

# Self-organizing traffic lights at multiple-street intersections

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## Abstract

The elementary cellular automaton following rule 184 can mimic particles flowing in one direction at a constant speed. This automaton can therefore model highway traffic. In a recent paper, we have incorporated intersections regulated by traffic lights to this model using exclusively elementary cellular automata. In such a paper, however, we only explored a rectangular grid. We now extend our model to more complex scenarios employing an hexagonal grid. This extension shows first that our model can readily incorporate multiple-way intersections and hence simulate complex scenarios. In addition, the current extension allows us to study and evaluate the behavior of two different kinds of traffic light controller for a grid of six-way streets allowing for either two or three street intersections: a traffic light that tries to *adapt* to the amount of traffic (which results in self-organizing traffic lights) and a system of synchronized traffic lights with coordinated rigid periods (sometimes called the “green wave” method). We observe a tradeoff between system capacity and topological complexity. The green wave method is unable to cope with the complexity of a higher-capacity scenario, while the self-organizing method is scalable, adapting to the complexity of a scenario and exploiting its maximum capacity. Additionally, in this paper we propose a benchmark, independent of methods and models, to measure the performance of a traffic light controller comparing it against a theoretical optimum.

## Nontechnical Abstract

Traffic light coordination is a complex problem. In this paper, we extend previous work on an abstract model of city traffic to allow for multiple street intersections.

We test a self-organizing method in our model, showing that it is close to theoretical optima and superior to a traditional method of traffic light coordination.

**Keywords:** self-organization, adaptation, traffic lights, elementary cellular automata.

## 1 Introduction

The purpose of any model is to simplify reasoning while illuminating reality. Elementary cellular automata (Wolfram, 1986; Wuensche and Lesser, 1992; Wolfram, 2002) are especially interesting models because of being at the simplicity end of the spectrum, and for at the same time exhibiting complex behavior. The elementary cellular automaton following rule 184, for example, can simulate particles moving in one direction at a constant speed. This property suggests using such an automaton for modeling vehicular traffic. Indeed, there have been several traffic models based on cellular automata (CA) (Cremer and Ludwig, 1986; Nagel and Schreckenberg, 1992; Biham et al., 1992; Nagel and Paczuski, 1995; Fukui and Ishibashi, 1996; Chopard et al., 1996; Simon and Nagel, 1997; Chowdhury and Schadschneider, 1999; Schadschneider et al., 1999; Brockfeld et al., 2001). We are interested in modeling intersections with traffic lights so as to compare different methods of controlling such intersections. In a previous work (Rosenbluth and Gershenson, 2011) we have shown how to model traffic-light intersections with elementary cellular automata following, in addition to rule 184, different rules depending on whether or not the light is allowing vehicles to flow. In that work, we only considered rectangular grids. Our purpose will be, first, to show that elementary cellular automata can also model hexagonal grids (allowing the possibility of incorporating 3-way intersections), and second, to evaluate different traffic-light systems. Our results show both the scalability of our approach and the capacity of our method to illuminate the relationships between street topology, intersection capacity, and traffic controllers.

We evaluate two different methods of traffic light coordination: a traditional “*green wave*” method that tries to optimize phases according to expected traffic flows, and a previously proposed *self-organizing* method (Gershenson, 2005; Cools et al., 2007; Gershenson and Rosenbluth, 2010) extended to a more complex scenario. We also compare results with theoretical optimality curves that our CA model of city traffic can provide straightforwardly and with a random assignment of phases as a worst-case scenario. It is to be expected that the adaptable, *self-organizing* method will be superior to the fixed-period, *green wave* method. A reason is that, unlike the fixed-period lights, the self-organizing lights have sensors. These sensors provide feedback which should pay off resulting in a better traffic-light method. What is interesting is that, unlike the *green wave* method, which has a central control, the *self-organizing* method has no such control: The traffic lights communicate with each other using as signals the vehicles traveling from one intersection to the next.

The simulations revealed, as expected, the superiority of the *self-organizing* method over the *green wave* approach, approaching or matching the optimality curves for a broad range of densities. Moreover, the simulations also showed a number of interesting characteristics of the self-organizing method, enabling us to discover up to 10 phase transitions and close-to-optimal performance.

In the next section, we review an extension of the rule 184 model of highway traffic where intersections can be incorporated, with the possibility of modeling city traffic at a very abstract level. We extend our previous work to compare traffic light controllers with theoretical optima. In section 3, the methods for coordinating traffic lights are introduced. Section 4 presents results of simple scenarios with only three streets, while section 5 shows results of more complex scenarios. Discussion follows in section 6 and conclusions in section 7. The *self-organizing* method is detailed in Appendix A.

## 2 An elementary model of city traffic

In CA models, time and space are discrete. Vehicles are represented within cells, and rules determine how and whether vehicles “move”. The simplest highway traffic model proposed is elementary cellular automaton “Rule 184” (Yukawa et al., 1994; Chowdhury et al., 2000; Maerivoet and De Moor, 2005).

Elementary cellular automata (ECA) (Wolfram, 1986; Wuensche and Lesser, 1992; Wolfram, 2002) are Boolean one dimensional CA, i.e. cells can take values 0 or 1 and are arranged in a one dimensional array, i.e. each cell has only two nearest neighbors. The state of a cell at time  $t$  depends on its state and the state of its nearest neighbors at time  $t-1$ . Thus, the state of each cell is determined by the states of three cells. There are  $2^3 = 8$  possible combinations of values (0 or 1). The ECA “rule” is a lookup table that determines the future state of cells depending on three cells. Thus, there are 256 possible rules, although many of them are somehow equivalent. In practice there are 88 equivalence classes citeWuenscheLesser1992. Rule 184 (shown in Table 1) happens to model highway traffic flowing to the right. If there is space to the right, vehicles move there. Otherwise, they stay in their current cell. The temporal evolution of rule 184 is shown in Figure 1.

We have previously proposed a model of city traffic based on elementary cellular automata (Rosenblueth and Gershenson, 2011) on a square grid. In this section, we briefly review the model and extend it to an hexagonal grid. This allows the modeling of three-way intersections. We consider single lane streets with periodic boundaries that can go in six different directions. Figure 2 shows a screenshot segment of a simulation implementing our city traffic model on an hexagonal grid.

The behavior of vehicles on streets is already modeled with rule 184. Depending on how neighbors are chosen in the hexagonal grid, the direction of the streets can be set. To model intersections and traffic lights, we consider several coupled non-homogeneous ECA, where rules change around the intersection, depending on the state of the traffic light. The CA system is conservative (Moreira, 2003), i.e. the number of vehicles (1’s) remains constant.

If a street has a green light, all its cells use rule 184. If there is a red light, all cells on the street also use rule 184, with two exceptions: The cell immediately before the intersection has to stop traffic from going into the intersection. This is achieved with rule 252. The cell immediately after the intersection has to allow vehicles to leave, but not to allow vehicles in the intersection (flowing in a different direction) to enter the cell. This is achieved by rule 136. Table 1 lists the transition tables for the three rules used by the model.

The intersection cell is a special case, as it has six neighbors. The rule never changes (184). What changes is the neighborhood, i.e. it takes as nearest neighbors only the two

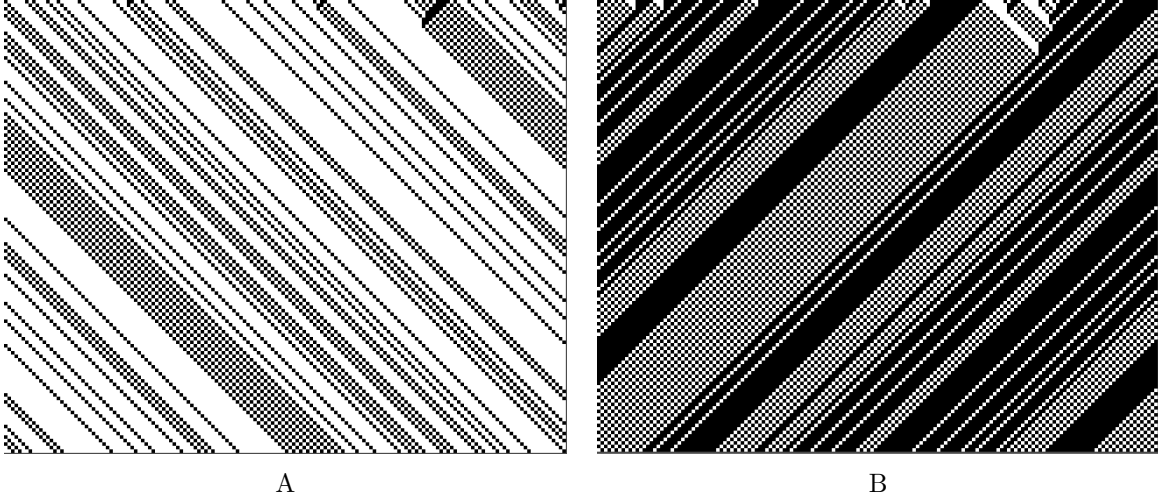


Figure 1: Evolution of rule 184. Black cells (1) represent vehicles, white cells (0) represent spaces. Traffic flows to the right, time flows to the bottom: (A) In the *free-flow* phase (density  $\rho \leq 0.5$ ,  $\rho = 0.25$  shown) all vehicles flow at a velocity of one cell per tick (time step). (B) In the *jammed* phase ( $\rho > 0.5$ ,  $\rho = 0.75$  shown) jams move to the left, as vehicles can only advance when there is a free space ahead of them.

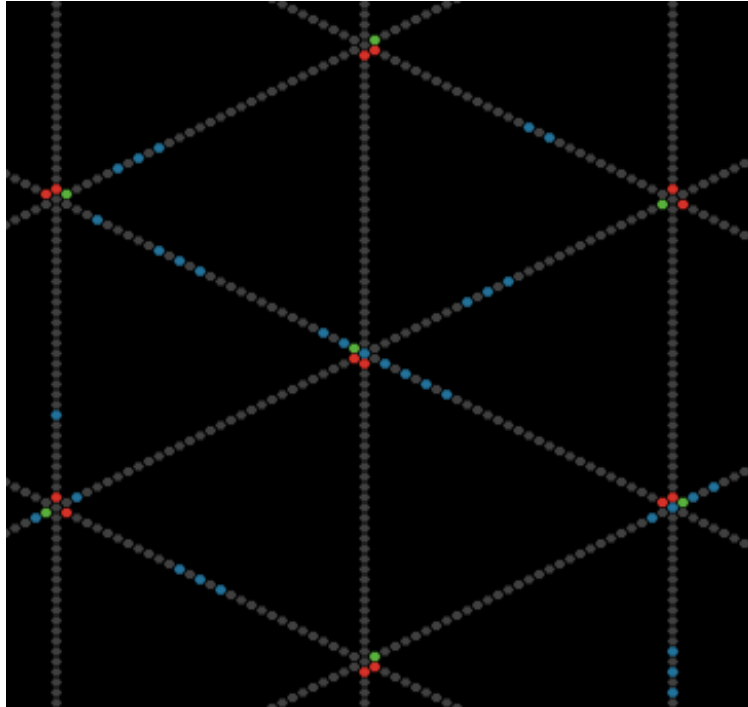


Figure 2: Screenshot segment of hexagonal grid (density  $\rho = 0.05$ ). Sky blue cells represent vehicles, while dark gray cells represent free spaces on streets. Black cells are not considered in the model.

cells in the street with a green light (also using rule 184). A diagram of the cells around an intersection is shown in Figure 3.

