Numerical Methods Lab 6

Nonlinear_equations

i. Open the colab file shared in BUX.
ii. Create a copy of that shared file.
iii. Rename the colab filename using the format **Name-ID-Lab Section**

Lab Introduction

Part 1: Polynomial Root Finding Using Bisection Method

One way to find out root's are to use bisection method. Here is the strategy, if α is a root between and interval $[a, b]$ then graph will cross the X -axis at α. So, $sign(f(\alpha - h)) = -sign(f(\alpha + h))$, for small value of h.

So, we can work our way up towards the root by taking average of a and b, as long as the signs are different.

we will start with a_0 and b_0, such that, $f(a_0)f(b_0) < 0$. Then we iterate as this,

$$m_k = \frac{a_k + b_k}{2}$$
$$\text{if, } f(a_k)f(m_k) < 0, \text{ then, } a_{k+1} = a_k \text{ and } b_{k+1} = m_k$$
$$\text{else, } a_{k+1} = m_k \text{ and, } b_{k+1} = b_k$$

We keep iterating until we find the root with sufficient precision. We usually use a formula like this,

$$\frac{|m_{k+1} - m_k|}{|m_{k+1}|} \leq \epsilon$$

Where, ϵ is a very small value, like $\epsilon < 10^{-6}$

[Task 1] – 4 marks

You have to complete the code to iterate and solve for a root of the following equation, between the interval, $[-0.5, 1.3]$:

$f(x) = 2 + 0.5x - 6x^2 - 2x^3 + 2.5x^4 + x^5$.

You will have to remove the “raise NotImplementedError()” .

Part 2: Fixed Point Iteration

A number ξ is called a **fixed point** to function $g(x)$ if $g(\xi) = \xi$. Using fixed points are a nice strategy to find roots of an equation. In this method if we are trying to find a root of $f(x) = 0$, we try to write the function in the form, $x = g(x)$. That is,

$$f(x) = x - g(x) = 0$$

So, if ξ is a fixed point of $g(x)$ it would also be a root of $f(x) = 0$, because,

$$f(\xi) = \xi - g(\xi) = \xi - \xi = 0$$

We can find a suitable $g(x)$ in any number of ways. Not all of them would converge; whereas, some would converge very fast. For example, consider $Eq. 6.1$.

$$\begin{aligned} & f(x) = x^5 + 2.5x^4 - 2x^3 - 6x^2 + x + 2 \\ \implies \quad & x - g(x) = x^5 + 2.5x^4 - 2x^3 - 6x^2 + x + 2 \\ \implies \quad & g(x) = -x^5 - 2.5x^4 + 2x^3 + 6x^2 - 2 \end{aligned} \tag{6.2}$$

[Task 2] – 4 marks

You have to complete the code by using a couple of g(x) functions to find out which one converges faster.

You will have to remove the “raise NotImplementedError()” .

[Task 3] – 2 marks

Problem related to Interval Bisection method: Consider the following function: f(x) = $x^3 + x^2 - 25x - 25$. Use interval bisection method to find the root, x∗ of f(x), on the interval [−4, 3], where the error bound, $\delta = 10^{-2}$.