CSE230 (Discrete Mathematics)

Assignment 1

Topics: Set, Function and Combinatorics

Set:

1. Write the expression in set builder notation. Also provide the number line.
(-10,3] ∩ [-5, 5)

2. A = {1, 3,5} B = {red, green}
Find out the power sets of set A and B. Also write down the cartesian product of A and B. What's the cardinality of this cartesian product?

3. Use set builder notation to establish the first De Morgan law
$A' \cap B' = (A \cup B)'$

4. A travel group has 105 travelers. Of them, 50 travelers already visited India, 30 Nepal, 20 Bhutan, 6 both India and Nepal, 1 both India and Bhutan, 5 Nepal and Bhutan, and one of them visited all the 3 countries.

Function:

5. Is the relation given by the following set of ordered pairs a function?
{ (1,2), (5,6), (8, 6), (7,2), (9,2), (8,6) }. Explain your reasoning.

6. For $f(x) = cos(4x - 1)$, find the range of f(x). What should be the domain of f(x)?

7. Find the domain of $f(x) = log\,(x^2 - 3)$

8. A student writes the following for the function $f(x) = \frac{x-2}{x^2 - 8x+8}$:

" The domain of f(x) is (-∞, -4) ∪ (-4 , +∞) "
Is this correct? If not, what is the correct domain of f(x) ?

Combinatorics:

9. Three young brother-sister pairs from different families need to take a trip in a van. These six children will occupy the second and third rows in the van, each of which has three seats. To avoid disruptions, the siblings may not sit right next to each other in a row and no child may sit directly in front of his or her sibling. How many seating arrangements are possible for this trip?

10. Every positive integer greater than 1 has at least two divisors and can be written as a unique product of some prime number/s with exponents. For example,

$5 = 5^1$ has two divisors (1 and 5 itself)

$6 = 2^1 \times 3^1$ has four divisors (1, 2, 3 and 6)

$16 = 2^4$ has five divisors (1, 2, 4, 8 and 16).

If a number $n = p_1^{\alpha_1} \times p_2^{\alpha_2} \times p_3^{\alpha_3} \times \ldots \times p_{k-1}^{\alpha_{k-1}} \times p_k^{\alpha_k}$ where $p_1,\ p_2,\ p_3 \ldots\ p_{k-1},\ p_k$ are prime numbers and $\alpha_1,\ \alpha_2,\ \alpha_3 \ldots\ \alpha_{k-1},\ \alpha_k$ are the corresponding exponents of the prime numbers, how many divisors does n have ?

11. How many 5 digit positive integers are divisible by 5 and have at least one 6 as their digit?

12. [Bonus] In how many ways can 7 people A, B, C, D, E, F and G be seated at a round table so that no two of A, B and C sit next to each other?