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Adaptive self-organization vs static optimization

A qualitative comparison in traffic light coordination

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Abstract

Purpose – The purpose of this paper is to compare qualitatively two methods for coordinating traffic lights: a static optimization "green wave" method and an adaptive self-organizing method.

Design/methodology/approach – Statistical results were obtained from implementing a recently proposed model of city traffic based on elementary cellular automata in a computer simulation.

Findings – The self-organizing method delivers considerable improvements over the green-wave method. Seven dynamical regimes and six phase transitions are identified and analyzed for the self-organizing method.

Practical implications – The paper shows that traffic light coordination can be improved in cities by using self-organizing methods.

Social implications – This improvement can have a noticeable effect on the quality of life of citizens.

Originality/value – Understanding how self-organization obtains adaptive solutions for complex problems can contribute to building more efficient systems.

Keywords Self-organization, Adaptation, Traffic lights, Elementary cellular automata, Phase transitions

Paper type Research paper

1. Introduction

There have been several models of vehicular traffic proposed in the literature (Prigogine and Herman, 1971; Wolf *et al.*, 1996; Schreckenberg and Wolf, 1998; Helbing *et al.*, 2000; Helbing, 1997; Helbing and Huberman, 1998; Chowdhury *et al.*, 2000; Di Febbraro and Sacco, 2004; Maerivoet and De Moor, 2005; Maroto *et al.*, 2006). These models have increased our understanding of vehicular traffic phenomena. One of the problems traffic engineers face is that of traffic light coordination. Given the interactions and feedbacks among flows exiting and entering intersections,

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it is not possible to generalize the properties of a single intersection model to a city grid. It is known that the optimal coordination of traffic lights is an EXP-complete problem (Papadimitriou and Tsitsiklis, 1999; Lämmer and Helbing, 2008). This implies that it is intractable. There have been several methods proposed over recent decades to solve this problem. We can distinguish two main approaches. One tries to optimize the (fixed) green phases of lights to maximize the movement of vehicles traveling at a certain velocity (Federal Highway Administration, 2005; Robertson, 1969; Gartner *et al.*, 1975; Sims and Dobinson, 1980; Török and Kertész, 1996; Brockfeld *et al.*, 2001). The other one tries to adapt – manually or automatically – phases depending on current traffic flows (Federal Highway Administration, 2005; Henry *et al.*, 1983; Mauro and Di Taranto, 1990; Robertson and Bretherton, 1991; Faieta and Huberman, 1993; Gartner *et al.*, 2001; Diakaki *et al.*, 2003; Fouladvand *et al.*, 2004; Mirchandani and Wang, 2005; Bazzan, 2005; Helbing *et al.*, 2005; Gershenson, 2005; Cools *et al.*, 2007)[1]. We implemented two methods, one corresponding to each approach: a green-wave method that tries to optimize phases for an expected traffic flow, and a self-organizing method that adapts to the current traffic conditions.

In this paper, we use a recently proposed model of city traffic based on elementary cellular automata (Rosenblueth and Gershenson, 2011) to study the problem of traffic light coordination and to compare the abovementioned methods. In the next section, the model is briefly reviewed. We detail the mechanisms of a green-wave method in Section 3 and of a self-organizing method in Section 4. This is followed by results and a discussion of experiments carried out in a Manhattan-style simulation in Section 5. Concluding remarks close the paper.

2. A city traffic model based on elementary cellular automata

An elementary cellular automaton is a collection of cells arranged on a one-dimensional array. A cell in such an automaton has only two possible states (0 or 1, say). Time is discrete and all cells' states are updated synchronously. Moreover, the state of a cell in the next time step, or "tick", depends only on the present states of that cell and those of its nearest neighbors. As a result, the behavior of an elementary cellular automaton can be described by a table specifying the state a given cell will have in the next "generation" based on the state of the cell to its left, the state of the cell itself, and the state of the cell to its right. Such a table has as input these three current states and as output the state of a cell in the next generation (Wolfram, 1986, 2002) names each elementary cellular automaton with the binary numeral, called "rule", resulting from reading the output of the table when the inputs are lexicographically ordered.

Rule 184 is one of the simplest highway traffic models (Yukawa *et al.*, 1994; Chowdhury *et al.*, 2000; Maerivoet and De Moor, 2005), where 1 represents a vehicle and 0 represents a space. Traffic flows to the right freely for low densities, while at higher densities traffic jams are formed, moving to the left. In its simplicity, there is a duality between vehicles and spaces. Even when this is not quantitatively realistic[2], it has been shown that rule 184 qualitatively reproduces real highway statistics (Kanai, 2010).

In Rosenblueth and Gershenson (2011) we extended the rule 184 model to a city grid configuration, considering coupled non-homogeneous ECA, where rules change around the intersection, depending on the state of the traffic light. If a street has a green light, all its cells use rule 184. If there is a red light, all cells also use rule 184, with two exceptions: the cell immediately before and the cell immediately after the intersection.

Let L be the cell immediately before the intersection. L has to stop traffic from going into the intersection. If there is a vehicle in L (cases 010, 011, 110, 111, where the value of L is in the middle), it will stay there, so the next state of L will continue to be 1. If there is no vehicle in L and there is a vehicle in the previous cell (100, 101), such a vehicle will advance, so the next state of L will also be 1. Only if there are no vehicles in L or its previous cell will the next state remain 0. This is rule 252. The cell immediately after the intersection has to allow vehicles to leave, but must not allow vehicles in the intersection cell (flowing in a different direction) to enter such a cell. Thus, the next state will be 1 only if the cell ahead is blocked (011, 111). This is rule 136. Table I lists the transition tables for the three rules used by the model.

The intersection cell is a special case, as it has four potential neighbors. The rule never changes (184). What changes is the neighborhood, i.e. it takes as nearest neighbors only the two cells in the street with a green light (also using rule 184). A diagram of the cells around an intersection is shown in Figure 1. For details of the model, we refer the reader to Rosenblueth and Gershenson (2011).

