BRAC UNIVERSITY

CSE230

Discrete Mathematics

Assignment 2

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Section: 02

Inspiring Excellence

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CSE230 Assignment-2

Lecture 4

$\boxed{1.}$ if a binomial expression is $(x+y)^n$ then

it's $(r+1)^{th}$ term is ${}^nc_r\, x^{n-r} y^r$.

$(r+1)^{th}$ term of $(2x^3+\frac{1}{x})^{29}$ is

$${}^{29}c_r (2x^3)^{29-r} (\frac{1}{x})^r$$

Now, $x^{3(29-r)-r} = x^{47}$

So, $x^{87-3r-r} = x^{47}$

$\therefore r = 10$

So, 11th term is for x^{47}.

$${}^{29}c_{10} (2x^3)^{29-10} (\frac{1}{x})^{10}$$

$$\Rightarrow {}^{29}c_{10}\, 2^{19}\, x^{47}$$

$\Rightarrow {}^{29}c_{10}\, 2^{19}$ is the coefficient of x^{47}

if we compare it with ${}^{29}c_a * b^k$ then

$(a+b+k) = 10+2+19 = 31$ (Ans)

[2] 3rd term of $\left(y^2+\frac{2}{y}\right)^{21}$ is

$$^{21}C_2\,(y^2)^{21-2}\left(\frac{2}{y}\right)^2 = {}^{21}C_2\, y^{38-2}\cdot 4$$

$$= {}^{21}C_2\cdot 4\cdot y^{36}$$

and 3rd term of $\left(y+\frac{1}{3}\right)^{37}$ is

$$^{37}C_2\,(y)^{37-2}\left(\frac{1}{3}\right)^2 = {}^{37}C_2\, y^{35}\,\frac{1}{9}$$

$$= {}^{37}C_2\cdot\frac{1}{9}\cdot y^{35}$$

Now $\quad {}^{21}C_2\cdot 4\cdot y^{36} = {}^{37}C_2\cdot\frac{1}{9}\cdot y^{35}$

$$\Rightarrow \frac{y^{36}}{y^{35}} = \frac{{}^{37}C_2\cdot\frac{1}{9}}{{}^{21}C_2\cdot 4} = \frac{{}^{37}C_2}{{}^{21}C_2\cdot 36}$$

$$\therefore y = \frac{{}^{37}C_2}{{}^{21}C_2\cdot 36} \quad \text{(Ans)}$$

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3| $(z^3+3z+1)^6$

the general term of the expression is

$$\frac{6!}{x_1!\,x_2!\,x_3!} \times (z^3)^{x_1} \cdot (3z)^{x_2} \cdot (1)^{x_3}$$

to get the coefficient of z^4,

$(z^3)^{x_1} \cdot z^{x_2}$ needs to be equal to z^4

$$x^{3x_1+x_2} = x^4$$

$$\Rightarrow 3x_1 + x_2 = 4$$

as, $x_1, x_2 \le 6$ so, it can be

$x_1 = 1,\ x_2 = 1$ or,

$x_1 = 0,\ x_2 = 4$

if, $x_1 = 1$ and $x_2 = 1$, then, coefficient is,

$$\frac{6!}{1! \cdot 1! \times 4!} \times 3 = 90$$

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if $x_1 = 0$ and $x_2 = 4$ then coefficient is,

$$\frac{6!}{0!\,4!\,2!} \times (3)^4 = 1215$$

So the co-efficient of z^4 in the expression $(z^3+3z+1)^6$ is $1215 + 90 = 1305$ (Ans)

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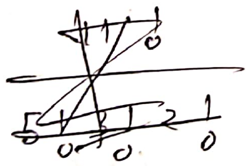
9) Given Expression $(370a + 285b + 99c)^{11}$

coefficient of $a^5b^3c^2$ is,



in the expression the power is 11 but if we want to calculate the co-efficient of $a^5b^3c^2$ we see that the sum of all power is $5+3+2 = 10$, but to be a part of any term of the expression total sum must be equal to 11. So, the coefficient of $a^5b^3c^2$ ~~is either 0 or~~ not exist, or 0.

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Lecture 7

1) if we tossed two dice then

$n(S) = 6 \times 6 = 36$

let E = event that the sum of two dice greater than 3 and prime number

$E = \{(1,4),(1,6),(2,3),(2,5),(3,2),(3,4),$
$(4,1),(4,3),(5,2),(5,6),(6,1),(6,5)\}$

$n(E) = 12$

$P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$ (Ans)

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2) $P(\text{none in defective}) = \frac{{}^{16}C_2}{{}^{20}C_2}$

$= \frac{12}{19}$

so, $P(\text{atleast 1 defective}) = \left(1 - \frac{12}{19}\right)$

$= \frac{7}{19}$ (Ans)

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3) First 3 cards were spades so

Now total spades are $13-3=10$. And total remaining cards are $(52-3)=49$

$P(\text{4th card is spade}) = \frac{10}{49}$

$\therefore P(\text{4th card is not spade}) = \left(1-\frac{10}{49}\right)$

$= \frac{39}{49}$ (Ans)

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4) $n(S) = \frac{9!}{5!\,4!} = 126$

$n(E) = n(\text{same publisher will be put together}) = 2$

$\therefore P(\text{Same publisher will be put together}) = \frac{n(E)}{n(S)}$

$= \frac{2}{126}$

$= \frac{1}{63}$ (Ans)

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Lecture 8

1. we know $\sigma^2 = E(x^2) - (E(x))^2$

the random variable x can be $1, 2, 3 \ldots n$

with same probability.

So, $E(x) = \frac{1}{n} \times 1 + \frac{1}{n} \times 2 \ldots \frac{1}{n} \times n$

$$= \frac{1}{n}(1 + 2 + \ldots n)$$

$$= \frac{1}{n}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{n+1}{2}$$

and $E(x^2) = \frac{1}{n} \times 1^2 + \frac{1}{n} \times 2^2 \ldots + \frac{1}{n} \times n^2$

$$= \frac{1}{n}(1^2 + 2^2 + \ldots + n^2)$$

$$= \frac{1}{n} \times \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6}$$

So, $\sigma^2 = E(x^2) - (E(x))^2$

$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$

$= \frac{n^2-1}{12}$ (Ans)

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[2] $n(x) = n$

and $n(S) = n$

So, $P_r(x \le n) = \frac{n(x)}{n(S)} = \frac{n}{n} = 1$

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[3] $n(S) = 10 \times 10 = 100$

Let E = sum of two numbers that are prime number.

$E = \{(1,1), (1,2), (1,4), (1,10) \ldots\ldots \}$

$n(E) = 37$

$\therefore P(E) = \frac{37}{100}$ (Ans)

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4] $P(\text{divisible by } 3) = \dfrac{\frac{n}{3}}{n}$

$= \dfrac{n}{3n}$

$P(\text{divisible by } 2) = \dfrac{\frac{n}{2}}{n} = \dfrac{n}{2n}$

P(doesn't divisible by 3) and P(doesn't divisible by 2)

$= \left(1 - \dfrac{n}{3n}\right)\left(1 - \dfrac{n}{2n}\right)$

$= \dfrac{2n}{3n} \times \dfrac{n}{2n}$

$= \dfrac{2}{3} \times \dfrac{1}{2}$

$= \dfrac{2}{6}$ (Ans)