BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for Spring 2023 Semester

Student ID: 2010534

Section: 10

Quiz 3

Full Marks: 10

Name: md. Daniel Islam

10

Duration: 20 minutes

1. [CO2] Consider the following dataset:

x	2.1	2.3	2.5	2.7
f(x)	18.2	22.3	24.5	30.6

a) Compute $f^{(1)}(2.5)$ upto 4 significant figures using the **central** difference method. [2 marks]

b) Compute $f^{(1)}(2.3)$ upto 4 significant figures using the **forward** difference method. [2 marks]

c) For the interval **[2.1, 2.4]**, compute the **upper bound** of truncation error if the above data is generated by the function, $f(x) = 4x^2 + 3e^{2x}$ using the method used in Part-(a). [3 marks]

d) Compute $D^{(1)}_{0.2}$ at $x_0 = 2.5$ using **Richardson extrapolation** method up to 4 significant figures if the actual function, $f(x)=6e^{-4x}$ and also compute truncation error. [3 marks]

(a) in central $f'(x) = \dfrac{f(x+h) - f(x-h)}{2h}$

here, $h = |2.3 - 2.1| = 0.2$

$\therefore f'(2.5) = \dfrac{f(2.5+0.2) - f(2.5-0.2)}{0.2 \times 2}$

$= \dfrac{f(2.7) - f(2.3)}{0.2 \times 2}$

$= \dfrac{30.6 - 22.3}{0.2 \times 2} = \dfrac{8.3}{0.4} = 20.75$

$\therefore f'(2.5) = 20.75$.

(b) in forward,

$$f'(x) = \frac{f(x+h) - f(x)}{h}$$

Q here, $h = 0.2$

$$f'(2.3) = \frac{f(2.3+0.2) - f(2.3)}{0.2}$$

$$= \frac{f(2.5) - f(2.3)}{0.2}$$

$$= \frac{24.5 - 22.3}{0.2} = 11.0000$$

$\therefore f'(2.3) = 11.00$

(c) here $\xi = [2.1, 2.4]$

$f(x) = 4x^2 + 3e^{2x}$

upper bound of truncation error in central method

is $\frac{f^3(\xi_1)}{3!} \times (h)^2$

$f'(x) = 8x + 6e^{2x}$

$f^2(x) = 8 + 12e^{2x}$

$f^3(x) = 0 + 24e^{2x}$

here, $f^3(2.1) = 24e^{2\times 2.1} \Rightarrow 1600.4219$

$f^3(2.4) = 24e^{2\times 2.4} \Rightarrow 2916.2500$

as $f^3(2.4) > f^3(2.1)$

$\therefore$ upper bound

$\Rightarrow \frac{f^3(2.4)}{3!} \times (0.2)^2$

$\Rightarrow \frac{2916.2500}{3!} \times (0.2)^2$

$\Rightarrow 29.44$ (Ans)

$$D'_{0.2} = \frac{2^2 D_{0.1} - D_{0.2}}{2^2 - 1}$$

at, $x_0 = 2.5$

$$D_{0.1} = \frac{f(2.5+0.1) - f(2.5-0.1)}{2 \times 0.1}$$

$$= \frac{f(2.6) - f(2.4)}{0.2}$$

$$= \frac{6e^{-4\times2.6} - 6e^{-4\times2.4}}{0.2} = -1.11888 \times 10^{-3}$$

$$D_{0.2} = \frac{f(2.5+0.2) - f(2.5-0.2)}{2 \times 0.2}$$

$$= \frac{f(2.7) - f(2.3)}{0.4}$$

$$= \frac{6e^{-4\times2.7} - 6e^{-4\times2.3}}{0.4} = -1.20959 \times 10^{-3}$$

$$\therefore D'_{0.2} = \frac{2^2 \times (-1.11888 \times 10^{-3}) - (-1.20959 \times 10^{-3})}{2^2 - 1}$$

$$= -1.088652 \times 10^{-3}$$

$f(x) = 6e^{-4x}$

$f'(x) = -24e^{-4x}$

$f'(2.5) = -24e^{-4\times2.5}$

$= -1.08959 \times 10^{-3}$

truncation error

$\Rightarrow |\text{actual} - \text{numerical}|$

$\Rightarrow |-1.08959 \times 10^{-3} + 1.088652 \times 10^{-3}|$

$\Rightarrow |-9.38 \times 10^{-7}| \Rightarrow 9.380 \times 10^{-7}$