Table I.
ECA rules used in model

$t-1$	t_{184}	t_{252}	t_{136}
000	0	0	0
001	0	0	0
010	0	1	0
011	1	1	1
100	1	1	0
101	1	1	0
110	0	1	0
111	1	1	1

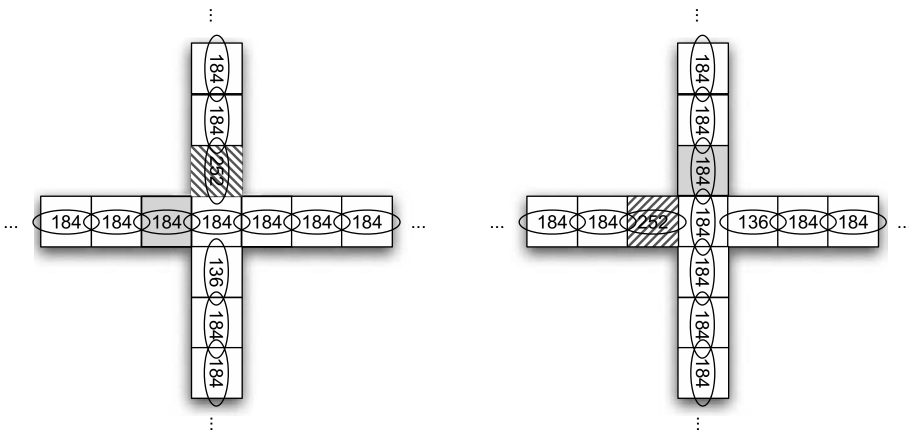


Figure 1.
Diagram for different rules (shown within cells) and neighborhoods (indicated by ovals) used around intersections, depending on the state of the traffic light

Notes: For green lights (indicated by a gray cell), rule 184 is used, and the intersection cell has as neighbors cells in the street with the green light; for red lights (indicated by a diagonally striped cell), rule 252 is used for the cell before the intersection and rule 136 for the cell after the intersection; the rest of the cells use rule 184

3. The green-wave method

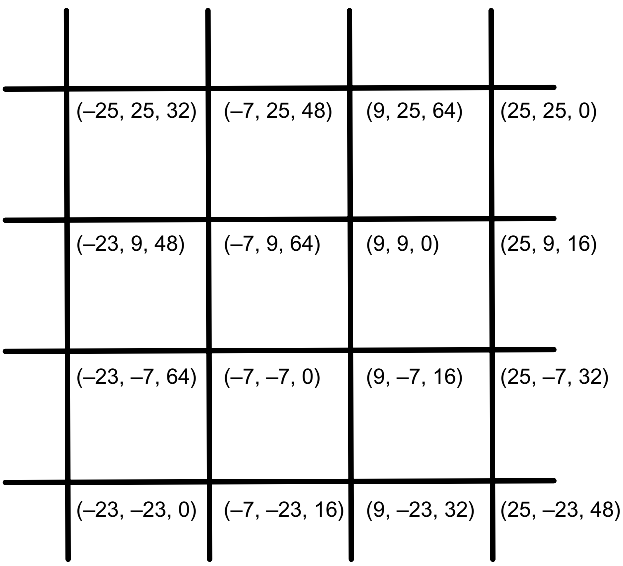
The idea behind the green-wave method is the following: if the consecutive traffic lights switch with an offset equivalent to the expected vehicle travel time between intersections, vehicles should not have to stop. Thus, waves of green light move through the street at the same velocity as the vehicles.

This method – also known as coordinated signal control – has advantages, e.g. when most of the traffic flows in the direction of the green wave at low densities. However, since only two directions can have green waves, the vehicles flowing in the opposite direction of the green wave will be delayed. Also, if traffic is flowing at velocities lower than expected, the green waves will go faster than the vehicles and these will be delayed.

To implement the green-wave method in our CA model, we consider a fixed period T for all traffic lights, and an individual offset ω_j for each traffic light. This offset is determined by the coordinates x, y of the intersection cells as follows:

$$\omega_j = \left\lfloor \left((x - y) \mathrm{mod} \frac{T}{2} \right) + 0.5 \right\rfloor \tag{1}$$

Equation (1) sets the offset by rounding to the nearest integer the x coordinate minus the y coordinate, modulus half a period. This offset creates green waves to the south and east, for regular or irregular grids. Lights are switched twice per period T, when ω_j is equal to a global phase φ that cycles from zero to $T/2$. Figure 2 shows a diagram of how ω_j depends on coordinates x and y. To allow vehicles to flow without stopping, the street lengths need to be a multiple of period T. The main idea behind setting the offset is to have intersections



Notes: The figure shows 16 intersections with values (x, y, w) for T = 160; notice that w values are the same for skew diagonals; lights are switched when ω is equal to a counter that cycles from zero to $T/2$

Figure 2.
Example of how equation (1) sets the offsets ω_j depending on coordinates x and y

lying on the skew diagonals to have the same offset value, so as to switch their lights at the same time. In this way, free-flowing vehicles going either eastbound or southbound will be able to reach an intersection with a green light. To change the direction of the green wave, only the sign of the x or y coordinates has to be changed.

The lights need to be initialized properly for the green-wave method to work. They can be in a state of a green light horizontally (eastbound or westbound) and thus red light vertically (northbound or southbound), or a state of a green light vertically and thus a red light horizontally. The state σ_j of the traffic lights is initialized as follows:

$$\sigma_j = \begin{cases} green_{vertical} & \text{if } \lfloor((x-y) \mod T) + 0.5\rfloor \geq \frac{T}{2} \\ green_{horizontal} & \text{otherwise} \end{cases} \tag{2}$$

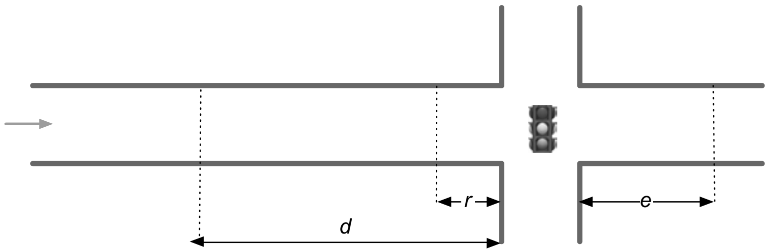
Equation (2) initializes the state of a traffic light to have a green vertical light if the nearest integer of the x coordinate minus the y coordinate, modulus one period T, is greater than or equal to half a period. Otherwise, the state is set as green horizontal. With this equation, 50 percent of contiguous intersections of every street (vertical and horizontal) will have a green vertical state and the other half a green horizontal state. The states of intersections are arranged in such a way that the transitions from green horizontal to green vertical lie on skew diagonals.

4. The self-organizing method

With the self-organizing method, each intersection independently follows the same set of rules, based only on local traffic information. There are only six rules (not related to ECA rules), with higher-numbered rules overriding lower-numbered ones. The full rule set is shown in Figure 3. This method is an improvement over previous work. The one reported in Ball (2004) considered only rules 1 and 2, while Gershenson (2005) and Cools *et al.* (2007) considered only rules 1-3.

