Energy Analysis of Sensor Nodes in WSN Based on Discrete-Time Queueing Model with a Setup

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Abstract: Wireless sensor network is an energy-constrained system. Energy efficiency and data latency are two important performance measures needed to be considered when designing a wireless sensor network. Based on the power management mode in IEEE 802.15.4, taking into account the switching procedure from the sleep mode to the active mode, a discrete-time queueing model with a setup is built, and the analysis of the queueing model in steady state is given. Moreover, we present the performance measures in terms of the handover ratio, the average response time of data frames and the average energy consumption, and we also discuss the relationships between the system performance measures and the constellation size. Finally, we construct a cost function and obtain the optimal constellation size.

Key Words: Wireless Sensor Network, Power Management Mode, Energy Efficiency, Data Latency, Constellation Size, Discrete-Time Queueing Model with Setup

1 INTRODUCTION

With the development of the technology for the micro sensor, the microelectronic and the wireless communication, the research of Wireless Sensor Network (WSN) has got more and more extensive attention [1]. For the self-organization, micro, low-cost, flexibility and other characteristics, WSN has great applications in many areas, such as military, environmental science, medical and health, space exploration, commercial applications and so on [2].

WSN is an energy-constrained system. Sensor node in WSN is a micro-embedded device, which typically carries only limited battery power [3]. Large number of sensor nodes are densely deployed in a wide region for monitoring the complex environment. In some regions, replacement or recharge of the battery is impossible, this requires designing a low-power mechanism to minimize the energy consumption, so IEEE 802.15.4 offers a power management mode. On the other hand, data latency is also an important factor could not be neglected when designing a real-time system.

In [4], an analytical model is constructed to describe the relationships between timeliness, energy and the degree of aggregation. Feedback control is used to adapt data aggregation dynamically in response to network load while meeting application deadlines.

In [5], for satisfying a given throughput and delay requirement, considering the delay and peak-power constraints, a new modulation strategy is given to minimize the total energy consumption.

In [6], the performance measures in terms of the average power consumption and data delay are given based on a continuous-time queueing model. Unfortunately, the switching procedure from the sleep mode to the active mode is neglected to simplify the analysis process.

Considering the digital nature and the setup procedure of the Phase-Locked Loop (PLL) from the sleep mode to the active mode, we build a discrete-time queueing model with a setup to describe the working principle of the power management mode of WSN in this paper. We analyze the relationships between the constellation size and the performance measures. Finally, numerical results are given to demonstrate the influence of the constellation size on the system performance.

The rest of the paper is organized as follows: In Section 2, a queueing model with a setup for the system is built, then the analysis of the queueing model is presented in Section 3. In Section 4, the performance measures and in Section 5, numerical results are given. A cost function is constructed in Section 6. Conclusions are shown in Section 7.

2 MODEL BUILDING

In order to reduce energy consumption of sensor nodes, power management mode is introduced in IEEE 802.15.4. When there is a data frame to be sent, both the source sensor node and the destination sensor node are in active mode. When there is no data frame to be sent, all these sensor nodes will be in sleep mode. Taking the communications between the sensor nodes as a research object, we build a

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queueing model with a setup to describe the working principle of the power management mode. Some assumptions for this queueing model are given as follows:

(1) Let sensor node A be a source node, sensor node C be a destination node, the distance between the sensor node A and the sensor node C be d. Let B be the bandwidth and k $(k = 1, 2, ...)$ be the modulation constellation size. We assume the length of a data frame follow a geometric distribution with the average L bits, and a square wave pulse is used.

Taking into account the Additive White Gaussian Noise (AWGN) channel and Pulse Amplitude Modulation (PAM), we assume the channel between the sensor nodes be Gaussian channel and the error-correcting codes are not used in the communication.

(2) Due to the short duration and less energy consumption when the system switches from the active mode to the sleep mode, the duration and energy consumption for this kind of switch are negligible.

(3) There is a setup procedure of the PLL in the frequency synthesizer when the system switches from the sleep mode to the active mode. We regard this procedure as a setup period. We take into account the energy consumption in the setup period, and we also not neglect the duration of the setup period.

