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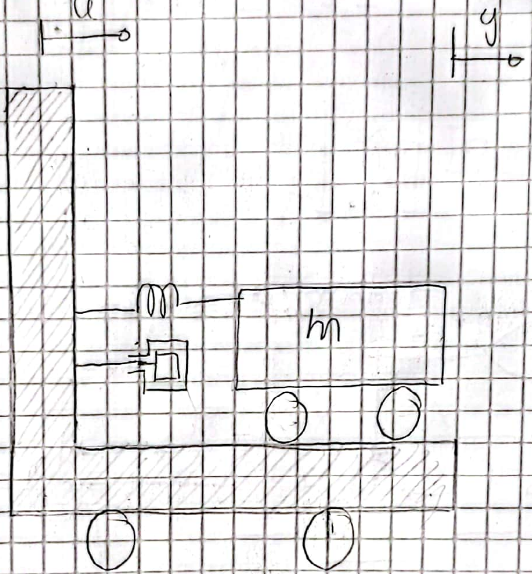
20192005057

# Punto 1. Parcial 2 Sistemas Dinamicos

1) Ejemplo 3,3 libro Ogata $\qquad y > u$




→ $\sum F = ma \rightarrow m\frac{\partial^2 y}{\partial t} = -b\left(\frac{\partial y}{\partial t} - \frac{\partial u}{\partial t}\right) - k(y-u)$

→ Aplicando $\mathcal{L}^{-1} \rightarrow (ms^2+bs+k)Y(s) = (bs+k)U(s)$

→ $G(s) = \frac{Y(s)}{U(s)} \rightarrow \frac{Y(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k}$

→ Sacamos las variables de estado del sistema

→ $\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{b}{m}\dot{u} + \frac{k}{m}u \rightarrow \ddot{y}+a_1\dot{y}+a_2 y = b_0\ddot{u}+b_1\dot{u}+b_2 u$

→ $a_1 = \frac{b}{m}$, $a_2 = \frac{k}{m}$, $b_0 = 0$, $b_1 = \frac{b}{m}$, $b_2 = \frac{k}{m}$

→ $\beta_0 = b_0 = 0 \quad \wedge \quad \beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{k}{m} - \left(\frac{b}{m}\right)^2$

$\beta_1 = b_1 - a_1\beta_0 = \frac{b}{m}$

→ $\dot{x}_1 = x_2 + \frac{b}{m}u$

$\dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right]u$

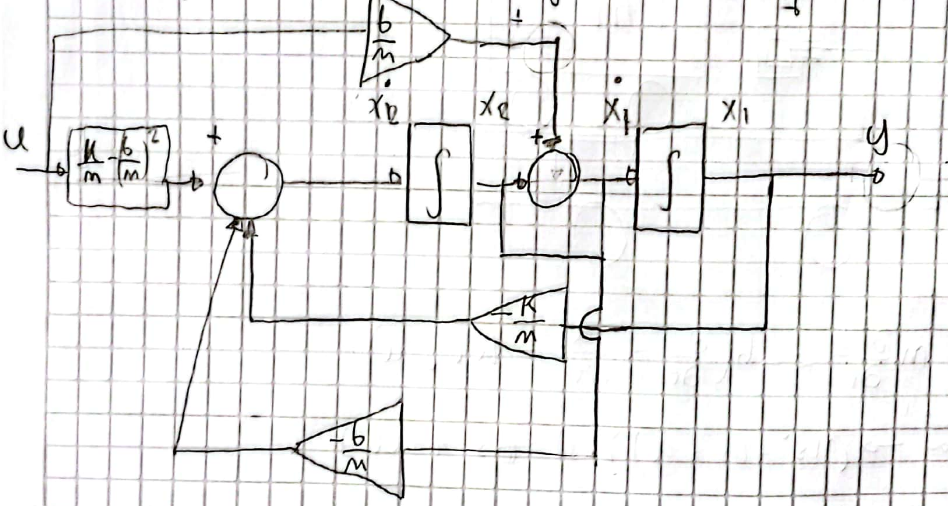
$y = x_1$

→ la matriz de espacio de estado es la siguiente

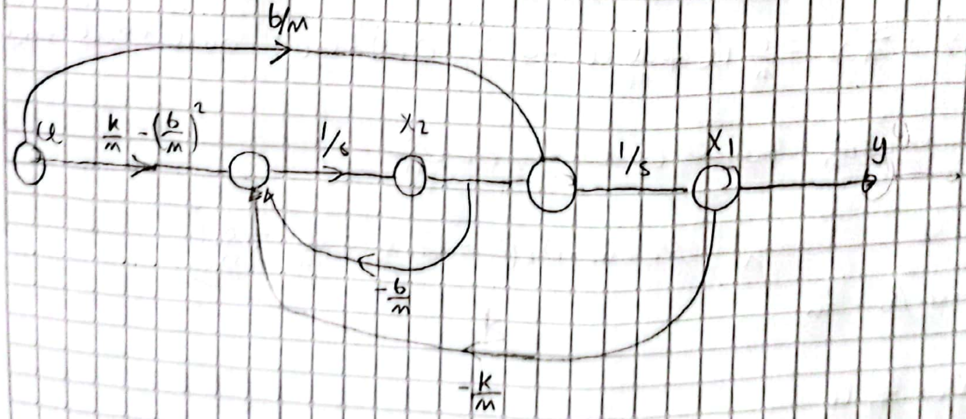
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{b}{m} \\ \frac{k}{m} - \left(\frac{b}{m}\right)^2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

su representacion en diagrama de bloques es




su representacion en diagrama de flujo de señal es



2) A(3-9) Representación en diagrama de bloques

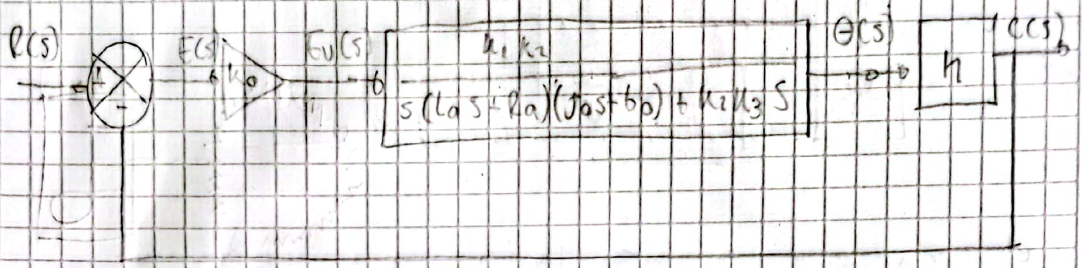
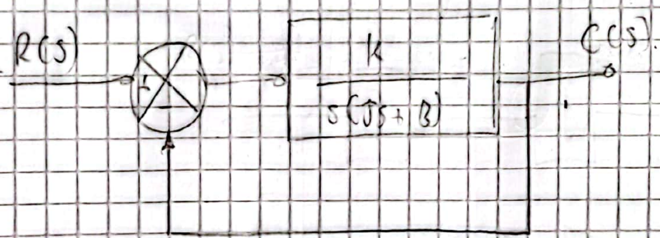



