Optimal monetary policy with menu costs is nominal wage targeting

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Latest version: February 2022

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Abstract

We show analytically that ensuring stable nominal wages is the optimal monetary policy in a multisector economy with fixed menu costs for adjusting goods prices, even when wages are completely flexible. This *nominal wage targeting* contrasts with inflation targeting, the optimal policy prescribed by the textbook one-sector New Keynesian model in which firms can only randomly adjust their prices. Menu costs, unlike the conventional model, induce state-dependent pricing and are less tractable to work with. We build an analytical model to show that optimal monetary policy is exactly nominal wage targeting for shocks that are not too small, and approximately so otherwise.

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We thank Marios Angeletos and Adrien Auclert for invaluable guidance. We also thank the Washington Center for Equitable Growth for generous support.

1 Introduction

Many central banks around the world have adopted some form of inflation targeting over the past decades. The theoretical foundation for inflation targeting, the Calvo-Yun formulation of the New Keynesian model, assumes a one-sector economy in which firms are randomly given the opportunity to change prices. Multisector versions of this model also find that the optimal monetary policy is some version of inflation targeting. While the Calvo-Yun assumption is mathematically convenient, an arguably more realistic model allows firms to choose when to change their prices by paying a "menu cost". What then does optimal monetary policy look like in a multisector world with menu costs?

This paper studies optimal monetary policy in a multisector economy in which prices are sticky but firms can choose to change their prices by paying a fixed "menu cost". With these realistic assumptions **we show analytically and without approximation techniques that optimal monetary policy is nominal wage targeting, not inflation targeting.** We do so by developing a model in which the economy is made up of sectors where firms are subject to sector-specific productivity shocks and can change their price at any point by paying a menu cost. Optimally, nominal wages should be stabilized, and inflation should *not* be stable but move inversely with output. This is despite wages themselves being completely flexible.

We show analytically and without approximation techniques that optimal monetary policy is nominal wage targeting, not inflation targeting. The economy is made up of sectors where firms are subject to sector-specific productivity shocks and can change their price at any point by paying a menu cost. Optimally, nominal wages should be stabilized, and inflation should *not* be stable but move inversely with output. This is despite wages themselves being completely flexible.

The intuition behind this result is that stabilizing nominal wages minimizes the number of firms needing to adjust their price and incurring a menu cost, while still achieving undistorted relative prices. Consider for example a positive productivity shock affecting only firms in sector 1. If the shock is sufficiently large, then it is efficient and desirable for firms in this sector to cut their *relative* prices, compared to other firms in other sectors of the economy. Under constant nominal wages, firms outside of sector 1 have no desire to adjust their prices since their nominal marginal costs are unchanged. Meanwhile, firms in sector 1 choose to adjust their nominal prices because of the productivity shock. As a result, relative prices between sector-1 firms and other firms are undistorted, and *only* this one sector has incurred wasteful menu costs. Alternatively, if the productivity shock is small, the welfare gain from firms in sector 1 updating their price – ensuring correct relative prices – will not outweigh the welfare loss due to the menu costs they need to pay to do so. In this case, it is optimal to ensure that no firm changes price, thus avoiding menu costs altogether. Given the small size of the shock, however, nominal wages nonetheless remain approximately constant.

Inflation targeting by contrast, in order to achieve undistorted relative prices, always requires *every* sector to pay a wasteful menu cost. Following the productivity shock, the relative price of sector 1 needs to fall to achieve efficiency. Inflation targeting requires that the overall average price level be unchanged. To simultaneously have the relative price fall *and* the price level be stable requires that both sector 1 firms cut their nominal prices and that firms in all other sectors raise their nominal prices. As a result, all firms are forced to adjust their prices and pay a menu cost. This is unnecessarily wasteful because the same efficient allocation can be achieved via a nominal wage target with only sector 1 firms paying the menu cost.[1]

The welfare loss of inflation targeting is precisely due to the excess menu costs that optimal policy avoids, which we also quantify in a dynamic version of the model. This model is calibrated to US data and, on top of sectoral shocks, allows for idiosyncratic shocks, another major source of price changes. We compare inflation and nominal wage targeting and show that nominal wage targeting would lead to a significant welfare improvement over inflation targeting. This result follows from the large estimates of menu costs in the empirical literature. For example, Nakamura and Steinsson (2010) estimate menu costs to be 0.5% of total firm revenues annually (an order of magnitude larger than the famous "welfare loss of business cycles" estimate of 0.05% of Lucas 1987).[2] More generally, in this paper we think of menu costs not only as the physical costs of adjusting prices, but as a reduced-form way of capturing any economic or welfare cost of price adjustment, such as the mental attention costs suffered by price setters.

Optimal policy can also be seen as a form of nominal income targeting. In the example above, the positive productivity shock in sector 1 implies that output in sector 1 goes up, and firms in this sector cut prices. Other sectors are unaffected, so these changes in sector 1 pass through to overall aggregates: aggregate output Y rises and the aggregate price level P falls, i.e. inflation moves inversely with output. Under baseline functional forms for preferences, this implies exactly nominal income targeting – the central bank should stabilize nominal income, $P \times Y$, but more generally this policy of nominal wage targeting can also be thought of as a form of generalized nominal income targeting, where P and Y

[1] In multisector models under the textbook Calvo-Yun friction, Rubbo (2020), Aoki (2001), and Benigno (2000) show that inflation targeting of a properly-specified price index of inflation is optimal. Kreamer (forthcoming) also studies optimal monetary policy in a sectoral model, with fixed prices and durable goods.

[2] See also Levy et al. (1997); Nakamura et al. (2018); and further discussion in section 3.3.

move inversely, but not necessarily one-for-one.