The first rule is designed to ensure that when traffic is waiting at a red light, or many vehicles are approaching an intersection, the light will switch to green. Since we are concerned about the idle time at intersections, a possible quantity to use for triggering lights is the accumulated waiting time of vehicles. However, it is even better to include the vehicles approaching a light, so they may not need to slow down or stop at all. Thus, every light has a counter that records the cumulative amount of vehicle time within a set distance d from the light since it last changed to red, adding to this value every tick. All incoming vehicles within distance d are counted, whether stationary or moving. When the counter exceeds a threshold n, the light is switched (subject to override by subsequent rules). Thus, for example, if one vehicle waits for 40 ticks, five vehicles wait for eight ticks, or ten vehicles wait for four ticks, and then the light will be ready to switch. If there are many incoming vehicles approaching a red light, rule 1 will tend to switch their light to green before they reach the intersection, so they will not need to stop. Vehicles waiting at an intersection may be joined by others to form a platoon before the light switches. As the platoon flows through the system, its approach switches other lights to green, creating an emergent green wave.

When platoons approach the same intersection from conflicting directions, rule 2 prevents vehicles from triggering repeated switching that would immobilize traffic. Such a rule sets a minimum time before a platoon can request a light change. The integrity of platoons is promoted by rule 3, which prevents the “tails” of platoons from being cut, but allows the division of long platoons. Rule 4 allows rapid switching of



1. On every tick, add to a counter the number of vehicle sapproaching or waiting at a red light within distance *d*. When this counter exceeds a threshold *n*, switch the light. Whenever the light switches, reset the counter to zero.
2. Lights must remain green for a minimum time *u*.
3. If a few vehicles (*m* or fewer, but more than zero) are left to cross a green light at a short distance *r*, do not switch the light.
4. If no vehicle is approaching a green light within a distance *d*, and at least one vehicle is approaching the red light within a distance *d*, then switch the light.
5. If there is a vehicle stopped in the street a short distance *e* beyond a green traffic light, then switch the light.
6. If there are vehicles stopped on both directions at a short distance *e* beyond the intersection, then switch both lights to red. Once one of the directions is free, restore the green light in that direction.

Note: Inset: schematic of an intersection, indicating distances *d*, *r* and *e* used for self-organizing lights

Figure 3. Self-organizing traffic light rules

lights for low traffic densities, so lone vehicles can trigger lights to switch as they approach an intersection without needing to wait for platoons to be formed. Rules 5 and 6 prevent gridlock otherwise caused by halting vehicles before they might block an intersection due to stopped vehicles on the other side of the intersection. Rule 5 changes the light if there is a blockage ahead of a green light, while rule 6 sets both lights to red if both directions are blocked. A formal description and further explanation of the self-organizing method is presented in the Appendix.

A variation of rule 1 has been in used in the UK for more than 20 years (Vincent and Young, 1986), but only at a limited number of isolated intersections. The technology to implement this method and similar methods – also known as vehicle-actuated signal control – has already been on the market for several years, while the sensor accuracy and sophistication has been improving constantly. Still, it is not obvious that independent intersections following local rules will self-organize and produce emergent green waves by their sole response to local densities, so this approach has not been implemented in cities to coordinate traffic lights.

5. Simulations of city traffic

We developed a computer simulation in NetLogo (Wilensky, 1999). The reader is invited to access the simulation via web browser at the web site: http://tinyurl.com/trafficCA

To test the model and to compare the abovementioned traffic light controllers, we constructed a ten-by-ten homogeneous grid, with alternating flow directions and cyclic boundaries. We used a street distance of 160 cells, equivalent to 800 m (Rosenblueth and

Gershenson, 2011). Thus, there are 80 simulated meters between streets[3]. There are 3,100 cells in the simulation, 100 intersections and 20 streets: five in each cardinal direction.

For the experiments, each run consisted of the following: half hour (5,400 ticks) was simulated for random initial conditions. Since the vehicles are placed randomly, one street may have a slightly higher density than another. After these initial 5,400 ticks, the system is considered to have stabilized, i.e. gone through a transient, so another half hour is simulated. At the end of the simulation, the velocities of the second half hour are averaged to obtain the average velocity $\langle v \rangle$ and average flux $\langle J \rangle$, which is the velocity multiplied by the density $\rho \in [0,1]$. The results are shown in Figure 4, which includes results of single runs for several densities (intervals of $\rho = 0.01$) on the left subfigures and boxplots[4] for fewer densities (intervals of $\rho = 0.05$) on the right subfigures.

For the green-wave method, at densities $\rho < 0.25$ half of the streets (southbound and eastbound) have a free-flow regime, i.e. $v = 1$. However, on the other half of the streets (northbound and westbound), the velocity is half the one shown in Figure 4 ($v \approx 0.35$). In these streets, vehicles stop every three blocks, leading to large delays due to the anti-correlated traffic lights they encounter. We found only two phases: an intermittent phase, where some vehicles have to stop at traffic lights and a gridlock phase, i.e. $v = 0$. The phase transition lies at $\rho \approx 0.3$, where there is a maximum flux of

