MAIN TOPICS:

1. Logic
2. Sets&Operations on sets
3. Relations&Their Properties
4. Functions
5. Sequences&Series
6. Recurrence Relations
7. Mathematical Induction
8. Loop Invariants
9. Combinatorics
10. Probability
11. Graphs and Trees

Lecture No.1

Course Objective:

1.Express statements with the precision of formal logic
2.Analyze arguments to test their validity
3.Apply the basic properties and operations related to sets
4.Apply to sets the basic properties and operations related to relations and function
5.Define terms recursively
6.Prove a formula using mathematical induction
7.Prove statements using direct and indirect methods
8.Compute probability of simple and conditional events
9.Identify and use the formulas of combinatorics in different problems
10.Illustrate the basic definitions of graph theory and properties of graphs
11.Relate each major topic in Discrete Mathematics to an application area in computing

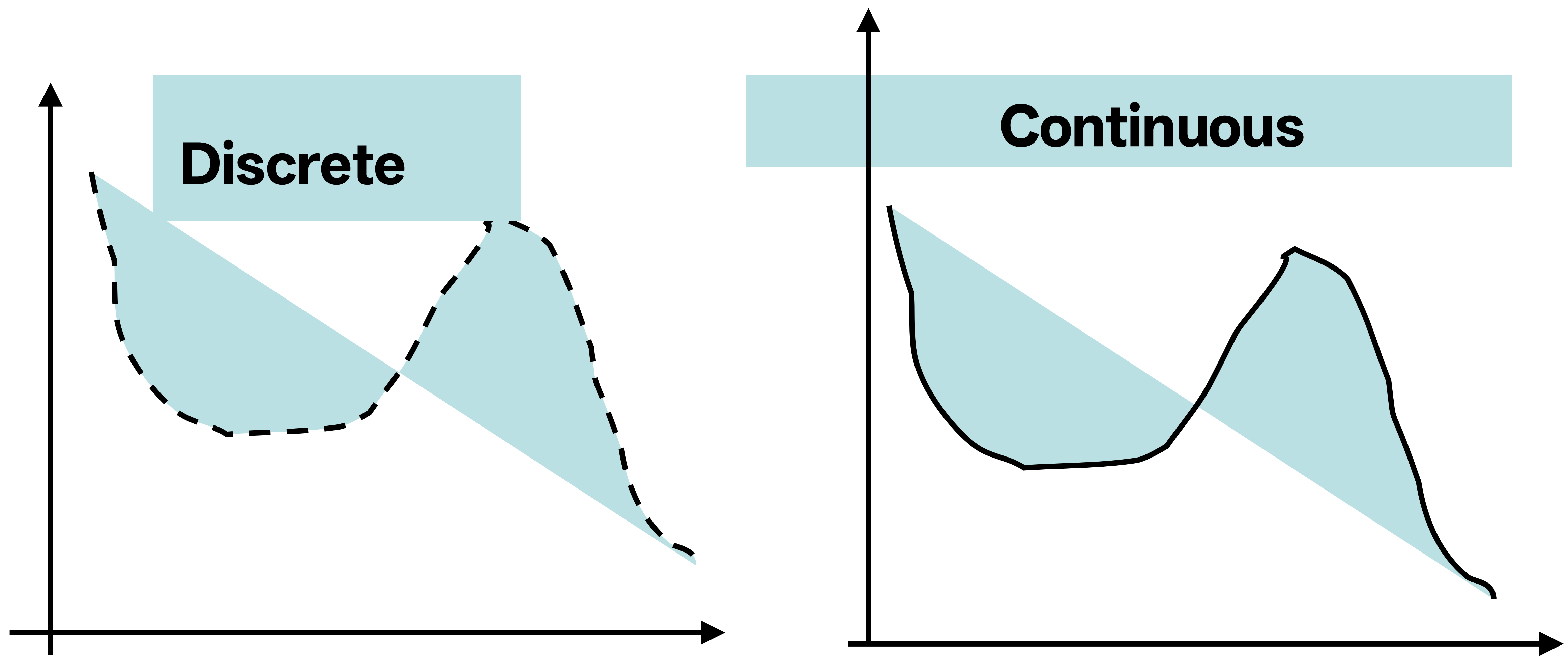
1.Recommended Books:

1.Discrete Mathematics and Its Applications (fourth edition) by Kenneth H. Rosen
1.Discrete Mathematics by Ross and Wright

What is quantitative reasoning?