Diagrama de bloques simplificado




Plantea las ED

$$L\frac{dI}{dt} + R I_a + e_b = e_a \rightarrow L\frac{\partial I}{\partial t} + R_a I_a + k_3 \frac{\partial \theta}{\partial t} = k_1 e_v$$

→ la ecuación de torque es → $J_0 \frac{\partial^2\theta}{\partial t^2} + b_0 \frac{\partial\theta}{\partial t} = T = k_2 I_a$

- A partir del diagrama de bloques simplificado se saca el modelo de espacio de estado

$$G(s) = \frac{k}{s(Js+B)} \rightarrow \frac{R(s)}{C(s)} = \frac{k}{s(Js+B)} \rightarrow R(s)[s(Js+B)] = kC(s)$$

$$\rightarrow R(s)[s^2J + Bs] = kC(s) \rightarrow \mathcal{L}^{-1} \rightarrow J\ddot{R} + B\dot{R} = kC \rightarrow \ddot{R} = \frac{k}{J}C - \frac{B}{J}\dot{R}$$

→ $R_1 = R$

$\boxed{\dot{R}_1 = R_2 = \dot{R}}$ → $\boxed{\dot{R}_2 = \frac{k}{J}C - \frac{B}{J}R_2}$

$\boxed{\dot{R}_2 = \ddot{R}_1 = \ddot{R}}$

Ec planta a Matriz

$$\begin{bmatrix} \dot{R}_1 \\ \dot{R}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} + \begin{bmatrix} 0 \\ u/J \end{bmatrix} C$$

$$R = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}$$

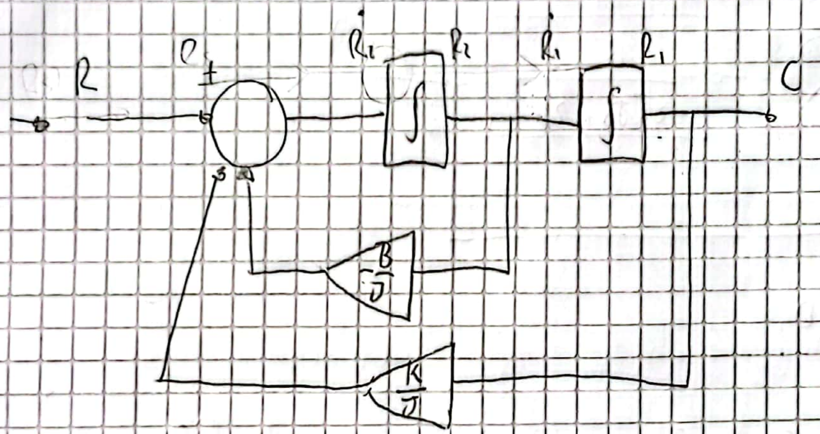
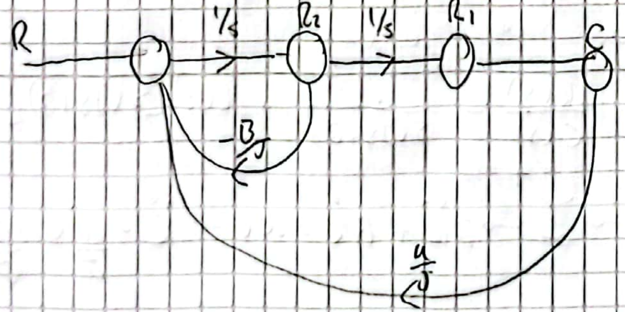


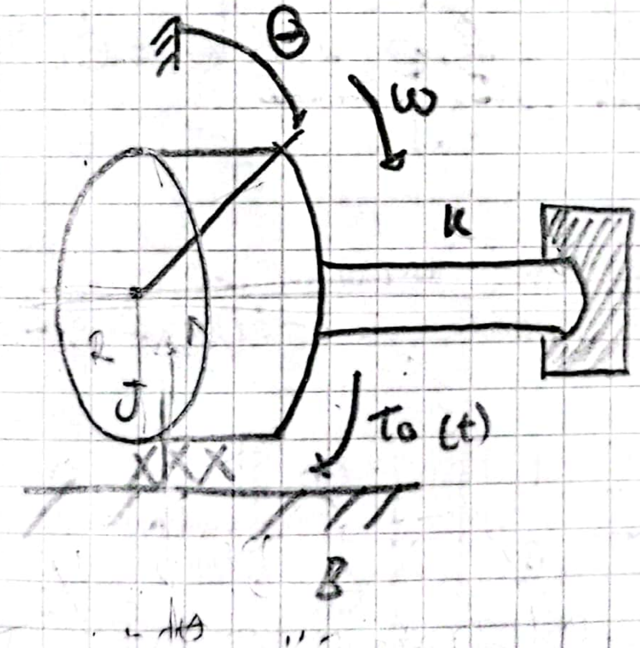

Diagrama de flujo de señal



# Parcial 2

2) Para el sistema rotacional de la figura determinar

a) representación en espacio de estados
b) diagrama de bloques
c) diagrama de flujo de señal
d) Función de transferencia




Para la matriz de espacio de estados se tiene lo siguiente

$$\sum \tau = I\alpha \rightarrow \tau_o(t) - B\omega + K\theta = I\frac{\partial^2\theta}{\partial t^2}$$

$$\rightarrow \omega = \frac{\partial\theta}{\partial t} \rightarrow \tau_o(t) - B\frac{\partial\theta}{\partial t} + K\theta = I\frac{\partial^2\theta}{\partial t^2}$$

$$\rightarrow \frac{\tau_o(t)}{I} - \frac{B}{I}\frac{\partial\theta}{\partial t} + \frac{K}{I}\theta = \frac{\partial^2\theta}{\partial t}$$

