BRAC UNIVERSITY

STA201

Elements of Statistics and Probability

Assignment 05

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Date: ..../......../........

STA201 Assignment 5

1 a) Given that,

PMF, $P(x=x) = \begin{cases} 3kx & x = 1,3,5 \\ k(x^2+0.5) & x = 7 \\ 0 & \text{otherwise} \end{cases}$

we know,

$\Sigma f(x) = 1$

so, $(3k \cdot 1) + (9k \cdot 3) + (15k \cdot 5) + (k \cdot 49.5) = 1$

$\Rightarrow 76.5k = 1$

$\therefore k = \frac{1}{76.5} = \frac{2}{153}$ (showed)

(b) $P(3 < x \leq 7) = P(x=5) + P(x=7)$

$= \frac{10}{51} + \frac{11}{17} = \frac{43}{51}$

(c) $P(3 < x < 5) = 0$,

as no sample space is between 3 and 5.

d) $E(x) = \sum x \cdot f(x)$

$= 1 \times \frac{2}{51} + 3 \cdot \frac{2}{17} + 5 \cdot \frac{10}{51} + 7 \cdot \frac{11}{17}$

$= \frac{301}{51}$

$= 5.9019$

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e) $var(x) = E(x^2) - (E(x))^2$

so, $E(x^2) = \sum x^2 \cdot f(x) = 1^2 \cdot \frac{2}{51} + 3^2 \cdot \frac{2}{17} + 5^2 \cdot \frac{10}{51}$

$+ 7^2 \cdot \frac{11}{17}$

$= 37.706$

from (d) we know, $E(x) = 5.9019$

$\therefore var(x) = 37.706 - (5.9019)^2$

$= 2.8735$ (Ans)

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f) $\text{Var}\left(-\frac{1}{3}x+5\right)$

$\Rightarrow \text{Var}\left(-\frac{1}{3}x\right)+\text{Var}(5)$

$\Rightarrow \text{Var}\left(-\frac{1}{3}x\right)+0$

$= \text{Var}\left(-\frac{1}{3}x\right) = \left(-\frac{1}{3}\right)^2 \cdot \text{Var}\,x$

$= 0.319$ (Ans)

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$\boxed{2}$ (a) $f(Y\leq 6) = \int_0^4 \frac{1}{20}y\,dy + \int_4^6 \frac{1}{3} - \frac{1}{30}y\,dy$

$= \frac{1}{20}\left[\frac{y^2}{2}\right]_0^4 + \frac{1}{3}\times[y]_4^6 - \frac{1}{3}\times\left[\frac{y^2}{2}\right]_4^6$

$= \frac{11}{15} = 0.7333$ (Ans)

(b) $f(y<3)+f(y>7) = \int_0^3 \frac{1}{20}y\,dy + \int_7^{10} \frac{1}{3} - \frac{1}{30}y\,dy$

$= \frac{1}{20}\left[\frac{y^2}{2}\right]_0^3 + \frac{1}{3}\times[y]_7^{10} - \frac{1}{30}\times\left[\frac{y^2}{2}\right]_7^{10}$

$= \frac{3}{8} = 0.375$

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(c) $E(y) = \int_0^4 y \cdot \frac{1}{20} y \, dy + \int_4^{10} y \times \left(\frac{1}{3} - \frac{1}{30} y\right) dy$

$= \frac{1}{20} \times \left[\frac{y^3}{3}\right]_0^4 + \frac{1}{3} \times \left[\frac{y^2}{2}\right]_4^{10} - \frac{1}{30} \times \left[\frac{y^3}{3}\right]_4^{10}$

$= \frac{14}{3} = 4.667$

(d) $E(y^2) = \int_0^4 y^2 \times \frac{1}{20} y \, dy + \int_4^{10} y^2 \times \left(\frac{1}{3} - \frac{1}{30} y\right) dy$