Position in literature. To our knowledge, we are the first to fully characterize optimal monetary policy in the face of fixed menu costs when firms *also* have a motive to adjust relative prices.[3] On the one hand, without changing productivity differences between firms, there is no motive for relative-price changes and so optimal policy under menu costs is trivially zero inflation as prices never need to move and price stickiness is irrelevant (see e.g. Nakov and Thomas (2014)). On the other hand, several papers allow for relative-price movements but take as *given* that the central bank targets inflation, and simulate numerically how the presence of menu costs affects the optimal level of inflation (e.g. Blanco (2021), Nakov and Thomas (2014) section 5, Adam and Weber (2019), Wolman (2011)). A larger literature makes assumptions on monetary policy – i.e. does not analyze optimal policy – and asks how the presence of menu costs affects macroeconomic dynamics (among others, Caplin and Spulber (1987), Golosov and Lucas (2007), Gertler and Leahy (2008),[4]; Nakamura and Steinsson (2010); Midrigan (2011); Alvarez, Lippi and Paciello (2011)). That is, while these papers conduct a positive analysis, we conduct a normative analysis. There is also a large empirical literature on menu costs that stresses the importance of endogenous price setting to match key aspects of the data.[5]

We see our paper as helping unify the literature on optimal monetary policy. In the last decade a number of papers across a variety of classes of models have found that policies of "approximate nominal income targeting" are optimal; however, sticky price models – the workhorse model of modern monetary economics – had conspicuously held out for the optimality of inflation targeting. Koenig (2018) and Sheedy (2014) show in heterogeneous agent models that when financial markets are incomplete and debt is written in nominal, non-state contingent terms, then nominal income targeting is optimal and

[3]We emphasize *fixed*, nonconvex menu costs – as pioneered by Barro (1972) and Sheshinki and Weiss (1977) – to distinguish from *convex* menu costs that scale with the size of the price change, such as the Rotemberg (1982) quadratic cost of price adjustment. The Rotemberg model is isomorphic in its structural equations, to a first-order approximation, to the textbook New Keynesian model; and isomorphic to a second-order approximation in its optimal policy implications (Nistico 2007), i.e. inflation targeting not wage targeting is optimal. The difference with our model in optimal policy comes directly from the convex nature of the Rotemberg menu costs: it is better to have all sectors adjust prices a little than to have one sector do all of the adjustment. With the nonconvex menu costs of our model, it is instead optimal to minimize the number of sectors which choose to adjust at all.

[4]Gertler and Leahy analytically derive a log-linearized model with menu costs and observation costs isomorphic to the textbook three-equation New Keynesian model. To do so, they make use of a series of extremely clever simplifying assumptions. However, critically they make assumptions on monetary policy to ensure that monetary shocks alone are "small" enough to never induce a firm to adjust its price. As we will see, inflation targeting requires such shocks.

[5]Among many others: Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer (2019); Ascari and Haber (2021); Nakamura, Steinsson, Sun, and Villar (2018); Klenow and Krystov (2008).

inflation targeting is suboptimal. Werning (2014) shows that this result extends approximately when including heterogeneity to Sheedy's model, that is P and Y should move inversely but not necessarily one-for-one. This also echoes our result below. Angeletos and La'o (2020) show that in a world where agents have incomplete information about the economy, under similar functional forms to the ones we use below, nominal income targeting is the optimal monetary policy; and more generally that the price level P and real output Y should move in opposite directions. Despite these results for two highly important classes of models, it may have been easy to set them aside and nonetheless consider inflation targeting as the proper baseline for optimal monetary policy due to its optimality in the workhorse sticky price model (e.g. Woodford and Walsh (2005)). We hope our paper helps to conceptually integrate these results from across the incomplete market, information friction, and sticky price models.

Our model formalizes and extends the insightful, literary argument made by Selgin (1997) (chapter 2, section 3) that nominal income targeting, or something like it, is optimal in a world with menu costs.[6] Relative to Selgin's elegant informal discussion, we are able to introduce the role of state dependence, which is natural in the context of menu costs and does affect optimal policy, as well as to formalize and be precise about the argument in the context of a standard macro model.[7] This formalization allows us to distinguish between nominal wage targeting versus nominal income targeting, to connect our results to prior modeling work, and to take the model to the data to quantify the welfare costs of inflation targeting.

Outline. We first illustrate the results in sections 2-3 in a baseline setting as described above: an off-the-shelf sectoral model augmented with menu costs hit by an unanticipated sectoral productivity shock. We also discuss the welfare loss of inflation targeting. In section 4, we show that nominal wage targeting continues to hold under a number of extensions: functional form generalizations; velocity shocks; heterogeneity in sector size and menu costs; and multiple productivity shocks. These extensions are also useful for illustrating the mechanism and logic of our main proposition in the baseline setting. In section 5, we generalize by building a quantitative model in order to incorporate dynamics and calculate the welfare gains of adopting nominal wage targeting. Section 6 concludes.

[6] The nominal contracts and incomplete information literatures cited above were both also preceded and discussed brilliantly and lucidly by Selgin (1997).

[7] See also the blog post by Rowe (2015), which frames the role of menu cost-induced state dependence in the context of coordination problems, and how this matters for optimal policy.

2 Baseline model

Our baseline framework is a two-period, multisector model. There are S sectors, each consisting of a continuum of monopolistically competitive intermediate firms which are aggregated into a sectoral good by a competitive sectoral packager. A competitive final goods producer combines the output of each of the S sectors into a final good, sold to the household. The model and the functional forms we use are the same as Golosov and Lucas (2007), except that productivity shocks are sectoral rather than firm-specific and that we analyze optimal monetary policy instead of exogenous monetary shocks.

The two key assumptions are the presence of (1) productivity shocks that move relative prices, and of (2) fixed menu costs which intermediate firms must pay if they choose to adjust their nominal price.

2.1 Household

The household's preferences are given by

$$\mathbb{W} = \left[\ln C - N + \ln\left(\frac{M}{P}\right)\right] - \chi\psi \tag{1}$$

where the terms in brackets are the value derived from consumption C, labor N, and real money holdings $\frac{M}{P}$.[8] The second term $\chi\psi$ is a menu cost penalty term that will be explained in detail in section 2.4. Because this menu cost penalty term is not controlled by choices of the household, the household's problem is equivalent to:

$$\max_{C,M,N} \ \ln(C) - N + \ln\left(\frac{M}{P}\right)$$
$$\text{s.t. } PC + M = WN + D + M_{-1} + T$$

To fund expenditures the household uses labor income from wages W, firm dividends D, previous period money balances M_{-1}, and government transfers T. The first order conditions imply:

$$PC = M \tag{2}$$
$$W = M \tag{3}$$

[8]We follow Woodford (1998) in ignoring the welfare effects of real balances when analyzing optimal monetary policy.