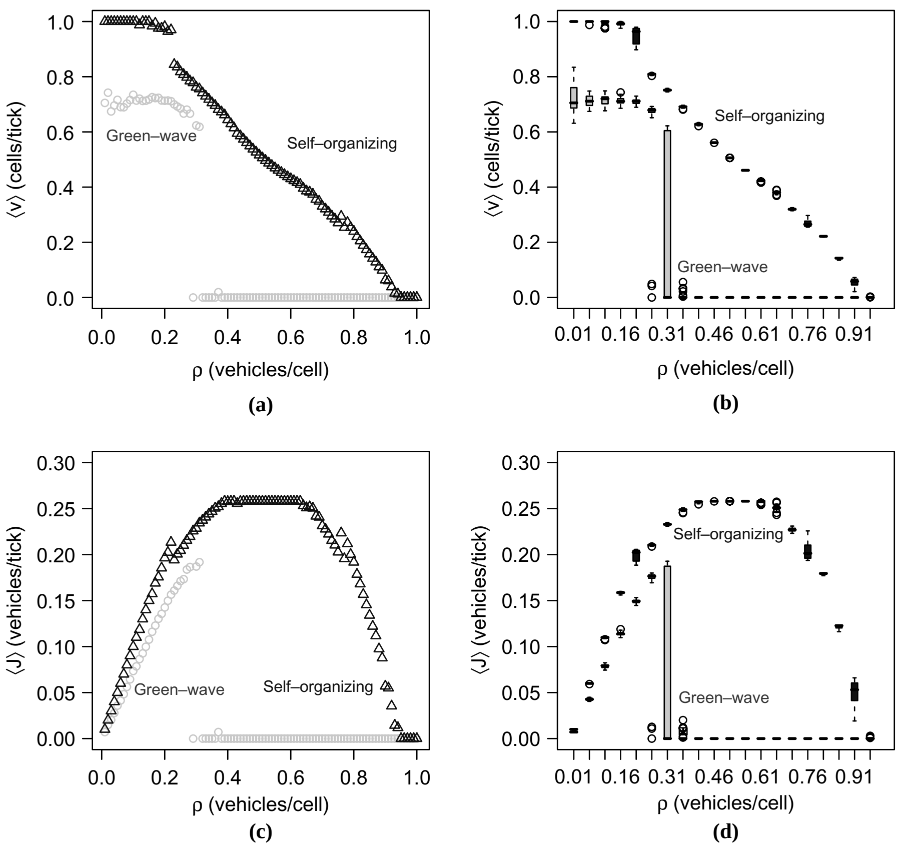


Figure 4.
Simulation results for a ten-by-ten city grid: (a, b) average velocity $\langle v \rangle$ and (c, d) average flux $\langle J \rangle$ for different densities ρ, green-wave (o) and self-organizing (Δ) methods: (a, c) single runs and (b, d) box plots of 50 runs per density

$J \approx 0.19$. For densities $0.25 < \rho \lessapprox 0.3$, vehicles flowing on streets with green wave (southbound and eastbound) cannot keep the speed of the green wave, so traffic jams from that move in the opposite direction of the traffic. At densities $\rho \gtrapprox 0.3$, the queues on directions opposite to the green wave grow and block intersections upstream, leading to gridlocks at medium and high densities. In Figure 4(b) and (d) there is a high variance around the phase transition because the gridlock outcome depends on which percentage of the randomly placed vehicles lie on streets with anti-correlated lights. Screenshots of the phases for the green-wave method can be seen in Figure 5.

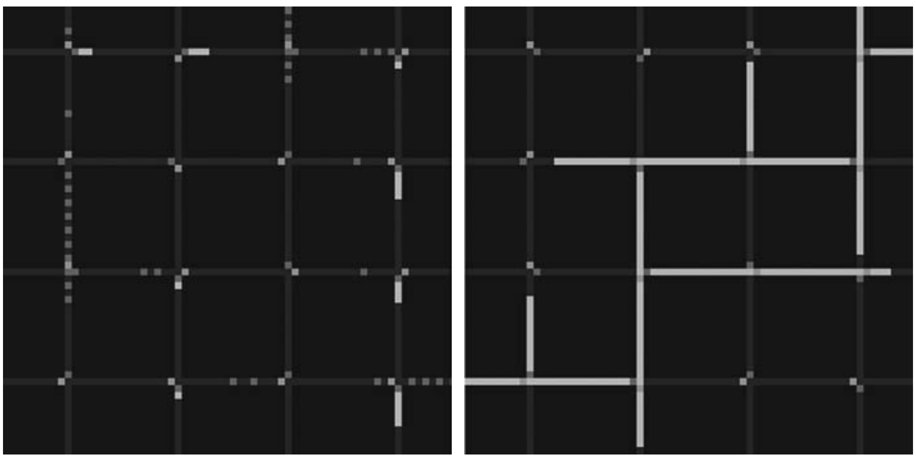
For the self-organizing method, five phases were found, one of which had three subphases:

(1) For low densities there is a free-flow phase ($v = 1$) and no vehicle has to stop.

(2) There is a phase transition at $\rho \approx 0.15$ into a quasi-free-flow phase, where very few vehicles stop for very little time. Most intersections have only one platoon requesting a green light, so this one is able to flow without having to wait for other platoons.

(3) At $\rho \approx 0.22$ there is another phase transition into an intermittent phase, where few or some vehicles stop behind traffic lights. Within the intermittent phase, we can distinguish three subphases depending on the flux diagram (Figure 4(c)):

- Until $\rho \approx 0.38$, there is an underutilized intermittent subphase, where intersections are idling some of the time, i.e. no vehicle uses them. The difference with the quasi-free-flow phase is that in the underutilized intermittent subphase there are two platoons requesting a green light in most of the cases. Thus, one platoon has to wait until the other one crosses.



(a) (b)

Notes: Four-by-four sections of ten-by-ten simulations shown; dark gray cells indicate moving vehicles, light gray cells indicate stopped vehicles; (a) intermittent phase (ρ = 0.1): vehicles flow freely on streets with green waves, but vehicles going in the opposite direction have to stop frequently; (b) gridlock phase (ρ = 0.3): intersection blockages propagate and all vehicles stop, even when the density is not high

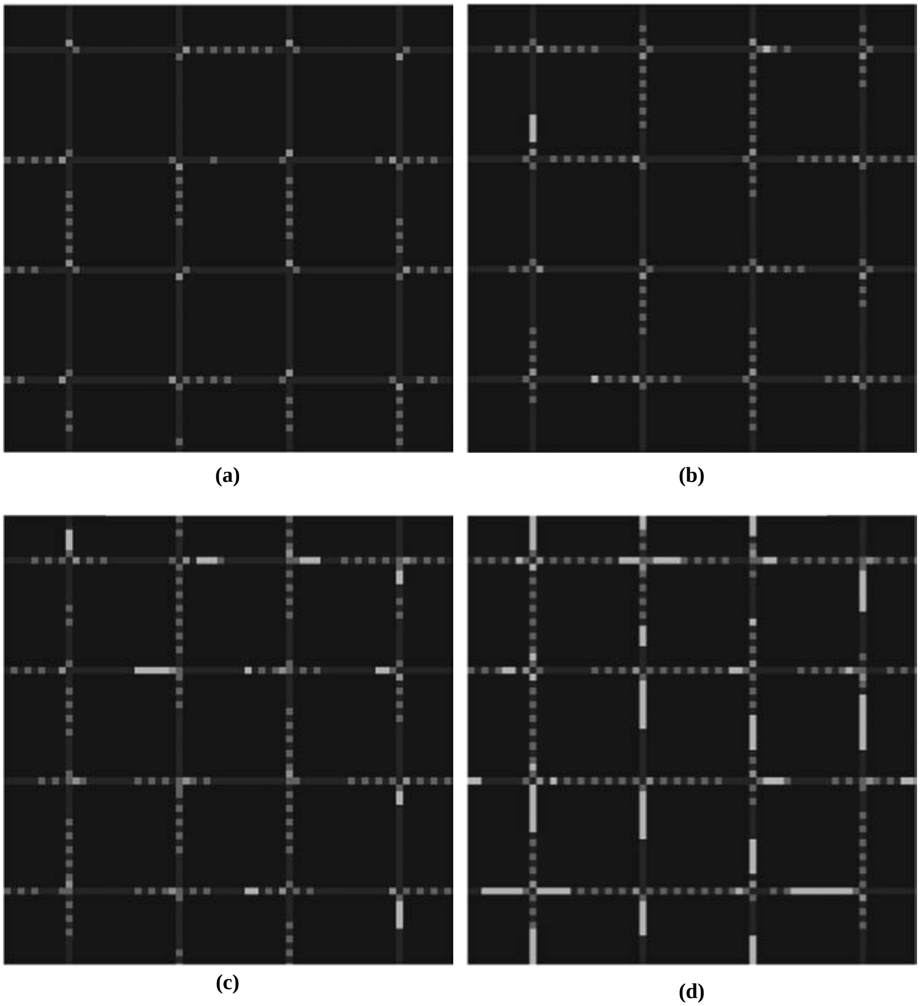
Figure 5. Screenshots of different phases for green-wave method

