

BRAC UNIVERSITY

STA201

ELEMENTS OF STATISTICS AND PROBABILITY

Assignment 3

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SECTION: 01



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Date :

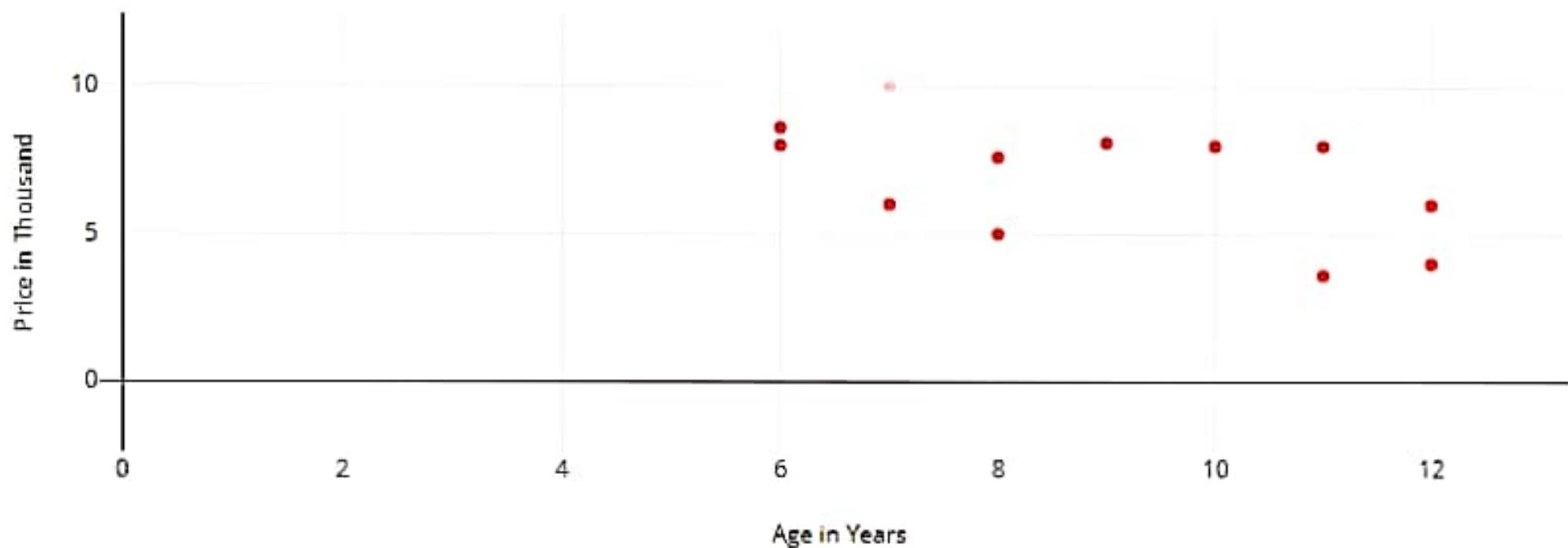
STA201 Assignment 3

Problem 1

(a)

Scatter Graph of Price vs Age

$R_s = -0.521$ $p = 0.10$ (90% statistical significance level) $df = 0$



~~28351.1 + 28723 - 100~~

from the diagram we can see that
there is a negative correlation between
age and price.

~~from the~~

b)

23.4.973 - 20.4.

