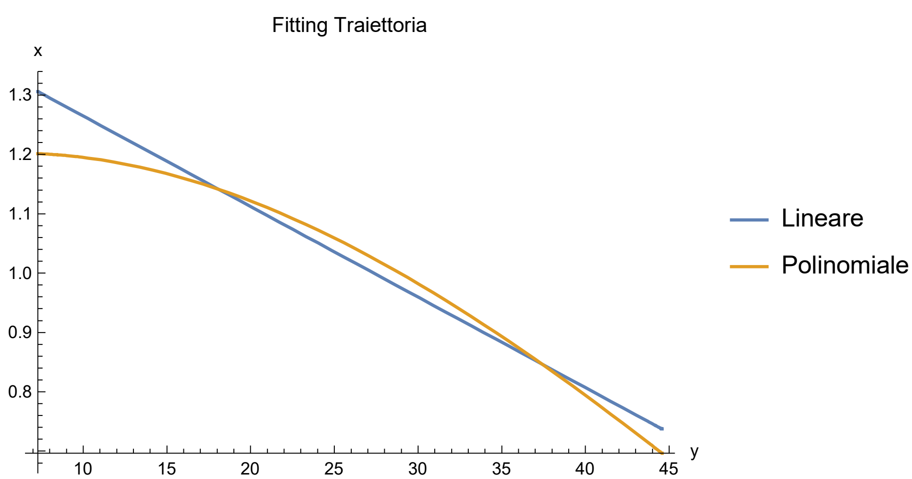
Out[ ]= FittedModel[$1.41737 - 0.0152597\,x$]

Out[ ]= $1.18586 + 0.00560664\,x - 0.000497271\,x^2 + 2.80518\,10^{-6}\,x^3$

In[ ]:=
```
Block[{Lv1 = L11, Lv2 = L22, Lv3 = L33,
   Lv4 = L44, Lv5 = L55, Lv6 = L66, LvP = L1P , xvD = xDD, yvD = yDD},
  Plot[Pol3[q][[2, 1]], {q, qmin, qmax}, PlotLabel → "Traiettoria", AxesLabel → {"y", "x"}]]
```

Out[ ]=


In[ ]:=
```
Plot[{lm[x], poli}, {x, Last@Fdx, First@Fdx}, PlotLabel → "Fitting Traiettoria",
  AxesLabel → {"y", "x"}, PlotLegends → {"Lineare", "Polinomiale"}]
```

Out[ ]=


In[ ]:=
```
range = Range[Last@Fdx, First@Fdx, (First@Fdx - Last@Fdx) / 100]
```

Out[ ]=
```
{7.29908, 7.67181, 8.04455, 8.41728, 8.79001, 9.16275, 9.53548, 9.90821, 10.2809, 10.6537,
 11.0264, 11.3991, 11.7719, 12.1446, 12.5173, 12.8901, 13.2628, 13.6355, 14.0083,
 14.381, 14.7537, 15.1265, 15.4992, 15.8719, 16.2447, 16.6174, 16.9901, 17.3629,
 17.7356, 18.1083, 18.4811, 18.8538, 19.2266, 19.5993, 19.972, 20.3448, 20.7175,
 21.0902, 21.463, 21.8357, 22.2084, 22.5812, 22.9539, 23.3266, 23.6994, 24.0721,
 24.4448, 24.8176, 25.1903, 25.563, 25.9358, 26.3085, 26.6812, 27.054, 27.4267, 27.7994,
 28.1722, 28.5449, 28.9176, 29.2904, 29.6631, 30.0358, 30.4086, 30.7813, 31.154,
 31.5268, 31.8995, 32.2722, 32.645, 33.0177, 33.3904, 33.7632, 34.1359, 34.5086,
 34.8814, 35.2541, 35.6268, 35.9996, 36.3723, 36.745, 37.1178, 37.4905, 37.8632,
 38.236, 38.6087, 38.9814, 39.3542, 39.7269, 40.0996, 40.4724, 40.8451, 41.2178,
 41.5906, 41.9633, 42.336, 42.7088, 43.0815, 43.4542, 43.827, 44.1997, 44.5724}
```

In[ ]:=
```
fFxl[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Pol3[Table[lm[x], {x, range}]][[2, 2]];
```

Applico dunque la funzione penalità sui valori ottenuti tramite la fuzione linearizzata.