- Between $\rho \approx 0.38$ and $\rho \approx 0.63$ there is a full capacity intermittent subphase, where there is a maximum flux $J = 0.25$. This implies that the intersections are being used at their maximum capacity, i.e. there is a vehiclecrossing every other tick, so there are no "wasted" resources of free space[5]. Notice that this subphase occupies one fourth of the density space $\rho \in [0,1]$.
- After $\rho \approx 0.63$ and until $\rho \approx 0.77$ there is an overutilized intermittent subphase, where the density is such that rule 6 sometimes forces both directions to stop, thus reducing the flux of the intersections. This subphases similar to the underutilized intermittent subphase, in the sense that the intersections cannot be used at their full flux capacity. In the underutilized intermittent case, this is because there are not enough vehicles. In the overutilized intermittent case, this is because there are too many vehicles and intersections need to wait before one street can get a green light.

(4) Between $\rho \approx 0.77$ and $\rho \approx 0.95$ there is a quasi-gridlock phase. Most vehicles are stopped, but free spaces "move" in the direction opposite of the vehicles between traffic jams at a speed of one cell per tick. Adaptive green waves also move in that direction, i.e. free spaces always trigger a green light. Just like the self-organizing method promotes the formation of platoons and these coordinate the traffic lights using mainly rule 1 for low densities, the method promotes the formation of free spaces that coordinate traffic lights using mainly rule 6 for high densities. These free spaces allow vehicles to advance for longer distances before stopping. Their coordination implies that there will be little interference between free spaces traveling in different streets, i.e. free spaces rarely have to stop at intersections, and they do so only to allow other free spaces to finish crossing. The difference with the overutilized intermittent phase is that in the quasi-gridlock phase most traffic lights are switched by rule 6, while in the overutilized intermittent phase other rules also play a role, i.e. free spaces are long enough to meet at intersections, so vehicles approaching intersections can trigger traffic lights.

(5) There is a final transition at $\rho \approx 0.95$ into a gridlock phase, i.e. $v = 0$. This is due to initial conditions, where some streets are blocked before the self-organizing method can prevent this.

There is some symmetry in the phases and in the flux diagram (Figure 4(c) and (d)) of the self-organizing method. On one extreme (low density) there is the free-flow phase, where no vehicle stops ($v = 1$). On the other extreme (very high density) there is the gridlock phase, where no vehicle moves ($v = 0$). As the density moves towards the middle, on the low-density side there is formation of platoons that trigger green lights in the quasi-free-flow phase. On the high-density side there is formation of free spaces that also trigger green lights in the quasi-gridlock phase. For medium densities we have the intermittent phase with its three subphases. In the center there is the full capacity subphase, where there is always a vehicle crossing an intersection, leading to a maximum flux $J = 0.25$. Notice that vehicles are separated by spaces, i.e. spaces are also always crossing an intersection. On the low-density side, the underutilized subphase cannot reach the full capacity because some intersections are idle. On the high-density side, the overutilized subphase cannot reach full capacity because some intersections are stopping traffic from both directions. Screenshots of the different phases for the self-organizing method can be seen in Figures 6 and 7.



Notes: Four-by-four sections of ten-by-ten simulations shown; dark gray cells indicate moving vehicles, light gray cells indicate stopped vehicles; see also Figure 7; (a) free-flow phase (ρ = 0.1): vehicles flow freely on all streets; notice that all platoons have a green light ahead of them; (b) quasi-free-flow phase (ρ = 0.2): very few vehicles stop, and those that do only stop briefly; (c) underutilized intermittent subphase (ρ = 0.3): some vehicles stop, there is some free space left at intersections; full capacity intermittent subphase (ρ = 0.5): some vehicles stop, intersections are used at full capacity (J = 0.25); notice that all intersections are being utilized

Figure 6.
Screenshots of different phases for self-organizing method

The average velocity over all densities in the experiments shown in Figure 4(b) for the green-wave method was $\langle v \rangle \approx 0.22$. For the self-organizing method it was $\langle v \rangle \approx 0.55$, i.e. a 150 percent improvement over the green-wave method. However, it is unrealistic to average over all densities, especially when the green-wave method reaches a gridlock for 70 percent of them. To have a better comparison, we can average the velocities for densities where

