The More You Learn, the *Fewer* Places You'll Go: The Rise in Education and the Decline in Worker Mobility

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Abstract

Why has worker mobility in the United States declined so much over the past decades? While previous work attributes this decline to reduced labor market dynamism, this paper reveals that one third of this decline is due to increased educational attainment among workers. Higher education affects labor mobility in two ways. First, having a larger share of young workers in school rather than in the labor market precludes these very workers, who are typically the most mobile, from switching jobs and occupations. Second, education provides workers an alternative to learning about their "type" making educated workers less reliant on experimenting with new jobs.

JEL Codes: J62, J22, J24, I23, I26

The more that you read, the more things you will know. The more that you learn, the more places you'll go.

Dr. Seuss

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1 Introduction

Labor mobility is a key driver of earnings growth for workers (Topel and Ward, 1992). Over the past 30 years, workers in the United States have become substantially less mobile: the monthly job-switching rate, the rate at which workers switch to a new employer without an intervening period of unemployment, fell from 2.90% in 1994 to 2.36% by the end of 2019. Understanding the causes of these trends is critical to evaluating the overall health of the labor market.

An influential argument in the literature suggests that the decline in job mobility reflects a broader decline in "economic dynamism" (Davis and Haltiwanger, 2014; Decker et al., 2016; Bilal et al., 2021) and therefore is a cause for concern. Our paper suggests a more positive connotation. We argue that between one third to one half of the decline in job mobility is due to increased educational attainment on the part of workers, driven by a rising skill premium. Two forces drive this trend. First, a greater share of young workers, who generally transition between jobs at a higher rate, are in school and hence no longer in the workforce, reducing the average transition rate. Second, we argue that spending time in school allows people to learn about the type of job that they are best suited for: by learning what coursework they enjoy, students implicitly learn what jobs will suit them best in the future. Thus, young workers with more schooling find better initial matches. This reduces their need to switch jobs over the course of their careers relative to workers with less schooling. By allowing workers to form better matches, schooling provides a benefit beyond human capital accumulation that people deciding whether or not to stay in school trade-off against the costs of forgone labor market experience. These costs, which are well established in the literature, include lost income and missed human capital accumulation on the job.

Empirics. In the first part of the paper we study the mobility of the US workforce in the aggregate and across different demographic characteristics. We document that both job- and occupation-switching rates have declined in the United States since the 1990s. Second, we document large differences in mobility by age and by education: younger and less educated workers are more mobile than older and more educated ones. Third, we conduct two exercises to investigate what the job- and occupation-switching rates would have been if (i) the share of working-age people engaged in school had remained fixed to what it was in 1996, and (ii) the share of college and non-college educated workers had been fixed to their 1996 values. These shift-share exercises imply that roughly one third of the decrease in labor mobility can be attributed to increased educational attainment.

Evidence for the mechanism. In the second part of the paper, we provide direct evidence that more years of schooling translate to lower job-switching rates for young workers. We use the Survey of Income and Program Participation (SIPP) to calculate the propensity of workers to make E-E transitions, and show that this propensity is negatively related to years of schooling. To circumvent endogeneity concerns related to unobserved worker attributes like ability, we use an instrumental variables strategy, exploiting minimum wage changes as an exogenous shifter of the opportunity cost of staying in school. We show that an extra year of schooling reduces the expected number of E-E transitions over one year by almost 0.3, roughly one standard deviation of the number of transitions that occur in a given year. Our results are robust to focusing on young workers only.

Model. (In progress). To interpret the evidence above and study the strength of our proposed mechanism, we construct a model in which agents choose how long to stay in school and, on entering the labor market, when to switch jobs. Agents begin in school with different innate levels of human capital, which schooling allows them to augment. When agents decide to graduate, they transition irreversibly to working in the labor market. The labor market features two-sided heterogeneity, with the output of a given firm-agent pair determined by how well-matched the two are. The longer an agent stays in school, the more information they accumulate about their *job type* – the job best suited for them – helping them decide what job to apply to upon graduation. While working, agents receive signals of their job type, which induce them to switch to better matched jobs à la Jovanovic (1979). Agents thus trade off the opportunity cost of an extra period in school, the foregone wage income, against the benefits of higher human capital and finding a better initial match in the labor market. We calibrate the model to match the age profile of the mobility rate and moments of the distribution of wages in 1996.

Counterfactual Exercises. (In progress). We use the model to conduct two exercises. First, we ask how much of the decrease in job mobility experienced in the US over the past decades is due to the increase in worker education. Starting from the baseline 1990s calibration, we raise the skill premium – the returns to skill in our model – to mimic the large increase in the returns to years of higher education that occurred up to the 2000's (Katz and Murphy, 1992). This allows us to evaluate how much of the decline in mobility is due to increased educational attainment. Second, we extend the model to study the impacts of two classes of policies. First, we study the implication of education requirements imposed by employers by introducing minimum years of schooling restrictions on workers applying to high quality jobs. Second, we study the implications of policies that reduce the costs of accessing college. Our model reveals a previously unexplored gain for society from raising access to college, which is that workers with more years of schooling pay the costs of switching jobs less frequently.

2 Literature

Our paper contributes to the following literatures. First, we contribute to a literature documenting and trying to establish causes of declining job transition rates (Davis and Haltiwanger, 2014; Decker et al., 2016). Hyatt and Spletzer (2013) show that the decline in worker mobility is associated with a decline in the share of low-tenure jobs. Bosler and Petrosky-Nadeau (2016) show that the decline in job mobility is concentrated among young workers. Our model provides a mechanism for the decline in job mobility consistent with both of these findings. Eeckhout and Weng (2023) construct a model in which the decline in labor mobility arises due to a combination of rising costs of mismatch between workers and jobs, a lower variance and arrival rate of productivity shocks, and an increase in the costs of searching. Hedtrich (2022) emphasizes the role of labor market polarization, which reduces the ability of workers to rise in their job ladders due to the fall in demand for middle-skilled routine-intensive workers. We see our paper as being complementary to these forces.

Closest to our work is Mercan (2017), who rationalizes the decline in job-to-job mobility as the outcome of better information availability about job quality, which reduces the need for workers to search for better matches. In our paper, we provide an interpretation of this "information channel" - rising educational attainment leads workers to possess superior information about the types of jobs they initially obtain. Unlike Mercan (2017), we also study the direct composition effects arising out of the optimal decisions by young workers to delay labor market entry as a result of the rising returns to years of schooling.

Second, we contribute to a large literature on the determination of wages and wage growth over the life cycle. Becker (1962), Mincer (1974) and Heckman, Lochner and Taber (1998) emphasize the role of private human capital investments, usually associated with schooling. Jovanovic (1979), Topel and Ward (1992), Papageorgiou (2014) and Bagger et al. (2014) emphasize the role of job-to-job moves. Pastorino (Forthcoming) shows that learning on the job about one's ability is an important determinant of transitions across jobs. Relative to this literature, we highlight an *informational* benefit to spending more time in school. While spending more time in school implies foregone income and a missed opportunity to gain on-the-job and firm-specific human capital early in life, it gives agents the opportunity to learn about the jobs that suit them best, reducing the number of costly job switches they undertake over their careers.

3 Mobility, age and education: a shift-share exercise

In this section, we document heterogeneity in worker mobility across age and education groups. We show that young workers are, all else equal, more mobile, and that better educated workers are, all else equal, less mobile. In section 4 we will provide some evidence that this latter correlation is not driven by composition effects or unobserved attributes of workers in education, such as ability. We then document that educational attainment has been rising for all age groups in the US, but disproportionately so for the young.