Our particularly simple assumptions on preferences – again matching those of Golosov and Lucas (2007) – result in two simple optimality conditions: an equation of exchange (2) and an equation (3) stating that in equilibrium the nominal wage W is directly determined by the money supply M. In section 4.1 we generalize the functional forms here and show that the optimal policy implications are entirely unchanged.[9]

2.2 Final good producer

The representative final good producer aggregates sectoral goods y_i of price p_i across S sectors, using Cobb-Douglas technology, into the final good Y consumed by the household. Operating under perfect competition, its problem is:

$$\max_{\{y_i\}_{i=1}^S} PY - \sum_{i=1}^S p_i y_i$$
$$\text{s.t. } Y = \prod_{i=1}^S y_i^{1/S} \tag{4}$$

The resulting demand for sectoral goods is:

$$y_i = \frac{1}{S}\frac{PY}{p_i} \tag{5}$$

The zero profit condition gives the price P for the final good:

$$P = S\prod_{i=1}^S p_i^{1/S} \tag{6}$$

Again, in section 4.1 we discuss how generalizing the functional form used here has no impact on the optimal policy result.[10]

2.3 Sectoral goods producers

In a sector i, a representative sectoral goods producer packages the continuum of intermediate goods, $y_i(j)$, produced within the sector using CES technology. Note that for notational clarity, we will consistently use j to identify an intermediate firm and i to iden-

[9]For now, observe from equating (2) and (3) that $W = PC$, so that with the functional forms used here, stabilizing nominal wages and stabilizing nominal income is equivalent.

[10]In particular, it is *not* the case that the the unit elasticity of the Cobb-Douglas technology used in (4) is necessary for nominal income targeting to be exactly optimal.

tify a sector. The problem of the sectoral packager for sector i is:

$$\max_{[y_i(j)]_{j=0}^1} p_i y_i - \int_0^1 p_i(j) y_i(j) dj$$
$$\text{s.t. } y_i = \left[\int_0^1 y_i(j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{7}$$

This results in a demand function $y_i(j)$ and a sectoral price index p_i:

$$y_i(j) = y_i \left(\frac{p_i(j)}{p_i}\right)^{-\eta} \tag{8}$$
$$p_i = \left[\int_0^1 p_i(j)^{1-\eta} dj\right]^{\frac{1}{1-\eta}} \tag{9}$$

2.4 Intermediate goods producers

In each sector there is a unit mass of monopolistically competitive firms each producing a different variety of the sectoral good. Their technology is linear, and all firms within a sector i share a common productivity level A_i.[11]

Intermediate firms are subject to menu costs: they first choose whether or not to pay a fixed menu cost $\psi \in (0, 1)$ to adjust their price, and if so choose their new price. We model this menu cost as a *utility penalty* rather than a real resource cost.[12] With menu costs as a real resource cost, the firm loses a fraction of its profits if it adjusts its price, lowering household income. The resulting income effect complicates the exposition of the model. With a utility penalty, on the other hand, the household directly suffers a utility cost if the firm adjusts its price, without any income effect.[13]

Firm j in sector i maximizes an objective function consisting of profits $D_i(j)$ scaled down by a penalty if choosing to adjust its price, subject to its demand curve and its

[11]Note that it is standard in the optimal policy literature on sectors and networks to only consider the optimal policy response to *sector*-level productivity shocks: see for example Aoki (2001), Benigno (2000), Woodford and Walsh (2005), Rubbo (2020). In particular, these papers do *not* consider idiosyncratic, firm-level productivity differences. In contrast, in the separate literature on menu costs, it is common to consider such idiosyncratic shocks (Golosov and Lucas 2007). We analyze the case of both sectoral and idiosyncratic shocks using the quantitative model in section 5.

[12]As we discuss below in section 3.4 on the welfare costs of inflation targeting, our menu cost specification also admits a "behavioral" interpretation of consumers disliking price changes, for reasons outside the model. See also e.g. Eyster, Madarasz, and Michaillat (2021).

[13]In appendix XXX, we solve analytically the same model with resource costs and show that our results carry through. See e.g. Auclert, Rognlie, and Straub (2018) for use of this modeling device.

production technology:

$$
\begin{aligned}
&\max_{p_i(j),\chi_i(j)} D_i(j)\left[1-\psi\chi_i(j)\right] \\
&\text{s.t. } D_i(j)=p_i(j)y_i(j)-Wn_i(j)(1-\tau) \\
&\quad p_i(j)=p_{[-1]} \quad \text{if} \quad \chi_i(j)=0 \\
&\quad y_i(j)=y_i\left(\frac{p_i(j)}{p_i}\right)^{-\eta} \\
&\quad y_i(j)=A_i n_i(j)
\end{aligned}
\tag{10}
$$

$\chi_i(j)\in\{0,1\}$ is a dummy indicating whether or not the firm chooses to adjust its price. If it does, it incurs a penalty of a fraction ψ of its profits $D_i(j)$.[14] This penalty passes directly to household welfare (equation 1) as the previously-mentioned menu cost utility penalty, $\psi\chi$, where $\chi\equiv\sum_i\int_j\chi_i(j)$ is the fraction of all firms in the economy which adjust their price. Again, this is a modeling device used to abstract from the complication of the income effect that would occur if we modeled menu costs as reducing physical profits; all of our results would still carry through if menu costs physically reduced profits (see appendix XXX). $p_{[-1]}$ indicates the price inherited from the previous period (dropping firm and sector subscripts for notational convenience).

To undo the markup distortion from monopolistic competition, the fiscal authority provides the standard labor subsidy $\tau=\frac{1}{\eta}$, so that conditional on adjustment the optimal reset price equals marginal cost:

$$
p_i(j)=\frac{W}{A_i} \tag{11}
$$

Notice that, because productivity is sector-specific, firms' problems within a sector are completely symmetric: all make the same decision on adjustment and choose the same reset price. Because of this equivalence, we will often refer interchangeably to firm-specific versus sector-specific prices and quantities, e.g. $p_i(j)$ versus p_i.