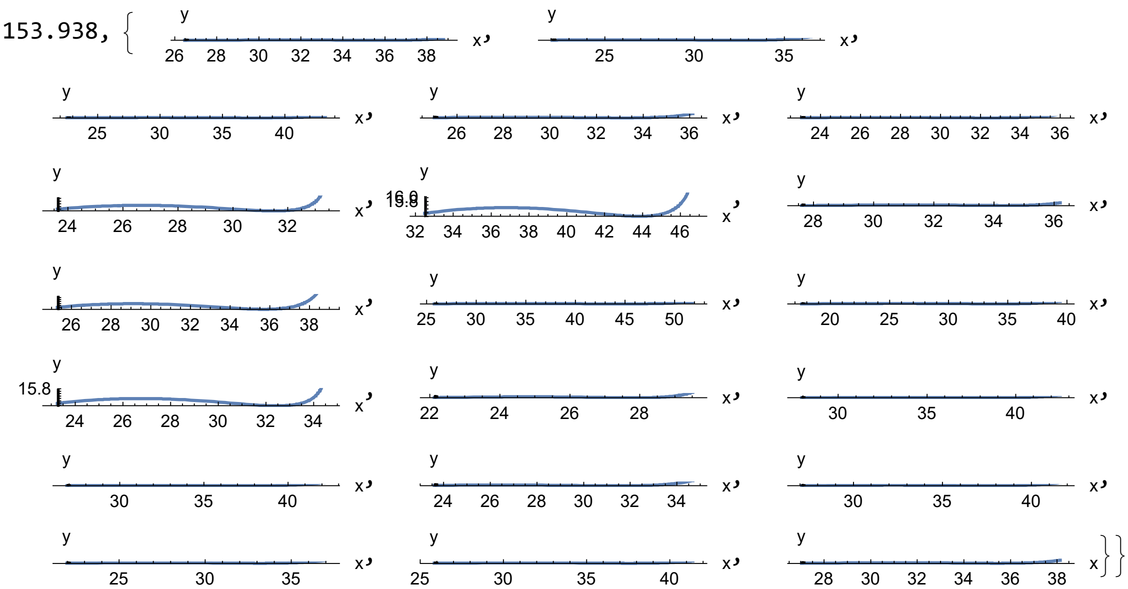
In[ ]:=
```
penaltyfFxl[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Total[Abs[fFxl[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD] - Fdy]];
```

