Representation of Numbers and Characters

Outline

- Number systems
- Conversion between number systems
- Addition and subtraction
- Integer representation in computer
- Character representation

Number Systems

- Decimal Number System
  - There are ten basic symbols(digits): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9
  - The base ten is represented in decimal as 10
  - $3932 = 3 \times 10^3 + 9 \times 10^2 + 3 \times 10^1 + 2 \times 10^0$
- Binary Number System
  - There are two digits: 0 and 1
  - The base 2 is represented in binary as 10
  - $11010 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
- Hexadecimal Number System
  - There are sixteen digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
  - The base is sixteen is represented in hex by 10
  - $1A = 1 \times 16^1 + 10 \times 16^0$

Converting Binary and Hex to Decimal

- Hex Number, $8A2Dh = 8 \times 16^3 + A \times 16^2 + 2 \times 16^1 + D \times 16^0$
  $= 8 \times 16^3 + 10 \times 16^2 + 2 \times 16^1 + 13 \times 16^0$
  $= 35373d$
- Binary Number, $1101b = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
  $= 13d$

Converting Decimal to Binary and Hex

- Decimal Number, 11172d

$$11172 = 698 \times 16 + 4$$
$$698 = 43 \times 16 + 10(\mathrm{A})$$
$$43 = 2 \times 16 + 11(\mathrm{B})$$
$$2 = 0 \times 16 + 2$$
$$= 2\mathrm{BA}4\mathrm{h}$$

- Decimal Number, 95d

$$95 = 47 \times 2 + 1$$
$$47 = 23 \times 2 + 1$$
$$23 = 11 \times 2 + 1$$
$$11 = 5 \times 2 + 1$$
$$5 = 2 \times 2 + 1$$
$$2 = 1 \times 2 + 0$$
$$1 = 0 \times 2 + 1$$
$$= 1011111\mathrm{b}$$

Conversion Between Hex and Binary

- Hex Number, $2B3Ch = 0010101100111100b$

2	B	3	C
0010	1011	0011	1100

- Binary Number, $1110101010b = 3AAh$

0011	1010	1010
3	A	A

Addition

- Hex Addition

5	B	3	9	h
7	A	F	4	h
D	6	2	D	h

- Binary Addition

1	0	0	1	0	1	1	1	1
			1	1	0	1	1	0
1	0	1	1	0	0	1	0	1

Subtraction

- Hex Subtraction

D	2	6	F	h
B	A	9	4	h
1	7	D	B	h

- Binary Subtraction

1	0	0	1
0	1	1	1
0	0	1	0

One's Complement Representation

- Representation of 5 and one's complement of 5

0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0

- If we add 5 and it's one's complement we will get 1111111111111111

Two's Complement Representation

- Representation of 5 and two's complement of 5

1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0
															1
1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1

- If we add 5 and it's two's complement we will get 10000000000000000
- So the two's complement of a number represents its negative form
- Two's complement of two's complement of 5 is 5

Integer Representations

- Unsigned Integers
  - Represents a magnitude so it is never negative
  - Largest unsigned integer stored in a byte is 11111111b = FFh =255d
  - Biggest unsigned integer stored in a word is 1111111111111111b = FFFFh = 65535d
  - If least significant bit is 0 then number is even otherwise number is odd
  - Example: Addresses of memory locations, ASCII character codes
- Signed Integers
  - It can either be positive or negative
  - Most significant bit is reserved for sign: 0 for positive and 1 for negative
  - Negative integers are stored in a computer as two's complement

Decimal Interpretations

- Unsigned Decimal Interpretation
  - Binary to decimal conversion

- Signed Decimal Interpretation
  - If MSB is 0 then signed decimal is same as unsigned decimal
  - If MSB is 1 take two's complement and convert it to decimal

Decimal Interpretation

- Most significant bit of a positive signed integer is 0. So the leading hex digit of a positive signed integer is 0-7h. Integers beginning with 8-Fh have 1 in their sign bit so they are negative
- For a word largest positive signed integer is 7FFFh=32767 and smallest negative signed integer is 8000h=-32768
- For a byte largest positive signed integer is 7Fh=127 and smallest negative signed integer is 80h=--128
- For 0000h-7FFFh and 00h-7Fh, signed decimal=unsigned decimal
- For 8000h-FFFFh, signed decimal =unsigned decimal-65536
- For 80h-FFh,signed decimal =unsigned decimal-256

Character Representation

- ASCII Code
  - Most popular encoding scheme for characters
  - Uses seven bits to code each character so there are total of 128 ASCII codes
  - Only 95 ASCII codes from 32-126 are considered to be printable
  - Others are used for communication control purposes
- Keyboard
  - Identifies a key by generating an ASCII code when the key is pressed
  - For IBM-PC each key is assigned an unique number called scan code