2.5 The intermediate firm's adjustment decision

We now turn to the question of whether a given intermediate firm will pay the menu cost to adjust its price. The firm compares profits $D_i(j)$ under the new optimal price

[14] To emphasize, profits themselves $D_i(j)$ are unaffected by the menu cost; the household receives the entirety of $D_i(j)$, as seen in the household's problem in section 3.1 above.

$p_i(j) = \frac{W}{A_i}$, net of the penalty ψ, to those under the inherited price $p_{[-1]}$:

$$D_i(j)^{\text{adjust}}(1-\psi) \overset{?}{\lesseqgtr} D_i(j)^{\text{no adjust}}$$

where, plugging in the optimal price or the inherited price and substituting out constraints,

$$\begin{aligned} D_i(j)^{\text{adjust}} &= y_i p_i^{\eta} \left[\frac{W}{A_i}\right]^{1-\eta} \left(\frac{1}{\eta}\right) \\ D_i(j)^{\text{no adjust}} &= y_i p_i^{\eta} \left[p_{[-1]}\right]^{1-\eta} \left(1 - \frac{1}{p_{[-1]}} \frac{W}{A_i} \frac{\eta - 1}{\eta}\right) \end{aligned}$$

Thus, the firm will choose to update its price if and only if the follow condition holds:

$$\left[\frac{W}{A_i}\right]^{1-\eta} \left(\frac{1}{\eta}\right) [1-\psi] > \left[p_{[-1]}\right]^{1-\eta} \left(1 - \frac{1}{p_{[-1]}} \frac{W}{A_i} \frac{\eta - 1}{\eta}\right) \tag{12}$$

This nonlinear adjustment condition implies an inaction region as a function of the desired price: a firm in sector i will not adjust if and only if

$$\frac{W}{A_i} \in p_{[-1]} \cdot \left(\underline{\lambda}, \overline{\lambda}\right) \tag{13}$$

where $0 < \underline{\lambda} < 1$ and $\overline{\lambda} > 1$ are constants that are functions only of the menu cost ψ and the elasticity η. That is, if the desired price $\frac{W}{A_i}$ is close to the inherited price $p_{[-1]}$, then it is not worthwhile to adjust. Conversely, if the desired price is sufficiently high *or* sufficiently low, it is worthwhile to adjust. It is also immediate from (12) that the larger is menu cost ψ, the smaller is the inaction region.

Importantly, the extent to which the desired price has moved "a lot" depends on two factors exclusively:

1. The sectoral productivity A_i, which is exogenous.

2. The level of nominal wages W, which we saw from (3) is completely determined by the central bank, $W = M$, in equilibrium.

Thus, firms are more likely to adjust after either a large productivity shock or a large monetary action.

In order to get a closed form solution to the adjustment condition (12) we make the following assumption whenever considering the analytic model:

Assumption 1 (Elasticity of substitution)**.** Assume $\eta = 2$.

Lemma 1 describes the inaction region under this assumption.

Lemma 1 (Inaction region with $\eta = 2$)**.** Under assumption 1, the inaction region is characterized by $\underline{\lambda} = 1 - \sqrt{\psi}$ and $\overline{\lambda} = 1 + \sqrt{\psi}$. That is, the firm adjusts if and only if

$$\left\{ \begin{array}{ll} \frac{W}{A_i} & < (1 - \sqrt{\psi})\, p_{[-1]} \\ \text{or} & \\ \frac{W}{A_i} & > (1 + \sqrt{\psi})\, p_{[-1]} \end{array} \right\} \tag{14}$$

Proof: follows immediately from setting $\eta = 2$ and solving (12) using the quadratic formula. □

The inaction region when $\eta = 2$ is symmetric around the inherited price, with the width of the band determined solely by the size of the menu cost, ψ.

2.6 Government budget constraint and market clearing

The government finances the labor subsidy with seigniorage, $M - M_{-1}$, and the lump sum tax on the household, $-T$:

$$-T + (M - M_{-1}) = \tau W N \tag{15}$$

Denote aggregate productivity as:

$$A \equiv \frac{1}{S} \prod_i A_i^{1/S} \tag{16}$$

Market clearing ensures that aggregate variables add up: $N = \sum_i \int_j N_i(j)$ and $D = \sum_i \int_j D_i(j)$. Finally, the aggregate resource constraint implies that consumption equals aggregate output:

$$C = Y \tag{17}$$

2.7 Steady state ($t = 0$)

The economy begins at $t = 0$ in a symmetric, flexible-price steady state (steady state variables are denoted with a superscript ss) in which sectoral productivities A_i^{ss} for $i \in \{1, \ldots, S\}$ are taken as given and nominal wages are normalized to $W^{ss} = 1$.

The money supply from (3) is then $M^{ss} = 1$. Firms set prices at their flexible levels (11), $p_i^{ss} = \frac{1}{A_i^{ss}}$. The aggregate price level (6) is $P^{ss} = (A^{ss})^{-1}$. From money demand (2), consumption and therefore output are equal to aggregate productivity, $C^{ss} = Y^{ss} = M^{ss}/P^{ss} = A^{ss}$. From demand equations (8) and (5), sectoral output is $y_i^{ss} = \frac{1}{S}A_i$. From intermediate production technology (10) we recover labor in sector i as $n_i^{ss} = \frac{1}{S}$ and aggregate labor as $N^{ss} = 1$.

3 Optimal policy after a productivity shock

As our baseline exercise, we consider the optimal response to a one-time, unexpected shock to sector 1 alone. For concreteness, consider a positive productivity shock which we denote as $\gamma > A_1^{ss}$. How should monetary policy optimally set the money supply M?

To simplify the analysis, assume that in the initial steady state all sectors have the same productivity, normalized to one, $A_i^{ss} = 1$ for $i = 1, ..., S$. This means that, after the shock to sector 1, firms in sectors $i = 2, ..., S$ all face precisely the same problem and make the same decision on whether and how to adjust.[15] We will consistently identify these sectors with a k – i.e. there are effectively two sectors of different sizes, sector 1 and sector k. As highlighted with many of our previous simplifying assumptions, this assumption is not of significant consequence economically, but it makes working with the harsh discontinuities of the sS pricing rule significantly simpler by reducing the number of cases which need be considered. We consider the case of shocks to multiple sectors in section 4.4, where we show the results carry through but the number of cases grows faster-than-exponentially more complicated.

