بسم الله الرحمن الرحیم

نظریه زبان‌ها و ماشین‌ها

جلسه ۹

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مثال

- عبارت منظم برای همه رشته‌هایی که سومین حرف از آخر برابر ۱ است (الفبای باینری).

$$(0 + 1)^*1(0 + 1)(0 + 1) = \Sigma^*1\Sigma\Sigma$$

مثال

- عبارت منظم برای همه رشته‌هایی که تعداد ۱ ها بر ۳ بخش‌پذیر باشد(الفبای باینری).

$$0^* + (0^*10^*10^*10^*)^*$$

عبارت‌های معادل/هم ارز

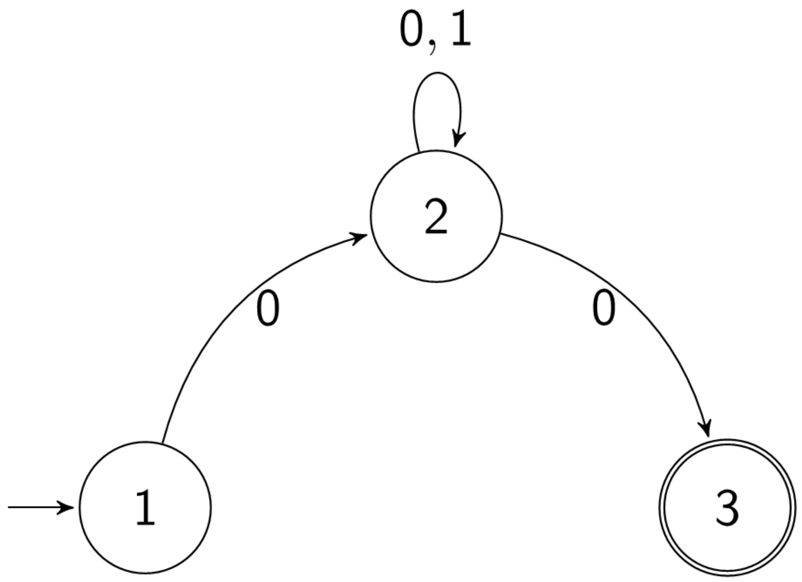
- دو عبارت منظم را معادل گوییم اگر هر دو یک زبان را توصیف کنند. مثال:

$$(a^*b^*)^* = (a+b)^* = \Sigma^*$$

عبارت منظم/اتوماتا

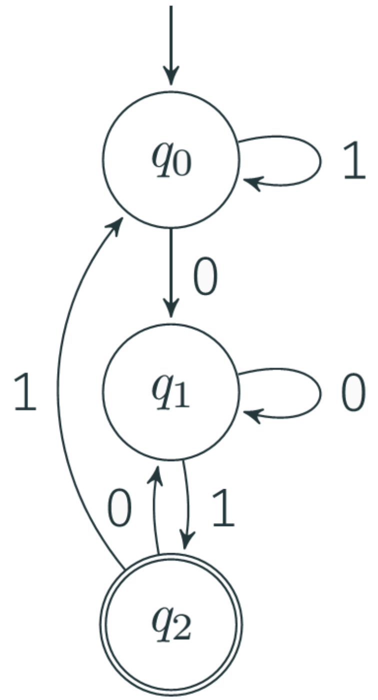
- ارتباط بین RE و DFA/NFA چیست؟

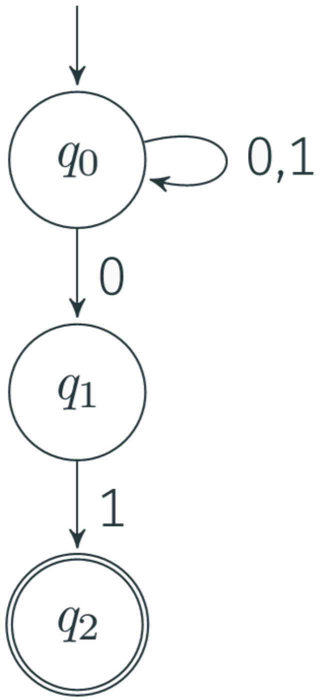
$0(0 \cup 1)^*0$:



عبارت منظم/اتوماتا

- زبانی شامل همه رشته‌های ختم به 01





$$(0+1)^*01$$

عبارت منظم/اتوماتا

- ارتباط بین RE و DFA/NFA چیست؟
  - آیا همه RE ها توسط DFA/NFA قابل نمایش هستند؟
  - آیا همه DFA/NFA ها توسط RE قابل توصیف هستند؟

عبارت‌های منظم/زبان منظم/اتوماتای متناهی

Regular expressions and finite automata are equivalent in their descriptive power. This fact is surprising because finite automata and regular expressions superficially appear to be rather different. However, any regular expression can be converted into a finite automaton that recognizes the language it describes, and vice versa. Recall that a regular language is one that is recognized by some finite automaton.