Table 1: ECA rules used in model

| $t - 1$ | $t_{184}$ | $t_{252}$ | $t_{136}$ |
|---------|-----------|-----------|-----------|
| 000     | 0         | 0         | 0         |
| 001     | 0         | 0         | 0         |
| 010     | 0         | 1         | 0         |
| 011     | 1         | 1         | 1         |
| 100     | 1         | 1         | 0         |
| 101     | 1         | 1         | 0         |
| 110     | 0         | 1         | 0         |
| 111     | 1         | 1         | 1         |

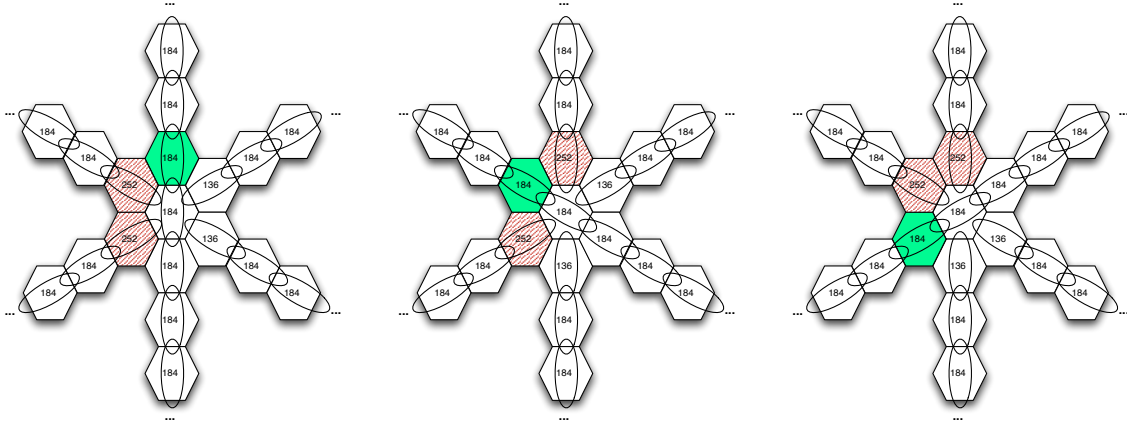


Figure 3: Diagram for different rules (shown within cells) and neighborhoods (indicated by ovals) used around intersections, depending on the state of the traffic light. For green lights (indicated by **green** solid cells), rule 184 is used, and the intersection cell has as neighbors cells in the street with the green light. For red lights (indicated by diagonally **red** striped cells), rule 252 is used for the cell immediately before the intersection and rule 136 for the cell immediately after the intersection. The rest of the cells use rule 184.

If at time  $t$  a traffic light is meant to switch, the model needs to ensure that the intersection cell is empty. Otherwise, the vehicle in the intersection would “turn” into the crossing street. To avoid this situation, the actual switching of rules and neighborhood is made only when the intersection is cleared.

## 2.1 Measures

The behavior of the model will depend strongly on the vehicle density  $\rho \in [0, 1]$ . The density can be easily calculated by dividing the number of cells with a 1 (i.e. total number of vehicles,  $\sum s_i$ ) by the total number of cells ( $|S|$ ):

$$\rho = \frac{\sum s_i}{|S|} \quad (1)$$

The performance of the system can be measured with velocity  $v \in [0, 1]$ , which is simply the number of cells that changed from 0 to 1 over the total number of vehicles:

$$v = \frac{\sum (s'_i > 0)}{\sum s_i} \quad (2)$$

where  $s'_i$  is the derivative of state  $s_i$ .

The flux of the system represents how much of the space is used by moving vehicles. It can be obtained by multiplying the vehicle density by the velocity:

$$J = \rho v \quad (3)$$

The vehicular flow capacity of a triple intersection ( $J_{max} = 1/6$ ) is 50% lower than that of a double intersection ( $J_{max} = 1/4$ ), since intersection time has to be shared with a third street.

## 2.2 Theoretical Optima

When coordinating traffic lights, the best performance that can be theoretically achieved would be a system in which each intersection has the best performance possible of an isolated intersection. A lower performance implies that there is *interference* between traffic lights. Given the properties of our model of city traffic, equations 4 and 5 describe the optimal curves for single intersections of velocity and flux, that depend on the maximum flux  $J_{max}$  allowed by an intersection :

$$v_{optim} = \begin{cases} 1 & \text{if } \rho \leq J_{max} \\ \frac{1-\rho}{\rho} & \text{if } 1 - J_{max} \leq \rho \\ J_{max}/\rho & \text{if } J_{max} < \rho < 1 - J_{max} \end{cases} \quad (4)$$

$$J_{optim} = \begin{cases} \rho & \text{if } \rho \leq J_{max} \\ 1 - \rho & \text{if } 1 - J_{max} \leq \rho \\ J_{max} & \text{if } J_{max} < \rho < 1 - J_{max} \end{cases} \quad (5)$$

If  $\rho \leq J_{max}$ , then the intersection can support a maximum velocity of 1 cells/tick. Following equation 3, the flux will then be equal to the density  $\rho$ , as all vehicles are moving. If  $J_{max} < \rho < 1 - J_{max}$ , then the flux of the intersection will be restricted by the maximum capacity of the intersection, i.e.  $J_{max}$ . This implies that vehicles will be using the intersection at all times, and the average velocity will be  $J_{max}/\rho$ . If  $1 - J_{max} \leq \rho$ , the density of the streets is so high that it restricts the flow of vehicles on streets, reducing the flux to  $1 - \rho$  and the velocity to  $\frac{1-\rho}{\rho}$ .

Notice that the flux optimum  $J_{optim}$  is symmetric, since there is a symmetry in our city traffic model between vehicles (1's) moving in one direction and spaces (0's) moving in the opposite direction. This is also observed in the rule 184 model of highway traffic (Kanai, 2010).

The “interference”  $\Phi$  between traffic lights can thus be measured with the difference of the integrals of the optimal and experimental curves:

$$\Phi_v = \int_{\rho_{min}}^{\rho_{max}} v_{optim} - v \quad (6)$$

for velocity, and

$$\Phi_J = \int_{\rho_{min}}^{\rho_{max}} J_{optim} - J \quad (7)$$

for flux.

Note that these equations are not normalized, i.e. different values of  $J_{max}$  will yield different values for  $\int v_{optim}$  and  $\int J_{optim}$ .

The Greek capital letter  $\Phi$  was chosen because the proposed measure of interference is related to the concept of “friction”  $\varphi$  (Gershenson, 2007, 2011), which measures the negative interaction between components of a system. This was proposed in the context of a general methodology for the design and control of self-organizing systems. Interference  $\Phi$  is a global average measuring different negative interactions between all intersections that affect the system performance.

## 2.3 Scales

Even when the time and space are abstract and discrete, it can be assumed that one cell represents five meters, roughly the space occupied by a vehicle. Thus, one kilometer of a street is represented by 200 cells. If each tick, i.e. time step, represents one third of a second, then a velocity of one cell per tick is equivalent to 15 m/s, i.e. 54 km/h, equivalent to the speed limit in cities. A maximum density of  $\rho = 1$  is equivalent to 200 vehicles per kilometer.

## 3 Methods for coordinating traffic lights

We briefly present in this section two methods for controlling traffic lights, that are more fully described in Gershenson and Rosenblueth (2010).

The optimal coordination of traffic lights is an EXP-complete problem (Papadimitriou and Tsitsiklis, 1999; Lämmer and Helbing, 2008). This implies that it is intractable. There have been several methods proposed to solve this problem. We can distinguish two main approaches. One establishes fixed periods and phases that would maximize the flux for an expected traffic flow (Federal Highway Administration, 2005; Robertson, 1969; Gartner et al., 1975; Sims and Dobinson, 1980; Török and Kertész, 1996; Brockfeld et al., 2001). The other one tries to *adapt*—manually or automatically—periods and phases depending on current traffic flows (Federal Highway Administration, 2005; Henry et al., 1983; Mauro and Di Taranto, 1990; Robertson and Bretherton, 1991; Faieta and Huberman, 1993; Gartner et al., 2001; Diakaki et al., 2003; Fouladvand et al., 2004; Mirchandani and Wang, 2005;

Bazzan, 2005; Helbing et al., 2005; Gershenson, 2005; Cools et al., 2007; Gershenson and Rosenblueth, 2010). A third conceivable approach would simply change the lights with fixed periods but random phases, i.e. no coordination between traffic lights. We implemented three methods, one corresponding to each approach: a *green-wave* method that tries to maximize the flux by controlling the phases, for an expected traffic flow, a *self-organizing* method that adapts to the current traffic conditions, and a *random* method.

### 3.1 The *green-wave* method

The idea behind the *green-wave* method (Török and Kertész, 1996) is the following: if the consecutive traffic lights switch with an offset (i.e. delay) equivalent to the expected vehicle travel time between intersections, vehicles should not have to stop. Thus, waves of green light move through the street at the same velocity as vehicles. This is the most commonly used method for coordinating traffic lights.

This method has advantages, e.g. when most of the traffic flows in the direction of the green wave at low densities. On an hexagonal grid, only one direction can have vehicles without stopping ( $v = 1$ ). Vehicles on two other directions stop briefly each time around the cyclic boundaries. Vehicles on the three other directions have to stop considerably. Moreover, if traffic is flowing at velocities lower than expected, the green waves will go faster than vehicles and these will be delayed.

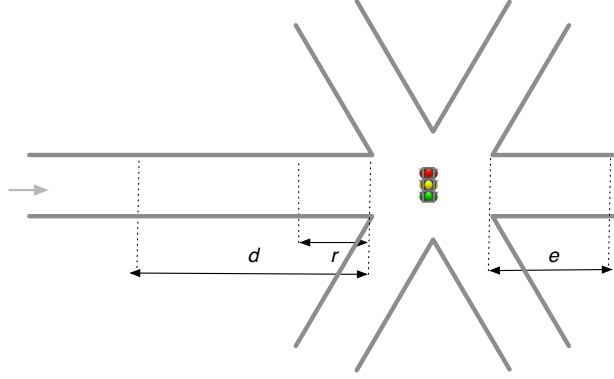
### 3.2 The *self-organizing* method

With the *self-organizing* method, each intersection independently follows the same set of rules, based only on local traffic information. There are only six rules (not related to ECA rules), with higher-numbered rules overriding lower-numbered ones. The full rule set is given in Table 2. This method is an improvement over previous work. The one reported in (Ball, 2004) considered only rules 1 and 2, while (Gershenson, 2005; Cools et al., 2007), considered only rules 1–3. For a detailed description of the model, please refer to Gershenson and Rosenblueth (2010). Here the rules are generalized naturally for several incoming streets per intersection. Details of the algorithm are presented in Appendix A.

