Optimal Monetary Policy Under Menu Costs

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Abstract

We analytically characterize optimal monetary policy in a multisector economy with menu costs and contrast it with the textbook Calvo model. Following a sectoral productivity shock, the textbook model prescribes stable inflation, providing a formal justification for inflation targeting. In contrast, under menu costs, policy should "look through" such shocks and allow inflation to move inversely with output. We provide sharp intuition: stabilizing inflation causes shocks to spill over across sectors, forcing firms to pay menu costs unnecessarily. In a quantitative model, a simple look-through policy – stabilizing nominal wages rather than inflation – improves welfare via lower menu costs.

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1 Introduction

Many central banks around the world have adopted some form of inflation targeting over the past three decades. The textbook formulation of the New Keynesian model provides theoretical grounding for such policies: in the Calvo formulation of the New Keynesian model, where firms are only randomly given the opportunity to change prices, optimal policy in response to efficient shocks is strict inflation targeting. This is true in the textbook one-sector New Keynesian model (Woodford 2003) as well as in heterogeneous multisector versions of the model, for an appropriately-defined price index (Rubbo 2023). The Calvo assumption of random price changes upon which these models are built is mathematically convenient, but arguably comes at the cost of realism. A natural but notoriously less tractable alternative is the "menu cost" model in which firms can *choose* to change their prices at any time but must pay a fixed menu cost to do so.

We analytically and without linearization characterize optimal monetary policy in a multisector economy with menu costs and show that optimal policy ensures that inflation and output move inversely after sectoral productivity shocks. That is, following negative productivity shocks, inflation should be allowed to rise, and vice versa. We show this by developing a multisector model where firms are subject to sector-specific productivity shocks and can change their price at any point by paying a menu cost. Under baseline assumptions, optimal policy in response to such shocks is precisely nominal wage targeting: nominal wages should be stabilized, but inflation should not be. This is despite wages themselves being completely flexible. More generally, the optimal policy response ensures that neither sectoral productivity shocks nor any monetary policy response spills over to affect the nominal marginal costs of unshocked firms.

Intuition. The key intuition is that stabilizing aggregate inflation causes shocks to spill over across sectors and therefore leads the economy to incur unnecessary menu costs. Consider, for example, a positive productivity shock affecting only firms in sector 1. If the shock is sufficiently large, then it is efficient and desirable for firms in sector 1 to cut their *relative* prices, compared to firms in other sectors of the economy. Under a policy of inflation targeting, the overall price level must be unchanged. To simultaneously have the relative price fall *and* the price level be stable requires not only that sector-1 firms cut their nominal prices, but also that firms in all other sectors raise their nominal prices. As a result, *all* sectors are forced to adjust their prices and pay a menu cost.

A natural alternative, which we show to be optimal, is instead to simply allow firms in sector 1 to cut their nominal prices and to ensure that firms in other sectors do not want to adjust their prices. As a result, relative prices are correct, and *only* sector-1 firms must pay a menu cost. Thus optimal policy economizes on wasteful menu costs compared to inflation targeting, while still achieving the efficient allocation. Optimal policy "looks through" the shock in the sense that *aggregate* inflation is allowed to adjust, instead of the central bank acting to ensure aggregate inflation is unaffected.

Firms only want to adjust their prices if their *nominal marginal costs* change, and so optimal policy seeks to ensure that nominal marginal costs do not change in unshocked sectors. In a baseline model where wages and productivity are the only factors affecting marginal cost, optimal policy stabilizes nominal wages, since this ensures marginal costs are unchanged for those firms whose productivity does not change. More generally, optimal policy causes inflation and output to move inversely: the positive productivity shock causes output to rise, and the price decrease in sector 1 causes aggregate inflation to fall. In contrast, in the Calvo version of this example, inflation targeting is optimal (Rubbo 2023; Woodford 2003; Aoki 2001; Benigno 2004).

Analytical model. We begin in sections 2 and 3 with an off-the-shelf, two-period, multisector menu cost model and analyze a one-sector shock, as described above, which is the minimum necessary machinery to highlight the core economic logic. The framework is general and can allow "menu costs" to capture a broad conception of *fixed* costs of price adjustment, whether they be physical costs (workers walking around a store replacing price tags), optimization costs (a business owner thinking about what price to set), or behavioral costs (consumers with a distaste for price changes).

We go on to show in sections 4 and 5 that the analytical logic generalizes to several extensions. We characterize optimal policy when shocks affect multiple sectors simultaneously and show how the same intuition generalizes. We also extend the model to allow for production networks or for other forms of heterogeneity, and we show that optimal policy continues to feature countercyclical inflation. Countercyclical inflation comes from the underlying microeconomic intuition that, in response to productivity or other "supply" shocks, prices and quantities should move in opposite directions.

Comparison to Calvo. Our results shed subtle new light on the standard Calvo model result: inflation targeting is optimal in the multisector Calvo model after sectoral shocks because *welfare losses from relative price dispersion are convex in the degree of dispersion*. This convexity of welfare costs makes it optimal to *smooth* price dispersion across sectors. It is better to have every sector change prices a little, rather than to have one sector change prices a lot. In contrast, the standard menu cost model has fixed, *nonconvex* costs of adjustment, so it is optimal to minimize the number of firms who do any price adjustment.

As a concrete example, consider again the case of a positive productivity shock affecting only sector 1. The relative price of sector 1 should fall. In the menu cost world with optimal monetary policy, sector-1 firms pay a menu cost and cut nominal prices, while other sectors leave prices unchanged: relative prices are correct and only one sector pays the menu cost. In the Calvo model under the same monetary policy, some firms within sector 1 cut prices, but the rest of sector-1 firms are exogenously prevented from cutting prices. Thus, there is relative price dispersion within sector 1. In other sectors, prices are unchanged, and there is zero within-sector dispersion.

But optimal monetary policy under Calvo is different: instead, the central bank optimally *loosens* monetary policy. The loosening causes firms in unshocked sectors to want to raise their prices. Because some firms in these sectors are exogenously prevented from adjusting prices, this creates relative price dispersion in these unshocked sectors, whereas previously there was zero dispersion. However, this new dispersion comes alongside a benefit: by actively loosening monetary policy, firms in the shocked sector 1 *do not want to cut their prices as deeply*, reducing the degree of dispersion within sector 1. Precisely because of the convexity of the welfare loss from price dispersion, this tradeoff is worthwhile. It is better to disrupt and create a little price dispersion in *every sector*, rather than to have a lot of price dispersion in one sector.

Quantitative model. We extend our results to a richer, dynamic model to study how the results generalize and to quantify the welfare loss of inflation targeting. This model is estimated to match US data, with particular attention to salient moments related to firms' price-setting behavior. In addition to the sectoral shocks that are present in the baseline model, the quantitative model features idiosyncratic, firm-level shocks. It also adopts the "CalvoPlus" framework of Nakamura and Steinsson (2010) in which occasionally firms get a chance to adjust prices for free.

We show that stabilizing nominal wages outperforms stabilizing inflation in this richer environment. Following a perfect foresight sectoral shock that is 10% on impact and decays over time, welfare is 0.08% higher under nominal wage targeting compared to inflation targeting, in consumption-equivalent terms. As the analytical intuition suggests, this improvement comes in part from a reduction in unnecessary menu cost expenditure.

These results are related to the fact that the empirical literature estimates menu costs to be fairly large. Based on directly-measured costs alone, the existing literature finds that between 0.6% and 1.2% of firm revenue is spent per year on costs related to price adjustment (Levy et al. 1997; Dutta et al. 1999; Zbaracki et al. 2004). We calibrate the quantitative model to the findings of this literature.

Position in literature. To our knowledge, we are the first to fully characterize optimal monetary policy in the face of fixed menu costs when firms have a motive to adjust relative prices. On the one hand, without changes in productivity between firms, there is no motive for relative-price changes and so optimal policy under menu costs is simply zero inflation: prices never need to move and price stickiness is irrelevant (Nakov and Thomas 2014). On the other hand, several papers allow for relative-price movements under menu costs but take as given that the central bank targets inflation, and simulate numerically how the presence of menu costs affects the optimal level of inflation (Blanco 2021, Nakov and Thomas 2014, Wolman 2011).[1]

Of most relevance, Adam and Weber (2023) study optimal monetary policy at steady state under menu costs with deterministic productivity trends, to a first-order approximation, which is complementary to our study of optimal policy in response to stochastic productivity shocks. Adam and Weber (2023) explicitly turn off consideration of minimizing resource costs (their Assumption 1), which our analytical results highlight as an important factor in optimal policy. Contemporaneously, Karadi et al. (2025) quantitatively study optimal monetary policy under menu costs in a single-sector environment with idiosyncratic shocks. In that environment, like in our model and as discussed above, optimal policy in response to a productivity shock that is *perfectly* uniform across all firms is zero inflation. Relative to their other quantitative analysis, our analytical results characterize and provide intuition for the specific economic forces driving optimal policy.

Our model formalizes and extends the insightful, literary argument made by Selgin (1997) (chapter 2, section 3) that nominal income targeting, or something like it, is optimal in a world with menu costs. Relative to Selgin's elegant informal discussion, we are able to introduce the role of state dependence, characterize precisely the nature of optimal policy, and quantify the welfare costs of inflation targeting.

A larger literature makes assumptions on monetary policy – i.e. does not analyze optimal policy – and asks how the presence of menu costs affects macroeconomic dynamics.[2] There is also a large empirical literature on menu costs.[3]

[1] Away from menu costs, other papers that analyze optimal monetary policy in a sectoral setting, i.e. a setting with relative price movements, besides those already cited include Huang and Liu (2005) and Cox et al. (2024), again under the Calvo friction; Kreamer (2022) as well as Erceg and Levin (2006), who study optimal monetary policy in sectoral models with fixed prices and durable goods; and Guerrieri et al. (2021), who study optimal monetary policy in a sectoral model with downward nominal wage rigidity.

[2] Among others: Caplin and Spulber (1987); Golosov and Lucas (2007); Gertler and Leahy (2008); Nakamura and Steinsson (2010), Midrigan (2011), Alvarez, Lippi and Paciello (2011), Auclert et al. (2023), Blanco et al. (2024*b*), Guerreiro et al. (2024), and Afrouzi et al. (2024).

[3] Among others:Alvarez et al. (2019); Nakamura et al. (2018); Cavallo (2018); Cavallo and Rigobon (2016); Klenow and Kryvtsov (2008); Gautier and Le Bihan (2022); Cavallo, Lippi and Miyahara (2024); Gagliardone et al. (2025).

The bigger picture. We see our paper as helping unify the literature on optimal monetary policy. In the last decade a number of papers across a variety of classes of models have found that optimal policy should cause inflation to be countercyclical after efficient shocks, not constant: the price level P should move inversely with real output Y. However, sticky price models – the workhorse model of modern macro – had conspicuously held out for the optimality of inflation targeting.

First, Koenig (2013) and Sheedy (2014) show in heterogeneous agent models that when financial markets are incomplete and debt is written in nominal, non-state contingent terms, then nominal income targeting is optimal and inflation targeting is suboptimal. That is, $P \times Y$ should be stabilized, and therefore P and Y should move inversely.[4] Second, Angeletos and La'O (2020) show that in a world where agents have incomplete information about the economy, optimal policy should again ensure the price level P and real output Y move inversely, in order to minimize monetary misperceptions.[5] Third, when wages are sticky due to a Calvo-type friction, optimal monetary policy is to stabilize nominal wages (Erceg, Henderson and Levin 2000), a policy which also results in countercyclical price inflation.

Despite these results in three highly important classes of models – incomplete markets, information frictions, and sticky wages – it may have been easy to set them aside and nonetheless consider inflation targeting as the proper baseline for optimal monetary policy due to its optimality in the workhorse sticky price model (e.g. Woodford 2003).[6] We hope our paper helps integrate these results from across the literature on incomplete markets, information friction, sticky wages, and sticky prices. Our results suggest that *countercyclical inflation*, not stable inflation, is a robustly-optimal policy prescription.

Outline. We illustrate the optimal policy result in sections 2 and 3 in a baseline setting: an off-the-shelf sectoral model with menu costs, hit by an unanticipated sectoral productivity shock. In section 4, we use our setup to shed new light on the conventional New Keynesian model. In section 5, we show how the analytical intuition generalizes. In section 6, we generalize further by building a rich quantitative model and quantify the welfare gains of nominal wage targeting in response to a sectoral productivity shock. Section 7 concludes with a brief discussion of practical implementation.

[4]These ideas have been developed further in Bullard and DiCecio (2019) and Bullard et al. (2023). Werning (2014) notes that if additional heterogeneity is added to the model, P and Y should move inversely but not one-for-one, echoing our results.

[5]The nominal contracts and incomplete information literatures also were preceded and discussed by Selgin (1997).

[6]In the textbook sticky price model, countercyclical inflation can be optimal if there is a binding zero lower bound constraint on the nominal interest rate (Eggertsson and Woodford 2003; Werning 2011; Woodford 2012).

2 Baseline model

Our baseline framework is a two-period model starting at steady state. There are S sectors, each consisting of a continuum of monopolistically competitive intermediate firms which are aggregated into a sectoral good by a competitive sectoral packager. A competitive final goods producer combines the output of each of the S sectors into a final good, sold to the household. The model and the functional forms we use are the same as Golosov and Lucas (2007), except that productivity shocks are sectoral rather than firm-specific and we analyze optimal monetary policy instead of exogenous monetary shocks. In section 5, we generalize the functional forms.

2.1 Household

The representative household's utility function is given by

$$\mathbb{W} = \ln C - N + \ln \left(\frac{M}{P} \right) \tag{1}$$

Utility is a function of consumption C, labor N, and real money holdings $\frac{M}{P}$.[7] The household chooses C, N and M to maximize its utility, subject to its budget constraint:

$$PC + M = WN + D + M_{-1} - T$$

To fund expenditures, the household uses labor income from wages W, firm dividends D, and previous period money balances M_{-1}, less taxes T. The first order conditions imply:

$$PC = M \tag{2}$$

$$W = M \tag{3}$$

Our Golosov-Lucas assumption on preferences results in two simple optimality conditions: an equation of exchange (2) and an equation (3) stating that in equilibrium the nominal wage W is directly determined by the money supply M.

2.2 Final good producer

The representative final good producer aggregates sectoral goods y_i of price p_i across S sectors, using Cobb-Douglas technology, into the final good Y consumed by the house-

[7] Following Woodford (1998), we ignore the welfare effects of real balances when analyzing optimal monetary policy.

hold. Operating under perfect competition, its problem is:

$$\max_{\{y_i\}_{i=1}^{S}} PY - \sum_{i=1}^{S} p_i y_i$$
$$\text{s.t. } Y = \prod_{i=1}^{S} y_i^{1/S} \tag{4}$$

The resulting demand for sectoral goods is:

$$y_i = \frac{1}{S}\frac{PY}{p_i} \tag{5}$$

The zero profit condition gives the price P for the final good:

$$P = S\prod_{i=1}^{S} p_i^{1/S} \tag{6}$$

In section 5.1 we discuss how generalizing the Cobb-Douglas functional form used here has no impact on the optimal policy result.

2.3 Sectoral goods producers

In sector i, a representative sectoral goods producer packages the continuum of intermediate goods, $y_i(j)$, produced within the sector using CES technology. Note that for notational clarity, we will consistently use j to identify an intermediate firm and i to identify a sector. The problem of the sectoral packager for sector i is:

$$\max_{[y_i(j)]_{j=0}^{1}} p_i y_i - \int_0^1 p_i(j) y_i(j) dj$$
$$\text{s.t. } y_i = \left[\int_0^1 y_i(j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{7}$$

This results in a demand function $y_i(j)$ and a sectoral price index p_i:

$$y_i(j) = y_i\left(\frac{p_i(j)}{p_i}\right)^{-\eta} \tag{8}$$

$$p_i = \left[\int_0^1 p_i(j)^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{9}$$

2.4 Intermediate goods producers

In each sector there is a unit mass of monopolistically competitive firms, each producing a different variety of the sectoral good. Their technology is linear, and all firms within a sector i share a common productivity level A_i.[8] The linearity of technology significantly simplifies the exposition and is important for generating the optimality of nominal wage targeting; we generalize this in section 5.

Intermediate firms are subject to menu costs: if they choose to adjust their price, they must hire an extra ψ units of labor at the wage rate W. This fixed cost of price adjustment, $W\psi$, is what we refer to as a "menu cost". The menu cost itself is simply a transfer from firm profits to household labor income; the *welfare cost* of menu costs comes from the fact that households must supply extra labor in order for prices to be adjusted, and there is a disutility cost associated to this additional labor. This is motivated by the idea of firms needing to employ workers for extra hours to physically walk around and update price stickers in a store, but more generally can be thought of as a modeling device to stand in for *any* fixed costs of price adjustment. For example, if menu costs are optimization costs, $W\psi$ represents the opportunity cost of the labor time spent thinking about what the optimal price adjustment should be. Modeling menu costs in other ways does not affect the optimal policy conclusions.[9]

Firm j in sector i thus maximizes profits, including the menu cost if choosing to adjust its price, subject to its demand curve and its production technology, taking as given the inherited price from the previous period, $p_i^{\text{old}}(j)$:

$$\begin{aligned}
&\max_{p_i(j)} p_i(j)y_i(j) - Wn_i(j)(1-\tau) - W\psi\chi_i(j) \\
&\text{s.t. } y_i(j) = y_i\left(\frac{p_i(j)}{p_i}\right)^{-\eta} \\
&\qquad y_i(j) = A_i n_i(j)
\end{aligned} \tag{10}$$

The objective function defines firm profits, $D_i(j)$. The variable $\chi_i(j) \in \{0,1\}$ is a dummy indicating whether or not the firm chooses to adjust its price, $p_i(j)$. If it does, it incurs the menu cost $W\psi$. Otherwise, the price remains at the level inherited from the previous period, denoted $p_i^{\text{old}}(j)$. The term τ in the firm's problem is the standard labor subsidy provided by the fiscal authority to undo the markup distortion from monopolistic competition, $\tau = \frac{1}{\eta}$, for each unit of labor used in production, $n_i(j)$.

