Ejercicio de Video 1

$$G(s) = \frac{20(s+5)}{s(s+1)(s+4)}$$

- Overshoot de 9,5%
- $t_s = 0,74$ sg

$$U(s) \longrightarrow \boxed{\frac{1}{s^3+5s^2+4s}} \xrightarrow{X_1(s)} \boxed{0s^2+20s+100} \longrightarrow Y(s)$$

Analiso la función de transferencia del primer bloque

$$\frac{X_1(s)}{U(s)} = \frac{1}{s^3+5s^2+4s} \rightarrow (s^3+5s^2+4s)X_1(s) = U(s)$$

$\mathcal{L}^{-1}$

$$\rightarrow \underbrace{\cancel{\dddot{x}_1}}_{x_3} + 5\underbrace{\cancel{\ddot{x}_1}}_{x_3} + 4\underbrace{\cancel{\dot{x}_1}}_{x_2} = u(t) \rightarrow x_1 = x_1$$

$$x_2 = \dot{x}_1$$

$$x_3 = \dot{x}_2 = \ddot{x}_1$$

$$\dot{x}_3 = \dddot{x}_1$$

$$\rightarrow \boxed{\dot{x}_3 = -5x_3 - 4x_2 + u(t)} \quad ① \rightarrow Y(s) = (b_2 s^2 + b_1 s + b_0)X_1($$

$\mathcal{L}^{-1}$

$$\rightarrow (0s^2+20s+100)X_1(s) = (20s+100)X_1(s) \rightarrow 20\underbrace{\cancel{\dot{x}_1}}_{x_2} + 100x_1$$

$$\rightarrow \boxed{Y = 20x_2 + 100x_1} \quad ②$$

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→ Se plantea la matriz

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Para hallar el OS del 9,5% (Overshoot)

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \cdot 100$$

$$0,095 = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \rightarrow \ln(0,095) = \ln\left(e^{-\zeta\pi/\sqrt{1-\zeta^2}}\right)$$

$$\rightarrow -2,3539 = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \rightarrow \cancel{-}2,3539\left(\sqrt{1-\zeta^2}\right) = \cancel{-}\zeta\pi$$

$$\rightarrow \left[2,3539\left(\sqrt{1-\zeta^2}\right)\right]^2 = (\zeta\pi)^2 \rightarrow 5,5407(1-\zeta^2) = \zeta^2\pi^2$$

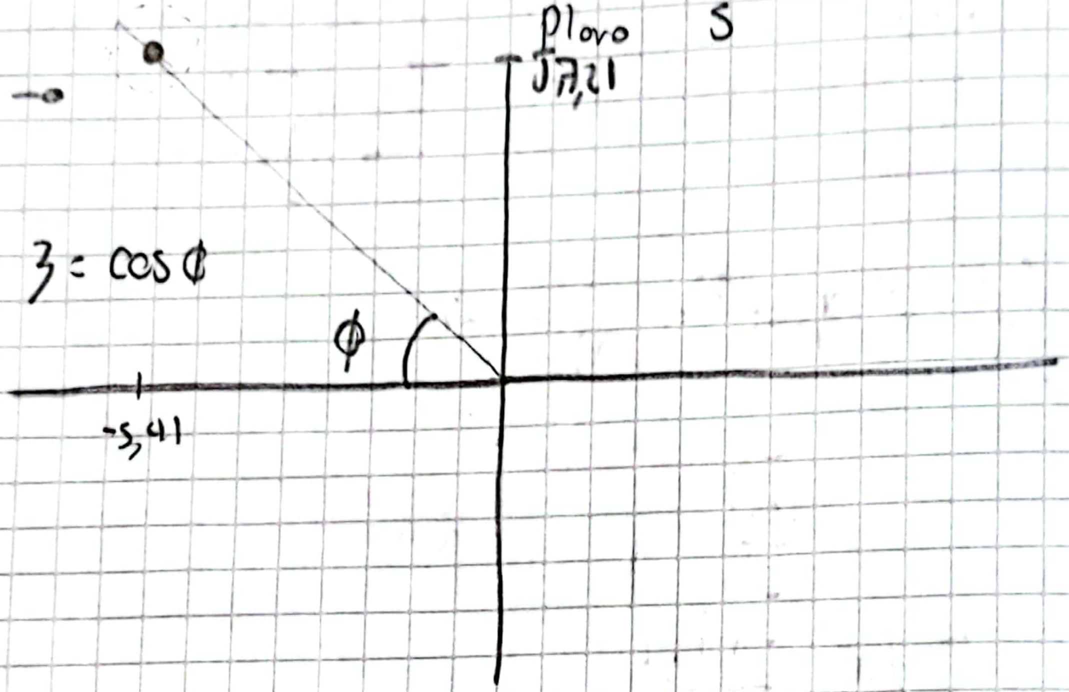
$$\rightarrow 5,5407 - 5,5407\zeta^2 = \zeta^2\pi^2$$

