



*Doctoral Thesis*

WORD-METRIC-BASED GREEDY ROUTING  
FOR CAYLEY GRAPHS

DANIELA AGUIRRE GUERRERO

2018





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FOR CAYLEY GRAPHS

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2018

Doctoral Program in Technology

*Supervised by:*

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Thesis submitted to the University of Girona in fulfillment of the  
requirements for the degree of Doctor of Philosophy



## CERTIFICAT DE DIRECCIÓ DE TESI

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Dr. Pere Vilà Talleda and Dr. Lluís Fàbrega Soler, del Departament d'Arquitectura i Tecnologia de Computadors de la Universitat de Girona,

DECLAREM:

Que el treball titulat *Word-Metric-based Greedy Routing for Cayley Graphs*, que presenta Daniela Aguirre Guerrero per a l'obtenció del títol de doctor, ha estat realitzat sota la nostra direcció i que compleix els requisits per poder optar a Menció Internacional.

I, perquè així consti i tingui els efectes oportuns, signem aquest document.

*Girona, Juliol 2018*

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Dr. Pere Vilà Talleda

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Dr. Lluís Fàbrega Soler



## ACKNOWLEDGMENTS

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## PUBLICATIONS

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The work developed in this thesis led to the following publications:

### JOURNAL ARTICLES

- [1] **D. Aguirre-Guerrero**, M. Camelo, P. Vilà and Ll. Fàbrega, "WMGR: A Generic and Compact Routing Scheme for Data Center Networks," in *IEEE/ACM Transactions on Networking*, vol. 26, no. 1, pp. 356-369. Feb. 2018. DOI: 10.1109/TNET.2017.2779866

### PEER-REVIEWED CONFERENCES AND WORKSHOPS

- [1] **D. Aguirre-Guerrero**, M. Camelo, P. Vilà and Ll. Fàbrega, "Compact Greedy Routing in Large-scale Networks using Word-metric Spaces," *1st. International Workshop on Elastic Networks Design and Optimization*, Cartagena, Spain, 2016.

### OTHER CONFERENCES AND WORKSHOPS

- [1] **D. Aguirre-Guerrero**, P. Vilà and Ll. Fàbrega, "Encaminamiento de Información en Redes Comunicación de Gran Escala," *6to. Simposio de Becarios CONACyT en Europa*, Strasbourg, France 2017.
- [2] **D. Aguirre-Guerrero**, P. Vilà and Ll. Fàbrega, "Greedy Geometric in Word Metric Spaces," *1st. Conference of Pre-doctorals Researches*, Girona, Spain, 2016. ISBN: 978-8-48458-502-2

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## LIST OF ALGORITHMS

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## ACRONYMS

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WMGR word-metric based greedy routing

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## ABSTRACT

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Since word-metric based greedy routing ([WMGR](#)) *a priori* (e.g., exploring confined natural environments like underwater caves). In these scenarios, [WMGRs](#)





## RESUMEN

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## RESUM

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## Part I

### PRELIMINARIES

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## INTRODUCTION

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1.1 MOTIVATION

1.2 PROBLEM STATEMENT

1.3 OBJECTIVES

1.4 CONTRIBUTIONS

1.5 HOW TO READ THIS THESIS

## THEORETICAL FRAMEWORK

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### 2.1 GRAPH THEORY

### 2.2 FINITE STATE AUTOMATA

#### 2.2.1 *Regular Languages*

#### 2.2.2 *2-variable Finite State Automata*

#### 2.2.3 *Existential Quantification*

### 2.3 GROUP THEORY

#### 2.3.1 *Finitely Presented Groups*

#### 2.3.2 *Group Homomorphism*

#### 2.3.3 *Abelian Groups*

#### 2.3.4 *Cayley Graphs*

### 2.4 AUTOMATIC GROUPS

#### 2.4.1 *Automatic Structures*

#### 2.4.2 *Words as Nodes and Paths*

#### 2.4.3 *Solving the Shortest Path Problem in Cayley Graphs*

## CAYLEY GRAPHS: APPLICATIONS AND ROUTING ALGORITHMS

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### 3.1 TOPOLOGICAL PROPERTIES OF CAYLEY GRAPHS

### 3.2 APPLICATIONS

#### 3.2.1 *Interconnection Networks*

##### 3.2.1.1 *Processor Interconnection Networks*

##### 3.2.1.2 *Data Center Networks*

##### 3.2.1.3 *Wireless Sensor Networks*

#### 3.2.2 *Error Correcting Codes*

#### 3.2.3 *Hash Functions*

### 3.3 ROUTING ALGORITHMS

#### 3.3.1 *Performance Metrics*

#### 3.3.2 *Sims Factoring Algorithm*

#### 3.3.3 *Permutation-sort Algorithm*

#### 3.3.4 *Routing Algorithm for Boreal Cayley Graphs*



## Part II

### WORD-METRIC-BASED GREEDY ROUTING

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## PATH COMPUTATION IN CAYLEY GRAPHS

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4.1 DISTRIBUTED CONFIGURATION

4.2 COMPUTING THE SHORTEST PATH

4.3 COMPUTING THE K-SHORTEST PATHS

4.4 COMPUTING THE SHORTEST NODE-DISJOINT SHORTEST PATHS

4.5 COMPUTING THE SHORTEST EDGE-DISJOINT SHORTEST PATHS

## FAULT-TOLERANT ROUTING IN CAYLEY GRAPHS

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### 5.1 MULTIPLE-NODE FAILURES

#### 5.1.1 *Computing the Shortest Path Avoiding a Set of Nodes*

#### 5.1.2 *Node-failure Notification*

#### 5.1.3 *Node-failure Recovery*

### 5.2 MULTIPLE-EDGE FAILURES

#### 5.2.1 *Computing the Shortest Path Avoiding a Set of Edges*

#### 5.2.2 *Edge-failure Notification*

#### 5.2.3 *Edge-failure Recovery*

## PERFORMANCE EVALUATION

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### 6.1 STRETCH

### 6.2 SPACE REQUIREMENTS

#### 6.2.1 *Node Label*

##### 6.2.1.1 *Source Routing*

##### 6.2.1.2 *Hop-by-hop Routing*

#### 6.2.2 *Automatic Structures*

### 6.3 FORWARDING DECISION TIME

#### 6.3.1 *Source Routing*

#### 6.3.2 *Hop-by-hop Routing*

### 6.4 CONVERGENCE TIME OF FAULT-TOLERANT ROUTING

### 6.5 MESSAGE COMPLEXITY

#### 6.5.1 *Node Label Assignment*

#### 6.5.2 *Failures Notification*

#### 6.5.3 *Recovery Notification*

## CONCLUSIONS

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### 7.1 SUMMARY OF COMPLETED WORK

This thesis has addressed the problem of

### 7.2 REVIEW OF CONTRIBUTIONS

### 7.3 FUTURE WORK

## APPENDIX