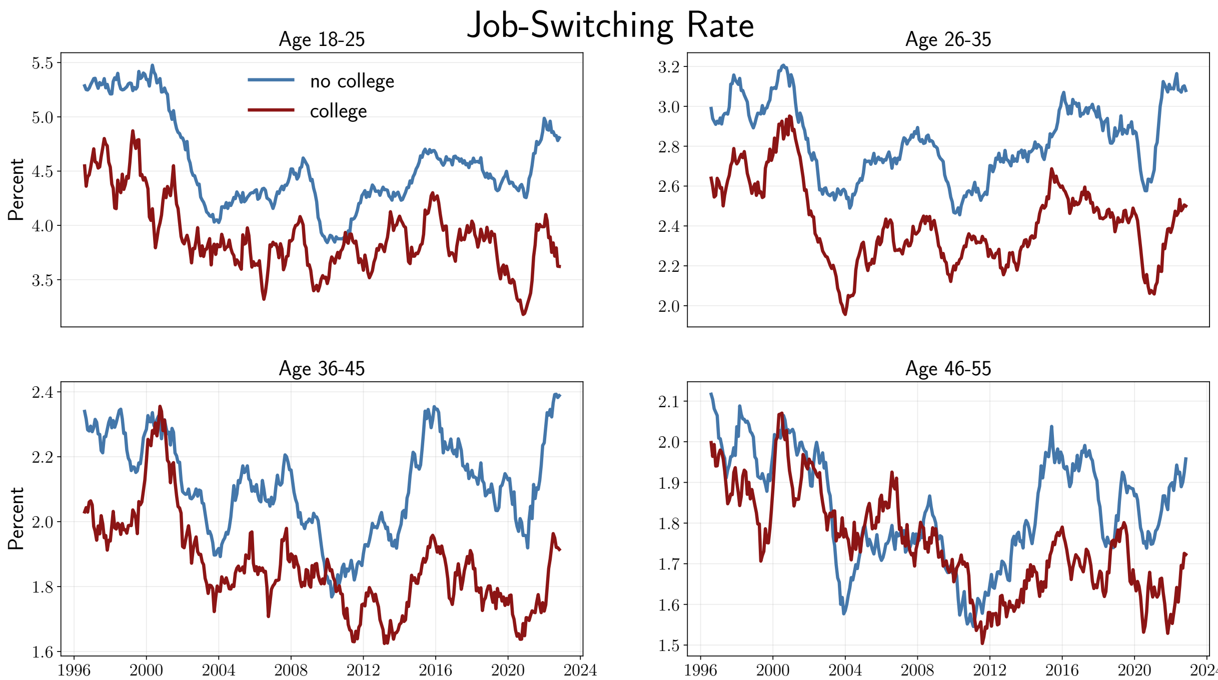
3.1 Facts about Mobility by Age and Education

Workers are very mobile when they first enter the labor market and become less and less so with age. More than 4 percent of workers aged 18-25 and roughly 2.6 percent of workers aged 26-35 switch jobs in a given month month. By contrast, only 2 percent of workers aged 36-45 and 1.7 percent of workers aged 46-55 switch jobs in any given month. The differences across age are even more striking when it comes to occupational switching: around 2.5 percent of workers aged 18-25 and roughly 1.2 percent of workers aged 26-35 switch occupations every month. By contrast, only 0.8 percent of workers aged 36-45 and 0.6 percent of workers aged 46-55 switch occupations in any given month[1]. Figure 1 shows, job and occupation-switching rates vary not only across age groups but also across educational groups: consistently, across all age groups, college educated workers (blue line) are less likely to switch jobs and occupations than workers without a college degree (red line).

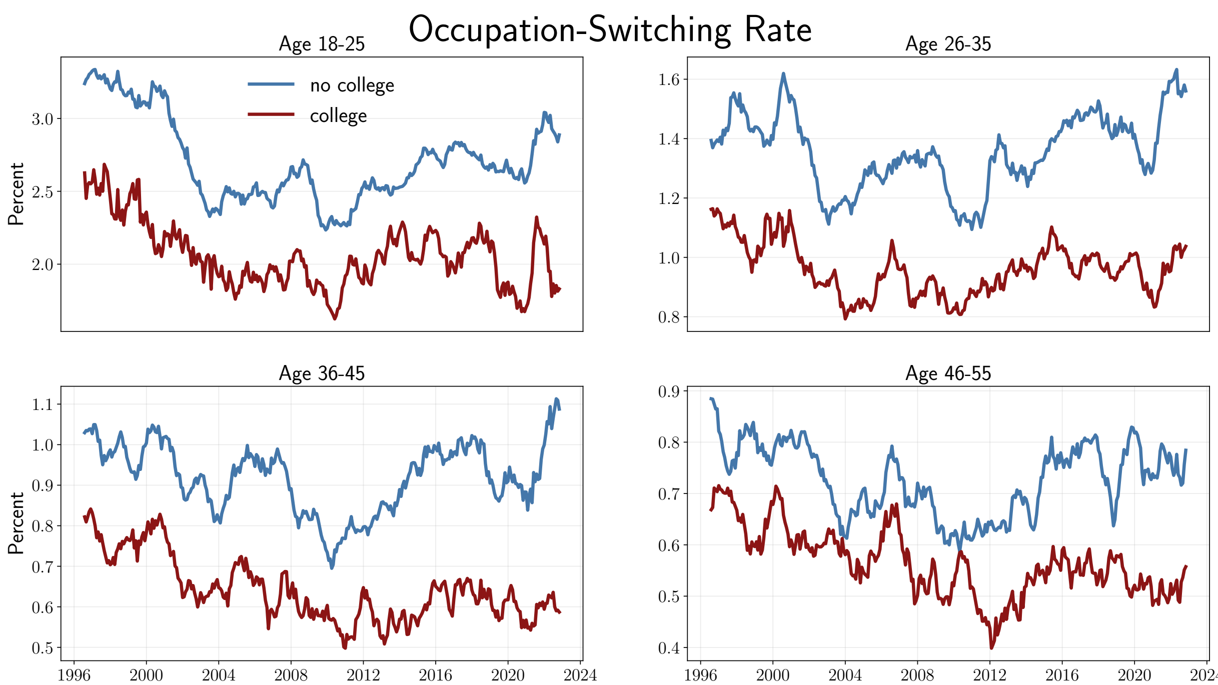
Over the last thirty years, the US has seen a substantial expansion in access to higher education, with the share of workers with a four-year college degree or greater rising from 26.5% in 1992 to 38.9% by 2016. Figure 2 plots the share of hours worked by workers with a given level of educational attainment, showing that between 1995 and 2019, the share of hours worked by workers with a four year degree or more has risen by 9.6 percentage points. Panel B of figure 3 decomposes the rise in educational attainment by age, and shows that the share of college-educated workers has increased within all age groups.

The differences in labor market mobility by education and age we documented earlier, combined with this first-order trend in the share of workers with a college degree, suggest that the overall trend in labor market mobility can be explained at least in part by the effect of rising educational attainment. We argue that there are two ways in which this

[1] Our job-switching results are consistent with those of Bosler and Petrosky-Nadeau (2016).



(a) Job-switching rate by age and education



(b) Occupation-switching rate by age and education

Figure 1: Declining labor market mobility (job and occupation transition rates) in the United States. Sources: CPS and SIPP.