In[ ]:=
```
sol2 =
 Timing@Table[NMinimize[{penaltyfFxl[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD], 5 < Lv1,
     5 < Lv2, 10 < Lv3, 2 < Lv4, 20 < Lv5, 2 < Lv6, 15 < LvP, -15 < xvD, 5 < yvD
    }, {Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD} ∈ Reals, AccuracyGoal → 5,
    PrecisionGoal → 5, Method → {"NelderMead", "RandomSeed" → i}], {i, 1, 20}]
```

```
Out[•]= {794.094,
  {{0.859755, {Lv1 → 9.8948, Lv2 → 12.9321, Lv3 → 15.7662, Lv4 → 3.99714, Lv5 → 22.7924,
      Lv6 → 5.32685, LvP → 19.2951, xvD → -8.25417, yvD → 6.06746}},
   {1.3066, {Lv1 → 10.0102, Lv2 → 11.6284, Lv3 → 17.2254, Lv4 → 4.03197,
      Lv5 → 20.8599, Lv6 → 6.28298, LvP → 15.9084, xvD → -6.43881, yvD → 8.50478}},
   {1.13125, {Lv1 → 12.8586, Lv2 → 12.5648, Lv3 → 20.4068, Lv4 → 3.79778,
      Lv5 → 25.7026, Lv6 → 7.74371, LvP → 18.9715, xvD → -6.49692, yvD → 9.34966}},
   {1.21296, {Lv1 → 8.49046, Lv2 → 15.1646, Lv3 → 15.5297, Lv4 → 6.15158,
      Lv5 → 20.6948, Lv6 → 5.01927, LvP → 17.8522, xvD → -10.2055, yvD → 9.0544}},
   {1.1223, {Lv1 → 9.01267, Lv2 → 9.09779, Lv3 → 16.8799, Lv4 → 3.03384,
      Lv5 → 20.3005, Lv6 → 5.61935, LvP → 16.3476, xvD → -5.00492, yvD → 5.88213}},
   {9.3736, {Lv1 → 10.3602, Lv2 → 15.2316, Lv3 → 11.9244, Lv4 → 7.91486,
      Lv5 → 20.1435, Lv6 → 6.58279, LvP → 16.5338, xvD → -13.9331, yvD → 6.48724}},
   {20.6327, {Lv1 → 14.2037, Lv2 → 8.97541, Lv3 → 14.6811, Lv4 → 10.9617,
      Lv5 → 20.0776, Lv6 → 16.0753, LvP → 25.0019, xvD → -14.1953, yvD → 6.28857}},
   {0.895241, {Lv1 → 6.59086, Lv2 → 13.9176, Lv3 → 14.102, Lv4 → 6.10068,
      Lv5 → 20.1772, Lv6 → 3.1258, LvP → 20.5382, xvD → -10.5929, yvD → 5.89547}},
   {13.4507, {Lv1 → 8.94949, Lv2 → 9.21393, Lv3 → 17.8092, Lv4 → 8.2378,
      Lv5 → 20., Lv6 → 6.32415, LvP → 18.4266, xvD → -8.09067, yvD → 10.1233}},
   {1.43652, {Lv1 → 17.2436, Lv2 → 13.692, Lv3 → 21.8515, Lv4 → 5.46047,
      Lv5 → 24.2357, Lv6 → 11.6534, LvP → 24.2258, xvD → -5.2474, yvD → 14.0173}},
   {1.74038, {Lv1 → 16.6461, Lv2 → 17.2469, Lv3 → 15.4915, Lv4 → 7.15214,
      Lv5 → 20.2988, Lv6 → 11.2351, LvP → 15.9123, xvD → -10.4119, yvD → 18.0209}},
   {14.3016, {Lv1 → 7.36651, Lv2 → 9.60465, Lv3 → 16.9163, Lv4 → 8.07755,
      Lv5 → 20., Lv6 → 6.56152, LvP → 16.2903, xvD → -10.3927, yvD → 7.71738}},
   {1.18543, {Lv1 → 5.31182, Lv2 → 17.5212, Lv3 → 13.5284, Lv4 → 7.32873,
      Lv5 → 21.0538, Lv6 → 2.40964, LvP → 15.5012, xvD → -14.3041, yvD → 6.53587}},
   {1.21362, {Lv1 → 5.37353, Lv2 → 7.22937, Lv3 → 23.2136, Lv4 → 5.01604,
      Lv5 → 20.4055, Lv6 → 7.44795, LvP → 20.9967, xvD → -6.35974, yvD → 9.53042}},
   {1.37219, {Lv1 → 5.00471, Lv2 → 8.94903, Lv3 → 24.2366, Lv4 → 5.51656,
      Lv5 → 20.3237, Lv6 → 8.78941, LvP → 20.273, xvD → -7.9514, yvD → 11.5319}},
   {1.37563, {Lv1 → 9.29503, Lv2 → 14.1116, Lv3 → 14.5383, Lv4 → 5.59827,
      Lv5 → 20., Lv6 → 5.0442, LvP → 16.5439, xvD → -9.55575, yvD → 8.75717}},
   {1.42391, {Lv1 → 6.23037, Lv2 → 5.93719, Lv3 → 22.1306, Lv4 → 2.79785,
      Lv5 → 23.9012, Lv6 → 3.35567, LvP → 20.2491, xvD → -3.27081, yvD → 5.}},
   {1.39959, {Lv1 → 8.02545, Lv2 → 8.1045, Lv3 → 19.7813, Lv4 → 3.24691,
      Lv5 → 20.8692, Lv6 → 5.04133, LvP → 16.0359, xvD → -3.76346, yvD → 7.33935}},
   {1.28667, {Lv1 → 7.57357, Lv2 → 7.94572, Lv3 → 21.6881, Lv4 → 6.8115,
      Lv5 → 20., Lv6 → 15.6246, LvP → 19.4614, xvD → -14.3854, yvD → 13.1145}},
   {1.1504, {Lv1 → 6.71456, Lv2 → 11.9324, Lv3 → 17.4817, Lv4 → 5.58058,
      Lv5 → 20.5008, Lv6 → 3.76408, LvP → 19.7181, xvD → -7.91293, yvD → 7.82226}}}}
```

