ECON 165, Review Section # 6

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Plan for Today

- Eaton-Gersovitz Model of Default - Review
- Practice

Old Model vs. New Model

Old Model:

- 2 periods
- SOE
- Consume (C), Borrows (B)

New Model:

- 2 periods
- SOE
- Consume (C), Borrows (B)

Old Model vs. New Model

Old Model:

- 2 periods
- SOE
- Consume (C), Borrows (B)
- Household is main agent
- HH always repays debt
- Endowment/Production is known

New Model:

- 2 periods
- SOE
- Consume (C), Borrows (B)
- **Government** is main agent
- Possibility of **Default**
- **Uncertain Tax Revenue**

Eaton-Gersovitz: Budget Constraint pg. 1

Debt:

- Notation is flipped: government borrows $q_t B_t$ a certain amount and promises to repay B_t at $t+1$
- What is the sign of q_t?

Eaton-Gersovitz: Budget Constraint pg. 1

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- q_t is the price at which foreign lenders buy the government bond. Another way to think of it is as the *discount* at which borrowers borrow (the higher the discount the more they have to pay back). Specifically, $\frac{1}{q_t} = 1 + r_1$

$t = 1$

- Suppose the tax revenue is Y_1, the government starts with debt B_0 and repays it in $t = 1$, what is the govt. budget constraint?

Eaton-Gersovitz: Budget Constraint pg. 1

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- $$G_1 + B_0 = Y_1 + q_1 B_1$$

Eaton-Gersovitz: Budget Constraint pg. 2

$t = 2$

- Suppose the tax revenue is $Y_2(s)$ (where $s \in \{L, H\}$), the government starts with debt B_1 and repays it in $t = 2$, what is the govt. budget constraint?

Eaton-Gersovitz: Budget Constraint pg. 2

$t = 2$

- Suppose the tax revenue is $Y_2(s)$ (where $s \in \{L, H\}$), the government starts with debt B_1 and repays it in $t = 2$, what is the govt. budget constraint?
- $$G_2(s) + B_1 = Y_2(s)$$

Eaton-Gersovitz: Budget Constraint pg. 2

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Q: Why B_1 not $B_1(s)$?

Eaton-Gersoitz: Default

- If govt. defaults at t ($D_t = 1$):
  - Does not have to pay back the debt incurred before (i.e. B_{t-1})
  - Cannot access debt market in any future period (i.e. $B_{t+j} = 0$ for $j > 0$)
  - Is subject to tax revenue loss τ
- What are the $t = 1$ and $t = 2$ budget constraints assuming default?

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$$\begin{aligned} G_1 &= (1-\tau)Y_1 \\ G_1(s) &= (1-\tau)Y_2(s) \end{aligned}$$

Eaton-Gersoitz: Solution

- Government problem

$$\max \qquad u\left(G_1\right) + \beta u\left(G_2(s)\right)$$

  - What is the govt. choosing?
  - What is wrong with this expression?

- What is the period $t = 2$ budget constraint?

Eaton-Gersoitz: Solution

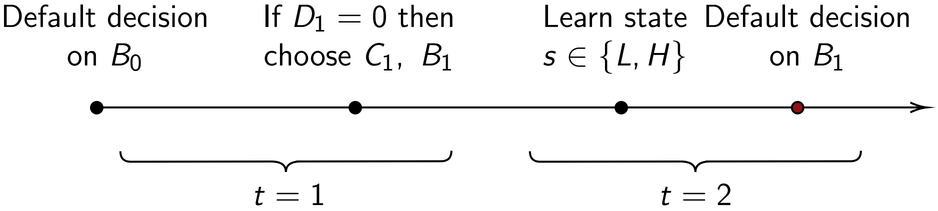
- Government problem

$$\max_{G_1, B_1, G_2, D_1, D_2} \quad u\left(G_1\right) + \beta \mathbb{E}\left[u\left(G_2(s)\right)\right]$$

- What is the period $t = 2$ budget constraint?

$$G_2(s) = \begin{cases} Y_2(s)(1-\tau) & \text{if } D_1 = 0, D_2 = 1 \\ Y_2(s)(1-\tau) & \text{if } D_1 = 1, D_2 = 0 \\ Y_2(s) - B_1 & \text{if } D_1 = 0, D_2 = 0 \end{cases}$$

Solving the Problem Backwards



$t = 2$ Default decision:

- G_1, D_1, B_1 have been chosen, what determines no-defaul rather than default?

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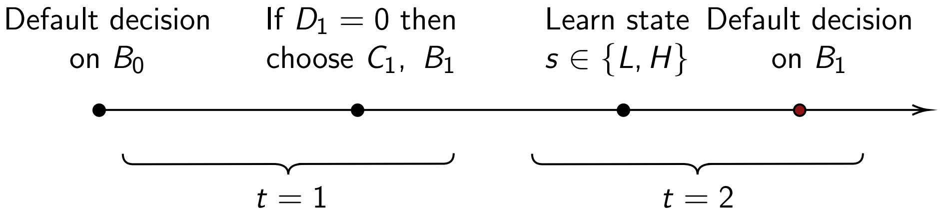


$t = 2$ Default decision:

- G_1, D_1, B_1 have been chosen, what determines no-defaul rather than default?

$$
\begin{aligned}
u\left(G_2(s)\right) &> u\left(Y_2(1-\tau)\right) \\
\iff Y_2 - B_1 &> Y_2(1-\tau) \\
\iff \tau Y_2 &> B_1
\end{aligned}
$$

Pricing schedule q

- How is q determined?