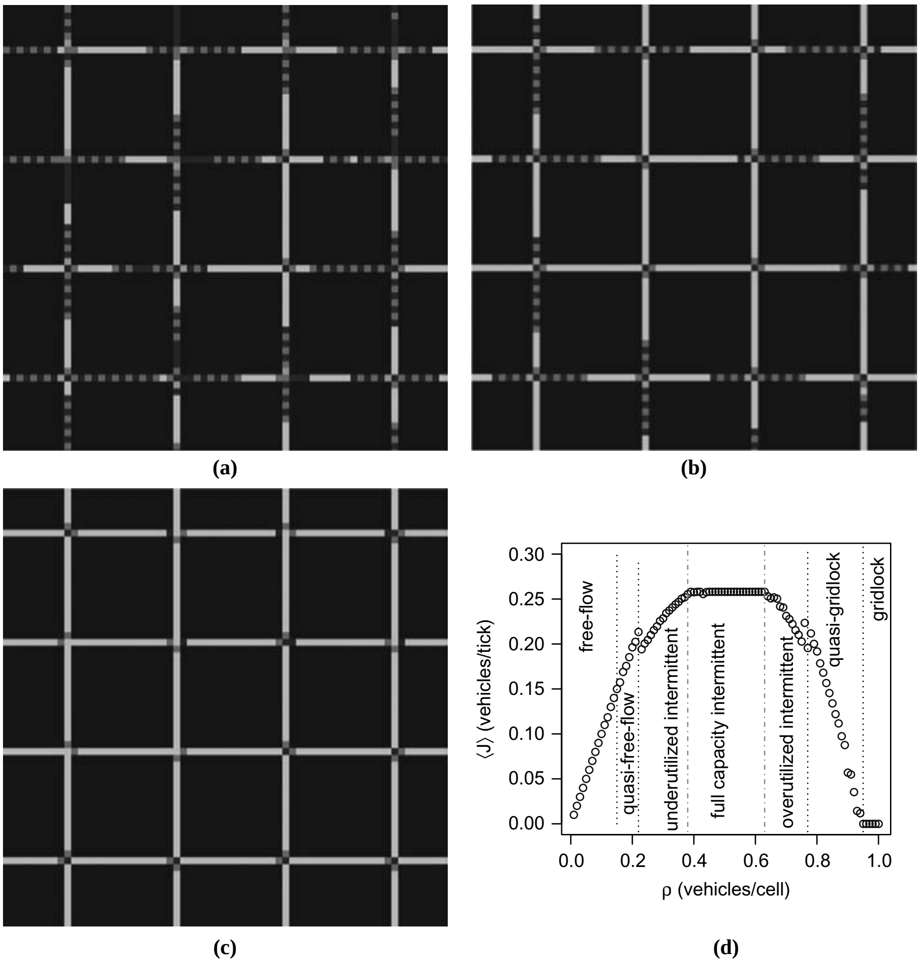


Figure 7.
Screenshots of different phases for self-organizing method (a-c)

Notes: Four-by-four sections of ten-by-ten simulations shown; dark gray cells indicate moving vehicles, light gray cells indicate stopped vehicles; see also Figure 6; (a) overutilized intermittent subphase ($\rho = 0.7$): some vehicles stop, some intersections have to stop traffic in both directions due to high density; this subphase is analogous to the underutilized intermittent subphase (Figure 6(c)), where intersections are not used at their full capacity because of low density; (b) quasi-gridlock phase ($\rho = 0.8$): almost all vehicles are stopped, but free spaces self-organize and flow in the opposite direction, triggering green lights when they are about to reach an intersection; notice that all intersections with free space have a green light; this phase is analogous to the quasi-free-flow phase (Figure 6(b)), where vehicles self-organize in platoons and trigger green lights when they are about to reach an intersection; (c) gridlock phase ($\rho = 0.95$): at extremely high densities, initial conditions lead to a flow standstill ($v = 0$); this phase is analogous to the free-flow phase (Figure 6(a)), where no vehicle has to stop ($v = 1$); (d) flux diagram (same as Figure 4(c)) indicating different phases and their transitions) neither method has reached gridlocks, i.e. $\rho < 0.3$. The results for the green-wave method were $\langle v \rangle_{\rho \leq 0.26} \approx 0.7$ and for the self-organizing method $\langle v \rangle_{\rho \leq 0.26} \approx 0.95$, i.e. an improvement of 35 percent over the green-wave method, which is considerable. Even when the aim of this study is to obtain a qualitatively comparison, the relative results (the differences between methods) are quantitatively comparable with our previous results (Gershenson, 2005; Cools *et al.*, 2007), which included more realistic simulations (non-periodic boundaries, probabilistic turns, non-homogeneous inter-street distances, different driving behaviors and speeds, and pedestrians).

As for the flux (Figure 4(d)), the total average for the green-wave method was $\langle J \rangle \approx 0.03$. For the self-organizing method it was $\langle J \rangle \approx 0.18$. This is a 470 percent improvement overall densities. For lower densities, i.e. $\rho < 0.3$, $\langle J \rangle_{\rho \leq 0.26} \approx 0.09$ for the green-wave method and $\langle J \rangle_{\rho \leq 0.26} \approx 0.12$ for the self-organizing method, i.e. a 33 percent improvement. Notice also that the maximum flux for the green-wave method $J_{max} \approx 0.19$ (for a particular density) and for the self-organizing method $J_{max} = 0.25$ (for a broad range of densities), i.e. a 31 percent improvement in maximum flux.

The green-wave method is good for vehicles going in the direction of the green wave at low densities. However, it performs poorly overall because of the slow flow of vehicles going on the opposite direction. The lights are anti-correlated in such a way that long queues are formed for densities $\rho > 0.3$, blocking intersections upstream and leading to gridlocks. Moreover, it has been shown that this method is remarkedly sensitive to the value of T (Brockfeld *et al.*, 2001). Our self-organizing method achieves good performance for all densities compared with the green-wave method. It responds to the current traffic demand, so vehicles have to wait only if there are vehicles crossing at that moment. Rule 1 promotes the formation of platoons, which leave free space for other platoons to cross without interference. Together with rule 4, this achieves free-flow in four directions – for random initial conditions – for low densities. The performance at medium densities is also good, reaching the maximum possible flux for a broad range of densities. Traffic lights respond to the demand of platoons, and these do not have to wait for long, reducing the probability of long queues, which would interfere with intersections upstream. Rules 3, 5 and 6 alleviate the interference caused by long queues. Rule 3 prevents platoons from growing too much if there is a demand on the intersecting street, whereas rules 5 and 6 prevent vehicles from blocking an intersection upstream. Experiments varying the parameters of the self-organizing method showed that the method is robust to changes in these values.

1. Different city sizes

We performed simulations varying the number of cyclic streets (all streets 160 cells long). The results for two-by-two, six-by-six, ten-by-ten and fourteen-by-fourteen configurations can be seen in Figure 8.