Se plantean variables de estado

$$\theta_1 = \theta$$

$$\theta_2 = \dot{\theta}_1 = \dot{\theta} \rightarrow \frac{\tau_o}{I} - \frac{B}{I}\theta_2 + \frac{K}{I}\theta_1 = \dot{\theta}_2$$

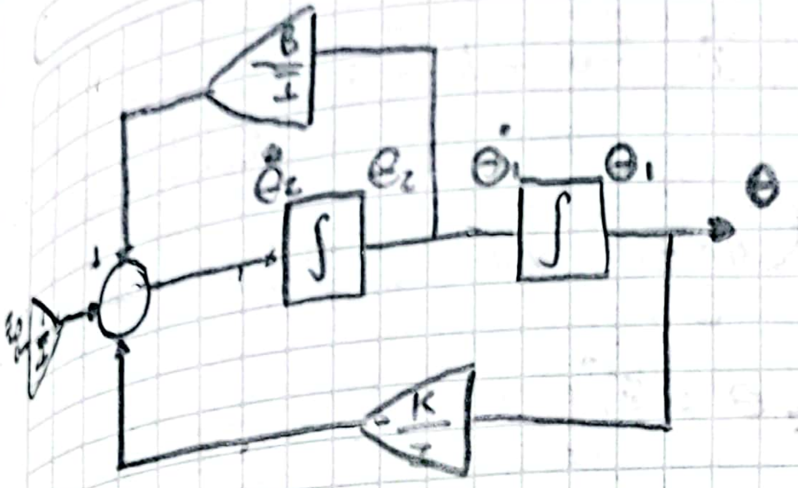
$$\dot{\theta}_2 = \ddot{\theta}_1 = \ddot{\theta}$$

Se plantea la matriz de espacio de estados

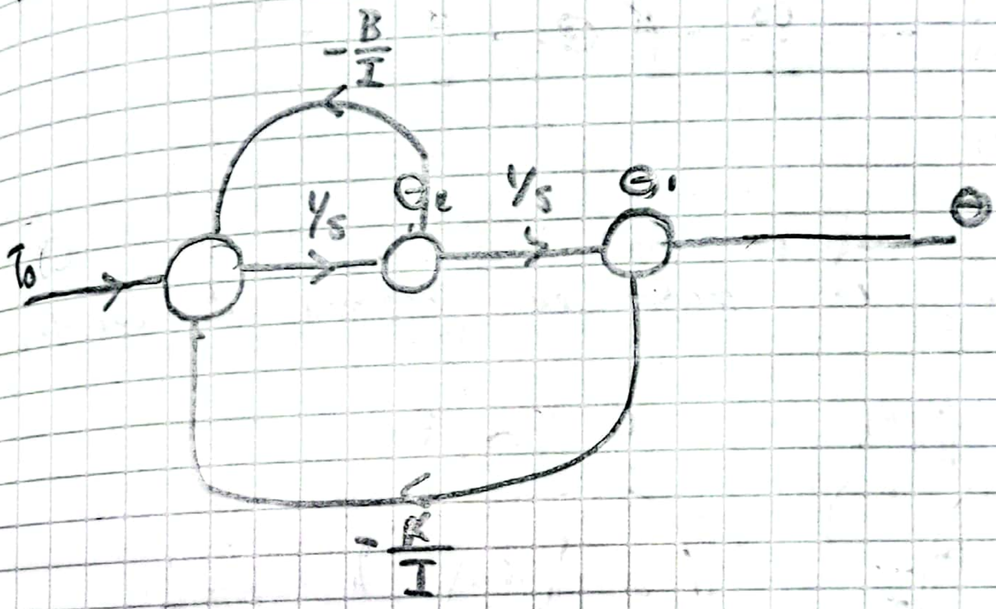
$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{I} & -\frac{B}{I} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/I \end{bmatrix} \tau_o$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

Para el diagrama de bloques




Para el diagrama de flujo de señal




Para la funcion de transferencia

$$T_0(t) = I\frac{\partial^2 \theta}{\partial t} - K\theta + B\omega$$

$$T_0(t) = I\ddot{\theta} + B\dot{\theta} - K\theta$$

$$\rightarrow \frac{\tau_o}{I} = \ddot{\theta} + \frac{B}{I}\dot{\theta} - \frac{k}{I}\theta \rightarrow \mathcal{L}^{-1} \rightarrow$$

$$\rightarrow \frac{\tau_o(s)}{I} = \left[s^2 + \frac{B}{I}s - \frac{k}{I}\right]\theta(s)$$

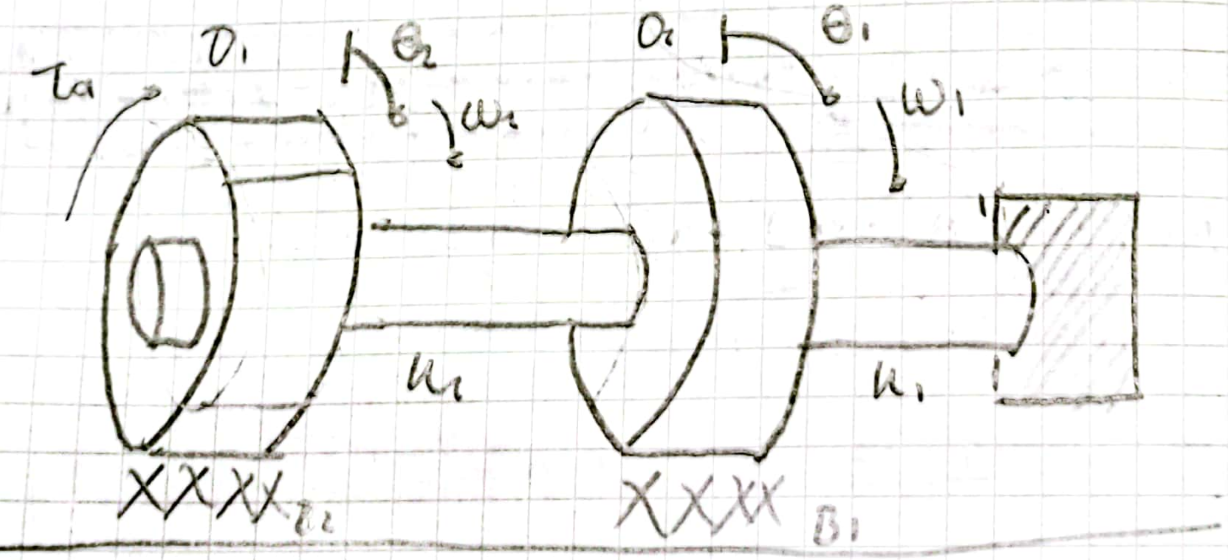
$$\rightarrow \frac{1}{I} = \frac{\theta(s)}{\tau_o(s)}\left[s^2 + \frac{B}{I}s - \frac{k}{I}\right] \rightarrow \frac{\theta(s)}{\tau_o(s)} = \frac{1}{Is^2 + Bs - k}$$