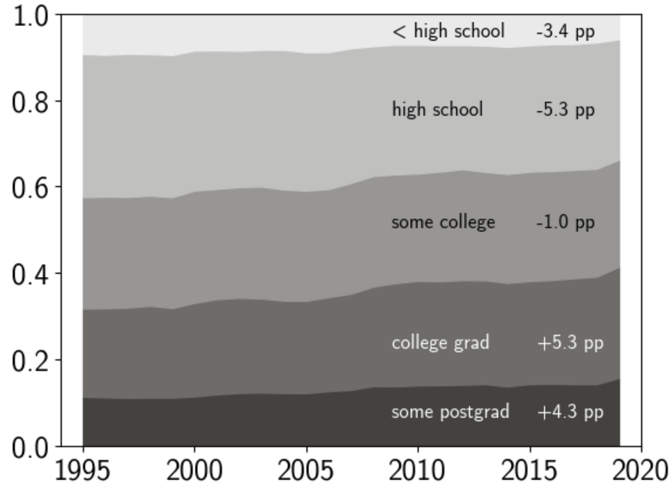


Figure 2: Shares of different educational groups in total hours worked, constructed following Autor (2019). Source: CPS

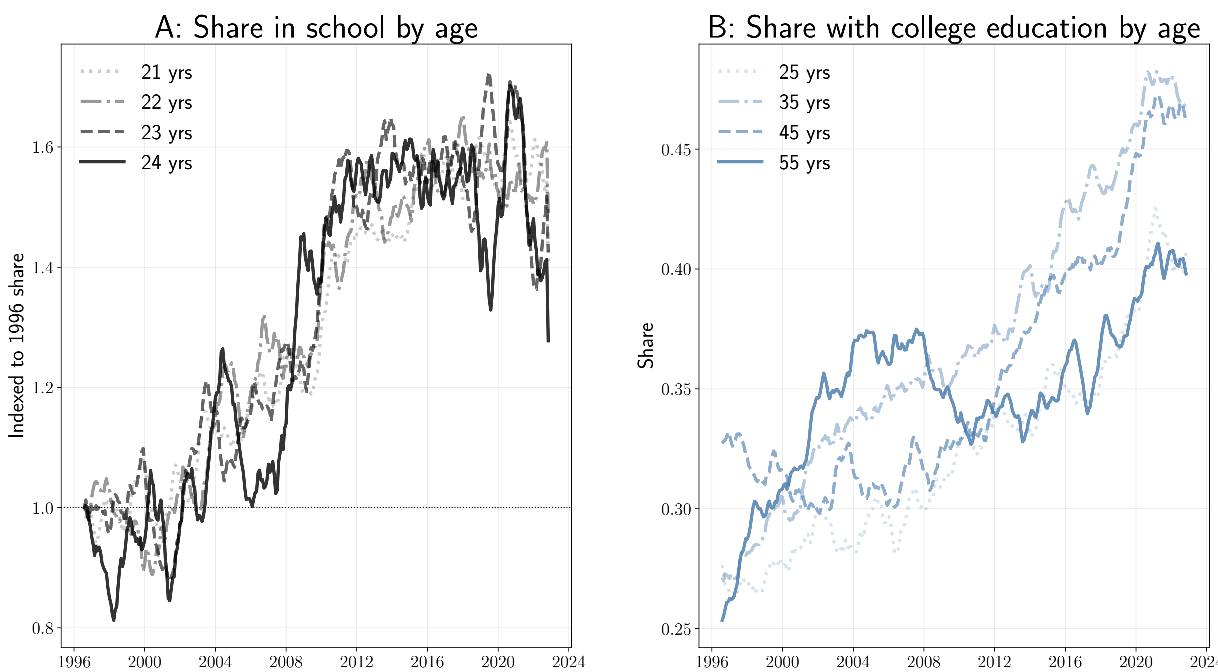


Figure 3: Shares of workers in education and with a completed college degree.

may occur. First, as panel A of figure 3 shows, rising educational attainment implies that at any given point of time, young individuals spend more time in school and postpone their labor market entry. Since these workers are more mobile on average, a decline in their share in the labor force will reduce aggregate mobility via a composition effect. Second, since college educated workers are less mobile on average, a rising share of college educated workers should directly decrease aggregate job mobility in the economy.

To investigate how much of the decline in mobility in the US can be explained by changes in educational attainment, we now run a shift-share exercise. We use data from the Current Population Survey to collect evidence on job and occupational mobility between 1996 and 2023. The definition of job mobility exactly follows the methodology in Fujita, Moscarini and Postel-Vinay (2023). Similarly, we define an occupational switch to be a job switch in which the occupational code provided in the CPS also changes across jobs. CPS contains a panel dimension that allows us to continuously track individuals over short periods of time. Each housing unit is interviewed for four months, then dropped for eight months, and finally re-interviewed for four more months. This leads to one significant limitation: we can observe a maximum of 6 job transitions. Nonetheless, CPS has characteristics that make it useful to us: it has a relatively long time series, it tracks other individual characteristics of importance, namely education, it is at the monthly frequency,[2] and it samples a large number of individuals. Similar results hold when we use the Survey of Income and Program Participation (SIPP) which is ideal on all fronts except that it has a relatively short time series (the survey at high-frequency is run only up to 2013).

To describe our counterfactuals, we introduce some notation. Denote the aggregate job mobility date at date t by x_t. In what follows, let $x_t^{a,i}$ denote the job mobility rate for workers of age a and educational status i at date t. Let $\omega_t^{a,i}$ be the share of the labor force consisting of workers of age group a and educational status $i \in \{\text{college, no college}\}$ at date t. Let $\tilde{\omega}_t^a$ be the share of the population in the corresponding age group. Then, $\tilde{\omega}_t^a$ and $\omega_t^{a,i}$ are related by the identity

$$\omega_t^a \equiv \omega_t^{a,\text{college}} + \omega_t^{a,\text{no college}} = \tilde{\omega}_t^a \left(1 - p_t^{\text{school}} - p_t^{\text{no school+NE}}\right)$$

where $p_t^{a,\text{school}}$ is the share of people in age group a that are in school, and $p_t^{a,\text{no school+NE}}$ is the share of people in age group a that are not in school and not employed. Now note

[2] The lower the frequency the harder job-transitions are to identify.

that the following identity must hold at all dates.

$$x_t = \sum_a \tilde{\omega}_t^a \left(1 - p_t^{\text{school}} - p_t^{\text{no school+NE}}\right) \left[\omega_t^{a,\text{college}} x_t^{a,\text{college}} + \omega_t^{a,\text{no college}} x_t^{a,\text{college}}\right] \tag{1}$$

In the first step, we hold college enrolment rates constant at their 1996 levels, and ask how mobility rates would have evolved in the absence of higher enrolment rates. These counterfactuals capture how changes in the composition of the workforce by educational status have affected aggregate mobility via a pure composition effect. Denote these counterfactual mobility rates by $\widehat{x}_t^{\text{1996 college}}$, where $x \in \{EE, OO\}$. We compute them using the formula

$$x_t = \sum_a \tilde{\omega}_t^a \left(1 - p_t^{\text{school}} - p_t^{\text{no school+NE}}\right) \left[\omega_{1996}^{a,\text{college}} x_t^{a,\text{college}} + \omega_{1996}^{a,\text{no college}} x_t^{a,\text{college}}\right] \tag{2}$$

The green lines in figures 4a and 4b show the counterfactual job-switching, $\widehat{EE}_t^{\text{1996 college}}$, and occupation-switching, $\widehat{OO}_t^{\text{1996 college}}$, rates relative to the actual data x_t in black. Relative to the 40 bps fall in the true E-E transition rate, the counterfactual declines by only 30 bps. Even more significant, relative to the 27 bps fall in the true O-O transition rate, the counterfactual declines by only 18 bps.

