Optimal Monetary Policy with Menu Costs is Nominal Wage Targeting

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Abstract

We show analytically that ensuring stable nominal wage growth is optimal monetary policy in a multisector economy with menu costs. This *nominal wage targeting* contrasts with inflation targeting, the optimal policy prescribed by the textbook New Keynesian model in which firms are permitted to adjust their prices only randomly and exogenously. The intuition is that stabilizing nominal wages minimizes the number of firms which need to adjust their prices, and therefore minimizes resources wasted on menu costs. We show that the analytical result that nominal wage targeting is superior to inflation targeting carries over in a rich quantitative model.

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1 Introduction

Many central banks around the world have adopted some form of inflation targeting over the past three decades. The textbook formulation of the New Keynesian model provides theoretical grounding for such policies: in the Calvo-Yun formulation of the New Keynesian model, where firms are only randomly given the opportunity to change prices, optimal policy is strict inflation targeting. This is true in the textbook one-sector New Keynesian model (Woodford 2003) as well as in multisector versions of the model, for an appropriately-defined price index (Rubbo 2022). The Calvo-Yun assumption of random price changes is mathematically convenient, but arguably comes at the cost of realism. A natural – but notoriously less tractable – alternative is "menu cost" models in which firms can *choose* to change their prices at any time, but must pay a fixed "menu" cost to do so.

We show analytically and without linearization that optimal monetary policy is nominal wage targeting in a multisector economy with menu costs, not inflation targeting. We do so by developing a model in which the economy is made up of sectors where firms are subject to sector-specific productivity shocks and can change their price at any point by paying a menu cost. Optimally, nominal wages should be stabilized, and inflation should *not* be stable but move inversely with output. This is despite wages themselves being completely flexible.[1]

The intuition for this result is that stabilizing nominal wages minimizes the number of firms adjusting their price and incurring a menu cost, while still achieving undistorted relative prices. Consider for example a positive productivity shock affecting only firms in sector 1. If the shock is sufficiently large, then it is efficient and desirable for firms in this sector to cut their *relative* prices, compared to other firms in other sectors of the economy. Under a policy of constant nominal wages, firms outside of sector 1 have no desire to adjust their prices: firms wish to adjust their prices only if their nominal marginal costs change, and their nominal marginal costs are unchanged because their productivity is unchanged and nominal wages are kept stable. Meanwhile, firms in sector 1 choose to adjust their nominal prices because of the productivity shock. As a result, relative prices between sector-1 firms and other firms are undistorted – because firms in sector 1 updated their prices – and *only* this one sector has incurred wasteful menu costs.

Alternatively, if the productivity shock is small, then the welfare gain from firms in sector 1 updating their prices – ensuring correct relative prices – will not outweigh the welfare loss due to the menu costs they need to pay to do so. In this case, it is optimal to

[1] This policy of targeting constant nominal wages can be generalized to a policy of targeting constant nominal wage *growth* by allowing firms to index, analogous to a 2% inflation target rather than 0% in the baseline New Keynesian model (Yun 1996).

ensure that no firm changes price, thus avoiding menu costs altogether. However, since the shock is small, nominal wages nonetheless remain approximately constant.

Inflation targeting by contrast, in order to achieve undistorted relative prices, always requires *every* sector to pay a wasteful menu cost. Following the productivity shock, the relative price of sector 1 needs to fall to achieve efficiency. Inflation targeting requires that the overall price level be unchanged. To simultaneously have the relative price fall *and* the price level be stable requires that both sector 1 firms cut their nominal prices and that firms in all other sectors raise their nominal prices. As a result, all firms are forced to adjust their prices and pay a menu cost. This is unnecessarily wasteful because the same efficient allocation can be achieved via a nominal wage target with only sector 1 firms paying the menu cost. As noted above, such a policy of inflation targeting would be optimal in the same New Keynesian setting with exogenous price stickiness, for an appropriately-defined price index.[2]

The welfare loss of inflation targeting is precisely due to the excess menu costs that optimal policy avoids, which we also quantify in a dynamic version of the model. This model is calibrated to US data and, on top of sectoral shocks, allows for idiosyncratic shocks, another major source of price changes. We compare inflation and nominal wage targeting and show that nominal wage targeting would lead to a significant welfare improvement over inflation targeting. This result follows from the large estimates of menu costs in the empirical literature. For example, Nakamura and Steinsson (2010) estimate menu costs to be 0.5% of total firm revenues annually (an order of magnitude larger than the famous "welfare loss of business cycles" estimate of 0.05% of Lucas 1987). More generally, in this paper we think of menu costs not only as the physical costs of adjusting prices, but as a reduced-form way of capturing any economic or welfare cost of price adjustment, such as the mental attention costs suffered by price setters.

Optimal policy can also be seen as a form of nominal income targeting. In the example above, the positive productivity shock in sector 1 implies that output in sector 1 goes up while prices in sector 1 fall. Other sectors are unaffected, so these changes in sector 1 pass through to overall aggregates: aggregate output Y rises and the aggregate price level P falls, i.e. inflation moves inversely with output. Under baseline functional forms for preferences, under which nominal income and nominal wages are equal, this implies that nominal income targeting is exactly optimal – the central bank should stabilize nominal

[2] Rubbo (2022) shows this in a multisector model with a general input-output structure and the textbook Calvo-Yun friction. Woodford (2003), Aoki (2001), and Benigno (2004) show the same in models without the general network structure, as in the environment presented here, again under the Calvo-Yun friction. Kreamer (2022) also studies optimal monetary policy in a sectoral model, with fixed prices and durable goods; Guerrieri et al. (2021) also study optimal monetary policy in a sectoral model, with downward nominal wage rigidity.

income $P \times Y$. More generally, the policy of nominal wage targeting can also be thought of as a form of generalized nominal income targeting, where P and Y move inversely, but not necessarily one-for-one.

Position in literature. To our knowledge, we are the first to characterize optimal monetary policy in the face of fixed menu costs when firms *also* have a motive to adjust relative prices.[3] On the one hand, without changes in productivity between firms, there is no motive for relative-price changes and so optimal policy under menu costs is trivially zero inflation: prices never need to move and price stickiness is irrelevant (see e.g. Nakov and Thomas 2014). On the other hand, several papers allow for relative-price movements under menu costs but take as *given* that the central bank targets inflation, and simulate numerically how the presence of menu costs affects the optimal level of inflation (e.g. Blanco 2021, Nakov and Thomas 2014, Adam and Weber 2019, Wolman 2011). A larger literature makes assumptions on monetary policy – i.e. does not analyze optimal policy – and asks how the presence of menu costs affects macroeconomic dynamics (among others, Caplin and Spulber 1987; Golosov and Lucas 2007; Gertler and Leahy 2008[4]; Nakamura and Steinsson 2010; Midrigan 2011; Alvarez, Lippi and Paciello 2011; Auclert et al. 2022). That is, these papers conduct a positive analysis, while we conduct a normative analysis. There is also a large empirical literature on menu costs that stresses the importance of endogenous price setting to match key aspects of the data.[5]

We see our paper as helping unify the literature on optimal monetary policy. In the last decade a number of papers across a variety of classes of models have found that policies of "approximate nominal income targeting" are optimal; however, sticky price models – the workhorse model of modern monetary economics – had conspicuously held out for the optimality of inflation targeting. First, Koenig (2013) and Sheedy (2014) show in heterogeneous agent models that when financial markets are incomplete and debt is written in nominal, non-state contingent terms, nominal income targeting is optimal and inflation targeting is suboptimal. Werning (2014) notes that if heterogeneity is added to

[3] We emphasize *fixed*, nonconvex menu costs – as pioneered by Barro (1972) and Sheshinski and Weiss (1977) – to distinguish from *convex* menu costs that scale with the size of the price change, such as the Rotemberg (1982) quadratic cost of price adjustment. Section 4.2 contrasts nonconvex and convex menu cost models in detail.

[4] Gertler and Leahy analytically derive a log-linearized model with menu costs and observation costs isomorphic to the textbook three-equation New Keynesian model. To do so, they make use of a series of extremely clever simplifying assumptions. However, critically they make assumptions on monetary policy to ensure that monetary shocks alone are "small" enough to never induce a firm to adjust its price. As we will see, inflation targeting requires such shocks.

[5] Among many others: Alvarez et al. (2019); Ascari and Haber (2021); Nakamura et al. (2018); Klenow and Kryvtsov (2008).

the model, then "approximate" nominal income targeting is optimal: P and Y should move inversely but not one-for-one. This also echoes our result below. Second, Angeletos and La'O (2020) show that in a world where agents have incomplete information about the economy, under similar functional forms to the ones we use below, nominal income targeting is optimal monetary policy; and more generally that the price level P and real output Y should move in opposite directions. Third, it has long been known that when wages are sticky, optimal monetary policy is to stabilize nominal wages. Despite these results for three highly important classes of models – incomplete markets, information frictions, and sticky wages – it may have been easy to set them aside and nonetheless consider inflation targeting as the proper baseline for optimal monetary policy due to its optimality in the workhorse sticky price model (e.g. Woodford 2003). We hope our paper helps to conceptually integrate these results from across the incomplete market, information friction, sticky wage, and sticky price models.

Our model formalizes and extends the insightful, literary argument made by Selgin (1997) (chapter 2, section 3) that nominal income targeting, or something like it, is optimal in a world with menu costs.[6] Relative to Selgin's elegant informal discussion, we are able to introduce the role of state dependence, which is natural in the context of menu costs and does affect optimal policy, as well as to formalize and be precise about the argument in the context of a standard macro model. This formalization allows us to distinguish between nominal wage targeting versus nominal income targeting, to connect our results to prior modeling work, and to take the model to the data to quantify the welfare costs of inflation targeting.

Outline. We first illustrate the optimal policy result in sections 2 and 3 in a baseline setting as described above: an off-the-shelf sectoral model, augmented with menu costs, hit by an unanticipated sectoral productivity shock. In section 4, we discuss the welfare loss of inflation targeting and use our setup to shed new light on the conventional New Keynesian model. In section 5, we show that the optimality of nominal wage targeting continues to hold under a number of extensions: functional form generalizations; velocity shocks; heterogeneity in sector size and menu costs; and multiple productivity shocks. In section 6, we generalize further by building a quantitative model in order to incorporate dynamics and calculate the welfare gains of adopting nominal wage targeting. Section 7 concludes.

[6] The nominal contracts and incomplete information literatures cited above were both also preceded and discussed very clearly by Selgin (1997).

2 Baseline model

Our baseline framework is a two-period model starting at steady state. There are S sectors, each consisting of a continuum of monopolistically competitive intermediate firms which are aggregated into a sectoral good by a competitive sectoral packager. A competitive final goods producer combines the output of each of the S sectors into a final good, sold to the household. The model and the functional forms we use are the same as Golosov and Lucas (2007), except that productivity shocks are sectoral rather than firm-specific and that we analyze optimal monetary policy instead of exogenous monetary shocks. In section 5, we describe how all of the functional forms here can be made quite general.

The two key assumptions are (1) productivity shocks that move relative prices, and (2) fixed menu costs which intermediate firms must pay if they choose to adjust their nominal price.

2.1 Household

The household's preferences are given by

$$\mathbb{W} = \ln C - N + \ln\left(\frac{M}{P}\right) \tag{1}$$

as a function of consumption C, labor N, and real money holdings $\frac{M}{P}$.[7] The household's problem is:

$$\max_{C,N,M} \ln(C) - N + \ln\left(\frac{M}{P}\right)$$
$$\text{s.t. } PC + M = WN + D + M_{-1} - T$$

To fund expenditures the household uses labor income from wages W, firm dividends D, and previous period money balances M_{-1}, less taxes T. The first order conditions imply:

$$PC = M \tag{2}$$
$$W = M \tag{3}$$

Our particularly simple assumptions on preferences – again matching those of Golosov and Lucas (2007) – result in two simple optimality conditions: an equation of exchange

[7] We follow Woodford (1998) in ignoring the welfare effects of real balances when analyzing optimal monetary policy.

(2) and an equation (3) stating that in equilibrium the nominal wage W is directly determined by the money supply M. In section 5.1 we discuss how the utility function can be generalized to any other function in the class of Greenwood, Hercowitz and Huffman (1988) forms without affecting the optimal policy result.