> **Quantitative reasoning** is the branch of Mathematics devoted to study of discrete objects.
> **Quantitative reasoning** concerns processes that consist of a sequence of individual steps

>Integers (aka whole numbers), rational numbers (ones that can be expressed as the quotient of two integers), automobiles, houses, people etc. are all discrete objects.
>On the other hand real numbers which include irrational as well as rational numbers are not discrete.
>Discrete Means Distinct, Separate
> **Quantitative reasoning** is the Study of Distinct Objects
> **Quantitative reasoning** represent the Discrete Objects



Why Study Discrete Structures?

> To Develop Mathematical Maturity
>To develop an ability to create mathematical arguments
>To understand the techniques of Computing and Counting
>Analyze the number of ways to solve a problem

What are Components of Discrete Structures?

> Sets, which are collection of objects

> Combinations, which are unordered collection of objects used extensively in counting
> Relations, which are sets of ordered pairs, which represent relationship between objects
> Graphs, which are the sets of vertices and the edges connecting the vertices
> Functions, which is special kind of relation with assigns precisely one element of a set to one element of the other set

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What are Applications of quantitative reasoning?

- Formal Languages (computer languages)
- Compiler Design
- Data Structures
- Computability
- Automata Theory
- Algorithm Design
- Relational Database Theory
- Complexity Theory (counting)

Set of Integers:

• • • • • •

3 -2 -1 0 1 2

Set of Real Numbers:

• • • • • • •

-3 -2 -1 0 1 2

Lecture .2

LOGIC:

Logic is the study of the principles and methods that distinguishes between a valid and an invalid argument.

Logic means understanding, reasoning

In Discrete Structures logic is the understanding of mathematical reasoning

Logical Rules are important in circuit design, Program Verification and Correctness

SIMPLE STATEMENT:

A statement is a declarative sentence that is either true or false but not both.

A statement is also referred to as a proposition
Example: 2+2 = 4, It is Sunday today
If a proposition is true, we say that it has a **truth value** of "**true**" .
If a proposition is false, its truth value is "**false**".
The truth values **"true"** and **"false"** are, respectively, denoted by the letters **T** and **F**.

<u>EXAMPLES:</u>
1.Grass is green.
2.4 + 2 = 6
2.4 + 2 = 7
3.There are four fingers in a hand.
are propositions

Not Propositions
- Close the door.
- x is greater than 2.
- He is very rich

are not propositions.

<u>Rule:</u>
If the sentence is preceded by other sentences that make the pronoun or variable reference clear, then the sentence is a sta

<u>Example:</u>
$x = 1$
$x > 2$
$x > 2$ is a statement with truth-value FALSE.

Example
Bill Gates is an American
He is very rich
He is very rich is a statement with truth-value TRUE.

UNDERSTANDING STATEMENTS:

1. $x + 2$ is positive.	Not a statement
2. May I come in?	Not a statement
3. Logic is interesting.	A statement
4. It is hot today.	A statement
5. -1>0	A statement
6. $x + y = 12$	Not a statement

<u>COMPOUND STATEMENT:</u>
Simple statements could be used to build a compound statement.
<u>EXAMPLES:</u> **<u>LOGICAL CONNECTIVES</u>**

1. "3 + 2 = 5" **and** "Lahore is a city in Pakistan"
2. "The grass is green" or " It is hot today"
3. "Discrete Mathematics is **not** difficult to me"

AND, OR, NOT are called LOGICAL CONNECTIVES.