Importante tener en cuenta $I = \frac{mR}{2}$

M : masa del disco

R : Radio del disco

3) Para el sistema Rotacional en la figura, asumir $\theta_2 > \theta_1$



Planteo la suma de torques

Para disco 1

$$T_a - k_2(\theta_2 - \theta_1) - B_2(\dot{\theta}_2 - \dot{\theta}_1) = I_2\ddot{\theta}_2$$

$$T_a - k_2\theta_2 + k_2\theta_1 - B_2\dot{\theta}_2 + B_2\dot{\theta}_1 = I_2\ddot{\theta}_2$$

$$\boxed{T_a = I_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + k_2\theta_2 - B_2\dot{\theta}_1 + k_2\theta_1} \quad (1)$$

Para disco 2

$$k_2(\theta_2 - \theta_1) - B_1(\dot{\theta}_2 - \dot{\theta}_1) - k_1\theta_1 - B_2(\dot{\theta}_2 - \dot{\theta}_1) = I_1\ddot{\theta}_1$$

$$k_2\theta_2 - k_2\theta_1 - B_1\dot{\theta}_2 + B_1\dot{\theta}_1 - k_1\theta_1 - B_2\dot{\theta}_2 + B_2\dot{\theta}_1 - I_1\ddot{\theta}_1 = 0$$

$$\boxed{k_2\theta_2 + B_2\dot{\theta}_2 + (B_1 + B_2)\dot{\theta}_1 - (k_1 + k_2)\theta_1 - I_1\ddot{\theta}_1 = 0} \quad (2)$$

Aplicando Laplace a ① y ②

$$Ta(s) = [s^2 I_2 + B_2 s + k_2]\Theta_2(s) - [B_2 s + k_2]\Theta_1(s) \quad ③$$

$$[k_2 + s B_2]\Theta_2(s) + [-I_1 s^2 - (B_1 + B_2)s - (k_1 + k_2)]\Theta_1(s) = 0$$

Aplicando cramer

$$A \begin{bmatrix} -B_2 s - k_2 & I_2 s^2 + B_2 s + k_2 \\ -I_1 s^2 - (B_1 + B_2)s - (k_1 + k_2) & B_2 s + k_2 \end{bmatrix} \begin{bmatrix} \Theta_1 \\ \Theta_2 \end{bmatrix} =$$

$$\det(A) = -(B_2 s + k_2)^2 + (I_2 s^2 + B_2 s + k_2)(I_1 s^2 - (B_1 + B_2)s - (k_1 + k_2$$

sustituyendo

$$\Delta_{\Theta 1} = Ta \cdot (B_2 s + k_2)$$

$$\Delta_{\Theta 2} = Ta \cdot (T_1 s^2 + (B_1 + B_2)s + (k_1 + k_2))$$

$$\Theta_1 = \frac{\Delta_{\Theta 1}}{A} = \frac{Ta(B_2 s + k_2)}{(I_2 s^2 + B_2 s + k_2)(I_1 s^2 + (B_1 + B_1)s + k_1 + k_2}$$

(5) $$\theta_2 = \frac{A_{\theta_2}}{A} = \frac{T_a\left(I_1 s^2 + (B_1+B_2)s + k_1 + k_2\right)}{(I_2 s^2 + B_2 s + k_2)(I_1 s^2 + (B_1+B_2)s + k_1 + k_2) - (B_2 s + k_2)^2}$$

de (5) se obtiene $\frac{\theta_2}{T_a}$

$k_1 = k \quad k_2 = k$

$I_1 = J \quad B_1 = B$

$I_2 = J \quad B_2 = B$

(6) $$\frac{\theta_2}{T_a} = \frac{I_1 s^2 + (B_1+B_2)s + k_1 + k_2}{(I_2 s^2 + B_2 s + k_2)(I_1 s^2 + (B_1+B_2)s + k_1 + k_2) - (B_2 s + k_2)^2}$$

simplificando el denominador

$$\frac{\theta_2}{T_a} = \frac{I_1 s^2 + (B_1+B_2)s + (k_1+k_2)}{J_1 J_2 s^4 + (B_1 J_2 + B_2 J_1 + B_2 J_2)s^3 + (B_1 B_2 + J_1 k_2 + J_2 k_1 + J_2 k_2)s^2 + (B_1 k_2 + B_2 k_1)s + k_1 k_2}$$

$a_1 = J_1 J_2$

$a_2 = B_1 J_2 + B_2 J_1 + B_2 J_2$

$a_3 = (B_1 B_2 + J_1 k_2 + J_2 k_1 + J_2 k_2)$

$a_4 = B_1 k_2 + B_2 k_1$

$a_5 = k_2 k_1$

$a_6 = B_1 + B_2$

$a_7 = k_1 + k_2$

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$$\frac{\theta_2}{\theta_a} = \frac{I_1 s^2 + a_6 s + a_7}{a_1 s^4 + a_2 s^3 + a_3 s^2 + a_4 s + a_5} \cdot \frac{x_1}{x_1}$$

$$\frac{\theta_2}{\tau_a} = \frac{I_1 s^2 x_1 + a_6 s x_1 + a_7 x_1}{a_1 s^4 x_1 + a_2 s^3 x_1 + a_3 s^2 x_1 + a_4 s x_1 + a_5 x_1}$$

$$\boxed{x_2 = s x_1} \rightarrow \frac{\theta_2}{\tau_a} = \frac{I_1 s x_2 + a_5 x_2 + a_7 x_1}{a_1 s^3 x_2 + a_2 s^2 x_2 + a_3 s x_2 + a_4 x_2 + a_5 x_1}$$

$$\boxed{x_3 = s x_2} \rightarrow \frac{\theta_2}{\tau_a} = \frac{I_1 x_3 + a_5 x_2 + a_7 x_1}{a_1 s^2 x_3 + a_2 s x_3 + a_3 x_3 + a_4 x_2 + a_5 x_1}$$

$$\boxed{x_4 = s x_3} \rightarrow \frac{\theta_2}{\tau_a} = \frac{I_1 x_3 + a_5 x_2 + a_7 x_1}{a_1 s x_4 + a_2 x_4 + a_3 x_3 + a_4 x_2 + a_5 x_1}$$

$$\boxed{\theta_2 = I_1 x_3 + a_5 x_2 + a_7 x_1}$$

$$a_1 s x_4 = a_2 x_4 + a_3 x_3 + a_4 x_2 + a_5 x_1 - \tau_a$$

$$\boxed{s x_4 = \frac{a_2}{a_1} x_4 + \frac{a_3}{a_1} x_3 + \frac{a_4}{a_1} x_2 + \frac{a_5}{a_1} x_1 - \tau_a}$$