The main idea is the following: each intersection counts how many vehicles are behind red lights (approaching or waiting). Each time interval, the vehicular count is added to a counter which represents the integral of vehicles over time approaching the intersection on each direction with a red light. When one of these counters reaches a certain threshold, the red light switches to green (rule 1 in Table 2). If there are few vehicles approaching, the counter will take longer to reach the threshold. This increases the probability that more vehicles will aggregate behind those already waiting, promoting the formation of “platoons”. The more vehicles there are, the faster they will get a green light. Like this, platoons of a certain size might not have to stop at intersections. There are other simple rules to ensure a traffic smooth flow. The method adapts to the current traffic density and responds to it efficiently: For low densities, almost no vehicle has to stop. For medium densities, intersections are used at their maximum capacity, i.e. there are always vehicles crossing intersections, there is no wasted time. For high densities, most vehicles are stopped, but



Table 2: *Self-organizing* traffic light rules, independently followed by each intersection. Inset: Schematic of a triple intersection, indicating for the horizontal street distances  $d$ ,  $r$ , and  $e$  used for self-organizing lights.



1. On every tick, add to counters  $k_j$  the number of vehicles approaching or waiting at red lights within distance  $d$ . When the first counter exceeds a threshold  $n$ , switch the light. (There are separate counters for each incoming direction  $j$ . Whenever the light switches, reset the counter for that direction to 0.)
2. Lights must remain green for a minimum time  $u$ .
3. If a few vehicles ( $m$  or fewer, but more than zero) are left to cross a green light at a short distance  $r$ , do not switch the light.
4. If no vehicle is approaching a green light within a distance  $d$ , and at least one vehicle is approaching a red light within a distance  $d$ , then switch that light to green.
5. If there is a vehicle stopped on the road a short distance  $e$  beyond a green traffic light, i.e. traffic is blocked upstream, then switch that light to red. Switch to green the direction with a highest value in counter  $k_j$  and no blockage upstream.
6. If there are vehicles stopped on all directions at a short distance  $e$  beyond the intersection, then switch all lights to red. Once one of the directions is free, restore the green light in that direction.

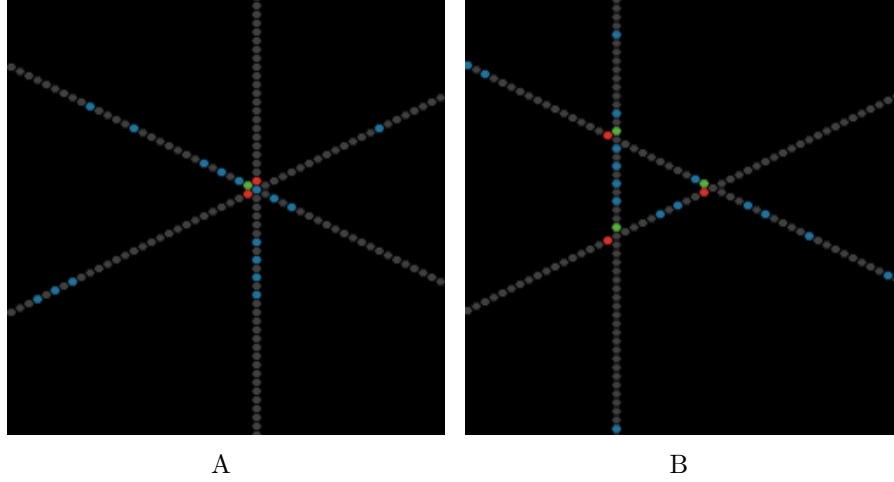


Figure 4: Studied scenarios involving three streets: (A) one triple intersection and (B) three double intersections. For both scenarios, the length of cyclic streets is 180 cells. Sky blue cells represent vehicles, while dark gray cells represent free spaces on streets.

gridlock is avoided with simple rules that coordinate the flow of “free spaces” in the opposite direction of traffic, allowing vehicles to advance (rules 5 and 6).

Intersections with only two incoming streets will have one green and one red light, or both lights red. The algorithm was extended naturally for more than two incoming streets. Only one street (or none) will have a green light. In this way, every direction  $j$  with a red light will keep a counter  $k_j$  quantifying incoming vehicles. The first one to reach the threshold will request the green light and be reset. Other directions will not reset their counter  $k_j$ , so they will request a green light soon if there is the demand for it. For other rules, the street with highest demand (measured with  $k_j$ ) will usually get the preference, unless traffic is blocked upstream. Notice that if there are e.g. three incoming directions A, B, and C, there is no prefixed sequence of which direction will get a green light. This will depend entirely on the actual traffic conditions. An example switching pattern could be A, B, A, C, B, A, C, B, C, B, A, B... The periods are also variable (with the constraint of a minimum green time), and depend on the traffic demand. Each switching usually has a different green period.

## 4 Three streets: one or three intersections?

In this section, we compare a scenario with a single triple intersection with another scenario with three double intersections. We developed a computer simulation in NetLogo (Wilensky, 1999). The reader is invited to access the simulation via web browser at the URL <http://turing.iimas.unam.mx/~cgg/NetLogo/4.1/trafficHexCA.html> (for short, <http://tinyurl.com/tHexCA>). The environment consists of three cyclic streets, i.e. with periodic boundaries. Each street has a length of 180 cells. We explore two scenarios: one where the three streets meet at a single cell, i.e. one triple intersection, and another one consisting of three double intersections with a distance of 11 cells between intersections. A section of the simulation showing the intersection arrangements of both scenarios can be seen in Figure 4.

For the experiments, each run consisted of the following: Half an hour of virtual time

(as opposed to real time), corresponding to 5,400 ticks (see subsection 2.3) was simulated for random initial conditions. Since the vehicles are placed randomly, one street may have a slightly higher density than another. After this initial half an hour of virtual time, the system is considered to have stabilized, i.e. gone through a transient, so another half an hour is simulated, of which the average velocity of all vehicles is measured at every tick (see equation 2). At the end of the simulation, the velocities of the second half an hour are averaged to obtain the average velocity  $\langle v \rangle$  and average flux  $\langle J \rangle$  for a given density  $\rho$  (see equation 3). The results are shown in Figure 5. Since successive simulations show low standard deviations—apart from phase transitions—only single runs per density are shown for clarity.

For both scenarios, we compared a *fixed-period* method with the *self-organizing* method. The fixed method used a period of  $T = 180$  ticks. For the triple intersection scenario, this period  $T = 180$  implies that each direction will have a green phase of 60 ticks. Moreover, since the length of streets is equal to the period, a vehicle can in principle *free flow* in any direction, i.e.  $v = 1$ . This is the case for densities  $\rho \leq 1/6$ . Afterwards, there is no possibility to let all vehicles flow freely, so there is a phase transition into *intermittent* flow, where some vehicles have to stop. This phase is characterized by a maximum flux  $J = 1/6$ , given by the triple intersection. After a density of  $\rho \approx 0.8$ , the waiting queues grow long enough to block the intersection while another street should have a green light. This is noticeable in the abrupt decrease in velocity and flux. We call this phase “*interfered*”, since the queue of one street interferes with and blocks another street.

For the scenario with the three double intersections, the period  $T = 180$  implies that each direction will have a green phase of 90 ticks. This increases the capacity of the intersections by 50%. However, the coordination of the traffic lights now has an effect on the flux. In these simulations, all three traffic lights changed at the same time, i.e. there was no green wave. This restricts the *free flow* phase for very small densities ( $\rho \lesssim 0.05$ , depending on initial conditions). The *intermittent phase* does not reach a maximum possible flux of  $J = 0.5$ , although the flux is slightly higher than for the triple intersection scenario. Still, the transition into the *interfered* phase occurs at a lower density.

For the *self-organizing* method (subfigures 5B and 5D), the scenario with a single triple intersection performs for most densities as the *fixed-period* method, since there is no coordination of traffic lights involved, and the maximum flux is determined by the intersection, not so much by the controller method. There is a *free-flow* phase for  $\rho \leq 1/6$ , and then an *intermittent* phase until  $\rho \leq 5/6$ . However, for high densities ( $\rho \geq 5/6$ ), the intersection is not blocked due to rules 5 and 6. Therefore, there is no abrupt decrease in flux. When a street is blocked ahead, another one without blockage is given a green light. Thus, for densities  $\rho \geq 5/6$  there is a “*quasi-gridlock*” phase, where most vehicles are stopped, but flow is not stopped. In fact, “free-spaces” move in the direction opposite of the vehicles with a velocity close to  $-1$  cells/tick.

For the scenario with three double intersections, it can be seen that the *self-organizing* method manages to coordinate the traffic flows, and the performance is very close to optimal. There is *free-flow* until  $\rho \approx 1/4$  and then *intermittent* flow until  $\rho \approx 3/4$ . For higher densities ( $\rho \gtrsim 3/4$ ), the *interfered* phase is also avoided, giving place to *quasi-gridlock*.

It is interesting to note that there is a symmetry in the flux diagram for both scenarios. The flux diagram of the rule 186 model of highway traffic is also symmetric (Kanai, 2010).