2.2 Final good producer

The representative final good producer aggregates sectoral goods y_i of price p_i across S sectors, using Cobb-Douglas technology, into the final good Y consumed by the household. Operating under perfect competition, its problem is:

$$\max_{\{y_i\}_{i=1}^{S}} PY - \sum_{i=1}^{S} p_i y_i$$
$$\text{s.t. } Y = \prod_{i=1}^{S} y_i^{1/S} \tag{4}$$

The resulting demand for sectoral goods is:

$$y_i = \frac{1}{S}\frac{PY}{p_i} \tag{5}$$

The zero profit condition gives the price P for the final good:

$$P = S \prod_{i=1}^{S} p_i^{1/S} \tag{6}$$

In section 5.1 we discuss how generalizing the Cobb-Douglas functional form used here has no impact on the optimal policy result.

2.3 Sectoral goods producers

In a sector i, a representative sectoral goods producer packages the continuum of intermediate goods, $y_i(j)$, produced within the sector using CES technology. Note that for notational clarity, we will consistently use j to identify an intermediate firm and i to identify a sector. The problem of the sectoral packager for sector i is:

$$\max_{[y_i(j)]_{j=0}^{1}} p_i y_i - \int_0^1 p_i(j) y_i(j) dj$$

$$\text{s.t. } y_i = \left[\int_0^1 y_i(j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{7}$$

This results in a demand function $y_i(j)$ and a sectoral price index p_i:

$$y_i(j) = y_i \left(\frac{p_i(j)}{p_i}\right)^{-\eta} \tag{8}$$

$$p_i = \left[\int_0^1 p_i(j)^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{9}$$

2.4 Intermediate goods producers

In each sector there is a unit mass of monopolistically competitive firms each producing a different variety of the sectoral good. Their technology is linear, and all firms within a sector i share a common productivity level A_i.[8]

Intermediate firms are subject to menu costs: if they choose to adjust their price, they must hire an extra ψ units of labor in order to do so. This fixed cost of price adjustment, $W\psi$, is what we refer to as a "menu cost". The menu cost itself is simply a transfer from firm profits to household labor income; the *welfare cost* of menu costs comes from the fact that households must supply extra labor in order for prices to be adjusted, and there is a disutility cost to labor. Modeling menu costs in other ways does not affect the optimal policy conclusions.[9]

Firm j in sector i thus maximizes profits, including the menu cost if choosing to adjust its price, subject to its demand curve and its production technology, taking as given the inherited price from the previous period $p_i^{\text{old}}(j)$:

[8]Note that it is standard in the optimal policy literature on sectors and networks to only consider the optimal policy response to *sector*-level productivity shocks: see for example Rubbo (2022), Woodford (2003), Aoki (2001), or Benigno (2004). In particular, these papers do *not* consider idiosyncratic, firm-level productivity differences. In contrast, in the separate literature on menu costs, it is common to consider such idiosyncratic shocks (Golosov and Lucas 2007). We analyze the case of both sectoral and idiosyncratic shocks using the quantitative model in section 6.

[9]One example of an alternate modeling method is when menu costs burn real resources (as in Alvarez, Lippi and Paciello 2011), and therefore lower the level of profits transferred to households. Another, more behavioral, modeling method would be to model menu costs as directly inflicting a utility penalty on households, as in Auclert, Rognlie and Straub (2018) and as could be motivated by the literature on fairness in pricing (e.g. Eyster, Madarász and Michaillat 2021).

It is straightforward to show that optimal policy is the same if menu costs are modeled in either of these ways. This is because the core intuition remains unchanged: optimal policy still seeks to stabilize the desired price of unshocked firms in order to minimize menu costs. In fact the main result in proposition 1 can become even more general than how it is presented here – under a utility penalty, we no longer need assume GHH preferences, and the proposition holds under any functional form for preferences. We use a labor cost instead of the other possibilities to remain closer to most of the existing literature.

$$
\begin{aligned}
&\max_{p_i(j)} p_i(j) y_i(j) - W n_i(j)(1-\tau) - W\psi\chi_i(j) \\
&\text{s.t. } \chi_i(j) = \begin{Bmatrix} 1 & \text{if } p_i(j) \neq p_i^{\text{old}}(j) \\ 0 & \text{else} \end{Bmatrix} \\
&\quad\; y_i(j) = y_i \left(\frac{p_i(j)}{p_i}\right)^{-\eta} \\
&\quad\; y_i(j) = A_i n_i(j)
\end{aligned} \tag{10}
$$

The objective function defines firm profits $D_i(j)$. The variable $\chi_i(j) \in \{0, 1\}$ is a dummy indicating whether or not the firm chooses to adjust its price, $p_i(j)$. If it does, it incurs the menu cost $W\psi$. Otherwise, the price is forced to remain at the inherited level from the previous period, denoted $p_i^{\text{old}}(j)$. The term τ in the firm's problem is the standard labor subsidy provided by the fiscal authority to undo the markup distortion from monopolistic competition, $\tau = \frac{1}{\eta}$, for each unit of labor used in production, $n_i(j)$. Equation (10) assumes a linear technology for producing output $y_i(j)$ for simplicity, an assumption which again can be generalized without affecting the conclusion.

If the firm chooses to pay the menu cost and adjust its price, then – from the firm's first order condition – the optimal reset price equals the nominal marginal cost:

$$
p_i(j) = \frac{W}{A_i} \tag{11}
$$

Notice that, because productivity is sector-specific, all firms j within a sector i face the same decision problem, and thus all make the same decision on whether to adjust and choose the same reset price. Because of this equivalence, we will often refer interchangeably to firm-specific versus sector-specific prices and quantities, e.g. $p_i(j)$ versus p_i and $n_i(j)$ versus n_i.[10]

2.5 The intermediate firm's adjustment decision

We now turn to the question of whether a given intermediate firm will pay the menu cost to adjust its price. The firm makes it decision to adjust by comparing profits under the new optimal price $\frac{W}{A_i}$ net of the menu cost $W\psi$ versus profits under the inherited price p_i^{old} without the loss from menu costs. Plugging in the respective prices as well as constraints into the profit function and comparing, we arrive at the price-adjustment

[10]Where sectoral labor is defined naturally as $n_i \equiv \int_0^1 n_i(j)dj$.

condition: firm j in sector i will adjust if and only if

$$\left(\frac{W}{A_i}\right)^{1-\eta} p_i^{\eta} y_i \left[\frac{1}{\eta}\right] - W\psi > \left(p_i^{\text{old}}\right)^{1-\eta} p_i^{\eta} y_i \left[1 - \frac{W/A_i}{p_i^{\text{old}}} \cdot \frac{\eta - 1}{\eta}\right] \tag{12}$$

This nonlinear adjustment condition implies an inaction region Λ – a standard result in menu cost models – as the following lemma describes.

Lemma 1 (Inaction region)**.** There exists an inaction region Λ in (W, A_i) space such that a firm in sector i will not adjust its price if and only if the value of (W, A_i) remains within this inaction region:

$$(W, A_i) \in \Lambda \tag{13}$$

The larger is menu cost ψ, the larger is this inaction region. The locus of points that result in the new optimal price equaling the inherited price, $W/A_i = p_i^{\text{old}}$, always lies within the inaction region. The inaction region is a connected set.

Proof: See Appendix A.1. □

To interpret this, note that the desired reset price W/A_i depends on two factors:

1. The sectoral productivity A_i, which is exogenous.
2. The level of nominal wages W, which we saw from (3) is completely determined by the central bank, $W = M$, in equilibrium.

Thus, firms are more likely to adjust after either a large productivity shock or a large monetary action, all else equal.

2.6 Market clearing

Labor market clearing implies that total labor supplied by the household, N, equals labor demanded in production, $\sum_i n_i$, plus the amount of labor required to adjust prices, which is $\psi \sum_i \chi_i$:

$$N = \sum_{i=1}^{S} n_i + \psi \sum_{i=1}^{S} \chi_i \tag{14}$$

This market clearing condition is key to the welfare costs of menu costs. Since labor supply N enters the household utility function negatively, then, larger menu costs ψ requiring the household to work more to adjust prices will lower household welfare.

The rest of market clearing is standard. Denote aggregate productivity as:

$$A \equiv \frac{1}{S} \prod_{i=1}^{S} A_i^{1/S} \tag{15}$$

The government budget constraint is: $T + (M - M_{-1}) = \tau W \sum_{i=1}^{S} n_i$. Market clearing ensures that dividends add up, $D = \sum_i D_i$. Finally, the aggregate resource constraint implies that consumption equals aggregate output:

$$C = Y \tag{16}$$

2.7 Steady state

The economy begins in a symmetric, flexible-price steady state (steady state variables are denoted with a superscript ss) in which sectoral productivities A_i^{ss} for $i \in \{1, \ldots, S\}$ are taken as given and nominal wages are normalized to $W^{ss} = 1$. Without loss of generality, we can set $A_i^{ss} = 1$ for all i, so that $A^{ss} = 1/S$.

The money supply from (3) is then $M^{ss} = 1$. Firms set prices at their flexible levels (11), $p_i^{ss} = 1$. The aggregate price level (6) is $P^{ss} = S$. From money demand (2), consumption and therefore output are equal to aggregate productivity, $C^{ss} = Y^{ss} = M^{ss}/P^{ss} = 1/S$. From demand equations (8) and (5), sectoral output is $y_i^{ss} = \frac{1}{S}$. From intermediate production technology (10) we recover labor in sector i as $n_i^{ss} = \frac{1}{S}$ and aggregate labor from market clearing (14) as $N^{ss} = 1$.

3 Optimal policy after a productivity shock

As our baseline exercise, we consider the optimal response to an unexpected shock to sector 1 alone. For concreteness, consider a positive productivity shock, which we denote as $\gamma > A_1^{ss} = 1$. How should monetary policy optimally set the money supply M?

Because in the initial steady state all sectors have the same productivity normalized to one, firms in sectors $i = 2, ..., S$ all face precisely the same problem after the shock to sector 1 and make the same decision on whether and how to adjust. As a result, for our purposes in this section there are effectively two sectors of different sizes, sector 1 (with productivity $\gamma > 1$ and size 1) and sectors k (with productivity $A_k = 1$ and size $S - 1$). Section 5.3 discusses how this would generalize. We will consistently identify variables for these unshocked sectors with a k. The relative price between the shocked and unshocked sectors, p_1/p_k, will be a key object of analysis.

Proposition 1, which is our core result, characterizes optimal monetary policy in response to this shock.

Proposition 1 (Optimal monetary policy)**.** For a fixed level of menu costs ψ, there exists a threshold level of productivity $\overline{\gamma} > 1$, such that:

1. If the productivity shock to sector 1 is above the threshold, $\gamma \geq \overline{\gamma}$, then optimal policy is exactly nominal wage targeting: monetary policy should ensure $W = W^{ss}$. This is implemented by having only firms in sector 1 adjust their price, while firms in other sectors k leave prices unchanged.
2. If the shock is below the threshold, $\gamma \in [1, \overline{\gamma})$, then optimal policy is to ensure that prices remain unchanged and no firm in any sector adjusts.

Additionally, the productivity threshold $\overline{\gamma}$ is increasing in the size of menu costs ψ.

Proof: Lemma 2 and lemma 3 below directly imply the proposition. □

The economic intuition for this result is exactly as previewed in the introduction. For a sufficiently large productivity shock $\gamma \geq \overline{\gamma}$, it is efficient for the relative price of sector 1, p_1/p_k, to update. To achieve this while simultaneously minimizing the number of sectors which must incur a wasteful menu cost, it is only necessary that firms in sector 1 update their price p_1 – firms in other sectors do not need to update p_k. To ensure that firms in other sectors have no desire to update, the central bank wants to stabilize the level of nominal wages, W, so that the nominal marginal cost of firms in these other sectors have no motive to adjust their prices. On the other hand, for a small productivity shock $\gamma < \overline{\gamma}$, the benefit of updating the relative price p_1/p_k does not outweigh the welfare loss from the menu cost necessary to do so; and it is therefore optimal to ensure that prices remain unchanged across all sectors.

We now step through the math behind this intuition in more detail. We build up to lemma 2 and lemma 3, which together prove proposition 1.