- se aplica $\mathcal{L}^{-1}$ →

- $x_2 = \dot{x}_1$

$x_3 = \dot{x}_2$

$x_4 = \dot{x}_3$

$\theta_2 = I_1 x_3 + a_6 x_2 + a_7 x_1$

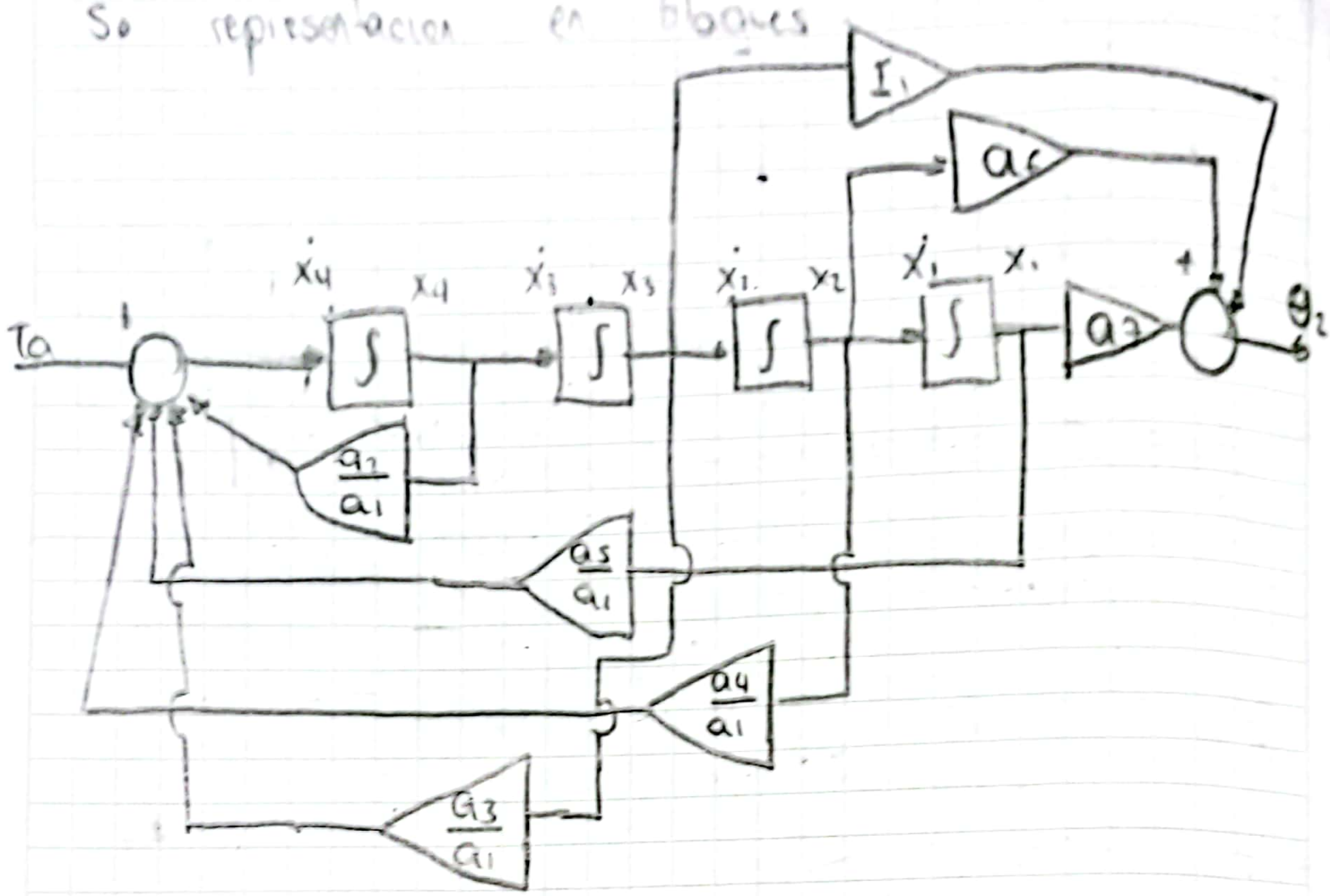
$\dot{x}_4 = \frac{a_2}{a_1} x_4 + \frac{a_3}{a_1} x_3 + \frac{a_4}{a_1} x_2 + \frac{a_5}{a_1} x_1 - T_a(t)$

