Section # 1

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To Do

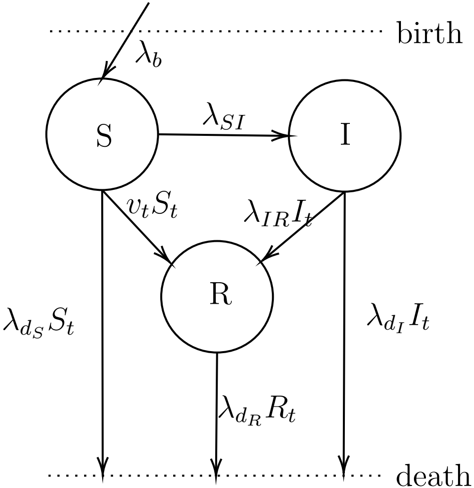
- Logistics
- Basics of the SIR model
- Extending the SIR model
  - solving the transition equations
  - solving for steady state

Logistics

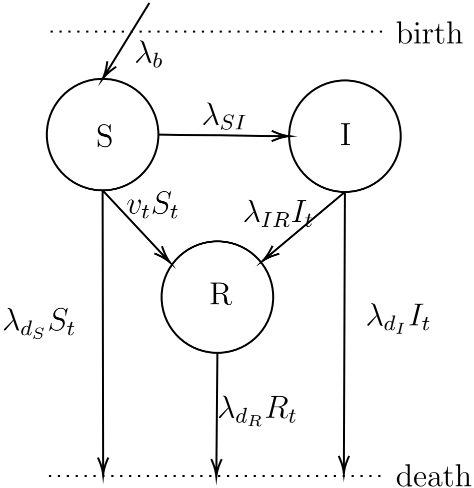
- Economics of Health and Medical Care
  - 4 problem sets. Grade is simply Complete/Incomplete
  - 2 Op-eds to write at the middle and end of the quarter
- Mine
  - danicara@stanford.edu.
  - OH Thursdays 9am-11am.
  - TA section Fridays 11.30am-12.30pm.

SIR: the basics

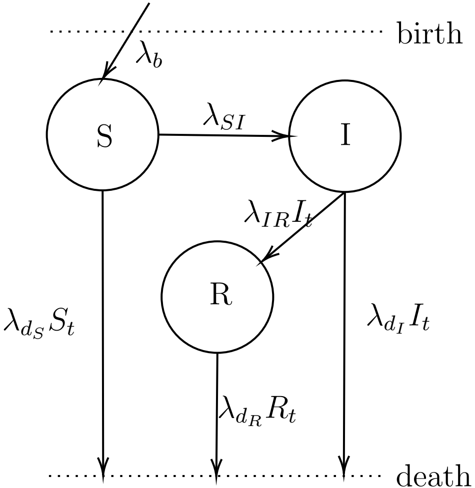
$$\begin{aligned} \text{S} &\longrightarrow \text{Susceptible} \\ \text{I} &\longrightarrow \text{Infected} \\ \text{R} &\longrightarrow \text{Recovered} \end{aligned}$$



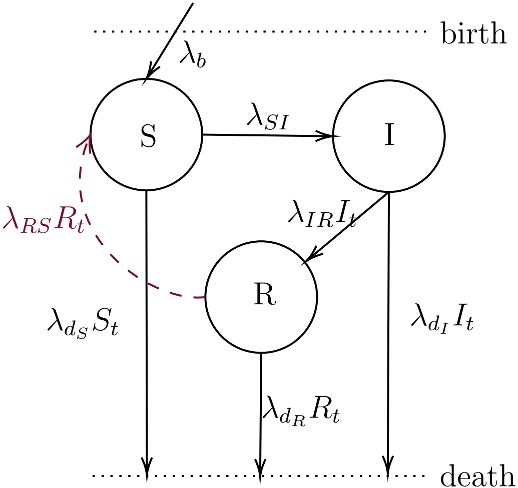
Extending the model to COVID19



Extending the model to COVID19



Extending the model to COVID19



Transition Equations

- **Transition equations:** derive how each population evolves over time.

