Answer to the question no 1

(a) from the equations we can easily converts to A, x and b

here,

$$A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

the matrix equation will be,

$$A.x = b$$

$$\begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

(b) $A = \begin{bmatrix} 1 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$

$\downarrow$

$A = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & -19 & -6 \end{bmatrix}$ $r_2' = r_2 - \frac{3}{1} r_1$

$r_3' = r_3 - \frac{4}{1} r_1$

$\therefore F^{(1)} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix}$ $r_3'' = r_3' - \frac{19}{16} r_2$

$F^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix}$

So these F^1 & F^2 are the Frobenius Matrices

(c) We know, $L = (F^{(1)})^{-1} . (F^{(2)})^{-1}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}^{-1} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -\frac{19}{16} & 1 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{19}{16} & 1 \end{bmatrix}$$

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix} \text{ (Ans)}$$

(d) we know

$$L . y = b$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 4 & \frac{19}{16} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

$\therefore y_1 = 10$ — (i)

$3y_1 + y_2 = 6$ — (ii)

$4y_1 + \frac{19}{16} y_2 + y_3 = 9$ — (iii)

from (ii)

$3 \times 10 + y_2 = 6$

$\therefore y_2 = 6 - 30$

$= -24$

from (iii)

$4 \times 10 + \frac{19}{16} \times -24 + y_3 = 9$

$\therefore y_3 = -\frac{5}{2}$

$$\therefore y = \begin{bmatrix} 10 \\ -24 \\ -\frac{5}{2} \end{bmatrix}$$

again,

$$U.x = y \qquad \Big| \quad U = \text{converted } A$$

$$\Rightarrow \begin{bmatrix} 1 & 6 & 2 \\ 0 & -16 & -5 \\ 0 & 0 & -\frac{1}{16} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ -24 \\ -\frac{5}{2} \end{bmatrix}$$

$$\therefore x_1 + 6x_2 + 2x_3 = 10 \quad -(i)$$

$$-16x_2 - 5x_3 = -24 \quad -(ii)$$

$$-\frac{1}{16}x_3 = -\frac{5}{2} \quad -(iii)$$

from, (iii) $x_3 = \frac{5 \times 16}{2} = 40$

from (ii) $-16x_2 - 5 \times 40 = -24$

$$\therefore x_2 = \frac{-24 + 200}{-16} = -11$$

from (i)

$$x_1 + 6x_2 + 2x_3 = 10$$

$$\Rightarrow x_1 = 10 - 6 \times -11 - 2 \times 40$$

$$\therefore x_1 = 10 + 66 - 80 \Rightarrow -4$$

$$\therefore x = \begin{bmatrix} -4 \\ -11 \\ 40 \end{bmatrix}$$

(Ans)

Answer to the question no 2

(a) from the equations we get

$$A = \begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$b = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

where,

$$A.x = b$$

$$\begin{bmatrix} 0 & 6 & 2 \\ 3 & 2 & 1 \\ 4 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 6 \\ 9 \end{bmatrix}$$

(b) the matrix A has a pivoting problem as in row 1 the column 1 is 0 so it doesn't have any pivot on in diagonal elements there is a 0.

As the first diagonal element is zero so the row multiplier will be 0 in the denominator so it'll be undefined. that's why it has pivoting problem but if we shift the first row with another one it'll not face any pivoting problem.

(c) the Augmented Matrix will is

$$\cdot\ Aug(A) = \left[\begin{array}{ccc:c} 0 & 6 & 2 & 10 \\ 3 & 2 & 1 & 6 \\ 4 & 5 & 2 & 9 \end{array}\right]$$

$$= \left[\begin{array}{ccc:c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 4 & 5 & 2 & 9 \end{array}\right] \quad \begin{array}{l} r_1' = r_2 \\ r_2' = r_1 \\ \text{changing the rowe} \end{array}$$

$$= \left[\begin{array}{ccc:c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & \frac{7}{3} & \frac{2}{3} & 1 \end{array}\right] \quad \begin{array}{l} r_2' = r_2 - \frac{0}{3} r_1 \\ r_3' = r_3 - \frac{4}{3} r_1 \\ m_{21} = 0 \,;\ m_{31} = \frac{4}{3} \end{array}$$

$$= \left[\begin{array}{ccc:c} 3 & 2 & 1 & 6 \\ 0 & 6 & 2 & 10 \\ 0 & 0 & -\frac{1}{9} & -\frac{26}{9} \end{array}\right] \quad \begin{array}{l} r_3' = r_3 - \frac{\frac{7}{3}}{6} r_2 \\ m_{32} = \frac{7}{18} \end{array}$$

So, the upper triangular Matrix

$$U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & -\frac{1}{9} \end{bmatrix}$$

(d) we get, $U = \begin{bmatrix} 3 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & -\frac{1}{9} \end{bmatrix}$

we know

$$\begin{bmatrix} 3 & 2 & 1 \\ 0 & 6 & 2 \\ 0 & 0 & -\frac{1}{9} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ -\frac{26}{9} \end{bmatrix}$$

$\therefore 3x_1 + 2x_2 + x_3 = 6$ — (i)

$6x_2 + 2x_3 = 10$ — (ii)

$-\frac{1}{9}x_3 = -\frac{26}{9}$ — (iii)

$\therefore$ from (iii)

$x_3 = -\frac{26}{9} \times -\frac{9}{1}$

$= 26$

from (ii)

~~$6 \times 26 + 2x_3 = 10$~~

~~$\therefore x_3 = \frac{10 - 26 \times 6}{2} = $~~

$6x_2 + 2 \times 26 = 10$

$\Rightarrow x_2 = \frac{10 - 26 \times 2}{6}$

$= -7$

from (iii)

$3x_1 + 2 \times -7 + 26 = 6$

$\therefore x_1 = \frac{6 - 26 + 14}{3} = -2$

$\therefore x = \begin{bmatrix} -2 \\ -7 \\ 26 \end{bmatrix}$

(Ans)