Planteo la matriz

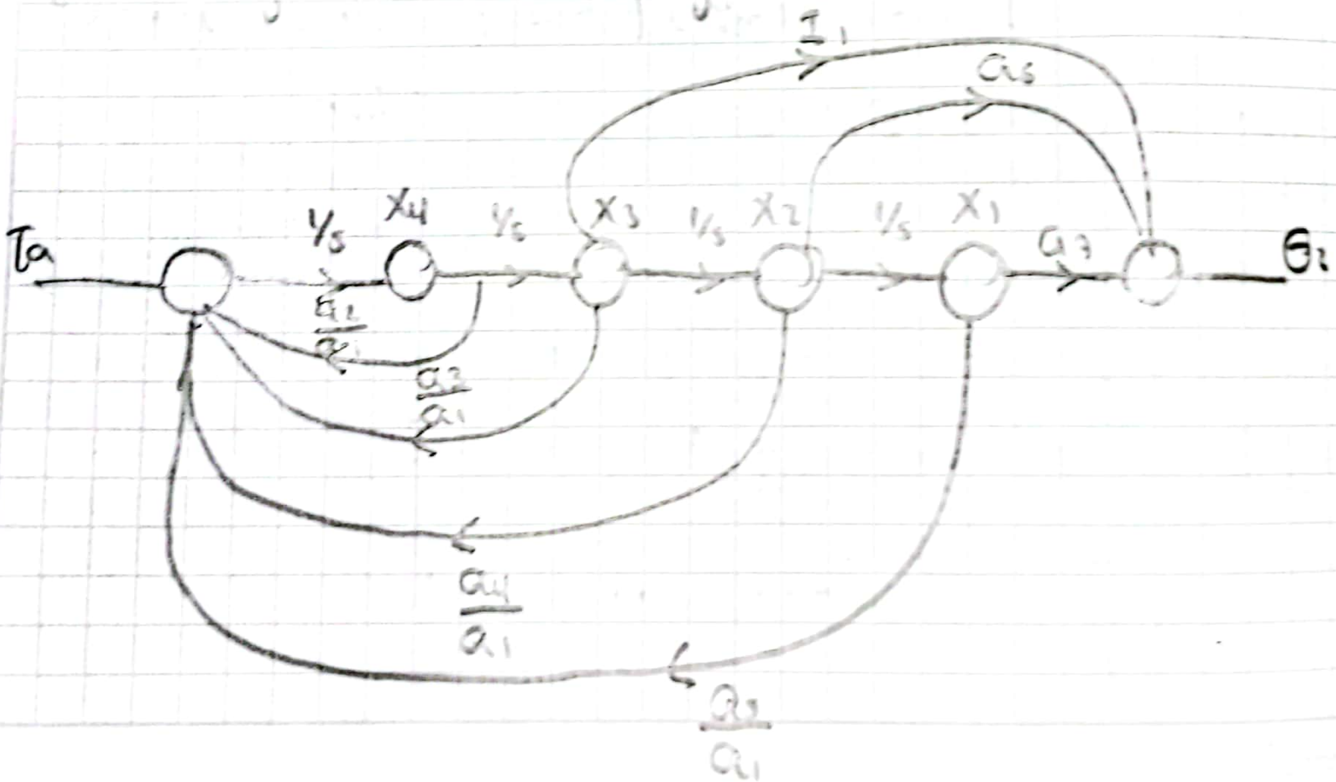
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{a_5}{a_1} & \frac{a_4}{a_1} & \frac{a_3}{a_1} & \frac{a_2}{a_1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} T_a$$

$$\theta_2 = \begin{bmatrix} a_7 & a_6 & I_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Su representación en bloques




El diagrama de flujo de señal



3) Para el sistema del item anterior, mismos requerimientos considerando $k_1 = 0$

Usando ① con $k_1 = 0$

$$\boxed{k_2\theta_2 + B_2\dot{\theta}_2 + (B_1+B_2)\dot{\theta}_1 - k_2\theta_1 - I_1\ddot{\theta}_1 = 0}$$

aplicando laplace a la ④ con $k_1 = 0$

$$[k_2 + sB_2]\,\theta_2(s) + [-I_1 s^2 - (B_1+B_2)s - k_2]\,\theta_1 = 0$$

Tomando $k_1 = 0$ tenemos lo siguiente en ⑥

$$\frac{\theta_2}{Ta} = \frac{I_1 s^2 + (B_1+B_2)s + k_2}{I_1 I_2 s^4 + (B_1 J_2 + B_2 J_1 + B_2 J_2)s^3 + (B_1 B_2 + I_1 k_2 + J_2 k_2)s^2 + B_1 k_2 s}$$

sustitucion

$C_1 = I_1$ $\quad C_2 = B_1 + B_2$ $\quad C_3 = k_2$

$I_1 I_2 = C_4$ $\quad B_1 J_2 + B_2 J_1 + B_2 J_2 = C_5$

$C_6 = B_1 B_2 + I_1 K_2 + J_2 k_2)$ $\quad C_7 = B_1 k_2$

$$\frac{\theta_2}{T_a} = \frac{C_1 s^2 + C_2 s + C_3}{C_4 s^4 + C_5 s^3 + C_6 s^2 + C_7 s} \cdot \frac{X_1(s)}{X_1(s)}$$

$$\rightarrow \frac{\theta_2}{T_a} = \frac{C_1 s^2 x_1 + C_2 s x_1 + C_3 x_1}{C_4 s^4 x_1 + C_5 s^3 x_1 + C_6 s^2 x_1 + C_7 s X_1} \rightarrow$$

$$\rightarrow \boxed{s x_1 = x_2} \rightarrow \frac{C_1 s x_2 + C_2 x_2 + C_3 x_1}{C_4 s^3 x_2 + C_5 s^2 x_2 + C_6 s x_2 + C_7 x_2} = \frac{\theta_2}{T_a}$$

$$\rightarrow \boxed{s x_2 = x_3} \rightarrow \frac{C_1 x_3 + C_2 x_2 + C_3 x_1}{C_4 s^2 x_3 + C_5 s x_3 + C_6 x_3 + C_7 x_2} = \frac{\theta_2}{T_a}$$

$$\rightarrow \boxed{s x_3 = x_4} \rightarrow \frac{C_1 x_3 + C_2 x_2 + C_3 x_1}{C_4 s x_4 + C_5 x_4 + C_6 x_3 + C_7 x_2} = \frac{\theta_2}{T_a}$$

$$\rightarrow \theta_2 = C_1 x_3 + C_2 x_2 + C_3 x_1$$

$$\boxed{s x_4 = \frac{C_5}{C_4} x_4 + \frac{C_6}{C_4} x_3 + \frac{C_7}{C_4} x_2 - T_a}$$