3.1 Allocations in four possible regimes

Because there are two types of firms (those in sector 1 hit with productivity shock γ, and those in other sectors k with unchanged productivity) and each type has a binary choice (adjusting or not adjusting its price), there are 2×2 possible regimes which may occur in equilibrium:

1. Both sector 1 and sectors k adjust prices
2. Only sector 1 adjusts its price; sectors k do not adjust

3. Only sectors k adjusts their prices; sector 1 does not adjust

4. Neither sector 1 nor sectors k adjusts price

Furthermore, the central bank helps to determine which of these regimes occurs in equilibrium. This is because a firm in some sector i decides to adjust if its target price, $\frac{W}{A_i}$, is outside its inaction region (13). Since the central bank can move nominal wages W by its choice of money supply M, the central bank helps to determine whether a firm is pushed outside its inaction region or not. (Note that there is always a unique equilibrium for a given choice of M – this is not a choice of *equilibrium* selection, but a choice by monetary policy of how much to increase aggregate demand.)

The optimal policy problem thus consists of:

1. Considering each of these regimes individually, and choosing M to maximize welfare *conditional* on the given regime;

2. Then, choosing the regime among the four which has the highest welfare, and implementing the associated optimal M.

This optimal policy problem, formalized in (42) in appendix A.2, is necessarily piecewise due to the sharp discontinuities created by the discontinuous pricing rules of (13), themselves the result of the fixed menu costs.

We now consider each of these four regimes individually, after discussing a benchmark of flexible prices. The ensuing subsection compares across the four.

Flexible price benchmark. As a benchmark, first consider the flexible price allocation, where the menu cost $\psi = 0$. Nominal wages are determined by the money supply, $W = M$, so that the flexibly-adjusted prices are $p_1 = \frac{M}{\gamma}$ and $p_k = M$. Observe that under flexibility, the relative price across types $\frac{p_1}{p_k}$ is:

$$\left(\frac{p_1}{p_k}\right)_{\text{flex}} = \frac{1}{\gamma}$$

This is an important object. This flexible relative price results in aggregate output and consumption equal to $Y = C = \frac{\gamma^{1/S}}{S}$. Total labor, as in steady state, is $N = 1$. Plugging these quantities into the household utility function (1), we have a flexible-price benchmark for welfare of:

$$\mathbb{W}_{\text{flex}} = \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1 \tag{17}$$

The flexible-price level of welfare is the first-best, efficient benchmark to which policy should be compared.

All sectors adjust. Next, return to the world where there are nonzero menu costs, and consider the case where all sectors pay the menu cost to adjust. Because all firms adjust to the flexible levels of $p_1 = \frac{M}{\gamma}$ and $p_k = M$, the relative price achieves the flexible price level:

$$\left(\frac{p_1}{p_k}\right)_{\text{all adjust}} = \frac{1}{\gamma} = \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

However, despite prices flexibly adjusting, the equilibrium differs from the flexible-price equilibrium because additional labor is required to pay the menu costs of price adjustment. This is where the assumption of GHH preferences plays a useful simplifying role: GHH preferences ensure that there are no income effects on labor supply, so that the additional labor required for menu costs has no effect on equilibrium *except* to increase the amount of labor used.[11] That is, prices and quantities are the same as the flexible price equilibrium, *except* for the additional labor which must be hired to pay for the menu costs: $N = 1 + S\psi$, where $S\psi$ reflects that there are S sectors which must hire ψ units of labor each to adjust price.

All together, this means that – conditional on all sectors adjusting – welfare is independent of monetary policy because all prices flexibly adjust, and it is equal to the flexible-price level minus the S sectors' worth of menu costs:

$$\begin{aligned} \mathbb{W}_{\text{all adjust}} &= \ln\left(\frac{\gamma^{1/S}}{S}\right) - [1 + S\psi] \\ &= \mathbb{W}_{\text{flex}} - S\psi \end{aligned} \tag{18}$$

Only sector 1 adjusts. Next consider if only sector 1 updates to $p_1 = \frac{M}{\gamma}$ and sectors k leave their prices unchanged at the steady state level of $p_k = 1$. This results in aggregate output of $Y = \frac{\gamma^{1/S}}{S} M^{\frac{S-1}{S}}$. The total level of labor is $N = \left[\frac{1}{S} + (S-1)\frac{M}{S}\right] + \psi$, reflecting

[11] Without GHH preferences and therefore allowing for income effects on labor supply, optimal policy would need to account for the fact that the labor required for menu costs affects the marginal rate of substitution between consumption and leisure. Under optimal policy, production would therefore be slightly tilted away from the flexible-price level. If one thinks, reasonably, that income effects on labor supply aren't relevant at business cycle frequencies (c.f. Angeletos and La'O 2010) then it is safe to ignore this effect. An alternative approach would be to model menu costs as a utility penalty affecting the household directly (c.f. Auclert, Rognlie and Straub 2018 among others), in which case the flexible-price allocation is replicated exactly; we model this in appendix B.

one sector's worth of menu costs ψ, since only sector 1 is adjusting. Thus, household welfare is a function of the money supply decision:

$$\mathbb{W}_{\text{only 1 adjusts}}(M) = \ln\left(\frac{\gamma^{\frac{1}{S}}}{S} M^{\frac{S-1}{S}}\right) - \left[\frac{1}{S} + (S-1)\frac{M}{S} + \psi\right]$$

Conditional on being in this regime, optimal monetary policy chooses M to maximize this expression, which can be found from the first order condition to be:

$$M^*_{\text{only 1 adjusts}} = 1$$

where asterisks will denote objects under optimal policy. The optimal money supply in this case is left unchanged at the steady state level, $M^{ss} = 1$. Importantly, this ensures that the relative price across sectors, $\frac{p_1}{p_k} = \frac{M}{\gamma}$, equals the flex-price level:

$$\left(\frac{p_1}{p_k}\right)^*_{\text{only 1 adjusts}} = \frac{1}{\gamma} = \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

Why does this policy result in the efficient relative price? Setting $M = 1$ ensures nominal wages are $W = 1$, since $M = W$ from (3), which means that nominal wages are unchanged from steady state $W^{ss} = 1$. As a result, *even though* firms in sectors k leave their prices unchanged at $p_k^{ss} = 1$, if they *were* to adjust to the flex-price level of $\frac{W}{A_k} = W$, they would leave their prices unchanged.

Thus, optimal monetary policy is able to replicate the flexible-price allocation by ensuring that all prices are at the correct level despite sectors k not adjusting, aside from the extra labor required for menu costs. As a result, welfare under optimal policy is equal to the flexible-price level, minus one sector's worth of menu costs from sector 1 adjusting:

$$\begin{aligned}\mathbb{W}^*_{\text{only 1 adjusts}} &= \ln\left(\frac{\gamma^{1/S}}{S}\right) - [1+\psi] \\ &= \mathbb{W}_{\text{flex}} - \psi \qquad (19)\end{aligned}$$

Only sectors k adjust. If only sectors $k = 2, ..., S$ adjust, the logic is similar to the prior case, except equilibrium has sectors k adjusting, of which there are $S-1$ in number, instead of only one sector adjusting. The flexible-price allocation is again achievable aside from the extra labor required to pay for menu costs, this time by ensuring that the desired price in sector *1* equals the inherited price. The optimized level of welfare is thus the flexible-price level minus $S-1$ sectors' worth of menu costs:

$$\mathbb{W}^*_{\text{only } k \text{ adjust}} = \mathbb{W}_{\text{flex}} - (S-1)\psi \tag{20}$$

No sector adjusts. Finally consider the possibility that no firm in any sector adjusts. Sectoral prices are thus unchanged from steady state, $p_i = p_i^{ss} = 1 \ \ \forall i$, and consequently so is the aggregate price level, $P = P^{ss} = S$. Within this regime, this is as if all prices were fully rigid: aggregate output is determined by monetary policy, $Y = C = \frac{M}{S}$. Total labor is $N = \frac{1}{\gamma}\frac{M}{S} + (S-1)\frac{M}{S}$, noting no labor is required for menu costs because no prices are adjusted. Household welfare as a function of the chosen level of the money supply M is:

$$\mathbb{W}_{\text{none adjust}}(M) = \ln\left(\frac{M}{S}\right) - \left[\frac{1}{\gamma}\frac{M}{S} + (S-1)\frac{M}{S}\right]$$

Conditional on being in this regime, optimal monetary policy chooses M to maximize this expression, which can be found from the first order condition to be $M^*_{\text{none adjust}} = \left[\frac{1}{\gamma}\frac{1}{S} + \frac{S-1}{S}\right]^{-1}$. Under this, the optimized level of welfare is:

$$\mathbb{W}^*_{\text{none adjust}} = -\ln\left(S - 1 + \frac{1}{\gamma}\right) - 1 \tag{21}$$

To understand this, note that the relative price $\frac{p_1}{p_k}$ is stuck at the steady state level of 1 instead of being updated to the flexible price level of $\frac{1}{\gamma}$:

$$\left(\frac{p_1}{p_k}\right)_{\text{none adjust}} = 1 \neq \left(\frac{p_1}{p_k}\right)_{\text{flex}}$$

It is because this relative price is stuck at a distorted level that monetary policy is unable to achieve the flexible-price allocation for output and production labor – in contrast to the prior two cases, where by setting this relative price at the flexible level, the central bank could do so.

3.2 Comparing across regimes

Comparing welfare across the four possible outcomes we can immediately observe that only two of the four are worth considering for optimal policy.

Lemma 2 (If adjusting, only the shocked sector should adjust)**.** Welfare when only sector 1 adjusts $\mathbb{W}^*_{\text{only 1 adjusts}}$ is always strictly higher than welfare when all sectors adjust

$\mathbb{W}^*_{\text{all adjust}}$ and strictly higher than welfare when only sectors k adjust $\mathbb{W}^*_{\text{only } k \text{ adjust}}$.

Proof: This follows immediately from comparing (19) with (18) and (20). $\square$

Lemma 2 follows from the idea that it is better to have fewer firms incur menu costs, together with the fact that optimal policy can implement the efficient relative price $\left(\frac{p_1}{p_k}\right)_{\text{flex}}$ by having *either* sector 1 only adjust, *or* sectors k only adjust, *or* all sectors adjust. Thus, if any firms at all are going to adjust, it is best to have sector-1 firms only adjust.

What remains is to compare welfare if "only sector 1 adjusts" versus if "none adjust". The next lemma compares these two.

Lemma 3 (Only adjust prices beyond a threshold)**.** There is a threshold $\overline{\gamma}$ such that $\mathbb{W}^*_{\text{only 1 adjusts}}$ dominates $\mathbb{W}^*_{\text{none adjust}}$ if and only if $\gamma \geq \overline{\gamma}$. Furthermore, the threshold $\overline{\gamma}$ is increasing in the menu cost ψ.

Proof: Define $f(\gamma) \equiv \mathbb{W}^*_{\text{none adjust}} - \mathbb{W}^*_{\text{only 1 adjusts}}$. Observe that if $\gamma = 1$, then $f(\gamma) = \psi > 0$. Additionally, as $\gamma \to \infty$, then $f(\gamma) \to -\infty$. Finally, f is strictly monotonically decreasing in γ, with $f'(\gamma) = \frac{1}{\gamma}\left[\frac{1}{\gamma(S-1)+1} - \frac{1}{S}\right] < 0$. Since f is continuous in γ, by the intermediate value theorem there exists a $\overline{\gamma} > 1$ such that $f(\overline{\gamma}) = 0$. To see that $\overline{\gamma}$ is increasing in ψ, observe that increasing ψ shifts the entire $f(\gamma)$ curve up, i.e. $\frac{\partial f}{\partial \psi} > 0$. $\square$

Lemma 3 says that there is an important threshold level $\overline{\gamma}$ for the productivity shock. Below this threshold, household welfare is maximized by ensuring that no firm in any sector adjust; above this threshold, it is maximized by ensuring that sector-1 firms adjust. The intuition for this, as emphasized, is that the welfare loss from menu costs is fixed in size. For a sufficiently small improvement in productivity, the benefit to adjusting prices does not outweigh the fixed welfare loss from the menu cost that is required to adjust. It is only worthwhile to pay this fixed cost above the threshold. The proof follows this same logic.

Additionally, the productivity threshold $\overline{\gamma}$ is increasing in the size of the menu cost ψ. The intuition for this is that for a larger menu cost, the productivity shock must be bigger for it to be worthwhile to adjust.

In the case where none adjust, the level of nominal wages is $W_{\text{none adjust}} = M^*_{\text{none adjust}} = \left[\frac{1}{\gamma}\frac{1}{S} + \frac{S-1}{S}\right]^{-1}$. Observe that for $\gamma = 1$, then nominal wages are exactly unchanged from the steady state level of $W^{ss} = 1$. For small shocks, $1 < \gamma < \overline{\gamma}$, nominal wages are also approximately unchanged. In the quantitative model of section 6, we discuss how close this approximation is.

