

BRAC UNIVERSITY

STA201

ELEMENTS OF STATISTICS AND PROBABILITY

Assignment 4

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SECTION: 01



Date: 10 December 2021

STA 201 Assignment 4

1. Sample spaces, $S = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

(a) $E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$

$$\text{So, } P(\text{A sum of 8}) = P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

(b) $E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

$$\text{So, } P(\text{A doublet}) = P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

- (c) $E = \{(1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4),$
 $(3, 5), (3, 6), (4, 2), (4, 3), (4, 4), (4, 5),$
 $(4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6),$
 $(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$$\therefore P(\text{a sum greater than } 5) = P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{26}{36} = \frac{13}{18}$$

- (d) $E = \{(1, 1), (1, 2), (2, 1), (3, 6), (4, 5), (4, 6), (5, 4),$
 $(5, 5), (5, 6), (6, 3), (6, 4), (6, 5), (6, 6)\}$

$$P(\text{A sum less than 4 or greater than 8})$$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{13}{36}$$

- (e) $E = \{(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2),$
 $(4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3),$
 $(6, 4), (6, 5), (6, 6)\}$

$$P(\text{An even number on the 1st die}) = P(E) = \frac{n(E)}{n(S)}$$

$$= \frac{18}{36} = \frac{1}{2}$$

$$(g) E = \{(1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2), (3, 4), (3, 6), (4, 1), (4, 3), (4, 5), (5, 2), (5, 3), (5, 6), (6, 1), (6, 3), (6, 5)\}$$

$P(\text{An odd number on one and an even number on the other}) = P(E) = \frac{n(E)}{n(S)} = \frac{18}{36} = \frac{1}{2}$

$$(g) E = \{(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$$

$$P(\text{At least one } 6) = P(E) = \frac{n(E)}{n(S)} = \frac{11}{36}$$

$$(h) E = \{(1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5)\}$$

$P(\text{At least one } 6, \text{ if the two faces are different})$

$$\Rightarrow P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

(a) As two balls are drawn with replacement then total number of even balls and total numbers of ball will not be changed.

(i) from 1 to 30 there are 15 even numbers.

$$P(\text{Two success}) = \frac{15}{30} \times \frac{15}{30} = \frac{1}{4} = 0.25$$

(ii) Case 1: 1st one is success and second one is failure.

$$\Rightarrow \frac{15}{30} \times \frac{15}{30}$$

Case 2: 1st one is failure and second one is success.

$$\Rightarrow \frac{15}{30} \times \frac{15}{30}$$

$$P(\text{at least exactly one success}) = \frac{15}{30} \times \frac{15}{30} + \frac{15}{30} \times \frac{15}{30}$$

$$\Rightarrow \frac{1}{4} + \frac{1}{4}$$

$$\Rightarrow \frac{2}{4} = 0.5$$

(iii) Case 1: 1st success and 2nd failure

Case 2: 1st failure and 2nd success

Case 3: 1st success and 2nd success

$$P(\text{at least one success}) = \frac{15}{30} \times \frac{15}{30} + \frac{15}{30} \times \frac{15}{30} + \frac{15}{30} \times \frac{15}{30}$$

$$(= \frac{3}{4} = 0.75 \text{ (iv)})$$

(v) If no success then both will be failure,

$$P(\text{No success}) = \frac{15}{30} \times \frac{15}{30}$$

$$= \frac{1}{4} = 0.25$$

(b) if without replacement the total ball will be decreased by one after each ball is drawn.

$$(i) P(\text{two success}) = \left(\frac{15}{30}\right) \times \left(\frac{14}{29}\right) = \frac{2}{29}$$

$$(ii) P(\text{Exactly one success}) = \frac{15}{30} \times \frac{15}{29} + \frac{15}{30} \times \frac{15}{29}$$

$$= \frac{15}{29}$$

(iii) $P(\text{at least one success})$

$$\Rightarrow \left(\frac{15}{30} \times \frac{15}{29} \right) + \left(\frac{15}{30} \times \frac{15}{29} \right) + \left(\frac{15}{30} \times \frac{14}{29} \right)$$

$$\Rightarrow \frac{22}{29}$$

(iv) $P(\text{No success})$

$$\Rightarrow \frac{15}{30} \times \frac{14}{30} = \frac{7}{29}$$

3 Sample space is in the question no 1.

for rolling two regular six-sided dice

Sample space for rolling two 4 sided dice

$$(1,1), (1,2), (1,3), (1,4)$$

$$(2,1), (2,2), (2,3), (2,4)$$

$$(3,1), (3,2), (3,3), (3,4)$$

$$(4,1), (4,2), (4,3), (4,4)$$

A = sum of two regular sided dice

B = sum of two n sided dice

we have to get $A \times B = 12$

Case 1 : $A = 2, B = 6$

Case 2 : $A = 3, B = 4$

Case 3 : $A = 4, B = 3$

Case 4 : $A = 6, B = 2$

$$\text{Prob} P(B=6) = ?$$

Case 1 :

$$P(A=2) = \frac{1}{36} \quad P(B=6) = \frac{3}{16}$$

as $E = \{(1,1)\}$ $\text{Prob} = \frac{1}{36}$ $\text{an, } E = \{(2,4), (3,3), (4,2)\}$
 $\text{Prob} = \frac{3}{36}$ $\text{an, } E = \{(2,4), (3,3), (4,2)\}$
 $\text{Prob} = \frac{3}{36}$ $\text{an, } E = \{(2,4), (3,3), (4,2)\}$
 $\text{Prob} = \frac{3}{36}$ $\text{an, } E = \{(2,4), (3,3), (4,2)\}$