If the firm chooses to pay the menu cost and adjust its price, then, from the firm's first

[8]It is typical in the analytical multisector literature to study the optimal policy response to *sector*-level productivity shocks (e.g. Rubbo (2023), Aoki (2001), or Benigno (2004)) and to abstract away from idiosyncratic shocks. We analyze the case of both sectoral and idiosyncratic shocks quantitatively in section 6.

[9] One alternative is menu costs as burning real resources (as in Alvarez, Lippi and Paciello 2011), and thus lowering the level of profits transferred to households. Another more behavioral alternative is menu costs as a direct utility penalty to households. It is straightforward to show that optimal policy is the same if menu costs are modeled in either of these ways, as we show in the behavioral case in appendix B.

order condition, the optimal reset price equals the nominal marginal cost:

$$MC_i(j) = \frac{W}{A_i} \tag{11}$$

Notice that, because productivity is sector-specific, all firms j within a sector i face the same decision problem, and thus all make the same decision on whether to adjust and choose the same reset price. Because of this equivalence, we will often refer interchangeably to firm-specific versus sector-specific prices and quantities, e.g. $p_i(j)$ versus p_i and $n_i(j)$ versus $n_i \equiv \int_0^1 n_i(j)dj$.

2.5 The intermediate firm's adjustment decision

We now turn to the question of whether a given intermediate firm will pay the menu cost to adjust its price. The firm makes its decision to adjust by comparing profits under the new optimal price $\frac{W}{A_i}$ net of the menu cost $W\psi$, versus profits under the inherited price p_i^{old} without the loss from menu costs. Plugging in the respective prices as well as constraints into the profit function, we arrive at the price-adjustment condition: firm j in sector i will adjust if and only if

$$\left(\frac{W}{A_i}\right)^{1-\eta} p_i^{\eta} y_i \left[\frac{1}{\eta}\right] - W\psi > \left(p_i^{\text{old}}(j)\right)^{1-\eta} p_i^{\eta} y_i \left[1 - \frac{W/A_i}{p_i^{\text{old}}(j)} \cdot \frac{\eta-1}{\eta}\right] \tag{12}$$

The condition implies an inaction region Λ, a standard result in menu cost models.

Lemma 1 (Inaction region)**.** There exists an inaction region Λ in (W, A_i) space such that a firm in sector i will not adjust its price if and only if the value of (W, A_i) remains within this inaction region, i.e. $(W, A_i) \in \Lambda$. The larger the menu cost ψ, the larger is this inaction region. The locus of points that result in the new optimal price equaling the inherited price, $\{(W, A_i) | \frac{W}{A_i} = p_i^{\text{old}}\}$, always lies within the inaction region. The inaction region is a connected set.

Proof: See Appendix A.1. □

To interpret this, note that the desired reset price W/A_i depends on two factors: (1) sectoral productivity A_i, which is exogenous; (2) nominal wages W, which from (3) is completely determined by the central bank, $W = M$. Thus, firms are more likely to adjust after either a large productivity shock or a large monetary action, all else equal.

2.6 Market clearing

Labor market clearing implies that total labor supplied by the household, N, equals labor demanded in production, $\sum_i n_i$, plus labor required to adjust prices, $\psi \sum_i \chi_i$:

$$N = \sum_{i=1}^{S} n_i + \psi \sum_{i=1}^{S} \chi_i \tag{13}$$

This market clearing condition is key to the welfare costs of menu costs. Since labor supply N enters the household utility function negatively, larger menu costs ψ requiring the household to work more to adjust prices will lower household welfare.

The remaining equilibrium conditions are standard. The government budget constraint is: $T + (M - M_{-1}) = \tau W \sum_{i=1}^{S} n_i$. Finally, the aggregate resource constraint implies that consumption equals aggregate output, $C = Y$.

2.7 Steady state

The economy begins in a symmetric, flexible-price steady state (steady state variables are denoted with a superscript ss) in which sectoral productivities A_i^{ss} for $i \in \{1, \ldots, S\}$ are taken as given and the money supply is normalized to $M^{ss} = 1$. Without loss of generality, we can set $A_i^{ss} = 1$ for all i.

Nominal wages from (3) are then $W^{ss} = 1$. Firms set prices at their flexible levels (11), $p_i^{ss} = 1$. The aggregate price level (6) is $P^{ss} = S$. From money demand (2), consumption and therefore output are equal to aggregate productivity, $C^{ss} = Y^{ss} = M^{ss}/P^{ss} = 1/S$. From demand equations (8) and (5), sectoral output is $y_i^{ss} = \frac{1}{S}$. From intermediate production technology (10) we recover labor in sector i as $n_i^{ss} = \frac{1}{S}$ and aggregate labor from market clearing (13) as $N^{ss} = 1$.

3 Optimal policy after a sectoral productivity shock

As our baseline exercise, we consider the optimal response to an unexpected shock to sector 1 alone. For concreteness, consider a positive productivity shock, which we denote as $\gamma > A_1^{ss} = 1$. How should monetary policy optimally set the money supply M?

Because in the initial steady state all sectors have the same productivity normalized to one, firms in all unshocked sectors $i > 1$ face precisely the same problem after the shock to sector 1 and make the same decision on whether and how to adjust. As a result, for our purposes in this section there are effectively two sectors of different sizes, sector 1 (with productivity $A_1 = \gamma > 1$ and size 1) and sectors k (with productivity $A_k = 1$ and size

$S-1$). Section 5.4 discusses how this generalizes to shocking multiple sectors. We will consistently identify variables for these unshocked sectors with a k. The relative price between the shocked and unshocked sectors, p_1/p_k, will be a key object of analysis.

Proposition 1, our benchmark result, characterizes optimal monetary policy in response to this shock.

Proposition 1 (Optimal monetary policy)**.** For a fixed level of menu costs ψ, there exists a threshold level of productivity $\overline{\gamma} > 1$, such that:

1. If the productivity shock to sector 1 is above the threshold, $\gamma \geq \overline{\gamma}$, then optimal policy is exactly nominal wage targeting: monetary policy should ensure $W = W^{ss}$. This results in firms in sector 1 adjusting their prices, while firms in other sectors k leave prices unchanged. This is implemented by leaving the money supply unchanged, $M = M^{ss}$.

2. If the shock is below the threshold, $\gamma \in [1, \overline{\gamma})$, then optimal policy is to ensure that prices remain unchanged and no firm in any sector adjusts.

Additionally, the productivity threshold $\overline{\gamma}$ is increasing in the size of menu costs ψ.

Proof: Lemma 2 and lemma 3 below directly imply the proposition. □

Before proving the proposition, we review the economic intuition, which was previewed in the introduction. For a sufficiently large productivity shock $\gamma \geq \overline{\gamma}$, it is efficient for the relative price of sector 1, p_1/p_k, to update. To achieve this while simultaneously minimizing the number of sectors which must incur a menu cost, it is only necessary that firms in sector 1 update their price p_1 – firms in other sectors do not need to update p_k. To ensure that firms in other sectors have no desire to update, the central bank wants to stabilize the level of nominal wages, W, so that the nominal marginal cost of firms in these other sectors is unchanged and these firms have no motive to adjust their prices. On the other hand, for a small productivity shock $\gamma \in [1, \overline{\gamma})$, the benefit of updating the relative price p_1/p_k does not outweigh the welfare loss from the menu cost necessary to do so. It is therefore optimal to ensure that prices remain unchanged across all sectors.

We next step through the math behind this intuition in more detail. We build up to lemma 2 and lemma 3, which together prove proposition 1.

3.1 Allocations in four possible regimes

In this subsection, we characterize the four possibilities for equilibrium that monetary policy can implement. In the next subsection, we will compare welfare across them.

From symmetry, there are two types of firms (those in sector 1 hit with productivity shock γ, and those in other sectors k with unchanged productivity) and each type has a binary choice (adjusting or not adjusting its price), and therefore there are 2×2 possibilities for what may occur in equilibrium:

1. Both sector 1 and sectors k adjust prices
2. Only sector 1 adjusts its price; sectors k do not adjust
3. Only sectors k adjust their prices; sector 1 does not adjust
4. Neither sector 1 nor sectors k adjust

The central bank determines which of these regimes occurs in equilibrium by manipulating the money supply, M. Whether a firm in some sector i decides to adjust its price depends solely on whether its target price, $\frac{W}{A_i}$, is outside its inaction region Λ. Because the central bank can move nominal wages W by its choice of money supply M, it controls which equilibrium is implemented. (Note that there is always a unique equilibrium for a given choice of M – this is not a choice of *equilibrium* selection, but a choice by monetary policy of how much to increase aggregate demand.)

The optimal policy problem thus consists of:

1. Considering each of these regimes individually, and choosing M to maximize welfare *conditional* on the given regime;
2. Then, choosing the regime among the four which has the highest welfare, and implementing the associated optimal M.

This optimal policy problem, formalized in (43) in appendix A.2, is necessarily piecewise due to the sharp discontinuities created by the discontinuous pricing rules of lemma 1, themselves the result of the fixed menu costs.

We now consider each of these four regimes individually, after discussing the flexible price benchmark. The ensuing subsection compares across the four.

Flexible price benchmark. As a benchmark, first consider the flexible price allocation, where the menu cost $\psi = 0$. Nominal wages are determined by the money supply, $W = M$, so that from (11) the flexibly-adjusted prices are $p_1 = \frac{M}{\gamma}$ and $p_k = M$. Observe that under flexibility, the relative price across types $\frac{p_1}{p_k}$ is:

$$\left(\frac{p_1}{p_k}\right)_{\text{flex}} = \frac{1}{\gamma}$$

This is an important object. This flexible relative price results in aggregate output and consumption equal to $Y = C = \frac{\gamma^{1/S}}{S}$. Total labor is $N = 1$. Plugging these quantities into the household utility function (1), we have a first-best, efficient benchmark for welfare to which policy should be compared:

$$\mathbb{W}_{\text{flex}} = \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1 \tag{14}$$

All sectors adjust. Next, return to the world with nonzero menu costs and consider the case where all sectors pay the menu cost to adjust. Because all firms adjust to the flexible levels of $p_1 = \frac{M}{\gamma}$ and $p_k = M$, the relative price achieves the flexible price level:

$$\left(\frac{p_1}{p_k}\right)_{\text{all adjust}} = \frac{1}{\gamma} = \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

However, despite prices adjusting, the equilibrium differs from the flexible-price equilibrium because additional labor is required to pay the menu costs of price adjustment. This is where the assumptions on preferences plays a useful simplifying role: the fact that the Golosov-Lucas preferences are quasilinear in labor ensure that all income effects affect labor supply. As a result, the additional labor required for menu costs has no effect on the equilibrium *except* to increase the amount of labor used.[10] That is, prices and quantities are the same as the flexible price equilibrium, *except* for the additional labor which must be hired to pay for the menu costs: $N = 1 + S\psi$, where $S\psi$ reflects that there are S sectors which must hire ψ units of labor each to adjust prices.

Thus, conditional on all sectors adjusting, welfare is independent of monetary policy and is equal to the flexible-price level minus the S sectors' worth of menu costs expended:

$$\mathbb{W}_{\text{all adjust}} = \ln\left(\frac{\gamma^{1/S}}{S}\right) - [1 + S\psi] \tag{15}$$

$$= \mathbb{W}_{\text{flex}} - S\psi \tag{16}$$

Only sector 1 adjusts. Next consider if only sector 1 updates to $p_1 = \frac{M}{\gamma}$ and sectors k leave their prices unchanged at the steady state level of $p_k = 1$. This results in aggregate output of $Y = \frac{\gamma^{1/S}}{S} M^{\frac{S-1}{S}}$. The total level of labor is $N = \left[\frac{1}{S} + (S-1)\frac{M}{S}\right] + \psi$, reflecting one sector's worth of menu costs ψ, since only sector 1 is adjusting. Thus, household

[10]Without quasilinear Golosov-Lucas preferences suppressing income effects on consumption, the labor required for menu costs affects the marginal rate of substitution between consumption and leisure. Under optimal policy, production therefore differs from the flexible-price level. Alternatively, if menu costs were a utility penalty affecting the household directly, then the flexible-price allocation is replicated exactly even with more general preferences that display income effects, as modeled in appendix B.

welfare is a function of the money supply decision:

$$\mathbb{W}_{\text{only 1 adjusts}}(M) = \ln\left(\frac{\gamma^{\frac{1}{S}}}{S} M^{\frac{S-1}{S}}\right) - \left[\frac{1}{S} + (S-1)\frac{M}{S} + \psi\right]$$

Conditional on being in this regime, optimal monetary policy chooses M to maximize this expression, which can be found from the first order condition to be:

$$M^*_{\text{only 1 adjusts}} = 1$$

where asterisks denote objects under optimal policy.[11] The optimal money supply in this case is left unchanged at the steady state level, $M^{ss} = 1$. Importantly, this ensures that the relative price across sectors, $\frac{p_1}{p_k} = \frac{M}{\gamma}$, equals the flex-price level:

$$\left(\frac{p_1}{p_k}\right)^*_{\text{only 1 adjusts}} = \frac{1}{\gamma} = \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

Why does this policy result in the efficient relative price? Setting $M = 1$ ensures nominal wages are $W = 1$, since $M = W$ from (3), which means that nominal wages are unchanged from steady state $W^{ss} = 1$. As a result, the optimal reset price for unshocked sectors, $\frac{W}{A_k} = 1$, *coincides with* the inherited price, $p_k^{ss} = 1$, and the optimal pricing is achieved without a need to adjust.

Thus, optimal monetary policy is able to replicate the flexible-price allocation by ensuring that all prices are at the correct level despite sectors k not adjusting, aside from the extra labor required for menu costs. As a result, welfare under optimal policy is equal to the flexible-price level, minus one sector's worth of menu costs from sector 1 adjusting:

$$\begin{aligned}\mathbb{W}^*_{\text{only 1 adjusts}} &= \ln\left(\frac{\gamma^{1/S}}{S}\right) - [1+\psi] \\ &= \mathbb{W}_{\text{flex}} - \psi \qquad (17)\end{aligned}$$

Only sectors k adjust. If only sectors $k = 2, ..., S$ adjust, the logic is similar to the prior case, except $S-1$ sectors adjust, instead of only one sector adjusting. The flexible-price allocation is again achievable aside from the extra labor required to pay for menu costs, this time by ensuring that the desired price in sector 1 equals the inherited price. This is implemented by the central bank increasing the money supply, inflating nominal wages to the point where firms in sector 1 have no desire to adjust, $W = \gamma$, and causing firms in other sectors to have a motive to adjust. The optimized level of welfare is thus the

[11] In subsection 3.3, we discuss the implementability of this choice of money supply.

flexible-price level minus $S-1$ sectors' worth of menu costs expended:

$$\mathbb{W}^*_{\text{only } k \text{ adjust}} = \mathbb{W}_{\text{flex}} - (S-1)\psi \tag{18}$$

No sector adjusts. Finally consider the possibility that no firm in any sector adjusts. Sectoral prices are thus unchanged from steady state, $p_i = p_i^{ss} = 1 \ \ \forall i$, and consequently so is the aggregate price level, $P = P^{ss} = S$. Within this regime, this is as if all prices were fully rigid: aggregate output is determined by monetary policy, $Y = C = \frac{M}{S}$. Total labor is $N = \frac{1}{\gamma}\frac{M}{S} + (S-1)\frac{M}{S}$, noting no labor is required for menu costs because no prices are adjusted. Household welfare as a function of the chosen level of the money supply M is:

$$\mathbb{W}_{\text{none adjust}}(M) = \ln\left(\frac{M}{S}\right) - \left[\frac{1}{\gamma}\frac{M}{S} + (S-1)\frac{M}{S}\right]$$

Conditional on being in this regime, optimal monetary policy chooses M to maximize this expression, which can be found from the first order condition to be $M^*_{\text{none adjust}} = \left[\frac{1}{\gamma}\frac{1}{S} + \frac{S-1}{S}\right]^{-1}$. Under this, the optimized level of welfare is:

$$\mathbb{W}^*_{\text{none adjust}} = -\ln\left(S - 1 + \frac{1}{\gamma}\right) - 1 \tag{19}$$

To understand this, note that the relative price $\frac{p_1}{p_k}$ is stuck at the steady state value of 1 instead of being updated to the flexible price value of $\frac{1}{\gamma}$:

$$\left(\frac{p_1}{p_k}\right)_{\text{none adjust}} = 1 \neq \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

It is because this relative price is stuck at a distorted level that monetary policy is unable to achieve the flexible-price allocation.

3.2 Comparing across regimes

In this subsection, we compare welfare across the four possible regimes just derived. We can immediately observe that only two of the four are worth considering for optimal policy.

Lemma 2 (If adjusting, only the shocked sector should adjust). Welfare when only sector 1 adjusts, $\mathbb{W}^*_{\text{only 1 adjusts}}$, is strictly higher than welfare when all sectors adjust, $\mathbb{W}^*_{\text{all adjust}}$, and welfare when only sectors k adjust, $\mathbb{W}^*_{\text{only } k \text{ adjust}}$.

Proof: This follows immediately from comparing (17) with (16) and (18). □

Lemma 2 follows from the idea that it is better to have fewer firms incur menu costs, together with the fact that optimal policy can implement the efficient relative price $\left(\frac{p_1}{p_k}\right)_{\text{flex}}$ by having *either* sector 1 only adjust, *or* sectors k only adjust, *or* all sectors adjust. Thus, if any firms at all are going to adjust, it is best to have sector-1 firms only adjust.