From the results, it can be seen that both methods perform better for fewer intersections in the simulation. This is because it is easier to coordinate four intersections (two-by-two case, $|x|,|y| = 2$) than 196 (fourteen-by-fourteen case, $|x|,|y| = 14$). Actually, the green-wave method can reach free-flow for the two-by-two case. Having only two intersections per street and cyclic boundaries makes the problem symmetric, and vehicles can flow freely in four directions (Figure 8(a)) and the system can reach a maximum flux $J = 0.25$ (Figure 8(c)). However, even for the six-by-six case ($|x|,|y| = 6$) the performance of the green-wave method is quite poor, and decreases as the number of streets increases. For the

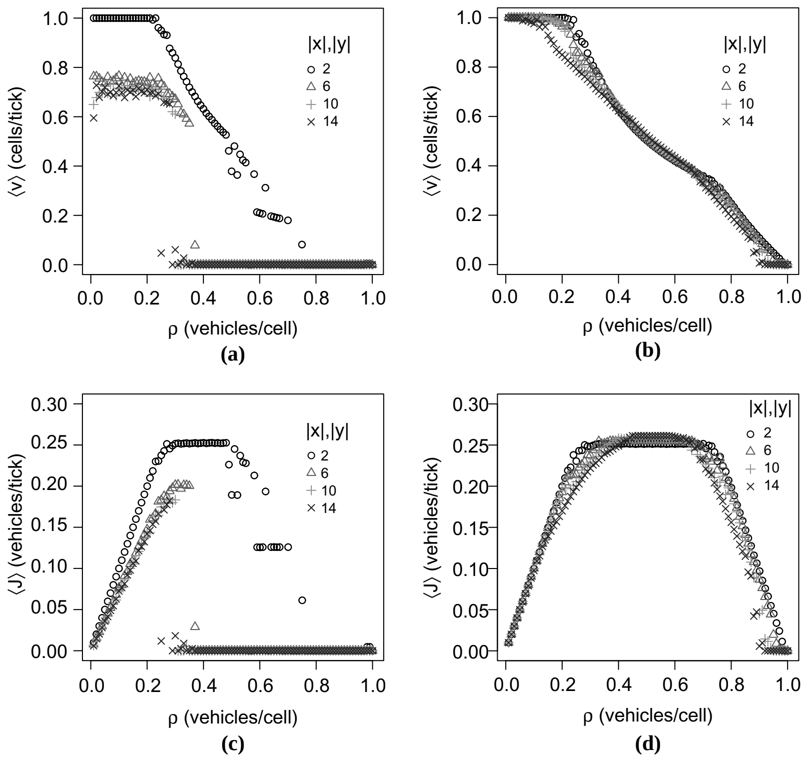


Figure 8. Simulation results varying the number of streets in the x and y directions: (a, b) average velocity $\langle v \rangle$ and (c, d) average flux $\langle J \rangle$ for different densities ρ: (a, c) green-wave method and (b, d) self-organizing method)

self-organizing method, the performance also decreases, although increasing the number of streets does not change the dynamical properties of the method. When $|x|,|y| \rightarrow \infty$ there is no free-flow phase, since there is no opportunity for platoons to encounter repeatedly and coordinate, as it is the case with a finite torus. Further simulations with very large grids ($|x|,|y| > 100$) should be made to better understand this behavior. Will the performance decrease constantly? Or will it converge to a limit when $|x|,|y| \rightarrow \infty$? Other relevant experiments would be with open or non-orientable boundary conditions. In any case, real cities have a limited number of streets, but it is necessary to know how dependent the performance of traffic controllers is on the city grid size. In Figure 8(d) the maximum capacity of the system seems to increase with $|x|,|y|$. This is an artifact of the model, since a vehicle can go into the intersection when a vehicle crossing it in the perpendicular street just left it, i.e. there can be two vehicles in contiguous cells on different streets, but not on the same street.

Since in the above simulations all cities had a length of 160 cells, more intersections implied shorter inter-street distances. To check whether the variation on the performance was due to the number of intersections and not due to the inter-street distances, we performed simulations of three different ten-by-ten scenarios, varying the street lengths. Results are shown in Figure 9. It can bee seen that the inter-street distances have no noticeable effect on the performance of both methods.

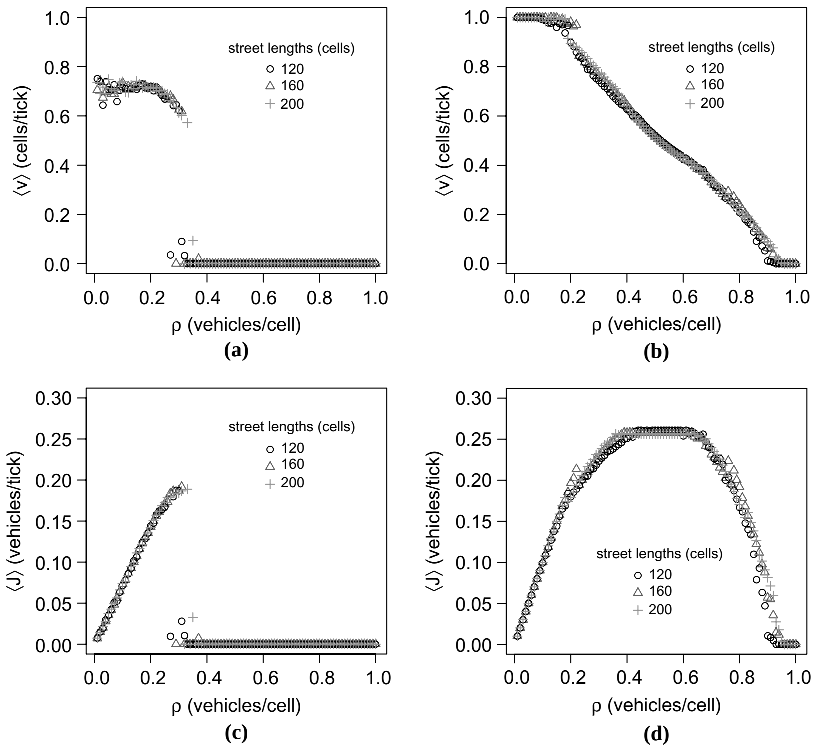


Figure 9. Simulation results varying the street lengths in ten-by-ten scenarios: (a, b) average velocity $\langle v \rangle$ and (c, d) average flux $\langle J \rangle$ for different densities ρ: (a, c) green-wave method and (b, d) self-organizing method

6. Conclusions

There are several advantages of simple traffic models. Such models are easy to implement and reproduce and are computationally cheap. Also, by abstracting most details from real traffic, one can observe properties more clearly. For example, the phase transitions we found were not visible in our previous, more realistic multi-agent simulations. The phases can be identified, but the stochastic elements of those models smoothened the transitions, which are difficult to find analytically. With our simple model it is not possible to make realistic predictions. However, it was possible to find better explanations of why the green-wave method is not efficient and why the upgraded self-organizing method delivers such a considerable improvement. Moreover, it has been shown that rule 184 realistically reproduces the main features of the flow-density diagram for highway traffic (Kanai, 2010). It is very probable that these results apply to our city traffic model, although this has to be further explored.