As a result, we need only consider the decision problems of a representative firm in sector 1 (with productivity $\gamma > 1$) and a representative firm in sector k (with productivity $A_k = 1$). There are, in effect, only two relative prices that the central bank seeks to ensure are correct: the relative price of sector 1, p_1/W, and that of any other sector k, p_k/W – where we have taken the nominal price relative to the nominal wage.

We now characterize optimal monetary policy. To do so as clearly as possible, we first assume that menu costs are not unrealistically large:

Assumption 2 (Menu cost condition)**.** Define $\overline{\psi} \in (0, 1]$, independent of the productivity shock γ, as implicitly defined by the value of ψ solving:

$$-\ln\left[S - 1 + \underline{\lambda}(\psi)\right] = -\ln(S) - \frac{1}{S}\ln\left[\underline{\lambda}(\psi)\right] - \frac{\psi}{S} \tag{18}$$

[15] In symmetric equilibrium across sectors.

Assume menu costs are below this threshold which is easy to show is slightly above 0.47, that is $\psi \leq 0.47 \leq \overline{\psi}$.

This condition that the menu cost not be too large holds for all remotely plausible calibrations, as discussed further in section 5. Under assumption 1 that $\eta = 2$, then $\overline{\psi}$ is (depending on S) never smaller than approximately 47% – ten times as large as any empirical estimate of the size of menu costs.[16] For the sake of completeness, in section 3.5 we also discuss optimal monetary policy in the empirically-irrelevant case where $\psi > \overline{\psi}$, where we describe the existence of "adjustment externalities". An individual firm choosing whether or not to adjust its price does not internalize the effect on aggregate demand; so prices may be either too sticky or too flexible, depending on the size of menu costs and the size of the shock. The purpose of assumption 2 is precisely to shut off this effect in a way that allows for clarity.

We now present the main result of the paper, Theorem 2, characterizing optimal monetary policy: for large enough productivity shocks γ above a threshold level $\overline{\gamma}$, optimal policy is exactly nominal wage targeting; below the threshold, prices should remain unchanged.

Theorem 2 (Optimal monetary policy)**.** Suppose assumption 2 holds. For any fixed level of menu costs ψ, there exists a threshold level of productivity $\overline{\gamma} > 1$, such that:

1. If the productivity shock to sector 1 is above the threshold $\gamma \geq \overline{\gamma}$, then optimal policy is exactly nominal wage targeting: $W = W^{ss}$. This is implemented by the central bank not reacting to the shock – it leaves the money supply unchanged at $M = M^{ss}$ – and only firms in sector 1 adjust their price.

2. If the shock is below the threshold $\gamma \in [1, \overline{\gamma})$, then optimal policy is to ensure that prices remain unchanged and no firm in any sector adjusts.

Additionally, the threshold level of productivity $\overline{\gamma}$ is increasing in the size of menu costs ψ.

Proof. See appendix A.

While the formal proofs are in Appendix A, the intuition behind this result is straightforward.

The central bank faces one fundamental trade-off when choosing the money supply: achieving undistorted relative prices versus minimizing the total menu costs expended. While undistorted relative prices are *good* for welfare allowing the economy to mimic the

[16] Indeed, for large portions of the parameter space, $\overline{\psi} = 1$: that is, this condition never binds whatsoever.

flexible-price allocations, this scenario cannot be achieved without having some firms pay a menu costs which drags down welfare by burning real resources.

In the case of a relatively small productivity shock, $\gamma \in [1, \overline{\gamma})$, it is just not worth achieving undistorted relative prices because the welfare gains are outweighed by the drag menu costs have on welfare. In other words, the menu cost is just too high to justify any change in price. In this scenario the central bank chooses the money supply such that all firms in all sectors decide not to reset their price. This covers the second case of theorem 2.

In the case of a relatively large productivity shock, $\gamma \geq \overline{\gamma}$, the welfare gains coming from undistorted relative prices exceed the welfare loss due to menu costs. There are three ways of achieving undistorted relative prices. The first way is to have all firms reset their prices, this scenario is, ex-post, the same as the flexible price environment except for a total menu cost expenditure of ψ. The second way is to have only firms in sector 1 reset their price, in particular all these firms would lower their price with a total menu cost expenditure of $\frac{\psi}{S}$. The third and final way is to have only firms in sector $s > 1$ reset their price, in particular all these firms would increase their price with a total menu cost expenditure of $\frac{S-1}{S}\psi$. While the ultimate allocation in these three scenarios is identical because all prices are undistorted, the total menu cost expenditures and consequently welfare is not. The central bank should set the money supply so as to minimize the total menu costs expended, that is it should follow the second approach by having only firms in sector 1 reset their prices.

But why does having only sector 1 firms reset their prices correspond to a policy of nominal wage targeting as the theorem suggests? To see this it is easiest to think of firms in sectors $s > 1$. To prevent these firms from wanting to change their price, their nominal marginal cost, $\frac{W}{A_s}$, must not change. As these firms did not face a change in productivity, A_s, the nominal wage, W, must be stabilized. Thus, targeting W prevents all firms in sectors $s > 1$ from wanting to update their price while firms in sector 1 will want to lower their price to reflect a lower marginal cost due to the increase in A_1. If $\gamma \geq \overline{\gamma}$ this results in having *only* firms in sector 1 reset their prices leading to undistorted relative prices while minimizing the menu costs expended. Thus, nominal wage targeting is here the welfare-maximizing policy, as the first case of theorem 2 indicates.

3.1 The welfare cost of inflation targeting

It is worthwhile to briefly contrast contrast this result to inflation targeting, the result in the standard New Keynesian model as well as the policy actually implemented by many central banks around the world.