What remains is to compare welfare if "only sector 1 adjusts" versus if "none adjust". The next lemma compares these two.

Lemma 3 (Only adjust prices beyond a threshold). There is a threshold $\overline{\gamma}$ such that $\mathbb{W}^*_{\text{only 1 adjusts}}$ dominates $\mathbb{W}^*_{\text{none adjust}}$ if and only if the productivity shock exceeds the threshold, $\gamma \geq \overline{\gamma}$. Furthermore, the threshold $\overline{\gamma}$ is increasing in the menu cost ψ.

Proof: Define $f(\gamma) \equiv \mathbb{W}^*_{\text{none adjust}} - \mathbb{W}^*_{\text{only 1 adjusts}}$. Observe that if $\gamma = 1$, then $f(\gamma) = \psi > 0$. Additionally, as $\gamma \to \infty$, then $f(\gamma) \to -\infty$. Finally, f is strictly monotonically decreasing in γ, with $f'(\gamma) = \frac{1}{\gamma}\left[\frac{1}{\gamma(S-1)+1} - \frac{1}{S}\right] < 0$. Since f is continuous in γ, by the intermediate value theorem there exists a $\overline{\gamma} > 1$ such that $f(\overline{\gamma}) = 0$. To see that $\overline{\gamma}$ is increasing in ψ, observe that increasing ψ shifts the entire $f(\gamma)$ curve up, i.e. $\frac{\partial f}{\partial \psi} > 0$. □

Lemma 3 says that there is a threshold level $\overline{\gamma}$ for the productivity shock. Below this threshold, household welfare is maximized by ensuring that no firm in any sector adjusts; above this threshold, it is maximized by ensuring that sector-1 firms adjust. The intuition for this, as emphasized, is that the welfare loss from menu costs is fixed in size. For a sufficiently small improvement in productivity, the benefit to adjusting prices does not outweigh the fixed welfare loss from the menu cost that is required to adjust. It is only worthwhile to pay this fixed cost above the threshold. The proof follows this same logic.

Additionally, the productivity threshold $\overline{\gamma}$ is increasing in the size of the menu cost ψ. The intuition for this is that for a larger menu cost, the productivity shock must be bigger for it to be worthwhile to adjust.

In the case where none adjust, the level of nominal wages is $W_{\text{none adjust}} = M^*_{\text{none adjust}} = \left[\frac{1}{\gamma}\frac{1}{S} + \frac{S-1}{S}\right]^{-1}$. Observe that for $\gamma = 1$, nominal wages are exactly unchanged from the steady state level of $W^{ss} = 1$. For small shocks, $1 < \gamma < \overline{\gamma}$, nominal wages are also approximately unchanged.

Denote welfare under optimal policy as $\mathbb{W}^*$, where lemma 2 and lemma 3 together imply $\mathbb{W}^* = \max\left\{\mathbb{W}^*_{\text{only 1 adjusts}}, \mathbb{W}^*_{\text{neither adjust}}\right\}$. Lemma 2 and lemma 3 together also prove proposition 1.

3.3 Adjustment externalities

When discussing the regime where only sector 1 adjusts, we derived equilibrium household welfare as a function of the money supply choice, $\mathbb{W}_{\text{only 1 adjusts}}(M)$, by *assum-*

ing that only firms in sector 1 adjusted prices. We then found the optimal $M^*_{\text{only 1 adjusts}}$ by simply taking the first order condition of this function.

More precisely, however, a central bank would choose the money supply M that maximizes welfare $\mathbb{W}_{\text{only 1 adjusts}}(M)$ *subject to the implementability constraint* that such a choice of M *induces* sector 1 to adjust and other sectors k not to adjust. We term this as "constrained" optimal. The choice of M would need to be incentive-compatible with the assumption on who is adjusting price. The same is true for the case where none adjust: the choice of optimal M must not push any firm outside its inaction region. (The same is true of the case where only sectors k adjust, though this is less important because of lemma 2.) These constraints can be written formally as in equations (36)-(38) in appendix A.2.

In proposition 1 and throughout the body of this paper, we have endowed the social planner with the power to force firms to adjust prices – or equivalently, to subsidize price adjustment – so that these implementability constraints are always nonbinding. This is written out explicitly in (40)-(42) in appendix A.2. In appendix B, however, we show that if the planner does not have this instrument, then it is possible for these implementability constraints to bind.

We term the case where the unconstrained-optimal choice of M is not feasible as "adjustment externalities", and discuss these in detail in appendix B. It may be the case that it is socially optimal for firms in sector 1 to adjust their prices, but it may not be privately optimal to adjust: prices are "too sticky", and there is a positive externality to price adjustment. It is also possible, however, that it is socially optimal for firms in either sector 1 or in sectors k to leave their price unchanged, but it is privately optimal to adjust: prices are "too flexible", and there is a negative externality to price adjustment. The nature of the externality depends on the size of the shock and the size of menu costs, as detailed in appendix B.

This issue does not arise in the Calvo literature since there is no cost of adjusting prices (when adjustment is possible). Therefore, these adjustment externalities do not arise. As a result, there is limited precedent in the literature, with a handful of important exceptions. Ball and Romer (1989*a*) find that menu costs create negative externalities after a *monetary policy shock*. Our setting instead studies whether *efficient* (productivity) shocks create externalities when monetary policy is set optimally and finds the possibility of not just negative externalities but also the possibility of positive adjustment externalities. Other related studies include Ball (1987) on negative externalities in the length of labor contracts; Ball and Romer (1989*b*) on externalities in the timing of staggered price setting; and Ball and Romer (1991) on the possibility of menu cost-induced multiple equilibria. All of these papers study economies where monetary policy is not set optimally; our results show that, *even* when monetary policy is set optimally, adjustment externalities may arise.

Finally, Angeletos and La'O (2020) study optimal monetary policy under information frictions, and in the case of endogenous information acquisition studied in their online appendix A, they find that there are no externalities to information acquisition in price-setting as long as both technology has a constant elasticity of substitution and monetary policy is set optimally.[12] We find the possibility of externalities even under CES technology and optimal monetary policy.

3.4 Discussion: "Menu costs" in the model can be interpreted broadly

The term "menu costs" originates with the *physical* resource costs of updating posted prices: restaurants needing to print new menus, or retailers needing to pay workers to replace price stickers on their shelves. These costs are sizable and underappreciated; we review the literature in section 4.2, where direct measurement shows these to be between 0.6% and 1.2% of firm revenues in key industries of the economy. However, menu costs can be conceptualized more broadly than physical resource costs through the lens of our model. Consider four possible sources of menu costs:

1. **Physical adjustment costs.** The baseline interpretation of menu costs are physical costs, such as retail firms needing to pay to print new price stickers and to employ workers in updating these stickers on store shelves.

2. **Optimization costs.** "Menu costs" may represent fixed costs of *optimization* or *information processing*, such as time thinking about what price to set (Morales-Jiménez and Stevens 2024). These costs also require additional labor and thus operate through the same mechanism as in the model above.

3. **Behavioral costs.** "Menu costs" can be interpreted more behaviorally. Consumers may have an intrinsic *preference* for stable prices, and changing prices has a direct psychological cost on consumers. In appendix B, we model menu costs as directly impinging on household welfare. Under thsi interpretation, optimal policy and the core intuition are unchanged.[13]

4. **Zero-sum backlash.** Optimal policy would change if "menu costs" are zero sum, for

[12] Gorodnichenko (2008) studies a model with both menu costs and information frictions and numerically studies an information externality that results from their interaction. Caplin and Leahy (2010) also speculate informally about such a phenomenon; see also Caplin and Leahy (1994).

[13] If these psychological costs are asymmetric, as the field experiment of Anderson and Simester (2010) suggests, then optimal policy would be altered and could include stabilizing the prices of *shocked* firms.

example as a reduced-form representation of a behavioral *backlash* to price changes, in which consumers shift their purchases from one firm to another after a price change. In this case, there need not be *aggregate* consequences of "menu costs", since one firm's loss is another firm's gain. If this were the *only* source of menu costs, this interpretation leads to the unlikely conclusion that optimal policy is to hyperinflate every period, ensuring that firms reset prices and relative prices are correct.

The general principle is that any fixed cost of price adjustment which is not zero sum operates through the same logic as proposition 1. In reality, "menu costs" likely are a combination of both zero-sum and non-zero-sum costs. To the extent that menu costs at all have a non-zero-sum component, the logic of proposition 1 goes through.

4 Comparison with the Calvo model

In this section, we discuss the welfare loss from inflation targeting, review the literature measuring the magnitude of menu costs, and contrast our optimal policy results under menu costs to those of the canonical, Calvo-based New Keynesian model.[14]

4.1 The welfare loss of inflation targeting under menu costs

The standard Calvo-based New Keynesian model implies that a policy of zero inflation is optimal after any efficient shock. Inflation targeting in response to a sectoral shock requires all firms to adjust: e.g. in our setting, shocked firms in sector 1 must lower their prices, and unshocked firms in other sectors must raise their prices, so that the aggregate price level is unchanged.[15] However, we showed above that, under menu costs, forcing all sectors to adjust prices results in unnecessary price adjustment after a sectoral shock.

Thus, the welfare loss of inflation targeting relative to optimal policy is directly captured by the welfare loss caused by the unnecessary menu costs paid by the $S-1$ unshocked sectors, that is, $(S-1)\psi$. The size of menu costs ψ together with the number of unshocked sectors $S-1$ are sufficient statistics for the welfare gains of transitioning from inflation targeting to nominal wage targeting. Proposition 3 in appendix C states and proves this formally.

[14] Thsroughout the paper, we refer to "Calvo" sticky pricing (Calvo 1983). A fuller accounting would refer to the "Calvo-Yun assumption", in reference to the important work of Yun (1996).

[15] To implement inflation targeting where $P = P^{ss} = 1$, the central bank can: (1) force *all firms* to adjust and set M to ensure that the increase in price in sectors k to $p_k = M$ exactly offsets the fall in price in sector 1 to $p_1 = M/\gamma$, or (2) ensure *no firm* in any sector adjusts. We only refer to the case in which all firms adjust as inflation targeting because, while the policy ensuring no firm adjusts also leads to no change in the price level, it would also mean that relative prices never change, shutting down the price system.

4.2 Empirical estimates of the size of menu costs are sizeable

Given that the welfare loss from inflation targeting is proportional to the magnitude of menu costs, it is important to measure this quantity empirically.

First, a priori, the literature on menu costs often builds on the idea that 'second-order menu costs can result in first-order output fluctuations' (Mankiw 1985), in which case the welfare loss of inflation targeting compared to optimal policy would be second-order. However, it is important to note that in the textbook New Keynesian model with the exogenous Calvo friction, the welfare loss from price stickiness is *also* only second-order.

Additionally, estimates of the real resource cost of menu costs from the empirical literature are in fact sizable: at least 0.5% of total firm revenues annually. These estimates come from two sources: calibrated models and direct measurement.[16]

Calibrated models. One method for estimating the size of menu costs is to build a menu cost model and calibrate the magnitude of menu costs to match the frequency of price adjustment and other salient moments from the microdata. A number of papers perform this exercise, such as Nakamura and Steinsson (2010), who estimate the size of menu costs to be around 0.5% of revenue per year (their Table II).[17]

Direct measurement. A more direct and model-free approach to measuring menu costs is to measure them directly. However, because measuring all forms of menu costs – physical adjustment costs, optimization costs, psychological costs – is difficult, the existing measurement literature largely focuses on physical adjustment costs alone. These numbers therefore can be interpreted as a lower bound on the total size of "menu costs" for the relevant firms.

To our knowledge, only three papers directly measure menu costs. Levy et al. (1997) measures the physical costs of price adjustment for five large grocery store chains across the US. They directly measure the time spent by workers manually changing price stickers on grocery store shelves, using a stopwatch. Such time maps directly the menu cost parameter ψ in our model. They find such menu costs to be 0.7% of firm revenue on average. Dutta et al. (1999) use a similar approach to examine a large drugstore chain, with a narrower conception of menu costs, and find menu costs to be 0.6% of firm revenue. Finally, Zbaracki et al. (2004) examine an industrial manufacturer and, using a broader conception of menu costs, find such costs to be 1.2% of firm revenue.[18]

[16] Guerreiro et al. (2024) offers a third methodology: asking via surveys. Their survey estimates that menu costs *on wages* are 1.75% of wages.

[17] Blanco et al. (2024*b*) find that a baseline menu cost model predicts 8.3% of firm revenues are paid as menu costs, and argue that a richer multiproduct menu cost model is needed to fit the data.

[18] Any measurement of the level of expenditure on menu costs is endogenous to the existing monetary

In short, the literature has found that menu costs, even when only examining their most measurable manifestation, are no small matter.

4.3 Nonconvex menu costs vs. convex (Rotemberg) menu costs

Why does optimal policy differ in the menu cost world compared to the canonical New Keynesian model based on the Calvo friction? To answer this question, it is helpful to first compare our menu cost model to the Rotemberg model of quadratic menu costs.

Throughout this paper, we have used a model of "nonconvex menu costs": the cost of price adjustment is *fixed* and does not scale with the size of a price change (Barro 1972; Sheshinki and Weiss 1977). That is, the menu cost facing a firm is a *nonconvex* function of the size of its price change. Contrast this model with models of *convex* menu costs, where the cost of price adjustment depends on the size of the price change in a convex way.

In the canonical model of convex menu costs, the Rotemberg (1982) model of quadratic menu costs, the menu cost scales with the square of the size of the price change:

$$\psi \cdot (p_i - p_i^{ss})^2$$

In contrast, in the model we present above, the menu cost is constant as a function of the size of the price change: where $\mathbb{I}$ is the indicator function,

$$\psi \cdot \mathbb{I}\{p_i \neq p_i^{ss}\}$$

It is well known that the single-sector Rotemberg convex menu cost model is isomorphic in its structural equations, to a first-order approximation, to the New Keynesian model built on the Calvo time-dependent friction.[19] Furthermore, the two models are isomorphic to a second-order approximation in their optimal policy implications (Nisticò 2007), i.e. inflation targeting not nominal wage targeting is optimal in both.

Why do Rotemberg *convex* menu costs imply inflation targeting is optimal, while our *nonconvex* menu costs imply nominal wage targeting is optimal?

The difference comes directly from the convex nature of the Rotemberg menu costs: due precisely to the convexity, it is better to have all sectors adjust prices a little than to have one sector do all of the adjustment. With the nonconvex menu costs of our model, it is instead optimal to minimize the number of sectors which choose to adjust at all.

The key intuition is in the labor market clearing condition. The labor market clearing

policy regime, as proposition 1 emphasizes.

[19]The *mechanism* of the Rotemberg model is very different from that of Calvo. Under Calvo, the welfare loss results from relative price dispersion, and total factor productivity is effectively lower. Under Rotemberg and in our model the loss comes from the real resource cost of menu costs.

condition in the multisector Rotemberg model is:

$$N = \sum_{i=1}^{S} n_i + \psi \sum_{i=1}^{S} (p_i - p_i^{ss})^2 \tag{20}$$

This contrasts with the labor market clearing condition under nonconvex menu costs, our equation (13): $N = \sum_{i=1}^{S} n_i + \psi \sum_{i=1}^{S} \mathbb{I}\{p_i \neq p_i^{ss}\}$.

Under both Rotemberg and nonconvex menu costs, it is desirable to minimize the amount of menu costs because of the disutility of labor they create. Due to the *convex* nature of the Rotemberg menu costs in (20), it is better to *smooth* the price changes over all sectors: it is better to have a small price change in every sector, rather than a large price change in one sector. Under nonconvex menu costs, it is instead better to minimize the *number* of sectors which experience any price change. It is this difference – convex versus nonconvex costs – which explains the differing optimal policy prescriptions.[20]

4.4 Reexamining optimal policy in the Calvo model

Finally, we come to why optimal policy in the menu cost setting differs from optimal policy in the textbook Calvo model (Woodford 2003; Rubbo 2023).

Consider the analogy to the Rotemberg model. As discussed in the prior subsection, it is known that optimal policy under the Rotemberg and Calvo frictions is the same: inflation targeting. We also explained that the Rotemberg model and the fixed menu cost model have differing optimal policy implications because Rotemberg assumes a *convex* menu cost, implying that price changes should be spread over many sectors rather than concentrated in one sector.

Similarly, in the Calvo model, *the welfare cost of price dispersion is convex*. The intuition can be seen in the standard definition of price dispersion:

$$\Delta \equiv \sum_{i=1}^{S} \int_0^1 \left[\frac{p_i(j)}{p_i} \right]^{-\eta} dj \tag{21}$$

Here, $\eta > 1$ continues to be the within-sector elasticity of substitution. Nominal prices $p_i(j)$ are now heterogeneous within a sector thanks to the Calvo friction.

The convexity of welfare costs of price dispersion can be seen mathematically from the fact that $\eta > 0$. It is better to have many goods with slightly distorted prices, rather than to have few goods with highly distorted prices.[21]

[20] Blanco et al. (2024*a*) develop a model where each monopolistically competitive firm produces a continuum of products, chooses the fraction of products for which prices are updated each period, and faces a menu cost that is quadratic in this fraction of prices which are updated.

[21] In the two-period, symmetric multisector Calvo model studied here, the welfare loss function has three

Illustration. This discussion is illustrated in figure 1. The figure depicts the level of sectoral prices p_i in a three-sector Calvo model where production technology is constant returns to scale and sectors have symmetric parameters. Marginal cost is, as in (11), W/A_i.