Case 2 :

$$P(A=3) = \frac{2}{36} = \frac{1}{18} \quad P(B=4) = \frac{3}{16}$$

as $E = \{(1,2), (2,1)\}$ $\text{Prob} = \frac{2}{36}$ $\text{an, } E = \{(1,3), (2,2), (3,1)\}$
 $\text{Prob} = \frac{3}{36}$ $\text{an, } E = \{(1,3), (2,2), (3,1)\}$

Case 3 :

$$P(A=4) = \frac{3}{36} = \frac{1}{12} \quad P(B=3) = \frac{2}{16}$$

as $E = \{(1,3), (2,2), (3,1)\}$

as $E = \{(1,2), (2,1)\}$

Case 4:

$$P(A = 6) = \frac{5}{36}$$

$$P(B = 2) = \frac{1}{16}$$

$$\therefore E = \{(1, 5), (3, 4), (3, 3), (4, 2), (5, 1)\}$$

$$\text{or } E = \{(1, 1)\}$$

$$P = \frac{1}{36} \times \frac{3}{16} + \frac{2}{36} \times \frac{3}{16} + \frac{3}{36} \times \frac{2}{16} + \frac{5}{36} \times \frac{1}{16}$$

$$= \frac{5}{144} \quad (\text{Ans})$$

14

$$P(\text{enrolled in photography}) = \frac{48}{250} = P(P)$$

$$P(\text{enrolled in Swimming}) = \frac{34}{250} = P(S)$$

$$P(\text{both photography and swimming}) = \frac{12}{250} = P(P \cap S)$$

$$\text{enrolled in any classes} = 48 + 34 - 12$$

$$= 70$$

$$\therefore P(P \cap S) = \frac{70}{250}$$

\therefore Not enrolled in any

$$P(P \cap S) = 1 - \frac{70}{250} = \frac{18}{25}$$

5

Let, $A = \text{treated with meditation}$

$B = \text{treated with drugs}$

$C = \text{has high blood pressure}$

given that,

$$P(A) = P(B) = 0.5 \quad \text{and} \quad P(C) = 60\% = 0.6$$

$$P(C|A) = 0.6 \times 0.5$$

$$= 0.33$$

also,

$$P(C|B) = 0.6 \times 0.5$$

$$= 0.3$$

$$\therefore P(C) = P(C|A) \times P(A) + P(C|B) \times P(B)$$

$$= (0.33 \times 0.5) + (0.3 \times 0.5) = 0.3$$

$P(A|C')$ = ~~not~~ given meditation and does not suffer from high blood pressure

$$= \frac{P(C'|A) \times P(A)}{P(C')} = \frac{(1 - (P(C|A)) \times P(A))}{1 - P(C)} = \frac{(1 - 0.33) \cdot 0.5}{1 - 0.3}$$

$$= \frac{0.67}{1.4} = 0.4786 (\text{Ans})$$

6

Bag A,

Red \rightarrow 6

Black \rightarrow 7

Bag B,

Red \rightarrow 9

Black \rightarrow 6

Q) Express \therefore

$$P(\text{Black drawn}) = \frac{7}{13}$$

$$P(\text{Black transferred from } A \text{ to } B) = \frac{7}{13}$$

$$P(\text{Red } " \text{ transferred from } A \text{ to } B) = \frac{6}{13}$$

Date: _____ / _____ / _____

P(Drawing black ball from Bag B when transferred ball is Red)

$$\Rightarrow \frac{6}{16} = 0.375 \text{ or } 37.5\%$$

P(Drawing black ball from Bag B when transferred ball is Black)

$$\Rightarrow \frac{x}{16}$$

P(transferred ball was ~~Red~~)

$$\Rightarrow \frac{\frac{6}{16} \times \frac{6}{13}}{\frac{6}{16} \times \frac{6}{13} + \frac{x}{16} \times \frac{x}{13}}$$

$$\Rightarrow \frac{36}{85}$$

$$\Rightarrow 0.4235 \text{ (Ans)}$$

Let,

A = two headed coin chosen

B = fair coin is chosen

C = biased coin is chosen

D = Head is shown.

Given that

$$P(A) = P(B) = P(C) = \frac{1}{3} = 0.33$$

P(getting head from the two headed coin)

$$\Rightarrow P(D|A) = 1$$

P(getting head from the fair coin)

$$\Rightarrow P(D|B) = \frac{1}{2}$$

P(getting head from the biased coin)

$$\Rightarrow P(D|C) = \frac{3}{4}$$

$$P(D) = P(D|A) \times P(A) + P(D|B) \times P(B) + P(D|C) \times P(C)$$

$$= 1 \times \frac{1}{3} + \frac{1}{2} \times \frac{1}{3} + \frac{3}{4} \times \frac{1}{3} = \frac{3}{4}$$

$P(\text{Head is chosen and two headed coin})$

$$\Rightarrow P(A|D) = \frac{P(D|A) \times P(A)}{P(D)}$$

$$\text{no heads in tail} = \frac{1 \times \frac{1}{3}}{\frac{3}{4}} = \frac{4}{9} \quad (\text{Ans})$$

heads in tail = 4

$$E(X) = \frac{1}{3} = (2)2 + (2)9 = 18, 9$$

(also balanced binomial mark board problem)

$$E = (1)(2)9 <$$

(comes with 30 marks based on $B(11, 2)$)

$$E = (8)(2)9 <$$

(also based on different approach mark board problem)

$$E = (9)(2)9 <$$

$\rightarrow (2)(2)9 + (8)(2)9 + (9)(2)9 = (4)(2)9$

$$E = \frac{1}{3} \times 2 + \frac{1}{3} \times 8 + \frac{1}{3} \times 9 = 7$$