Second, we ask what the mobility rates would be had the shares of workers in school at each age remained as they were in 1996. These counterfactuals capture how much of the decline in mobility is due to more workers being in school rather than in employment. Denote the counterfactual mobility rates we construct by $\widehat{x}_t^{\text{1996 school}}$. We compute them using the formula

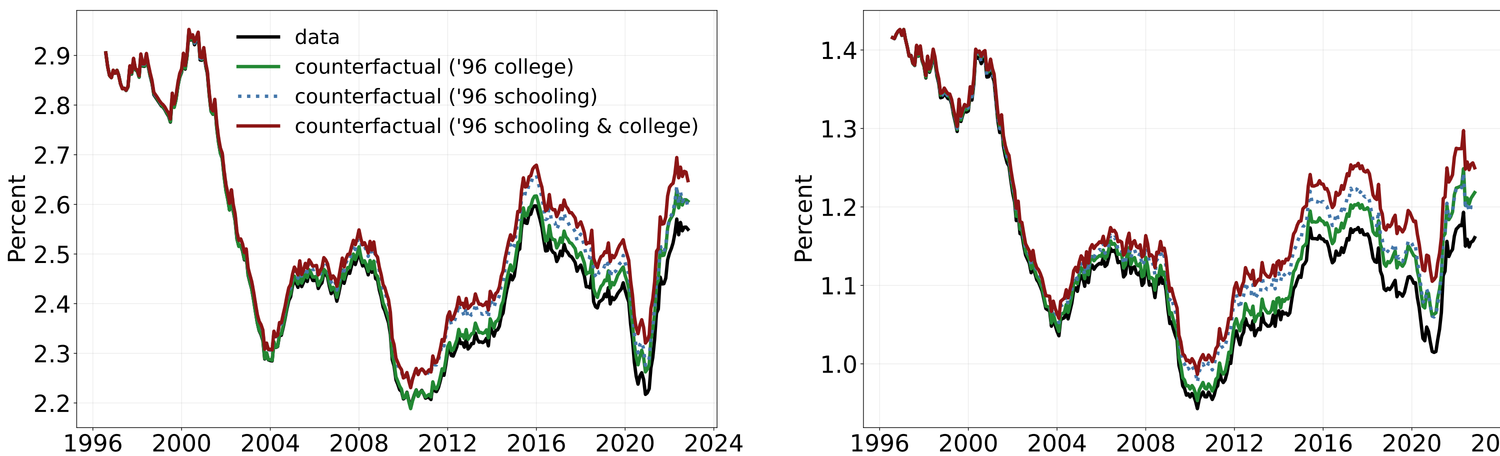
$$\widehat{x}_t^{\text{1996 school}} = \sum_a \tilde{\omega}_t^a \left(1 - p_{1996}^{\text{school}} - p_{1996}^{\text{no school+NE}}\right) \left[\omega_t^{a,\text{college}} x_t^{a,\text{college}} + \omega_t^{a,\text{no college}} x_t^{a,\text{college}}\right] \tag{3}$$

The blue dotted lines in figures 4a and 4b show the counterfactual job-switching and occupation-switching rates relative to the actual data in black.

Finally, the red lines in figures 4a and 4b show the counterfactual job and occupation-switching rates in the case both the schooling and college education shares are fixed to their 1996 levels. This counterfactual rates, denoted $\hat{x}_t^{1996}$ for $x \in \{EE, OO\}$, is computed as

$$\hat{x}_t^{1996} = \sum_a \tilde{\omega}_t^a \left(1 - p_{1996}^{a,\text{school}} - p_{1996}^{a,\text{no school+NE}}\right) \left(\omega_{1996}^{a,\text{college}} x_t^{a,\text{college}} + \omega_{1996}^{a,\text{no college}} x_t^{a,\text{no college}}\right) \tag{4}$$

These exercises suggest that changes in schooling and college rates experienced over the past 30 years contributed to more than one third of the overall decline in the job-switching rate and almost half of the overall decline in the occupation-switching rate from 1996 to today. A significant chunk of the gap between the counterfactual series and the data is explained by the fact that college educated workers switch jobs less than non-college educated workers do. In the following section, we provide direct evidence for this.



(a) Job-switching rate counterfactuals

(b) Occupation-switching rate counterfactuals

Figure 4: Counterfactual mobility rates. Source: CPS.

4 Direct evidence for the mechanism

In this section, we provide some direct evidence for the central component of our idea: that more time spent in school reduces the propensity to switch jobs on labor market entry. To do this, we exploit changes in state-level minimum wages. Intuitively, an increase in the minimum wage raises the returns to working, particularly so for low-skilled individuals. By raising the opportunity cost of time spent in schooling, this can induce fewer individuals to continue in higher education (Neumark, 1995; Neumark and Wascher, 1995). Alessandrini and Milla (2021) show, using individual-level panel data from Canada, that minimum wage increases affect enrolment in four-year college degrees adversely, with a 10% increase in minimum wages reducing four-year college enrolment by 5%. In line with our model, less time spent in school implies less learning about one's own type and hence higher rates of job mobility in the labor market.

Our identifying assumption is that higher minimum wages do not directly affect the probability than a given individual switches jobs, other than by influencing how much time an individual spends in school. We believe this assumption is defensible. Changes in minimum wages are driven by a complex legislative process aggregating the preferences of a wide range of stakeholders, not just individuals at the margin between entering the workforce and going to college. Further, changes in minimum wages should symmetrically affect the returns to working at one's existing workplace and the returns to working in a different firm. Thus, theoretically, there is no reason to expect changes in minimum wages to directly affect worker *mobility* at the worker level.

In what follows, we use data from the Survey of Income and Program Participation (SIPP). Following the work of Nagypál (2008), we define a worker's employment status in a given month to be the employment status in the final week of that month. If the worker has multiple jobs, the *main* job is the one with the most hours. Following the definition in Caratelli (2022), a worker switches jobs when they are employed full time (i.e. they work more than 35 hours a week) in both the current and past month, their main job identifier changes from one month to the next, and the new main job is not in the history of main jobs at which an individual has been employed in the past. This last restriction is appropriate given the role mobility plays in this paper as previously held jobs do not provide new information to worker. We further exclude workers with annual income less than $\$1,000$. We consider four SIPP surveys: the 1996, 2001, 2004, and 2008. These allow our analysis to stretch from 1996 to 2014 with only a few months of interruptions within the sample.[3]

Our main outcome of interest is the number of job switches individual i makes within one year of a given month t, $NumTrans_{i,t\to t+12}$. We aim to check whether higher years of schooling achieved by a given date t translate to a lower number of transitions in a given year. Our coefficient of interest is β in the regression

$$NumTrans_{i,t\to t+12} = \alpha + \gamma_i + \gamma_t + \beta YearsEduc_{it} + \varepsilon_{it} \tag{5}$$

The key threat to identification is the existence of time-varying unobserved individual circumstances which might affect both years of education and the number of transitions individuals undertake in subsequent periods. One example of this might be heterogeneity in individual ability - high ability individuals might face lower costs of schooling (à la Spence 1973) *and* also face lower costs of learning in school, allowing them to acquire more information on their individual types for a given number of years of schooling. This

[3] As many other users of SIPP, we do not use pre-1996 and post-2014 waves because of lower quality of the data in those years.

kind of variation would, all else equal, bias the coefficient towards zero. To get around this, we use an instrumental variables strategy, using local minimum wages as a shifter of the returns to entering the labor market. We obtain data on local minimum wages from Vaghul and Zipperer (2022).

Table 1 shows our results. The first two columns show OLS estimates of equation 5, first with a time trend and second with a state-specific time trend. The coefficient estimates are negative, precisely estimated and small in magnitude - the estimate in the second column suggests that an additional year of education reduces the number of job switches expected in a year by 0.009, or about 10% of its mean value. In column 3, we restrict the sample to only keep dependents: mostly young people for whom the minimum wage increase and the schooling decision is most relevant.[4]. Our results are virtually unchanged. The IV estimates are substantially larger, and strongly negative - the estimate in the fifth column suggests that an additional year of schooling translates to an increase in the expected number of job switches by 0.28, about 9 times the mean number of transitions and about 0.8 standard deviations of the mean number of transitions. When we restrict attention to dependents (column 6), our estimate is almost equal to one standard deviation of the number of job switches in a given year.

Why do additional years of schooling imply a lower propensity to switch jobs? Our argument is that while in school, agents learn about the specific types of jobs and careers that they wish to pursue, ensuring that the types of jobs they apply to on graduation are better aligned with their intrinsic motivation and abilities. This is in line with an extensive literature in sociology which documents the emergence of the college experience as a key part of "identity exploration" by young adults aged 18-29, a process by which they identify the specific careers they will potentially commit to for a substantial portion of their lives (Arnett, Žukauskienė and Sugimura, 2014; Arum and Roksa, 2014). Additionally, college attendance allows for a wide range of experiences and experimentation across careers, such as temporary work assignments and internships, as well as targeted guidance on career choices provided by university placement services.