Aplicando laplace

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = x_3$$

$$\dot{x}_3 = x_4$$

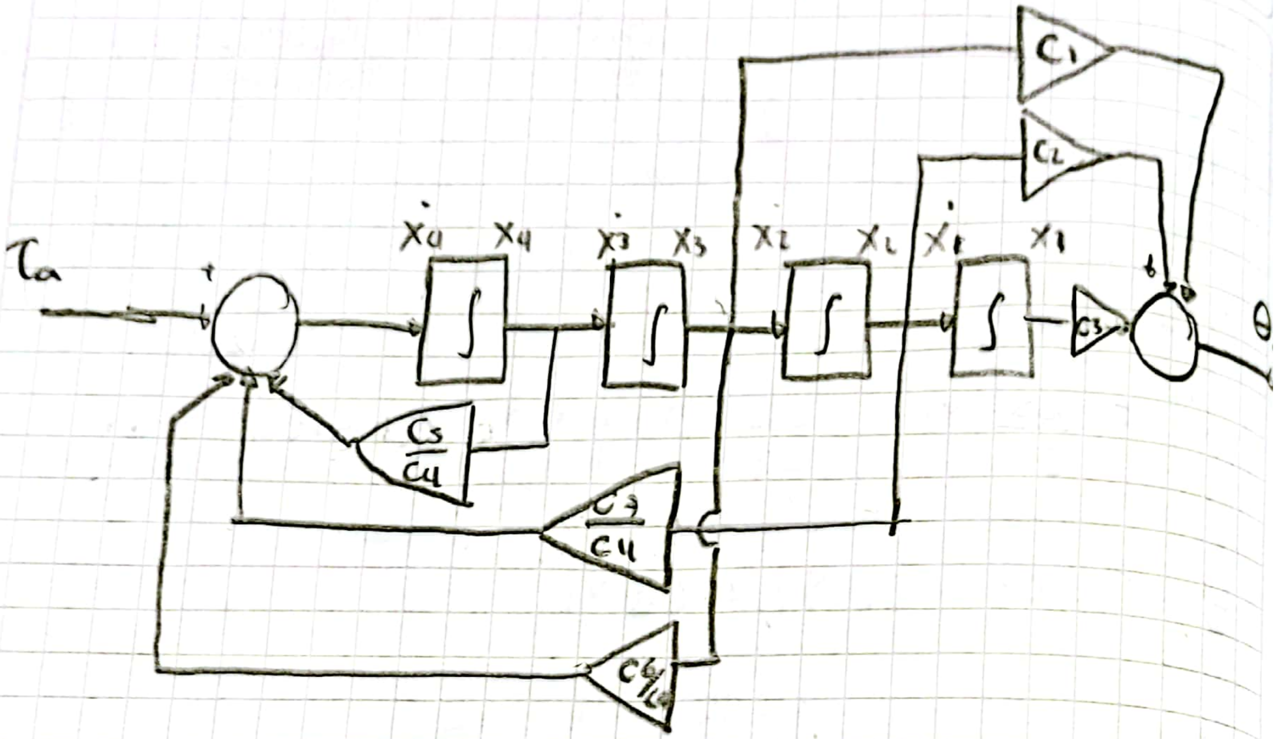
$$\dot{x}_4 = \frac{C_5}{C_4} x_4 + \frac{C_6}{C_4} x_3 + \frac{C_7}{C_4} x_2 - \tau_a$$

$$\theta_2 = C_1 x_3 + C_2 x_2 + C_3 x_1$$

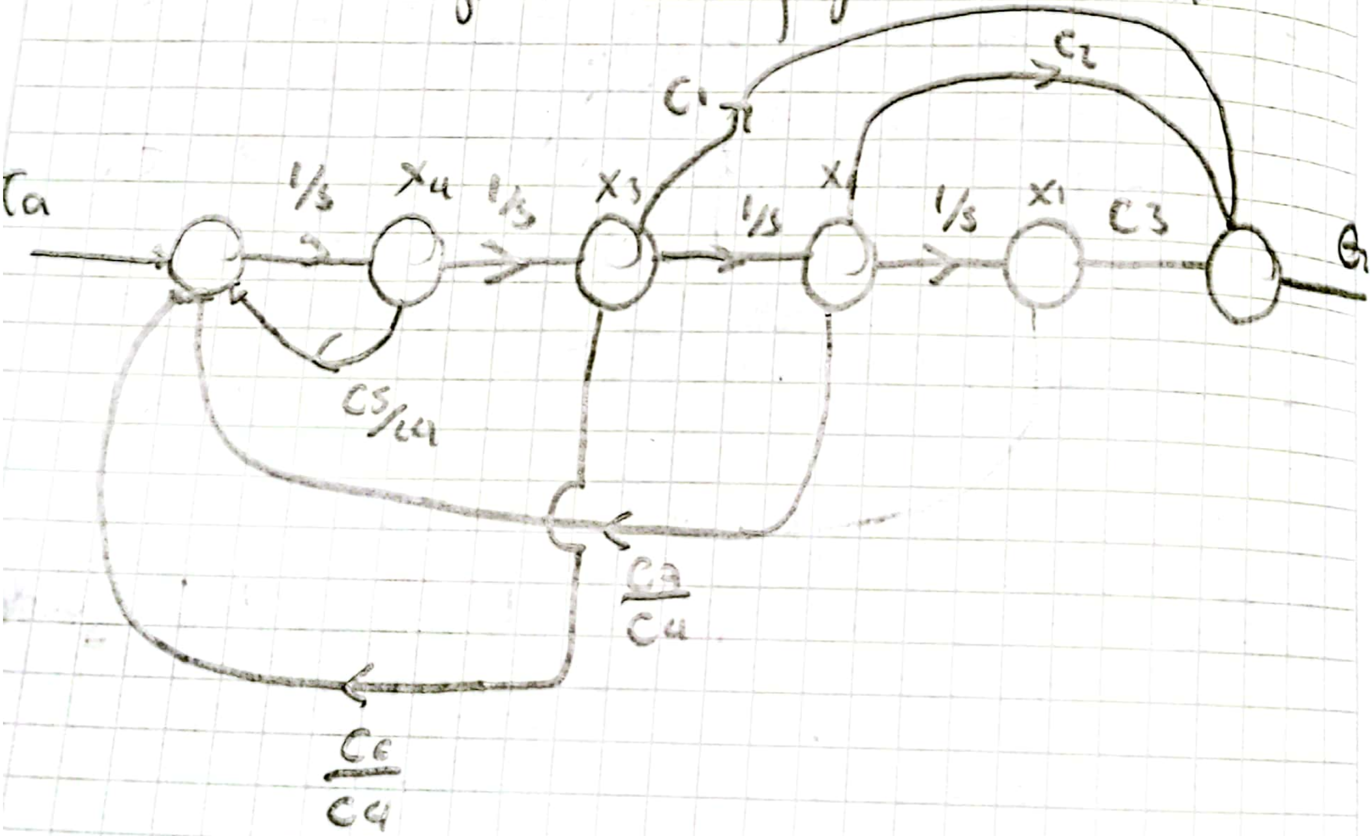
se plantea la matriz

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{C_7}{C_4} & \frac{C_6}{C_4} & \frac{C_5}{C_4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} \tau_a$$

$$\theta_2 = \begin{bmatrix} C_3 & C_2 & C_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$