Pricing schedule q

- How is q determined? $\rightarrow$ *no-arbitrage condition*

$$\underbrace{\pi(L)\frac{(1-D_2)\,(B_1,Y_2(L))\times 1+0}{q_1}+(1-\pi(L))\,\frac{(1-D_2\,(B_1,Y_2(H)))\times 1+0}{q_1}}_{\text{if lend to govt.}} \;=\; \underbrace{1+r^*}_{\text{outside option}}$$

- What happens if RHS is $>$ than LHS? What if LHS is $>$ than RHS?

Pricing schedule q

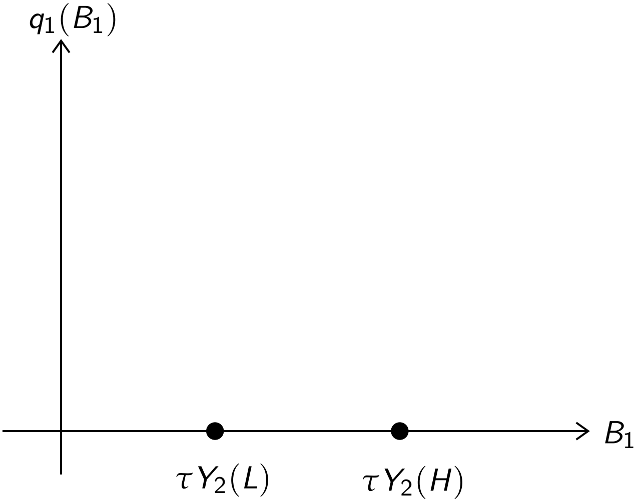
- How is q determined? $\rightarrow$ *no-arbitrage condition*

$$\underbrace{\pi(L)\frac{(1-D_2)\left(B_1, Y_2(L)\right)\times 1+0}{q_1}+(1-\pi(L))\,\frac{\left(1-D_2\left(B_1, Y_2(H)\right)\right)\times 1+0}{q_1}}_{\text{if lend to govt.}} \quad = \quad \underbrace{1+r^*}_{\text{outside option}}$$

- What happens if RHS is $>$ than LHS? What if LHS is $>$ than RHS?

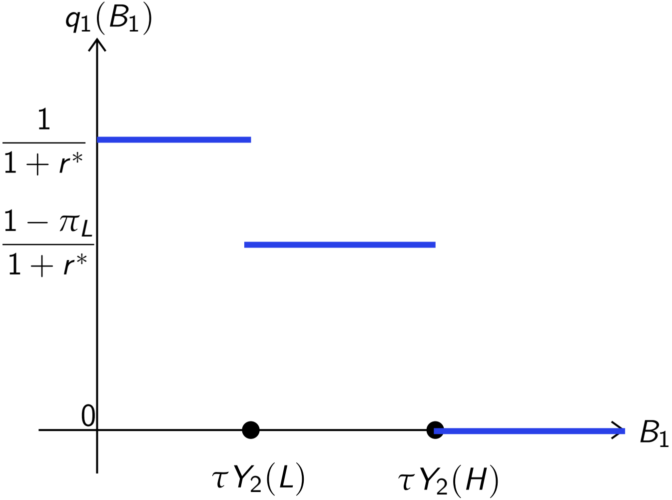
$$q_1=\frac{\sum_s \pi(s)\left(1-D_2\left(B_1, Y_s(s)\right)\right)}{1+r^*}$$

Pricing Schedule



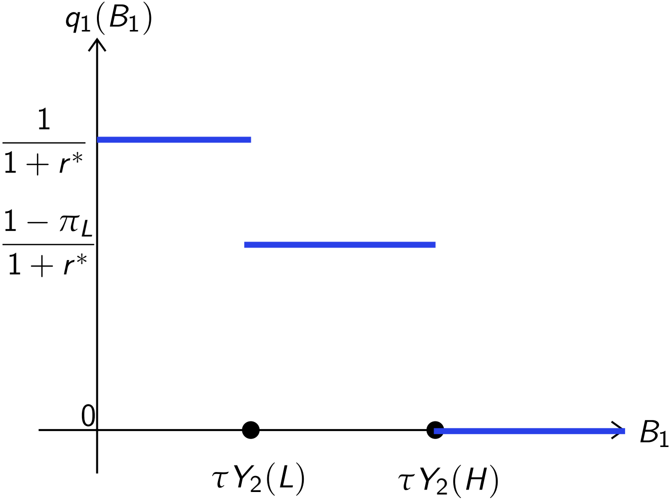
Pricing Schedule

- What happens when $r^* \uparrow$?



Pricing Schedule

- What happens when $\tau \uparrow$?



Pricing Schedule

- What if there are S states with prob. $\pi_1, \ldots \pi_S$ and $\pi_S = 1 - \sum_{s=1}^{S-1} \pi_s$?

Speed Round

- Do we need uncertainty for default to occur?
- Why do we assume that the government loses τY_t?
- Does a country have to issue in domestic currency?

Speed Round

- Do we need uncertainty for default to occur?
- Why do we assume that the government loses τY_t?
- Does a country have to issue in domestic currency?
  - Why issue in foreign currency?