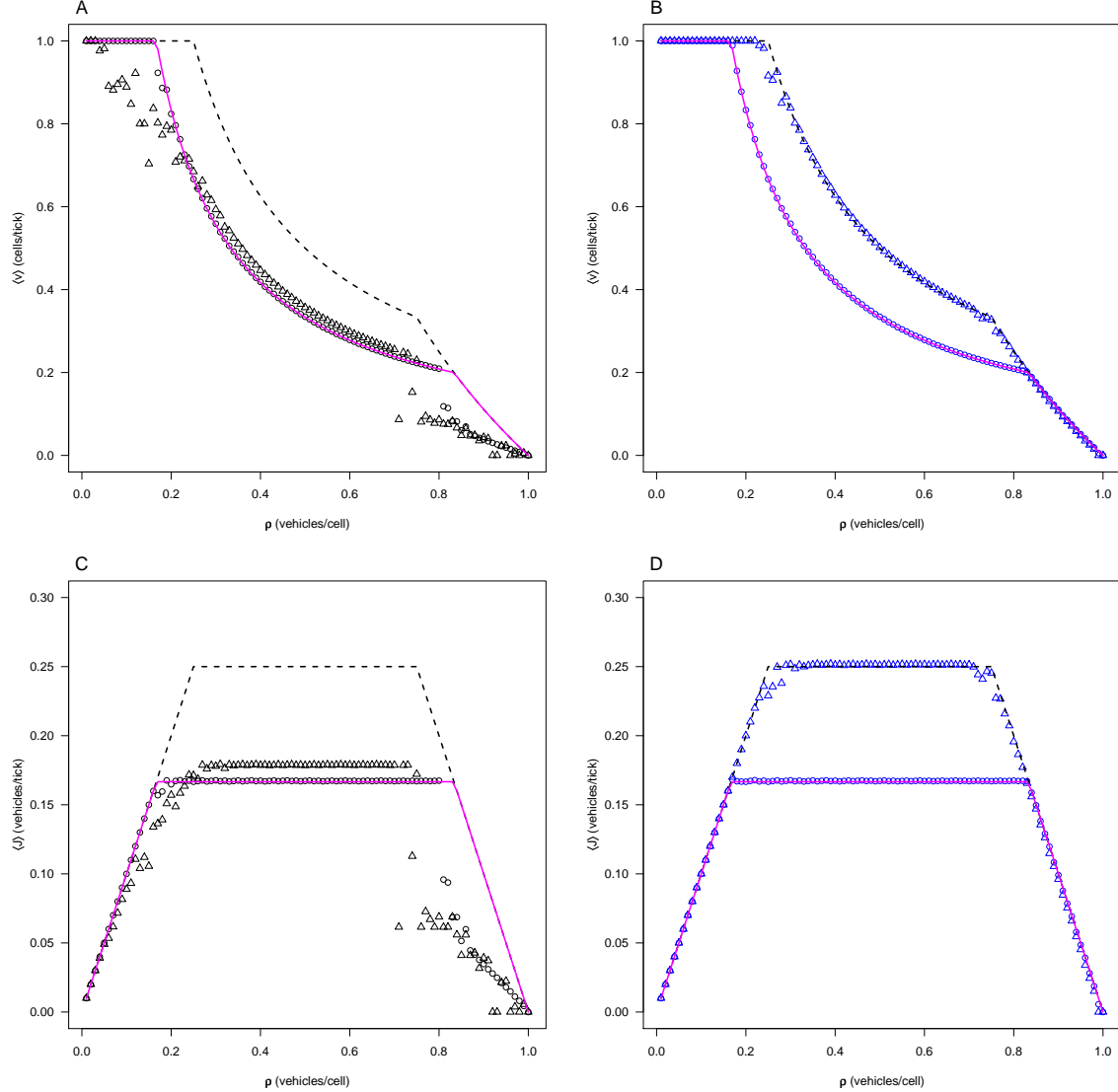


Figure 5: Simulation results for a single intersection: (A,B) average velocity  $\langle v \rangle$  and (C,D) average flux  $\langle J \rangle$  for different densities  $\rho$ . Black points indicate fixed traffic lights (A,C), while blue points indicate self-organizing traffic lights (B,D), in scenarios with one triple intersection ( $\circ$ ) and three double intersections ( $\triangle$ ). Optimality curves are shown with a magenta continuous line (for single triple intersection scenario) and a with black dashed line (for three double intersections scenario).

This is given by the fact that there is a certain duality between vehicles (1) and spaces (0) in these models.

For the *self-organizing* method, the flux is the same for both scenarios at  $J < 1/6$ . This further suggests that the restrictions on traffic flow depend on the street topology and not on the *self-organizing* method, which adapts to both scenarios.

In the next section, we study more complicated scenarios, where a large number of intersections has to be coordinated.

## 5 18 streets: 36 or 108 intersections?

To study the coordination of several traffic lights, we simulated 18 cyclic streets of 180 cells each, three of which have traffic flowing in each of six directions. In one scenario, shown in Figure 6, the streets were setup to cross at 36 triple intersections, having 30 cells between intersections, i.e. one block is 30 cells. In a second scenario, shown in Figure 7, the setup was such that streets crossed at 108 double intersections, with heterogeneous block distances of 11 or 19 cells.

The same simulation developed in NetLogo and experimental setup as the one described in the previous section were used. The simulation is available at the URL <http://tinyurl.com/tHexCA>. Results are shown in Figure 8. As a worst case scenario, a *random* method is included in these simulations, where each traffic light has a fixed time period with randomly assigned offsets, i.e. no correlation between traffic lights.

It can be seen that the *green-wave* method offers a poor performance. Having six directions, only one can be fully coordinated with the green wave. Two other directions are partially coordinated, but three are anti-coordinated with the green-wave. Even when theoretically the scenario with double intersections should be more efficient—given the fact that intersections have 50% more capacity—the performance difference with the triple intersection scenario is minimal for low densities. Moreover, for densities  $\rho \approx 0.2$ , the performance in the triple intersection scenario is more efficient. Still, the *green-wave* method collapses into gridlock for medium densities ( $\rho \gtrsim 0.3$ ). This is because traffic flowing against the green wave accumulate in long queues that block intersections upstream, eventually leading to partial or complete gridlocks. This performance is comparable with that of a *random* method, where no traffic light is correlated.

For the *self-organizing* method, at low densities ( $\rho \leq 0.05$ ) free flow is reached in both scenarios, i.e. vehicles are coordinated in six directions and  $v = 1$ , i.e. no vehicle has to stop. This is of course an artifact of the cyclic boundaries, but it illustrates the coordination capacity of the *self-organizing* method. As the density increases, the average velocity decreases gradually (Figure 8A). Gridlock is reached only for very high densities, where initial conditions already block intersections.

There is a noticeable difference in the performance of the *self-organizing* method in both scenarios. In both of them a maximum flux is reached according to the intersection type:  $J = 1/6$  for triple intersections and  $J = 1/4$  for double intersections (Figure 8B)<sup>1</sup>. A

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<sup>1</sup>The supraoptimal results ( $J > J_{max}$ ) are an artifact of the city traffic model (Rosenblueth and Gershenson, 2011). Rule 184 assumes a  $J_{max} = 0.5$  for single streets since a free space is required between occupied cells for vehicles to flow. However, in intersections, this requirement can be relaxed once a light is switched,

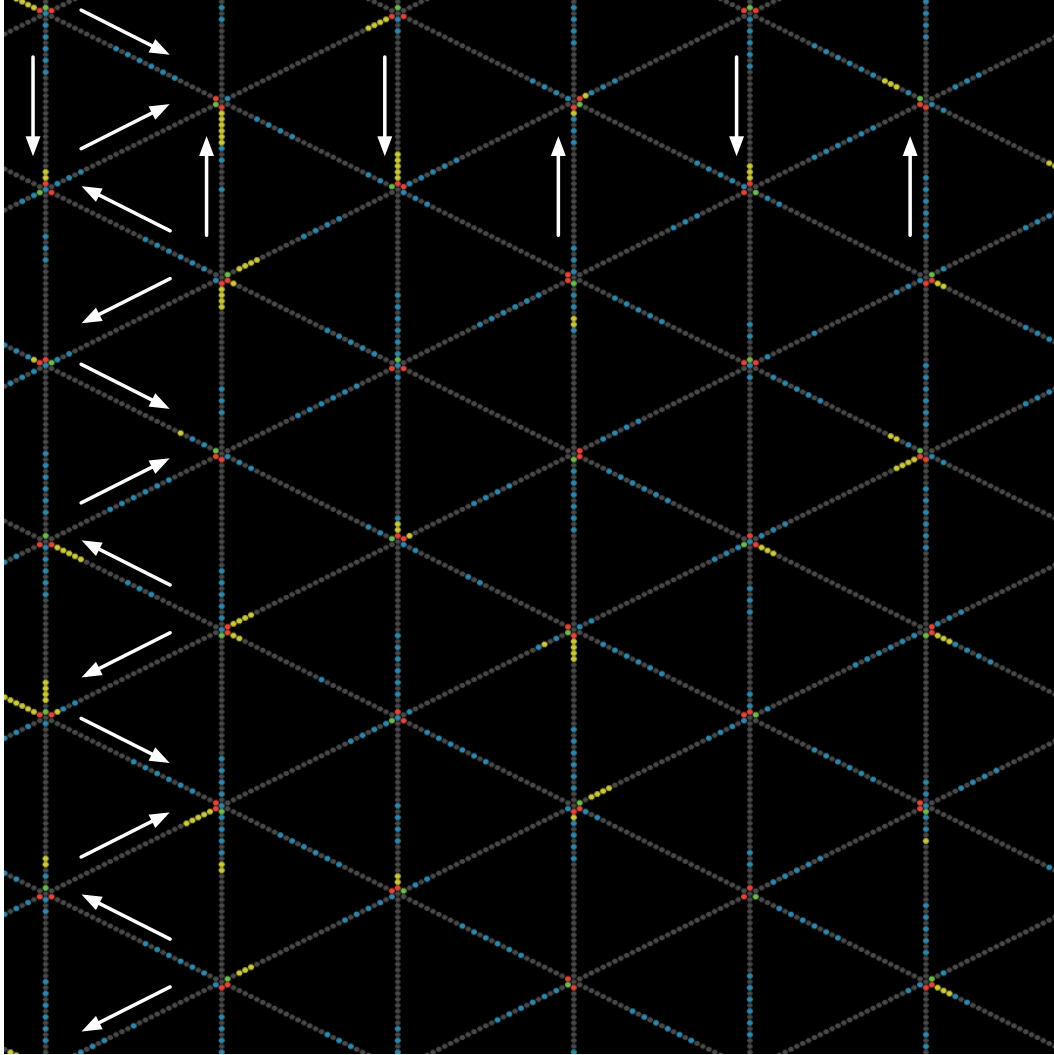


Figure 6: Screenshot of scenario with 18 streets and 36 triple intersections,  $\rho = 1/5$ . Moving vehicles are colored with sky blue and stopped vehicles are colored in yellow.