Denote welfare under optimal policy as $\mathbb{W}^*$, where lemma 2 and lemma 3 together imply $\mathbb{W}^* = \max\left\{\mathbb{W}^*_{\text{only 1 adjusts}}, \mathbb{W}^*_{\text{neither adjust}}\right\}$.

3.3 Adjustment externalities

When discussing the regime where only sector 1 adjusts, we derived equilibrium household welfare as a function of the money supply choice, $\mathbb{W}_{\text{only 1 adjusts}}(M)$ by *assuming* that only firms in sector 1 adjusted prices. We then found the optimal $M^*_{\text{only 1 adjusts}}$ by simply taking the first order condition of this function.

More precisely, however, what we term as a "constrained" central bank would choose the money supply M that maximizes welfare $\mathbb{W}_{\text{only 1 adjusts}}(M)$ *subject to the implementability constraint* that such a choice of M *induces* sector 1 to adjust and other sectors k not to adjust. The choice of M would need to be incentive-compatible with the assumption on who is adjusting price. The same is true for the case where none adjust: the choice of optimal M must not push any firm outside its inaction region. (The same is true of the case where only sectors k adjust, though this is less important because of lemma 2.) These constraints can be written formally as in equations (35)-(37) in appendix A.2.

In proposition 1 and throughout the body of this paper, we have endowed the social planner with the power to subsidize menu costs – or equivalently, can force firms to adjust prices – so that these implementability constraints are always nonbinding. This is written out explicitly in (39)-(41) in appendix A.2. In appendix B, however, we show that if the planner does not have this instrument, then it is possible for these implementability constraints to bind.

We term the case where the unconstrained-optimal choice of M is not feasible as "adjustment externalities", and discuss these in detail in appendix B. It may be the case that it is socially optimal for firms in sector 1 to adjust their prices, but it may not be privately optimal to do so: prices are "too sticky", and there is a positive externality to price adjustment. It is also possible, however, that it is socially optimal for firms in either sector 1 *or* in sectors k to leave their price unchanged, but it is privately optimal to do so: prices are "too flexible", and there is a negative externality to price adjustment.

This issue does not arise in the Calvo literature since there is no *choice* of whether or not to adjust – if given the chance to freely adjust under Calvo, a firm will always do so – and therefore these adjustment externalities do not arise. As a result, there is limited precedent in the literature, with a handful of important exceptions. Ball and Romer (1989*a*) find that menu costs create negative externalities after a *monetary policy shock*. Our setting instead studies whether *efficient* (productivity) shocks create externalities, when monetary policy is set optimally, and finds the possibility of not just negative externalities but also the possibility of positive adjustment externalities. Other related studies include Ball (1987) on negative externalities in the length of labor contracts; Ball and Romer (1989*b*) on externalities in the timing of staggered price setting; and Ball and Romer (1991) on the possibility of menu cost-induced multiple equilibria. All of these papers study economies where monetary policy is not set optimally; our results show that, *even* when monetary policy is set optimally, adjustment externalities may arise.

Finally, Angeletos and La'O (2020) studies optimal monetary policy under information frictions, and in the case of endogenous information acquisition studied in online appendix A, find that there are no externalities to information acquisition in price-setting if technology is specified as Dixit-Stiglitz as long as monetary policy is set optimally. In our setting with optimal policy under menu costs, rather than information frictions, we show the possibility of externalities even under the Dixit-Stiglitz specification.

In appendix B, in an analytically-tractable variant of the baseline model presented above, we derive a maximum level for menu costs $\overline{\psi}$ such that as long menu costs are below this threshold, $\psi < \overline{\psi}$, then adjustment externalities cannot occur in the exercise above. We show that this maximum level of menu costs for any configuration of parameters is an order of magnitude larger than empirical estimates of menu costs in the literature: menu costs would need to be on the order of 50% of profits to induce adjustment externalities. Thus, for the purposes of proposition 1 and for our other analytical results, we have endowed the planner with the choice to overcome adjustment externalities by selectively subsidizing menu costs. We leave further study of the quantitative importance of adjustment externalities under menu costs to future work.

4 The welfare loss of inflation targeting & comparison with the NK model

In this section, we contrast our optimal policy results under menu costs to the canonical sticky price New Keynesian model, where prices are sticky exogenously due to the Calvo-Yun assumption. We first discuss how stabilizing inflation under menu costs generates a welfare loss, in contrast to the standard model where stabilizing inflation is optimal. This is also directly policy-relevant, because leading central banks today describe their policy goals in terms of inflation targeting. Second, we contrast optimal policy under *nonconvex* menu costs (as in our setting) with optimal policy in a model of *convex* menu costs, another setting where inflation targeting is optimal.

4.1 The welfare loss of inflation targeting under menu costs

The standard New Keynesian model of sticky prices, built on the Calvo-Yun exogenous sticky pricing framework, implies that a policy of zero inflation is optimal policy;

but in our menu cost setting, such inflation targeting would be strictly suboptimal. In order to implement inflation targeting – i.e. in order to ensure that the price level is unchanged with $P = P^{ss} = 1$ – the central bank has two possibilities:

1. "All adjust": It may force all firms to adjust, and set M to ensure that the increase in price in sectors k to $p_k = M$ exactly offsets the fall in price in sector 1 to $p_1 = M/\gamma$.

2. "None adjust": It may ensure that no firm in any sector adjusts.

Although per proposition 1 it is optimal to ensure no sector adjusts in the case of small productivity shocks, $\gamma < \overline{\gamma}$, it would be unusual to conceptualize a policy of inflation targeting as aiming for a world in which relative prices *never* change. If maintained indefinitely, such a policy of aiming to prevent all relative price changes would seem to be self-evidently unreasonable, since it would shut down the price system. Thus it is more natural to characterize "inflation targeting" in this context as referring to the policy that would ensure all firms adjust.

We saw above, though, that forcing all sectors to adjust price incurs unnecessary menu costs. Thus, the welfare loss of inflation targeting relative to optimal policy is directly captured by the welfare loss caused by the unnecessary menu costs paid by the $S-1$ unshocked sectors. The empirical size of menu costs ψ together with the number of unshocked sectors $S-1$ are sufficient statistics for the welfare gains that would come from moving from inflation targeting to nominal wage targeting.

Proposition 2 (The welfare loss of inflation targeting)**.** Suppose $\gamma \geq \overline{\gamma}$. Then:

1. A policy of inflation targeting aiming for $P = P^{ss}$, if ensuring correct relative prices, requires that all sectors adjust their prices and increasing the money supply to $M = \gamma^{1/S} > M^{ss}$.

2. Welfare under inflation targeting $\mathbb{W}_{\text{IT}}$ is strictly less than welfare under the optimal policy described in proposition 1, $\mathbb{W}^*$. The welfare loss is summarized by the size of menu costs ψ and the number of sectors unaffected by the shock $S-1$.

$$\mathbb{W}_{\text{IT}} - \mathbb{W}^* = (S-1)\psi$$

Proof: The second claim comes from formulas (18) and (19). For the first claim, suppose the central bank tried to both achieve correct relative prices by only having sector 1 adjust – in which case, $P = SM^{1/S}\gamma^{-1/S}$ – and simultaneously setting M such that the price level was unchanged, $P = P^{ss} = S$. This would require $M = \gamma$. However, if $M = \gamma$ then the optimal price for firms in sector 1 is $p_1 = W/\gamma = 1$, which would mean that firms in sector 1 leave prices unchanged, a contradiction. Similarly, if the central bank tried to achieve correct relative prices while only having sectors k adjust, in which case $P = SM^{\frac{S-1}{S}}$, then this would require $M = 1$, which would cause firms in sectors k to not adjust, again a contradiction. Finally, if no sector adjusts, then it is impossible to achieve correct relative prices, since $p_1/p_k = 1$. It is only by having firms in all sectors adjust, in which case $P = SM\gamma^{-1/S}$, that the central bank can achieve both correct relative prices and ensure that $P = P^{ss}$, by setting $M = \gamma^{1/S}$. □

A reaction may be that the welfare cost of menu costs ψ – the welfare costs imposed directly from updating prices – could be relatively small. The literature on menu costs often builds on the idea that 'second-order menu costs can result in first-order output fluctuations' (Mankiw 1985), in which case the welfare loss of inflation targeting compared to optimal policy would be second-order. However, it is important to note that in the textbook New Keynesian model with the exogenous Calvo-Yun friction, the welfare loss of price stickiness is *also* only second-order (see e.g. Gali 2008). We also highlight that estimates of the real resource cost of menu costs from the empirical literature are sizeable, on the order of 0.5% of total firm revenues annually (an order of magnitude larger than the famous "welfare loss of business cycles" estimate of 0.05% of Lucas 1987).[12] More generally, in this paper and in our model, we think of menu costs as representing not merely any physical costs of adjusting prices, but being a reduced-form way of capturing any economic or welfare cost of price adjustment, such as the mental attention costs suffered by price setters. In section 6, we directly estimate the impact of menu costs on welfare in the context of a dynamic, quantitative version of our model, and we compare inflation targeting versus nominal wage targeting.

4.2 Nonconvex menu costs vs. convex menu costs

Throughout this paper, we have used a model of nonconvex menu costs: the cost of price adjustment is *fixed* and does not scale with the size of a price change. We now contrast this model with models of *convex* menu costs, where the cost of price adjustment depends on the magnitude of the desired price change, and this cost grows at an increasing rate.

Consider the canonical model of convex menu costs, the Rotemberg (1982) model of quadratic menu costs, where the menu cost scales with the square of the size of the price change: $\psi\left(p_i - p_i^{ss}\right)^2$. It can be shown that the single-sector Rotemberg model is isomorphic in its structural equations, to a first-order approximation, to the textbook New

[12]See e.g. Nakamura and Steinsson (2010); Nakamura et al. (2018); and Levy et al. (1997).

Keynesian model built on the Calvo time-dependent friction.[13] Furthermore, the model is isomorphic to a second-order approximation in its optimal policy implications to the Calvo-Yun model (Nisticò 2007), i.e. inflation targeting not wage targeting is optimal.

The difference with our model in optimal policy comes directly from the convex nature of the Rotemberg menu costs: it is better to have all sectors adjust prices a little than to have one sector do all of the adjustment. With the nonconvex menu costs of our model, it is instead optimal to minimize the number of sectors which choose to adjust at all.

The key intuition is in the labor market clearing condition. The labor market clearing condition in the multisector Rotemberg model is:

$$N = \sum_{i=1}^{S} n_i + \psi \sum_{i=1}^{S} (p_i - p_i^{ss})^2 \tag{22}$$

This contrasts with the labor market clearing condition under nonconvex menu costs, equation 14:

$$N = \sum_{i=1}^{S} n_i + \psi \sum_{i=1}^{S} \mathbb{I}\{p_i \neq p_i^{ss}\}$$

where for clarity we've explicitly written the expression in terms of an indicator for whether a sector adjusts its price, rather than in terms of the variable $\chi_i \equiv \mathbb{I}\{p_i \neq p_i^{ss}\}$.

Under both Rotemberg and under nonconvex menu costs, it is desirable to minimize the amount of menu costs because of the disutility of labor they create. Due to the *convex* nature of the Rotemberg menu costs in (22), it is better to *smooth* the price changes over all sectors: it is better to have a small price change in every sector, rather than a large price change in one sector. Under nonconvex menu costs, it is instead better to minimize the *count* of sectors which experience any price change. It this difference – convex versus nonconvex costs of adjustment – which explains the differing optimal policy prescriptions.

5 Extensions to the benchmark model

We now consider several natural extensions to the model, which we continue to solve analytically. These showcase the robustness of our results, and are useful for reinforcing

[13] Do note however that the *mechanism* of the Rotemberg model is very different from the Calvo-Yun model. Under Calvo-Yun, the welfare loss of monetary instability is the resulting relative price dispersion: total factor productivity is effectively lower. Under Rotemberg – and in our model – instead, the loss comes from the real resource cost of menu costs. If the quadratic menu cost requires extra labor, this comes from the additional labor required to adjust prices. If the quadratic menu cost is a real resouce cost, then this is a wedge between consumption and output.

the intuition built above on the mechanism of our results.