[4]We define dependents as those with person number `epppnum=103`.

	(1) **OLS**	(2) **OLS**	(3) **OLS**	(4) **IV**	(5) **IV**	(6) **IV**
Years in School	-0.009*** (0.0006)	-0.009*** (0.0006)	-0.006*** (0.001)	-0.221** (0.0689)	-0.278*** (0.0535)	-0.35** (0.146)
Indiv FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
State FE	Yes	No	No	Yes	No	No
Trend	Yes	No	No	Yes	No	No
State x Trend	No	Yes	Yes	No	Yes	Yes
N	7,780,682	7,780,682	610,254	7,780,682	7,780,682	610,254
Sample	All	All	Dep.	All	All	Dep.

Table 1: Estimates of equation 5. In all specifications, the dependent variable is the expected number of job-job transitions occurring within a year of the observation. Robust standard errors in parentheses.
$* \ p < 0.05$, $** \ p < 0.01$, $*** \ p < 0.001$

5 Model

We construct a partial equilibrium model of schooling and employment. The economy is populated by overlapping generations of risk-neutral agents. Agents start their lives in school with a known ability[5], a_0. While in school, students accumulate ability at the cost of foregone labor market earnings. They can choose to irreversibly transition into employment at any age. If so, agents become workers and draw a type, ϕ^w, which indicates the job they are best suited for. Workers earn more if employed at jobs consistent with their type. While in the labor market, workers slowly learn about their true type by observing noisy signals. Workers can switch jobs in pursuit of a better match.

Life cycle. Let $j = j_0, \ldots, J_R, \ldots, J_D$ denote the age of an agent. At j_0, agents draw ability $a_0 \sim \mathcal{N}\left(0, \sigma_a^2\right)$ and can, for the first time, choose to join the labor market. At age $J_R + 1$, agents retire deterministically and receive an annuity proportional to their final wage until their certain death at age J_D. Next, we consider each stage of agents' lives.

Schooling. At j_0 agents have to decide whether to continue their education and go to college or to end their education and enter the labor market. An agent with ability a will then make the following choice at j_0:

$$V_{j_0}(a) = \max\left\{V_{j_0}^{\text{school}}(a), V_{j_0}^{\text{work}}(a)\right\} \tag{6}$$

If the agent chooses to remain in school for the next four years (J^{school} =16 quarters), they accrue ability according to the low of motion

$$a' = f(a) \tag{7}$$

where $f(\cdot)$ satisfies $f_1, f_2 \geq 0$, $f_{12} = f_{21} \geq 0$, $f_{11}, f_{22} < 0$, and $\lim_{x\to\infty} f_1 = \lim_{a\to\infty} = 0$. Thus, an agent choosing to stay in school has a value defined by

$$\begin{aligned} V_{j_0}^{\text{school}}(a) &= -x + \beta^{J^{school}} V_{j+J^{school}}^{\text{work}}\left(a'\right) \\ \text{s.t.} \quad & a' = f(a) \end{aligned} \tag{8}$$

If the agent chooses to enter the workforce, they can expect value $V_{j_0}^{\text{work}}(a)$ which we

[5] Structural models of college attendance (Hendricks and Leukhina, 2014, 2017) show that students who attend college are, more often than not, well-informed about their own higher ability relative to non-attenders, which contributes to their ability to complete college and attain the benefits of the skill premium.

discuss next.

Entering the labor market. Agents who decide to start work at $j_s \in \{j_0, J^{school}\}$ enter the labor market by searching for a job in a particular labor market. Before searching, agents observe a noisy signal, $\hat{\phi}$, of their true ideal job type, ϕ:

$$\hat{\phi} = \phi + s$$

where the distribution of true types $\phi \sim \mathcal{N}\left(0, \sigma_\phi^2(j_s)\right)$ and the distribution of the signal is $s \sim \mathcal{N}\left(0, \sigma_s^2\right)$. Importantly, the variance $\sigma_\phi^2(j_s)$ is decreasing in j_s. In other words, an agent who spends longer in school is more certain of their type.

Thus, at j_s, the agent expects a value from starting work given by

$$V_{j_s+1}^{\text{work}}(a) \equiv \mathbb{E}_\phi \left\{ \mathbb{E}_{\hat{\phi}|\sigma_\phi^2(j_s)} \left[V_{j_s}^{\text{search}}\left(a, \hat{\phi}\right) \middle| \phi \right] \right\} \tag{9}$$

where $V_{j_s}^{\text{search}}$ is the value of searching for a job for a worker with ability a, conditional mean type $\hat{\phi}$, and with j_s years of schooling.

Working life. Working agents match with firms in a frictional labor market. There is a continuum of labor markets, each indexed by the type of job ϕ^f that is offered in it. In labor market ϕ^f the job finding rate faced by searchers is $\lambda\left(\phi^f\right)$. Workers search for the "best" labor market to work in given their current beliefs

$$V_{J_s+1}^{\text{search}}\left(a, \hat{\phi}, J_s\right) = -c + \max_{\phi^f} \lambda\left(\phi^f\right) \underbrace{W_{J_S+1}\left(a, \hat{\phi}, \phi^f, J_S\right)}_{\text{employment}} + \left(1 - \lambda\left(\phi^f\right)\right) \underbrace{U_{J_S+1}\left(a, \hat{\phi}, J_S\right)}_{\text{unemployment}} \tag{10}$$

If they match to an employer, they earn a flow wage

$$w\left(a, \hat{\phi}, \phi^f\right) = \exp\left\{-\frac{\left(\hat{\phi} - \phi^f\right)^2}{2}\right\} \cdot \kappa \cdot \exp\{a\}$$

Workers who fail to match to an employer become unemployed.

After the employment/unemployment realization, employed agents draw a signal $s \sim \mathcal{N}\left(0, \sigma_s^2\right)$ which they use to update their beliefs about their type according to Bayes' rule. The normal posterior distribution of their type is

$$\hat{\phi}\prime \sim \mathcal{N}\left(\hat{\phi} + \frac{\hat{\sigma}_{\phi}^{2}(J_S)}{\sigma_s^2 + \hat{\sigma}_{\phi}^{2}(J_S)}\left(s - \hat{\phi}\right), \hat{\sigma}_{\phi}^{2}(J_S) - \frac{\hat{\sigma}_{\phi}^{4}(J_S)}{\sigma_s^2 + \hat{\sigma}_{\phi}^{2}(J_S)}\right)$$