Para el diagrama de flujo de señal

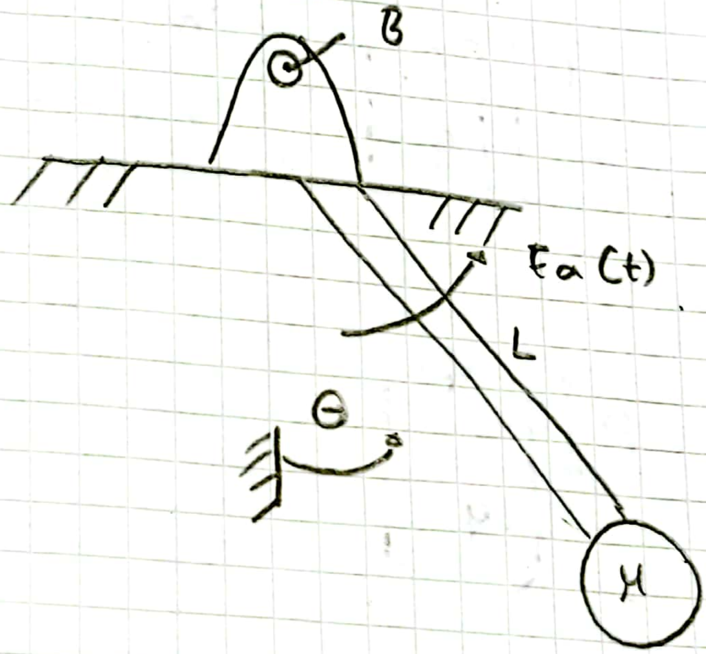


4) Para el sistema rotacional de la figura, determine:

a) La funcion de transferencia relacionando con $\theta$ y $\tau_a$

b) La representacion en el espacio de estado

c) junto a su diagrama de bloques y su diagrama de flujo de señal




Tomando como referencia la EO con aproximacion lineal $\sin\theta = \theta$ la EO queda

$$\ddot{\theta} + \frac{g}{l}\theta = \frac{\tau_a}{ml^2}$$

$$q_1 = \theta$$

$$q_2 = \dot{q}_1 = \dot{\theta}$$

$$\dot{q}_2 = \ddot{\theta}$$

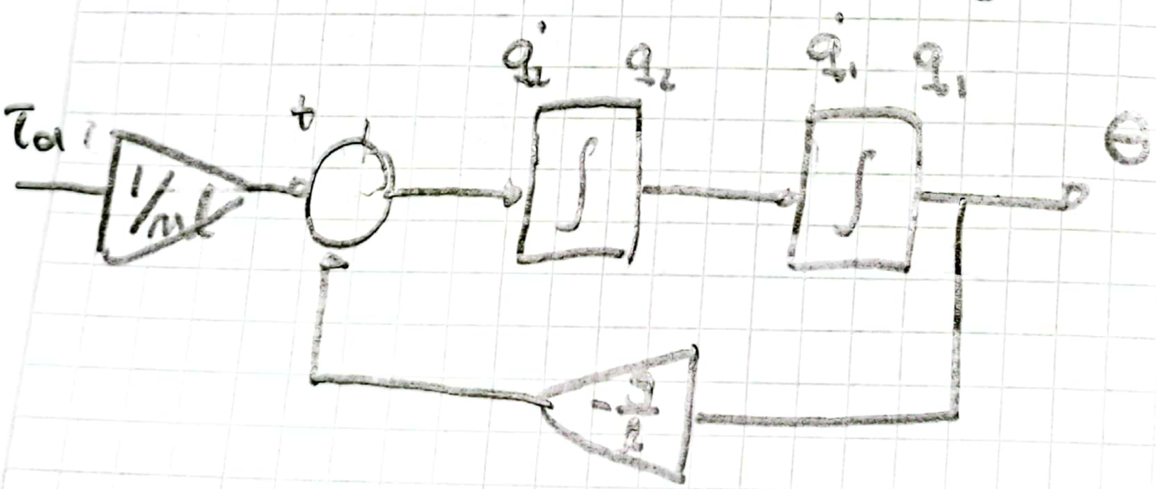
sustituyendo

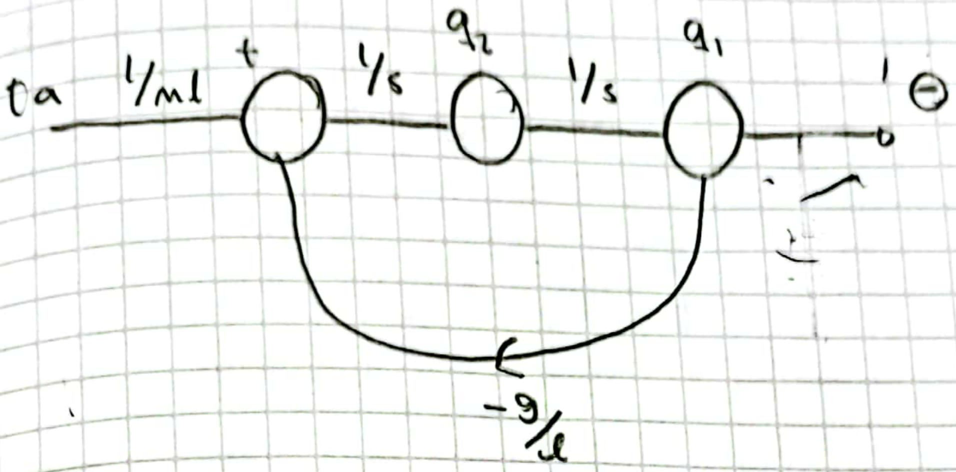
$$\dot{q}_2 = -\frac{g}{l} q_1 + \frac{\tau_a}{ml^2}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/l & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ml \end{bmatrix} \tau_a$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Para el diagrama de bloques






Para la funcion de transferencia

$$\frac{Ta}{ml^2} = \ddot{\Theta} + \frac{g}{l}\Theta \longrightarrow \mathcal{L}^{-1} \longrightarrow \frac{1}{ml^2} Ta(s) = \left[s^2 + \frac{s}{l}\right]\Theta(s)$$

$$\longrightarrow \frac{\Theta(s)}{Ta(s)} = \frac{1}{ml^2\left(s^2 + \frac{s}{l}\right)}$$

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