Let the setup period U (in slots) follow a general distribution. The probability distribution, the PGF (Probability Generating Function) $U(z)$ and the average $E[U]$ of U are given as follows:

$$u_j = P\{U = j\}, \quad j = 1, 2, ...,$$

$$U(z) = \sum_{j=1}^{\infty} u_j z^j, \quad E[U] = \sum_{j=1}^{\infty} j z^j.$$

(4) Suppose the data frame arrival interval follow a geometric distribution with arrival ratio p. The transmission time of a data frame is assumed to be an independent and identically distributed random variables denoted by S. S is geometric distribution with the parameter of μ. The number of bits transmitted per slot is kB, so we can give the probability distribution, the PGF $S(z)$ and the average $E[S]$ of S as follows:

$$P\{S = j\} = \bar{\mu}^{j-1}\mu, \quad \bar{\mu} = 1 - \mu, j = 1, 2 \dots,$$

$$S(z) = \sum_{j=1}^{\infty} z^j P\{S = j\}, \quad E[S] = \frac{1}{\mu} = \frac{L}{kB}.$$

Let A_S be the number of data frames arrived in the transmission time S of a data frame, the PGF $A_S(z)$ and the average $E[A_S]$ of A_S are given as follows:

$$A_S(z) = S(\bar{p}+pz), \quad \bar{p} = 1-p, \quad E[A_S] = pE[S]. \tag{1}$$

In addition, we assume that the arrival interval be independent of the transmission time.

(5) The channel is abstracted as a server and one server is assumed. The data frames are transmitted on a First In First Out (FIFO) discipline. The capacity of the system cache is infinite. Therefore, a Geo/Geo/1 queueing model with a setup is built.

When the system load is $\rho = \dfrac{p}{\mu} < 1$, the system will arrive at a state of equilibrium.

3 MODEL ANALYSIS

We consider a discrete-time queueing system where the time axis is segmented into a sequence of equal time intervals (called slots). It is assumed that all queueing activities (arrivals and departures) occur at the slot boundaries. For mathematical clarity, we suppose that the departures occur at the moment immediately before the slot boundaries and the arrivals occur at the moment immediately after the slot boundaries.

3.1 Queueing Length and Waiting Time

Let Q_B be the number of data frames at the instant when a busy period begins, the PGF $Q_B(z)$ and the average $E[Q_B]$ of Q_B are given as follows:

$$Q_B(z) = zU(\bar{p} + pz), \quad E[Q_B] = 1 + pE[U]. \tag{2}$$

Let L^+ be the queueing length at the transmission completion instant. Obviously $L^+ = L_0 + L_d$, L_0 is the queueing length for the classical Geo/Geo/1 queue without setup period, and L_d is the additional queueing length introduced by setup period. The PGF $L_0(z)$ and the average $E[L_0]$ of L_0 are given as follows:

$$L_0(z) = \frac{p\bar{\mu}}{\bar{p}\mu - p\bar{\mu}z}, \quad E[L_0] = \frac{p\bar{\mu}}{\mu - p}. \tag{3}$$

Using the boundary state variable theory in [7], the PGF $L_d(z)$ and the average $E[L_d]$ of L_d are given as follows:

$$L_d(z) = \frac{1 - zU(\bar{p} + pz)}{(1 + pE[U])(1 - z)}, \tag{4}$$

$$E[L_d] = \frac{2pE[U] + p^2U''(1)}{2(1 + pE[U])}. \tag{5}$$

Combining Eqs. (3)-(5), the PGF $L^+(z)$ and the average $E[L^+]$ of L^+ can be written as follows:

$$L^+(z) = L_0(z)L_d(z) = \frac{p\bar{\mu}}{\bar{p}\mu - p\bar{\mu}z} \times \frac{1 - zU(\bar{p} + pz)}{(1 + pE[U])(1 - z)},$$

$$E[L^+] = E[L_0] + E[L_d] = \frac{p\bar{\mu}}{\mu - p} + \frac{2pE[U] + p^2U''(1)}{2(1 + pE[U])}.$$

Let W be the waiting time of a data frame, by using the Little formula, the average $E[W]$ of W is given by

$$E[W] = \frac{E[L^+]}{p} = \frac{L - kB}{kB - pL} + \frac{2E[U] + pU''(1)}{2(1 + pE[U])}.$$

3.2 Busy Cycle

The busy cycle R is defined as the period from the instant in which a busy period of the system completes to the instant in which the next busy period of the system ends. The busy cycle R consists of a sleep period V_S, a setup period U and a busy period Θ.