After earning their wage and observing the signal, agents separate into unemployment at an exogenous rate δ. If they do not separate into unemployment, they can choose to search for a better match in a different labor market $\tilde{\phi}^f$ at cost c. The Bellman equation characterizing the employed worker's problem for $J_S < j \leq J$ is

$$\begin{aligned}
W_j\left(a, \hat{\phi}, \phi^f, J_S\right) &= \exp\left\{-\frac{\left(\hat{\phi} - \phi^f\right)^2}{2}\right\} \cdot \kappa \cdot \exp\{a\} \qquad (11) \\
&\quad + \beta \mathbb{E}_{\phi'}\Big\{\delta U_{j+1}\left(a, \hat{\phi}', J_S\right) + (1-\delta)\Big[\max_{\tilde{\phi}^f} s\lambda\left(\tilde{\phi}^f\right) W_j\left(a, \hat{\phi}', \tilde{\phi}^f, J_S\right) \\
&\quad + \left(1 - s\lambda\left(\tilde{\phi}^f\right)\right) W_j\left(a, \hat{\phi}', \phi^f, J_S\right) - c \cdot \mathbb{I}\left(\tilde{\phi}^f = \phi^f\right)\Big]\Big\} \\
&\quad \text{s.t.} \\
\hat{\phi}'(J_S) &= \hat{\phi} + \frac{\hat{\sigma}_{\phi}^{2}(J_S)}{\sigma_s^2 + \hat{\sigma}_{\phi}^{2}(J_S)}\left(s - \hat{\phi}\right) \text{ and } \hat{\sigma}_{\phi,j+1}^{2}(J_S) = \hat{\sigma}_{\phi}^{2}(J_S) - \frac{\hat{\sigma}_{\phi}^{4}(J_S)}{\sigma_s^2 + \hat{\sigma}_{\phi}^{2}(J_S)}
\end{aligned}$$

where the value of being unemployed U_j is

$$\begin{aligned}
U_j\left(a, \hat{\phi}, J_S\right) &= b + \beta \mathbb{E}\left\{\max_{\phi^f} \lambda\left(\phi^f\right) \cdot W_j\left(a, \hat{\phi}', \phi^f, J_S\right) + \left(1 - \lambda\left(\phi^f\right)\right) U_j\left(a, \hat{\phi}', J_S\right)\right\} \\
&\quad \text{s.t.} \qquad (12) \\
\hat{\phi}'(J_S) &= \hat{\phi} + \frac{\hat{\sigma}_{\phi}^{2}(J_S)}{\sigma_s^2 + \hat{\sigma}_{\phi}^{2}(J_S)}\left(s - \hat{\phi}\right) \text{ and } \hat{\sigma}_{\phi,j+1}^{2}(J_S) = \hat{\sigma}_{\phi}^{2}(J_S) - \frac{\hat{\sigma}_{\phi}^{4}(J_S)}{\sigma_s^2 + \hat{\sigma}_{\phi}^{2}(J_S)}
\end{aligned}$$

Retirement. At $j = J_R + 1$ the worker must retire and gets an annuity proportional to their wage at J_R (or UI benefits in case they were unemployed at J_R). The present value of retirement from the point of view of the age $J_R + 1$ worker, given the previous wage / UI w earned at age J_R, is given by the expression

$$R(w) = \frac{1 - \beta^{J_D - J}}{1 - \beta} w \qquad (13)$$

which means the values of work and unemployment at age J are

$$W_J\left(a,\hat{\phi},\phi^f,J_S\right) = \frac{1-\beta^{J_D+1-J_R}}{1-\beta}\exp\left\{-\frac{\left(\hat{\phi}-\phi^f\right)^2}{2}\right\}\cdot\kappa\cdot\exp\{a\} \tag{14}$$

$$U_J\left(a,\hat{\phi},J_s\right) = \frac{1-\beta^{J_D+1-J_R}}{1-\beta}b \tag{15}$$

5.1 Identifying Job-to-Job Transitions in the Model

We define a job-switch in the model as anytime in which a currently employed worker chooses to search for a different firm type $\tilde{\phi}^f \neq \phi^f$ and successfully matches to that firm. Let $\mathbf{1}^{Trans}(a,\hat{\phi},\phi^f,J_S,j)$ be an indicator equal to 1 if a worker with J_S years of schooling, age $j > J_S$, ability a, currently employed in job type ϕ^f and with conditional mean worker type $\hat{\phi}$ transitions into a new job ϕ'. The job-switching rate we construct from the model is the unconditional mean of this indicator in the population.

6 Calibration [Preliminary]

Moments and Parameters:

moments to match:

- σ_ϕ: 2 parameters (level and curvature)
  - idea 1: "claim the trend" and calibrate it to best approximate aggregate job switching rates
  - idea 2: our wage function associates wages to the degree of mismatch, so changes in wage dispersion is informative about learning. Match dispersion in wages at young vs middle age.
  - idea 3: match initial level + regression coefft from causal evidence on effect of college education on job switch rates one year out
  - idea 4: match initial level + diff between data moments and treat regression coefft as moment to target
- $f(\cdot)$: returns to college, depends on one parameter k. : College share in 1996.
- κ: cost of
- c: cost of switching

Table 2 displays a calibration intended to demonstrate the model's mechanisms. A rigorous quantification exercise is currently in progress.

Parameter	Definition	Value
J_D	Age at death	75
J_R	Age at retirement	55
j_0	Age of first schooling/working decision	16
β	Discount Factor	0.987
$\log(a_0), \sigma_{a,0}$	Mean and Std. Dev. of ability in population	0, 0.2
$\lambda\left(\phi^f\right)$	Job finding rates	$0.55\ \forall\ \phi^f$
s	Search prob. for the employed	0.5
b	Home Production for unemployed	1
κ	Scaling for returns to human capital	1
c	Cost of job switching	0.15
σ_s	Std. Dev. of signal	8
$\sigma_{\phi,0}$	Std. Dev. for type in population	15
$\sigma_{\phi,0}^{curv}$	Curvature in Std. Dev. function	1
$\sigma^2_\phi(J_s)$	Variance of ability signal	$\sigma^2_\phi(J_s) = \sigma_{\phi,0}\left(\frac{1}{J_s - j_0 + 1}\right)^{\sigma_{\phi,0}^{curv}}$

Table 2: Quarterly calibration for 1996 steady state.

The calibration is quarterly. Agents make schooling and labor market decisions starting at age $j_0 = 16$ until retirement at age $J_R = 55$. They die deterministically at age $J_D = 75$. The discount factor is set to match a 5% annual interest rate.

We exogenously set the job-finding rate to be $\lambda^f = 0.55$, equal across all labor markets. The probability of search for those already employed is $s = 0.5$ and it scales their actual probability of finding a new job. Home production is normalized to $b = 1$ and, for the 1996 steady state, the returns to human capital scale is set to $\kappa = 1$.

The rest of the parameters are calibrated to match the job-switching rate life profile of workers in 1996. This leads to a cost of switching $c = 0.15$, a standard deviation of the signals $\sigma_s = 8$, and of the type in the population $\sigma_{\phi,0} = 15$. Finally, the curvature in the profile of standard deviations depending on number of periods in schooling is $\sigma_{\phi,0}^{curv} = 1$. This parametrization leads to the pattern in job-switching shown in figure 5.

The job-switching rates in the model and the data are both elevated at early ages and are greatly reduced by age 30. The empirical job-switching rate is overall than that of the model at later ages. This is in part because there are other motives for switching jobs not related to learning. Importantly, switching jobs facilitates earnings increases and hence

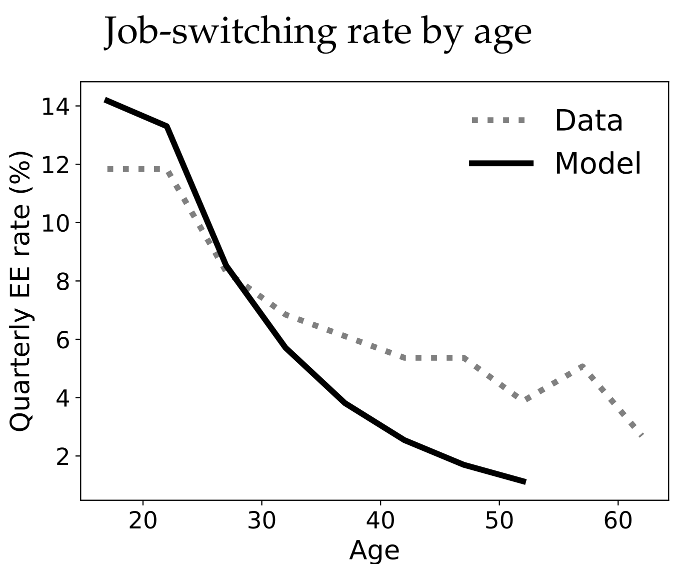


Figure 5: Job-switching over the life cycle: model and data.