In subfigure 1a, the economy is at steady state, where by assumption all sectoral productivities and prices are equal ($W^{ss} = A_i^{ss} = p_i^{ss} = 1$), as in section 2.7. We then consider an increase in productivity in sector 1 to $A_1 = \gamma > 1$. Subfigure 1b shows what would happen under flexible prices: firms in sector 1 would cut prices, while firms in other sectors remain unchanged. That is, $p_1 = 1/\gamma < 1$ and $p_k = 1$.[22]

Subfigure 1c shows what would happen to sectoral prices in the Calvo sticky-price world under nominal wage targeting. Firms in sector 1 want to cut their prices to the flexible-price level. However, only some fraction of firms in that sector may do so, thanks to the Calvo friction. Other firms remain stuck at the steady state price, and have the wrong price. This creates within-sector price dispersion – the blue area in sector 1 is not uniformly the same height – as well as incorrect relative prices between the unchanged sector-1 firms and firms in other sectors. However, there is no within-sector price dispersion in other sectors: the green area in sector 2 has a uniform height, as does the purple area in sector 3. Firms in unshocked sectors are thus not affected by the shock.

Subfigure 1d shows what happens to sectoral prices in the Calvo world under inflation targeting, which is the optimal policy in this setting. Now, monetary policy seeks to ensure that on average prices are unchanged. This requires the central bank to induce an increase in nominal wages. The result is that firms in sector 1 want to cut their prices, but not as much as under stable nominal wages; and firms in other sectors want to increase their prices. Because of the Calvo friction, only a fraction of firms in each sector is able to set the optimal price. As a result, there is within-sector dispersion in *all* sectors: every sector contains firms with differing prices.

As described above, the benefit of disturbing sectors 2 and 3 under inflation targeting is that *the sector-1 price dispersion is lessened*. That is, the gap in heights between the two blue bars is lessened compared to nominal wage targeting, as indicated by the red line. Because of the convexity of the welfare cost of this gap, the welfare benefit from this decrease outweighs the incorrect prices induced in other sectors.

In short, the central bank under Calvo does not "look through" the shock. Instead, it chooses to actively distort prices in unshocked sectors in order to reduce distortions in

terms: (1) within-sector misallocation; (2) cross-sector misallocation; (3) the output gap. The within-sector misallocation reflects the convex costs of (21). The cross-sector misallocation is invariant to monetary policy. The output gap is proportional to the within-sector misallocation. Thus, the overall welfare loss function that a central bank seeks to minimize is proportional to (21).

[22] These flexible-price nominal formulas hold up to an unimportant scaling factor.

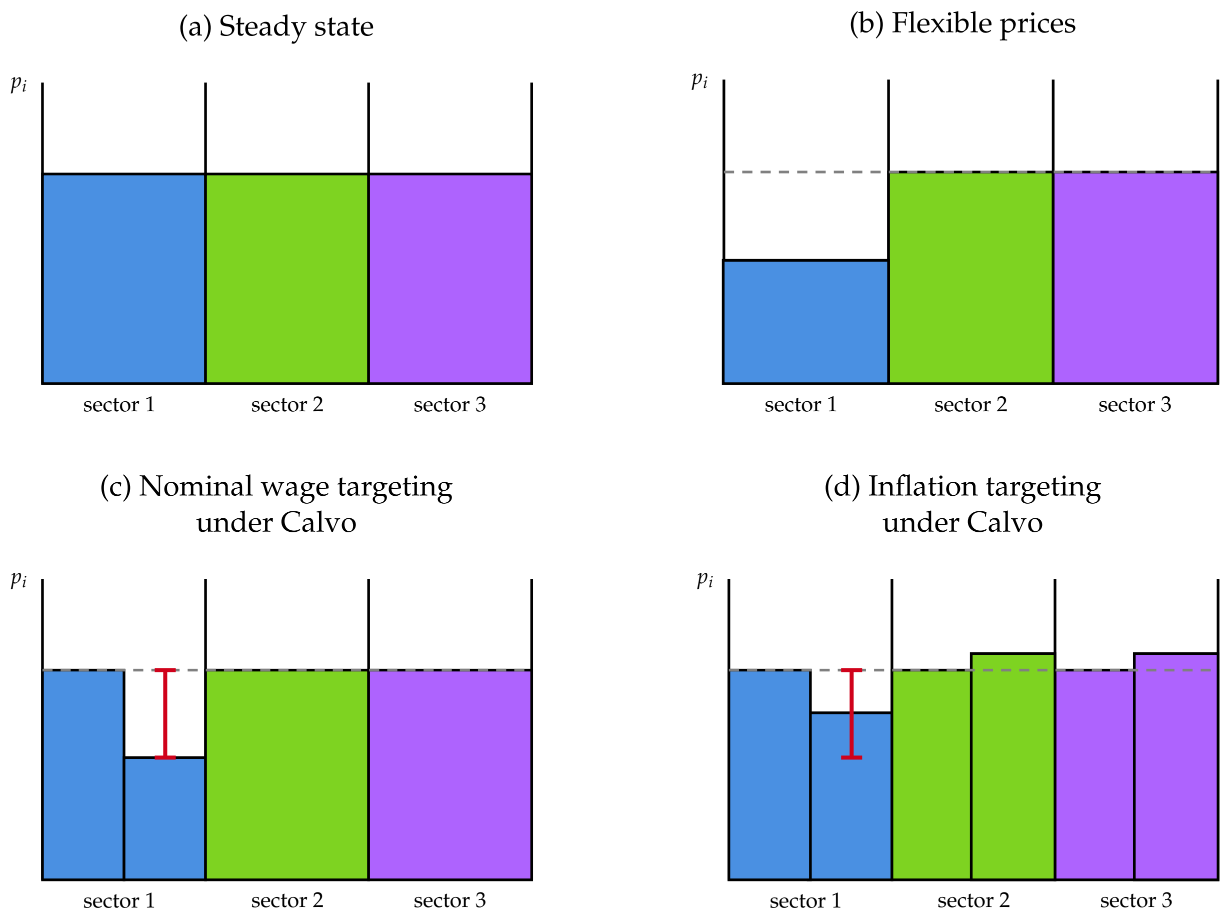

Figure 1: A stylized depiction of sectoral prices under Calvo in steady state (a), under flexible prices (b), under nominal wage targeting (c), and under inflation targeting (d). Nominal wage targeting (c) ensures that the shock to sector 1 does not spill over across sectors. Inflation targeting (d) minimizes total price dispersion by inducing unshocked sectors 2 and 3 to raise prices and thus is optimal.

the shocked sector. The key to this logic is that the welfare loss is convex in the degree of distortion.

Future work. This suggests an important target for empirical work: *how convex are the costs of price changes* as a function of the size of the change? To our knowledge, this question has received no quantitative attention in the empirical literature cited in section 4.2.[23] Additionally, future work could consider coordination frictions that dampen the ability of monetary policy to smooth shocks across sectors. The exogenous nature of the Calvo friction means that firms in unshocked sectors respond symmetrically to a movement in nominal wages induced by monetary policy as do firms in the shocked sector in response

[23] Zbaracki et al. (2004) contains an informal discussion.

to the shock. More realistically, firms may more easily adjust prices following a productivity shock to their own sector, since they are more likely to be aware of it. This would be natural if productivity changes are in fact *endogenous*, rather than an exogenous shock; or in models of rational inattention where sectoral shocks typically receive more attention, endogenously, than aggregate shocks (Maćkowiak, Matějka and Wiederholt 2023).

5 Extensions to the benchmark model

We now consider several natural extensions to the analytical model and demonstrate how the baseline intuition generalizes.

5.1 Functional forms

Consider a generalized version of the baseline model in section 2 allowing for the following functional forms: (i) any returns to scale production technology for final goods, with (4) becoming $Y = F(y_1, ..., y_S)$ where F is homogenous of degree 1; (ii) decreasing returns to scale in production, with (10) becoming $y_i(j) = A_i n_i(j)^{1/\alpha}$ with $1/\alpha \in (0, 1]$; (iii) preferences quasilinear in labor, with (1) becoming $\mathbb{W} = U\left(C, \frac{M}{P}\right) - N$. [24]

Equilibrium nominal marginal costs are:

$$MC_i(j) = \left[\alpha \frac{W}{A_i^{\alpha}} \left(y_i p_i^{\eta}\right)^{\alpha-1}\right]^{1/(1-\eta(1-\alpha))} \tag{22}$$

Observe that if α were equal to one, then $MC_i(j) = W/A_i$, as in equation (11) of the baseline model. In this general model, the marginal cost depends not just on wages and productivity, but also on demand, due to decreasing returns to scale.

As a result, optimal policy in this general model does not exactly stabilize nominal wages, but instead stabilizes nominal marginal costs (22) for unshocked sectors. Proposition 4 in appendix C states this formally. The intuition remains the same: ensuring that only the shocked sector adjusts minimizes the menu costs incurred in the economy.

As an example, consider isoelastic utility $U(C, M/P) = \frac{1}{1-\gamma}[C^{1-\gamma} + (M/P)^{1-\gamma}]$, Cobb-Douglas aggregation as in (4), and decreasing returns to scale production. Then equilib-

[24] In the generalized model, the functional form for utility remains quasilinear in labor, as with Golosov-Lucas preferences. As discussed in section 3.1 and footnote 10, this is necessary to ensure that the income effects induced by menu costs do not distort the consumption-leisure margin. In the model of appendix B, where menu costs are modeled as a utility penalty and therefore do not reduce household income, preferences need not be quasilinear in labor and can be fully general. Alternatively, consider if menu costs are modeled as a loss of profits not paid out through the labor market (as in e.g. Alvarez et al. 2019). Then, we would want to ensure there is no income effect on *labor supply* rather than no income effect on consumption. In other words, preferences would have to be of the Greenwood, Hercowitz and Huffman (1988) form.

rium nominal marginal costs in sector i are given by a weighted average of wages and prices: $MC_i(j) = k\frac{W^\zeta P^{1-\zeta}}{A_i}$, where k and ζ are constants related to technology and preferences. Following a shock to sector-1 productivity, stabilizing the nominal marginal costs of unshocked sectors means stabilizing a weighted average of nominal wages and prices, $W^\zeta P^{1-\zeta}$. This policy still implements countercyclical inflation, due to the same underlying microeconomic intuition discussed earlier.

5.2 Production networks

We can also generalize the model to allow for a production network, where intermediate firms use not just labor as a factor of production, but also use other goods as an input. We consider the symmetric roundabout economy of Basu (1995), where the production technology of firm j in sector i is:

$$y_i(j) = A_i n_i(j)^\beta I_i(j)^{1-\beta} \tag{23}$$
$$I_i(j) = \prod_{k=1}^{S} I_{ki}(j)^{1/S}$$

Here, $I_i(j)$ is the composite intermediate good used by firm j in sector i. It has weight $1-\beta$ in production compared to β for labor; setting $\beta = 1$ returns us to the baseline model. The composite intermediate good is a bundle of outputs from every other sector: $I_{ki}(j)$ is output from sector k purchased by the firm j in sector i.

With input-output linkages, the marginal cost of a given firm now depends not just on wages but also on the price of every other good in the economy. Nominal marginal costs in this economy are a weighted average of nominal wages and the aggregate price index:

$$MC_i(j) = k\frac{W^\beta P^{1-\beta}}{A_i} \tag{24}$$

where $k = \frac{1}{(1-\alpha)^{1-\alpha}\alpha^\alpha}$. The marginal cost has a weight β on wages, reflecting labor's share in production; and a weight $1-\beta$ on the price level, reflecting an intermediate share of $1-\beta$ and that the average price of intermediates is precisely the price index.

In response to the baseline shock affecting only sector 1, optimal policy thus is to stabilize this weighted average of wages and prices, with the weights determined by intermediate share β. Proposition 5 in appendix C states this formally.

5.3 Sectoral heterogeneity and the "least-cost avoider" principle

We now consider two kinds of heterogeneity, in sector size and in menu cost magnitude, that lead us to a "least-cost avoider" interpretation of optimal monetary policy.

Suppose first that sectors differ in size, with sector i having mass S_i (rather than a unit mass). Heterogeneity in sector size only alters the model through labor market clearing (13), where larger sectors require hiring more labor to adjust prices: $N = \sum_i n_i + \psi \sum_i S_i \chi_i$.

Optimal policy is unchanged as long as sector 1 is not larger than all other sectors combined, $S_1 < \sum_{k>1} S_k$. In the case where sector 1 is larger than every other sector, economizing on menu costs in response to a sector-1 shock is best done by having the *unshocked* sectors adjust, and inflation would be *pro*cyclical. Although this is a perverse case, it illustrates the logic of minimizing menu cost expenditure.

Introducing heterogeneity in the size of menu costs by sector is similar. If the menu cost of sector i is ψ_i, labor market clearing becomes: $N = \sum_i n_i + \sum_i S_i \psi_i \chi_i$. Heterogeneity in menu costs has two effects. First, analogous to the previous perverse possibility, if the size-weighted menu cost of sector 1 is larger than that of all other sectors combined, $S_1 \psi_1 > \sum_{k>1} S_k \psi_k$, then it is again optimal to have firms outside sector 1 adjust rather than those in sector 1. Second, heterogeneity in ψ_i does affect the value of the threshold $\overline{\gamma}$ from proposition 1, i.e. when it is optimal to allow prices to go unchanged.

A monetary "least-cost avoider" principle. These results on sectoral heterogeneity are formalized in proposition 6 in appendix C and can be interpreted as a "least-cost avoider" theory of optimal monetary policy. In the economic analysis of law, the least-cost avoider principle states that when assigning liability between parties, it is efficient to assign liability to the party who has the lowest cost of avoiding harm (Calabresi 1970). Similarly, the generalized principle of optimal monetary policy under menu costs is: *the agents for whom it is least costly to adjust their price are the agents who should adjust*.

In the monetary economics literature, this is also strongly related to the idea that "monetary policy should target the stickiest price" (e.g. Mankiw and Reis 2003; Aoki 2001; Eusepi, Hobijn and Tambalotti 2011). Under menu costs, the central bank should minimize adjustment by the firms with the most expensive menu costs – i.e. it should stabilize the stickiest prices. Importantly however, under menu costs the stickiest price is in part *endogenous* to the shock hitting the economy.

5.4 Multiple shocks

In this section, we consider the case where an arbitrary set of sectors is shocked, including possibly every sector. Start again at steady state, where every sector has productivity $A_i^{ss} = 1$. We consider the exercise of shocking every sector to productivity A_i, where A_i could be above, below, or equal to 1.

Equilibrium. It is illustrative to consider a generic equilibrium when some fixed subset of sectors $\Omega \subseteq \{1, ..., S\}$ adjusts, while the remaining sectors do not adjust. Denote the cardinality of Ω as $\omega \equiv |\Omega|$. Sectors which adjust update their price to $p_i = M/A_i$, whereas others remain at the steady state value of $p_i^{ss} = 1$. The price level thus aggregates from (6) to $P = \frac{SM^{\omega/S}}{\prod_{i\in\Omega} A_i^{1/S}}$. Using the quantity equation (2) gives consumption and output as $C = Y = \frac{1}{S}\left[\prod_{i\in\Omega} A_i^{1/S}\right] M^{\frac{S-\omega}{S}}$. For comparison, the flexible-price level of output is $Y_{\text{flex}} = \frac{1}{S}\prod_{i=1}^{S} A_i^{1/S}$. Using sectoral demand, production technology, and labor market aggregation, aggregate labor is $N = \frac{\omega}{S} + \frac{M}{S}\sum_{i\notin\Omega}\frac{1}{A_i} + \psi\omega$. Thus welfare conditional on the set Ω adjusting is, as a function of the money supply:

$$\mathbb{W}_\Omega(M) = \ln\left[\frac{1}{S}\left[\prod_{i\in\Omega} A_i^{1/S}\right] M^{\frac{S-\omega}{S}}\right] - \left[\frac{\omega}{S} + \frac{M}{S}\sum_{i\notin\Omega}\frac{1}{A_i} + \psi\omega\right] \tag{25}$$

As before allowing the social planner to overcome adjustment externalities, the optimal choice of money supply follows from the first order condition:

$$M_\Omega^* = \frac{S-\omega}{\sum_{i\notin\Omega}\frac{1}{A_i}} \tag{26}$$

Thus welfare under optimal policy, conditional on the set Ω of sectors adjusting, is:

$$\mathbb{W}_\Omega^* = \ln\left[\frac{1}{S}\left[\prod_{i\in\Omega} A_i^{1/S}\right]\left(\frac{S-\omega}{\sum_{i\notin\Omega}\frac{1}{A_i}}\right)^{\frac{S-\omega}{S}}\right] - [1+\psi\omega] \tag{27}$$

Replicating the flexible-price allocation. It is only possible to replicate the flexible-price allocation – net of the extra labor required for menu costs – in three cases. First, as before, if all sectors adjust, i.e. $\Omega = \{1, ..., S\}$ and $\omega = S$, then naturally this ensures the flexible-price allocation. This comes at the cost of all S sectors paying the menu cost. Second, if the shock is uniform across sectors; we return to this case below.

Third, the flexible-price allocation can be achieved also if $\omega = S - 1$, so all but one sector adjust. The one non-adjusting sector simply may be any arbitrary sector r. In this case, the central bank would set the money supply at $M = \frac{1}{A_r}$. This ensures that the desired price of sector r, that is $\frac{M}{A_r}$, equals the steady-state level of $p_r^{ss} = 1$, so that despite not changing its price, sector r has the price that it would choose if it were to adjust. This comes at the cost of $S-1$ sectors paying the menu cost.

An immediate implication is that it is never optimal to have all S sectors adjust, since the same allocation can be achieved if $S-1$ sectors adjust, but with less labor required for menu costs. In other words, it is always best to peg at least one sector and ensure that one sector does not adjust its price. This sector may be arbitrarily chosen in the baseline model. If sectors were of heterogeneous size or had heterogeneous menu costs, as in proposition 6, then it would be optimal to choose the sector with the largest size-weighted menu costs. This reinforces the "least-cost avoider principle" interpretation, or the "stabilize the stickiest price" interpretation described in that proposition.