In[ ]:=
```
Timing@Table[ParametricPlot[
  {{Pol3[q][[2, 1]] /. Last@sol2[[2, i]], Pol3[q][[2, 2]] /. Last@sol2[[2, i]]}},
  {q, qmin, qmax}, AxesLabel → {"x", "y"}, PlotLegends → Automatic], {i, 1, 20}]
```

Out[ ]= {153.938, {



}}

In[ ]:=
```
paramsol2 = 10;
```

In[ ]:=
```
Manipulate[
  Show[
    ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]} /. Last@sol2[[2, paramsol2]],
     {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}} /. Last@sol2[[2, paramsol2]], {q, qmin, qmax},
   PlotStyle → Directive[Black, Dashed], PlotRange → {{-30, 70}, {0, 50}}],
    ListPlot[{Pol1[q][[3]] /. Last@sol2[[2, paramsol2]],
    Pol2[q][[3]] /. Last@sol2[[2, paramsol2]],
    Pol3[q] /. Last@sol2[[2, paramsol2]], Pol4[q] /. Last@sol2[[2, paramsol2]]},
   Joined → True, PlotMarkers → {Style["○", Red], Medium},
   PlotRange → {{-30, 70}, {0, 50}}]], {q, qmin, qmax}]
```

Out[ ]=

In[ ]:=
```
fFxpoli[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Pol3[Table[poli, {x, range}]][[2, 2]];
```

Applico dunque la funzione penalità sui valori ottenuti tramite la fuzione fittata nella polinomiale.