| Age (years) (x) | Price (\$) (y) | x^2 | y^2 | xy |
|--------------------|-------------------|-------|--------|-------|
| 9 | 8.1 | 81 | 65.61 | 72.9 |
| 10 | 6 | 100 | 36 | 60 |
| 11 | 3.6 | 121 | 12.96 | 39.6 |
| 12 | 4 | 144 | 16 | 48 |
| 8 | 5 | 64 | 25 | 40 |
| 10 | 10 | 100 | 100 | 100 |
| 8 | 7.6 | 64 | 57.76 | 60.8 |
| 11 | 8 | 121 | 64 | 88 |
| 10 | 8 | 100 | 64 | 80 |
| 12 | 6 | 144 | 36 | 72 |
| 6 | 8.6 | 36 | 73.96 | 51.6 |
| 6 | 8 | 36 | 64 | 48 |
| 102 | 82.9 | 1009 | 615.29 | 712.9 |

$$\bar{x} = \frac{102}{12} = 8.5167$$

$$\bar{y} = \frac{82.9}{12} = 6.9083$$

$$712.9 - (12 \times 8.5167 \times 6.9083)$$

$$r = \frac{\sum_{i=1}^{12} (x_i, y_i) - n \bar{x} \cdot \bar{y}}{\sqrt{\left(\sum_{i=1}^{12} x_i^2 - n \bar{x}^2 \right) \left(\sum_{i=1}^{12} y_i^2 - n \bar{y}^2 \right)}} = \sqrt{\frac{\left(102 - (12 \times 8.5167)^2 \right) \times (615.29 - 712.9)}{12 \times 6.9083}}$$

$$\therefore r = -0.5436$$

as $r = -0.5436$ there is a moderate negative correlation between age and price.

$$r^2 = 0.2956 = 0.2956 \times \frac{100}{100} \\ = 29.56\%$$

only 29.56% of the total variation in the price of car can be explained by the age of the car.

Problem 2

(a)

| Judge 1(x) | Judge 2(y) | Rank x | Rank y | $R_x - R_y = d$ | d^2 |
|------------|------------|--------|--------|-----------------|-------|
| 650 | 900 | 8 | 4 | 4 | 16 |
| 260 | 220 | 2 | 9 | -7 | 49 |
| 240 | 690 | 3 | 11.5 | -8.5 | 72.25 |
| 200 | 850 | 5 | 6 | -1 | 1 |
| 590 | 920 | 11 | 2.5 | 8.5 | 72.25 |
| 620 | 800 | 9 | 7 | 2 | 4 |
| 200 | 890 | 5 | 5 | 0 | 0 |
| 690 | 920 | 7 | 2.5 | 4.5 | 20.25 |
| 900 | 1050 | 1 | 1 | 0 | 0 |
| 500 | 690 | 12 | 11.5 | 0.5 | 0.25 |
| 610 | 200 | 10 | 10 | 0 | 0 |
| 200 | 260 | 5 | 8 | -3 | 9 |
| Total | | | | | 244 |

$$r_s = 1 - \frac{6 \sum_{i=1}^{12} d_i^2}{n(n^2-1)} = 1 - \frac{6 \times 244}{12 \times 143} = 0.1469$$

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b) as $r_s = 0.1469$ so the scores of judges have a very weak (+ve) correlation.

Problem 3

(a)

| Numbers of Rooms | Energy Consumption (thousand kWh) | x^2 | y^2 | xy |
|------------------|-----------------------------------|-------|-------|------|
| 12 | 9 | 144 | 81 | 108 |
| 9 | 8 | 81 | 64 | 63 |
| 14 | 10 | 196 | 100 | 140 |
| 6 | 5 | 36 | 25 | 30 |
| 10 | 8 | 100 | 64 | 80 |
| 8 | 6 | 64 | 36 | 48 |
| 10 | 8 | 100 | 64 | 80 |
| 20 | 10 | 400 | 100 | 200 |
| 5 | 8 | 25 | 64 | 40 |
| 10 | 8 | 100 | 64 | 80 |
| Total | 91 | 895 | 584 | 718 |

$$\begin{aligned}
 b_1 &= \frac{n \left(\sum_{i=1}^{10} x_i y_i \right) - \left(\sum_{i=1}^{10} x_i \right) \left(\sum_{i=1}^{10} y_i \right)}{n \left(\sum_{i=1}^{10} x_i^2 \right) - \left(\sum_{i=1}^{10} x_i \right)^2} \\
 &= \frac{(10 * 18) - (91 * 24)}{(10 * 895) - 91^2} \\
 &= 0.6667
 \end{aligned}$$