If more than one sector leaves their price unchanged, i.e. $\omega < S-1$, then it is usually not possible to achieve the flexible-price allocation. This is the standard result that when relative prices change, if there is sufficient nominal rigidity, the flexible-price allocation cannot be achieved: there is more than one target (the many relative prices), but only one instrument, M (Poole 1970).

Interpreting conditionally-optimal policy. Since it is still the case that nominal wages are determined by monetary policy, $W = M$, it follows from (26) that nominal wages *conditional on sectors* Ω *adjusting* are:[25]

$$W^*_\Omega = \frac{S-\omega}{\sum_{i\notin\Omega}\frac{1}{A_i}} \tag{28}$$

In other words, the equilibrium nominal wage under optimal policy depends on the central bank's choice of the set of adjusting firms Ω.

From (28), we can see that optimal policy will stabilize nominal wages, $W^*_\Omega = W^{ss}$, if all firms who do not adjust are unshocked (since $A_i = 1$ for unshocked sectors, and the cardinality of Ω is $S-\omega$).

Thus we can summarize optimal policy under multiple shocks as follows.

Proposition 2 (Optimal policy with multiple shocks)**.** Consider an arbitrary set of productivity shocks to the baseline model, $\{A_1, ..., A_S\}$.

1. Conditional on sectors $\Omega \subseteq \{1, ..., S\}$ adjusting, optimal policy is given by setting $M = M^*_\Omega$ defined in (26).
2. The optimal set of sectors that should adjust, Ω^*, is given by comparing welfare under the various possibilities for Ω, using $\mathbb{W}^*_\Omega$ defined in (27).

[25] For any Ω with $\omega < S$. With $\omega = S$, nominal wages are indeterminate and may be anything. We may then for simplicity choose the level given in (28).

3. Nominal wage targeting is exactly optimal if the set of sectors which should not adjust are unshocked: $A_i = 1 \ \ \forall i \notin \Omega^*$.

Even when optimal policy does not exactly stabilize nominal wages, it may nonetheless be considered to approximately do so. For $A_i \approx 1$, it is the case that $\frac{1}{A_i} \approx 1$; this implies that $\sum_{i \notin \Omega} \frac{1}{A_i} \approx \sum_{i \notin \Omega} 1 = S - \omega$, and so by (28) nominal wages are approximately unchanged, $W_\Omega^* \approx \frac{S-\omega}{S-\omega} = 1$. As in proposition 1, it will only be optimal to adjust for sectors which experience larger shocks. As a result, for unadjusting sectors, $i \notin \Omega$, it is particularly true that $A_i \approx 1$.

Ultimately, the performance of exact nominal wage targeting in the face of multiple shocks is in part an empirical question that depends on the distribution of shocks and how tightly centered around 1 they are, and in part a quantitative question that depends on how well exact nominal wage targeting performs compared to the analytical optimal policy. We answer this question in the quantitative model of section 6.

The case of perfectly uniform aggregate shocks. While we emphasize that inflation targeting is not optimal in response to heterogeneous sectoral shocks, stabilizing aggregate inflation is optimal in the special case when the productivity shocks are *perfectly* uniform across sectors: $\frac{A_i}{A_i^{ss}} = \frac{A_j}{A_j^{ss}}$ for all i, j. In this case, relative prices do not need to change, so optimal policy ensures that no sector adjusts its price, which both maximizes allocative efficiency and minimizes menu cost expenditure.

5.5 Further extensions

Online appendix C considers further extensions. Section C.2 studies optimal policy in the "CalvoPlus" variant of the menu cost model, where a random fraction of firms across all sectors has an opportunity to adjust prices for free. Optimal policy is unchanged, modulo the threshold $\overline{\gamma}$, because under optimal policy unshocked firms already have no desire to adjust prices. Section C.3 considers optimal policy in an economy with segmented labor markets, where each sector has its own labor market with its own wage. Optimal policy still seeks to ensure that unshocked sectors have their nominal marginal costs unchanged. Section C.4 studies an economy where not only are prices sticky due to menu costs but so are nominal wages. Optimal policy continues to stabilize nominal marginal costs of unshocked sectors to both maximize allocative efficiency and minimize menu costs.

6 Quantitative model

In this section, we develop a dynamic version of our baseline model, augmented with idiosyncratic shocks and calibrated to the US economy.

6.1 Model description and solution method

Our dynamic multisector model of menu costs is similar to that in Nakamura and Steinsson (2010). The main difference is that we include sector-specific productivity shocks on top of idiosyncratic, firm-level shocks.

The household chooses paths for consumption, C_t, labor, N_t, money balances, M_t, and bonds, B_t, to maximize the present discounted value of utility. The problem faced by the household is:

$$\begin{aligned} &\max_{\{C_t,N_t,B_t,M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \theta \frac{N_t^{1+\varphi}}{1+\varphi} + \ln\left(\frac{M_t}{P_t}\right) \right] \\ \text{s.t.} \quad & P_tC_t + B_t + M_t \leq R_tB_{t-1} + W_tN_t + M_{t-1} + D_t - T_t \end{aligned} \tag{29}$$

This problem represents a dynamic version of the static household problem in section 2.1, with more general preferences. To consume and save (the left hand side of the budget constraint) the household uses gross wealth, money holdings, labor earnings, and firm dividends net of the lump sum tax imposed by the government (the right hand side of the budget constraint). R_t is the nominal interest rate on bonds.

The final good producer and sectoral good producers behave the same as in the baseline analytical model of section 2. These firms operate in competitive environments and aggregate the goods produced by the corresponding lower-tier firms according to the same technologies as the baseline model, (4) and (7).

A first key difference with the baseline model is that intermediate firms are now subject not only to sector-level productivity shocks, A_{it}, but also to idiosyncratic, firm-level shocks, $a_{it}(j)$. Log firm-level idiosyncratic productivity shocks follow a mixture between an AR(1) process and a uniform distribution:

$$\log\left(a_{it}(j)\right) = \begin{cases} \rho_{\text{idio}} \log\left(a_{it-1}(j)\right) + \varepsilon_{it}^{\text{idio}}(j) & \text{with prob. } 1-\varsigma \\ \mathcal{U}\left[-\log\left(\underline{a}\right), \log\left(\overline{a}\right)\right] & \text{with prob. } \varsigma \end{cases} \tag{30}$$

where the innovations in the AR(1) process are Gaussian, $\varepsilon_{it}^{\text{idio}}(j) \sim \mathcal{N}\left(0, \sigma_{\text{idio}}^2\right)$. The mixture with uniform shocks creates a fat-tailed distribution, which is important to match the presence of large price changes in the data Midrigan (2011).[26]

[26] Alternatively, Yang (2022) shows that menu costs in conjunction with rational inattention can endoge-

A second key difference is that menu costs $\psi_{it}(j)$ are stochastic: while firms usually have to pay a menu cost $\psi > 0$, with probability ν each period they receive the opportunity to adjust for free (the "CalvoPlus" model). If a firm chooses not to adjust in period t, $\chi_{it}(j) = 0$, it inherits its price $p_{i,t-1}(j)$ from the previous period. The problem facing intermediate firms is:

$$\max_{p_{it}(j)} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \left[\frac{1}{R^t P_t} \left\{ p_{it}(j)\, y_{it}(j) - W_t n_{it}(j)\,(1-\tau) - \chi_{it}(j)\psi_{it}(j) W_t \right\} \right] \tag{31}$$

$$\text{s.t.} \quad y_{it}(j) = y_{it} \left(\frac{p_{it}(j)}{p_{it}} \right)^{-\eta}$$

$$y_{it}(j) = A_{it} a_{it}(j)\, n_{it}(j)^{\alpha} \tag{32}$$

where τ is again the labor subsidy and $R^t = \prod_{z=0}^{t} R_z$. Note that production technology (32) may have decreasing returns to scale, $\alpha \in (0, 1]$.

As in the baseline model, labor market clearing requires that labor supplied by the household equals labor used in production plus labor used in menu costs:

$$N_t = \sum_i \int_0^1 n_{it}(j) dj + (1-\nu)\psi\chi_t \tag{33}$$

where χ_t is the mass of firms that adjust prices, $\chi_t \equiv \sum_i \int_0^1 \chi_{it}(j) dj$.

To solve the model numerically, we use the sequence-space Jacobian method of Auclert et al. (2021). This method provides linearized general equilibrium responses with respect to perfect-foresight shocks to aggregate and sectoral variables, while allowing agents' decision rules to be nonlinear with respect to idiosyncratic variables. Note that, by certainty equivalence, the resulting linearized perfect-foresight transition paths are equal to the first-order perturbation solution of the model with aggregate risk.

6.2 Calibration

The model is calibrated to match salient micro and macro moments of the US economy at the monthly frequency. There are two sets of parameters. The first set of parameters is standard and taken from the macroeconomics literature. The second is calibrated to match price-adjustment behavior by US firms. The model parameters are listed in table 1.

The preference parameters are set to standard values. The discount factor $\beta = 0.99835$ is chosen to match a 2% annualized interest rate. The disutility of labor is $\theta = 1$, the inverse Frisch elasticity is $\varphi = 0$ as in Golosov and Lucas (2007), and the inverse elasticity of intertemporal substitution is $\gamma = 2$.

nously generate a distribution of price changes that is fat-tailed.

	Parameter (monthly frequency)	Value	Target
β	Discount factor	0.99835	2% annual interest rate
θ	Disutility of labor	1	
φ	Inverse Frisch elasticity	0	Golosov and Lucas (2007)
γ	Inverse EIS	2	
S	Number of sectors	6	Nakamura and Steinsson (2010)
η	Elasticity of subst. between sectors	5	
α	Returns to scale	0.6	
τ	Labor subsidy	0.2	$1/\eta$
σ_{idio}	Standard deviation of idio. shocks	0.058	menu cost expenditure / revenue 1.0 (1.1%)
ρ_{idio}	Persistence of idio. shocks	0.992	share of price changers 9.7 (10.1%)
ψ	Menu cost	0.1	median absolute price change 8.3 (7.9%)
ν	Calvo parameter	0.09	Q1 absolute price change 4.2 (5.6%)
ς	Fat tail parameter	0.001	Q3 absolute price change 12.0 (12.5%)
			kurtosis of price changes 5.4 (5.1)

Table 1: Model parameters and calibration (monthly frequency). The last row of the third column shows the model-implied values of targets, with empirical values in parentheses.

Following Nakamura and Steinsson (2010), we choose $S = 6$ sectors. We assume the sectors are identical in their structural parameters. The elasticity of substitution across goods within sector is $\eta = 5$ and the decreasing return to scale parameter is $\alpha = 0.6$. The labor subsidy $\tau = \frac{1}{\eta} = 0.2$ is set to offset the markup distortion.

We estimate the five key parameters of the firm's problem to target important price-setting moments in the data. These estimated parameters are the standard deviation and persistence of idiosyncratic productivity shocks, σ_{idio} and ρ_{idio}; the size of the menu cost ψ; the probability a firm can choose its price for free, ν; and the fat tail parameter, ς. These parameters are set to match six targets. First, per the literature cited in section 4.2, we target a menu cost expenditure as a share of firm revenue of 1%. Second, following Nakamura et al. (2018), we target the share of firms changing prices any given month (excluding sales) of 10.1%. Additionally, we target the first, second, and third quartiles of the absolute price change distribution, at 5.6%, 7.9%, and 12.5%, . Finally, we target the kurtosis of the price change distribution of 5.1 from Nakamura and Steinsson (2008).

The last cell in table 1 shows the targeted moments and, in parentheses, the model moments. The model matches the targeted moments well.

6.3 A one-sector shock

Our goal is to compare the performance of nominal wage targeting versus inflation targeting. We begin with the dynamic analog to the exercise considered in section 3: a positive productivity shock to sector 1, of an arbitrary size.

Prices and quantities. Starting from the stationary distribution, consider a shock to sector-1 productivity A_1 of 1% which decays exponentially. Sectoral productivity is unshocked for all other sectors, while idiosyncratic shocks continue for all firms. Figure 2 displays the time paths of A_1, consumption, labor, and prices in percentage point deviations from steady state. Three cases are depicted: nominal wage targeting, inflation targeting, and flexible prices.

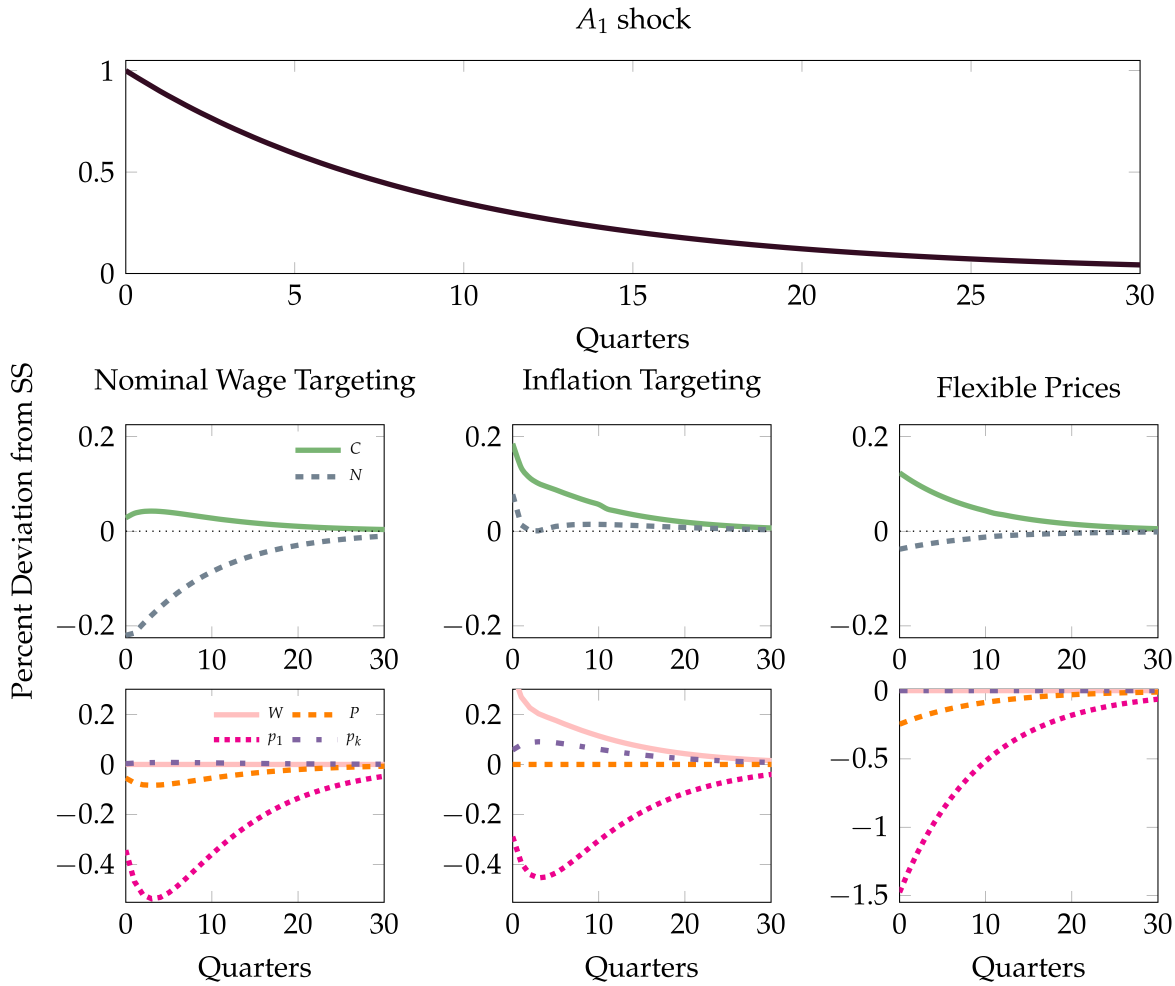

Figure 2: Sector-1 productivity is increased by a 1% shock which decays at an exponential rate. The first column shows outcomes under nominal wage targeting; the second under inflation targeting; and the third under the flexible-price benchmark.

Observe in the first column that under nominal wage targeting, besides nominal wages W remaining stable, the aggregate price level P falls in response to the positive productivity shock, driven almost entirely by the fall in sector-1 price p_1. Meanwhile, under inflation targeting in the second column, P is constant and W rises, driven by a fall in p_1 and an increase in all other sectoral prices p_k. What matters for welfare, however, are consumption and labor. Labor falls under nominal wage targeting and rises under inflation targeting, a difference that is largely explained by differences in menu cost expenditure, discussed in the next subsection. As a benchmark for comparison, the third column shows the impulse responses under flexible prices, in which $\psi = 0$.[27]

Menu cost expenditure. The total quantity of labor N depicted above can be decomposed into labor used in production and menu cost expenditure. The first panel of figure 3 shows total labor (solid) and labor directed towards changing prices (dashed) under inflation (blue) and nominal wage (red) targets following the shock. The second panel shows real menu cost expenditures, that is, labor scaled by real wages.