3.1.1 Menu Cost Model

Above we showed that inflation targeting is not the optimal monetary policy in the presence of menu costs and sectoral shocks. What are then the welfare consequences of inflation targeting? In this model, targeting a 0 rate of inflation means targeting a constant price level, P. While if $\gamma \in [1, \overline{\gamma})$ the optimal policy does imply a constant price level because no firm resets its price, if $\gamma \geq \overline{\gamma}$ the optimal policy does not. If only sector 1 firms change their prices and no other firms do, it must be that the aggregate price level moves in whatever direction sector 1 firms revise their prices. A similar argument applies when considering the third strategy in which only firms in sector $s > 1$ reset their prices: the aggregate price level moves in whatever direction these firms update their prices in. The only strategy consistent with both undistorted relative prices and a constat price level targeting is the first in which all firms update their prices. This world is, after the menu cost expenditure, a flexible price world, and so monetary neutrality allows the central bank to achieve any price level desired. As the following proposition makes clear, the welfare loss of inflation targeting relative to the optimal policy of nominal wage targeting is therefore directly captured by the welfare loss caused by the unnecessary menu costs expended by sectors $s > 1$. Furthermore, the size of the menu cost is a sufficient statistic for the welfare gain achieved moving from inflation targeting to nominal wage targeting.

Proposition 3 (The welfare loss of inflation targeting)**.** Suppose assumption 2 holds and the productivity shock is sufficiently large relative to the menu cost, $\gamma > \overline{\gamma}$. Then:

1. Inflation targeting requires increasing the money supply to $M^{IT} = \gamma^{1/S} > M^{ss} = 1$, which results in all firms adjusting and paying the menu cost.

2. Welfare under inflation targeting $\mathbb{W}^{IT}$ is strictly less than welfare under the optimal policy of nominal wage targeting $\mathbb{W}^*$. The difference is summarized by the size of menu costs ψ and the fraction of firms unaffected by the shock $\frac{S-1}{S}$. Denoting $\mathbb{W}^f$ the level of welfare under flexible prices, then:

$$\begin{aligned} \mathbb{W}^{IT} &= \mathbb{W}^f - \psi \\ \mathbb{W}^* &= \mathbb{W}^f - \frac{1}{S}\psi > \mathbb{W}^{IT} \end{aligned}$$

Proof. Proof: See appendix XXX. □

An economic interpretation of this result depends on how one conceptualizes menu costs. In the model above, we model menu costs as directly impinging on household welfare. If menu costs represent the direct attention and psychological costs to firm managers of adjusting prices or a direct welfare loss to consumers who (for whatever reason) have a preference for stable prices, this is reasonable. Additionally, if menu costs are actual, physical costs associated with changing price (e.g. printing new menus), then this is also reasonable: higher physical adjustment costs means lower profits, which means fewer resources are produced in the economy and available to consume (see Levy et al. (1997)).[17] The same is true for managerial costs associated with communicating price changes to consumers (see Zbaracki et al. (2004)).

However, if menu costs are a reduced-form way of representing *zero-sum* competition among firms, then *in aggregate* summing across firms it would be wrong to interpret menu costs as having a welfare cost. For example, if a firm is averse to changing prices for fear of losing customers to rival firms, then one's firm loss is another firm's gain and in aggregate there is no direct welfare cost of menu costs. Note that taking this seriously as the only reason for menu costs would entail strong and unlikely conclusions: optimal policy would be to hyperinflate every period, to ensure that firms are induced to adjust, so that relative prices are always correct – without any cost. While the question of to what in reality menu costs map remains largely an open question, it seems likely that at least *some* portion of what they represent is a real welfare cost, in which case the mechanism identified here is relevant.[18]

A concern may be that the welfare cost of menu costs – resources burned maintaining correct pricing – is relatively small: the literature on menu costs often builds on the idea that 'second-order menu costs can result in first-order output fluctuations' (Mankiw 1985). However, it is important to note that in the textbook model with the Calvo-Yun friction, the welfare loss of price stickiness is *also* only second-order.[19] We also highlight that estimates of the real resource cost of menu costs from the empirical literature are sizeable, on the order of 0.5% of total firm revenues annually (an order of magnitude larger than the famous "welfare loss of business cycles" estimate of 0.05% of Lucas 1987).[20]. More generally, in this paper and in our model, we think of menu costs as representing not merely any physical costs of adjusting prices, but being a reduced-form way of capturing any economic or welfare cost of price adjustment, such as the mental attention costs suffered by price setters.

[17]Or more labor is required to do so.

[18]See also Rotemberg (2005); Anderson and Simester (2003); Nakamura and Steinsson (2011); or Eyster, Madarasz, and Michaillat (2021), among others.

[19]See e.g. Gali (2008) chapter 5.

[20]See e.g. Nakamura and Steinsson (2010); Nakamura, Steinsson, Sun, and Villar (2018); and Levy et al. (1997)

3.1.2 Calvo Model(s)

The analysis above makes clear by now why inflation targeting is suboptimal in a multisector model in which the nominal rigidity is characterized using menu costs. A natural question is whether the same results hold true in a world in which price stickiness is modeled following Calvo (1983) and Yun (1982). The answer is generally not, except in a very special circumstance. While the details pertaining to optimal policy with Calvo rather than menu costs in the models are deferred to section C, here we outline how these models diverge from our optimal policy prescription.

One Sector. The standard result presented in Woordford (2003) is that, in a one sector economy with Calvo price stickiness, and symmetry across firms, it is optimal for the central bank to target 0 inflation. The intuition is that, because only certain firms can reset their price at any given period and because price dispersion, i.e. having identical firms set different prices, is inefficient, the optimal central bank policy is to target no inflation so that no firm, even the ones that could, wants to reset its price. This prevents any price dispersion and mimics the flexible-price world.

Homogeneous Multisector. Key to our own result is that there is a motive for price changes. In a model in which different sectors of the economy are subject to different productivity shocks, relative price adjustments can be good, in fact that is what you would see in the flexible price world. In a multisector Calvo model with homogeneous price-stickiness across firms, inflation targeting is still the optimal policy as we show in Appendix (**??**). Importantly, this policy *does not* stabilize nominal wages. Using the experiment we used to characterize optimal policy, nominal wages increase after the positive productivity shock to sector 1.

Heterogeneous Multisector. There is one case, however, in which a multisector Calvo model implies the same optimal policy as our own menu cost model. That is the case in which sector 1, the shocked sector, is completely flexible. This scenario is described in Aoki (2001) and the prescription here is to target the "core" price inflation, that is inflation in the sticky sectors. But that is identical to our own logic in which, in order to prevent any price changes in sectors $s > 1$, the central bank ensured that nominal wages (and hence prices) in those sectors stayed fixed. Of course, this is a very particular case, and assuming that sector 1 is fully flexible is a strong assumption.

