

Instructions for preparing the solution script:

- Write your name, ID#, and Section number clearly in the very front page.
 - Write all answers sequentially.
 - Start answering a question (not the part of the question) from the top of a new page.
 - Write legibly and in orderly fashion maintaining all mathematical norms and rules. Prepare a single solution file.
 - Start working right away. There is no late submission form. If you miss the deadline, you need to use the make-up assignment to cover up the marks.
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1. Let $f(x) = \tan(x)$. In the following we would like to calculate the truncation errors.
 - (a) (3 marks) First write down the approximate polynomial, $p_3(x)$, for the function $f(x)$ and identify the Taylor coefficients, a_0, \dots, a_3 .
 - (b) (2 marks) Compute the percentage relative error at $x = \pi/4$ if $f(x)$ is approximated by $p_3(x)$ polynomial.
 - (c) (5 marks) Use the Lagrange remainder form to evaluate the upper bound of truncation error at $x = \pi/4$ for some $\xi \in [0, \pi/4]$.
2. Consider the function $f(x) = e^x - e^{-x}$ and the nodes are at -1, 0, and 1. Now answer the following questions using 3 significant figures:
 - (a) (1 mark) Write down the matrices b and V used in Vandermonde method.
 - (b) (2 marks) Compute the determinant of the Vandermonde matrix V .
 - (c) (3 marks) Using The results of the previous two parts, calculate the Taylor coefficients a_0, a_1 and a_2 ; and finally find the interpolating polynomial.
 - (d) (4 marks) Evaluate the upper bound of interpolation error for the given function for the interval $\xi \in [-2.1, 2.1]$.
3. Consider the function $f(x) = e^x + e^{-x}$ and the nodes are at -1, 0, and 1. Now answer the following questions using 3 significant figures:
 - (a) (4 marks) Evaluate the Lagrange bases for the given function and nodes.
 - (b) (3 marks) Compute the Lagrange interpolation polynomial for the given function, and express the result in the natural basis. Also use this polynomial to find an approximate value of $f(6)$.
 - (c) (3 marks) Evaluate the relative error in percentage form at $x = 1.5$.
4. Consider the function $f(x) = e^x - e^{-x}$ and the nodes are at -2, 0, and 2. Now answer the following questions using 3 significant figures:
 - (a) (4 marks) Evaluate the Newton coefficients $a_k = f[x_0, \dots, x_k]$ using Newton's divided-difference method for the given function and nodes.
 - (b) (3 marks) Compute the Newton interpolation polynomial for the given function, and express the result in the natural basis. Also use this polynomial to find an approximate value of $f(6)$.
 - (c) (3 marks) Evaluate the relative error in percentage form at $x = 1.5$.