ECON 165, Section # 2

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Plan for Today

- Consumption Smoothing
- Terms of Trade
- Production and Investment

Reminder: Pset #1 due Sunday.

Terminology: Consumption Smoothing, pg. 1

- What does this mean?

- This applies to households (Milton Friedman, Robert Hall), but also to countries!

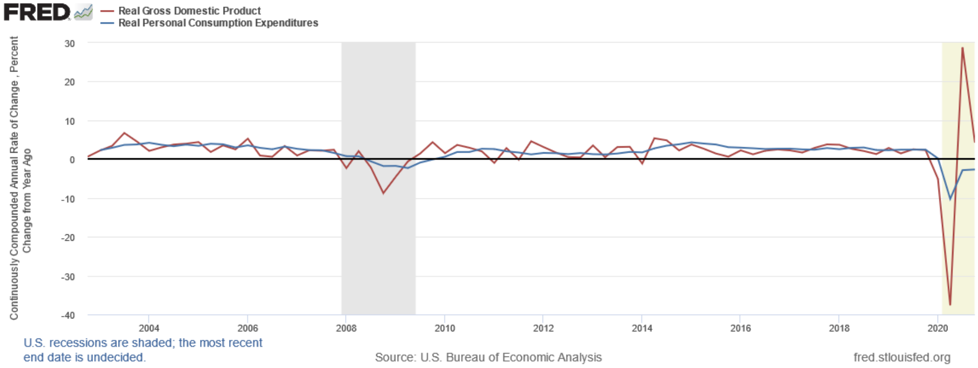

Figure: Output (red) and consumption (blue) change since 2002.

Terminology: Consumption Smoothing, pg. 2

- What allows countries and households to smooth consumption?

  → Borrowing and saving!

- Under what types of shocks does this happen?

  → Temporary shocks, not permanent ones (the theory says ...).

Terminology: Terms of Trade

- Definition?
  - → $\frac{\text{Export price}}{\text{Import price}}$ and in words...?
- Why is this more relevant for Norway or Saudi Arabia than it is for the United States?
  - → Small, non-diversified (in terms of production) economies such as Norway or Saudi Arabia depend highly on just a few exports (e.g. oil) with which they fund their internal consumption

Problem (Anticipated Terms-of-Trade Shocks)

We are given: lifetime utility $U(C_1, C_2) = \sqrt{C_1 C_2}$, endowments $Q_1 = Q_2 = 10$, terms of trade $TT_1 = TT_2 = 1$, initial net foreign wealth $B_0^* = 0$, and interest rate $r_0 = r_1 = r^* = 0.05$.

1. Compute the equilibrium consumption and trade balance.

Same problem as usual but different budget constraints:

$t=1$ budget constraint : $C_1 + B_1 = Q_1 \cdot TT_1 + \cancel{(1+r) B_0}^{\to 0}$

$t=2$ — — : $C_2 + B_2 = Q_2 \cdot TT_2 + (1+r) B_1$

$\Big\} \; C_1 + \frac{C_2}{1+r} = Q_1 TT_1 + \frac{Q_2 TT_2}{1+r}$

Because of No-Ponzi & Transversality $B_2 = 0$

$$\max_{C_1, C_2, B_1} \sqrt{C_1 C_2} \quad \text{s.t.} \quad C_1 + \frac{C_2}{1+r} = Q_1 TT_1 + \frac{Q_2 TT_2}{1+r}$$

$$\mathcal{L} = \sqrt{C_1 C_2} + \lambda \left[Q_1 TT_1 + \frac{Q_2 TT_2}{1+r} - C_1 - \frac{C_2}{1+r} \right]$$

$[C_1]$ $\frac{C_2}{2\sqrt{C_1 C_2}} = \lambda$ $\quad [C_2]$ $\frac{C_1}{2\sqrt{C_1 C_2}} = \frac{\lambda}{1+r}$ $\Rightarrow$ $C_2 = (1+r) C_1$

using the budget constraints $C_1 = 4.75$ $\quad C_2 = 10.24$ $\quad TB_1 = Q_1 TT_1 - C_1 = 0.25$

Problem (Anticipated Terms-of-Trade Shocks) - Solution

1. Compute the equilibrium consumption and trade balance.

<u>Solution:</u>

See previous page

Sample Problem (Anticipated Terms-of-Trade Shocks)

We are given: lifetime utility $U(C_1, C_2) = \sqrt{C_1 C_2}$, endowments $Q_1 = Q_2 = 10$, terms of trade $TT_1 = TT_2 = 1$, initial net foreign wealth $B_0^* = 0$, and interest rate $r_0 = r_1 = r^*$.

2. Let the terms of trade in period 2 increase by 50%. Calculate the effect of this anticipated terms of trade improvement on consumption and the trade balance. Provide intuition.

$\rightsquigarrow$ see next page

Problem (Anticipated Terms-of-Trade Shocks) - Solution

2. Let the terms of trade in period 2 increase by 50%. Calculate the effect of this anticipated terms of trade improvement on consumption and the trade balance. Provide intuition.