عبارت‌های منظم/زبان منظم

THEOREM 1.54

A language is regular if and only if some regular expression describes it.

اثبات (ادامه)

LEMMA 1.55

If a language is described by a regular expression, then it is regular.

- اثبات:

R عبارت منظم $\longrightarrow$ NFA N

$L(R) = A$ $\qquad$ $L(N) = A$

اثبات (ادامه)

LEMMA 1.55

If a language is described by a regular expression, then it is regular.

1. a for some $a \in \Sigma$
2. ϵ
3. ϕ
4. $(R_1 \cup R_2)$, where R_1 and R_2 are regular expressions
5. $(R_1 \circ R_2)$, where R_1 and R_2 are regular expressions
6. (R_1^*), where R_1 is a regular expression

اثبات با استقرا

۱-۳ به عنوان base case

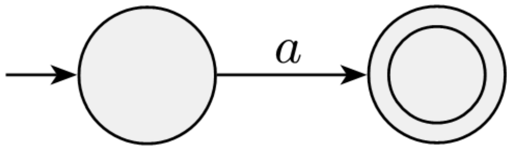
فرض استقرا: R1 و R2 عبارت منظم هستند و dfa معادل دارند.

۴-۶ گامهای استقرا هستند.

اثبات (ادامه)

PROOF Let's convert R into an NFA N. We consider the six cases in the formal definition of regular expressions.

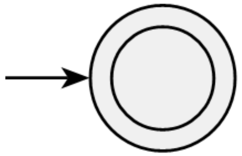
1. $R = a$ for some $a \in \Sigma$. Then $L(R) = \{a\}$, and the following NFA recognizes $L(R)$.



Formally, $N = (\{q_1, q_2\}, \Sigma, \delta, q_1, \{q_2\})$, where we describe δ by saying that $\delta(q_1, a) = \{q_2\}$ and that $\delta(r, b) = \emptyset$ for $r \neq q_1$ or $b \neq a$.

اثبات (ادامه)

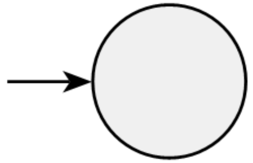
2. $R = \varepsilon$. Then $L(R) = \{\varepsilon\}$, and the following NFA recognizes $L(R)$.



Formally, $N = \big(\{q_1\}, \Sigma, \delta, q_1, \{q_1\}\big)$, where $\delta(r, b) = \emptyset$ for any r and b.

اثبات (ادامه)

3. $R = \emptyset$. Then $L(R) = \emptyset$, and the following NFA recognizes $L(R)$.



Formally, $N = \big(\{q\}, \Sigma, \delta, q, \emptyset\big)$, where $\delta(r, b) = \emptyset$ for any r and b.

اثبات (ادامه)

4. $R = R_1 \cup R_2$.

5. $R = R_1 \circ R_2$.

6. $R = R_1^*$.

For the last three cases, we use the constructions given in the proofs that the class of regular languages is closed under the regular operations. In other words, we construct the NFA for R from the NFAs for R_1 and R_2 (or just R_1 in case 6) and the appropriate closure construction.

مثال

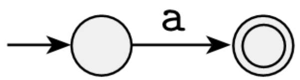
EXAMPLE 1.56

We convert the regular expression $(\texttt{ab} \cup \texttt{a})^*$ to an NFA in a sequence of stages.

