

# Models of Supercapacitors

- Overpotential in electrode:  $\eta = \phi_{solid} - \phi_{liquid} - U_{eq}$
- $\gamma$  = conductivity ratio of solid and liquid
- $\xi, \tau$  = normalized space and time
- $I(\tau)$  = applied current

## High Fidelity model

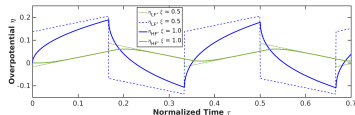
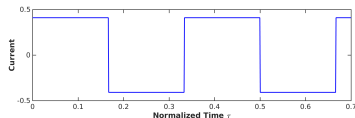
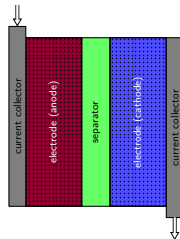
$$\frac{\partial \eta_{HF}}{\partial \tau} = \frac{\partial^2 \eta_{HF}}{\partial \xi^2}$$

$$\begin{cases} \frac{\partial \eta_{HF}}{\partial \xi} \big|_{\xi=0} = -\frac{\gamma}{1+\gamma} I(\tau) \\ \frac{\partial \eta_{HF}}{\partial \xi} \big|_{\xi=1} = \frac{1}{1+\gamma} I(\tau) \\ \eta_{HF} \big|_{\tau=0} = \eta_0(\xi) \end{cases}$$

## Low Fidelity model

$$\eta_{LF}(\xi, \tau) = \frac{1}{2} I \xi^2 - I \frac{\gamma}{1+\gamma} \xi + \eta^{avg}(\tau) - I \frac{2\gamma - 1}{6(1+\gamma)}$$

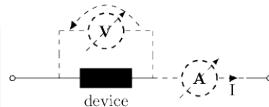
where  $\frac{\partial \eta^{avg}}{\partial \tau} = I$



## Quantity of Interest

Potential drop across the system

$$V^{cell}(\tau) = \phi_{collector}^L - \phi_{collector}^R = \frac{1+2\gamma}{1+\gamma} \eta \big|_{\xi=1} - \frac{\gamma}{1+\gamma} \eta \big|_{\xi=0} - \frac{\gamma}{(1+\gamma)^2} I$$



# Model Inadequacy

## Features of Models

- The high fidelity model accounts for the time history of the current. This feature is hidden in the low fidelity model.
- Solution of low fidelity model converge to high fidelity over time i.e. modeling error is larger for higher frequency current.
- Given what we know about high fidelity model  $\eta_{HF}$ , one can formulate inadequacy representation.

$$\text{Error in QoI: } \epsilon = V_{HF}^{cell} - V_{LF}^{cell}$$

## Inadequacy representation

Stochastic ODE:

$$\frac{\partial \epsilon}{\partial \tau} = -\lambda \epsilon + \alpha \frac{\partial I}{\partial \tau}$$

where  $\lambda$  is a stochastic process with following time evolution:

$$\frac{\partial \lambda}{\partial \tau} = -c(\lambda - \lambda_{mean}) + \beta \frac{\partial W}{\partial \tau}$$

where  $W(t)$  is a Wiener process and  $(\alpha, \beta, c, \lambda_{mean})$  are parameters of inadequacy representation.