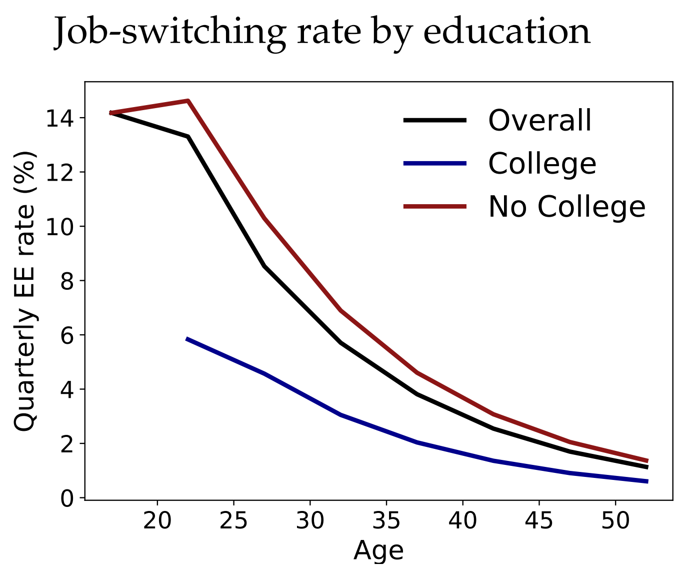


Figure 6: Model job-switching by education over the life cycle.

workers may switch between two equally good jobs just to earn more. While this goes beyond the scope of this paper, it can explain some of the divergence between the model and the data in later years of life. Importantly, as the data suggests, the calibrated model implies a lower job-switching rate for workers who take on more education, as displayed in figure 6.

6.1 Model Implications for Educational Choices

The left panel of figure 7 shows the distribution of workers across ability. In black is the distribution before any schooling decision is made; in the pink dotted area is the part of the ability distribution that benefits from investing in schooling (i.e. $V^{\text{school}}(a) > V^{\text{work}}(a)$); the pink dashed line indicates where these agents who invest in schooling end up in the ability distribution after their time in school. While those who opt not to invest in schooling do not see a change in ability, those who invest in schooling see an increase in their ability.

To study the model's ability to account for the changes in the job-switching rate, we consider an alternative value for the parameter κ, which governs returns to ability in the model. In the benchmark calibration $\kappa = 1$, a normalization. We consider a second steady state in which we hold all other parameters unchanged and raise κ to $\kappa = 1.8$ to match an 80% increase in the returns to skill. Panel (a) 8 shows that the model generates a substantial increase in the share of workers choosing to remain in education (blue vs.

Ability Distribution

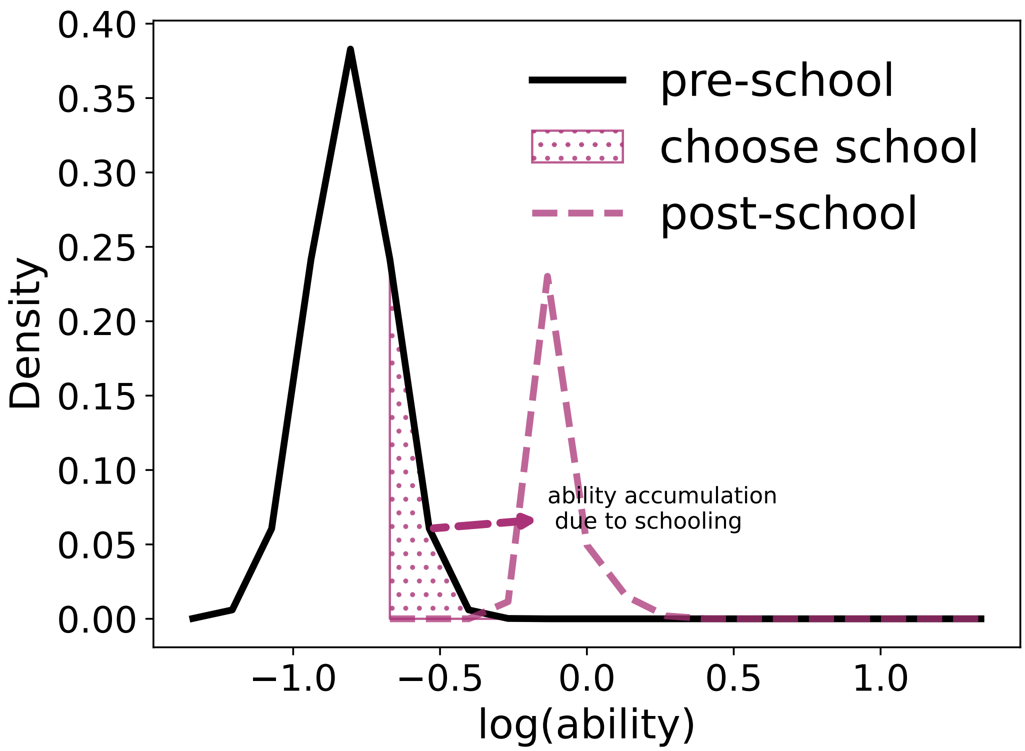
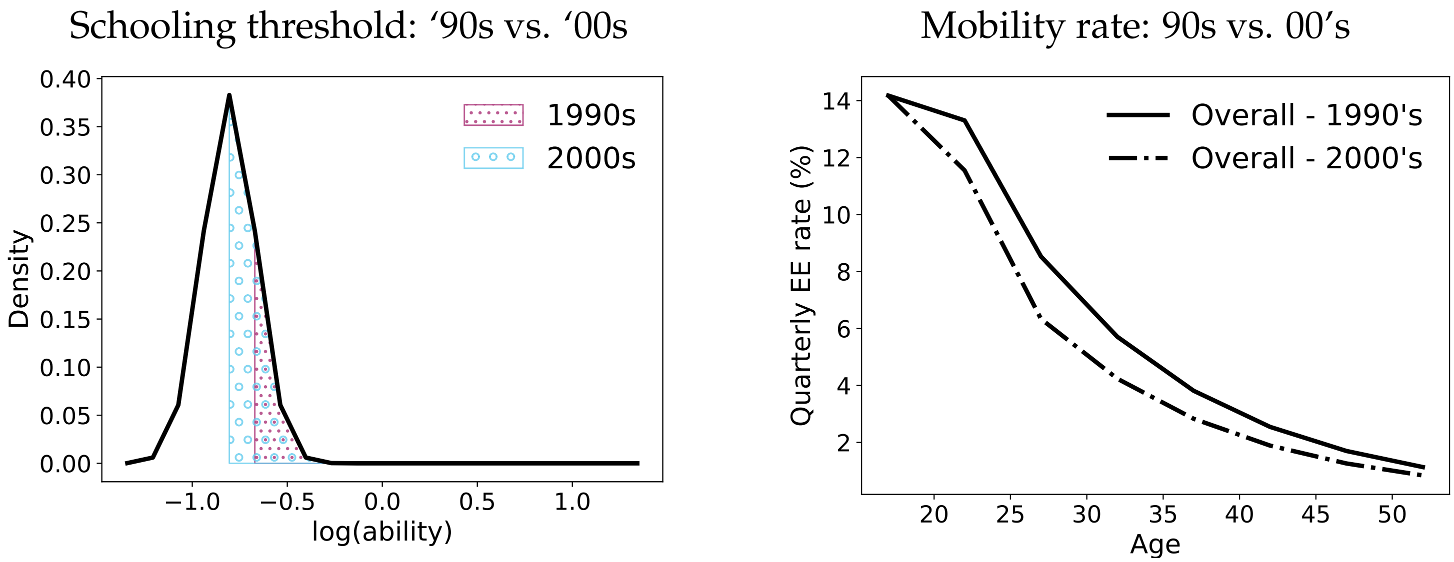


Figure 7: Model-implied distribution of workers pre and post schooling decision.



(a) Shares of workers who choose to go to college by ability in '90s and '00s steady states.

(b) Mobility life profile in '90s and '00s steady states.