4 Extensions to the benchmark model

We now consider several natural extensions to the model, which we solve analytically. These showcase the robustness of our results, and are useful for reinforcing the intuition built above on the mechanism of our results.

4.1 Functional form generalizations and stabilizing W vs. PY vs. M

The core intuition argued above is that monetary policy seeks to ensure that nominal marginal costs are unchanged for firms who do not receive a productivity shock, so that they have no desire to adjust their price. In section 3, we assumed Cobb-Douglas technology at the aggregate level, CES technology at the sector level, and linear technology at the intermediate firm level. However, any (stationary) functional forms could have been used without any implication for optimal policy. For example, CES technology at the aggregate level $Y = \left[\sum_i y_i^\theta\right]^{1/\theta}$ would not alter theorem 2 or any of the discussion above at all because of this intuition. Introducing decreasing returns at the intermediate level $y_i(j) = A_i n_i(j)^{1/\alpha}$ does not affect the statement of theorem 2, but would affect the value of the threshold $\overline{\gamma}$, since decreasing returns affects the size of firms' inaction regions and therefore interacts with the adjustment externalities. The robustness of the theorem to these and other changes *on the firm side* of the model is because stabilizing nominal marginal cost of unaffected firms still means stabilizing W, which would still be equivalent to stabilizing M or stabilizing PY due the optimality conditions *from the household's problem*, (2) and (3).

However, we describe optimal policy in this paper as nominal wage targeting, not as money supply targeting or as nominal income targeting, because of the robustness to changes on the household side.

First consider why optimal policy is not a k-percent money growth rule (a la Friedman 1960): velocity shocks. Under velocity shocks, nominal wage targeting and nominal income targeting are still equivalent. Nominal income targeting can be thought of as a *velocity-adjusted* money supply targeting rule. From the equation of exchange (2), velocity V is implicitly defined as $MV = PY$. A money supply rule fixes M, whereas nominal income targeting fixes PY – thus a nominal income target is equivalent to "velocity-adjusting" the money supply target. In our benchmark model, velocity is constant at 1. If we introduced velocity shocks – e.g. by introducing a preference shifter affecting the household's utility for real balances – the optimal monetary policy response to a negative velocity shock would be to increase the money supply. For additional, informal discussion, see e.g. Sumner (2012), Beckworth (2019), and Binder (2020).

Second, it is more accurate to describe optimal policy as nominal wage targeting rather than nominal income targeting. This is because nominal wage stabilization continues to be optimal when household preferences are generalized – but this is not so for nominal income. Consider for example if household preferences (1) are generalized to be separably isoelastic:

$$\mathbb{W} = \left[\frac{1}{1-\sigma_C}C^{1-\sigma_C} - \frac{1}{1+\sigma_N}N^{1+\sigma_N} + \frac{1}{1-\sigma_M}\left(\frac{M}{P}\right)^{1-\sigma_M}\right] - \chi\psi$$

Under these preferences, (2) and (3) do not hold, and so nominal wages and nominal GDP are not equivalent, $W \neq PY$. The core intuition for nominal wage targeting however continues to hold: by stabilizing nominal wages W, the central bank ensures that firms outside sector 1 have no motive to adjust prices. Firms in sector 1 cut their prices and raise output, so that P and Y do continue to move inversely – "approximate nominal income targeting" – but exact nominal targeting is not precisely optimal. Exact nominal wage targeting is.

Finally, we note that while optimal policy here prescribes constant nominal wages, this could be generalized to a constant trend for nominal wage growth. This would be analogous to the generalization in the textbook Calvo-Yun framework from the prescription of a constant price level to the prescription for a 2% inflation target employed by many countries in actuality. This can be captured formally by allowing firms to adjust their prices at a constant trend for free, but to pay the menu cost in order to deviate from trend, i.e. indexing. See e.g. Woodford (2003) for the case of this indexing in the textbook model.[21]

4.2 Sectoral heterogeneity and a monetary "least-cost avoider" principle

We now return to the baseline setting and consider two kinds of heterogeneity, in sector size and menu cost magnitude, that lead us to a "least-cost avoider" interpretation of optimal monetary policy.

[21] An alternative to a trend growth of zero or 2%, suggested by Halperin (2017), would be to target a nominal income trend growth rate of $1 - 1/\beta$. In other words, nominal wages and nominal income would gradually decline at the rate of time preference. With this trend and under the baseline functional forms, nominal wage and income targeting would not only be the optimal *short-run* stabilization policy. It would also result in a nominal interest rate of zero and thus implement the Friedman rule for the *long-run* optimum quantity of money (and would do so without the need to pay interest on money).

4.2.1 Heterogeneity in sector size

Suppose that sectors are of different sizes. Instead of each sector being an equal size $\frac{1}{S}$, allow sector i to be size $\frac{1}{S_i}$, where $\sum_{i=1}^{S} \frac{1}{S_i} = 1$ to preserve constant returns to scale in finals goods production. The final goods technology (4) is now:

$$Y = \prod_{i=1}^{S} y_i^{1/S_i} \tag{19}$$

The resulting demand function is nearly the same as previously (5):

$$y_i = \frac{1}{S_i} \frac{PY}{p_i} \tag{20}$$

All that has changed in these two equations is the replacement of $\frac{1}{S}$ with $\frac{1}{S_i}$.

It is natural to assume that this also affects how menu costs are weighted in household welfare (1). The term χ which measures the fraction of sectors adjusting their price, previously defined as a simple average $\chi = \frac{1}{S} \sum_{i=1}^{S} \chi_i$, we now write as a weighted average with weights determined by sector size:

$$\chi = \sum_{i=1}^{S} \frac{1}{S_i} \chi_i$$

With this change, optimal policy is *nearly* the exact same as characterized in Theorem 2. What differs is only in the extreme case when sector 1 is larger than all other sectors put together, $\frac{1}{S_1} > \sum_{k>1} \frac{1}{S_k}$. Then if relative prices are to adjust it is actually optimal to have firms *outside* sector 1 adjust their price in response to a shock affecting sector 1 itself. That is because although it is only sector 1 which is affected by the shock, the combined mass of firms outside sector is smaller than sector 1 itself. Therefore the menu costs burned by having all other firms adjust price is less than the menu costs burned by having "just" sector 1 adjust.