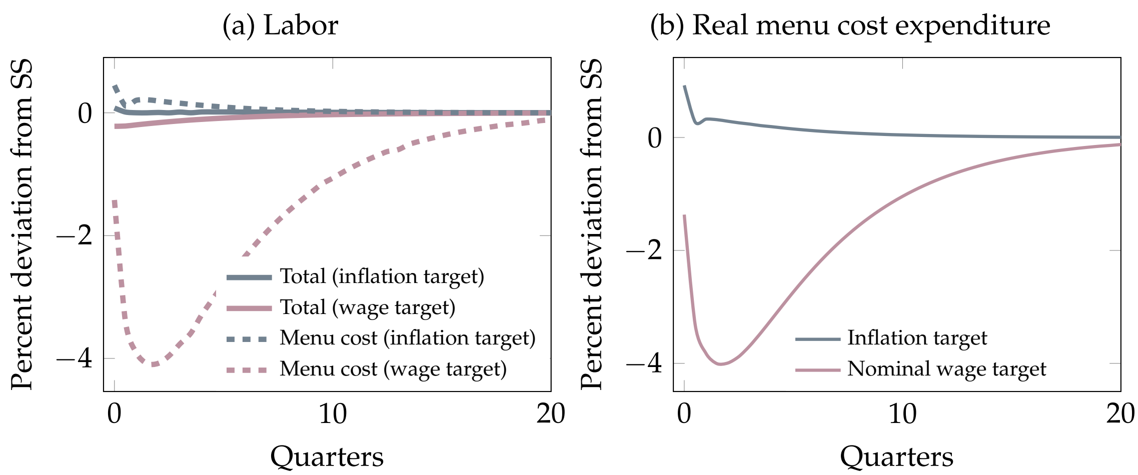

Figure 3: Panel (a) shows the percent deviation from steady state in labor used on menu costs (dashed) and overall (solid) after a persistent 1% increase in A_1, under nominal wage targeting (red) and inflation targeting (blue). Panel (b) shows the real menu cost expenditures under nominal wage (red) and inflation (blue) targeting.

The labor employed in price changes is notably larger under inflation targeting than under nominal wage targeting, indicating that, following the productivity shock, the menu costs expended under inflation targeting are larger than those expended under nominal wage targeting. This result is consistent with the intuition from the analytical model that nominal wage targeting economizes on menu cost expenditures.

Welfare. The welfare response to the productivity shock is shown in figure 4, in utility units. Nominal wage targeting dominates inflation targeting. To make the comparison

[27] In this economy, we also normalize steady state sectoral productivity, A_i^{ss} so that steady state welfare in this flex-price economy exactly matches steady state welfare in the menu cost model. $A_i^{ss} = 1$ in the menu cost economy and $A_i^{ss} = 0.9779$ in the flex-price economy.

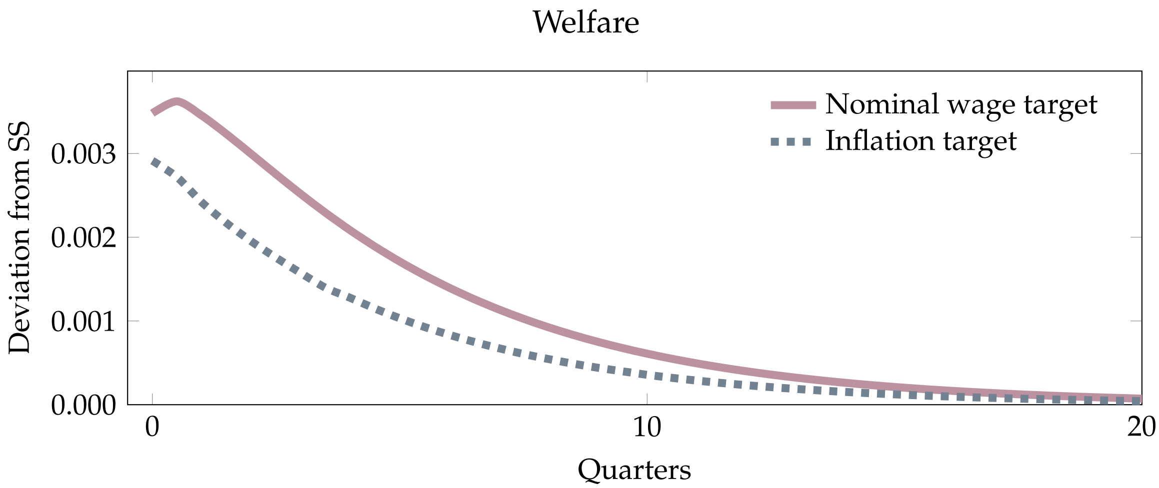

Figure 4: Deviation from steady state in welfare after the persistent 1% increase in A_1 under nominal wage targeting and under inflation targeting targeting.

interpretable, we convert the welfare differences to consumption units. Denote the paths for consumption and labor as $\{C_t^x\}$ and $\{N_t^x\}$, where x refers to either nominal wage or inflation targeting, $x \in \{W, P\}$. Denote the household period utility function from (29) as $U(C_t^x, N_t^x)$. We ask: how much higher would the path for consumption need to be such that lifetime household welfare under inflation targeting equals that under nominal wage targeting? That is, to a first-order approximation, what is the λ that solves:

$$\sum_{t=0}^{\infty} \beta^t U\left((1+\lambda) C_t^P, N_t^P\right) = \sum_{t=0}^{\infty} \beta^t U\left(C_t^W, N_t^W\right) \tag{34}$$

We find that $\lambda = 0.008\%$, for this arbitrary 1% shock to sector-1 productivity.

Summary. Taken together, figures 2-4 show the same spillover mechanism emphasized in the analytical model. Under inflation targeting, the sector-1 price decline must be offset by price increases in other sectors, inducing widespread price adjustment and higher menu cost expenditure. Under nominal wage targeting, unshocked sectors' incentives to reprice are muted, so menu cost expenditure is lower and welfare is higher.

7 Conclusion

This paper studies optimal monetary policy in a multisector, menu cost economy. We show analytically that stabilizing nominal marginal costs in unshocked sectors is optimal for sufficiently large shocks, resulting in countercyclical inflation. In a baseline model, optimal policy is nominal wage targeting. In a rich quantitative version of the model, targeting nominal wages dominates inflation targeting. These findings underscore the importance of allowing sector-specific shocks to transmit directly into aggregate inflation, a recommendation that aligns with recent policymaker perspectives on "looking through" sectoral shocks (Powell 2023; Brainard 2022; Schnabel 2022). Historical examples, such as central bank reactions to oil shocks, illustrate the inefficiency of rigidly pursuing inflation stabilization in the presence of significant sectoral disturbances (Bernanke et al. 1997).

A full analysis of how to implement optimal policy is beyond the scope of this paper. A first important question for policymakers is the choice of which empirical measure to track in order to best track nominal marginal costs. However, this challenge also applies under inflation targeting: headline or core inflation, the consumer price index or the personal consumption expenditure index? A second question may be the role of data revisions, although once again an analogous issue arises when tracking inflation since inflation revisions are often quite sizable (Audoly et al. 2023). A third challenge for implementation may be informational constraints, e.g. the ability to identify sectoral shocks.

The conclusion that optimal monetary policy in response to sectoral shocks should result in countercylical inflation resonates with other studies in the broader optimal monetary policy literature that depart from the traditional New Keynesian model, (Sheedy 2014; Angeletos and La'O 2020; Selgin 1997).[28] Integrating these varied approaches into a unified theory of optimal monetary policy is an open question for future research.

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A Additional proofs

A.1 Proof of lemma 1

Proof. Equation (12) showed the conditions under which a firm j in sector i with inherited price p_i^{old} adjusts. Define $f(W, A_i)$ such that the firm adjusts iff $f(W, A_i) \geq 0$:

$$f(W, A_i) \equiv \left(\frac{W}{A_i}\right)^{1-\eta} p_i^\eta y_i \left[\frac{1}{\eta}\right] - W\psi - \left(p_i^{\text{old}}\right)^{1-\eta} p_i^\eta y_i \left[1 - \frac{W/A_i}{p_i^{\text{old}}} \cdot \frac{\eta - 1}{\eta}\right] \tag{35}$$

Observe first that for the locus in (W, A_i) space where $W/A_i = p_i^{\text{old}}$, it is the case that $f(W, A_i) = -W\psi < 0$ and the firm will not adjust. That is, this locus is a subset of the inaction region $\Lambda \equiv \{(W, A_i) | f(W, A_i) < 0\}$. Thus Λ is nonempty.

In A_i space. Observe that $\frac{\partial f}{\partial A_i} = p_i^\eta y_i W(A_i^{-2}) \left(\frac{\eta-1}{\eta}\right) \left[\left(\frac{W}{A_i}\right)^{-\eta} - (p_i^{\text{old}})^{-\eta}\right]$. This is positive iff $W/A_i < p_i^{\text{old}}$. Additionally, $\lim_{A_i \to 0} f(\cdot, A_i) = \lim_{A_i \to \infty} f(\cdot, A_i) = \infty$ and $f(\cdot, A_i)$ is continuous in A_i on $(0, \infty)$.

Now consider any fixed W^0 such that there exists some A_i^0 with $f(W^0, A_i^0) < 0$. Then by the intermediate value theorem there exists an inaction interval $(\underline{\lambda}, \overline{\lambda})$ around W^0 such that $f(W^0, \underline{\lambda}) = f(W^0, \overline{\lambda}) = 0$, and $f(W^0, A_i) < 0$ iff $A_i \in (\underline{\lambda}, \overline{\lambda})$. To see that $\overline{\lambda}$ is increasing in ψ and $\underline{\lambda}$ is decreasing in ψ, observe that increasing ψ shifts the entire $f(x)$ curve down, i.e. $\frac{\partial f}{\partial \psi} < 0$. If for a fixed W^0 there is no A_i with $f(W, A_i) < 0$, then by construction there is no point in Λ along W^0.

In W space. Similarly, observe that $\frac{\partial f}{\partial W} = p_i^\eta y_i A_i^{-1} \left(\frac{\eta-1}{\eta}\right) \left[(p_i^{\text{old}})^{-\eta} - \left(\frac{W}{A_i}\right)^{-\eta}\right] - \psi$. This is zeroed for the locus of (W, A_i) such that $\left(\frac{W}{A_i}\right)^{-\eta} = (p_i^{\text{old}})^{-\eta} - \psi p_i^{-\eta} y_i^{-1} A_i \left(\frac{\eta-1}{\eta}\right)^{-1} \equiv \zeta^{-\eta}$. Observe that $f_1 < 0$ iff $W/A_i < \zeta$. Additionally, $\lim_{W \to 0} f(W, \cdot) = \infty$.[29] Additionally, $f(W, \cdot)$ is continuous in W on $(0, \infty)$. Thus, as above, consider any fixed A_i^0 such that there exists some W^0 with $f(W^0, A_i^0) < 0$. By the intermediate value theorem there exists (abusing notation) an inaction interval $(\underline{\lambda}, \overline{\lambda})$ around A_i^0 such that $f(\underline{\lambda}, A_i^0) = f(\overline{\lambda}, A_i^0) = 0$, and $f(W, A_i^0) < 0$ iff $W \in (\underline{\lambda}, \overline{\lambda})$, where $\overline{\lambda}$ is potentially infinite. To see that $\overline{\lambda}$ is increasing in ψ and $\underline{\lambda}$ is decreasing in ψ, observe that increasing ψ shifts the entire $f(x)$ curve down, i.e. $\frac{\partial f}{\partial \psi} < 0$. $\square$

[29] The second limit comes from L'Hopital's rule, together with the natural restriction that $\psi < \frac{1}{S\eta}$. Without this maximum bound on ψ, firms would *always* earn negative profits after adjusting and would *never* adjust.

A.2 Formal statement of planner's problem

(16)-(19) derived welfare in each regime as a function of the money supply choice, $\mathbb{W}_{\text{regime}}$ for regime $\in \{\text{only 1 adjusts}, \text{only } k \text{ adjusts}, \text{none adjust}, \text{all adjust}\}$ (where in the last case, when all adjust, welfare is independent of monetary policy).

Constrained planner's problem. We define the constrained planner as the planner who chooses M in each regime to maximize welfare, *constrained* by the fact that the choice of M must be incentive compatible with whether various sectors actually adjust or not:

$$M^{*,\text{ constrained}}_{\text{only 1 adjusts}} \equiv \arg\max_{M} \mathbb{W}_{\text{only 1 adjusts}}(M) \tag{36}$$

$$\text{s.t. } f(M,\gamma) \geq 0 \text{ and } f(M,1) \leq 0$$

$$M^{*,\text{ constrained}}_{\text{only } k \text{ adjust}} \equiv \arg\max_{M} \mathbb{W}_{\text{only } k \text{ adjust}}(M) \tag{37}$$

$$\text{s.t. } f(M,\gamma) \leq 0 \text{ and } f(M,1) \geq 0$$

$$M^{*,\text{ constrained}}_{\text{none adjust}} \equiv \arg\max_{M} \mathbb{W}_{\text{none adjust}}(M) \tag{38}$$

$$\text{s.t. } f(M,\gamma) \leq 0 \text{ and } f(M,1) \leq 0$$

where $f(M, A_i)$ refers to the function defined in (35) which is positive iff firms in sector i want to adjust. This defines, for example, $M^{*,\text{ constrained}}_{\text{only 1 adjusts}}$ as the level of money supply which maximizes welfare in equilibrium when only sector 1 adjusts $\mathbb{W}_{\text{only 1 adjusts}}(M)$, *subject to the constraint that* it is indeed incentive-compatible for sector-1 firms to adjust, $f(M,\gamma) \geq 0$, and incentive-compatible for firms in sectors k to not adjust, $f(M,1) \leq 0$. Denote the associated constrained-optimal levels of welfare in each regime as $\mathbb{W}^{*,\text{constrained}}_{\text{regime}} = \mathbb{W}_{\text{regime}}\left(M^{*,\text{ constrained}}_{\text{regime}}\right)$ for regime $\in \{\text{only 1 adjusts}, \text{only } k \text{ adjusts}, \text{none adjust}\}$. The constrained social planner's problem is then to select among these, or to implement the regime where all adjust (in which case the choice of M is irrelevant, as long as it is incentive-compatible):

$$\max \left\{ \mathbb{W}^{*,\text{ constrained}}_{\text{only 1 adjusts}}, \mathbb{W}^{*,\text{ constrained}}_{\text{only } k \text{ adjust}}, \mathbb{W}^{*,\text{ constrained}}_{\text{none adjust}}, \mathbb{W}_{\text{all adjust}} \right\} \tag{39}$$

which is implemented with the associated incentive-compatible choice of M.

Unconstrained social planner's problem. Here, as in the body of the paper, the planner is endowed with the instrument of forcing firms to adjust (equivalently, subsidizing menu costs funded by lump sum taxes), so the constraints on (36), (37), and (38) never bind:

$$M^{*}_{\text{only 1 adjusts}} \equiv \arg\max_{M} \mathbb{W}_{\text{only 1 adjusts}}(M) \tag{40}$$

$$M^*_{\text{only } k \text{ adjust}} \equiv \arg\max_M \mathbb{W}_{\text{only } k \text{ adjust}}(M) \tag{41}$$

$$M^*_{\text{none adjust}} \equiv \arg\max_M \mathbb{W}_{\text{none adjust}}(M) \tag{42}$$

Since the objective functions in all of these arg maxes are strictly concave, the solution is found from the first order condition, as presented in the text. We denoted the associated unconstrained-optimal levels of welfare in each regime in equations (17), (18), (19) as $\mathbb{W}^*_{\text{regime}} = \mathbb{W}_{\text{regime}}\left(M^*_{\text{regime}}\right)$ for regime $\in$ {only 1 adjusts, only k adjusts, none adjust}. The constrained social planner's problem is then to select among these, or to implement the regime where all adjust (in which case the choice of M is irrelevant):

$$\max\left\{\mathbb{W}^*_{\text{only 1 adjusts}}, \mathbb{W}^*_{\text{only } k \text{ adjust}}, \mathbb{W}^*_{\text{none adjust}}, \mathbb{W}_{\text{all adjust}}\right\} \tag{43}$$

It is this maximization problem that produces lemma 2 and lemma 3, which in turn produce proposition 1. □

Supplemental Appendix

B Adjustment externalities

In this section, we work with a slightly modified version of the baseline model, where menu costs are modeled as a utility penalty rather than as a labor cost. This highlights the way in which the results do not depend on how menu costs are modeled, and also facilitates an analysis of the role of adjustment externalities.

B.1 Model setup

The final goods producer and sectoral goods producer are exactly the same as the baseline model.

Intermediate goods producers. The intermediate goods producers again are a unit mass of monopolistically competitive firms in each sector with linear technology and productivity that is common to the sector. They again face a menu cost if adjusting prices. Here, unlike the baseline model, the adjustment cost is not ψ units of extra labor, but a penalty $(1-\psi)$ that scales down the firm manager's objective function (but not profits). Firm i in sector j faces the following maximization program:

$$\begin{aligned}
&\max_{p_i(j)} D_i(j)\left(1-\psi\chi_i(j)\right)\\
&\text{s.t. } D_i(j) = p_i(j)y_i(j) - Wn_i(j)(1-\tau)\\
&\qquad \chi_i(j) = \begin{Bmatrix} 1 & \text{if } p_i(j) \neq p_i^{\text{old}} \\ 0 & \text{else} \end{Bmatrix}\\
&\qquad y_i(j) = y_i\left(\frac{p_i(j)}{p_i}\right)^{-\eta}\\
&\qquad y_i(j) = A_i n_i(j)
\end{aligned}$$

This menu cost is a utility penalty that is passed on to households, but does not affect physical profits. As before, all firms within a sector face the same problem, and we drop the (j) notation when the context is clear.

Households. The representative household is precisely as in the baseline model, except that the utility function (1) is modified to be:

$$\mathbb{W} = \ln C - N + \ln\left(\frac{M}{P}\right) - \psi \sum_i \chi_i$$

The household has the same preferences over consumption, labor, and real balances; but now is directly penalized in terms of welfare when firms adjust prices. The benefit of this modeling technique is that it turns off the income effects caused by menu costs, as discussed in section 5.1, and is a technique that has been used by e.g. Auclert, Rognlie and Straub (2018) or Guerrieri et al. (2021).