5.1 Functional form generalizations and stabilizing W vs. PY vs. M

The core intuition argued above is that monetary policy seeks to ensure that nominal marginal costs are unchanged for firms who do not receive a productivity shock, so that they have no desire to adjust their prices. In section 3, we assumed Cobb-Douglas technology at the aggregate level, CES technology at the sector level, and linear technology at the intermediate firm level. However, any functional forms could have been used without any implication for optimal policy. For example, generalizing to CES technology at the aggregate level $Y = \left[\sum_i y_i^\theta\right]^{1/\theta}$ would not alter proposition 1 or any of the discussion above at all because of this intuition. Introducing decreasing returns at the intermediate level $y_i(j) = A_i n_i(j)^\alpha$ with $\alpha \in (0,1)$ does not affect the statement of proposition 1, but would affect the value of the threshold $\overline{\gamma}$, since decreasing returns affects the value of price adjustment. The robustness of the proposition to these and other changes *on the firm side* of the model is because stabilizing nominal marginal cost of unaffected firms still means stabilizing W.

Additionally, stabilizing nominal wages W in the baseline model is equivalent to stabilizing the money supply M or stabilizing nominal income PY. This equivalence would also be robust to any changes in the production side of the model, since the equivalence is due to the optimality conditions *from the household's problem*, (2) and (3).

However, we describe optimal policy in this paper as nominal wage targeting, not as money supply targeting or as nominal income targeting, because of the robustness to changes on the household side.

First consider the reason why optimal policy is not money supply targeting, such as the k-percent money growth rule of Friedman (1960): velocity shocks. In the context of our model, a velocity shock is anything that affects the marginal rate of substitution between consumption and real balances. For example, if preferences (1) were modified to be $\mathbb{W} = \ln C - N + \frac{1}{V}\ln\left(\frac{M}{P}\right)$ with V an exogenous preference shifter subject to shocks. Then it would no longer be the case that $M = W$ in condition (3), but instead $M = \frac{1}{V}W$. Meanwhile, (2) would become $PC = MV$. This expression is simply the equation of exchange; in the baseline model, velocity V was implicitly fixed at 1. A money supply rule would fix M, whereas optimal policy would stabilize W, which would require adjusting the money supply M to offset velocity shocks V. Hence why nominal income targeting is sometimes referred to as velocity-adjusted money supply targeting.[14]

[14]For additional discussion, see e.g. Sumner (2012), Beckworth (2019), and Binder (2020).

Other generalizations to household preferences break the equivalence between nominal income targeting and nominal wage targeting: it is only nominal wage targeting that is optimal. Consider for example if household preferences (1) are generalized to be generically GHH over consumption and labor:

$$\mathbb{W} = U\left[C - G(N)\right] - V^{-1} \cdot U\left[M/P\right] \tag{23}$$

where the usual properties hold, $U' > 0, U'' < 0, G' > 0, G'' > 0$. Under these preferences, (2) and (3) do not hold, and so nominal wages and nominal income are not equivalent, $W \neq PY$. Instead, nominal income is some complicated function of nominal wages. The core intuition for nominal wage targeting coming from the firm side of the model, however, continues to hold: by stabilizing nominal wages W, the central bank ensures that firms outside sector 1 have no motive to adjust prices. Firms in sector 1 cut their prices and raise output, so that P and Y do continue to move inversely – "approximate nominal income targeting" – but exact nominal targeting is not precisely optimal. Exact nominal wage targeting is.

Finally, we note that while optimal policy here prescribes constant nominal wages, this could be generalized to a constant trend for nominal wage growth. This would be analogous to the generalization in the textbook Calvo-Yun framework from the prescription of a constant price level to the prescription for a 2% inflation target employed by many countries in actuality. This can be captured formally by allowing firms to adjust their prices at a constant trend for free, but to pay the menu cost in order to deviate from trend, i.e. indexing. See Yun (1996) or Woodford (2003) for the case of this indexing in the textbook model.

We summarize these results in the following proposition.

Proposition 3 (Functional form generalizations)**.** Suppose the baseline model is modified to allow for:

1. Any constant returns to scale production technology for final goods, with (4) becoming $Y = F(y_1, ..., y_S)$ with F homogenous of degree 1
2. Potentially decreasing returns to scale in production technology, with (10) becoming $y_i(j) = A_i n_i(j)^\alpha$ with $\alpha \in (0, 1]$
3. Any household preferences of the GHH form (23) replacing (1), including with money demand shocks V

Then optimal monetary is exactly the same as characterized in proposition 1 modulo changes in the constant $\overline{\gamma}$.

Proof: The proof follows exactly as in the proof of proposition 1. □

5.2 Sectoral heterogeneity and a monetary "least-cost avoider" principle

We now return to the baseline setting and consider two kinds of heterogeneity, in sector size and menu cost magnitude, that lead us to a "least-cost avoider" interpretation of optimal monetary policy.

5.2.1 Heterogeneity in sector size

Suppose that sectors are of different sizes. Instead of each sector being an equal size $\frac{1}{S}$, allow sector i to be size $\frac{1}{S_i}$. Assume sector sizes sum to unity, $\sum_{i=1}^{S} \frac{1}{S_i} = 1$, to preserve constant returns to scale in finals goods production. The final goods technology (4) is now:

$$Y = \prod_i y_i^{1/S_i} \tag{24}$$

The resulting demand function is nearly the same as previously (5):

$$y_i = \frac{1}{S_i} \frac{PY}{p_i} \tag{25}$$

All that has changed in these two equations is the replacement of $\frac{1}{S}$ with $\frac{1}{S_i}$.

Heterogeneity in sector size also interacts with menu costs, since larger sectors require hiring more labor to adjust prices. This shows up in the modified labor market clearing condition (14):

$$N = \sum_i n_i + \psi \sum_i \frac{1}{S_i} \chi_i$$

With this change, optimal policy is *nearly* the exact same as characterized in proposition 1. What differs is only in the extreme case when sector 1 is larger than all other sectors put together, $\frac{1}{S_1} > \sum_{k>1} \frac{1}{S_k}$. Then if relative prices are to adjust it is actually optimal to have firms *outside* sector 1 adjust their price in response to a shock affecting sector 1 itself. That is because although it is only sector 1 which is affected by the shock, the combined mass of firms outside sector is smaller than sector 1 itself. Therefore the menu costs burned by having all other firms adjust price is less than the menu costs burned by having "just" sector 1 adjust.

Thus under the (extreme) assumption that sector 1 is larger than the combined mass of all other sectors, implementing the regime where 'only sectors k adjust' is preferable to implementing the regime where 'only 1 adjusts'. This has the same intuition that it achieves the correct relative prices while economizing on menu costs – where, here, economizing on menu costs means having sector 1 not adjust. This policy would not stabilize nominal wages (or inflation).

We interpret this case as an illustration of the logic of our results, rather than an empirically-relevant case in general. Only in the case where *more than half* of the economy is *homogeneously* affected by the same shock does this result carry through. Otherwise, the optimal policy prescriptions of proposition 1 carry through exactly.

5.2.2 Heterogeneity in menu cost size by sector

Introducing heterogeneity in menu cost size by sector is mostly similar. If the menu cost of sector i is ψ_i, the labor market clearing (14) becomes:

$$N = \sum_i n_i + \sum_i \frac{1}{S_i} \psi_i \chi_i$$

Observe that the direct effect of heterogeneity in menu costs (ψ_i) on welfare is isomorphic to that of heterogeneity in sector size (S_i). But heterogeneity in menu cost size, unlike that in sector size, *also* affects the size of inaction regions given in (13). However, this additional complication has somewhat limited impact.

First – analogous to the possibility just discussed that sector 1 is very large in size – if weighted the menu costs of sector 1 are extremely large relative to those of other sectors, $\frac{1}{S_1}\psi_1 > \sum_{k>1} \frac{1}{S_k}\psi_k$, then it again is optimal to have all firms outside sector 1 adjust rather than those in sector 1 if relative prices are to change. Second, variation in ψ_1 does affect when it is optimal to allow prices to go unchanged, i.e. affects the value of the threshold $\overline{\gamma}$.

5.2.3 Interpretation: a monetary "least-cost avoider" principle

We summarize both the above results in the following proposition.

Proposition 4 (Sectoral heterogeneity)**.** Suppose sector i is of size $1/S_i$ and has menu cost ψ_i. Suppose further that the size-weighted menu cost of sector 1 is smaller than the combined weighted average of menu costs for other sectors, $\frac{1}{S_1}\psi_1 < \sum_{k>1} \frac{1}{S_k}\psi_k$. Then optimal monetary is exactly the same as characterized in proposition 1 modulo changes in the constant $\overline{\gamma}$.

Proof: Under the assumption about the magnitude of weighted menu costs, the proof follows exactly as in the proof of proposition 1. □

These two results on sectoral size and menu cost heterogeneity, summarized in proposition 4, can be interpreted as a "least-cost avoider" theory of optimal monetary policy. In economic analysis of law, the least-cost avoider principal states that when considering assignment of liability between parties, it is efficient to assign liability to the party who has the lowest cost of avoiding harm (Calabresi 1970). Similarly, the generalized principle of optimal monetary policy under menu costs is: *the agents for whom it is least costly to adjust their price are the agents who should do so*.

More closely to the monetary economics literature, this is also very related to the idea that 'monetary policy should target the stickiest price' (e.g. Mankiw and Reis 2003 and Aoki 2001). Under menu costs, the central bank should minimize adjustment by the firms with the most expensive menu costs – i.e. it should stabilize the stickiest prices.

5.3 Multiple shocks

In the baseline exercise analyzed in proposition 1, we consider a shock to the productivity of sector 1 alone. One motivation for this is the idea that in reality productivity shocks arrive as Poisson shocks, separated by spans of time, with no two sectors ever being shocked at precisely at the same time. Such a motivation introduces dynamics which we do not study in the analytical model, however – though we will consider the role of dynamics in the quantitative results of section 6.

In this section, we consider the case where every sector is shocked. Start again at steady state, where every sector has productivity of $A_i^{ss} = 1$. We consider the exercise of shocking every sector to productivity $A_i = \gamma_i$, where γ_i could be potentially above, below, or equal to 1: it may be a positive shock, it may be a negative shock, or the sector may be unshocked.

Equilibrium. It is illustrative to consider a generic equilibrium when some fixed subset of sectors $\Omega \subseteq \{1, ..., S\}$ adjusts. Denote the cardinality of Ω as $\omega \equiv |\Omega|$. Sectors which adjust update their price to $p_i = M/A_i$, whereas others remain at the steady state value of $p_i^{ss} = 1$. The aggregate price level thus aggregates from (6) to:

$$P = \frac{SM^{\omega/S}}{\prod_{i\in\Omega} A_i^{1/S}}$$

From the quantity equation (2), this gives consumption and output as:

$$C = Y = \frac{1}{S}\left[\prod_{i\in\Omega} A_i^{1/S}\right] M^{\frac{S-\omega}{S}}$$

For comparison, the flexible-price level of output is $Y_{\text{flex}} = \frac{1}{S}\prod_{i=1}^{S} A_i^{1/S}$. Using sectoral demand, production technology, and labor market aggregation, the amount of aggregate labor is:

$$N = \frac{\omega}{S} + \frac{M}{S}\sum_{i\notin\Omega}\frac{1}{A_i} + \psi\omega$$

Thus, welfare – conditional on the set Ω of sectors adjusting – as a function of the choice of money supply, is:

$$\mathbb{W}_\Omega(M) = \ln\left[\frac{1}{S}\left[\prod_{i\in\Omega} A_i^{1/S}\right] M^{\frac{S-\omega}{S}}\right] - \left[\frac{\omega}{S} + \frac{M}{S}\sum_{i\notin\Omega}\frac{1}{A_i} + \psi\omega\right] \tag{26}$$

As before allowing the social planner to overcome adjustment externalities, the optimal choice of money supply (conditional on Ω sectors adjusting) is found directly from the first order condition of (26).[15] The first order condition gives:

$$M_\Omega^* = \frac{S-\omega}{\sum_{i\notin\Omega}\frac{1}{A_i}} \tag{27}$$

Welfare under optimal policy is:

$$\mathbb{W}_\Omega^*(M) = \ln\left[\frac{1}{S}\left[\prod_{i\in\Omega} A_i^{1/S}\right]\left(\frac{S-\omega}{\sum_{i\notin\Omega}\frac{1}{A_i}}\right)^{\frac{S-\omega}{S}}\right] - [1+\psi\omega] \tag{28}$$

Replicating the flexible-price allocation. Now, it is only possible replicate the flexible-price allocation – aside from the extra labor required for menu costs – in two cases. First,

[15]As long as the number of sectors adjusting is not all S sectors, i.e. $\omega < S$. If all sectors adjust, i.e. if $\omega = S$ and $\Omega = \emptyset$, then welfare is independent of the choice of the money supply M.

as before, if all sectors adjust, i.e. $\Omega = \{1, ..., S\}$ and $\omega = S$, then naturally this ensures the flexible-price allocation. This comes at the cost of S sectors' worth of menu costs.

