

Lecture Notes

Poisson Distribution

POISSON DISTRIBUTION:

A random variable is said to have Poisson distribution if the following condition are satisfied:

1. The event can occur any number of times during a time period
2. The events occur independently
3. The rate of occurrence is constant
4. The probability of an event occurring is proportional to the length of the time period.

Formula:

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Examples:

Q1. A small business receives, on average, 12 customers per day. What is the probability that the business will receive exactly 8 customers in a day?

Ans: $\lambda = 12$

$$P(X=x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(X=8) = \frac{e^{-12} 12^8}{8!} = 0.065523 = 6.55\%$$

Q2. A student receives on average 7 text message in a two hours period

- a. What is the probability that the student will receive exactly 9 text messages in two hours?
- b. What's the probability that the student will receive exactly 24 text messages in 8 hours?

Ans.

a.

$$\lambda = 7$$

$$x = 9$$

$$P(X=9) = \frac{e^{-7} 7^9}{9!} = 0.1014 \approx 10.14\%$$

b.

$$\lambda = 28$$

$$x = 24$$

$$P(X = 24) = \frac{e^{-28} 28^{24}}{24!} = 0.060095 \approx 6.01\%$$

Q3. A small business receives on average 8 calls per hour

- a. probability that it will receive 7 calls in an hour =?
- b. what is the probability that the business will receive, at most 5 calls in an hour?
- c. What is the probability that the business will receive more than 6 calls in an hour?

Ans:

a. $\lambda = 8, x = 7$

$$P(X = 7) = \frac{e^{-8} 8^7}{7!} = 0.1395865 \approx 13.96\%$$

b. $P(X \leq 5) = P(X = 0) + P(X = 1) + P(X = 2) + \dots + P(X = 5)$

$$P(X \leq 5) = e^{-8} \times \left[\frac{8^0}{0!} + \frac{8^1}{1!} + \dots + \frac{8^5}{5!} \right]$$

$$= 0.191236 \approx 19.12\%$$

c. $P(X > 6) = 1 - P(X \leq 6)$

$$= 1 - e^{-8} \times \left[\frac{8^0}{0!} + \frac{8^1}{1!} + \dots + \frac{8^6}{6!} \right]$$

$$= 0.686626 \approx 68.66\%$$

Practice Exercise: From Jay L. Davore's Probability & Statistics

Section 3.6 - 81 (a, b, c), 82, 83, 84