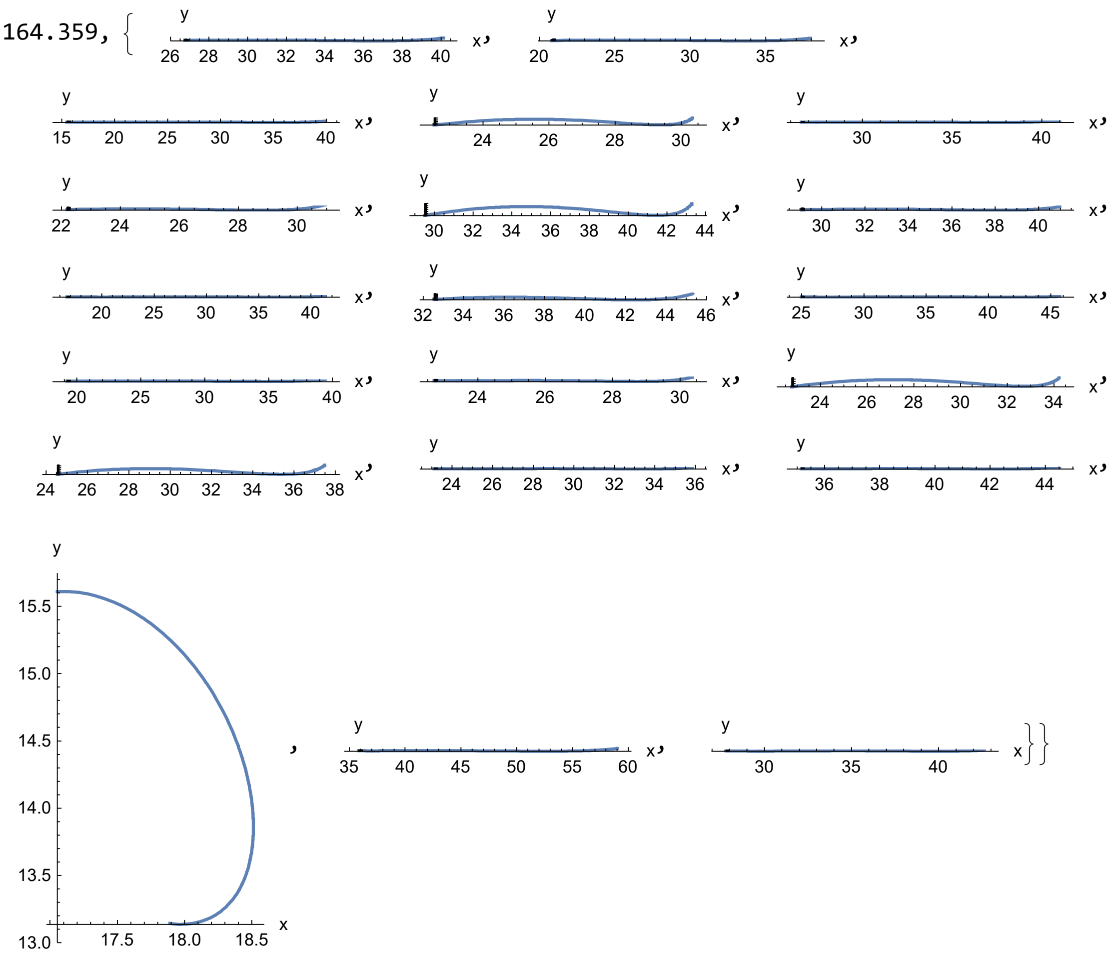
In[ ]:=
```
penaltyfFxpoli[Lv1_, Lv2_, Lv3_, Lv4_, Lv5_, Lv6_, LvP_, xvD_, yvD_] :=
  Total[Abs[fFxpoli[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD] - Fdy]];
```