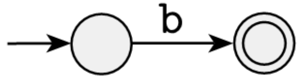
مثال

$(\mathtt{ab} \cup \mathtt{a})^*$

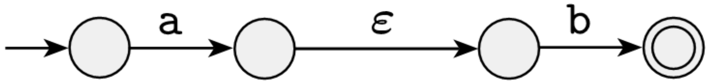
a



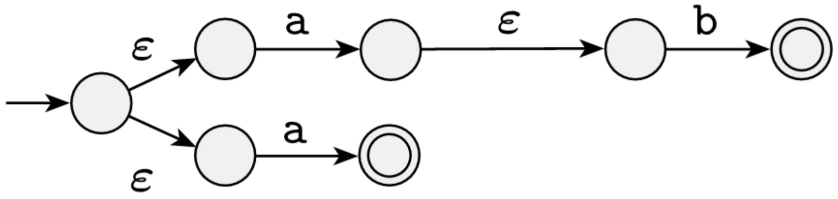
b



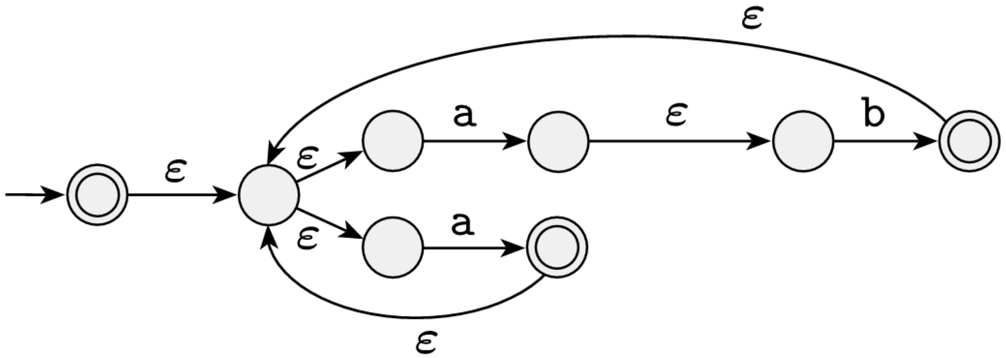
ab



ab ∪ a



(ab ∪ a)*



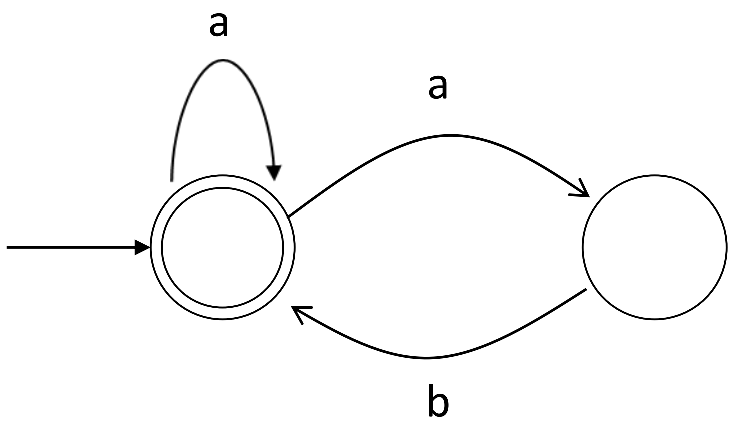
مثال

IUT-ECE

EXAMPLE 1.56

We convert the regular expression $(\mathtt{ab} \cup \mathtt{a})^*$ to an NFA in a sequence of stages.

