Combined Proof Methods for Multimodal Logic

Daniella Angelos

March 20, 2018

Cláudia Nalon — University of Brasilia

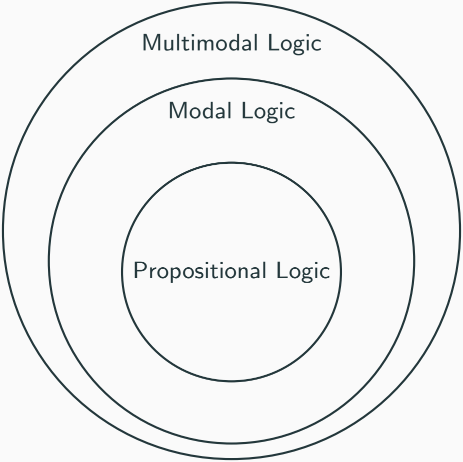
Introduction

Logic

in Computer Science:

Develop languages to model situations

Languages



Multimodal Logic (K_n)

- Propositional Symbols ($\mathcal{P} = \{p, q, r \ldots\}$)
- Operators ($\neg, \vee, \boxed{a}$) with $a \in \mathcal{A} = \{1, \ldots, n\}$

Multimodal Logic (K_n)

Well Formed Formulae (WFF)

- Propositional Symbols $\mathcal{P}$
- If φ and ψ are each a WFF then also are:
  1. $\neg\varphi$
  2. $\varphi \vee \psi$
  3. $\boxed{a}\varphi$

Semantics

Let W be a set of possible worlds and R_a a binary relation over these worlds for some $a \in \mathcal{A}$:

> Possibility: $\Diamond_a \varphi$ is true at $w \in W$ iff φ is true at some $w' \in W$ such that wR_aw'

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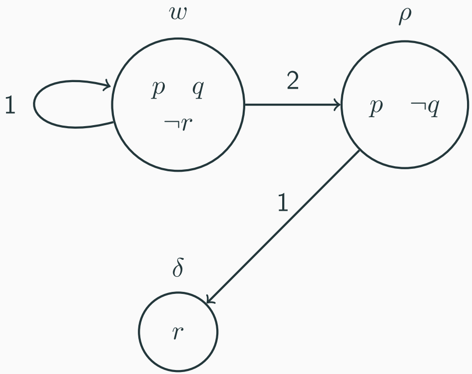
> Possibility: $\langle a \rangle \varphi$ is true at $w \in W$ iff φ is true at some $w' \in W$ such that wR_aw'
>
> Necessity: $\boxed{a}\varphi$ is true at $w \in W$ iff φ is true at all $w' \in W$ such that wR_aw'

Example



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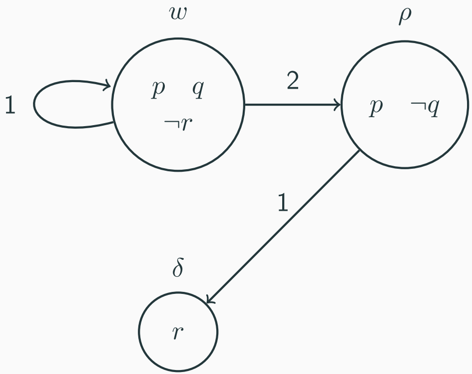

Formulae true in w:

$\boxed{1}\, p$

$\boxed{1}\, q \wedge \boxed{2}\, \neg q$

$\langle 2 \rangle \langle 1 \rangle r$

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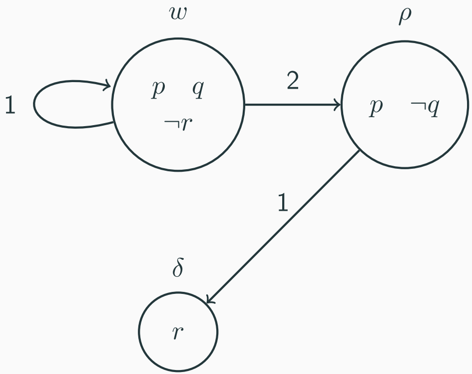
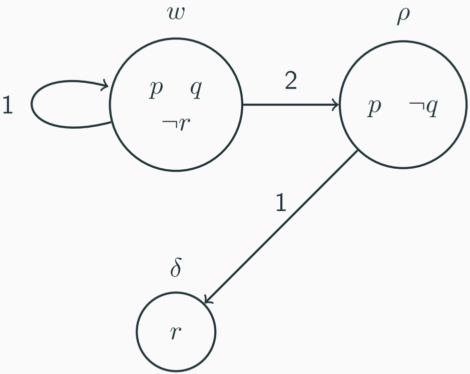


Formulae true in w:

- $\boxed{1}\, p$: modal level of $p = 1$
- $\boxed{1}\, q \wedge \boxed{2}\, \neg q$
- $\Diamond_2 \Diamond_1 r$

Example



Formulae true in w:

- $\boxed{1}\,p$: modal level of $p = 1$
- $\boxed{1}\,q \wedge \boxed{2}\,\neg q$
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Model - Kripke Structure

$$\mathfrak{M} = (W, w_0, R_1, \ldots, R_n, \pi)$$

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- A finite number of binary relations R_a over the worlds in W, $R_a \subseteq W \times W$, with $a \in \mathcal{A}$
- Valuation function to propositional symbols $\pi : W \times \mathcal{P} \rightarrow \{true, false\}$

Satisfiability Relation

Let φ be a WFF and $\mathfrak{M} = (W, w_0, R_1, \ldots, R_n, \pi)$ a model in K_n:

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- $\langle \mathfrak{M}, w \rangle \models \boxed{a}\,\psi$ iff for all $w' \in W$ such that $wR_a w'$, with $a \in \mathcal{A}$, we have that $\langle \mathfrak{M}, w' \rangle \models \psi$

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$\mathfrak{M} \models_G \varphi$

A set $\mathcal{F}$ of WFF is said to be (locally of globally) satisfiable, if the conjunction of each $\varphi \in \mathcal{F}$ is satisfiable.

Problem

Given any set Φ of well formed formulae, to determine if Φ is (locally or globally) satisfiable in K_n.

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- Local satisfiability: PSPACE-Complete (Halpern and Moses – 1992),
- Global satisfiability: EXPTIME-Complete (E. Spaan – 1993).

Layered Resolution

Clauses in $\mathsf{SNF}_{\mathsf{K}_n}$

$$\bigwedge_i ml : C_i$$

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$$ml : l' \Rightarrow \langle a \rangle\, l$$

where l is a propositional symbol or its negation

Calculus

$$[\text{LRES}] \quad \frac{\begin{array}{ll} ml: & D \vee l \\ ml: & D' \vee \neg l \end{array}}{\begin{array}{ll} ml: & D \vee D' \end{array}}$$

Calculus

[GEN1]

$$\begin{array}{rl}
ml: & l'_1 \Rightarrow \boxed{a}\neg l_1 \\
\vdots & \\
ml: & l'_m \Rightarrow \boxed{a}\neg l_m \\
ml: & l' \Rightarrow \Diamond_a \neg l \\
ml+1: & l_1 \vee \ldots \vee l_m \vee l \\
\hline
ml: & \neg l'_1 \vee \ldots \vee \neg l'_m \vee \neg l'
\end{array}$$

[GEN3]

$$\begin{array}{rl}
ml: & l'_1 \Rightarrow \boxed{a}\neg l_1 \\
\vdots & \\
ml: & l'_m \Rightarrow \boxed{a}\neg l_m \\
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K$_S$P

- A theorem prover that implements this modal resolution-based calculus
- It also implements several refinements and simplification techniques
- K$_S$P performs pretty well!
- But not so well when there is a large number of variables in one particular level

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Problem

As resolution relies on saturation, the performance of $\text{K}_\text{S}\text{P}$ for such entries deteriorates.

Hypotheses

We take advantage of the great theoretical and practical efforts that have been directed in improving the efficiency of CDCL SAT solvers, to reduce the time KςP spends during saturation.

CDCL SAT Solvers

SAT Problem

Given a formula in classical propositional logic, does this formula have a satisfying assignment?

SAT Solvers

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- Generic combinatorial tool and search platform

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- DPLL Procedure

SAT Solvers

- Generic combinatorial tool and search platform
- DPLL Procedure
- Watched literals, restart strategies, deletion mechanisms, branching heuristics, learning mechanisms etc

Example

$$\varphi = (p \vee t \vee \neg q) \wedge (p \vee \neg r) \wedge (q \vee r \vee s)$$

Example

$$\varphi = (p \vee t \vee \neg q) \wedge (p \vee \neg r) \wedge (q \vee r \vee s)$$

$$p = false$$

Example

$$\varphi = (p \lor t \lor \neg q) \land (p \lor \neg r) \land (q \lor r \lor s)$$

$$p = false \longrightarrow r = false$$

Example

$$\varphi = (p \vee t \vee \neg q) \wedge (p \vee \neg r) \wedge (q \vee r \vee s)$$