The optimal coordination of traffic lights is an EXP-complete problem (Papadimitriou and Tsitsiklis, 1999). Our results also showed that a successful alternative to optimization of complex problems lies in adaptation by self-organization. The self-organizing method is able to coordinate traffic flows – not necessarily optimally, but efficiently – without the intersections having any prior knowledge of the incoming vehicles. This flexibility is a great advantage in such a complex problem domain.

The potential benefits of implementing the self-organizing method are many. Improving traffic flow reduces the cost of transport, pollution and greenhouse gas emission, time lost during transit, and in general can improve the quality of life of citizens. However, caution should be taken. On the one hand, an improved traffic flow could motivate more drivers to use a personal vehicle, perhaps even worsening the traffic conditions in the long run. The solution to the traffic problem in cities lies not only better controllers. The main problem is that there are too many vehicles. Promoting alternative modes of transport and increasing the cost of personal vehicle usage are solutions that have been explored with different degrees of success in different scenarios. Nevertheless, our self-organizing method can alleviate traffic problems while better alternatives are found.

Notes

1. Only few of these are considered as self-organizing, whereas all self-organizing methods are adaptive.
2. In real highways the distance between vehicles changes with speed, while it remains constant with rule 184.
3. The inter-street distance in Manhattan is 75 m. Flow directions also alternate.
4. A boxplot is a non-parametric representation of a statistical distribution. Each box contains the following information: the median ($Q2 = x_{0.50}$) is represented by the horizontal line inside the box. The lower edge of the box represents the lower quartile ($Q1 = x_{0.25}$) and the upper edge represents the upper quartile ($Q3 = x_{0.75}$). The interquartile range ($IQR = x_{0.75} - x_{0.25}$) is represented by the height of the box. Data which are less than $Q1 - 1.5 \cdot IQR$ or greater than $Q3 + 1.5 \cdot IQR$ is considered an "outlier", and is indicated with circles. The "whiskers" (horizontal lines connected to the box) show the smallest and largest values that are not outliers.
5. This maximum capacity of intersections explains the constant flux J for different densities, as opposed to the highway case, where flux always varies with density.

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Appendix. Self-organizing algorithm

Algorithm A formally describes our self-organizing method. Each intersection uses this algorithm independently to regulate traffic, i.e. there is no direct communication between intersections. The values of the parameters used in the simulations are shown in Table AI. We performed several simulations varying these parameters without much difference, i.e. the system is robust, since its performance is not affected by modest parameter variations.

Algorithm1: self-organizing method

foreach (Δt) **do**
$t_i += \Delta t$; //local phase
$k_i += vehicles_{approachingRed}$ in d; //for rules 1 and 4
if ($vehicles_{stoppedAfterGreen}$ at e>0) **then**
if ($vehicles_{stoppedAfterRed}$ at e>0) **then**
switchBothRed$_i$(); //rule 6
end
else
switchlight$_i$(); //rule 5
end
end
else if $vehicles_{stoppedAfterRed}$ at $e == 0$ **then**
if *bothRed?* **then**
restoreSingleGreen$_i$(); // complement to rule 6
end
if ($k_i \geq 1$) **and** ($vehicles_{approachingGreen}$ in $d == 0$) **then**
switchlight$_i$(); //rule 4

Variable	Abstract value	Scaled value	Used by
Δt	One tick	1/3 s	Algorithm
n	40 vehicles · tick	13.33 vehicles · s	Rule 1
d	Ten cells	50 m	Rules 1 and 4
$t_{\min}$	Ten ticks	3.33 s	Rule 2
m	Two vehicles	Two vehicles	Rule 3
r	Five cells	25 m	Rule 3
e	Two cells	10 m	Rules 5 and 6

Table AI.
Parameters used by self-organizing method in simulations

```
18 end
19 else if not (0<vehicles_approachingGreen in r<m) then
     ;              //rule 3
20 if (t_i ≥ t_min) then
       ;            //rule 2
21 if (k_i ≥ n) then
22 switchlight_i();          //rule 1
23 end
24 end
25 end
26 end
27 end
28 switchlight_i() begin
29 k_i = 0;
30 t_i = 0;
31 switchTrafficLight_i();
32 end
```

On every tick (Δt), Algorithm A increases the phase t_i by the duration of Δt (line 2), and the counter k_i is increased by the number of vehicles approaching or waiting behind a red light within a certain distance em d (line 3). Rule 6 switches both lights to red if both streets are blocked ahead of the intersection (line 6). Rule 5 changes a green light to red if there are vehicles stopped ahead of green light at a distance e from the intersection (line 9). This prevents the accumulation of vehicles when they cannot advance, diminishing the probability of their blocking the intersection, while at the same time allowing vehicles in the crossing street (if any) to advance. Rules 5 and 6 are normally used at high vehicle densities. A single green light is restored if there are no vehicles stopped ahead of the intersection and both lights are red (line 14). All of the following rules, to determine whether the light will switch, also depend on the condition that there is free space ahead of the red light. With rule 4, if there are no vehicles approaching a green light within a distance d, and there is at least one vehicle approaching a red light ($k_i \geq 1$), this is switched, so that by the time the vehicle(s) reach(es) the intersection it(they) will not need to stop (line 17). This rule is normally used for low vehicle densities. Rule 3 prevents the "tails" of platoons from being cut, by delaying the switching of a green light when there are few vehicles (fewer than m) just about to cross, i.e. within a distance r (line 19). Still, rule 3 allows the division of long platoons, preventing the accumulation of vehicles waiting behind a red light. Rule 2 prevents the fast switching of traffic lights caused by high vehicle densities with a minimum phase $t_{\min}$ (line 20). If the conditions of rules 2 and 3 are satisfied, rule 1 changes a traffic light when the count k_i reaches a certain threshold n (lines 21-22). This makes single vehicles wait for some time, increasing the probability that more vehicles will join them and thus promoting the formation of platoons. Once platoons reach a certain size, they can request a green light before reaching the intersection, if all other conditions are met. Even if this does not occur, once the conditions are proper, the vehicles will get a green light. Thus, in principle vehicles have to wait very little time because of red lights. When a traffic light is switched (lines 28-32), the counter k_i and the phase t_i are reset (lines 29-30). The phase t_i keeps the time since the last light switch. Afterwards, the traffic lights are changed (line 31).

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