- به طور مستقیم؟



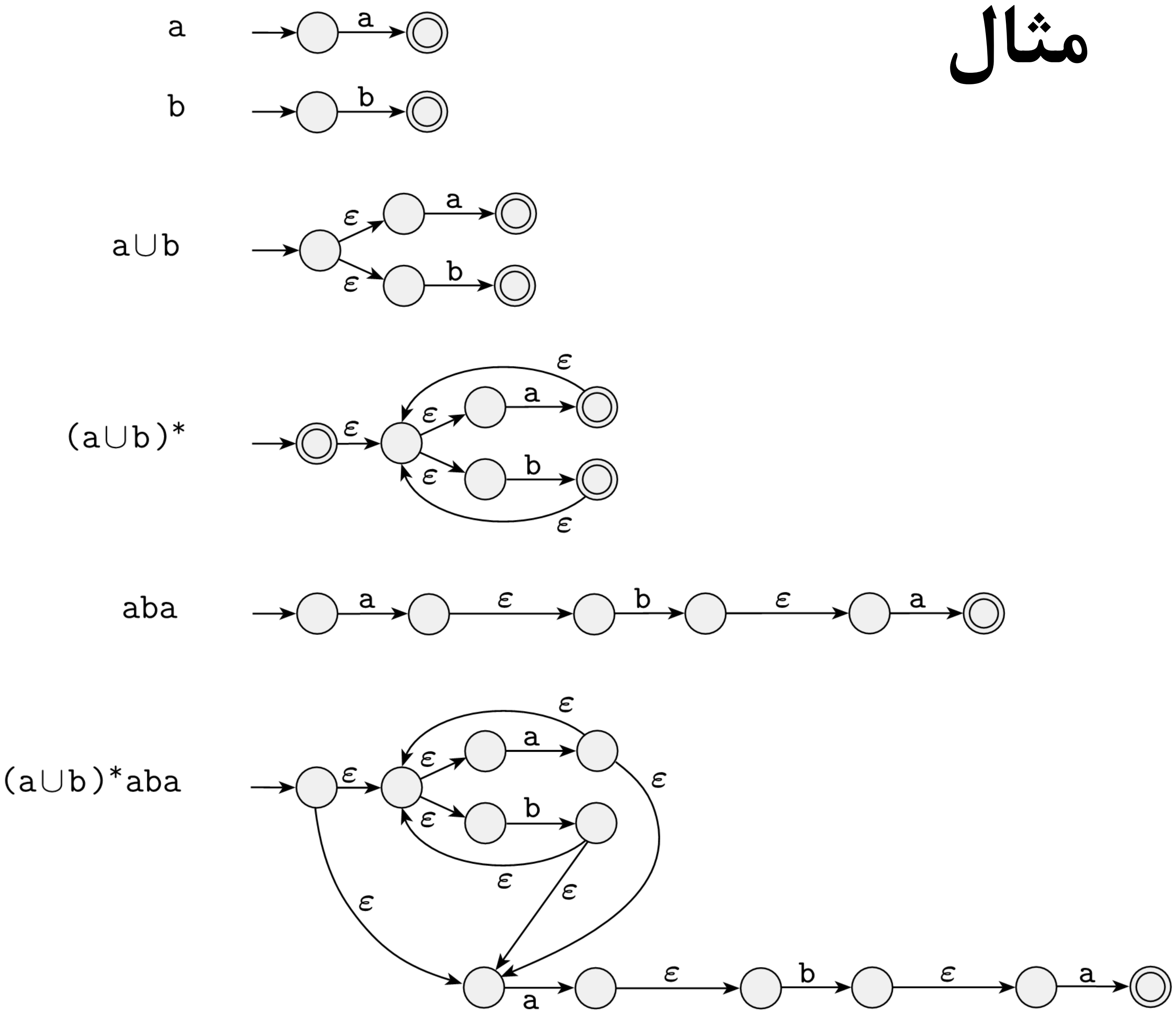
مثال

EXAMPLE 1.58

In Figure 1.59, we convert the regular expression $(\mathtt{a} \cup \mathtt{b})^*\mathtt{aba}$ to an NFA. A few of the minor steps are not shown.

مثال

IUT-ECE



$(\mathtt{a} \cup \mathtt{b})^* \mathtt{aba}$

اثبات (طرف دوم)

IUT-ECE

LEMMA 1.60 ....

If a language is regular, then it is described by a regular expression.

- اثبات:

زبان منظم A → DFA M → GNFA N → عبارت منظم R

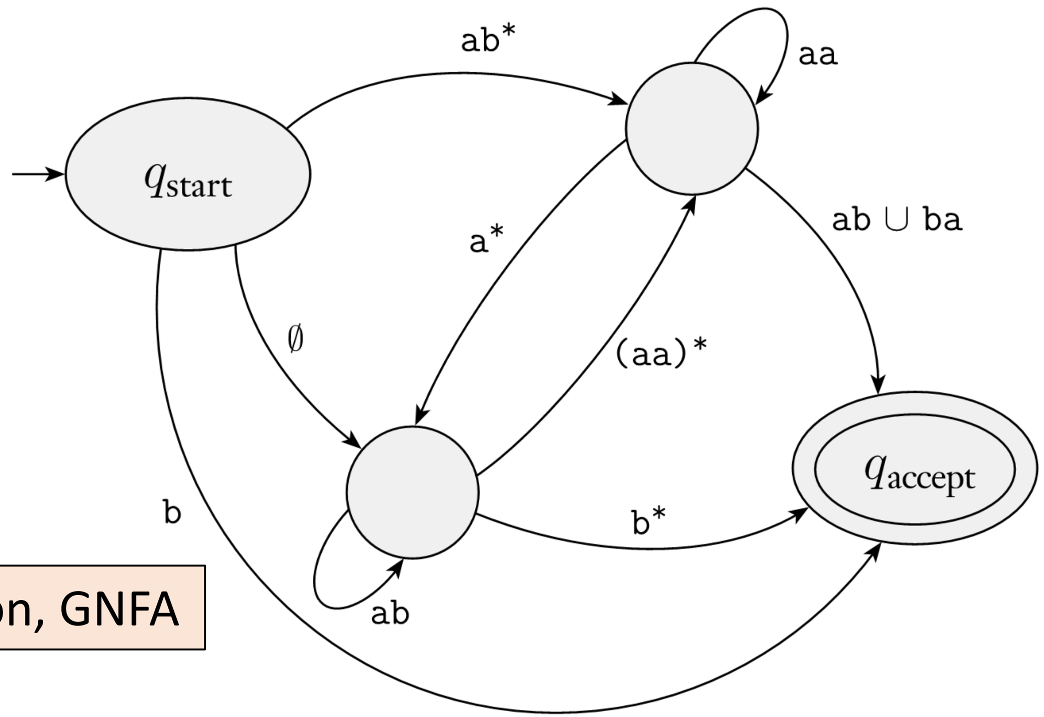
$L(M) = A$ $L(N) = A$ $L(R) = A$

اثبات (طرف دوم)

IUT-ECE

زبان منظم A → DFA M → GNFA N → عبارت منظم R

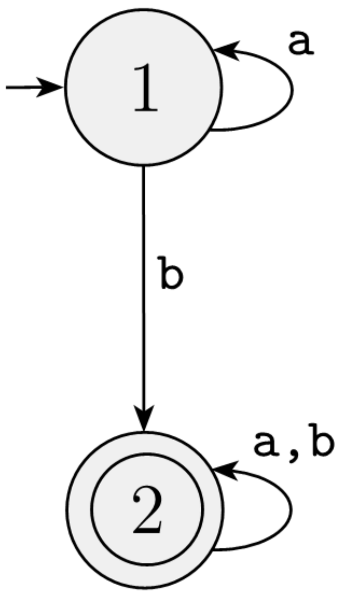
$L(M) = A$ $\quad$ $L(N) = A$ $\quad$ $L(R) = A$

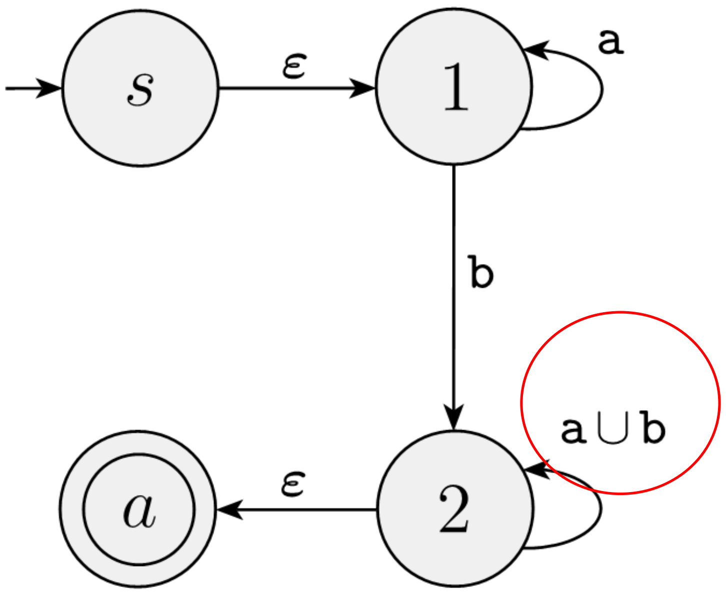


generalized nondeterministic finite automaton, GNFA

اثبات (طرف دوم)

- تبدیل DFA به GNFA:





اثبات (طرف دوم)

- تبدیل DFA به GNFA:

- The start state has transition arrows going to every other state but no arrows coming in from any other state.
- There is only a single accept state, and it has arrows coming in from every other state but no arrows going to any other state. Furthermore, the accept state is not the same as the start state.
- Except for the start and accept states, one arrow goes from every state to every other state and also from each state to itself.

اثبات (طرف دوم)

- تبدیل DFA به GNFA: