Correccion Parcial I

1) Dada la siguiente expresion de ED represente la en el espacio de estados y encontrar la funcion de transferencia

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t)$$

$q_1 = x$

$q_2 = \dot{q}_1 = \dot{x}$

$\dot{q}_2 = \ddot{q}_1 = \ddot{x}$

$q_3 = \dot{q}_2 = \ddot{x}$

$\dot{q}_3 = \dddot{x}$

$\rightarrow \dddot{x} = 2f(t) - \ddot{x} - 2\dot{x} - x \rightarrow \dot{q}_3 = 2f - q_3 - 2q_2 - q_1$

se plantea la matriz de espacio de estado

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} f(t)$$

$$[x] = [1 \quad 0 \quad 0] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Para la funcion de transferencia

$$\dddot{x} + \ddot{x} + 2\dot{x} + x = 2f(t)$$

$$\mathcal{L}\{\dddot{x} + \ddot{x} + 2\dot{x} + x\} = 2\mathcal{L}\{f(t)\}$$

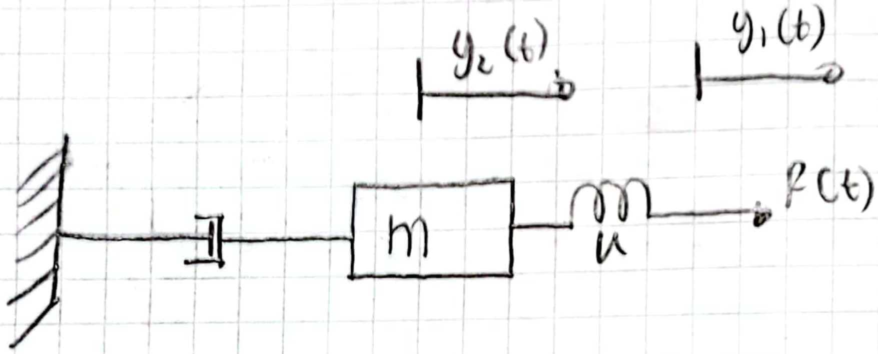
$$\to \mathcal{L}\{\dddot{x}\} + \mathcal{L}\{\ddot{x}\} + 2\mathcal{L}\{\dot{x}\} + \mathcal{L}\{x\} = 2F(s)$$

$$s^3 X(s) + s^2 X(s) + 2sX(s) + X(s) = 2F(s)$$

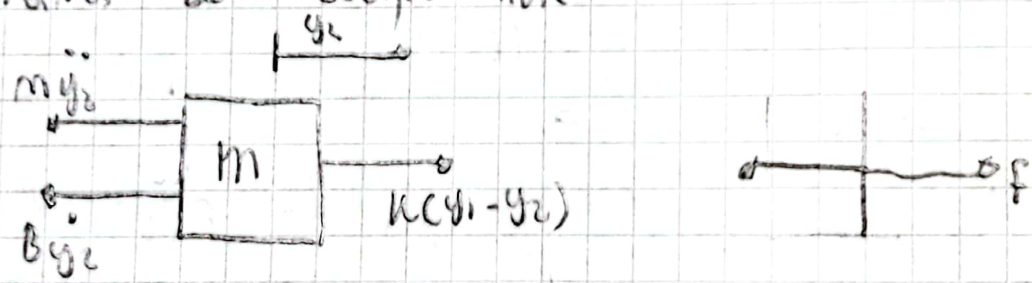
$$\to X(s)[s^3 + s^2 + 2s + 1] = 2F(s)$$

$$\to \frac{X(s)}{F(s)} = \frac{2}{s^3 + s^2 + 2s + 1}$$

2) Encontrar una expresión en el espacio de estados valida para el siguiente sistema