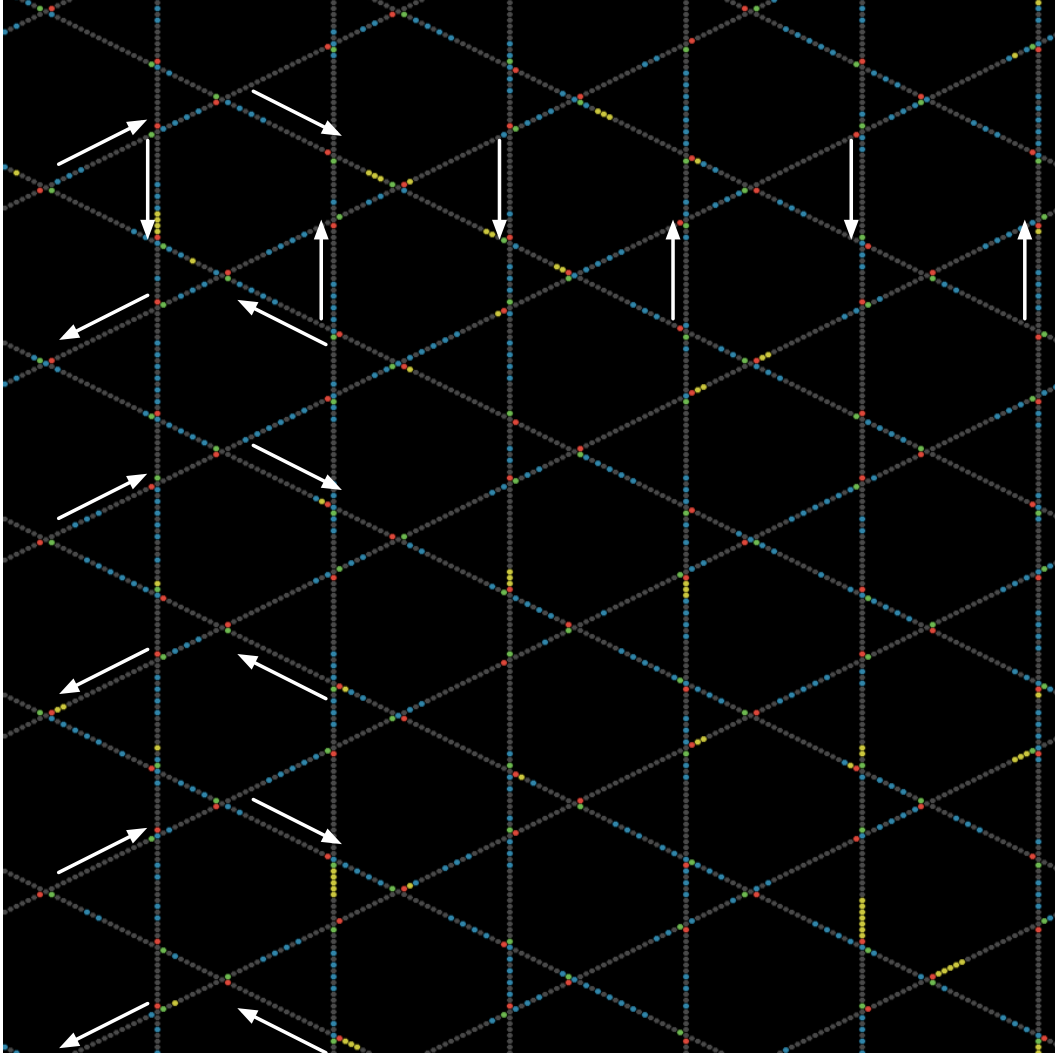


Figure 7: Screenshot of scenario with 18 streets and 108 double intersections,  $\rho = 1/5$ . Moving vehicles are colored with sky blue and stopped vehicles are colored in yellow.

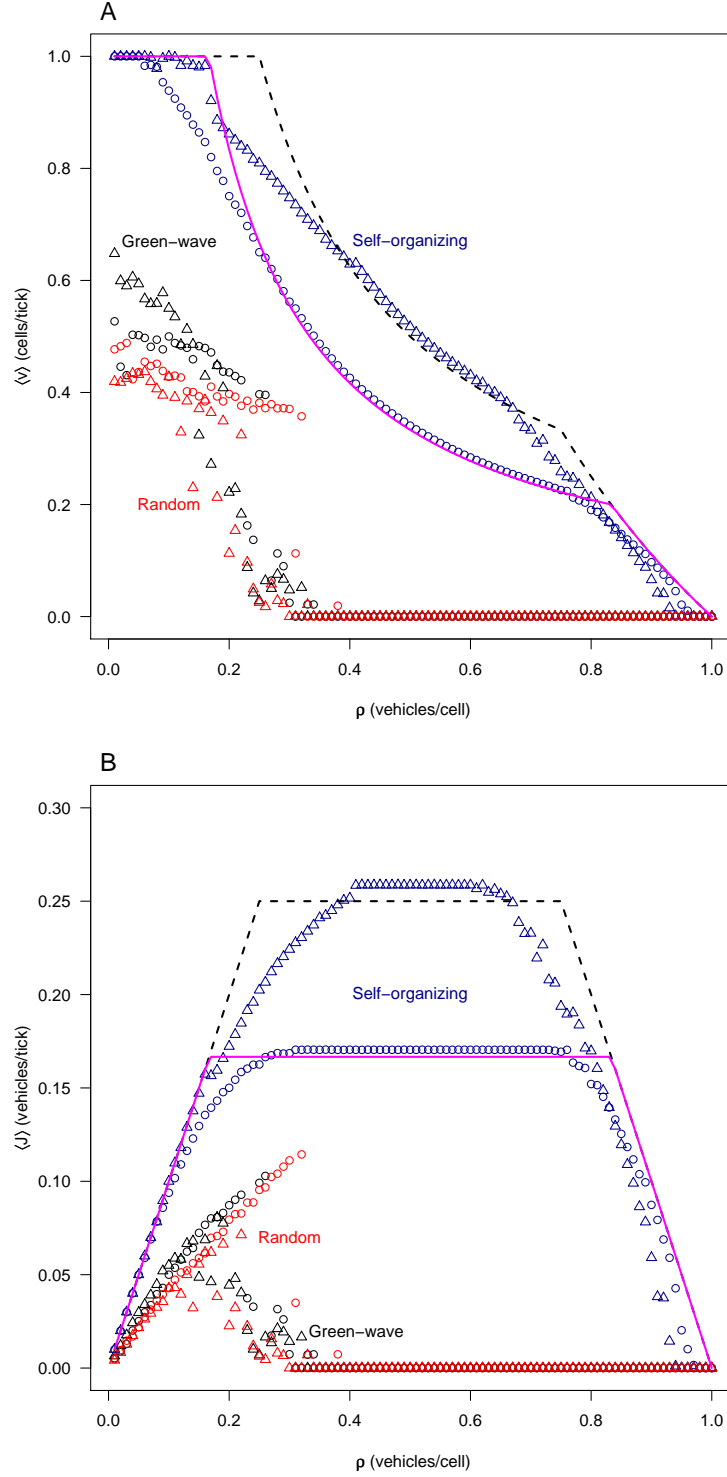


Figure 8: Simulation results for 18 street scenarios: (A) average velocity  $\langle v \rangle$  and (B) average flux  $\langle J \rangle$  for different densities  $\rho$ , *green-wave* (black), *self-organizing* (blue), and *random* (red) methods. Two scenarios are considered: 36 triple intersections ( $\circ$ ) and 108 double intersections ( $\triangle$ ). Optimality curves are shown with a magenta continuous line (for single triple intersection scenario) and a with black dashed line (for three double intersections scenario).



good performance is achieved, restricted by the street topology, as opposed to the *green-wave* method, where inefficient flow is produced independently of the street topology, i.e. it cannot exploit the increased capacity of double intersections because it cannot coordinate so many directions and intersections. In contrast, the *self-organizing* method is able to coordinate many directions and has shown to be highly scalable.

The triple intersection scenario is less deviated from the optimal curves, indicating that it is computationally easier to coordinate less intersections. Still, the double intersection scenario achieves a better performance, in spite of deviating more from the optimality curves, since it is more complicated to coordinate more intersections.

A clearer understanding of the performance of the *self-organizing* method will be obtained by analyzing the dynamical phases that are found as the density changes.

## 5.1 Dynamical Phases

The *green-wave* method has only two dynamical phases in both scenarios: *intermittent* (some vehicles move, some are stopped) and *gridlock* (all vehicles are stopped,  $v = 0$ ).

Phase transitions for the *self-organizing* method are indicated in the flux diagrams for both scenarios in Figure 9.

For the triple intersection scenario (Figure 9A), the same seven phases as with a square grid were observed (Gershenson and Rosenbluth, 2010): *free-flow*, *quasi-free-flow*, *underutilized intermittent*, *full capacity intermittent*, *overutilized intermittent*, *quasi-gridlock*, and *gridlock* (to be described below). Still, the phase transitions occur at different densities.

For the double intersection scenario (Figure 9B), apart from the phases already detected, further subphases appeared, with a total of ten phase transitions.

**Free-flow** ( $\rho \leq 0.06$ ). In this phase, all vehicles have a maximum velocity  $v = 1$ . This implies a perfect coordination between all six different flow directions.

**Quasi-free-flow** ( $0.07 \geq \rho \geq 0.16$ ). Most vehicles are flowing, but some have to stop momentarily, when platoons coincide at a given time at an intersection.

**Intermittent** ( $0.17 \geq \rho \geq 0.79$ ). Some vehicles stop while others flow. It can be subdivided into three subphases: underutilized, full capacity, and overutilized.

**Underutilized** ( $0.17 \geq \rho \geq 0.4$ ). This phase is characterized by a partial utilization of the intersection's full capacity, i.e.  $J < J_{max}$  ( $J_{max} = 1/6$  for triple intersections,  $J_{max} = 1/4$  for double intersections).  $J_{max}$  implies that at every intersection vehicles are crossing all the time. The underutilized intermittent phase has a flux  $J < J_{max}$  because there are not enough vehicles to reach  $J_{max}$ , i.e. intersections are not utilized at their maximum capacity. Still, flux  $J$  is higher than the maximum flux  $J_{max}$  for triple intersections. This subphase can be further subdivided in three subphases for the double intersection scenario: no propagation, inter-platoon propagation, and upstream propagation.

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i.e. if a light changes, a vehicle can occupy an intersection the tick after a vehicle going on another street crossed the intersection. This slightly increases the analytical maximum intersection capacity, depending on the traffic light switching frequency.

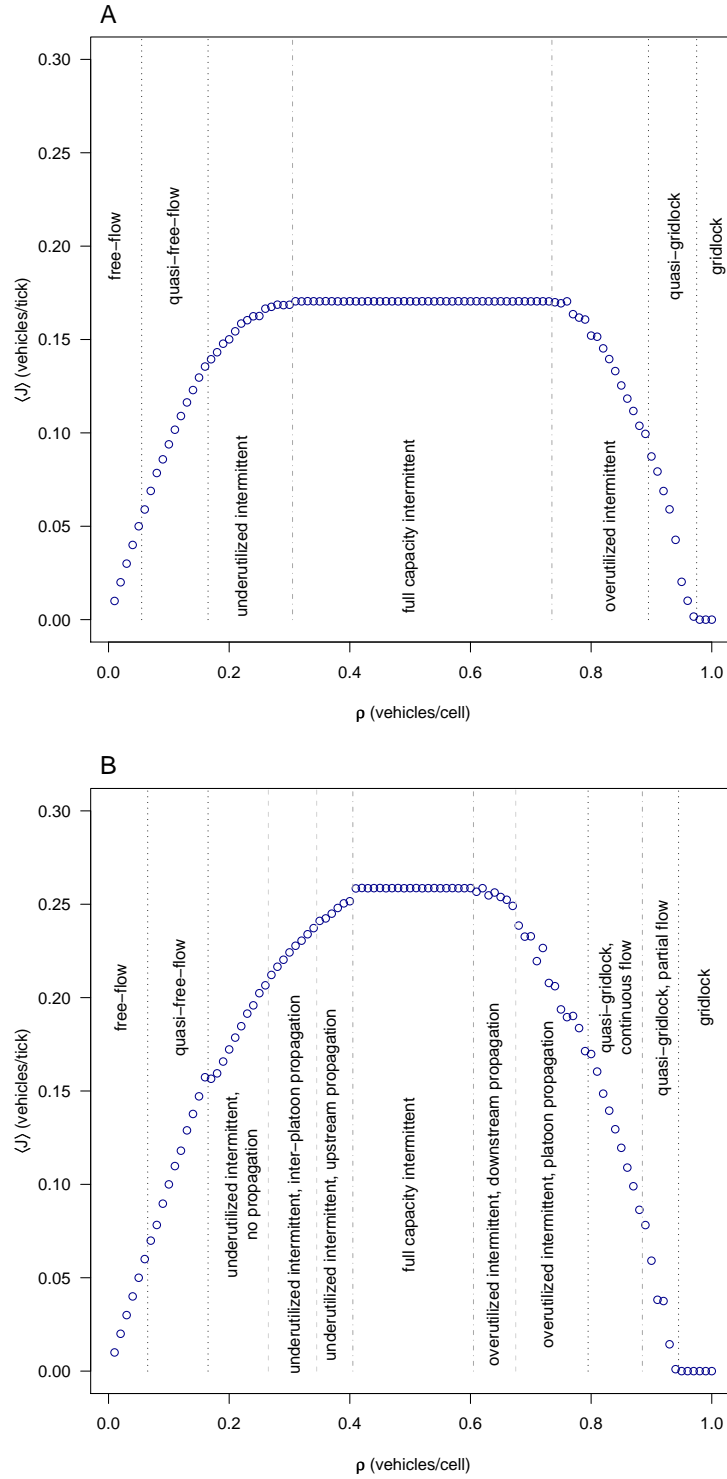


Figure 9: Phase transitions shown in flux diagrams: (A) 36 triple intersections scenario and (B) 108 double intersections scenario.