SYMBOLIC REPRESENTATION:

Statements are symbolically represented by letters such as ***p, q, r,...***

EXAMPLES:

p **=** "Islamabad is the capital of Pakistan"

q **=** "17 is divisible by 3"

CONNECTIV	MEANING	SYMBOL	CALLED
Negation	not	~	Tilde
Conjunction	and	$\wedge$	Hat
Disjunction	or	$\vee$	Vel
Conditional	if... then...	$\rightarrow$	Arrow
Biconditional	if and only if	$\leftrightarrow$	Double arrow

EXAMPLES:

p **=** "Islamabad is the capital of Pakistan"

q **=** "17 is divisible by 3"

$p \wedge q$ **=** "Islamabad is the capital of Pakistan and 17 is divisible by 3"

$p \vee q$ **=** "Islamabad is the capital of Pakistan or 17 is divisible by 3"

$\sim p$ **=** "It is not the case that Islamabad is the capital of Pakistan" or simply "Islamabad is not the capital of Pakistan"

TRANSLATING FROM ENGLISH TO SYMBOLS:

Let p = "It is hot" , and q = "It is sunny"

SENTENCE	**SYMBOLIC FORM**
1. It is **not** hot.	$\sim p$
2. It is hot **and** sunny.	$p \wedge q$
3. It is hot **or** sunny.	$p \vee q$
4. It is **not** hot **but** sunny.	$\sim p \wedge q$
5. It is **neither** hot **nor** sunny.	$\sim p \wedge \sim q$

EXAMPLE:

Let ***h*** = "Zia is healthy"

w = "Zia is wealthy"

s = "Zia is wise"

Translate the compound statements to symbolic form:

1. Zia is healthy and wealthy but not wise. $(h \wedge w) \wedge (\sim s)$
2. Zia is not wealthy but he is healthy and wise. $\sim w \wedge (h \wedge s)$
3. Zia is neither healthy, wealthy nor wise. $\sim h \wedge \sim w \wedge \sim s$

TRANSLATING FROM SYMBOLS TO ENGLISH:

Let m = "Ali is good in Mathematics"

c = "Ali is a Computer Science student"

Translate the following statement forms into plain English:

1. $\sim c$ Ali is not a Computer Science student
2. $c \vee m$ Ali is a Computer Science student or good in Maths.
3. $m \wedge \sim c$ Ali is good in Maths but not a Computer Science student

A convenient method for analyzing a compound statement is to make a truth table for it.

A **truth table** specifies the truth value of a compound proposition for all possible truth values of its constituent propositions.

NEGATION (~):

If ***p*** is a statement variable, then negation of ***p***, ***"not p"***, is denoted as **"~p"**

It has opposite truth value from p i.e.,

if p is true, ~p is false; if p is false, ~p is true.

TRUTH TABLE FOR ~ p

-	-

CONJUNCTION ($\wedge$):

If ***p*** and ***q*** are statements, then the conjunction of ***p*** and ***q*** is ***"p and q"***, denoted as

***"$p \wedge q$"*.**

It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

TRUTH TABLE FOR

$p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

DISJUNCTION ($\vee$)
or INCLUSIVE OR

If ***p*** & ***q*** are statements, then the disjunction of ***p*** and ***q*** is ***"p or q"***, denoted as ***"$p \vee q$"***. It is true when at least one of p or q is true and is false only when both p and q are false.

TRUTH TABLE FOR
$p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Note it that in the table F is only in that row where both p and q have F and all other values are T. Thus for finding out the truth values for the disjunction of two statements we will only first search out where the both statements are false and write down the F in the corresponding row in the column of $p \vee q$ and in all other rows we will write T in the column of $p \vee q$.

Remark:

Note that for Conjunction of two statements we find the T in both the statements, but in disjunction we find F in both the statements. In other words we will fill T first in the column of conjunction and F in the column of disjunction.

SUMMARY

1. What is a statement?

2. How a compound statement is formed.
3. Logical connectives (negation, conjunction, disjunction).
4. How to construct a truth table for a statement form.