Diagramas de cuerpo libre



Al aplicar la segunda ley de Newton

en y_2 $\rightarrow$ $-B\dot{y}_2 + k(y_1 - y_2) = m\ddot{y}_2$

$$\ddot{y}_2 = -\frac{B}{m}\dot{y}_2 + \frac{k}{m}(y_1 - y_2)$$

en y_1 $\rightarrow$ $-k(y_1 - y_2) + F = 0$

$$-ky_1 + ky_2 + F = 0$$

Se plantean variables de estado

$q_1 = y_1$

$q_2 = y_2$

$q_3 = \dot{q}_2 = \dot{y}_2$

$\dot{q}_3 = \ddot{q}_2 = \ddot{y}_2$

$\rightarrow \dot{q}_3 = -\frac{B}{m} q_3 + \frac{k}{m} q_1 - \frac{k}{m} q_2$ ①

$-k q_1 + k q_2 + f = 0 \rightarrow q_2 = q_1 - \frac{f}{k}$

$\rightarrow \dot{q}_3 = -\frac{B}{m} q_3 + \frac{k}{m} q_1 - \frac{k}{m} \left[q_1 - \frac{f}{k} \right]$

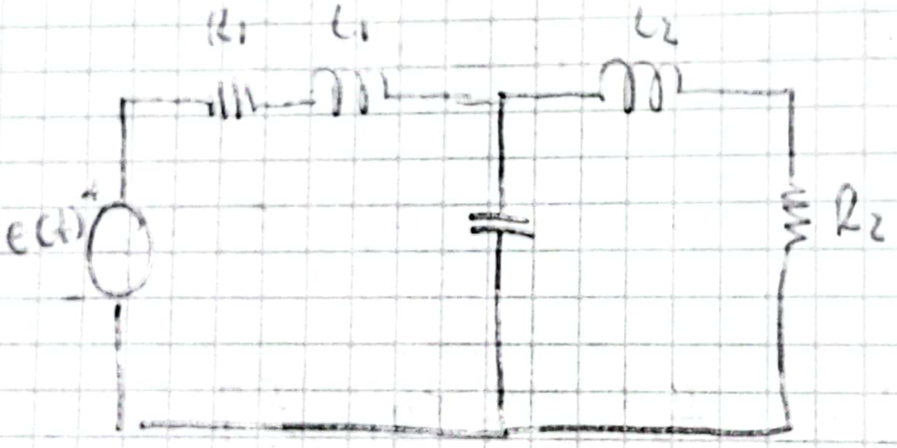
$\rightarrow \dot{q}_3 = -\frac{B}{m} q_3 + \frac{f}{m}$

Se plantea la matriz

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -B/M \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1/M \end{bmatrix} f$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

3)



$e(t) = V_{R1} + V_{L1} + V_C$

$V_{L1} = -V_C - V_{R1} + e(t)$

$V_{L1} = -V_C - I_{L1} + e(t)$

$i_1 = i_2 + i_3$

$V_C = V_{L2} + V_{R2}$

$V_C = V_{L2} + I_{L2} R_2$

$\rightarrow V_{L2} = V_C - I_{L2} R_2$

$L_2 \frac{\partial i_{L2}}{\partial t} = V_C - R_2 i_{L2}$

$$\boxed{\dot{i}_{L2} = \frac{1}{L_2} V_C - \frac{R_2}{L_2} i_{L2}}$$

$$\dot{I}_{L1} = -\frac{V_C}{L_1} - \frac{R_1}{L_1} I_{L1} + \frac{e(t)}{L_1}$$

$\rightarrow i_{L1} = I_C + I_{L2} \rightarrow C\frac{\partial V_C}{\partial t} = i_{L1} - i_{L2} \rightarrow C\dot{V} = i_{L1} - i_{L2}$

$$\rightarrow \boxed{\dot{V} = \frac{i_{L1}}{C} - \frac{i_{L2}}{C}}$$

Se plantea la matriz

$$\begin{bmatrix} \dot{V}_C \\ \dot{i}_{L1} \\ \dot{i}_{L2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} & -\frac{1}{C} \\ -\frac{1}{L_1} & -\frac{R_1}{L_1} & 0 \\ \frac{1}{L_2} & 0 & -\frac{R_2}{L_2} \end{bmatrix} \begin{bmatrix} V_C \\ i_{L1} \\ i_{L2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_1} \\ C \end{bmatrix} e(t)$$

$$V_{R2} = \begin{bmatrix} 0 & 0 & R_2 \end{bmatrix} \begin{bmatrix} V_C \\ i_{L1} \\ i_{L2} \end{bmatrix}$$