$t = false$

$p = false \longrightarrow r = false$

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$$\begin{tikzcd}
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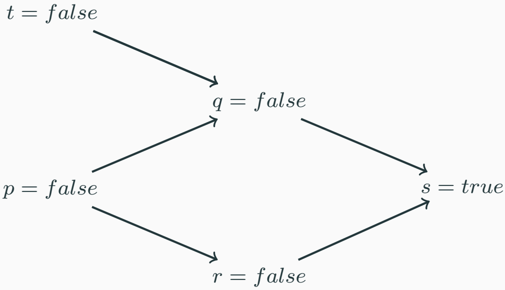
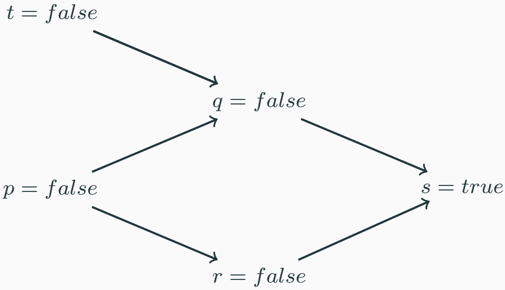


Example

$$\varphi = (p \vee t \vee \neg q) \wedge (p \vee \neg r) \wedge (q \vee r \vee s) \wedge (\neg s \vee \neg u) \wedge (y \vee x) \wedge (y \vee u \vee \neg x)$$



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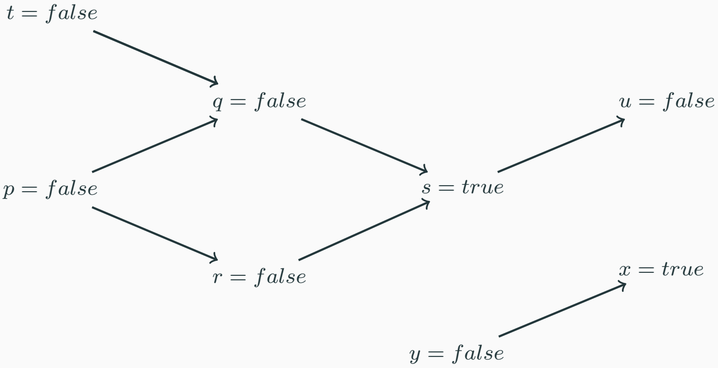

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Implication Graph

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- Edges are antecedent clauses

Conflict analysis

Analyses the conflict and learns a new clause

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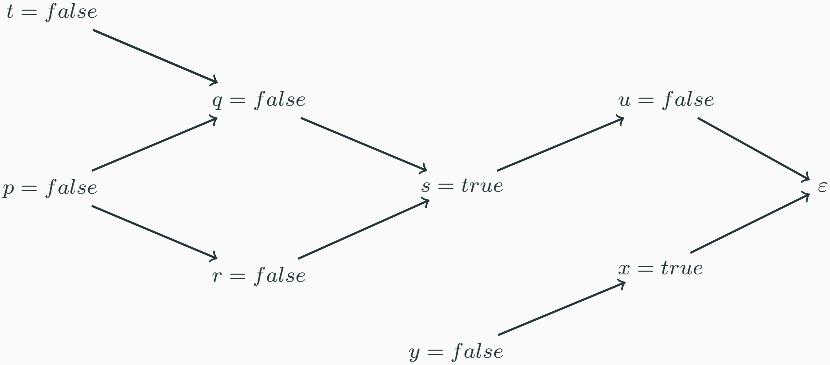
Conflict analysis

$$\varphi = (p \lor t \lor \neg q) \land (p \lor \neg r) \land (q \lor r \lor s) \land (\neg s \lor \neg u) \land (y \lor x) \land (y \lor u \lor \neg x)$$

new clause: $(p \lor t \lor y)$



Combination

K_SP + CDCL

During the main loop of K_SP, we will feed the SAT solver with the propositional clauses at a particular modal level.

K$_S$P + CDCL

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- The SAT solver finds a model for the set of clauses

K_SP + CDCL

During the main loop of K_SP, we will feed the SAT solver with the propositional clauses at a particular modal level.

- The SAT solver finds a model for the set of clauses
- The SAT solver returns one or more new clauses

Example

$$1 : l_1 \Rightarrow \Box \neg p$$
$$1 : l_2 \Rightarrow \Box \neg t$$
$$1 : l_3 \Rightarrow \Diamond \neg y$$
$$2 : \{p, t, \neg q\}$$
$$2 : \{p, \neg r\}$$
$$2 : \{q, r, s\}$$
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$2 : \{p, t, y\}$
$1 : \{\neg l_1, \neg l_2, \neg l_3\}$

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Conclusion

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We believe that calling the SAT solver from the main loop of KSP will significantly improve its performance.

Future Work

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Activities	Months									
	March	April	May	June	July	August	September	October	November	December
1	x	x								
2	x	x	x	x						
3				x	x					
4					x	x	x	x		
5							x	x	x	
6	x	x	x	x	x	x	x	x	x	x

Thank you!

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