STA 201 Lecture Notes

Continuous Probability Distribution – Exponential Distribution

Exponential Probability Distribution:

The exponential distribution is often concerned with the amount of time until some specific event occurs. For example, the amount of time (beginning now) until an earthquake occurs has an exponential distribution.

Other examples include the length of time, in minutes, of long-distance business telephone calls, and the amount of time, in months, a car battery lasts. It can be shown, too, that the value of the change that you have in your pocket or purse approximately follows an exponential distribution.

Values for an exponential random variable occur in the following way. There are fewer large values and more small values. For example, marketing studies have shown that the amount of money customers spend in one trip to the supermarket follows an exponential distribution. There are more people who spend small amounts of money and fewer people who spend large amounts of money.

Exponential distributions are commonly used in calculations of product reliability, or the length of time a product last.

The random variable for the exponential distribution is continuous and often measures a passage of time, although it can be used in other applications

A continuous random variable X is said to follow exponential distribution, if its pdf is,

$$f(x) = \lambda e^{-\lambda x}; \qquad x > 0, where\ \lambda > 0$$

We write as, $X \sim \exp(\lambda)$

Here, X is usually time until certain event occurs.

Mean, $E(X) = \frac{1}{\lambda}$

Variance, $V(X) = \frac{1}{\lambda^2}$

Standard deviation, $SD(X) = \frac{1}{\lambda}$

An alternative form of the exponential distribution formula-

$$f(x)= \frac{1}{\lambda} e^{-(\frac{1}{\lambda} * x)}\ ; \qquad x > 0, \text{ where } \lambda > 0$$

We write as, X ~ exp $(\frac{1}{\lambda})$

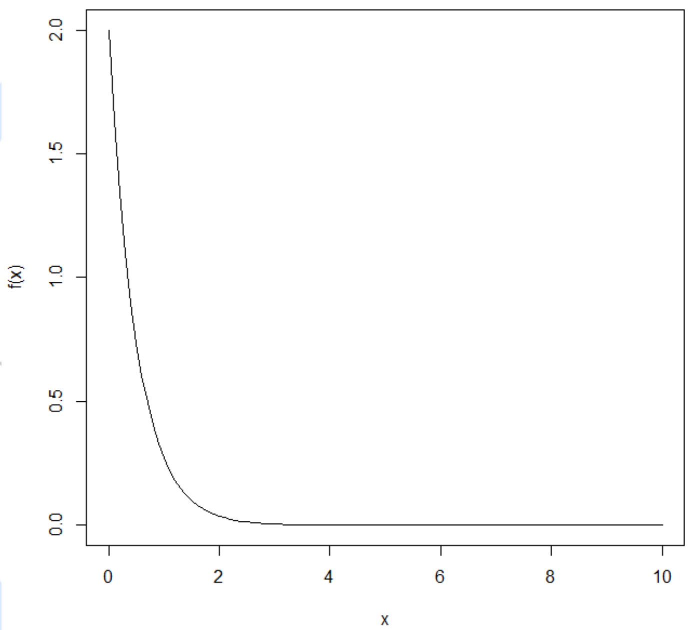
Here, X is usually time until certain event occurs.

Mean, μ = E(X) = $\frac{1}{\frac{1}{\lambda}}$ = λ

Variance, σ^2 = V(X) = $\frac{1}{\frac{1}{\lambda^2}}$ = λ^2

Thus, standard deviation, $\sigma = \lambda$

The exponential distribution can be represented graphically like the following form-



- X1= time to repair a machine
- X2= life time of an electronic component
- X3= time until certain event occurs.
- X4= time between transactions at an ATM Machine

These variables sometimes follow exponential distribution.

X ~ exponential (λ)

P (X≤x) = $\int_0^x \lambda e^{-\lambda u}$ du

= $\lambda \int_0^x e^{-\lambda u}$ du

= 1- $e^{-\lambda x}$

P (X>x) = $e^{-\lambda x}$

Example 1:

Average time required to repair a machine is 0.5 hours. What is the probability that the next repair will take more than 2 hours?

Solution:

Let, X= time required to repair the machine

$\therefore X \sim \exp(\lambda = 2) \qquad \left[since, E(X) = \frac{1}{\lambda} = 0.5 \Rightarrow \lambda = \frac{1}{0.5} = 2\right]$

$$\Pr[X > 2] = \int_2^\infty \lambda e^{-\lambda x} dx = \left[-e^{-\lambda x}\right]_2^\infty = \left[-0 + e^{-2\lambda}\right] = e^{-2\lambda} = e^{-2\times 2} = e^{-4} = 0.0183$$

Example 2:

On the average, a certain computer part lasts ten years. The length of time the computer part lasts is exponentially distributed.

a. What is the probability that a computer part lasts more than 5 years?

b. What is the probability that a computer part lasts between 9 and 11 years?

Solution:

Let, X= the amount of time (in years) a computer part lasts.

$$\therefore X \sim \exp(\lambda = 0.1)$$

$$\Pr[X > 5] = 1 - \Pr[X < 5] = 1 - e^{-0.1\times 5} = e^{-0.1*5} = e^{-0.5}$$

$$= 0.6065$$

$$\Pr[9 < X < 11] = \Pr[X < 11] - \Pr[X < 9] = (1 - e^{-0.1\times 11}) - (1 - e^{-0.1\times 9})$$

(1- 0.3329) – (1 – 0.4066) = 0.6671 − 0.5934 = 0.0737

Practice problem:

1. Waiting times to receive food after placing an order at the local Subway sandwich shop follow an exponential distribution with a mean of 60 seconds. Calculate the probability a customer wait:
   a. Less than 30 seconds.
   b. More than 120 seconds.
   c. Between 45 and 75 seconds.

[**Answer:** a. 0.3935; b.0.1353; c. 0.1859]

2. The Bureau of Labor Statistics' American Time Use Survey, www.bls.gov/data, showed that the amount of time spent using a computer for leisure varied greatly by age. Individuals age 75 and over averaged 0.3 hour (18 minutes) per day using a computer for leisure. Individuals ages 15 to 19 spend 1.0 hour per day using a computer for leisure. If these times follow an exponential distribution, find the proportion of each group that spends:

a. Less than 15 minutes per day using a computer for leisure.
b. More than 2 hours.
c. Between 30 minutes and 90 minutes using a computer for leisure.

[**Answer:** Age 75 and over group: a. 0.5654; b.0.0013; c. 0.1821 &

Age 15-19 group: a. 0.2212; b.0.1353; c. 0.3834]