The length of a sleep period is a residual arrival interval. Note that the arrival interval follow a geometric distribution, so we can get the average $E[V_S]$ of V_S as follows:

$$E[V_S] = \frac{1}{p}. \tag{6}$$

Each data frame at the beginning of a busy period Θ will introduce a sub-busy period θ. A sub-busy period θ of a data frame is composed of the transmission period S of this data frame and the sum of the sub-busy period θ incurred by all the data frames arriving during the transmission period S of this data frame.

All the sub-busy periods θ brought by the data frames at the beginning of the busy period combine to make a busy period Θ. We have that

$$\theta = S + \underbrace{\theta + \theta + \cdots + \theta}_{A_S}, \quad \Theta = \underbrace{\theta + \theta + \cdots + \theta}_{Q_B}.$$

Considering the Bernoulli arrival process in this system, the PGF $\theta(z)$ of θ can be obtained as follows:

$$\theta(z) = S(z(A_S(\theta(z)))),$$

which yields the average $E[\theta]$ of θ as follows:

$$E[\theta] = \frac{E[S]}{1-\rho} = \frac{1}{\mu - p}. \tag{7}$$

Therefore, we can obtain the PGF $\Theta(z)$ of Θ as follows:

$$\Theta(z) = Q_B(z)|_{z=\theta(z)} = \theta(z)U(\bar{p} + p\theta(z)). \tag{8}$$

Differentiating Eq. (8) with respect z at $z = 1$, the average $E[\Theta]$ of Θ is given as follows:

$$E[\Theta] = (1 + pE[U])E[\theta] = \frac{1 + pE[U]}{\mu - p}. \tag{9}$$

Combining Eq. (6) and Eq. (9), we can get the average $E[R]$ of R as follows:

$$\begin{aligned} E[R] &= E[V_S] + E[U] + E[\Theta] \\ &= \frac{\mu(1 + pE[U])}{p(\mu - p)}. \end{aligned} \tag{10}$$

4 PERFORMANCE MEASURES

In this section, we will present the performance measures for the sensor nodes.

The handover ratio β is defined as the number of switches from the sleep mode to the active mode per slot. There is a switch procedure in a busy cycle, so the handover ratio β is given as follows:

$$\beta = \frac{1}{E[R]} = \frac{p}{1 + pE[U]}\left(1 - \frac{pL}{kB}\right). \tag{11}$$

The average response time of data frames $E[T]$, which is also called data latency, is the time period in slots elapsed from the arrival of a data frame to the end of the transmission of this data frame. The average response time of data frames $E[T]$ is given as follows:

$$\begin{aligned} E[T] &= E[W] + E[S] \\ &= \frac{L - kB}{kB - pL} + \frac{2E[U] + pU''(1)}{2\,(1 + pE[U])} + \frac{L}{kB}. \end{aligned} \tag{12}$$

The average energy consumption $E[P]$ is defined as the sum of the energy consumed by two adjacent sensor nodes per slot. The energy is mainly consumed by circuit and amplifier. Let P_{CA} be the circuit power in the active mode, P_{CS} be the circuit power in the sleep mode, P_A be the amplifier power, P_{SA} be the energy consumption when the system switches from the sleep mode to the active mode. We get the average energy consumption as follows:

$$\begin{aligned} E[P] = &\frac{1}{1 + pE[U]}\left(1 - \frac{pL}{kB}\right)P_{CS} \\ &+ \frac{pL}{kB}(P_{CA} + P_A) + \beta\, P_{SA}, \end{aligned} \tag{13}$$

where $\frac{1}{1 + pE[U]}\left(1 - \frac{pL}{kB}\right)$ is the probability for a sensor node being in the sleep mode, $\frac{pL}{kB}$ is the probability for a sensor node being in the active mode.

From [6], the minimum power of the amplifier P_A is given as follows:

$$P_A = 8(\mathrm{M}^2 - 1)\pi^2 d^2 B\mathrm{N}_0 \left(Q^{-1}(T_0)\right)^2 (3G\Gamma^2)^{-1}, \tag{14}$$

where $Q(x) = (2\pi)^{-1/2}\int_x^\infty e^{-t^2/2}dt,\ x \geq 0$, T_0 is Bit Error Rate, M=2^k, k is the constellation size, G is a constant defined by the antenna gain and other system parameters, Γ is the carrier wavelength, f_c is the carrier frequency. The explanations for other parameters can be obtained from [6].

Substituting Eq. (14) into Eq. (13), then differentiating Eq. (13) with respect the constellation size k, we can get the derivative function as follows:

$$\begin{aligned} \frac{\partial E[P]}{\partial k} = &\frac{pL}{k^2 B} \times \frac{1}{1 + pE[U]}(P_{CS} + pP_{SA} - P_{CA}) \\ &+ \frac{pL}{k^2 B} \times 8\pi^2 d^2 B\mathrm{N}_0 \left(Q^{-1}(T_0)\right)^2 (3G\Gamma^2)^{-1} \\ &\times (4^k k \ln 4 - 4^k + 1). \end{aligned} \tag{15}$$

It can be found that $\frac{\partial E[P]}{\partial k}$ is always greater than zero at $P_{CA} \leq \frac{1}{1 + pE[U]}(P_{CS} + pP_{SA})$, so $E[P]$ is a increasing function for this case, therefore, we can obtain the minimum value of $E[P]$ at $k = 1$. On the other hand, under the condition that $P_{CA} > \frac{1}{1 + pE[U]}(P_{CS} + pP_{SA})$, there is a minimum value of $E[P]$ when the constellation size k is set to an optimal value k^*.

5 NUMERICAL RESULTS

We will numerically evaluate the system performance in this section. We set the system parameters as follows: a slot is 1 ms, $p = 0.05$, $d = 30$ m, $T_0 = 10^{-4}$, $G = 2$, $L = 16$ kb, $f_c = 10^8$ Hz, $B = 1$ MHz, $P_{CS} = 10^{-4}$ W, $P_{SA} = 5 \times 10^{-4}$ W, $\mathrm{N}_0 = 2 \times 10^{-16}$ W/Hz.

The influence of the constellation size k on the handover ratio β is plotted in Fig. 1.

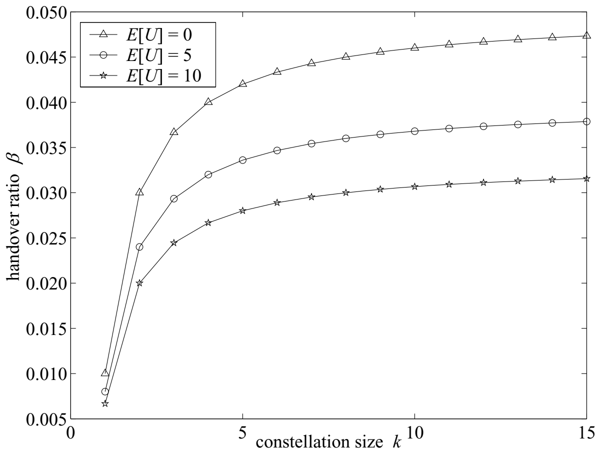

Figure 1: Handover Ratio vs. Constellation Size

Figure 1 shows that when the constellation size k take the same value, the handover ratio β will decrease as the average setup period $E[U]$ increases. This is because that the larger the average setup period is, the longer the busy cycle will be, so the handover ratio will decrease. On the other hand, the handover ratio β increases with the constellation size k for all the average setup period $E[U]$. The reason is that the larger the constellation size is, the shorter the transmission period of a data frame and the busy cycle are, so the more the handover ratio is.