The flexible-price allocation can, however, be achieved also if $\omega = S - 1$, so all but one sector adjust. The one non-adjusting sector simply may be any arbitrary sector r. In this case, the central bank would set the money supply at $M = \frac{1}{A_r}$. This ensures that the desired price of sector r, that is $\frac{W}{A_r}$, equals the steady-state level of $p_r^{ss} = 1$, so that despite not changing its price, sector r has the price that it would choose if it were to adjust. This comes at the cost of $S - 1$ sectors' worth of menu costs.

An immediate implication is that it is never optimal to have all S sectors to adjust, since the same allocation can be achieved if $S - 1$ sectors adjust, but with less labor required for menu costs. In other words, it is always best to peg at least one sector and ensure that at least that one sector need not adjust its price. This sector may be arbitrarily chosen in the baseline model. If sectors were of heterogeneous size or had heterogeneous menu cost sizes, as in proposition 4, then it would be optimal to choose the sector with the largest size-weighted menu costs. This reinforces the "least-cost avoider principle" interpretation, or the "stabilize the stickiest price" interpretation described in that proposition.

If more than one sector leaves their price unchanged, i.e. $\omega < S - 1$, then it is not possible achieve the flexible-price allocation. This is the standard result that when relative prices change, if there is sufficient nominal rigidity, the flexible-price allocation cannot be achieved: there is more than one target (the many relative prices), but only one instrument, M (Poole 1971, Angeletos and Sastry 2021).

Interpreting conditionally-optimal policy. Since it is still the case that nominal wages are determined by monetary policy, $W = M$, it follows from (27) that nominal wages *conditional on sectors Ω adjusting* are:[16]

$$W_\Omega^* = \frac{S - \omega}{\sum_{i \notin \Omega} \frac{1}{A_i}} \tag{29}$$

To emphasize, the equilibrium nominal wage under optimal policy depends on the central bank's choice of the set of adjusting firms Ω. However, for any fixed choice of Ω, this is the equilibrium nominal wage.

From (29), we can see that any form of optimal policy will not stabilize nominal wages, $W_\Omega^* \neq W^{ss}$. Optimal policy is not exactly nominal wage targeting.

[16] For any Ω with $\omega < S$. With $\omega = S$, nominal wages are indeterminate and may be anything. We may then for simplicity choose the level given in (29).

However, policy nonetheless may be considered to approximately stabilize nominal wages. For $A_i \approx 1$, it is the case that $\frac{1}{A_i} \approx 1$; this implies that $\sum_{i \notin \Omega} \frac{1}{A_i} \approx \sum_{i \notin \Omega} 1 = S - \omega$, and so nominal wages are approximately unchanged, $W_\Omega^* \approx 1$. As in proposition 1, it will only be optimal to adjust for sectors which experience larger shocks. As a result, for sectors $i \notin \Omega$, i.e. for sectors i not adjusting, it will particularly be the case that $A_i \approx 1$.

Ultimately, the performance of exact nominal wage targeting in the face of multiple shocks depends on an empirical question – the distribution of shocks, and how tightly centered around 1 they are – and a quantitative question – how well exact nominal wage targeting performs compared to the analytically optimal policy. We now answer this question in a quantitative, dynamic model.

6 Quantitative results [preliminary]

Though the intuition is entirely captured by the static model, we show that even in a quantitative dynamic model calibrated to the US economy, nominal wage targeting dominates inflation targeting.

6.1 Model description and solution method

Our dynamic multisector model of menu costs is the same as that of Nakamura and Steinsson (2010), except that we additionally include sector-specific productivity shocks on top of idiosyncratic, firm-level shocks.

Household. The household chooses paths for consumption, C_t, labor N_t, money balances M_t, and bonds B_t to maximize the present discounted value of utility. The problem faced by the household is:

$$\max_{\{C_t, N_t, B_t, M_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\gamma}}{1-\gamma} - \omega \frac{N_t^{1+\varphi}}{1+\varphi} + \ln\left(\frac{M_t}{P_t}\right) \right]$$

$$\text{s.t.} \qquad P_t C_t + B_t + M_t \leq R_t B_{t-1} + W_t N_t + M_{t-1} + \sum_{i=1}^{S} \int_0^1 D_{t,i}(j) dj - T_t$$

To consume and save (the left hand side of the budget constraint) the household uses previous savings, money holdings, labor earnings, the profits from the firms, net of the lump sum tax imposed by the government (the right hand side of the budget constraint). R_t is the nominal interest rate on bonds.

Firms. Final and sectoral good producing firms behave similarly to the baseline analytical model presented of section 2. In particular, these firms operate in competitive environments and aggregate the goods produced by lower-tier firms according to the technologies:

$$Y = \left(\sum_{i=1}^{S} y_i^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{30}$$

$$y_i = \left[\int_0^1 y_i(j)^{\frac{\eta-1}{\eta}}\, dj\right]^{\frac{\eta}{\eta-1}} \tag{31}$$

Note that we have generalized the final goods production technology from the baseline model from Cobb-Douglas to CES.

Intermediate firms. A key difference in the quantitative model versus the baseline model is that intermediate firms are subject to not only sector-level productivity shocks $A_{t,i}$ but also idiosyncratic, firm-level shocks $a_{t,i}(j)$. The firm's production technology is:

$$y_{t,i}(j) = a_{t,i}(j) A_{t,i} n_{t,i}(j)^{\alpha} \tag{32}$$

Idiosyncratic and sectoral shocks are i.i.d. Gaussian, have mean zero, and are not correlated with each other. Denote the volatility of the idiosyncratic and sectoral shocks as σ_{idio} and σ_{sect} respectively.

The firm maximizes the present discounted value of real profits. In any given period the firm chooses whether to update its price, $\chi_{t,i}(j) = 1$ or not. If the firm decides to change its price, $p_{t,i}(j)$, it must pay a menu cost equal to a share ψ of total profits. The problem faced by each intermediate firm then is

$$\begin{aligned}
&V\Big(\frac{p_{t-1,i}(j)}{P_t}, \{p_{t,i}\}_i, \{A_{t,i}\}_i, a_{t,i}(j)\Big) \\
&\quad = \max_{p_{t,i}(j), \chi_{t,i}(j)} \left\{ \frac{D_{t,i}(j)}{P_t} + \mathbb{E}\left[\frac{1}{R_t} V\left(\frac{p_{t,i}(j)}{P_{t+1}}, \{p_{t+1,i}\}_i, \{A_{t+1,i}\}_i, a_{t+1,i}(j)\right)\right] \right\} \\
&\qquad \text{s.t. } D_{t,i}(j) = \left[p_{t,i}(j) y_{t,i}(j) - W_t n_{t,i}(j)(1-\tau)\right](1 - \chi_{t,i}(j)\psi) \\
&\qquad\qquad y_{t,i}(j) = y_{t,i}\left(\frac{p_{t,i}}{p_{t,i}(j)}\right)^{\eta} \\
&\qquad\qquad y_{t,i}(j) = a_{t,i}(j) A_{t,i} n_{t,i}(j)^{\alpha}
\end{aligned}$$

The relevant state variables necessary for firm to make its decisions are: the real price it enters the period with, $\frac{p_{t-1,i}(j)}{P_t}$; the sectoral prices, $\{p_{t,i}\}_{i=1}^S$, noting P_t is derived from these; the sector-level shocks, $\{A_{t,i}\}$; and its idiosyncratic productivity shock $a_{t,i}(j)$. We have followed Nakamura and Steinsson (2010) in assuming that, to a first order around the steady state, $\hat{Y} = \hat{C}$ and $\hat{N} = \hat{Y}$. These assumption allow us to simplify the firm's problem and not add the aggregate wage, W_t and the aggregate labor supply, N_t as state variables.

Model solution. Because of the sector-level productivity shocks we must resort to a quantitative solution method that allows for aggregate shocks. As in Nakamura and Steinsson (2010) we use a version of Krusell and Smith (1998). Firms form expectations of the sector-level prices using a VAR(1), that is they predict the price of sector s at time t according to:

$$\log\left(p_{t,i}\right) = \phi_0\left(\{a_{t,i}\}_i\right) + \sum_{i=1}^{S} \phi_i\left(\{a_{t,i}\}_i\right) \log\left(p_{t,i}\right) \tag{33}$$

where ϕ_0 and ϕ_i depend on the realization of the states just as in the original Krusell and Smith (1998).

6.2 Calibration

We again closely follow Nakamura and Steinsson (2010) in calibrating the model. In particular, the main parameters are listed in table 1:

Table 1: Model parameters and baseline calibration

	Parameter (quarterly frequency)	Value	Target
β	Discount factor	0.988	5% annual interest rate
ω	Disutility of labor	1	aggregate labor $N = 1$
φ	Inverse Frisch elasticity	0.5	standard
γ	Inverse elasticity of intertemporal substitution	1	standard
S	Number of sectors	6	Nakamura and Steinsson (2010)
η	Elasticity of subst. between sectors	2	standard value
ϵ	Elasticity of subst. across sectors	10	standard value
α	Returns to scale	1	standard value
σ_{sect}	Volatility of sectoral shocks	0.0665	derived from Nakamura and Steinsson (2010)
σ_{idio}	Volatility of idiosyncratic shocks	0.0665	matching σ_{sect} (benchmark)
ψ	Menu cost	0.07	5% of revenue in menu costs

6.3 Policy comparison: nominal wage targeting vs. inflation targeting

All firms in all sectors are continuously hit by idiosyncratic productivity shocks, and we run the following experiment. Additionally, firms in sector 1 are hit by a common sector-level productivity shock, while firms in other sectors are not hit by a sector-level shock, as studied analytically in section 3. We then simulate the economy for 100 periods and keep track of how many times on average firms in sector 1 versus firms in sectors k change prices. This is shown in figure 1 for the periods in which sector 1 is hit by a negative productivity shock (red bars), a positive shock (green bars), and no shock at all (blue bars). In the benchmark calibration sectoral and idiosyncratic productivity shocks have the same standard deviation, $\sigma_{idio} = \sigma_{sect}$, with solid bars. In the counterfactual calibration, the standard deviation of idiosyncratic shocks is double that of sectoral shocks, $\sigma_{idio} = 2\sigma_{sect}$, with hatched bars).

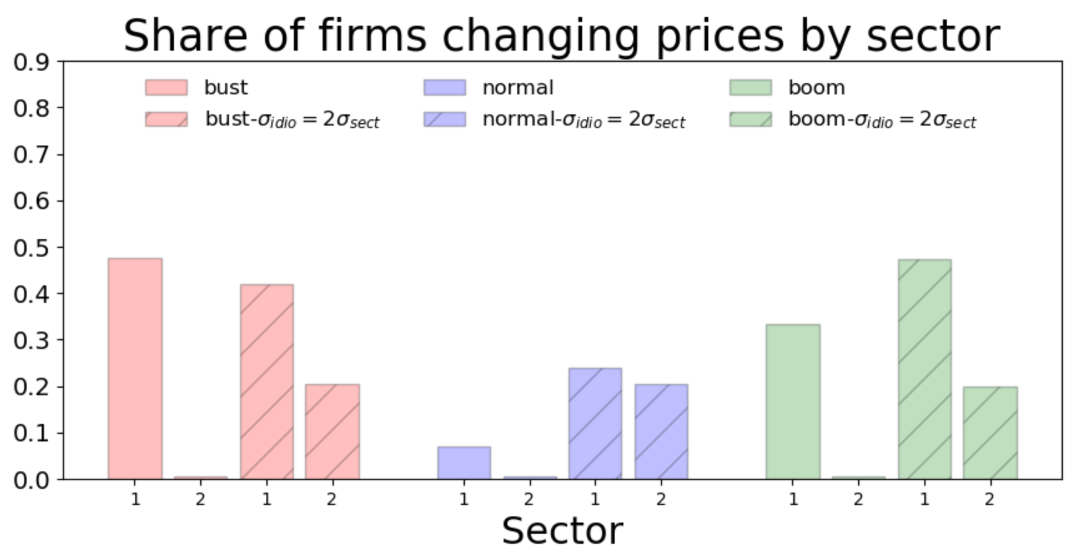

Figure 1: Share of firms changing prices by sector over a 100 periods simulation. Solid bars are for the calibration in which idiosyncratic and sectoral shocks have the same standard deviation; hatched bars are for the calibration in which sectoral shocks have twice the standard deviation of idiosyncratic shocks.