$= \frac{1}{20} \times \left[\frac{y^4}{4}\right]_0^4 + \frac{1}{3} \times \left[\frac{y^3}{3}\right]_4^{10} - \frac{1}{30} \times \left[\frac{y^4}{4}\right]_4^{10}$

$= 26$

$Var(Y) = E(y^2) - (E(y))^2$

$= 26 - \left(\frac{14}{3}\right)^2$ [from (c) we get $E(y) = \frac{14}{3}$

$= \frac{38}{9}$

standard Deviation $= \sqrt{var(y)} = \sqrt{\frac{38}{9}} = 2.055$

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3. (a) $P(A=B) = P(0,0) + P(1,1) + P(2,2) + P(3,3)$

$= 0.09 + 0.01 + 0.1 + 0.1$

$= 0.3$

(b) $P(A+B=4) = P(1,3) + P(3,1) + P(2,2)$

$+ P(4,0)$

$= 0.04 + 0.03 + 0.1 + 0.01$

$= 0.18$

(c) Marginal PMF,

$P_A(A=0) = 0.17$

$P_A(A=1) = 0.11$

$P_A(A=2) = 0.31$

$P_A(A=3) = 0.14$

$P_A(A=4) = 0.27$

$P_B(B=0) = 0.19$

$P_B(B=1) = 0.3$

$P_B(B=2) = 0.24$

$P_B(B=3) = 0.27$

and,

$E(B) = \sum B.P(B) = 0.0.19 + 1.0.3 + 2.0.24$

$+ 3.0.27 = 1.59$ (Ans)

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~~d) P(B=2 or A=3)~~

d) $P(B=2 \mid A=3) = \dfrac{P(B=2 \cap A=3)}{P(A=3)}$

$$= \frac{0.01}{0.14}$$

$$= \frac{1}{14} = 0.07143 \text{ (Ans)}$$

e) A random variable will be independent if and only if, $P(x,y) = P_x(x) \times P_y(y)$ for all values of A and B

Now,

$P(A,B) = P(0,0) = 0.09$

but, $P_A(A) \times P_B(B) = P_A(A=0) \times P_B(B=0)$

$$= 0.17 \times 0.19$$

$$= 0.0323$$

as, $P(A,B) \neq P_A(A) \times P_B(B)$ so, A and B are not independent

4 (a) $P(\text{getting Head}) = \frac{3}{3+7+2} = \frac{1}{4}$

$P(\text{getting Head in the 5th turn}) = \left(1 - P(\text{getting Head})\right)^4 \cdot P(\text{getting Head})$

$= \left(1 - \frac{1}{4}\right)^4 \cdot \frac{1}{4}$

$= \frac{81}{1024}$ (Ans)

(b) Let,

probability of getting a non spade card

$\Rightarrow P(S') \Rightarrow \frac{10}{12} = \frac{5}{6}$

we know $E(x) = \frac{1}{P}$

So, $E(S') = \frac{1}{P} = \frac{6}{5} = 1.2$

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c) Let, probability of getting a club $\Rightarrow P(C) = \frac{7}{12}$

and, $Var(x) = \frac{1 - P(C)}{(P(C))^2} = \frac{1 - \frac{7}{12}}{\left(\frac{7}{12}\right)^2} = 1.2245$

$\boxed{5}$ (a) Let,

Probability of drawing a club

$$P(C) = \frac{6}{12} = \frac{1}{2}$$

Probability of getting exactly 4 after 6 turns

$$^6C_4 \times \left(\frac{1}{2}\right)^4 \times \left(1-\frac{1}{2}\right)^2$$

$$\Rightarrow 0.2344 \text{ (Ans)}$$

(b) $P(C) = \frac{1}{2}$

Probability of getting more than 3 after 6 turns

$$P(x>3) = P(x=4) + P(x=5) + P(x=6)$$

$$= {^6C_4} \times \frac{1}{2}^4 \times \left(1-\frac{1}{2}\right)^2 + {^6C_5}\left(\frac{1}{2}\right)^5\left(1-\frac{1}{2}\right)^1$$