Lecture 4, 5,&6

Truth Tables

<u>Truth Tables for:</u>

1. **$\sim p \wedge q$**
2. **$\sim p \wedge (q \vee \sim r)$**
3. **$(p \vee q) \wedge \sim (p \wedge q)$**

Truth table for the statement form $\sim$ **$p \wedge q$**

p	q	~p	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

Truth table for $\sim p \wedge (q \vee \sim r)$

p	q	r	~ r	$q \vee \sim r$	~ p	$\sim p \wedge (q \vee \sim r)$
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	T
F	T	F	T	T	T	T
F	F	T	F	F	T	F
F	F	F	T	T	T	T

Truth table for **$(p \vee q) \wedge \sim (p \wedge q)$**

p	q	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Double Negative Property $\sim(\sim p) \equiv p$

p	~p	~(~p)
T	F	T
F	T	F

Example

"It is not true that I am not happy"

Solution:

Let p = "I am happy"

then ~ **p** = "I am not happy"

and ~**(~ p)** = "It is not true that I am not happy"

Since ~ (~**p**) ≡ **p**

Hence the given statement is equivalent to:

"I am happy"

$\sim(p \wedge q)$ and $\sim p \wedge \sim q$ are not logically equivalent

p	q	~p	~q	p∧q	~(p∧q)	~p ∧ ~q
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	F	T	T

Different truth values in row 2 and row 3

DE MORGAN' S LAWS:

1) The negation of an **and** statement is logically equivalent to the **or** statement in which each component is negated. Symbolically $\sim(p \wedge q) \equiv \sim p \vee \sim q$.

2) The negation of an **or** statement is logically equivalent to the **and** statement in which each component is negated. Symbolically: $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	~p	~q	$p \vee q$	$\sim(p \vee q)$	$\sim p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Same truth values

Application:

Give negations for each of the following statements:

a. The fan is slow **or** it is very hot.

b. Akram is unfit **and** Saleem is injured.

Solution

a. The fan is **not** slow **and** it is **not** very hot.

b. Akram is **not** unfit **or** Saleem is **not** injured.

INEQUALITIES AND DEMORGAN’ S LAWS:

Use DeMorgan’ s Laws to write the negation of

$-1 < x \leqslant 4$

for some particular real no. x

$-1 < x \leqslant 4$ means $x > -1$ **and** $x \leqslant 4$

By DeMorgan’ s Law, the negation is:

$x > -1$ or $x \leqslant 4$Which is equivalent to: $x \leqslant -1$ **or** $x > 4$

EXERCISE:

1. $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
2. Are the statements $(p \wedge q) \vee r$ and $p \wedge (q \vee r)$ logically equivalent?

TAUTOLOGY:

A tautology is a statement form that is always true regardless of the truth values of the statement variables. A tautology is represented by the symbol “**T**” ..

EXAMPLE: The statement form $p \vee \sim p$ is tautology

p	~ p	$p \vee \sim p$
T	F	T
F	T	T

$p \vee \sim p \equiv t$

CONTRADICTION:

A contradiction is a statement form that is always false regardless of the truth values of the statement variables. A contradiction is represented by the symbol "c".

So if we have to prove that a given statement form is **CONTRADICTION** we will make the truth table for the statement form and if in the column of the given statement form all the entries are F ,then we say that statement form is contradiction.

EXAMPLE:

The statement form $p \wedge \sim p$ is a contradiction.

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Since in the last column in the truth table we have F in all the entries so is a contradiction

$p \wedge \sim p \equiv c$

REMARKS:

- Most statements are neither tautologies nor contradictions.
- The negation of a tautology is a contradiction and vice versa.
- In common usage we sometimes say that two statements are contradictory.

By this we mean that their conjunction is a contradiction: they cannot both be true.

LOGICAL EQUIVALENCE INVOLVING TAUTOLOGY

1. Show that $p \wedge t \equiv p$

p	t	$p \wedge t$
T	T	T
F	T	F

Since in the above table the entries in the first and last columns are identical so we have the corresponding statement forms are Logically Equivalent that is

$p \wedge t \equiv p$

LOGICAL EQUIVALENCE INVOLVING CONTRADICTION

Show that $p \wedge c \equiv c$

p	c	$p \wedge c$
T	F	F
F	F	F

Same truth values in the indicated columns so $p \wedge c \equiv c$

EXERCISE:

Use truth table to show that $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ is a tautology.

SOLUTION:

Since we have to show that the given statement form is Tautology so the column of the above proposition in the truth table will have all entries as T.

As clear from the table below

p	Q	$p \wedge q$	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee (p \wedge \sim q)$	$(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$
T	T	T	F	F	F	F	T
T	F	F	F	T	T	T	T
F	T	F	T	F	F	T	T
F	F	F	T	T	F	T	T

Hence $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q)) \equiv t$

EXERCISE:

Use truth table to show that $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

SOLUTION:

Since we have to show that the given statement form is Contradiction so its column in the truth table will have all entries as F.

As clear from the table below

p	q	$\sim q$	$p \wedge \sim q$	$\sim p$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	T	T	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	T	F

USAGE OF "OR" IN ENGLISH

In English language the word **or** is sometimes used in an inclusive sense (p or q or both).

Example: I shall buy a pen or a book.

In the above statement, if you buy a pen or a book in both cases the statement is true and if you buy (both) pen and book then statement is again true. Thus we say in the above statement we use or in inclusive sense.

The word or is sometimes used in an exclusive sense (p or q but not both). As in the below statement

Example: Tomorrow at 9, I' ll be in Lahore or Islamabad.

Now in above statement we are using or in exclusive sense because both the statements are true then we have F for the statement.

While defining a disjunction the word or is used in its inclusive sense. Thus the symbol $\vee$ means the "inclusive or"

EXCLUSIVE OR:

When or is used in its exclusive sense, the statement "p or q" means "p or q but not both" or "p or q and not p and q" which translates into symbols as:

$(p \vee q) \wedge \sim (p \wedge q)$

Which is abbreviated as:

$p \oplus q$

or **p XOR q**

TRUTH TABLE FOR EXCLUSIVE OR:

p	q	$p\vee q$	$p\wedge q$	$\sim (p\wedge q)$	$(p\vee q) \wedge \sim (p\wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Note:

Basically

$$p \oplus q \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$$
$$\equiv [p \wedge \sim q) \vee \sim p] \wedge [(p \wedge \sim q) \vee q]$$
$$\equiv (p \vee q) \wedge \sim (p \wedge q)$$
$$\equiv (p \vee q) \wedge (\sim p \vee \sim q)$$

APPLYING LAWS OF LOGIC

Using law of logic, simplify the statement form

$p \vee [\sim(\sim p \wedge q)]$

Solution:

$p \vee [\sim(\sim p \wedge q)] \equiv p \vee [\sim(\sim p) \vee (\sim q)]$ DeMorgan's Law

$\equiv p \vee [p \vee (\sim q)]$ Double Negative Law

$\equiv [p \vee p] \vee (\sim q)$ Associative Law for $\vee$

$\equiv p \vee (\sim q)$ Indempotent Law

This is the simplified statement form.

EXAMPLE Using Laws of Logic, verify the logical equivalence

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv p$

$\sim(\sim p \wedge q) \wedge (p \vee q) \equiv (\sim(\sim p) \vee \sim q) \wedge (p \vee q)$ DeMorgan's Law

$\equiv (p \vee \sim q) \wedge (p \vee q)$ Double Negative Law

$\equiv p \vee (\sim q \wedge q)$ Distributive Law

$\equiv p \vee c$ Negation Law

$\equiv p$ Identity Law

SIMPLIFYING A STATEMENT:

"You will get an A if you are hardworking and the sun shines, or you are hardworking and it rains."

Rephrase the condition more simply.

Solution:

Let p = "You are hardworking'

q = "The sun shines"

r = "It rains" .The condition is then $(p \wedge q) \vee (p \wedge r)$

And using distributive law in reverse,

$\mathbf{(p \wedge q) \vee (p \wedge r) \equiv p \wedge (q \vee r)}$

Putting $p \wedge (q \vee r)$ back into English, we can rephrase the given sentence as

"You will get an A if you are hardworking and the sun shines or it rains.

EXERCISE:

Use Logical Equivalence to rewrite each of the following sentences more simply.

1. It is not true that I am tired and you are smart.

{I am not tired or you are not smart.}

2. It is not true that I am tired or you are smart.

{I am not tired and you are not smart.}

3. I forgot my pen or my bag and I forgot my pen or my glasses.

{I forgot my pen or I forgot my bag and glasses.

