BRAC UNIVERSITY

CSE230

Discrete Mathematics

Assignment 3 (Bonus)

Student Information:

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Section: 02

Date: 12 December 2021

Lecture 9:

[1] Given that,

$$P(B+) = 0.44$$

$$P(x=6) = (1 - P(B+))^{6-1} \, P(B+)$$

$$= (1 - 0.44)^{5} \cdot 0.44$$

$$= 0.0242 \text{ (Ans)}$$

$$E(x) = \mu = \frac{1}{p} = \frac{1}{0.44} = 2.2727 \text{ (Ans)}$$

$$\text{Standard Deviation} = \frac{\sqrt{1-p}}{p} = \frac{\sqrt{(1-0.44)}}{0.44}$$

$$= 1.7007 \text{ (Ans)}$$

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[2] Probability of getting Six in a die $P = \frac{1}{6}$

$$P(x=5) = (1-P)^{5-1} P$$

$$= \left(1-\frac{1}{6}\right)^{4} \cdot \frac{1}{6}$$

$$= 0.0804 \text{ (Ans)}$$

$$E(x) = \frac{1}{P} = \frac{1}{\frac{1}{6}} = 6 \text{ (Ans)}$$

Standard deviation of the number of rolls

$$= \frac{\sqrt{1-P}}{P} = \frac{\sqrt{1-\frac{1}{6}}}{\frac{1}{6}} = \sqrt{30}$$

$$= 5.477 \text{ (Ans)}$$

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Protivas

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3 Probability of being accepted to Harvard is .05 .

we know,

$$b(x; n, p) = {}^{n}C_{x}\, p^{x} (1-p)^{n-x}$$

so,

$$b(3; 20, 0.05) = {}^{20}C_{3} (0.05)^{3} (1-0.05)^{20-3}$$

$$= 1140 \times \frac{1}{8000} \times 0.4181$$

$$= 0.05958 \text{ (Ans)}$$

if $n = 10$,

$$E(x) = 10 \times 0.05 = 0.5 \text{ (Ans)}$$

Standard Deviation $= \sigma(x) = \sqrt{10 \times 0.05 \times (1-0.05)}$

$$= 0.6892 \text{ (Ans)}$$

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Protivas

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[4] Probability of people hav hazel eyes

$= 0.05$.

Probability that 10 people from 100 people have hazel eyes in

$${}^{100}C_{10}\,(0.05)^{10}\,(1-0.05)^{100-10}$$

$\Rightarrow 0.0167$ (Ans)

if $n = 150$

Mean $= E(x) = \cancel{n \times 15\ 100 \times}\ 150 \times 0.05$

$= 7.5$ (Ans)

Variance $= \sigma^2(x) = npq = np(1-p)$

$= 150 \times 0.05 \times (1-0.05)$

$= 7.125$ (Ans)

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Lecture 10 :

[1]. Given that,

$$\lambda = 4 \text{ per } 100$$

$$= 4\times 4 \text{ per } 100\times 4$$

$$= 16 \text{ per } 400$$

$$P(x \le 1) = e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!}\right]$$

$$= e^{-16}\left[\frac{16^0}{0!} + \frac{16}{1!}\right]$$

$$= 1.9130 \times 10^{-6} \text{ (Ans)}$$

[2]. (a) Given that,

$$\lambda = 180 \text{ per hour}$$

$$= 3 \text{ per minute}$$

$$P(x=0) = \frac{e^{-\lambda} \cdot \lambda^0}{0!} = 0.04979 \text{ (Ans)}$$

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b Given that,

$\lambda = 180$ per hour

$= 3$ per minutes

So, expected number of docking in 10 minutes are $(3 \times 10) = 30$ ships per 10 minutes

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3 Given that,

$P(A \text{ wins}) = 0.3$

$P(B \text{ wins}) = 0.5$

$P(\text{tie}) = 0.2$

Probability that all games end in a Tie is

$${}^{30}C_{30} \; P(\text{tie})^{30} \left(1 - P(\text{tie})\right)^{30-30}$$

$$\Rightarrow 1 \times (0.2)^{30} (1 - 0.2)^{0}$$

$$\Rightarrow 1.0737 \times 10^{-21} \text{ (Ans)}$$

4 (a) Given that,

$$P(\text{single}) = \frac{1}{16}$$

$$P(\text{douple}) = \frac{1}{4}$$

$$P(\text{tolple}) = \frac{1}{5}$$

$$P(\text{none}) = \frac{1}{24}$$

if 8 ~~balls~~ times is given,

$$^8C_4 \left(\frac{1}{16}\right) \cdot \left(\frac{1}{4}\right) \cdot \left(\frac{1}{5}\right) \cdot \frac{1}{24}$$

$$\Rightarrow \frac{7}{768} = 9.1145 \times 10^{-3}$$

(b) if 4 times is given

$$^4C_4 \times \frac{1}{16} \times \frac{1}{4} \times \frac{1}{5} \times \frac{1}{24}$$