**No propagation** ( $0.17 \geq \rho \geq 0.26$ ). Queues form, but these do not propagate, i.e. after the simulation has stabilized, queues do not propagate between platoons: when a platoon has to wait behind a red light, it will quickly get a green light and its last vehicle will be moving before another vehicle reaches it and stops.

**Inter-platoon propagation** ( $0.27 \geq \rho \geq 0.34$ ). Queues form, and in some cases platoons reach a stopped platoon, causing vehicles to shift platoons.

**Upstream propagation** ( $0.35 \geq \rho \geq 0.4$ ). Queues can be long enough to reach an upstream intersection, potentially delaying vehicles more than one block upstream.

**Full capacity** ( $0.41 \geq \rho \geq 0.6$ ). There is intermittent traffic flow but  $J = J_{max}$ , i.e. all intersections are utilized at their maximum capacity, with vehicles crossing constantly. Certainly, average velocities  $\langle v \rangle$  decrease as density  $\rho$  increases.

**Overutilized** ( $0.6 \geq \rho \geq 0.79$ ). This subphase of the intermittent phase is characterized also by a flux  $J < J_{max}$ , but for reasons opposite to those for the underutilized subphase. At these high densities, rule 6 sometimes blocks all directions of an intersection, preventing any vehicle from crossing. This is not due to lack of vehicles, but to excess of vehicles. In fact, more than half of vehicles are stopped at any given moment. Thus, it is useful to focus on the movement of “free space” between vehicles, which occurs in the opposite direction of traffic. Free spaces also group in “space platoons”. The dynamics of the space platoons are complementary to those of platoons in the underutilized intermittent subphase. In both subphases, flux  $J$  is greater than maximum flux  $J_{max}$  for triple intersections. The overutilized intermittent subphase can be further subdivided in two subphases for the double intersection scenario: downstream propagation and platoon propagation.

**Downstream propagation** ( $0.6 \geq \rho \geq 0.67$ ). In this subphase, space platoons can be long enough to cover more than one intersection. In this way, a space platoon can sometimes propagate traffic flow downstream, so that vehicles may cross two intersections without stopping.

**Platoon propagation** ( $0.68 \geq \rho \geq 0.79$ ). Space platoons are shorter in this subphase, so vehicles need to stop at least once before crossing any intersection.

**Quasi-gridlock** ( $0.8 \geq \rho \geq 0.94$ ). Almost all vehicles are stopped, but rule 6 prevents gridlock. Short space platoons are formed, and they rarely have to “stop” flowing opposite to the vehicles’ direction. This phase is complementary to the quasi-free flow phase. This phase can be further subdivided in two subphases for the double intersection scenario: continuous flow and partial flow.

**Continuous flow** ( $0.8 \geq \rho \geq 0.88$ ). Traffic flows on all streets.

**Partial flow** ( $0.89 \geq \rho \geq 0.94$ ). Traffic flows only on some streets, as extreme densities prevent flow on some roads. Considering that there are 12 intersections per street of 160 cells, a density  $\rho > 0.925$  for a particular street implies no flow  $J = 0$ .

**Gridlock** ( $\rho > 0.94$ ). In this phase, all traffic is stopped ( $J = v = 0$ ). Due to initial conditions, intersections are blocked and traffic can flow on no street. This is complementary to the free flow phase, where no vehicle is stopped ( $v = 1$ ).

For the *self-organizing* method, the triple intersection scenario offers *full capacity* for almost half of the density spectrum. However, the maximum flux  $J_{max} = 1/6$  is 50% less than for the double intersection scenario  $J_{max} = 1/4$ . It is clear that, for most densities, the double intersection scenario—even when there are three times more intersections to be coordinated—offers more efficient traffic flow than the triple intersection scenario.

## 5.2 Mixed scenario

A third scenario is considered, where double and triple intersections are used. There are 16 streets, three streets in each of four directions and two streets in each of other two directions. Streets are arranged in such a way that there are 12 triple and 48 double intersections, as shown in Figure 10. The triple intersections, as was already mentioned, have 50% less flow capacity than double intersections. Of the 16 streets, there are only two with no triple intersections, while there are only two with only triple intersections, i.e. there are 12 streets with both intersection types.

Intuitively, one could think that streets with at least one triple intersection would have their capacity limited by this intersection, i.e.  $J_{max} = 1/6$ . If 14 out of 16 streets (all of same length) have at least one triple intersection, we could assume that the average maximum flux of the scenario would be  $J_{max} = \frac{14 \cdot \frac{1}{6} + 2 \cdot \frac{1}{4}}{16} = \frac{17}{96} \approx 0.177$ . Still, the *self-organizing method* achieves  $J_{max} > 0.177$ , especially for  $\rho > 1/2$  ( $J_{max} \approx 0.19$ ). This implies that even in streets with some intersections with triple intersections, the traffic lights are coordinated in such a way that the average flow can be even higher than the expected one. This is because, even when the flux is limited by triple intersections, double intersections can take advantage of their higher capacity. If a triple intersection delays a double intersection upstream, the street crossing at the double intersection will take the opportunity to flow as much as possible. In this way, vehicles on streets with six double intersections and two triple intersections in practice will flow faster than vehicles on streets with six triple intersections. Certainly, vehicles on streets with twelve double intersections will flow even faster. The simulation results for this mixed scenario, together with the results of the previous two scenarios already discussed, are shown in Figure 11.

The *green-wave* method performs even worse in the mixed scenario than in the triple scenario. The reason is that the fixed coordination of mixed traffic lights becomes even more complicated, and there is no free flow on any street. Since the duration of green phases is the same for all intersections, triple intersections have a period 50% longer than double intersections. Again, the performance is comparable with that of the *random* method.

The *self-organizing* method, as could be expected, has a performance lying between the double and triple scenarios. However, as mentioned above, it reaches a maximum flux higher than the expected one, according to the number of streets with at least one triple intersection. There is free flow for low densities, showing that the *self-organizing method* manages to coordinate traffic lights in such a complex scenario. Nevertheless, given the mixed topology, there is no full capacity intermittent phase, which would imply a flux  $J = \frac{12 \cdot \frac{1}{6} + 48 \cdot \frac{1}{4}}{60} = \frac{7}{30} \approx 0.233$ ,

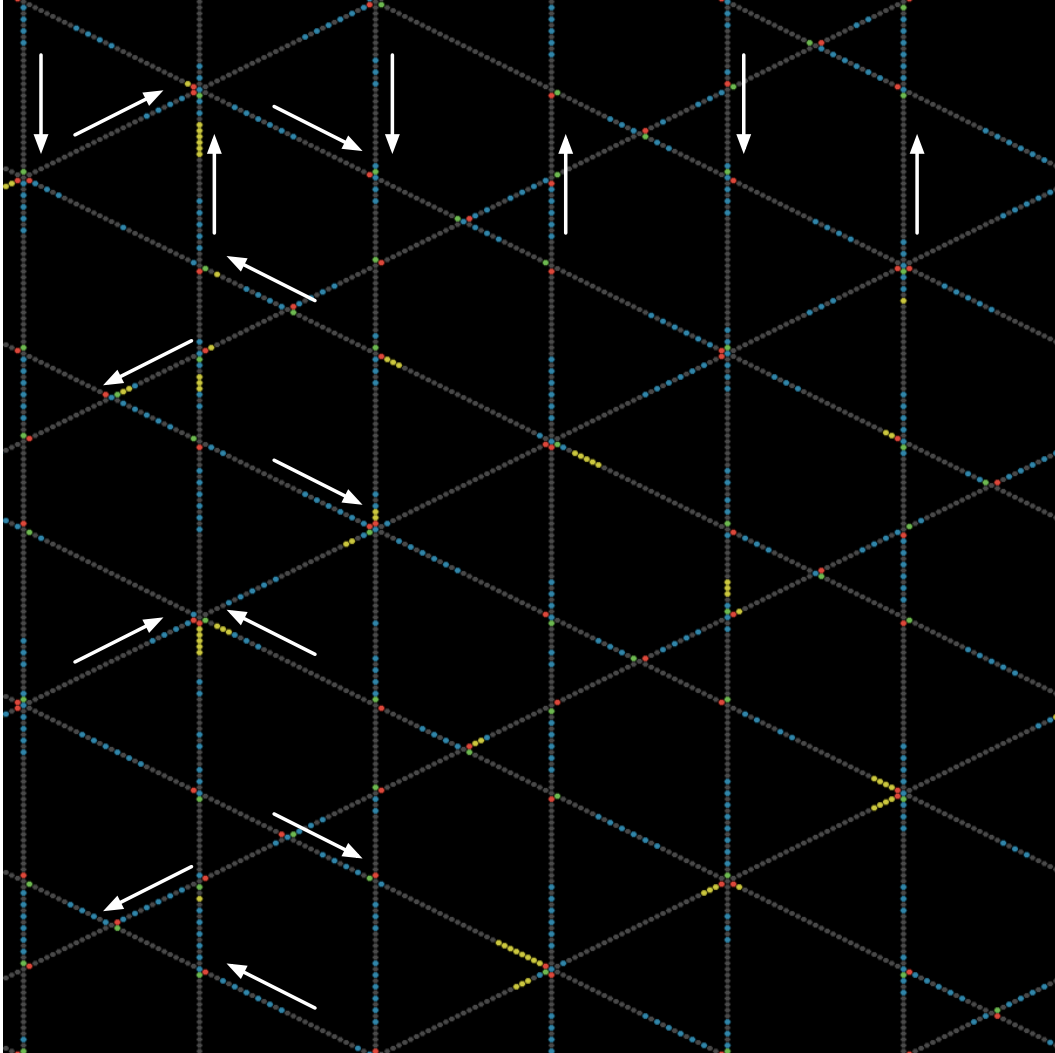


Figure 10: Screenshot of mixed scenario with 16 streets and 12 triple and 48 double intersections,  $\rho = 1/5$ . Moving vehicles are colored with [sky blue](#) and stopped vehicles are colored in [yellow](#).