Transition Equations

- Susceptible

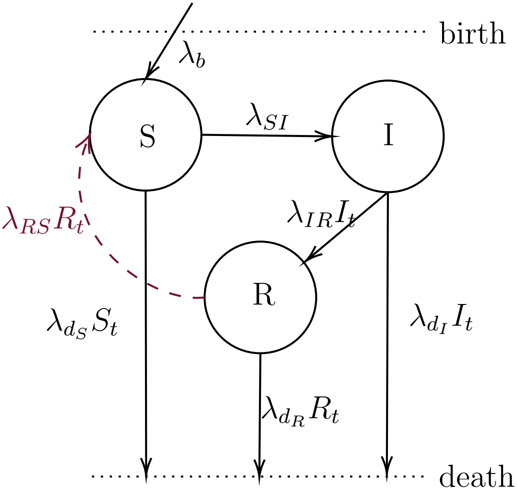
$$\frac{dS}{dt} =$$

- Infected

$$\frac{dI}{dt} =$$

- Recovered

$$\frac{dR}{dt} =$$



Transition Equations

- Susceptible

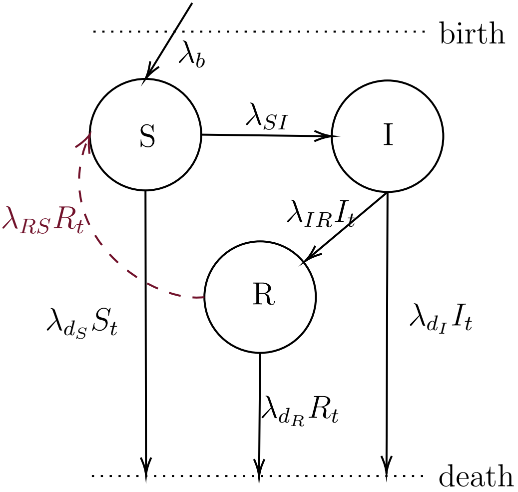
$$\frac{dS}{dt} = \lambda_b - \lambda_{SI} S_t - \lambda_{d_S} S_t + \lambda_{RS} R_t$$

- Infected

$$\frac{dI}{dt} = \lambda_{SI} S_t - \lambda_{IR} I_t - \lambda_{d_I} I_t$$

- Recovered

$$\frac{dR}{dt} = \lambda_{IR} I_t - \lambda_{d_R} R_t - \lambda_{RS} R_t$$



Steady State

- The steady state is defined by the system being *steady*.
- Does that mean nobody is moving around?
- No! Just that the moving around that is going on does not imply changes for each individual population, namely S_t, I_t and R_t.

Steady State

In steady state we have:

- Susceptible

$$\frac{dS}{dt} = 0$$

- Infected

$$\frac{dI}{dt} = 0$$

- Recovered

$$\frac{dR}{dt} = 0$$

Steady State

In steady state we have: Susceptible

$$S^* = \frac{\lambda_b\left(\lambda_{IR}\lambda_{RS} + \lambda_{IR}\lambda_{dR} + \lambda_{RS}\lambda_{dI} + \lambda_{dI}\lambda_{dR}\right)}{\lambda_{IR}\lambda_{RS}\lambda_{dS} + \lambda_{IR}\lambda_{SI}\lambda_{dR} + \lambda_{RS}\lambda_{SI}\lambda_{dI} + \lambda_{IR}\lambda_{dR}\lambda_{dS} + \lambda_{RS}\lambda_{dI}\lambda_{dS} + \lambda_{SI}\lambda_{dI}\lambda_{dR} + \lambda_{dI}\lambda_{dR}\lambda_{dS}}$$