4. It is raining and I have forgotten my umbrella, or it is raining and I have forgotten my hat.

{It is raining and I have forgotten my umbrella or my hat.}

CONDITIONAL STATEMENTS:

Introduction

Consider the statement:

"**If you earn an A in Math, then I'll buy you a computer**."
This statement is made up of two simpler statements:
p: "You earn an A in Math," and
q: "I will buy you a computer."
The original statement is then saying:
if p is true, then q is true, or, more simply, **if** p, **then** q.
We can also phrase this as p **implies** q, and we write $\mathbf{p \rightarrow q}$.

<u>CONDITIONAL STATEMENTS OR IMPLICATIONS:</u>
If p and q are statement variables, the conditional of q by p is "If p then q" or "p implies q" and is denoted $p \rightarrow q$.
It is false when p is true and q is false; otherwise it is true. The arrow "$\rightarrow$" is the **conditional** operator, and in $p \rightarrow q$ the statement **p** is called the **hypothesis (or antecedent)** and q is called the conclusion (or consequent).

TRUTH TABLE:

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>PRACTICE WITH CONDITIONAL STATEMENTS:</u>
Determine the truth value of each of the following conditional statements:

1. "If 1 = 1, then 3 = 3." **TRUE**
2. "If 1 = 1, then 2 = 3." **FALSE**
3. "If 1 = 0, then 3 = 3." **TRUE**
4. "If 1 = 2, then 2 = 3." **TRUE**
5. "If 1 = 1, then 1 = 2 and 2 = 3." **FALSE**
6. "If 1 = 3 or 1 = 2 then 3 = 3." **TRUE**

<u>ALTERNATIVE WAYS OF EXPRESSING IMPLICATIONS:</u>
The implication $\mathbf{p \rightarrow q}$ could be expressed in many alternative ways as:

- "if p then q"
- "not p unless q"
- "p implies q"
- "q follows from p"
- "if p, q"
- "q if p"
- "p only if q"
- "q whenever p"
- "p is sufficient for q"
- "q is necessary for p"

<u>EXERCISE:</u>
Write the following statements in the form "if p, then q" in English.

a) Your guarantee is good only if you bought your CD less than 90 days ago.

If your guarantee is good, then you must have bought your CD less than 90 days ago.

b) To get tenure as a professor, it is sufficient to be world-famous.

If you are world-famous, then you will get tenure as a professor.

c) That you get the job implies that you have the best credentials.

If you get the job, then you have the best credentials.

d) It is necessary to walk 8 miles to get to the top of the Peak.

If you get to the top of the peak, then you must have walked 8 miles.

TRANSLATING ENGLISH SENTENCES TO SYMBOLS:

Let p and q be propositions:

p = "you get an A on the final exam"
q = "you do every exercise in this book"
r = "you get an A in this class"

Write the following propositions using p, q, and r and logical connectives.

1. To get an A in this class it is necessary for you to get an A on the final.

<u>SOLUTION</u> $\mathbf{p \to r}$

2. You do every exercise in this book; You get an A on the final, implies, you get an A in the class.

<u>SOLUTION</u> $\mathbf{p \wedge q \to r}$

3. Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

<u>SOLUTION</u> $\mathbf{p \wedge q \to r}$

TRANSLATING SYMBOLIC PROPOSITIONS TO ENGLISH:

Let **p**, **q**, and **r** be the propositions:

p = "you have the flu"
q = "you miss the final exam"
r = "you pass the course"

Express the following propositions as an English sentence.

1. $\mathbf{p \to q}$

If you have flu, then you will miss the final exam.**2.** $\mathbf{\sim q \to r}$

If you don' t miss the final exam, you will pass the course.**3.** $\mathbf{\sim p \wedge \sim q \to r}$

If you neither have flu nor miss the final exam, then you will pass the course.