Solution:

Now $TT_2 \to 1.5$. Use the previously found relationships

$$C_2 = (1+r)C_1 \Rightarrow 2C_1 = Q_1 \cdot TT_1 + \frac{Q_2 \cdot TT_2}{1+r} = 10 + \frac{15}{1.05}$$

$C_1 = 12.14$ $\quad$ $C_2 = 12.75$

$TB_1 = Q_1 \cdot TT_1 - C_1 = -2.14$

Now you know you will be rich in the next period (because it's an anticipated increase in the value of what you produce). This means you consume more in both periods (consumption smoothing) and do so by borrowing against your future wealth ($TB_1 < 0$)

Sample Problem (Anticipated Terms-of-Trade Shocks)

We are given: lifetime utility $U(C_1, C_2) = \sqrt{C_1 C_2}$, endowments $Q_1 = Q_2 = 10$, terms of trade $TT_1 = TT_2 = 1$, initial net foreign wealth $B_0^* = 0$, and interest rate $r_0 = r_1 = r^*$.

3. Relate your findings to those discussed in the case study of Chile.

Production

- Instead of having an endowment economy we set up our problem so that consumption goods are *produced*.

- What are typical inputs to production?

  1. Labor (workers need to run machines or craft products)
  2. Land (to build a factory or to grow crops)
  3. **Capital/Investment** (need machines to make cars or teach remotely)

- Next we will think of production not only depending on investment (a flow) but on capital (a stock).

Production with Capital

Suppose the economy starts with capital $K_0 > 0$ in period $t = 0$ with which it can produce in period 1. In period $t = 1$ the economy can use capital K_1 which is the sum of the old (depreciated) capital $(1-\delta)K_0$ and the new capital invested I_1 (note this can be either positive - for savings - or negative - for borrowing). A country uses its output (Q) and whatever to either consume C or invest I. The country's objective is to maximize lifetime utility $\ln(C_1) + \beta \ln(C_2)$ and output is produced according to the production function $Q_t = F(K_{t-1}) = \sqrt{K_{t-1}}$.

1. What are the period $t = 1$ and $t = 2$ budget constraints?

First note that $K_1 = (1-\delta)K_0 + I_1$

$t=1$ Budget : $C_1 + I_1 = F(K_0)$

$t=2$ " : $C_2 + I_2 = F(K_1)$

$I_2 = 0$ for the usual reasons (No-Ponzi & transversality)

Production with Capital

Suppose that the economy starts with capital $K_0 > 0$ in period $t = 0$ with which it can produce in period 1. In period $t = 1$ it produces and can use that output or borrow in order to either consume (C_1) or invest (I_1) in capital accumulation. The country objective is to maximize lifetime utility $\ln(C_1) + \beta \ln(C_2)$ and output is produced according to the production function $Y_t = F(K_{t-1}) = \sqrt{K_{t-1}}$.

2. What are the optimal allocations for consumption C_1 and C_2, as well as the investment decision I_1? Suppose $K_0 = 1$.

Production with Capital

2. What are the optimal allocations for consumption C_1 and C_2, as well as the investment decision I_1? Suppose $K_0 = 1$.

$$\max_{C_1, C_2, I_1} \ln(C_1) + \beta \ln(C_2)$$

$$C_1 + I_1 = F(K_0) \quad \& \quad C_2 = F((1-\delta)K_0 + I_1)$$

Note $F(K_0) = \sqrt{1} = 1$

$\to$ consolidated as $C_2 = \sqrt{(1-\delta) + 1 - C_1}$

Thus $\longrightarrow \max_{C_1} \ln(C_1) + \beta \ln\left(\sqrt{(1-\delta) + 1 - C_1}\right)$

$$\frac{d}{dC_1}: \quad \frac{1}{C_1} - \frac{\beta}{2[(1-\delta) + 1 - C_1]} = 0 \implies 2(1-\delta) + 2 - 2C_1 = \beta C_1$$

$$C_1 = \frac{2(2-\delta)}{2+\beta}$$

$$C_2 = \sqrt{\frac{\cancel{2} - 2\delta + \beta - \beta\delta + \cancel{2} + \beta - \cancel{4} - 2\delta}{2+\beta}} =$$

$$I_1 = \frac{\beta + 2\delta - 4}{2+\beta}$$

$$C_2 = \sqrt{\frac{2\beta - \beta\delta - 4\delta}{2+\beta}}$$

Production with Capital

Suppose that the economy starts with capital $K_0 > 0$ in period $t = 0$ with which it can produce in period 1. In period $t = 1$ it produces and can use that output or borrow in order to either consume (C_1) or invest (I_1) in capital accumulation. The country objective is to maximize lifetime utility $\ln(C_1) + \beta \ln(C_2)$ and output is produced according to the production function $Y_t = F(K_{t-1}) = \sqrt{K_{t-1}}$.

3. What happens to consumption and investment if δ doubles? Provide intuition.

It's easy to see in the formulas in the previous page that consumption is DECREASING in δ.
Why? This is because as δ increases capital is harder to accumulate & so it's harder to produce (output falls). This in turn means consumption must also fall.

Q&A