B.2 Shock and equilibrium

We run the same exercise, shocking the productivity of sector 1 from $A_1 = 1$ to $A_1 = \gamma > 1$. The equilibrium allocations in the four regimes 3.1 is *exactly* the same as in the body of the paper, except that the level of aggregate labor in each of the four regimes is no longer affected by menu costs. The equilibrium level of welfare in each of the four regimes as a function of the choice of money supply is:

$$\mathbb{W}_{\text{flex}} = \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1$$

$$\mathbb{W}_{\text{all adjust}} = \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1$$

$$\mathbb{W}_{\text{only 1 adjusts}}(M) = \ln\left(\frac{\gamma^{1/S}}{S} M^{\frac{S-1}{S}}\right) - \frac{1}{S}\left[1 + M(S-1)\right] - \psi$$

$$\mathbb{W}_{\text{only } k \text{ adjust}}(M) = \ln\left(\frac{1}{S} M^{\frac{1}{S}}\right) - \frac{1}{S}\left[S - 1 + \frac{M}{\gamma}\right] - (S-1)\psi$$

$$\mathbb{W}_{\text{none adjust}}(M) = \ln\left(\frac{M}{S}\right) - \frac{M}{S}\left[S - 1 + \frac{1}{\gamma}\right]$$

B.3 Adjustment decision

The firm compares its objective function under price adjustment versus under the inherited price. The adjustment condition can be simplified to be written as: adjust if and only if

$$\frac{1}{\eta}(1-\psi) > \left[\frac{W/A_i}{p_i^{\text{old}}}\right]^{\eta}\left(\left[\frac{W/A_i}{p_i^{\text{old}}}\right]^{-1} - \frac{\eta - 1}{\eta}\right)$$

For additional analytical tractability, we make the following assumption in this section:

Assumption 1. The elasticity of substitution is $\eta = 2$.

This assumption allows for a closed form solution to the inaction region, using the quadratic formula: do not adjust if and only if

$$\frac{W}{A_i} \in \left(p_i^{\text{old}}(1-\sqrt{\psi}), p_i^{\text{old}}(1+\sqrt{\psi}) \right) \tag{44}$$

Clearly this has the same properties as the Λ inaction region described in lemma 1.

When starting from the steady state where p_i^{old} for all sectors i, and using the equilibrium $W = M$ condition, then we have the following. The inaction region for sector 1 is

$$M \in (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi}))$$

The inaction region for sectors k is

$$M \in (1-\sqrt{\psi}, 1+\sqrt{\psi})$$

B.4 The planner's problem

The planner's problem – importantly, without the ability to subsidize menu costs and so denoted "constrained" – written in full is:

$$\max_{\substack{\text{all adjust, only 1 adjusts} \\ \text{only } k \text{ adjust, none adjust}}} \left\{ \mathbb{W}_{\text{all adjust}}, \mathbb{W}^{*,\text{constrained}}_{\text{only 1 adjusts}}, \mathbb{W}^{*,\text{constrained}}_{\text{only } k \text{ adjust}}, \mathbb{W}^{*,\text{constrained}}_{\text{none adjust}} \right\} \tag{45}$$

s.t.

$$\mathbb{W}_{\text{all adjust}} = \left\{ \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1 \right\}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only 1 adjusts}} = \left\{ \begin{matrix} \max\limits_{M} \ln\left(\frac{\gamma^{1/S}}{S} M^{\frac{S-1}{S}}\right) - \frac{1}{S}[1+M(S-1)] - \psi \\ \text{s.t. } M \in (1-\sqrt{\psi}, 1+\sqrt{\psi}) \\ M \notin (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi})) \end{matrix} \right\}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only } k \text{ adjust}} = \left\{ \begin{matrix} \max\limits_{M} \ln\left(\frac{1}{S} M^{\frac{1}{S}}\right) - \frac{1}{S}\left[S-1+\frac{M}{\gamma}\right] - (S-1)\psi \\ \text{s.t. } M \notin (1-\sqrt{\psi}, 1+\sqrt{\psi}) \\ M \in (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi})) \end{matrix} \right\}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{none adjust}} = \left\{ \begin{matrix} \max\limits_{M} \ln\left(\frac{M}{S}\right) - \frac{M}{S}\left[S-1+\frac{1}{\gamma}\right] \\ \text{s.t. } M \in (1-\sqrt{\psi}, 1+\sqrt{\psi}) \\ M \in (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi})) \end{matrix} \right\}$$

B.5 Interior optima

The interior optima for each regime, found from the first order conditions, are the same as the baseline model:

$$M^{\text{interior}}_{\text{only 1 adjusts}} = 1$$
$$M^{\text{interior}}_{\text{only } k \text{ adjust}} = \gamma$$
$$M^{\text{interior}}_{\text{none adjust}} = \frac{S}{S - 1 + 1/\gamma}$$

B.6 Only sector 1 adjusts: The possibility of *positive* adjustment externalities

Suppose the unconstrained social planner – i.e., one who could subsidize menu costs and ignore the implementability constraints – would want to implement the regime where only sector 1 adjusts, and she therefore wants to set $M = 1$. We now examine whether this is incentive compatible: does it result in sector-k firms being within their inaction region and sector-1 firms being outside it?

First observe that $M = 1$ indeed ensures that sector-k firms are in their inaction region, since $M = 1 \in (1 - \sqrt{\psi}, 1 + \sqrt{\psi})$ always.

However, it is possible that $M = 1$ could leave sector-1 firms inside their inaction region, if the following condition holds:

$$\gamma < \frac{1}{1 - \sqrt{\psi}} \equiv \gamma_1 \tag{46}$$

As an existence proof, it is possible to come up with numerical examples for parameters satisfying the above where it would be, in fact, socially optimal to implement this regime if there were no implementability constraints. When this is the case, then the best the central bank can do within this regime is to set $M = \gamma(1 - \sqrt{\psi})$. This is a case of *positive* adjustment externalities: the social planner would prefer that sector 1 adjusts its prices, even though it is individually rational to not do so.

B.7 No sectors adjust: The possibility of *negative* adjustment externalities

Now suppose the unconstrained social planner would prefer that no sector adjusts (i.e. $\gamma < \overline{\gamma}$). The interior optimum level of the money supply, as previously noted, would be $M^{\text{interior}}_{\text{none adjust}} = \frac{S}{S-1+1/\gamma}$. Is this incentive-compatible?

To be incentive-compatible requires that both $\frac{S}{S-1+1/\gamma} > \gamma(1-\sqrt{\psi})$ and $\frac{S}{S-1+1/\gamma} < 1+\sqrt{\psi}$. Thus, there is a *negative* adjustment externality – where its privately optimal for firms in a sector to adjust even when its not socially optimal to do so – if either:

$$\gamma < \frac{1+\frac{1}{S-1}\sqrt{\psi}}{1-\sqrt{\psi}} \equiv \gamma_2 \tag{47}$$

or

$$\gamma > \frac{1+\sqrt{\psi}}{S-(S-1)(1+\sqrt{\psi})} \equiv \gamma_3 \tag{48}$$

As an existence proof, it is possible to come up with numerical examples for parameters satisfying either of the above where it would be, in fact, socially optimal to implement this regime if there were no implementability constraints. When this is the case, then the best the central bank can do within this regime is to set M at the respective boundary.

B.8 Summarizing the possibilities for welfare

A similar analysis the above can be done for the case when only sectors k adjust, where a constraint will bind if $\gamma > \gamma_4 \equiv 1+\sqrt{\psi}$. We summarize the results from above and this additional case in the following:

$$\mathbb{W}_{\text{all adjust}} = \mathbb{W}_{\text{flex}} - S\psi$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only 1 adjusts}} = \begin{Bmatrix} \mathbb{W}_{\text{flex}} - \psi & \text{if } \gamma \geq \gamma_1 \\ \ln\left(\frac{\gamma^{1/S}}{S}\gamma_1^{\frac{S-1}{S}}\right) - \frac{1}{S}\left[1+\gamma_1(S-1)\right] - \psi & \text{else} \end{Bmatrix}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only } k \text{ adjust}} = \begin{Bmatrix} \mathbb{W}_{\text{flex}} - (S-1)\psi & \text{if } \gamma \leq \gamma_4 \\ \ln\left(\frac{1}{S}\gamma_4^{1/S}\right) - \frac{1}{S}\left[S-1+\frac{\gamma_4}{\gamma}\right] - (S-1)\psi & \text{else} \end{Bmatrix}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{none adjust}} = \begin{Bmatrix} -\log\left[S-1+1/\gamma\right]-1 & \text{if } \gamma \in [\gamma_3, \gamma_2] \\ \ln\left(\frac{\gamma_2}{S}\right) - \frac{\gamma_2}{S}\left[S-1+\frac{1}{\gamma}\right] & \text{if } \gamma > \gamma_2 \\ \ln\left(\frac{\gamma_3}{S}\right) - \frac{\gamma_3}{S}\left[S-1+\frac{1}{\gamma}\right] & \text{if } \gamma < \gamma_3 \end{Bmatrix}$$

Optimal monetary policy considers which of these achieves the highest welfare, and sets the money supply M to implement.

C Details on extensions

C.1 Formal statement of propositions

Proposition 3 (The welfare loss of inflation targeting)**.** Denote a policy of "inflation targeting" as a rule for monetary policy ensuring that $P = P^{ss}$ while having correct relative prices, and suppose $\gamma \geq \overline{\gamma}$. Then:

1. Inflation targeting requires that all sectors adjust their prices. It is implemented by increasing the money supply to $M = \gamma^{1/S} > M^{ss}$.

2. Welfare under inflation targeting, denoted $\mathbb{W}^{\text{IT}}$, is strictly less than welfare under the optimal policy described in proposition 1, $\mathbb{W}^*$. The welfare loss is determined by size of menu costs ψ and the number of sectors unaffected by the shock, $S-1$:

$$\mathbb{W}^{\text{IT}} - \mathbb{W}^* = (S-1)\psi$$

Proof: The second claim comes from formulas (16) and (17). For the first claim, suppose the central bank tried to both achieve correct relative prices by only having sector 1 adjust – in which case, $P = SM^{1/S}\gamma^{-1/S}$ – and simultaneously setting M such that the price level was unchanged, $P = P^{ss} = S$. This would require $M = \gamma$. However, if $M = \gamma$ then the optimal price for firms in sector 1 is $p_1 = W/\gamma = 1$, which would mean that firms in sector 1 leave prices unchanged, a contradiction. Similarly, if the central bank tried to achieve correct relative prices while only having sectors k adjust, in which case $P = SM^{\frac{S-1}{S}}$, then this would require $M = 1$, which would cause firms in sectors k to not adjust, again a contradiction. Finally, if no sector adjusts, then it is impossible to achieve correct relative prices, since $p_1/p_k = 1$. It is only by having firms in all sectors adjust, in which case $P = SM\gamma^{-1/S}$, that the central bank can achieve both correct relative prices and ensure that $P = P^{ss}$, by setting $M = \gamma^{1/S}$. □

Proposition 4 (Functional form generalizations)**.** Consider again the positive productivity shock γ affecting sector 1, in the generalized model. For a fixed level of menu costs ψ, there exists a threshold level of productivity $\overline{\gamma} > 1$, such that:

1. If the productivity shock to sector 1 is above the threshold, $\gamma \geq \overline{\gamma}$, then optimal policy is to ensure the nominal marginal costs of firms outside sector 1 are unchanged. This results in firms in sector 1 adjusting their price, while firms in other sectors k leave prices unchanged.

2. If the shock is below the threshold, $\gamma \in [1, \overline{\gamma})$, then optimal policy is to ensure that prices remain unchanged and no firm in any sector adjusts.

Additionally, the productivity threshold $\overline{\gamma}$ is increasing in the size of menu costs ψ.

Proof: The proof follows exactly the same steps as in the proof of proposition 1. □

Proposition 5 (Roundabout economy)**.** Consider again the positive productivity shock γ affecting sector 1, in the baseline model augmented with the roundabout production technology of (23). For a fixed level of menu costs ψ, there exists a threshold level of productivity $\overline{\gamma} > 1$, such that:

1. If the productivity shock to sector 1 is above the threshold, $\gamma \geq \overline{\gamma}$, then optimal policy is to ensure the nominal marginal costs of firms outside sector 1 are unchanged. This is implemented by stabilizing $W^{\beta}P^{1-\beta}$. This results in firms in sector 1 adjusting their price, while firms in other sectors k leave prices unchanged.
2. If the shock is below the threshold, $\gamma \in [1, \overline{\gamma})$, then optimal policy is to ensure that prices remain unchanged and no firm in any sector adjusts.

Proof: The proof follows exactly the same steps as in the proof of proposition 1. □

Proposition 6 (Sectoral heterogeneity)**.** Suppose sector i is of size S_i and has menu cost ψ_i. Suppose further that the size-weighted menu cost of sector 1 is smaller than the combined weighted sum of menu costs for other sectors, $S_1\psi_1 < \sum_{k>1} S_k\psi_k$. Then optimal monetary is exactly the same as characterized in proposition 1 modulo changes in the constant $\overline{\gamma}$.

Proof: Under the assumption about the magnitude of weighted menu costs, the proof follows exactly as in the proof of proposition 1. □

C.2 Optimal policy without selection effects

The existence (or not) of selection effects in menu cost models is an important question in the literature, due to the argument that selection effects reduce monetary non-neutrality relative to models with time-dependent pricing like the Calvo model (Golosov and Lucas 2007; Caballero and Engel 2007; Carvalho and Kryvtsov 2021; Karadi, Schoenle and Wursten 2022; Gautier et al. 2022). The question this literature generally considers is:

in response to a *monetary policy shock*, how much is real output affected? On the other hand, under optimal monetary policy naturally there are no monetary shocks.

In this subsection, we show that the existence or not of selection effects plays little role. We demonstrate this by briefly characterizing a "CalvoPlus" variant of the baseline model (Nakamura and Steinsson 2010). In the CalvoPlus variant, the setup is precisely as in section 2, except that menu costs are now idiosyncratic at the firm level, $\psi_i(j)$. In particular, a random fraction $\zeta \in (0,1)$ of firms in each has an opportunity to adjust price for free, $\psi_i(j) = 0$; other firms face the nonzero menu cost, $\psi_i(j) = \psi$.

As the next proposition describes, optimal policy is in essence the same as in the baseline result of proposition 1, though the threshold $\overline{\gamma}$ is altered.

Proposition 7 (Optimal policy under CalvoPlus)**.** Consider the baseline model, modified so that a random fraction $\zeta \in (0,1)$ of firms in each sector face no menu cost, $\psi_i(j) = 0$, and remaining firms face $\psi_i(j) = 1$. For a fixed level of menu costs ψ, there exists a threshold level of productivity $\overline{\gamma}_{\text{CalvoPlus}} > 1$, such that:

1. If the productivity shock to sector 1 is above the threshold, $\gamma \geq \overline{\gamma}_{\text{CalvoPlus}}$, then optimal policy is exactly nominal wage targeting: monetary policy should ensure $W = W^{ss}$. This results in all firms in sector 1 adjusting their price, while all firms in other sectors k leave prices unchanged regardless of whether they have a free adjustment.

2. If the shock is below the threshold, $\gamma \in [1, \overline{\gamma}_{\text{CalvoPlus}})$, then optimal policy is to ensure that prices remain unchanged for firms with a nonzero menu cost, $\psi_i(j) > 0$.

3. The threshold $\overline{\gamma}_{\text{CalvoPlus}}$ is smaller than in the baseline model:

$$\overline{\gamma}_{\text{CalvoPlus}} < \overline{\gamma} \tag{49}$$

We sketch the proof to provide economic intuition. The logic follows very closely to the proof of proposition 1.

By stabilizing W, the central bank ensures that the nominal marginal cost of unshocked firms remains unchanged. Unshocked firms thus have no desire to change price, even if they have a free opportunity to do so. In this case, correct relative prices are achieved, at a welfare cost of $(1-\zeta)\psi$ quantity of menu costs. For large enough shocks, this is optimal, because the welfare gains from correct relative prices outweighs the loss from paying the $(1-\zeta)\psi$ menu cost.

For small shocks, however, menu costs are too costly to be worthwhile. In this case, firms with a free adjustment will adjust – creating within-sector price dispersion – but it is optimal to ensure that firms with a nonzero menu cost do not adjust. In fact, *within* this case, the equilibrium is as-if Calvo: an exogenous fraction of firms in each sector is allowed to update prices for free, and the remaining firms do not adjust – here, endogenously, unlike Calvo. Optimal policy thus precisely replicates the Calvo optimum, *if* $\gamma < \overline{\gamma}_{\text{CalvoPlus}}$.

Observe that in the case where firms in sector 1 adjust, the welfare loss from menu costs is $(1-\zeta)\psi$, which is smaller than the level of ψ in the non-CalvoPlus economy. Thus, the fixed welfare loss from menu costs in this case is smaller. As a result, the increase in productivity needed to make price adjustments overcome this welfare loss is correspondingly smaller. This explains the third part of the proposition.

C.3 Model with segmented labor markets

We extend the baseline model to account for segmented labor markets, where each sector is populated by households that can only work in that sector. Therefore, wages, consumption, labor, and money holdings may differ across sectors. We show that the key result still holds: when one sector is hit by a shock, restricting price adjustments to one sector leads to higher welfare than allowing all sectors to adjust.

C.3.1 Households

In the segmented labor market setting, workers are sector-specific and wages differ across sectors. The household in sector i derives utility from consumption C_i, sector-specific labor supply N_i, and real money balances M_i/P. The representative household in sector i maximizes:

$$\mathbb{W}_i = \ln C_i - N_i + \ln\left(\frac{M_i}{P}\right) \tag{50}$$

subject to the budget constraint:

$$PC_i + M_i = W_i N_i + D + M_{-1} - T \tag{51}$$

where dividends D, transfers T, and the final good's price P are all common across sectors. The household chooses C_i, M_i, and N_i to maximize utility subject to the budget constraint. The first-order conditions result in the usual conditions

$$PC_i = M_i \tag{52}$$

$$W_i = M_i \tag{53}$$

Note, consumption, labor, and money supply are here sector-specific.