$$b_0 = \bar{y} - b_1 \bar{x} = 2.4 - (0.6667 \times 9.1)$$

$$\hat{y} = b_0 + b_1 x$$

$$= 1.3333 + 0.6667 x \quad (\text{Ans})$$

(b) b_0 indicates the initial total energy consumption when the number of rooms are 0, so when number of rooms are 0 energy consumption will be 1.333.

b₁ means (the slope) or, the overall energy consumption will ~~not~~ increase by

$b_1 = 0.666x$ (thousand kWh) for increasing of 1 m^3

c) if $x = 6$

$$\hat{y} = 1.3333 + (0.666x^6)$$

$$= 5.335$$

if $1700 \text{ m}^3 = 6$ then energy consumption

will be 5.335 (thousand kWh)

d) we know,

$$r^2 = 1 - \frac{SSE}{SST}$$

$$= 1 - \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{\sum_{i=1}^n y_i^2}$$

$$= 1 - \frac{\sum y_i^2 - b_0 \sum y_i - b_1 \sum x_i y_i}{\sum y_i^2 - \frac{(\sum y_i)^2}{n}}$$

$$= 1 - \frac{6.6667}{36.4}$$

$$\approx 0.8168$$

$$r^2 = 0.8168 = 0.8168 \times \frac{100}{100} = 81.68\%$$

So, total 81.68% of the variation in energy consumption can be explained by

the number of rooms.

Problem 4

a) from the given data

$$b_0 = 356.12083$$

$$b_1 = -0.098x^4$$

$$b_2 = 122.86x^2$$

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2$$

$$= 356.12083 + 0.098x^4 x_1 + 122.86x^2 x_2$$

b) b_0 or 356.12083 indicates that

the price of a backpack will be

when both the capacity

356.12083

and comfort rating are 0 :

$b_1 = -0.098x^4$ indicates that the

price of a backpack will be $-0.098x^4$
less than the previous when the

capacity will increase 1 ct but comfort rating is fixed
 b_2 or $122.86x_2$ indicates that, the price of the backpack will be $122.86x_2$ more than previous unit cost when comfort rating increase by one unit but capacity is fixed.

c) from (a)

$$\begin{aligned}
 y &= 356.12083 - 0.098x_1 x_1 + 122.86x_2 y, \\
 &= 356.12083 - (0.098x_1, 4500) + (122.86x_2 x_2) \\
 &= 403.25962 \text{ (Ans)}
 \end{aligned}$$

d) From the data

$$\text{Adjusted R-squared} = 0.7838 \\ = 0.7838 \times \frac{100}{100} \\ = 78.38\%$$

\therefore Only 78.38% of the variation in the price of a backpack can be explained by taking into account of capacity and comfort rating.

Problem 5

a) From the data

$$\beta_0 = -471.44$$

$$\beta_1 = 6.394$$

$$\beta_2 = 1.368$$

$$\hat{y} = \frac{e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}{1 + e^{(\beta_0 + \beta_1 x_1 + \beta_2 x_2)}}$$

$$= \frac{e^{(-421.441 + 6.394 x_1 + 1.367 x_2)}}{1 + e^{(-421.441 + 6.394 x_1 + 1.367 x_2)}}$$

b) from the data,

$$\beta_1 = 6.394, \beta_2 = 1.367$$

$$\text{so, odds ratio} = e$$

$$= 598.2448$$

$$\text{and odds ratio} = e^{1.367}$$

$$= 3.8459$$

for β_1 , the odds of having another heart attack within one year is increased 598.2448 for every unit increase in the age.

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for b_2 ,
the odds of having a second heart
attack within 1 year is increased
3,8459 for every unit increase
in the score of anxiety.