Ultimately, however, the question is whether nominal wage targeting or inflation targeting results in higher welfare. To test this, we simulate the above model, where all sectors and all shocks are subject to both sectoral and idiosyncratic shocks, for 250 periods under the two different monetary policy regimes and evaluate welfare. Welfare under the nominal wage targeting regime is approximately 0.5% higher.

7 Conclusion

Consider an economy with S sectors and firms within each sector are subject to sector-specific productivity shocks. As an example, suppose firms in sector 1 are hit by a positive productivity shock. If the shock is sufficiently large, then it is efficient and desirable for firms in this sector to cut their *relative* prices, compared to other firms in other sectors of the economy. Under a policy of constant nominal wages, firms outside of sector 1 have no desire to adjust their prices: firms wish to adjust their prices only if their nominal marginal costs change, and their nominal marginal costs are unchanged because their productivity is unchanged and nominal wages are kept stable. Meanwhile, firms in sector 1 choose to adjust their nominal prices because of the productivity shock. As a result, relative prices between sector-1 firms and other firms are undistorted – because firms in sector 1 updated their prices – and *only* this one sector has incurred wasteful menu costs.

The logic of this analogy is formalized in our analytical model and explored quantitatively in our quantitative results. We revisit the question of optimal monetary under sticky prices, using a more realistic microfoundation for sticky prices – menu costs – than the benchmark New Keynesian model. Our analytical approach shows, without linearization, that the textbook prescription for inflation stabilization is not optimal under the more realistic foundation of menu costs. In our quantitative model, the welfare loss from implementing inflation targeting rather than nominal wage targeting is an order of magnitude larger than the Lucas (1987) estimate for the welfare costs of business cycles, due to the sizable estimates in the literature for the empirical magnitude of menu costs. The conclusion that optimal monetary policy should be approximate nominal income targeting – inflation should be countercylical – resonates with the results of other studies in the broader optimal monetary policy literature away from sticky prices (Sheedy 2014; Angeletos and La'O 2020; Selgin 1997). Integrating these varied approaches into a unified theory of optimal monetary policy is an open question for future research.

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A Additional proofs

A.1 Proof of lemma 1

Proof. Equation (12) showed that a firm j in sector i with inherited price p_i^{old} adjusts if and only if:

$$\left(\frac{W}{A_i}\right)^{1-\eta} p_i^\eta y_i \left[\frac{1}{\eta}\right] - W\psi > \left(p_i^{\text{old}}\right)^{1-\eta} p_i^\eta y_i \left[1 - \frac{W/A_i}{p_i^{\text{old}}} \cdot \frac{\eta - 1}{\eta}\right]$$

Define:

$$f(W, A_i) \equiv \left(\frac{W}{A_i}\right)^{1-\eta} p_i^\eta y_i \left[\frac{1}{\eta}\right] - W\psi - \left(p_i^{\text{old}}\right)^{1-\eta} p_i^\eta y_i \left[1 - \frac{W/A_i}{p_i^{\text{old}}} \cdot \frac{\eta - 1}{\eta}\right] \tag{34}$$

The firm will adjust iff $f(W, A_i) \geq 0$.

Observe first that for the locus of (W, A_i) such that $W/A_i = p_i^{\text{old}}$, it is the case that $f(W, A_i) = -W\psi < 0$ and the firm will not adjust. That is, this locus is a subset of the inaction region $\Lambda \equiv \{(W, A_i) | f(W, A_i) < 0\}$. Thus Λ is nonempty.

In A_i space. Observe that

$$\frac{\partial f}{\partial A_i} = p_i^\eta y_i W(A_i^{-2}) \left(\frac{\eta - 1}{\eta}\right) \left[\left(\frac{W}{A_i}\right)^{-\eta} - (p_i^{\text{old}})^{-\eta}\right]$$

This is positive iff $W/A_i < p_i^{\text{old}}$. Additionally, $\lim_{A_i \to 0} f(\cdot, A_i) = \lim_{A_i \to \infty} f(\cdot, A_i) = \infty$ and $f(\cdot, A_i)$ is continuous in A_i on $(0, \infty)$.

Now consider any fixed W^0 such that there exists some A_i^0 with $f(W^0, A_i^0) < 0$. Then by the intermediate value theorem there exists an inaction interval $(\underline{\lambda}, \overline{\lambda})$ around W^0 such that $f(W^0, \underline{\lambda}) = f(W^0, \overline{\lambda}) = 0$, and $f(W^0, A_i) < 0$ iff $A_i \in (\underline{\lambda}, \overline{\lambda})$. To see that $\overline{\lambda}$ is increasing in ψ and $\underline{\lambda}$ is decreasing in ψ, observe that increasing ψ shifts the entire $f(x)$ curve down, i.e. $\frac{\partial f}{\partial \psi} < 0$.

If for a fixed W^0 there is no A_i with $f(W, A_i) < 0$, then by construction there is no point in Λ along W^0.

In W space. Similarly, observe that

$$\frac{\partial f}{\partial W} = p_i^\eta y_i A_i^{-1} \left(\frac{\eta - 1}{\eta}\right) \left[(p_i^{\text{old}})^{-\eta} - \left(\frac{W}{A_i}\right)^{-\eta}\right] - \psi$$

This is zeroed for the locus of (W, A_i) such that

$$\left(\frac{W}{A_i}\right)^{-\eta} = (p_i^{\text{old}})^{-\eta} - \psi p_i^{-\eta} y_i^{-1} A_i \left(\frac{\eta - 1}{\eta}\right)^{-1} \equiv \zeta^{-\eta}$$

Observe that $f_1 < 0$ iff $W/A_i < \zeta$. Additionally, $\lim_{W \to 0} f(W, \cdot) = \infty$.[17] Additionally, $f(W, \cdot)$ is continuous in W on $(0, \infty)$. Thus, as above, consider any fixed A_i^0 such that there exists some W^0 with $f(W^0, A_i^0) < 0$. By the intermediate value theorem there exists (abusing notation) an inaction interval $(\underline{\lambda}, \overline{\lambda})$ around A_i^0 such that $f(\underline{\lambda}, A_i^0) = f(\overline{\lambda}, A_i^0) = 0$, and $f(W, A_i^0) < 0$ iff $W \in (\underline{\lambda}, \overline{\lambda})$, where $\overline{\lambda}$ is potentially infinite. To see that $\overline{\lambda}$ is increasing in ψ and $\underline{\lambda}$ is decreasing in ψ, observe that increasing ψ shifts the entire $f(x)$ curve down, i.e. $\frac{\partial f}{\partial \psi} < 0$. $\square$

[17] The second limit comes from using L'Hopital's rule, together with the natural assumption that $\psi < \frac{1}{S\eta}$. Without this maximum bound on ψ, firms would *always* earn negative profits after adjusting – i.e., firms would *never* adjust.

A.2 Formal statement of planner's problem

Recall we derived that equilibrium welfare in each of the four regimes as a (potentially constant) function of the social planner's choice of the money supply:

$$\begin{aligned}
\mathbb{W}_{\text{all adjust}} &= \ln\left(\frac{\gamma^{1/S}}{S}\right) - [1 + S\psi] \\
\mathbb{W}_{\text{only 1 adjusts}}(M) &= \ln\left(\frac{\gamma^{\frac{1}{S}}}{S} M^{\frac{S-1}{S}}\right) - \left[\frac{1}{S} + (S-1)\frac{M}{S} + \psi\right] \\
\mathbb{W}_{\text{only } k \text{ adjust}}(M) &= \ln\left(\frac{1}{S} M^{1/S}\right) - \left[\frac{S-1}{S} + \frac{1}{S}\frac{M}{S} + \frac{S-1}{S}\psi\right] \\
\mathbb{W}_{\text{none adjust}}(M) &= \ln\left(\frac{M}{S}\right) - \left[\frac{1}{\gamma}\frac{M}{S} + (S-1)\frac{M}{S}\right]
\end{aligned}$$

Constrained planner's problem. We define the constrained planner as the planner who chooses M in each regime to maximize welfare, *constrained* by the fact that the choice of M must be incentive compatible with whether various sectors actually adjust or not:

$$\begin{aligned}
M^{*,\text{ constrained}}_{\text{only 1 adjusts}} &\equiv \arg\max_{M} \mathbb{W}_{\text{only 1 adjusts}}(M) \\
&\text{s.t. } f(M,\gamma) \geq 0 \text{ and } f(M,1) \leq 0
\end{aligned} \tag{35}$$

$$\begin{aligned}
M^{*,\text{ constrained}}_{\text{only } k \text{ adjust}} &\equiv \arg\max_{M} \mathbb{W}_{\text{only } k \text{ adjust}}(M) \\
&\text{s.t. } f(M,\gamma) \leq 0 \text{ and } f(M,1) \geq 0
\end{aligned} \tag{36}$$

$$\begin{aligned}
M^{*,\text{ constrained}}_{\text{none adjust}} &\equiv \arg\max_{M} \mathbb{W}_{\text{none adjust}}(M) \\
&\text{s.t. } f(M,\gamma) \leq 0 \text{ and } f(M,1) \leq 0
\end{aligned} \tag{37}$$

where $f(M, A_i)$ refers to the function defined in (29) which is positive if and only if firms in sector i want to adjust. This defines, for example, $M^{*,\text{ constrained}}_{\text{only 1 adjusts}}$ as the level of money supply which maximizes welfare in equilibrium when only sector 1 adjusts $\mathbb{W}_{\text{only 1 adjusts}}(M)$, *subject to the constraint that* it is indeed incentive-compatible for sector-1 firms to adjust, $f(M,\gamma) \geq 0$, and incentive-compatible for firms in sectors k to not adjust, $f(M,1) \leq 0$. Denote the associated constrained-optimal levels of welfare in each regime as:

$$\mathbb{W}^{*,\text{ constrained}}_{\text{only 1 adjusts}} = \mathbb{W}_{\text{only 1 adjusts}}\left(M^{*,\text{ constrained}}_{\text{only 1 adjusts}}\right)$$

$$\mathbb{W}^{*,\text{ constrained}}_{\text{only }k\text{ adjust}} = \mathbb{W}_{\text{only }k\text{ adjust}}\left(M^{*,\text{ constrained}}_{\text{only }k\text{ adjust}}\right)$$
$$\mathbb{W}^{*,\text{ constrained}}_{\text{none adjust}} = \mathbb{W}_{\text{none adjust}}\left(M^{*,\text{ constrained}}_{\text{none adjust}}\right)$$

The constrained social planner's problem is then to select among these, or to implement the regime where all adjust (in which case the choice of M is irrelevant, as long as it is incentive-compatible):

$$\max\left\{\mathbb{W}^{*,\text{ constrained}}_{\text{only 1 adjusts}}, \mathbb{W}^{*,\text{ constrained}}_{\text{only }k\text{ adjust}}, \mathbb{W}^{*,\text{ constrained}}_{\text{none adjust}}, \mathbb{W}_{\text{all adjust}}\right\} \tag{38}$$

which she implements with the associated incentive-compatible choice for the money supply.

