BRAC UNIVERSITY

CSE230

DISCRETE MATHEMATICS

Assignment 04

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CSE 230 - Assignment 04

1. Given that,

$a_1 = 3$

$a_2 = 4$

and, $a_n = 2a_{n-1} + a_{n-2}$

$\Rightarrow a_n - 2a_{n-1} - a_{n-2} = 0$ —(i)

Let, $a_n = r^n$

So, from (i)

$r^n - 2r^{n-1} - r^{n-2} = 0$

$\therefore r^2 - 2r - 1 = 0$ [deviding by r^{n-2} both side]

$\therefore$

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4.1.(-1)}}{2.1}$$

$$= \frac{-(-2) \pm \sqrt{4+4}}{2}$$

$$= \frac{-(-2) \pm \sqrt{8}}{2}$$

$$= 1+\sqrt{2}, 1-\sqrt{2}$$

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So, $r_1 = 1+\sqrt{2}$, $r_2 = 1-\sqrt{2}$

Now solution,

$$a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$$

$$a_n = \alpha_1 (1+\sqrt{2})^n + \alpha_2 (1-\sqrt{2})^n$$

$$a_1 = 3 = \alpha_1 (1+\sqrt{2})^1 + \alpha_2 (1-\sqrt{2})^1$$

$$= \alpha_1 + \sqrt{2}\alpha_1 + \alpha_2 - \sqrt{2}\alpha_2$$

$$= (\alpha_1 + \alpha_2) + \sqrt{2}(\alpha_1 - \alpha_2) = 3 \quad -(ii)$$

$$a_2 = 4 = \alpha_1 (1+\sqrt{2})^2 + \alpha_2 (1-\sqrt{2})^2$$

$$= \alpha_1 (3+2\sqrt{2}) + \alpha_2 (3-2\sqrt{2})$$

$$= 3(\alpha_1 + \alpha_2) + 2\sqrt{2}(\alpha_1 - \alpha_2) = 4 \quad -(iii)$$

Solving (ii) and (iii) we get,

$$\alpha_1 - \alpha_2 = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2} \quad -(iv)$$

and $\alpha_1 + \alpha_2 = -2 \quad -(v)$

$$(iv+v) \Rightarrow 2\alpha_1 = \frac{5\sqrt{2}}{2} - 2 = \frac{5\sqrt{2}-4}{2}$$

$$\therefore \alpha_1 = \frac{5\sqrt{2}-4}{4} = \frac{1}{4}(5\sqrt{2}-4)$$

$$(iv-v) \Rightarrow -2\alpha_2 = \frac{5\sqrt{2}}{2} + 2 = \frac{5\sqrt{2}+4}{2}$$

$$\therefore \alpha_2 = -\frac{1}{4}(5\sqrt{2}+4)$$

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So, $a_n = \alpha_1(1+\sqrt{2})^n + \alpha_2(1-\sqrt{2})^n$

$= \frac{1}{4}(5\sqrt{2}-4)(1+\sqrt{2})^n + \{\frac{1}{4} \cdot -(5\sqrt{2}+4) \cdot (1-\sqrt{2})^n\}$

$= \frac{1}{4}\{(5\sqrt{2}-4)(1+\sqrt{2})^n - (5\sqrt{2}+4)(1-\sqrt{2})^n\}$

(Ans)

—o—

$\boxed{2}$ Given that,

$a_1 = 3$

$a_2 = 4$

$a_n = 2a_{n-1} + a_{n-2} + 5$

$\Rightarrow a_n - 2a_{n-1} - a_{n-2} = 5$

$\Rightarrow r^n - 2r^{n-1} - r^{n-2} = 0$ [homogenous part]

$\Rightarrow r^2 - 2r - 1 = 0$

$\therefore r_1 = 1+\sqrt{2}, \ r_2 = 1-\sqrt{2}$

we know, for non-homogenous

$a_n = a_n^{(h)} + a_n^{(p)}$

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$\therefore a_n^{(h)} = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$

here,

$f(n) = 5 = 5^1 \times n^0$

and, $a_n^P = \alpha_3 (b)^n$

$\therefore \cancel{a_n^{(P)} = A}$

$\therefore \cancel{A - 2A - A = 5}$ here, $b = 5$

$\therefore \cancel{-2A = 5}$ So, $a_n^P = \alpha_3 (5)^n$

$\therefore \cancel{A = -\frac{5}{2}}$

$\therefore a_n = \alpha_1 (1+\sqrt{2})^n + \alpha_2 (1-\sqrt{2})^n + \alpha_3 (5)^n$

given,

$a_1 = 3$

$a_2 = 4$

$\therefore a_3 = 2 a_{3-1} + a_{3-2} + 5$

$= 2 a_2 + a_1 + 5$

$= 2 \cdot 4 + 3 + 5$

$= 16$

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$\therefore a_1 = 3 = \alpha_1(1+\sqrt{2}) + \alpha_2(1-\sqrt{2}) + \alpha_3 \cdot 5$

$= \alpha_1 + \sqrt{2}\alpha_1 + \alpha_2 - \sqrt{2}\alpha_2 + 5\alpha_3$

$= (\alpha_1 + \alpha_2) + \sqrt{2}(\alpha_1 - \alpha_2) + 5\alpha_3$

$a_2 = 4 = \alpha_1(1+\sqrt{2})^2 + \alpha_2(1-\sqrt{2})^2 + 5\alpha_3$

$= \alpha_1(3+2\sqrt{2}) + \alpha_2(3-2\sqrt{2}) + 5\alpha_3$

$= 3(\alpha_1 + \alpha_2) + 2\sqrt{2}(\alpha_1 - \alpha_2) + 5\alpha_3$

$a_3 = 16 = \alpha_1(1+\sqrt{2})^3 + \alpha_2(1-\sqrt{2})^3 + 5\alpha_3$

$= \alpha_1(7+5\sqrt{2}) + \alpha_2(7-5\sqrt{2}) + 5\alpha_3$

$= 7(\alpha_1 + \alpha_2) + 5\sqrt{2}(\alpha_1 - \alpha_2) + 5\alpha_3$

Let, $\alpha_1 + \alpha_2 = x$

$\alpha_1 - \alpha_2 = y$

$\alpha_3 = z$

$\therefore$

$3 = x + \sqrt{2}y + 5z$ — (i)