In[ ]:=
```
sol3 = Timing@
  Table[NMinimize[{penaltyfFxpoli[Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD], 5 < Lv1,
     5 < Lv2, 10 < Lv3, 2 < Lv4, 20 < Lv5, 2 < Lv6, 15 < LvP, -15 < xvD, 5 < yvD
    }, {Lv1, Lv2, Lv3, Lv4, Lv5, Lv6, LvP, xvD, yvD} ∈ Reals, AccuracyGoal → 5,
    PrecisionGoal → 5, Method → {"NelderMead", "RandomSeed" → i}], {i, 1, 20}]
```

```
Out[•]= {747.641,
  {{1.49504, {Lv1 → 11.1615, Lv2 → 12.6937, Lv3 → 15.8221, Lv4 → 4.61788, Lv5 → 20.9747,
     Lv6 → 6.88273, LvP → 19.689, xvD → -7.6151, yvD → 7.96706}},
   {1.95998, {Lv1 → 11.3294, Lv2 → 10.825, Lv3 → 18.1617, Lv4 → 3.23851,
     Lv5 → 21.4899, Lv6 → 7.85829, LvP → 15.9542, xvD → -5.21524, yvD → 7.94078}},
   {1.42401, {Lv1 → 11.397, Lv2 → 9.70514, Lv3 → 21.6426, Lv4 → 3.27269,
     Lv5 → 22.9555, Lv6 → 7.82606, LvP → 15.9652, xvD → -3.72072, yvD → 9.51982}},
   {5.75871, {Lv1 → 5.44988, Lv2 → 16.2718, Lv3 → 13.9816, Lv4 → 8.82743,
     Lv5 → 20.1616, Lv6 → 3.00477, LvP → 15.7909, xvD → -14.7218, yvD → 6.61221}},
   {1.06277, {Lv1 → 8.73016, Lv2 → 9.8259, Lv3 → 19.4457, Lv4 → 2.77139,
     Lv5 → 23.2857, Lv6 → 6.22275, LvP → 19.776, xvD → -4.99181, yvD → 5.}},
   {1.92655, {Lv1 → 9.40278, Lv2 → 15.7214, Lv3 → 11.3819, Lv4 → 5.31334,
     Lv5 → 20.1681, Lv6 → 4.52743, LvP → 15.2555, xvD → -11.7429, yvD → 6.38542}},
   {15.3241, {Lv1 → 12.8218, Lv2 → 9.73621, Lv3 → 14.9298, Lv4 → 10.0185,
     Lv5 → 20., Lv6 → 14.9179, LvP → 22.3064, xvD → -14.6663, yvD → 6.17092}},
   {1.82511, {Lv1 → 12.2946, Lv2 → 17.0178, Lv3 → 13.1769, Lv4 → 7.23248,
     Lv5 → 20., Lv6 → 7.39618, LvP → 21.617, xvD → -11.5654, yvD → 10.4401}},
   {1.19525, {Lv1 → 11.4965, Lv2 → 11.6349, Lv3 → 22.3903, Lv4 → 3.74852,
     Lv5 → 25.8559, Lv6 → 7.17318, LvP → 16.9426, xvD → -5.65571, yvD → 10.1342}},
   {4.66265, {Lv1 → 5.00813, Lv2 → 10.2046, Lv3 → 21.9384, Lv4 → 8.36332,
     Lv5 → 20., Lv6 → 4.28502, LvP → 24.8849, xvD → -7.10736, yvD → 11.6673}},
   {0.979521, {Lv1 → 10.6265, Lv2 → 9.72515, Lv3 → 23.55, Lv4 → 6.94572,
     Lv5 → 20.4941, Lv6 → 19.3811, LvP → 20.8794, xvD → -14.8751, yvD → 15.7599}},
   {1.60863, {Lv1 → 8.82905, Lv2 → 9.15781, Lv3 → 22.7642, Lv4 → 3.27963,
     Lv5 → 22.4564, Lv6 → 6.10565, LvP → 16.2225, xvD → -2.92386, yvD → 9.18839}},
   {1.15082, {Lv1 → 5.30831, Lv2 → 17.5592, Lv3 → 13.9525, Lv4 → 7.1942,
     Lv5 → 21.7507, Lv6 → 2.32282, LvP → 15.9718, xvD → -14.2969, yvD → 6.49911}},
   {9.783, {Lv1 → 5.85756, Lv2 → 9.62701, Lv3 → 18.2436, Lv4 → 7.51687, Lv5 → 20.243,
     Lv6 → 5.58677, LvP → 15.8901, xvD → -9.99613, yvD → 8.03107}},
   {9.45689, {Lv1 → 7.37034, Lv2 → 9.07469, Lv3 → 18.6862, Lv4 → 7.86727,
     Lv5 → 20., Lv6 → 6.26777, LvP → 17.7136, xvD → -8.63481, yvD → 9.74345}},
   {0.783353, {Lv1 → 8.74429, Lv2 → 20.7753, Lv3 → 16.9542, Lv4 → 4.62799,
     Lv5 → 29.9042, Lv6 → 5.51501, LvP → 16.3537, xvD → -14.3258, yvD → 6.41946}},
   {0.686374, {Lv1 → 9.23285, Lv2 → 18.0751, Lv3 → 12.9077, Lv4 → 8.13227,
     Lv5 → 20.8719, Lv6 → 5.09754, LvP → 27.6899, xvD → -13.4333, yvD → 7.74228}},
   {239.388, {Lv1 → 23.2019, Lv2 → 10.8571, Lv3 → 11.1481, Lv4 → 16.1634,
     Lv5 → 38.1503, Lv6 → 7.74893, LvP → 15., xvD → 11.9875, yvD → 5.18908}},
   {2.26948, {Lv1 → 15.3443, Lv2 → 13.3728, Lv3 → 24.3191, Lv4 → 7.51636,
     Lv5 → 23.5303, Lv6 → 9.36337, LvP → 30.5935, xvD → -3.89769, yvD → 16.3747}},
   {0.967912, {Lv1 → 7.60953, Lv2 → 16.8697, Lv3 → 21.279, Lv4 → 6.78604,
     Lv5 → 26.7291, Lv6 → 4.07993, LvP → 20.9285, xvD → -11.1193, yvD → 10.9839}}}}
```