C.3.2 Firms

The production side remains identical to the baseline model.

C.3.3 Steady state

In steady state, all sectors are symmetric, and the following conditions hold:

$$A_i = 1, \quad W_i = \frac{1}{S}, \quad M_i = \frac{1}{S}, \quad p_i = \frac{1}{S}, \quad P = 1, \tag{54}$$

$$C_i = \frac{1}{S}, \quad C = Y = 1, \quad y_i = n_i = 1. \tag{55}$$

C.3.4 Welfare

We now analyze welfare under two different monetary policy regimes:

1. **Sector 1 adjusts:** The central bank sets M consistent with only sector-1 firms adjusting their prices in response to a productivity shock in sector 1.
2. **All sectors adjust and inflation targeting:** The central bank sets M consistent with all sectors adjusting their prices such that the aggregate price level remains constant.

Sector 1 adjusts. Suppose the central bank sets the money supply to M^* so as to make all firms in sector 1 but no firm in sectors $k > 1$ change price. This implies that $M_k = W_k = \frac{M}{S}$ after the shock. And, in turn,

$$M_1 = M^* - \frac{S-1}{S} M$$

In this first regime $p_k = p_k^* = \frac{1}{S}$ which, by the efficiency conditions equals PC_k while $p_1 = \frac{W_1}{\gamma} = \frac{M_1}{\gamma} = \frac{PC_1}{\gamma}$. The aggregate price level is

$$P = S \prod_i p_i^{\frac{1}{S}} = (Sp_1)^{\frac{1}{S}}$$

Including the price level in the above expressions pins down consumption in each sector:

$$\begin{cases} C_1 = \frac{\gamma}{S^{\frac{1}{S}}} p_1^{1-\frac{1}{S}} \\ C_k = S^{-\frac{S+1}{S}} p_1^{-\frac{1}{S}} = \frac{1}{S} \frac{1}{S^{\frac{1}{S}}} p_1^{-\frac{1}{S}} \end{cases}$$

and total consumption is simply

$$C = (Sp_1)^{-\frac{1}{S}} \left(\gamma p_1 + \frac{S-1}{S} \right)$$

The aggregate resource constraint is $Y = C$. In turn, $PY = PC = \left(\gamma p_1 + \frac{S-1}{S}\right)$. Sectoral demand then becomes

$$y_i = \frac{1}{S}\frac{PY}{p_i} = \frac{1}{S}\frac{\gamma p_1 + \frac{S-1}{S}}{p_i}$$

For $k > 1$, $p_k = \frac{1}{S}$ and so $y_k = (\gamma p_1 + \frac{S-1}{S})$, for sector 1, $y_1 = \frac{1}{S}\frac{\gamma p_1 + \frac{S-1}{S}}{p_1}$. In turn, labor demand is $n_k = y_k = \gamma p_1 + \frac{S-1}{S}$ and $n_1 = \frac{y_1}{\gamma} = \frac{1}{S}\frac{1}{\gamma}\frac{\gamma p_1 + \frac{S-1}{S}}{p_1}$. Labor supply in sector 1 is $N_1 = n_1 + \psi$, while in sectors k, just $N_k = n_k$. Rearranging the budget constraints of households in sector 1 and k, we have

$$PC_1 + M_1 - W_1 N_1 = D + M_{-1} - T$$

$$PC_k + M_k - W_k N_k = D + M_{-1} - T$$

The RHSs are identical and so must the LHSs. Substituting accordingly, gives

$$PC_1 + M_1 - W_1 N_1 = \gamma p_1 + \gamma p_1 + \gamma p_1 \cdot \left(\frac{1}{S}\frac{1}{\gamma}\frac{\gamma p_1 + \frac{S-1}{S}}{p_1} + \psi\right)$$

$$PC_k + M_k - W_k N_k = \frac{1}{S} + \frac{1}{S} + \frac{1}{S} \cdot \left(\gamma p_1 + \frac{S-1}{S}\right)$$

Solve for p_1 by denoting $x \equiv \gamma p_1$ and substituting in the previous two expressions and equate them

$$2x + x\left(\frac{1}{S}\frac{x + \frac{S-1}{S}}{x} + \psi\right) = \frac{2}{S} + \frac{x + \frac{S-1}{S}}{S}$$

which becomes

$$2x + \frac{x + \frac{S-1}{S}}{S} + \psi x = \frac{2}{S} + \frac{x + \frac{S-1}{S}}{S}$$

and so $(2+\psi)x = \frac{2}{S} \Rightarrow x = \frac{2}{S(2+\psi)}$ which means that

$$p_1 = \frac{2}{S\gamma}\frac{1}{2+\psi}$$

Note, if $\gamma = 1$ and $\psi = 0$, we get back to $p_1 = \frac{1}{S}$, as in steady state. With this price, we have that

$$P = \left(\frac{1}{\gamma} \cdot \frac{2}{2+\psi}\right)^{\frac{1}{S}}$$

$$C_1 = \frac{\gamma^{\frac{1}{S}}}{S}\left(\frac{2}{2+\psi}\right)^{1-\frac{1}{S}}$$

$$C_k = \frac{\gamma^{\frac{1}{S}}}{S}\left(\frac{2}{2+\psi}\right)^{-\frac{1}{S}}$$

$$M_1 = \frac{2}{S(2+\psi)}$$

$$M_k = \frac{1}{S}$$

$$n_1 = \frac{1}{S}\left(1 + \frac{2+\psi}{2}(S-1)\right) + \psi$$

$$n_k = \frac{1}{S}\left(\frac{2+\psi}{2} + S - 1\right)$$

And equally-weighted welfare is

$$\mathbb{W}^{\text{one adjusts}} = \ln C_1 - n_1 + \ln\left(\frac{M_1}{P}\right) + (S-1)\left[\ln C_k - n_k + \ln\left(\frac{M_k}{P}\right)\right]$$

which can be showns is equal to

$$\mathbb{W}^{\text{one adjusts}} = 2\left[\ln(\gamma) - S\ln(S)\right] - \left(\psi + \psi\frac{S-1}{S} + S\right) \tag{56}$$

Note that plugging in $\gamma = 1$ and $\psi = 0$ gives $\mathbb{W} = -2S\ln(S) - S$ which is steady state welfare.

All sectors adjust and inflation targeting. Suppose now that all sectors adjust in such a way that the price level stays fixed. This means $P = P^* = 1$. Symmetry across sectors $k > 1$ implies we can treat these sectors as identical and consider the money supply M_k for each sector. In turn $M_1 = M^* - \frac{S-1}{S}M_k$.

The price level will be

$$P = 1 = Sp_1^{\frac{1}{S}} \cdot p_k^{\frac{S-1}{S}}$$

from which we can derive the relationship between the prices of the sectors:

$$p_k = \left(S^S p_1\right)^{\frac{1}{1-S}} = p_1\left(Sp_1\right)^{-\frac{S}{S-1}}$$

We can then write consumption as

$$\begin{cases} C_1 = \gamma p_1 \\ C_k = p_1\left(Sp_1\right)^{-\frac{S}{S-1}} \end{cases}$$

Total consumption then is

$$C = p_1\left[\gamma + (S-1)\left(Sp_1\right)^{-\frac{S}{S-1}}\right]$$

This also equals PY and so sectoral demand is

$$y_1 = \frac{1}{S}\frac{PY}{p_1} = \frac{1}{S}\left[\gamma + (S-1)\left(Sp_1\right)^{-\frac{S}{S-1}}\right]$$

$$y_k = \frac{1}{S}\frac{PY}{p_k} = \frac{1}{S}\left[\gamma\left(Sp_1\right)^{\frac{S}{S-1}} + (S-1)\right]$$

This immediately lets us pin down labor demand into production $n_1 = \frac{1}{S}\frac{1}{\gamma}\left[\gamma + (S-1)\left(Sp_1\right)^{-\frac{S}{S-1}}\right]$ and $n_k = \frac{1}{S}\left[\gamma\left(Sp_1\right)^{\frac{S}{S-1}} + (S-1)\right]$. Because all sectors pay the menu cost, we have

$$N_1 = \frac{1}{S}\frac{1}{\gamma}\left[\gamma + (S-1)\left(Sp_1\right)^{-\frac{S}{S-1}}\right] + \psi$$

$$N_k = \frac{1}{S}\left[\gamma\left(Sp_1\right)^{\frac{S}{S-1}} + (S-1)\right] + \psi$$

Further note that $W_1 = p_1\gamma$ and $W_k = p_k = p_1\left(Sp_1\right)^{-\frac{S}{S-1}}$.

While we can solve for the relative price between sectors 1 and k, this is not necessary to compute welfare.

$$\mathbb{W}^{\text{all adjust}} = \ln(C_1) + (S-1)\ln(C_k) + \ln(M_1/P) + (S-1)\ln(M_k/P) - N_1 - (S-1)N_k$$

The first four terms boil down to

$$\ln(C_1) + (S-1)\ln(C_k) + \ln(M_1/P) + (S-1)\ln(M_k/P) = 2[\ln(C_1) + (S-1)\ln(C_k)]$$

plugging in the expressions for C_1 and C_k, we have

$$\begin{aligned} 2[\ln(C_1) + (S-1)\ln(C_k)] &= 2\left[\ln(\gamma p_1) + (S-1)\ln\left(p_1(Sp_1)^{-\frac{S}{S-1}}\right)\right] \\ &= 2\left[\ln(\gamma) + \ln(p_1) + (S-1)\ln(p_1) - S\ln(S) - S\ln(p_1)\right] \\ &= 2\left[\ln(\gamma) - S\ln(S)\right] \end{aligned}$$

which is the same exact term deriving from the consumption and real money balances from the one-sector adjusting only. This result is expected since the two economies are undistorted as relative prices have adjusted.

C.3.5 Comparison and optimal policy

All we need to do is compare the labor disutility in the two economies. In the "all-adjust" economy, we have

$$N^{\text{all adjust}} = N_1 + (S-1)N_k = \frac{1}{S} + \frac{S-1}{S\gamma}(Sp_1)^{-\frac{S}{S-1}} + \frac{(S-1)^2}{S} + \frac{S-1}{S}\gamma(Sp_1)^{\frac{S}{S-1}} + S\psi$$

$$
\begin{aligned}
&= S\psi + \frac{1+S^2-2S+1}{S} + \frac{S-1}{S\gamma}(Sp_1)^{-\frac{S}{S-1}} + \frac{S-1}{S}\gamma(Sp_1)^{\frac{S}{S-1}} \\
&= S\psi + (S-2) + \frac{2}{S} + \frac{S-1}{S}\underbrace{\left[\frac{1}{\gamma(Sp_1)^{\frac{S}{S-1}}} + \gamma(Sp_1)^{\frac{S}{S-1}}\right]}_{>2} \\
&> S\psi + (S-2) + \frac{2}{S} + 2\frac{S-1}{S} = S\psi + S
\end{aligned}
$$

while total labor in the "one-sector adjusts" economy is $N^{\text{one adjusts}} = \psi\left(\frac{2S-1}{S}\right) + S$.

Therefore:

$$
\begin{aligned}
\mathbb{W}^{\text{one adjusts}} - \mathbb{W}^{\text{all adjust}} &= -(N^{\text{one}} - N^{\text{all}}) = S\psi + S - \psi\left(\frac{2S-1}{S}\right) - S = \psi\frac{S^2-2S+1}{S} \\
&= \psi\frac{(S-1)^2}{S} > 0 \qquad (57)
\end{aligned}
$$

Therefore, in the version of the model in which labor markets are segmented, we still have that the central bank should aim to have only sector 1 change its price in response to a productivity shock to sector 1.

C.4 Sticky wages

Throughout the paper thus far, we have considered a model with sticky prices and flexible wages; we now consider the case of flexible prices and sticky wages. Optimal policy in response to the same sectoral productivity shock continues to stabilize nominal wages, again in order stabilize the nominal marginal cost of unshocked sectors and thus minimize menu cost expenditure.

Setup. Consider the baseline model of section 2, but now allow for heterogenous types of labor organized into a union with wage-setting power. Suppose there are S labor sectors, and each goods-producing sector $i = 1, ..., S$ hires labor exclusively from the corresponding labor sector $i = 1, ..., S$. Within each labor sector, there is a continuum of differentiated worker types, indexed on $[0, 1]$. Continue to denote the total amount of labor used by goods-producing firm j in sector i as $n_i(j)$, and continue to endow intermediate firms with technology $y_i(j) = A_i n_i(j)$.

Firm-level labor input $n_i(j)$ is now, unlike in the baseline model, composed of a CES bundle of workers from labor sector i:

$$
n_i(j) = \left[\int_0^1 \left(n_i^k(j)\right)^{\frac{\varepsilon-1}{\varepsilon}} dk\right]^{\frac{\varepsilon}{\varepsilon-1}}
$$

where $n_i^k(j)$ denotes the quantity of labor of type k in labor sector i hired by firm j in goods-producing sector i; and ε is the elasticity of substitution across labor types. Cost minimization produces the standard demand curve, $n_i^k(j) = \left(\frac{W_i(k)}{W_i}\right)^{-\varepsilon} n_i(j)$, where $W_i(k)$ is the nominal wage of labor type k in sector i and W_i is the wage index for sector i labor. This results in a profit function of $D_i(j) = p_i(j)y_i(j) - W_i n_i(j)$.

For firms in sector i, the optimal reset price (i.e. the nominal marginal cost) now depends on the *sector-specific* wage and sectoral productivity:

$$p_i^{\text{flex}}(j) = MC_i(j) = \frac{W_i}{A_i} \tag{58}$$

Because prices are now flexible – that is, firms face $\psi = 0$ – firms always set price equal to this optimal reset price.

The nominal wage for worker type j in sector i, $W_i(j)$, is set by a union which must pay a fixed cost for nominal wage changes: the "menu cost". This can be motivated by a fixed cost of contract renegotiation. The union's problem can be considered as part of the representative household's problem, which is written as:

$$\begin{aligned}
\max_{C,M,\{W_i(j)\}_{i,j}} \quad & \ln C + \ln\left(\frac{M}{P}\right) - \sum_{i=1}^{S}\int_0^1 N_i(j)dj \\
\text{s.t. } PC + M &= (1+\tau^W)\sum_{i=1}^{S}\int_0^1 W_i(j)N_i(j)dj + M_{-1} + D - T - \psi^W\chi^W \\
N_i(j) &= \left(\frac{W_i(j)}{W_i}\right)^{-\varepsilon} N_i
\end{aligned}$$

This utility maximization problem differs from that in section 2.1 in a few ways. The household sets nominal wages, given the demand curves for labor types, analogous to the price-setting problem of intermediate firms. The household receives a labor subsidy τ^W to offset the monopsony distortion from its wage-setting power, analogous to the labor subsidy that firms are given to offset the monopoly distortion from firm price-setting power. Finally, the household must pay a menu cost ψ^W if it wishes to change any wage, analogous to the menu cost facing firms. The variable χ^W measures the mass of wages which are changed:

$$\chi^W \equiv \sum_i \int_j \chi_i^W(j)dj$$

$$\chi_i^W(j) \equiv \mathbb{I}\{W_i(j) \neq W_i^{\text{old}}(j)\}$$

where $W_i^{\text{old}}(j)$ is the inherited nominal wage for type j in sector i, analogous to $p_i^{\text{old}}(j)$ in the firm's problem.

The optimality conditions of this optimization problem include the equation of exchange (2) but also a new condition for optimal wage-setting. Under the optimal labor subsidy of $\tau^W = \frac{1}{\varepsilon - 1}$, the optimal wage-setting condition *conditional on adjusting* is determined by the marginal rate of substitution between consumption and leisure:

$$W_i^{\text{flex}}(j) = PC = M \quad \forall i, j \tag{59}$$

Note that under flexible wages, wages across types and sectors are all equalized.

In equilibrium, the wage menu cost paid by the household creates a wedge between consumption and output:

$$C = Y - \psi^W \chi^W \tag{60}$$

This aggregate resource constraint is derived from the household budget constraint and market clearing conditions.

We summarize the important differences with the baseline model. First, the nominal marginal costs of the intermediate goods producers (58) now depend on a sector-specific wage, $MC_i(j) = W_i / A_i$. Firms always set price equal to this level because prices are flexible. Second, in the efficient flexible-wage equilibrium, all nominal wages are equalized from (59). Third, wage menu costs result in a wedge between consumption and output, from (60), and lower welfare.

Optimal policy after a sectoral shock. Consider again the same exercise: starting from a steady state with $A_i^{ss} = 1$ and $M = 1$, shock productivity of sector-1 goods producers, $A_1 = \gamma > 1$. For clarity, consider the case where γ is sufficiently large, so that we do not need to discuss an analog to the $\overline{\gamma}$ of proposition 1.

The optimal reset prices and wages – i.e., those that would prevail in the frictionless equilibrium – are:

$$\begin{aligned} p_1^{\text{flex}}(j) &= \frac{W_1}{\gamma} \\ p_k^{\text{flex}}(j) &= W_k \quad \forall k > 1 \\ W_i^{\text{flex}}(j) &= M \quad \forall i, j \end{aligned}$$

To minimize the amount of menu costs and simultaneously achieve correct relative prices, it is again desirable to leave M unchanged, thereby stabilizing nominal wages. This ensures that no wages need to be adjusted and no wage menu costs need to be paid: that is, $W_i = W_i^{ss}$ for all i. Meanwhile, given the shock was assumed to be sufficiently large, goods-producing firms in sector 1 can update their prices, thus ensuring all relative prices are correct.

In short, optimal policy stabilizes all nominal wages, which ensures correct relative prices and causes only sector-1 firms to adjust.