BRAC University (Department of Computer Science and Engineering)

CSE 330 (Numerical Methods) for Spring 2023 Semester

Quiz 2

Student ID: 20101534
Section: 10

Full Marks: 15

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14

Duration: 25 minutes

1. [CO2] An experiment is conducted to monitor the velocity change with respect to time. The table of results is given below:

Time(seconds)	Velocity(ms^-1)
x_0 1	$f(x_0)$ 10
x_1 2	$f(x_1)$ 15
x_2 3	$f(x_2)$ 20

a) Using **Lagrange basis**, construct a polynomial that goes through the above nodes. [5 marks]

b) Using **Newton's divided difference** method, construct a polynomial that goes through the above nodes. [4 marks]

c) Use any polynomial to find the approximate velocity at Time=6 seconds. [2 marks]

d) Let, **f(x) = 5 - ln(x)**. Evaluate the upper bound of interpolation error for the interval [4, 7]. [4 marks]

a) $P_n(x) = \sum_{n=0}^{K} l_k(x) f(x_k)$

here, $n+1=3$

$P_2(x) = l_0(x)f(x_0) + l_1(x)f(x_1) + l_2(x)f(x_2)$

$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{x^2-5x+6}{2}$

$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-1)(x-3)}{(2-1)(2-3)} = \frac{x^2-4x+3}{-1}$

$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-1)(x-2)}{(3-1)(3-2)} = \frac{x^2-3x+2}{2}$

$\therefore P_2(x) = \left(\frac{x^2-5x+6}{2}\times 10\right)+\left(\frac{x^2-4x+3}{-1}\times 15\right)+\left(\frac{x^2-3x+2}{2}\times 20\right)$

$= 5(x^2-5x+6) - 15(x^2-4x+3) + 10(x^2-3x+2)$ (Ans)

(b) $P_n(x) = \sum_{k=0}^{n} a_k \cdot n_k(x)$

$n_0(x) = 1$

$n_1(x) = (x - x_0) = (x-1)$

$n_2(x) = (x-x_0)(x-x_1) = (x-1)(x-2)$

$n_3(x) = (x-x_0)(x-x_1)(x-x_2) = (x-1)(x-2)(x-3)$

$a_0 = f[x_0]$ $\qquad x_0 = 1 \qquad f[x_0] = 10$

$a_1 = f[x_0, x_1]$ $\qquad x_1 = 2 \qquad f[x_1] = 15$

$a_2 = f[x_0, x_1, x_2]$ $\qquad x_2 = 3 \qquad f[x_2] = 20$

$f[x_0, x_1] = \frac{15-10}{2-1} = 5$

$f[x_1, x_2] = \frac{20-15}{3-2} = 5$

$f[x_0, x_1, x_2] = \frac{0}{2} = 0$

$P_2(x) = a_0 \cdot n_0 + a_1 n_1 + a_2 n_2$

$= 10 \cdot 1 + 5 \cdot (x-1) + 0 \cdot (x-1)(x-2)$

$\Rightarrow 10 + 5x - 5$

$= 5x + 10$ (Ans)

(c) ~~from (a)~~ using (a), given $x = 6$

$\therefore P_2(6) = 5(6^2 - 5.6 + 6) - 15(6^2 - 4.6 + 3) + 10(6^2 - 3.6 + 2)$

$= 5(12) - 15(15) + 10(20)$

$= 35$ (Ans)

$x^3 \quad 2\, 3\, x^2$

$x^2 - 3x \,/\, -2x + 6$

$x^2 - \,/\, 3x - x + 3$

$x^2 - 2x - x + 2$

(d) $f(x) = 5 - \ln(x)$ $\qquad f'(x) = -\frac{1}{x}$, $f''(x) = \frac{1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

interpolation error $= \dfrac{f^{n+1}(\xi)}{(n+1)!}(x-x_0)\cancel{(x-x_1)(x-x_2)}$
$(x-x_1)(x-x_2)$

$= \dfrac{f^3(\xi)}{3!}(x-1)(x-2)(x-3)$

3/

$\xi = [4, 7]$

$= \dfrac{2}{4^3} \times 2 = \dfrac{4}{4^3}$ (Ans)

$\dfrac{2}{4^3} > \dfrac{2}{7^3}$

$\therefore \max = f^3(4) = \dfrac{2}{4^3}$

$w_x = (x-1)(x-2)\,x-3)$

$= (x^2 - 3x + 3)(x-3)$

$= x^3 - 3x^2 + 3x^2 + 9x + 3x - 9$

$\Rightarrow x^3 - 6x^2 + 12x - 9$

$w'(x) = 3x^2 - 12x + 12 = 0$

$\therefore x = 2$

$x_1 = \dfrac{6-\sqrt{3}}{3}$

$x_2 = \dfrac{6+\sqrt{3}}{3}$