$5,5407 = \zeta^2 \pi^2 + 5,5407\zeta^2 \rightarrow 5,5407 = \zeta^2(\pi^2 + 5,5407)$

$\rightarrow \zeta^2 = \dfrac{5,5407}{\pi^2 + 5,5407} \rightarrow \boxed{\zeta = 0,5906}$

$s = \sigma + j\omega_d$

que es la transiente



$\zeta = \cos\phi$

$\rightarrow \phi = \cos^{-1}(0,5906) \rightarrow \phi = 53,80^\circ \rightarrow t_s = \dfrac{4}{\sigma}$

$\rightarrow 0,74 = \dfrac{4}{\sigma} \rightarrow \sigma = \dfrac{4}{0,74} \rightarrow \sigma = 5,405 \rightarrow \sigma = \zeta\omega_n$

$\rightarrow 5,405 = 0,5906\,\omega_n \rightarrow \omega_n = \dfrac{5,405}{0,5906} \rightarrow \omega_n = 9,02\ rad/s$

$\rightarrow \omega_d = \omega_n\sqrt{1-\zeta^2} \rightarrow \omega_d = 9,02\sqrt{1-0,5906^2}$

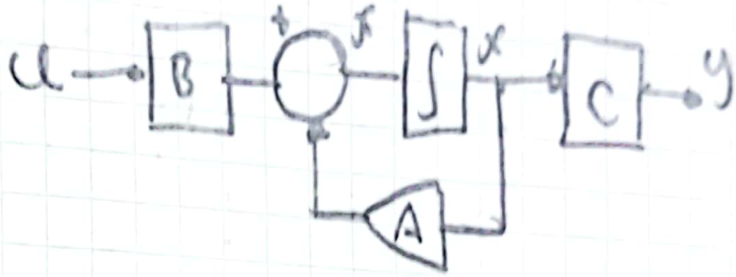
$\rightarrow \omega_d = 7,27\ rad$ ó $\rightarrow \quad = \tan(\phi) = \dfrac{\omega_d}{5,41}$

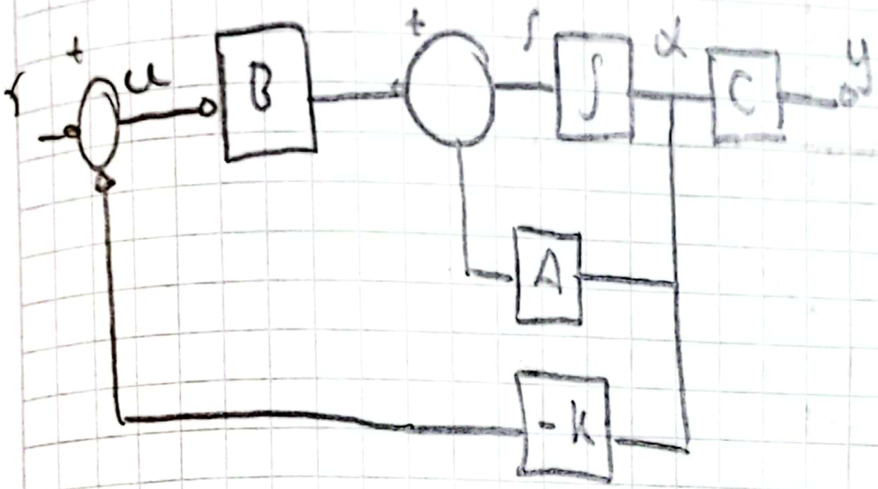
$\to \tan(53{,}16) \cdot 5{,}41 \Rightarrow 7{,}22 = \omega_d$