HIERARCHY OF OPERATIONS
FOR LOGICAL CONNECTIVES

~ (negation)

$\wedge$ (conjunction), $\vee$ (disjunction)

$\to$ (conditional)

Construct a truth table for the statement form $p \vee \mathbf{\sim q \to \sim p}$

p	q	~q	~p	$p \vee \sim q$	$\mathbf{p \vee \sim q \to \sim p}$
T	T	F	F	T	F
T	F	T	F	T	F
F	T	F	T	F	T

Construct a truth table for the statement form $(\mathbf{p \to q}) \wedge (\mathbf{\sim p \to r})$

p	q	r	$p \to q$	$\sim p$	$\sim p \to r$	$(p \to q) \wedge (\sim p \to r)$
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	F	F

LOGICAL EQUIVALENCE INVOLVING IMPLICATION

Use truth table to show $\mathbf{p \to q \equiv \sim q \to \sim p}$

p	q	$\sim q$	$\sim p$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	T	T

same truth values

Hence the given two expressions are equivalent.

IMPLICATION LAW

$\mathbf{p \to q \equiv \sim p \vee q}$

p	q	p→q	~p	~p∨q
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

same truth values

NEGATION OF A CONDITIONAL STATEMENT:

Since $p \to q \equiv \sim p \vee q$ therefore

$\sim (p \to q) \equiv \sim (\sim p \vee q)$

$\equiv \sim (\sim p) \wedge (\sim q)$ by De Morgan' s law

$\equiv p \wedge \sim q$ by the Double Negative law

Thus the negation of "**if p then q**" is logically equivalent to "**p and not q**".

Accordingly, the negation of an if-then statement does not start with the word if.

EXAMPLES

Write negations of each of the following statements:

1. If Ali lives in Pakistan then he lives in Lahore.
2. If my car is in the repair shop, then I cannot get to class.
3. If x is prime then x is odd **or** x is 2.
4. If n is divisible by 6, then n is divisible by 2 **and** n is divisible by 3.

SOLUTIONS:

1. Ali lives in Pakistan and he does not live in Lahore.
2. My car is in the repair shop and I can get to class.
3. x is prime but x is not odd **and** x is not 2.
4. n is divisible by 6 but n is not divisible by 2 **or** by 3.

INVERSE OF A CONDITIONAL STATEMENT:

The inverse of the conditional statement $\mathbf{p \to q}$ is $\mathbf{\sim p \to \sim q}$

A conditional and its inverse are not equivalent as could be seen from the truth table.

p	q	p→q	~p	~q	~p→~q
T	T	T	F	F	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

different truth values in rows 2 and 3

WRITING INVERSE:

1. ***If today is Friday, then 2 + 3 = 5.***

   If today is not Friday, then $2 + 3 \neq 5$.

2. ***If it snows today, I will ski tomorrow.***

   If it does not snow today I will not ski tomorrow.

3. ***If P is a square, then P is a rectangle.***

   If P is not a square then P is not a rectangle.

4. ***If my car is in the repair shop, then I cannot get to class.***

   If my car is not in the repair shop, then I shall get to the class.

CONVERSE OF A CONDITIONAL STATEMENT:

The converse of the conditional statement $\mathbf{p \rightarrow q}$ is $\mathbf{q \rightarrow p}$

A conditional and its converse are not equivalent.

i.e., $\rightarrow$ is not a commutative operator.

p	q	p→q	q→p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

not the same

WRITING CONVERSE:

1.If today is Friday, then 2 + 3 = 5.

If 2 + 3 = 5, then today is Friday.

2.If it snows today, I will ski tomorrow.

I will ski tomorrow only if it snows today.

3. ***If P is a square, then P is a rectangle.***

   If P is a rectangle then P is a square.

4. ***If my car is in the repair shop, then I cannot get to class.***

   If I cannot get to the class, then my car is in the repair shop.

CONTRAPOSITIVE OF A CONDITIONAL STATEMENT:

The contrapositive of the conditional statement $\mathbf{p \rightarrow q}$ is $\mathbf{\sim q \rightarrow \sim p}$

A conditional and its contrapositive are equivalent. Symbolically $\mathbf{\rightarrow q \equiv \sim q \rightarrow \sim p}$

1. If today is Friday, then 2 + 3 = 5.

If $2 + 3 \neq 5$, then today is not Friday.

2. If it snows today, I will ski tomorrow.

I will not ski tomorrow only if it does not snow today.

3. If P is a square, then P is a rectangle.

If P is not a rectangle then P is not a square.

4. If my car is in the repair shop, then I cannot get to class.

If I get to the class, then my car is not in the repair shop.