Unconstrained social planner's problem. In the body of the paper and in this subsection, we endow the planner with the instrument of subsidizing menu costs, so that the constraints on (35), (36), and (37) never bind. Because taxation is lump sum and wholly non-distortionary, subsidies to offset the menu cost are equivalent to endowing the planner with the power to change prices directly (but, if doing so, still incurring a menu cost for affected firms). The unconstrained social planner's problem is thus the same as the constrained planner's problem, but without any of the implementability constraints.

$$M^*_{\text{only 1 adjusts}} \equiv \arg\max_M \mathbb{W}_{\text{only 1 adjusts}}(M) \tag{39}$$

$$M^*_{\text{only }k\text{ adjust}} \equiv \arg\max_M \mathbb{W}_{\text{only }k\text{ adjust}}(M) \tag{40}$$

$$M^*_{\text{none adjust}} \equiv \arg\max_M \mathbb{W}_{\text{none adjust}}(M) \tag{41}$$

Since the objective functions in all of these arg maxes are strictly concave, the solution is found from the first order condition, as presented in the text. We denoted the associated unconstrained-optimal levels of welfare in each regime in equations (19), (20), (21) as:

$$\mathbb{W}^*_{\text{only 1 adjusts}} = \mathbb{W}_{\text{only 1 adjusts}}\left(M^*_{\text{only 1 adjusts}}\right)$$
$$\mathbb{W}^*_{\text{only }k\text{ adjust}} = \mathbb{W}_{\text{only }k\text{ adjust}}\left(M^*_{\text{only }k\text{ adjust}}\right)$$
$$\mathbb{W}^*_{\text{none adjust}} = \mathbb{W}_{\text{none adjust}}\left(M^*_{\text{none adjust}}\right)$$

The constrained social planner's problem is then to select among these, or to implement the regime where all adjust (in which case the choice of M is irrelevant):

$$\max\left\{\mathbb{W}^*_{\text{only 1 adjusts}}, \mathbb{W}^*_{\text{only } k \text{ adjust}}, \mathbb{W}^*_{\text{none adjust}}, \mathbb{W}_{\text{all adjust}}\right\} \tag{42}$$

It is this maximization problem that produces lemma 2 and lemma 3, which in turn produce proposition 1. $\square$

B Adjustment externalities

In this section, we work with a slightly modified version of the baseline model, which is yet more analytically tractable, to analyze the role of adjustment externalities.

B.1 Model setup

The final goods producer and sectoral goods producer are exactly the same as the baseline model.

Intermediate goods producers. The intermediate goods producers again are a unit mass of monopolistically competitive firms in each sector with linear technology and productivity that is common to the sector. They again face a menu cost if adjusting prices. Here, unlike the baseline model, the adjustment cost is not ψ units of extra labor, but a penalty $(1-\psi)$ that scales down the firm manager's objective function (but not profits). Firm i in sector j faces the following maximization program:

$$\begin{aligned}
&\max_{p_i(j)} D_i(j)\left(1-\psi\chi_i(j)\right) \\
&\text{s.t. } D_i(j) = p_i(j)y_i(j) - Wn_i(j)(1-\tau) \\
&\quad \chi_i(j) = \begin{Bmatrix} 1 & \text{if } p_i(j) \neq p_i^{\text{old}} \\ 0 & \text{else} \end{Bmatrix} \\
&\quad y_i(j) = y_i\left(\frac{p_i(j)}{p_i}\right)^{-\eta} \\
&\quad y_i(j) = A_i n_i(j)
\end{aligned}$$

This menu cost is a utility penalty that is passed on to households, but does not affect physical profits. As before, all firms within a sector face the same problem, and we drop the (j) notation when the context is clear.

Households. The representative household is precisely as in the baseline model, except that the utility function (1) is modified to be:

$$\mathbb{W} = \ln C - N + \ln\left(\frac{M}{P}\right) - \psi\sum_i \chi_i$$

The household has the same preferences over consumption, labor, and real balances; but now is directly penalized in terms of welfare when firms adjust prices. The benefit of this modeling technique is that it turns off the income effects caused by menu costs, as discussed in section 5.1, and is a technique that has been used by e.g. Auclert, Rognlie and Straub (2018) or Guerrieri et al. (2021).

B.2 Shock and equilibrium

We run the same exercise, shocking the productivity of sector 1 from $A_1 = 1$ to $A_1 = \gamma > 1$. The equilibrium allocations in the four regimes 3.1 is *exactly* the same as in the body of the paper, except that the level of aggregate labor in each of the four regimes is no longer affected by menu costs. The equilibrium level of welfare in each of the four regimes as a function of the choice of money supply is:

$$\begin{aligned}
\mathbb{W}_{\text{flex}} &= \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1 \\
\mathbb{W}_{\text{all adjust}} &= \ln\left(\frac{\gamma^{1/S}}{S}\right) - 1 \\
\mathbb{W}_{\text{only 1 adjusts}}(M) &= \ln\left(\frac{\gamma^{1/S}}{S} M^{\frac{S-1}{S}}\right) - \frac{1}{S}\left[1 + M(S-1)\right] - \psi \\
\mathbb{W}_{\text{only } k \text{ adjust}}(M) &= \ln\left(\frac{1}{S} M^{\frac{1}{S}}\right) - \frac{1}{S}\left[S - 1 + \frac{M}{\gamma}\right] - (S-1)\psi \\
\mathbb{W}_{\text{none adjust}}(M) &= \ln\left(\frac{M}{S}\right) - \frac{M}{S}\left[S - 1 + \frac{1}{\gamma}\right]
\end{aligned}$$

B.3 Adjustment decision

The firm compares its objective function under price adjustment versus under the inherited price. The adjustment condition can be simplified to be written as: adjust if and only if

$$\frac{1}{\eta}(1-\psi) > \left[\frac{W/A_i}{p_i^{\text{old}}}\right]^{\eta} \left(\left[\frac{W/A_i}{p_i^{\text{old}}}\right]^{-1} - \frac{\eta - 1}{\eta}\right)$$

For additional analytical tractability, we make the following assumption in this section:

Assumption 1. The elasticity of substitution is $\eta = 2$.

This assumption allows for a closed form solution to the inaction region, using the quadratic formula: do not adjust if and only if

$$\frac{W}{A_i} \in \left(p_i^{\text{old}}(1-\sqrt{\psi}), p_i^{\text{old}}(1+\sqrt{\psi})\right) \tag{43}$$

Clearly this has the same properties as the Λ inaction region described in lemma 1.

When starting from the steady state where p_i^{old} for all sectors i, and using the equilibrium $W = M$ condition, then we have the following. The inaction region for sector 1 is

$$M \in (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi}))$$

The inaction region for sectors k is

$$M \in (1-\sqrt{\psi}, 1+\sqrt{\psi})$$

B.4 The planner's problem

The planner's problem – importantly, without the ability to subsidize menu costs and so denoted "constrained" – written in full is:

$$\max_{\substack{\text{all adjust, only 1 adjusts}\\ \text{only } k \text{ adjust, none adjust}}} \left\{\mathbb{W}_{\text{all adjust}}, \mathbb{W}^{*,\text{constrained}}_{\text{only 1 adjusts}}, \mathbb{W}^{*,\text{constrained}}_{\text{only } k \text{ adjust}}, \mathbb{W}^{*,\text{constrained}}_{\text{none adjust}}\right\} \tag{44}$$

s.t.

$$\mathbb{W}_{\text{all adjust}} = \left\{\ln\left(\frac{\gamma^{1/S}}{S}\right) - 1\right\}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only 1 adjusts}} = \left\{\begin{matrix} \max\limits_M \ \ln\left(\frac{\gamma^{1/S}}{S} M^{\frac{S-1}{S}}\right) - \frac{1}{S}[1+M(S-1)] - \psi \\ \text{s.t. } M \in (1-\sqrt{\psi}, 1+\sqrt{\psi}) \\ M \notin (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi})) \end{matrix}\right\}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only } k \text{ adjust}} = \left\{\begin{matrix} \max\limits_M \ \ln\left(\frac{1}{S} M^{\frac{1}{S}}\right) - \frac{1}{S}\left[S-1+\frac{M}{\gamma}\right] - (S-1)\psi \\ \text{s.t. } M \notin (1-\sqrt{\psi}, 1+\sqrt{\psi}) \\ M \in (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi})) \end{matrix}\right\}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{none adjust}} = \left\{\begin{matrix} \max\limits_M \ \ln\left(\frac{M}{S}\right) - \frac{M}{S}\left[S-1+\frac{1}{\gamma}\right] \\ \text{s.t. } M \in (1-\sqrt{\psi}, 1+\sqrt{\psi}) \\ M \in (\gamma(1-\sqrt{\psi}), \gamma(1+\sqrt{\psi})) \end{matrix}\right\}$$

B.5 Interior optima

The interior optima for each regime, found from the first order conditions, are the same as the baseline model:

$$
\begin{aligned}
M^{\text{interior}}_{\text{only 1 adjusts}} &= 1 \\
M^{\text{interior}}_{\text{only } k \text{ adjust}} &= \gamma \\
M^{\text{interior}}_{\text{none adjust}} &= \frac{S}{S-1+1/\gamma}
\end{aligned}
$$

B.6 Only sector 1 adjusts: The possibility of *positive* adjustment externalities

Suppose the unconstrained social planner – i.e., one who could subsidize menu costs and ignore the implementability constraints – would want to implement the regime where only sector 1 adjusts, and she therefore wants to set $M = 1$. We now examine whether this is incentive compatible: does it result in sector-k firms being within their inaction region and sector-1 firms being outside it?

First observe that $M = 1$ indeed ensures that sector-k firms are in their inaction region, since $M = 1 \in (1 - \sqrt{\psi}, 1 + \sqrt{\psi})$ always.

However, it is possible that $M = 1$ could leave sector-1 firms inside their inaction region, if the following condition holds:

$$\gamma < \frac{1}{1-\sqrt{\psi}} \equiv \gamma_1 \tag{45}$$

As an existence proof, it is possible to come up with numerical examples for parameters satisfying the above where it would be, in fact, socially optimal to implement this regime if there were no implementability constraints. When this is the case, then the best the central bank can do within this regime is to set $M = \gamma(1 - \sqrt{\psi})$. This is a case of *positive* adjustment externalities: the social planner would prefer that sector 1 adjusts its prices, even though it is individually rational to not do so.

B.7 No sectors adjust: The possibility of *negative* adjustment externalities

Now suppose the unconstrained social planner would prefer that no sector adjusts (i.e. $\gamma < \overline{\gamma}$). The interior optimum level of the money supply, as previously noted, would be $M^{\text{interior}}_{\text{none adjust}} = \frac{S}{S-1+1/\gamma}$. Is this incentive-compatible?

To be incentive-compatible requires that both $\frac{S}{S-1+1/\gamma} > \gamma(1-\sqrt{\psi})$ and $\frac{S}{S-1+1/\gamma} < 1+\sqrt{\psi}$. Thus, there is a *negative* adjustment externality – where its privately optimal for firms in a sector to adjust even when its not socially optimal to do so – if either:

$$\gamma < \frac{1+\frac{1}{S-1}\sqrt{\psi}}{1-\sqrt{\psi}} \equiv \gamma_2 \tag{46}$$

or

$$\gamma > \frac{1+\sqrt{\psi}}{S-(S-1)(1+\sqrt{\psi})} \equiv \gamma_3 \tag{47}$$

As an existence proof, it is possible to come up with numerical examples for parameters satisfying either of the above where it would be, in fact, socially optimal to implement this regime if there were no implementability constraints. When this is the case, then the best the central bank can do within this regime is to set M at the respective boundary.

B.8 Summarizing the possibilities for welfare

A similar analysis the above can be done for the case when only sectors k adjust, where a constraint will bind if $\gamma > \gamma_4 \equiv 1+\sqrt{\psi}$. We summarize the results from above and this additional case in the following:

$$\mathbb{W}_{\text{all adjust}} = \mathbb{W}_{\text{flex}} - S\psi$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only 1 adjusts}} = \begin{Bmatrix} \mathbb{W}_{\text{flex}} - \psi & \text{if } \gamma \geq \gamma_1 \\ \ln\left(\frac{\gamma^{1/S}}{S}\gamma_1^{\frac{S-1}{S}}\right) - \frac{1}{S}\left[1+\gamma_1(S-1)\right] - \psi & \text{else} \end{Bmatrix}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{only } k \text{ adjust}} = \begin{Bmatrix} \mathbb{W}_{\text{flex}} - (S-1)\psi & \text{if } \gamma \leq \gamma_4 \\ \ln\left(\frac{1}{S}\gamma_4^{1/S}\right) - \frac{1}{S}\left[S-1+\frac{\gamma_4}{\gamma}\right] - (S-1)\psi & \text{else} \end{Bmatrix}$$

$$\mathbb{W}^{*,\text{constrained}}_{\text{none adjust}} = \begin{Bmatrix} -\log\left[S-1+1/\gamma\right]-1 & \text{if } \gamma \in [\gamma_3, \gamma_2] \\ \ln\left(\frac{\gamma_2}{S}\right) - \frac{\gamma_2}{S}\left[S-1+\frac{1}{\gamma}\right] & \text{if } \gamma > \gamma_2 \\ \ln\left(\frac{\gamma_3}{S}\right) - \frac{\gamma_3}{S}\left[S-1+\frac{1}{\gamma}\right] & \text{if } \gamma < \gamma_3 \end{Bmatrix}$$

Optimal monetary policy considers which of these achieves the highest welfare, and sets the money supply M to implement.