Realimentación de estados

$\to \dot{x} = Ax + Bx$

$y = Cx$ $\to$





$\to \dot{x} = Ax + Bu \to$ $\dot{x} = Ax - BKx + Br)$

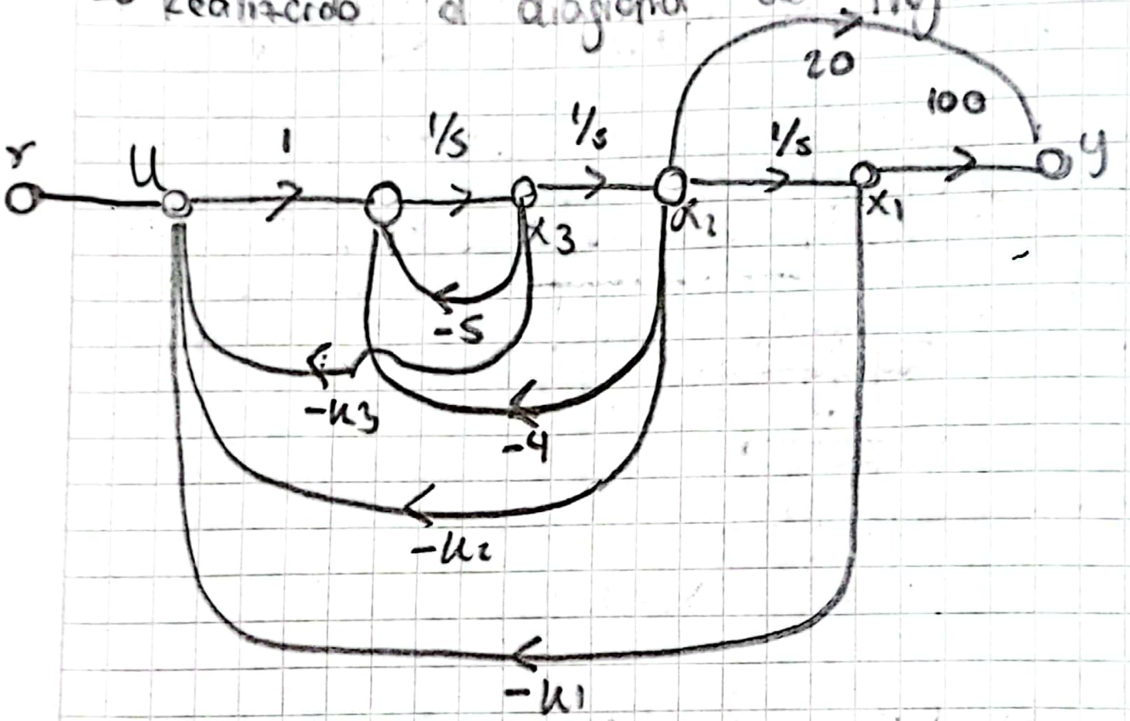
$\dot{x} = Ax + B(-Kx + r) \to$ $\dot{x} = (A - BK)x + Br$

Volviendo al ejercicio 1 del video

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 100 & 20 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

→ Realizando el diagrama de flujo de señal



→ $\dot{x}_3 = -4x_2 - 5x_3 + u$

$\dot{x}_3 = -4x_2 - 5x_3 + [-k_3x_3 - k_2x_2 - k_1x_1] + r$

→ $\dot{x}_3 = 4x_1 - 5x_3 - k_3x_3 - k_2x_2 - k_1x_1 + r$

→ $\dot{x}_3 = -k_1x_1 - x_2(4 + k_2) - (5 + k_3)x_3 + r$

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→ la nueva representacion en el espacio de estados

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_1 & -(4+k_2) & -(5+k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

→ $\det(sI-(A-BK)) = \boxed{s^3+(5+k_3)s^2+(4+k_2)s+k_1=0}$ (4)

Ecuacion Caracteristica del sistema

Se deben encontrar k_3, k_2, k_1

→ $T=(s+5{,}4-j7{,}21)(s+5{,}4+j7{,}21)(s+5{,}1)$

→ $T: \boxed{s^3+15{,}9s^2+136{,}22s+413{,}83=0}$ (5)

relacionando (4) con (5)

$$s^3+(5+k_3)s^2+(4+k_2)s+k_1 = s^3+15{,}9s^2+136{,}22s+413{,}83$$

→ $(5+k_3)\cancel{s^2} = 15{,}9\cancel{s^2}$ | $(4+k_2)\cancel{s} = 136{,}22\cancel{s}$ | $\boxed{k_1 = 413{,}83}$

$5+k_3 = 15{,}9$ | $4+k_2 = 136{,}22$

$\boxed{k_3 = 10{,}9}$ | $\boxed{k_2 = 132{,}22}$

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→ La nueva representación en el espacio de estado es

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -413{,}8 & -132{,}4 & -10{,}9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r$$

Sistemas de Segundo

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