$4 = 3x + 2\sqrt{2}y + 5z$ — (ii)

$16 = 7x + 5\sqrt{2}y + 5z$ — (iii)

Solving (i)(ii) and (iii)

$$x = -\frac{9}{2}$$
$$y = 5\sqrt{2}$$
$$z = -\frac{1}{2} = \alpha_3$$

$$\therefore \alpha_1 + \alpha_2 = -\frac{9}{2}$$
$$\alpha_1 - \alpha_2 = 5\sqrt{2}$$

$$\therefore 2\alpha_1 = \frac{-9+10\sqrt{2}}{2}$$
$$\therefore \alpha_1 = \frac{1}{4}(-9+10\sqrt{2})$$

$$2\alpha_2 = -\left(\frac{9+10\sqrt{2}}{2}\right)$$
$$\alpha_2 = -\frac{1}{4}\left(\frac{10\sqrt{2}+9}{1}\right)$$

$$\therefore \alpha_n = \frac{1}{4}(10\sqrt{2}-9)(1+\sqrt{2})^n \cancel{+} - \frac{1}{4}(10\sqrt{2}+9)(1-\sqrt{2})^n$$
$$+ 5 . -\frac{1}{2}$$
$$= \frac{1}{4}\left\{(10\sqrt{2}-9)(1+\sqrt{2})^n - (10\sqrt{2}+9)(1-\sqrt{2})^n\right\} - \frac{5}{2}$$

(Ans)

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[3] Given that,

$$a_1 = 3$$

$$a_2 = 4$$

and,

$$a_n = 2a_{n-1} + a_{n-2} + n^2 + 1$$

solution,

$$a_n = a_n^{(h)} + a_n^{(p)}$$

~~from,~~

$$r^n = 2r^{n-1} + r^{n-2}$$

~~or,~~

$\cancel{a_n = 2r^{n-1} + r^{n-2}}$

$$r^2 - 2r + 1 = 0$$

$$\therefore r = 1 \pm \sqrt{2}$$

for, $n^2 + 1$

$$a_n^{(p)} = A_0 + A_1 n + A_2 n^2$$

$$\therefore 2A_0 + 2A_1(n-1) + 2A_2(n-1)^2 + A_0 + A_1(n-2) + A_2(n-2)^2 + n^2 + 1$$

$$\Rightarrow A_0 + A_1 n + A_2 n^2 = 2A_0 + 2A_1 n - 2A_1 + 2A_2 + 2A_2 n^2 - 4A_2 n + A_0 + A_1 n - 2A_1 +$$

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$A_2 n^2 + 4A_2 - 4A_2 n + n^2 + 1$

$\Rightarrow A_0 + A_1 n + A_2 n^2 = (1 + 3A_0 - 4A_1 + 6A_2) + (3A_1 - 8A_2)n$
$+ (1 + 3A_2) n^2$

$\Rightarrow (1 + 2A_2) n^2 + (2A_1 - 8A_2) n + 1 + 2A_0 - 4A_1 + 6A_2) = 0$

$\therefore 1 + 2A_2 = 0$ or, $A_1 = 4A_2$

$\therefore A_2 = -\frac{1}{2}$ $\therefore A_1 = 4 \times (-\frac{1}{2})$

$\therefore A_1 = -3$

$\therefore a_n^{(P)} = -3 - 2n - \frac{n^2}{2}$

$\therefore a_n = \alpha_1 (1+\sqrt{2})^n + \alpha_2 (1-\sqrt{2})^n - 3 - 2n - \frac{n^2}{2}$

$a_1 = \alpha_1 (1+\sqrt{2}) + \alpha_2 (1-\sqrt{2}) - 3 - 2 - \frac{1}{2} = 3$

$a_2 = \alpha_1 (1+\sqrt{2})^2 + \alpha_2 (1-\sqrt{2})^2 - 3 - 4 - 2 = 4$

$\Rightarrow 3(\alpha_1 + \alpha_2) + 2\sqrt{2}(\alpha_1 - \alpha_2) = 13$

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$\therefore (\alpha_1+\alpha_2)+\sqrt{2}(\alpha_1-\alpha_2)=\frac{17}{2}$ — (i)

$3(\alpha_1+\alpha_2)+2\sqrt{2}(\alpha_1-\alpha_2)=13$ — (ii)

solving (i) and (ii)

$\alpha_1+\alpha_2=-4$ — (iii)

$\alpha_1-\alpha_2=\frac{25\sqrt{2}}{4}$ — (iv)

solving (iii) and (iv)

$\alpha_1=\frac{25\sqrt{2}}{8}-2=\frac{16+25\sqrt{2}}{8}$

$\alpha_2=-2-\frac{25\sqrt{2}}{8}$

$a_n=\left(\frac{25\sqrt{2}-16}{8}\right)(1+\sqrt{2})^n-\left(\frac{16+25\sqrt{2}}{8}\right)(1-\sqrt{2})^n$

$-3-2n-\frac{n^2}{2}$ (Ans)

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