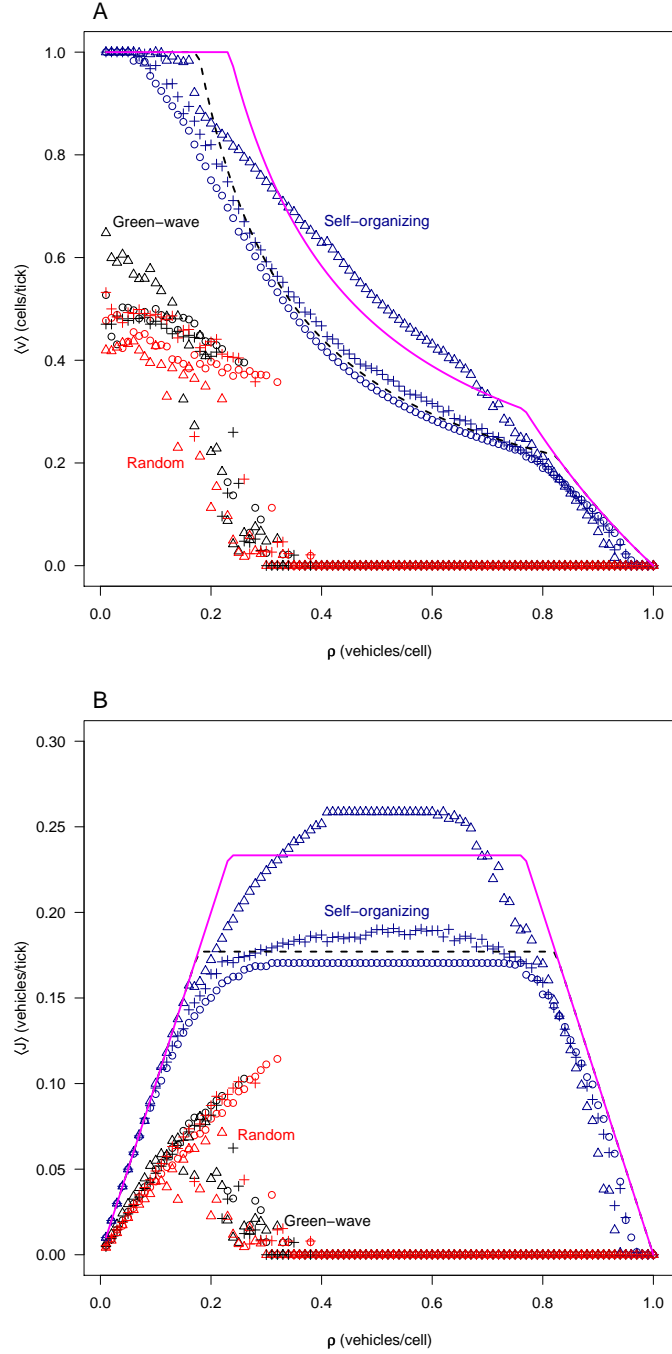


Figure 11: Simulation results for 18 and 16 street scenarios: (A) average velocity  $\langle v \rangle$  and (B) average flux  $\langle J \rangle$  for different densities  $\rho$ , *green-wave* (black), *self-organizing* (blue), and *random* (red) methods. Three scenarios are considered: 36 triple intersections (O), 108 double intersections ( $\Delta$ ), and mixed (+, 12 triple with 48 double intersections). Dashed lines indicate optimality curves for mixed scenario when street constraints are considered ( $J_{max} \approx 0.177$ ), while magenta continuous lines indicate optimality curves for mixed scenario when isolated intersections are considered ( $J_{max} \approx 0.233$ ).

i.e. all intersections being utilized at all times. If we take into account the criterion of isolated intersections, this yields a different set of optimality curves, where the *self-organizing* method is farther from the optima, i.e. interference  $\Phi$  is higher. This is because different capacities of double and triple intersections create more interferences between intersections. This capacity difference makes it impossible to reach a state of maximum flux for all intersections, i.e. full capacity intermittent phase. Nevertheless, it should be noted that the system performance is better in the mixed scenario than in the scenario with only triple intersections. This implies that the *self-organizing* method can take advantage of the increased capacity of some of the intersections, even if not optimally, in spite of the increased coordination complexity (more traffic lights in the mixed scenario (60) than in the triple scenario (36)).

### 5.3 Optimality and Interference

We can calculate how close to optimality different methods are comparing the optimality curves for isolated intersections with the experimental results for different scenarios. This difference was defined as interference  $\Phi$  in equations 6 and 7. The interference curves for different methods are shown in Figure 12, while the values of their integrals are shown in Tables 3 and 4.

For all methods, the higher velocity interference  $\Phi_v$  can be found near  $\rho = J_{max}$ . The reason is that this is the region where theoretically there is the highest possible velocity (free flow) for the highest possible density ( $J_{max}$ ). This is the region where more improvements can be made concerning velocity in the problem of coordinating traffic lights.

Since the *green-wave* and *random* methods reach a gridlock for relatively low densities, the flux interference  $\Phi_J$  is highest when the flux should be highest, i.e.  $J = J_{max}$ . For the *self-organizing* method, there is a higher flux interference  $\Phi_J$  within the underutilized intermittent and overutilized intermittent phases. There is low interference within the free-flow, quasi-free flow, and full capacity intermittent phases. In these regions of the density space, the *self-organizing method* is optimal or close to optimal. There is certain interference in the quasi-gridlock, partial flow and gridlock phases, since theoretically traffic still should flow in an isolated intersection, while in a city scenario initial conditions block the flow of some or all of the streets.

The large spikes near some phase transitions are due to single runs per density value. These are smoothened with a larger statistical sample, which has high standard deviations precisely near the same phase transitions where the spikes are present.

The interferences  $\Phi_v$  and  $\Phi_J$  (equations 6 and 7), shown in Tables 3 and 4, are similar for the *green-wave* and *random* methods. The *green-wave* method is more efficient for lower densities and for the double scenario, but worse than *random* for medium densities (it reaches gridlock earlier) and for the triple and mixed scenarios. The difference of values for the interferences for different scenarios is related more to differences of  $J_{max}$  than to differences of performance, since there is maximum interference ( $v = J = 0$ , i.e. gridlock) for a broad range of densities.

The interferences  $\Phi_v$  and  $\Phi_J$  for the *self-organizing* method are much lower in all scenarios. In the triple scenario, it is close to theoretical optimality, which is achieved when  $\Phi_J = 0$ . Still, the increased capacity of the double scenario gives a better performance, even when

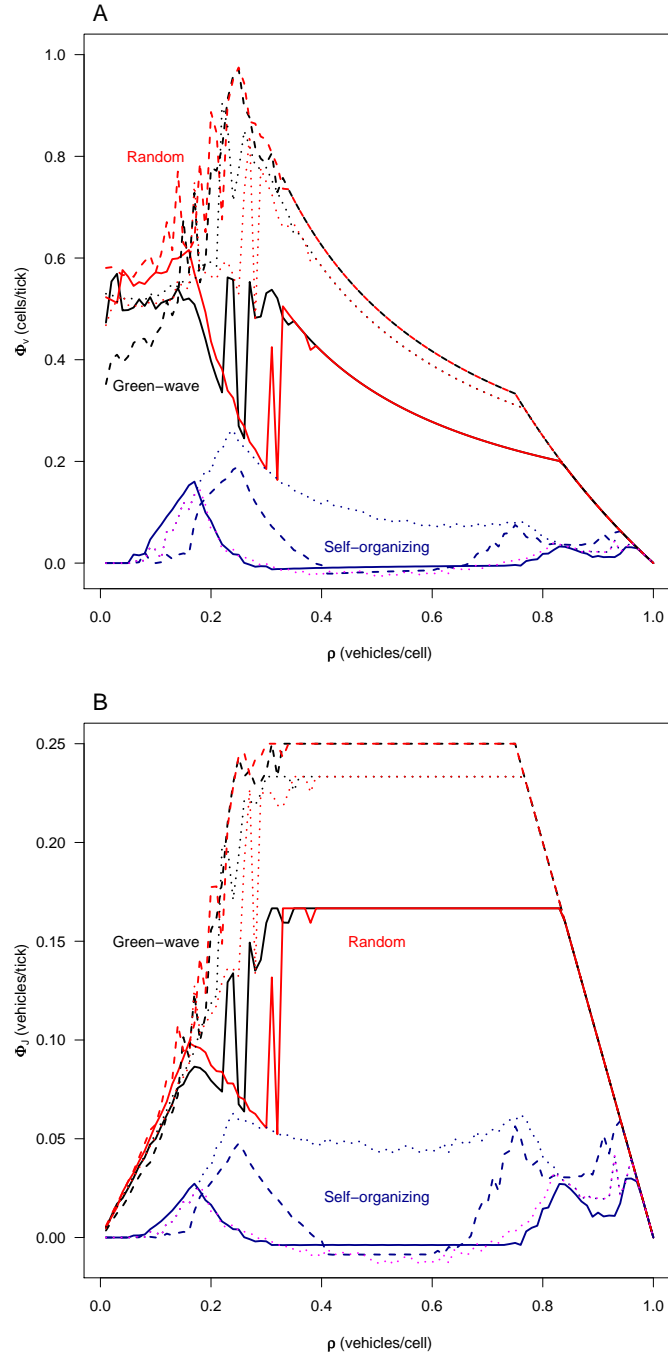


Figure 12: Interference curves for triple (continuous lines), double (dashed lines) and mixed (pointed lines) scenarios: (A) velocity interference  $\Phi_v$  and (B) flux interference  $\Phi_J$  for different densities  $\rho$ , *green-wave* (black), *self-organizing* (blue), and *random* (red) methods. magenta pointed lines indicate interference curves assuming a  $J_{max} \approx 0.177$  in the mixed scenario.



Table 3: Velocity interferences  $\Phi_v$  for different methods and scenarios. **Magenta** row assumes a  $J_{max} \approx 0.177$  in the mixed scenario.

| scenario                                         | $\Phi_v$ Self-organizing | $\Phi_v$ Green-wave | $\Phi_v$ Random |
|--------------------------------------------------|--------------------------|---------------------|-----------------|
| triple                                           | 0.01543474               | 0.3274843           | 0.3174297       |
| double                                           | 0.03256081               | 0.4484157           | 0.4759376       |
| mixed, $J_{max} \approx 0.233$                   | 0.08689782               | 0.4336379           | 0.416929        |
| <b>mixed, <math>J_{max} \approx 0.177</math></b> | 0.01056809               |                     |                 |

Table 4: Flux interferences  $\Phi_J$  for different methods and scenarios. **Magenta** row assumes a  $J_{max} \approx 0.177$  in the mixed scenario.

| scenario                                         | $\Phi_J$ Self-organizing | $\Phi_J$ Green-wave | $\Phi_J$ Random |
|--------------------------------------------------|--------------------------|---------------------|-----------------|
| triple                                           | 0.004418822              | 0.1234725           | 0.1190469       |
| double                                           | 0.01471438               | 0.1759046           | 0.1786202       |
| mixed, $J_{max} \approx 0.233$                   | 0.03700456               | 0.1664656           | 0.1625401       |
| <b>mixed, <math>J_{max} \approx 0.177</math></b> | 0.003842065              |                     |                 |

the interference is higher. This is also the case for the mixed scenario, where the interference is highest because the lack of a full-capacity intermittent phase.