Infected

$$I^* = \frac{\lambda_{SI}\lambda_b\left(\lambda_{RS} + \lambda_{dR}\right)}{\lambda_{IR}\lambda_{RS}\lambda_{dS} + \lambda_{IR}\lambda_{SI}\lambda_{dR} + \lambda_{RS}\lambda_{SI}\lambda_{dI} + \lambda_{IR}\lambda_{dR}\lambda_{dS} + \lambda_{RS}\lambda_{dI}\lambda_{dS} + \lambda_{SI}\lambda_{dI}\lambda_{dR} + \lambda_{dI}\lambda_{dR}\lambda_{dS}}$$

Recovered

$$R^* = \frac{\lambda_{IR}\lambda_{SI}\lambda_b}{\lambda_{IR}\lambda_{RS}\lambda_{dS} + \lambda_{IR}\lambda_{SI}\lambda_{dR} + \lambda_{RS}\lambda_{SI}\lambda_{dI} + \lambda_{IR}\lambda_{dR}\lambda_{dS} + \lambda_{RS}\lambda_{dI}\lambda_{dS} + \lambda_{SI}\lambda_{dI}\lambda_{dR} + \lambda_{dI}\lambda_{dR}\lambda_{dS}}$$

Steady State

- How does the level of infected in steady state increase as λ_{RS} increases?
    - intuition...
- Rigorous solutions?
    - → compute steady state I^*, check sign of $\frac{dI^*}{d\lambda_{RS}}$
    - → trick from problem 15(ish):
        1. differentiate transition equations evaluated at SS (i.e. $= 0$)
        2. solve system of differentiated transition equations to get $\frac{dI^*}{d\lambda_{RS}}$
    - → The second is often easier. In both approaches you have to solve a (complicated) system of equations, the differentiation step is likely easier.

Steady State: $\frac{dI^*}{d\lambda_{RS}}$

$$\frac{dS^*}{d\lambda_{RS}} = \frac{R^*(\lambda_{IR}\lambda_{dR}+\lambda_{dI}\lambda_{dR})}{\lambda_{IR}\lambda_{RS}\lambda_{dS}+\lambda_{IR}\lambda_{SI}\lambda_{dR}+\lambda_{RS}\lambda_{SI}\lambda_{dI}+\lambda_{IR}\lambda_{dR}\lambda_{dS}+\lambda_{RS}\lambda_{dI}\lambda_{dS}+\lambda_{SI}\lambda_{dI}\lambda_{dR}+\lambda_{dI}\lambda_{dR}\lambda_{dS}}$$

$$\frac{dI^*}{d\lambda_{RS}} = \frac{R^*\lambda_{SI}\lambda_{dR}}{\lambda_{IR}\lambda_{RS}\lambda_{dS}+\lambda_{IR}\lambda_{SI}\lambda_{dR}+\lambda_{RS}\lambda_{SI}\lambda_{dI}+\lambda_{IR}\lambda_{dR}\lambda_{dS}+\lambda_{RS}\lambda_{dI}\lambda_{dS}+\lambda_{SI}\lambda_{dI}\lambda_{dR}+\lambda_{dI}\lambda_{dR}\lambda_{dS}}$$

$$\frac{dR^*}{d\lambda_{RS}} = -\frac{R^*(\lambda_{IR}\lambda_{dS}+\lambda_{SI}\lambda_{dI}+\lambda_{dI}\lambda_{dS})}{\lambda_{IR}\lambda_{RS}\lambda_{dS}+\lambda_{IR}\lambda_{SI}\lambda_{dR}+\lambda_{RS}\lambda_{SI}\lambda_{dI}+\lambda_{IR}\lambda_{dR}\lambda_{dS}+\lambda_{RS}\lambda_{dI}\lambda_{dS}+\lambda_{SI}\lambda_{dI}\lambda_{dR}+\lambda_{dI}\lambda_{dR}\lambda_{dS}}$$

If time allows...

- Define an **externality** and give an example.
- In the US there are **open enrollments**, that is limited periods of time for people to choose their healthcare for the coming year. In particular most Americans have to make a decision between November and December of each year for healthcare starting on January 1st. Why? What is the problem with allowing people to change healthcare at any given time?