Figure 2 illustrates that how the average response time of data frames $E[T]$ change with the constellation size k.

It is observed that for the same the constellation size k, the average response time of data frames $E[T]$ will increase with the average setup period $E[U]$. The reason is that the larger the average setup period is, the longer the waiting time of data frames will be, so the average response time of data frames will increase. On the other hand, the average response time of data frames $E[T]$ will decrease as the constellation size k increases for all the average setup period $E[U]$. This is because that the larger the constellation size is, the shorter the transmission period of a data frame will be, so the less the average response time of data frames will be.

Figure 3 depicts the influence of the constellation size k on the average energy consumption $E[P]$.

It can be found that for all the same circuit power P_{CA} when the constellation size k take the same value, the average energy consumption $E[P]$ will decrease as the average setup period $E[U]$ increases. The reason is that the larger the average setup period is, the smaller the probability that sensor nodes being in the sleep mode and the

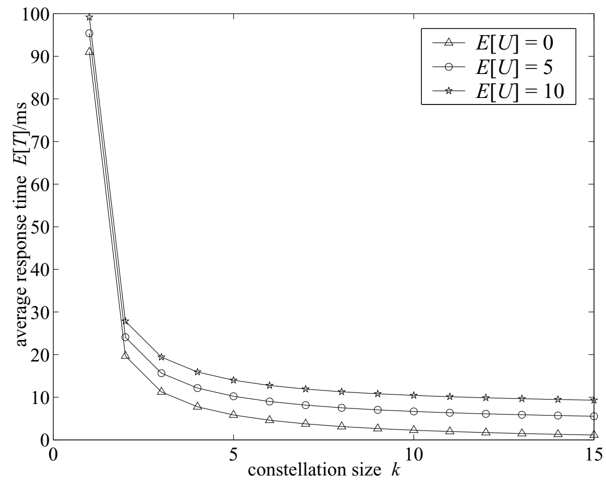

Figure 2: Average Response Time vs. Constellation Size

number of switches from the sleep mode to the active mode will be, so the average energy consumption will decrease. On the other hand, the average energy consumption $E[P]$ will increase as the constellation size k increases with $P_{CA} = 2 \times 10^{-5}$ W for all the average setup period $E[U]$. So the average energy consumption obtains minimum value at $k = 1$. In addition, the average energy consumption $E[P]$ experiences two trends as the constellation size k increases with $P_{CA} = 10^{-3}$ W for all the average setup period $E[U]$. When $1 \leq k \leq 12$, the average energy consumption $E[P]$ will decrease as the constellation size k increases, when $k > 12$, the average energy consumption $E[P]$ will increase as the constellation size k increases.

Therefore, we can conclude that there is an optimal value k^* for a minimum energy consumption, for example, when $E[U] = 0$ ms, the optimal k is $k^* = 12$ and the minimum value of the average energy consumption is $E[P_{min}] = 1.874 \times 10^{-7}$ W; when $E[U] = 10$ ms, the optimal k is $k^* = 12$ and the minimum value of the average energy

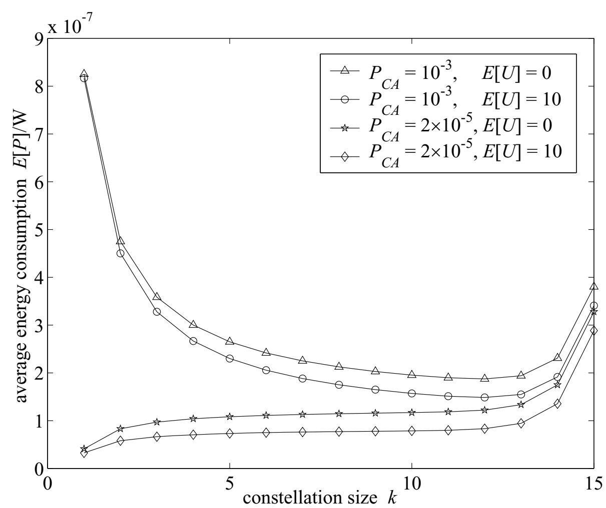

Figure 3: Average Energy Consumption vs. Constellation Size

consumption is $E[P_{min}] = 1.485 \times 10^{-7}$ W.