In[ ]:=
```
Timing@Table[ParametricPlot[
    {{Pol3[q][[2, 1]] /. Last@sol3[[2, i]], Pol3[q][[2, 2]] /. Last@sol3[[2, i]]}},
    {q, qmin, qmax}, AxesLabel → {"x", "y"}, PlotLegends → Automatic], {i, 1, 20}]
```

Out[ ]= {164.359, {



In[ ]:=
```
paramsol3 = 9;
```

In[ ]:=
```
Manipulate[
  Show[
    ParametricPlot[{{Pol3[q][[2, 1]], Pol3[q][[2, 2]]} /. Last@sol3[[2, paramsol3]],
    {Pol4[q][[2, 1]], Pol4[q][[2, 2]]}} /. Last@sol3[[2, paramsol3]], {q, qmin, qmax},
   PlotStyle → Directive[Black, Dashed], PlotRange → {{-30, 70}, {0, 50}}],
    ListPlot[{Pol1[q][[3]] /. Last@sol3[[2, paramsol3]],
    Pol2[q][[3]] /. Last@sol3[[2, paramsol3]],
    Pol3[q] /. Last@sol3[[2, paramsol3]], Pol4[q] /. Last@sol3[[2, paramsol3]]},
   Joined → True, PlotMarkers → {Style["○", Red], Medium},
   PlotRange → {{-30, 70}, {0, 50}}]], {q, qmin, qmax}]
```

Out[ ]=

Di seguito effettuo il plot dei migliori risutati ottenuti con ogni metodo applicato

In[ ]:=
```
ParametricPlot[{{Pol3[q][[2, 1]] /. Last@sol[[2, paramsol]],
    Pol3[q][[2, 2]] /. Last@sol[[2, paramsol]]},
   {Pol3[q][[2, 1]] /. Last@sol2[[2, paramsol2]],
    Pol3[q][[2, 2]] /. Last@sol2[[2, paramsol2]]}, {Pol3[q][[2, 1]] /.
     Last@sol3[[2, paramsol3]], Pol3[q][[2, 2]] /. Last@sol3[[2, paramsol3]]}
   , {Block[{Lv1 = L11, Lv2 = L22, Lv3 = L33, Lv4 = L44, Lv5 = L55,
      Lv6 = L66, LvP = L1P , xvD = xDD, yvD = yDD}, Pol3[q][[2, 1]]],
    Block[{Lv1 = L11, Lv2 = L22, Lv3 = L33, Lv4 = L44, Lv5 = L55, Lv6 = L66,
      LvP = L1P , xvD = xDD, yvD = yDD}, Pol3[q][[2, 2]]]}},
  {q, qmin, qmax}, AxesLabel → {"x", "y"}, PlotRange → {{0, 60}, {10, 20}},
  PlotLabel → "Traiettorie",
  PlotLegends → {"Ottimizzato con Correlazione",
    "Ottimizzato lineare", "Ottimizzato polinomiale", "Originale"}]
```

Out[ ]=