$$+ {^6C_6} \times \left(\frac{1}{2}\right)^6 \times \left(1-\frac{1}{2}\right)^0$$

$$= 0.3438$$

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(c) Let probability of getting a heart,

$$P(H) = \frac{4}{12} = \frac{1}{3}$$

mean number of hearts after 60 turns

$$\Rightarrow E x = n P(H) = 60 \times \frac{1}{3} = 20$$

(d) let, probability of getting a spade, $P(S) = \frac{1}{6}$

after 36 turns, $SD = \sigma = \sqrt{np(1-p)}$

$$\therefore \sigma = \sqrt{n(P(S))(1-P(S))}$$

$$= \sqrt{36 \cdot \frac{1}{6} \cdot \frac{5}{6}} = \sqrt{5} \text{ (Ans)}$$

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6 a) given that,

$\lambda = 14$ per week

So, $\lambda = 14 \times 4$ per month [or 4 week per month]

$= 56$ per month

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(b) $\lambda = 14$ per week

$= 28$ per two week

$$P(x=40) = e^{-\lambda} \cdot \frac{\lambda^{x}}{x!}$$

$$= e^{-28} \cdot \frac{28^{40}}{40!}$$

$= 0.0065$ (Ans)

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(c) $\lambda = 14$ per week

$$P(x \leq 9) = e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} \dots\dots + \frac{\lambda^9}{9!}\right]$$

$= 0.1094$ (Ans)

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$\boxed{7}$ (a) $mode = 100$

$Var = 36$

$\therefore SD = \sigma = \sqrt{Var} = 6$

also, Mode = Mean so, $\mu = 100$

$x \sim N(100, 36), z = \frac{x-\mu}{\sigma}$

(i) $P(100 < x < 105)$

$\Rightarrow P(0 < z < 0.83)$

$\Rightarrow 0.2967$

$E(x) = 2000 \times 0.2967$
$= 593.4$

(ii) $P(x > 110) = P(z > 1.67) = 1 - P(z \leq 1.67)$

$= 1 - 0.9525 = 0.0475$

$E(x) = 0.0475 \times 2000 = 95$

(iii) $P(x < 90) = P(z < -1.67) = 0.0475$

$E(x) = 2000 \times 0.0475 = 95.$

(iv) $P(x < 90 \cup x > 107) = P(z < -1.67) + P(z > 1.17)$

$= 0.0475 + 0.1210 = 0.1685$

$E(x) = 0.1685 \times 2000 = 337$

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(b) Let, $A = 85^{th}$ percentile value

$P(x < A) = 0.85$

$\therefore P\left(Z < \frac{A-100}{6}\right) = 0.85$

$\therefore \frac{A-100}{6} = 1.035$

$\therefore A = (1.035 \times 6) + 100 = 106.21$

$\boxed{8}$ $\lambda = 30$ customers per hour

$= 30 \times \frac{1}{60}$ per minute

$= \frac{1}{2}$ per minute

(a) $E(x) = \frac{1}{\lambda} = \frac{1}{0.5} = 2$ ~~customers per~~ minutes

(b) $P(x > 8) = e^{-\lambda x} = e^{-\frac{1}{2} \times 8} = 0.0183$

(c) $P(x < 2) = 1 - e^{-\lambda x} = 1 - e^{-\frac{1}{2} \times 2} = 0.6321$

(d) $P(x < a) = \frac{70}{100} = 0.70$

$1 - e^{-\lambda x} = 1 - e^{-\frac{1}{2} \times a} = 0.7$

$\therefore a = 2.408$ minutes

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(e) Let median = M

$P(x < M) = 50\% = 0.5$

So, $1 - e^{-\lambda x} = 1 - e^{-\frac{1}{2}M} = 0.5$

$e^{-\frac{1}{2} \times M} = 0.5$

$\therefore a = 1.386$ minutes

So, the mean is ~~long~~ longer.

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