$$\Rightarrow \frac{1}{7680} = 1.3020 \times 10^{-4} \text{ (Ans)}$$

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Lecture 11 & 12

$\boxed{1}$ Given that,

~~P(R)~~ Red balls = 6

Blue balls = 7

Green balls = 5

$$P(\text{different 2 balls}) = R.B + R.G + B.G = P(A)$$

$$= \frac{6C_1}{18C_1} \cdot \frac{7C_1}{18C_1} + \frac{6C_1}{18C_1} \cdot \frac{5C_1}{18C_1} + \frac{5C_1}{18C_1} \cdot \frac{7C_1}{18C_1}$$

$$= \frac{107}{324}$$

$$P(\text{Red, blue}) = \frac{6C_1}{18C_1} \times \frac{7C_1}{18C_1} = \frac{7}{54} = P(B)$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \qquad [\because P(B \cap A) = P(B)]$$

as all member of B is in A

$$= \frac{\frac{7}{54}}{\frac{107}{324}}$$

$$= \frac{42}{107} \text{ (Ans)}$$

Protivas

$\boxed{2}$ $v = 23 - 22$

$= 1$

$u + 2 + 23 + u - v = 40$

$\Rightarrow 2u + 25 - 1 = 40$

$\Rightarrow u = 8$

$5z - 5 + z + 3 = 40$

$\Rightarrow 6z - 2 = 40$

$\Rightarrow 6z = 42$

$\Rightarrow z = 7$

$22 + 2 + x = 5 \times 7 - 5$

$\Rightarrow x = 6$

$x + w = u - v$

$\Rightarrow 6 + w = 8 - 1$

$\Rightarrow w = 1$

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	Good	Average	Bad	Total
Regular	22	2	6	30
Irregular	1	8	1	10
total	23	10	7	40

$$P(\text{irregular}) = \frac{10}{40} = \frac{1}{4} = P(A)$$

$$P(\text{Bad}) = \frac{7}{40} = P(B)$$

$$P(A \cap B) = \frac{1}{40}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{40}}{\frac{1}{4}} = \frac{1}{10} \text{ (Ans)}$$

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$\boxed{3}$ $P(F) = .1$ $\qquad P(B|F) = .05$

$P(M) = .3$ $\qquad P(B|M) = .1$

$P(R) = .6$ $\qquad P(B|R) = .2$

$$P(\text{Broken}) = P(F).P(B|F) + P(M).P(B|M) + P(R).P(B|R)$$
$$= .1 \times .05 + .3 \times .1 + .6 \times .2$$
$$= 0.155 \text{ (Ans)}$$

$\boxed{4}$ Given that,

$P(\text{Good Risk}) = P(GR) = .3$

$P(\text{average Risk}) = P(AR) = .6$

$P(\text{Bad Risk}) = P(BR) = 1 - (.3 + .6)$
$= .1$

$P(\text{Accident} \mid \text{Good Risk}) = .1$

$P(\text{Accident} \mid \text{Arerage Risk}) = .2$

$P(\text{Accident} \mid \text{Bad Risk}) = .3$

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P(Accident of a Random Policy holder)

$\Rightarrow .3 \times .1 + .6 \times .2 + .1 \times .3$

$= 0.18$ (Ans)

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5 Bag A,

Red → 6

Black → 7

Bag B,

Red → 9

Black → 6

~~P(Black drawn) = $\frac{7}{13}$~~

P(Black transferred from A to B) = $\frac{7}{13}$

P(Red " " A to B) = $\frac{6}{13}$

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P(Drawing black ball from Bag B when transferred ball is Red)

$\Rightarrow \frac{6}{16}$

P(Drawing black ball from Bag B when transferred ball is Black)

$\Rightarrow \frac{7}{16}$

P(transferred ball was ~~Black~~ Red)

$$\Rightarrow \frac{\frac{6}{16} \times \frac{6}{13}}{\frac{6}{16} \times \frac{6}{13} + \frac{7}{16} \times \frac{7}{13}}$$

$$\Rightarrow \frac{36}{85}$$

$\Rightarrow 0.4235$ (Ans)

6. Probability of getting Head in unfair coin $= \frac{3}{4}$

$P(\text{Head in fair coin}) = \frac{1}{2}$

let,

F = Event selecting a fair coin

U = " " unfair coin

x = Event of the coin landing heads exactly 7 time in 9 times

$$P(U|x) = \frac{P(x|U) * P(U)}{P(x)}$$

$$P(x) = P(x|F) \cdot P(F) + P(x|U) * P(U) \quad \text{—(i)}$$

and, $P(U|x) \cdot P(x) = P(x|U) \cdot P(U)$

$$P(U|x) = \frac{P(x|U) \cdot P(U)}{P(x|F) \cdot P(F) + P(x|U) \cdot P(U)}$$

$$P(x|U) = {}^9C_7 \cdot (.75)^7 \cdot \left(\frac{1}{4}\right)^2$$

$$= 0.3033$$

$$P(U) = \frac{12}{12+8} = .6$$

$$P(F) = .4$$

$$P(x|F) = {}^9C_7 \cdot (.5)^7 \cdot (.5)^2$$

$$= 0.07031$$

$$\therefore P(U|x) = \frac{0.3003 \times .6}{(.0703 \times .4) + (.3003 \times .6)}$$

$$= 0.8650 \text{ (Ans)}$$