## 6 Discussion

The experiments showed that the *self-organizing* method is highly scalable. It manages to coordinate flow in at least six different directions and prevent gridlocks. This suggests that for the traffic light coordination problem, an adaptive mechanism, in this case achieved with self-organization, will be remarkably more efficient than a fixed prediction-based mechanism. This is because the traffic coordination problem is non-stationary (Gershenson, 2007). The positions of vehicles are changing constantly and are interdependent. The optimal coordination of traffic lights changes with each vehicular configuration, which cannot be predicted. Many adaptive methods in the literature (Federal Highway Administration, 2005; Henry et al., 1983; Mauro and Di Taranto, 1990; Robertson and Bretherton, 1991; Faieta and Huberman, 1993; Gartner et al., 2001; Diakaki et al., 2003; Fouladvand et al., 2004; Mirchandani and Wang, 2005; Bazzan, 2005) adjust parameters at a timescale considerably larger than the one in which changes in the traffic configuration occur. Certainly, the adaptation will offer some improvements over static methods. However, methods that make adjustments at the seconds scale will offer a better performance, since the predictable horizon of city traffic is limited to a couple of minutes (Helbing et al., 2005; Gershenson, 2005; Cools et al., 2007; Lämmer and Helbing, 2008). An adaptive method at short timescales, as the one presented here, will be able to adjust and regulate the traffic conditions with considerable improvements over current fixed-cycle methods, such as the *green-wave* used here.

A clear insight from our simulations for city planning is that complex intersections should be avoided, since these function as bottlenecks. This was already known (Buchanon, 1963; de Jong et al., 2006). However, an interesting result presented here is that without proper light coordination, the benefit of an efficient topology is almost lost, and can even be counterproductive. This can be seen with the similar performances of the *green-wave* method in different scenarios. A fixed-cycle method cannot take advantage of an efficient topology. This is important especially for cities where there are already complex intersections and the street topology can be difficult to modify.

Building on our previous results (Gershenson, 2005; Cools et al., 2007; Gershenson and Rosenbluth, 2010), the simulations presented here showed that the *self-organizing* method is highly scalable, and has a graceful degradation of performance as density increases. Moreover, this scalability enables the method to exploit much better the city topology.

An important issue that we have not considered in the simulations presented here is that of turning vehicles. In principle, turning vehicles do not limit the *self-organizing* method, as shown in previous work (Gershenson, 2005; Cools et al., 2007). However, if a vehicle is turning on a blocked street, this blockage could propagate to crossing streets. To prevent this, dedicated turning lanes should be used, where a traffic light for turning vehicles will be set to red if the crossing street has a blockage ahead. This would be a simple extension of rule 5.

Another issue to be considered before implementing the *self-organizing* method in a city is that of pedestrians. Preliminary results considering pedestrians have been encouraging, and will be presented in future work.

## 7 Conclusions

By definition, models are caricatures of the world. The objective of such caricatures is to abstract away details so as to foil properties of interest. Hence, a model should be as simple as possible, provided such properties are highlighted. Different models will therefore be useful for different purposes.

In the case of vehicular traffic in intersections controlled with traffic lights, the more complex and therefore more realistic models can reproduce numerous details. An example is our own model based on agents (Gershenson, 2005; Cools et al., 2007; de Gier et al., 2011). Such a model does exhibit a behavior reproducing many features. However, if we are interested in identifying phases, this agent-based model is not useful. The reason is that the phase transitions are smoothed out.

We have consequently considered a simpler model based on cellular automata. Such automata are valuable because of their simplicity and for nonetheless exhibiting a similar behavior to that of more elaborated models. Moreover, our model employs only elementary automata, where the state of a cell in the next tick is a function only of the current state of such a cell and those of its nearest neighbors. This model is computationally cheap allowing the simulations of numerous vehicles. Furthermore, this model is superior to our previous multi-agent one, as the phase transitions are conspicuous.

Another advantage of our simple model of city traffic is that optimality can be deduced in a straightforward fashion, enabling the comparison of methods for coordinating traffic

lights with a theoretical optimum. This informs which regions of the density space have an already optimal behavior and which regions can still be improved, at least in theory. Moreover, we were able to show that our proposed self-organizing method is close to optimality, independently of the poor performance of the green-wave method. This is relevant, because there were no previous benchmarks for comparing the performance of different traffic light controllers.

The proposed optimality measurement—where the performance of a traffic light controller is compared against the capacity of an isolated single intersection—can be also used with other models of city traffic for benchmarking. If the optimality curves (equations 4 and 5) cannot be deduced analytically, experimental data can be fitted and used to calculate interference  $\Phi$ .

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# Appendix

## A *Self-organizing* algorithm

Algorithm 1 formally describes our *self-organizing* method. Each intersection uses this algorithm independently to regulate traffic, i.e. there is no direct communication between intersections. The values of the parameters used in the simulations are shown in Table 5. We performed several simulations varying these parameters, and they are quite robust, i.e. the performance of the system is not affected by small changes in the parameters.

```

1  foreach ( $\Delta t$ ) do
2       $t_i += \Delta t$  ; // local phase
3       $k_{ij} += \text{vehicles}_{\text{approachingRed}}$  in  $d_j$ ; // for rules 1 and 4
4      if ( $\text{vehicles}_{\text{stoppedAfterGreen}}$  at  $e > 0$ ) then
5          if ( $\text{vehicles}_{\text{stoppedAfterRed}}$  at  $e_j > 0, \forall j$ ) then
6               $\text{switchAllRed}_i()$  ; // rule 6
7          end
8          else
9               $\text{switchlight}_i(j)$  for  $j$  with  $\max(k_{ij})$ 
10             and not  $\text{vehicles}_{\text{stoppedAfterRed}}$  at  $e_j > 0$  ; // rule 5
11         end
12     end
13     else if  $\text{vehicles}_{\text{stoppedAfterRed}}$  at  $e == 0$  then
14         ; // no blockage upstream
15         if  $\text{allRed?}$  then
16              $\text{restoreSingleGreen}_i(j)$  for  $j$  with  $\max(k_{ij})$ 
17             and not  $\text{vehicles}_{\text{stoppedAfterRed}}$  at  $e_j > 0$  ; // complement to rule 6
18         end
19         if ( $k_{ij} \geq 1$ ) and ( $\text{vehicles}_{\text{approachingGreen}}$  in  $d == 0$ ) then
20              $\text{switchlight}_i(j)$  ; // rule 4
21         end
22         else if not ( $0 < \text{vehicles}_{\text{approachingGreen}}$  in  $r < m$ ) then
23             ; // rule 3
24             if ( $t_i \geq t_{\min}$ ) then
25                 ; // rule 2
26                 if ( $k_{ij} \geq n$ ) then
27                      $\text{switchlight}_i(j)$  ; // rule 1
28                 end
29             end
30         end
31     end
32      $\text{switchlight}_i(j)$  begin
33          $k_{ij} = 0$  ; // Switch light for direction  $j$ 
34          $t_i = 0$  ;
35          $\text{switchTrafficLight}_i(j)$ ;
36     end

```

**Algorithm 1:** *Self-organizing* method.

On every tick ( $\Delta t$ ), Algorithm 1 increases the phase  $t_i$  by the duration of  $\Delta t$  (line 2), and

the counters  $k_{ij}$  are increased by the number of vehicles approaching or waiting behind a red light  $j$  within a certain distance  $d_j$  (line 3). *Rule 6* switches all lights to red if every street is blocked ahead of the intersection (line 6). *Rule 5* changes a green light to red if there are vehicles stopped ahead of green light at a distance  $e$  from the intersection (lines 9–10). This prevents the accumulation of vehicles when they cannot advance, diminishing the probability of their blocking the intersection, while at the same time allowing vehicles in the crossing street (if any) to advance. The red light with the highest  $k_{ij}$  and no blockage upstream is switched to green. Notice that directions with a green light have no subindex  $j$ , since there can be at most one direction with a green light, while there can be several directions with red lights. Rules 5 and 6 are normally used at high vehicle densities. A single green light is restored if there are no vehicles stopped ahead of the intersection and all lights are red (lines 15–16). Again, the light with the highest  $k_{ij}$  and no blockage upstream is switched to green. All of the following rules, to determine whether the light will switch, also depend on the condition that there is free space ahead of the red light. With *rule 4*, if there are no vehicles approaching a green light within a distance  $d$ , and there is at least one vehicle approaching a red light ( $k_{ij} \geq 1$ ), this is switched, so that by the time the vehicle(s) reach the intersection it(they) will not need to stop (line 19). This rule is normally used for low vehicle densities. *Rule 3* prevents the “tails” of platoons from being cut, by delaying the switching of a green light when there are few vehicles (fewer than  $m$ ) just about to cross, i.e. within a distance  $r$  (line 21). Still, rule 3 allows the division of long platoons, preventing the accumulation of vehicles waiting behind a red light. *Rule 2* prevents the fast switching of traffic lights caused by high vehicle densities with a minimum phase  $t_{\min}$  (line 22). If rules 2 and 3 are satisfied, *rule 1* changes a traffic light to direction  $j$  when the count  $k_{ij}$  reaches a certain threshold  $n$  (lines 23–24). This makes single vehicles wait for some time, increasing the probability that more vehicles will join them and thus promoting the formation of platoons. Once platoons reach a certain size, they can request a green light before reaching the intersection, if all other conditions are met. Even if this does not occur, once the conditions are proper, the vehicles will get a green light. Thus, in principle vehicles have to wait very little time because of red lights. When a traffic light is switched (lines 30–34), the counter of the direction that will get the green light  $k_{ij}$  and the phase  $t_i$  are reset (lines 31–32). The phase  $t_i$  keeps the time since the last light switch, independently of the direction selected. Afterwards, the traffic lights are changed (line 33).

| Variable   | Abstract Value   | Scaled Value     | Used by       |
|------------|------------------|------------------|---------------|
| $\Delta t$ | 1 tick           | 1/3 s            | Algorithm     |
| $n$        | 40 vehicles·tick | 13.33 vehicles·s | Rule 1        |
| $d$        | 10 cells         | 50 m             | Rules 1 and 4 |
| $t_{\min}$ | 10 ticks         | 3.33 s           | Rule 2        |
| $m$        | 2 vehicles       | 2 vehicles       | Rule 3        |
| $r$        | 5 cells          | 25 m             | Rule 3        |
| $e$        | 2 cells          | 10 m             | Rules 5 and 6 |

Table 5: Parameters used by *self-organizing* method in simulations.