6 COST FUNCTION

From the numerical results, we can observe that there is a trade-off between the average response time of data frames and the average energy consumption when $P_{CA} \leq \frac{1}{1+pE[U]}(P_{CS} + pP_{SA})$, but when $P_{CA} > \frac{1}{1+pE[U]}(P_{CS}+pP_{SA})$, there is an optimal constellation size with a minimal average energy consumption without concerning the average response time of data frames.

Considering both data latency and energy efficiency requirement under a certain condition, we develop a cost function as follows:

$$F(k) = C_1 E[P] + C_2 E[T]$$

where

$C_1 =$ Cost of the average energy consumption,

$C_2 =$ Cost of the average response time of data frames.

It can be shown that how the cost function $F(k)$ change with the constellation size k in Fig. 4.

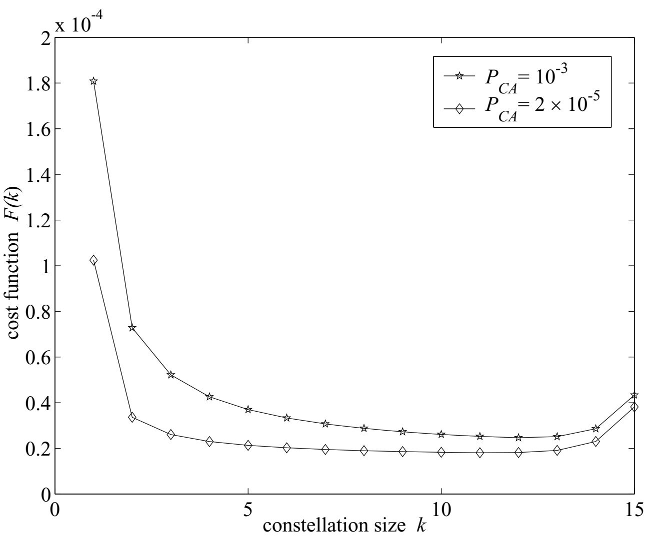

Figure 4: Cost Function vs. Constellation Size

From Fig. 4, we can conclude that the cost function $F(k)$ experiences two stages for all the circuit power P_{CA}. In the first stage, the cost function $F(k)$ will decrease along with the increase of the constellation size k. During this stage, the larger the constellation size is, the shorter the transmission period is, the less the average response time of data frames is, so the less the cost function will be. In the second stage, the cost function $F(k)$ will increase with the constellation size k. During this period, the larger the constellation size is, the greater the power consumption of the amplifier is, the more the average energy consumption will be, so the greater the cost function will be.

Therefore, there is a minimum cost for the cost function when the constellation size is set to an optimal value, for example, when $P_{CA} = 10^{-3}$ W, the optimal k is $k^* = 12$ and the minimum value of the cost function is $F(k)_{min} = 0.247 \times 10^{-4}$ W; when $P_{CA} = 2 \times 10^{-5}$ W, the optimal k is $k^* = 11$ and the minimum value of the cost function is $F(k)_{min} = 0.181 \times 10^{-4}$ W.

7 CONCLUSIONS

In wireless sensor networks, energy efficiency and data latency are two important performance measures needed to be considered. We built a discrete-time queueing model with a setup to capture the working principle of the power management mode offered in IEEE 802.15.4. We analyzed the queueing model in steady state, and proposed the formula for the system performance measures in terms of the handover ratio, the average response time of data frames and the average energy consumption. By numerical results, we demonstrated the influence of the constellation size on the system performance. Taking into account both the energy efficiency and the data latency, we constructed a cost function and gave the optimal value of the constellation size to minimize the cost function for certain distance between the source sensor node and the destination sensor node. This paper provides a theoretical basis for the optimal setting of the system parameters, and has potential applications in solving other energy conserving related problems in wireless sensor networks.

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