Figure 8: Ability threshold for schooling decision in '90s vs. '00s (a) and corresponding life profile of mobility rate (b).

pink area). While the job-switching policies of college and non-college workers are not affected, the greater share of college educated workers pushes down the aggregate job-switching rate through compositional effects as shown in panel (b) and in accordance with the data. Figure 8 implies an aggregate decline in job-mobility from 5.2 to 3.7%. That is, about one third of the decline in the job-switching rate can be accounted for by increased educational attainment due to higher returns to education.

7 Conclusion

This paper re-evaluates the causes behind the decline in job-mobility the United States has experienced over the past decades. Unlike the prevalent explanation in the literature, which argues lower economic dynamism is the cause of this decline, we argue that roughly one-third of the decline is due to increased educational attainment among workers. Using a shift-share exercise, we show that educational attainment affects job-mobility in two ways. First, the young, who are the most mobile workers, remain in school longer. Their withdrawal from the labor force mechanically lowers the measured job-mobility rate via a direct composition effect: if the most mobile workers are not working, they cannot switch jobs and occupations. Second, by spending more time in school workers acquire information on the type of job they are best suited for. All else equal, a young worker with more education ends up in a better match when first joining the labor market and switches jobs less frequently thereafter. We then provide causal evidence that higher educational attainment directly reduces the propensity of workers to switch jobs by considering the effect of increases in state minimum wage laws in the US. Finally, we develop a model that accounts for these dynamics by allowing workers to learn about the employment that best suits them both in school and on-the-job. When increasing returns to schooling the model predicts exactly the empirical facts: the young stay in school longer and switch jobs less frequently throughout their careers.

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A Appendix

$$
\begin{aligned}
\underbrace{\Pr(\phi \mid s, \hat{\phi})}_{\text{Posterior on seeing signal } s} &= \frac{\Pr(s \mid \phi)\underbrace{\Pr(\phi \mid \hat{\phi})}_{\text{Prior}}}{\int_{\phi} \Pr(s \mid \phi)\underbrace{\Pr(\phi \mid \hat{\phi})}_{\text{Prior}} d\phi} \\
&= \frac{\exp\left\{-\frac{1}{2}\left(\frac{s-\phi}{\sigma_{\phi}(J_s)}\right)^2\right\}\exp\left\{-\frac{1}{2}\left(\frac{\hat{\phi}-\phi}{\hat{\sigma}_{\phi,j}(J_s)}\right)^2\right\}}{\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left(\frac{s-\phi}{\sigma_{\phi}(J_s)}\right)^2\right\}\exp\left\{-\frac{1}{2}\left(\frac{\hat{\phi}-\phi}{\hat{\sigma}_{\phi,j}(J_s)}\right)^2\right\}d\phi} \\
&= \frac{\exp\left\{-\frac{1}{2}\left[\left(\frac{s-\phi}{\sigma_{\phi}(J_s)}\right)^2+\left(\frac{\hat{\phi}-\phi}{\hat{\sigma}_{\phi,j}(J_s)}\right)^2\right]\right\}}{\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left[\left(\frac{s-\phi}{\sigma_{\phi}(J_s)}\right)^2+\left(\frac{\hat{\phi}-\phi}{\hat{\sigma}_{\phi,j}(J_s)}\right)^2\right]\right\}d\phi} \\
&= \frac{\exp\left\{-\frac{1}{2}\left[\left(\frac{s^2+\phi^2-2s\phi}{\sigma_{\phi}^2(J_s)}\right)+\left(\frac{\hat{\phi}^2+\phi^2-2\phi\hat{\phi}}{\hat{\sigma}_{\phi,j}^2(J_s)}\right)\right]\right\}}{\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left[\left(\frac{s^2+\phi^2-2s\phi}{\sigma_{\phi}^2(J_s)}\right)+\left(\frac{\hat{\phi}^2+\phi^2-2\phi\hat{\phi}}{\hat{\sigma}_{\phi,j}^2(J_s)}\right)\right]\right\}d\phi} \\
&= \frac{\exp\left\{-\frac{1}{2}\left[\frac{\phi^2-2s\phi}{\sigma_{\phi}^2(J_s)}+\frac{\phi^2-2\phi\hat{\phi}}{\hat{\sigma}_{\phi,j}^2(J_s)}\right]\right\}}{\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left[\frac{\phi^2-2s\phi}{\sigma_{\phi}^2(J_s)}+\frac{\phi^2-2\phi\hat{\phi}}{\hat{\sigma}_{\phi,j}^2(J_s)}\right]\right\}d\phi} \\
&= \frac{\exp\left\{-\frac{1}{2}\left[\frac{\phi^2-2\phi\left(\frac{s\hat{\sigma}_{\phi,j}^2(J_s)+\hat{\phi}\sigma_{\phi}^2(J_s)}{\sigma_{\phi}^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right)+\left(\frac{s\hat{\sigma}_{\phi,j}^2(J_s)+\hat{\phi}\sigma_{\phi}^2(J_s)}{\sigma_{\phi}^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right)^2}{\frac{\sigma_{\phi}^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_{\phi}^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}}\right]\right\}}{\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left[\frac{\phi^2-\frac{2\phi s\hat{\sigma}_{\phi,j}^2(J_s)+2\hat{\phi}\phi\sigma_{\phi}^2(J_s)}{\sigma_{\phi}^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}+\left(\frac{s\hat{\sigma}_{\phi,j}^2(J_s)+\hat{\phi}\sigma_{\phi}^2(J_s)}{\sigma_{\phi}^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right)^2}{\frac{\sigma_{\phi}^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_{\phi}^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}}\right]\right\}d\phi}
\end{aligned}
$$

$$
= \underbrace{\frac{\frac{1}{\sqrt{2\pi\frac{\sigma_\phi^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}}}\exp\left\{-\frac{1}{2}\left[\frac{\phi-\left(\frac{s\hat{\sigma}_{\phi,j}^2(J_s)+\hat{\phi}\sigma_\phi^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right)}{\sqrt{\frac{\sigma_\phi^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}}}\right]^2\right\}}{\frac{1}{\sqrt{2\pi\frac{\sigma_\phi^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}}}\int_{-\infty}^{\infty}\exp\left\{-\frac{1}{2}\left[\frac{\phi-\left(\frac{s\hat{\sigma}_{\phi,j}^2(J_s)+\hat{\phi}\sigma_\phi^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right)}{\sqrt{\frac{\sigma_\phi^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}}}\right]^2\right\}d\phi}}_{=1}
$$

where in the second line we cancel the common term $\frac{1}{\sqrt{2\pi}\sigma_\phi(J_s)\hat{\sigma}_{\phi,j}}$ from numerator and denominator and in the fifth line we cancel the common term $\exp\left\{-\frac{1}{2}\left(\frac{s^2}{\sigma_\phi^2(J_s)}+\frac{\hat{\phi}^2}{\hat{\sigma}_{\phi,j}^2(J_s)}\right)\right\}$ from numerator and denominator. In the final line, the denominator equals 1 since it is the integral of a Normally-distributed random variable over the domain $\Re$. We have thus shown that

$$
\begin{aligned}
\Pr(\phi \mid s,\hat{\phi}) &\equiv \mathcal{N}\left(\frac{s\hat{\sigma}_{\phi,j}^2(J_s)+\hat{\phi}\sigma_\phi^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}, \frac{\sigma_\phi^2(J_s)\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right) \\
&\equiv \mathcal{N}\left(\hat{\phi}+\frac{\hat{\sigma}_{\phi,j}^2(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\left(s-\hat{\phi}\right), \hat{\sigma}_{\phi,j}^2(J_s)-\frac{\hat{\sigma}_{\phi,j}^4(J_s)}{\sigma_\phi^2(J_s)+\hat{\sigma}_{\phi,j}^2(J_s)}\right)
\end{aligned}
$$

where the second line is obtained by algebraic manipulation.