Thus under the (extreme) assumption that sector 1 is larger than the combined mass of all other sectors, implementing regime **D** where every sector except 1 adjusts is preferable to implementing **C** where 1 alone adjusts. This has the same intuition that it achieves the correct relative prices while economizing on menu costs – where, here, economizing on menu costs means having sector 1 not adjust. This policy would not stabilize nominal wages (or inflation).

We interpret this case mostly as an illustration of the logic of our results, rather than an empirically-relevant case in general. Otherwise, the optimal policy prescriptions of

Theorem 2 carry through exactly.

4.2.2 Heterogeneity in menu cost size by sector

Introducing heterogeneity in menu cost size by sector is mostly similar. If the menu cost of sector i is ψ_i, household welfare (1) should be written

$$\mathbb{W} = \left[\ln C - N + \ln \frac{M}{P}\right] - \sum_{i=1}^{S} \frac{1}{S_i} \chi_i \psi_i$$

Observe that the direct effect of heterogeneity in menu costs on welfare is isomorphic to that of heterogeneity in sector sizes. But heterogeneity in menu cost size, unlike that in sector size, *also* affects the size of inaction regions given in (13). The inaction region as a function of the money supply for any sector i becomes

$$M \in \left(A_i\left(1 - \sqrt{\psi_i}\right), A_i\left(1 + \sqrt{\psi_i}\right)\right)$$

However, this additional complication has somewhat limited impact.

First – analogous to the possibility just discussed that sector 1 is very large in size – if weighted the menu costs of sector 1 are extremely large relative to those of other sectors, $\frac{1}{S_1}\psi_1 > \sum_{k>1} \frac{1}{S_k}\psi_k$, then it again is optimal to have all firms outside sector 1 adjust (regime **D**) rather than those in sector 1 (regime **C**) if relative prices are to change. Second, variation in ψ_1 does affect when it is optimal to allow prices to go unchanged, i.e. to implement regime **A** versus regime **C**; but this impact is the same as varying the uniform ψ in the baseline model.

Third, where this variation in menu costs does introduce complications is by impacting the adjustment externalities. Specifically, the unconstrained optimum level of the money supply under regime **A** may be feasible for some shocked sectors but not others. This introduces additional dimensions of relative price distortions which the central bank must take into account when comparing regime **A** versus regime **C**.[22] While this affects the value of threshold value, $\overline{\gamma}$, determining how high the productivity shock γ must be for regime **C** to dominate regime **A**, optimal policy is still exactly as characterized in theorem 2. We summarize these results in the following proposition.

Proposition 4. Suppose sector i is of size $1/S_i$ and has menu cost ψ_i. Suppose further that the size-weighted menu cost of sector 1 is smaller than the combined weighted average of menu costs for other sectors, $\frac{1}{S_1}\psi_1 < \sum_{k>1} \frac{1}{S_k}\psi_k$. Then optimal monetary is exactly the

[22] As highlighted in the prior discussion, the empirical relevance of this externality is unclear.

same as characterized in Theorem 2 modulo changes in the constant $\overline{\gamma}$.

4.2.3 Interpretation: a monetary "least-cost avoider" principle

These two results on sectoral size and menu cost heterogeneity, summarized in proposition 4, can be interpreted as a "least-cost avoider" theory of optimal monetary policy. In law and economics, the least-cost avoider principal states that when considering assignment of liability between parties, it is efficient to assign liability to the party who has the lowest cost of avoiding harm (Calabresi 1972). Similarly, the generalized principle of optimal monetary policy under menu costs is: the agents for whom it is least costly to adjust their price are the agents who should do so.

More closely to the monetary economics literature, this is also very related to the idea that 'monetary policy should target the stickiest price' (e.g. Mankiw and Reis 2003 and Aoki (2001)). Under menu costs, the central bank should minimize adjustment by the firms with the most expensive menu costs – i.e. it should stabilize the stickiest prices.

4.3 Multiple shocks and time aggregation

[COMMENT: first subsection here requires significant revision: reframe proposition in terms of 'theorem still holds but thresholds change', drop table]

In the baseline model that we analyze above, only sector 1 is shocked. What lessons if any change if multiple sectors are shocked? In short: the problem is exponentially more complicated[23] but the lessons are the same.[24]

Consider starting again from steady state, and in addition to a *positive* shock to sector 1, $A_1 = \gamma > 1$, there is a shock to sector 2 of any sign, $A_2 = \delta > 0$. With one shock, we had four regimes: sector 1 could adjust or not adjust, and other sectors could adjust or not adjust. With two shocks, we have eight regimes by similar logic. Previously, we had two relative prices $(p_1/W, p_k/W)$ whereas now we have three (the addition of p_2/W, and k now refers to $\{3, ..., S\}$).

The intuition for the set of possible regimes is similar to that described above:

1. If both sector 1 and sector 2 adjust their nominal prices, that is sufficient to achieve correct relative prices, and only two sectors need pay a menu cost.

[23]See footnote **??**.

[24]In our model, sectors are independent of each other; a natural alternative would be to have a full input-output network structure. To the extent that a network structure simply transforms a productivity shock in sector 1 into a productivity shock in multiple sectors (with varying degrees of passthrough), this section can be thought of as a reduced-form way of analyzing optimal monetary policy in a networked economy with menu costs.

	Menu cost loss	Allocative inefficiency loss
Regime A (none change)	0	from γ, δ
Regime B (all change)	S	0
Regime C (1 changes)	1	from δ
Regime D (k changes)	S -2	from γ, δ
Regime E (2 changes)	1	from γ
Regime F (1 and 2 change)	2	0
Regime G (1 and k change)	S -1	from δ
Regime H (2 and k change)	S -1	from γ

Table 1: Different regimes when both sector 1 and sector 2 are shocked, and the result implications for welfare under within-regime optimal policy.

2. If only one of the two shocked sectors adjusts, allocative efficiency will be lower, but resources burned on menu costs will also be lower. It is always more efficient (if menu costs are the same for sector 1 and sector 2) to have the sector with the larger shock be the one to adjust[25]; and it is never efficient for the larger mass of the k unaffected sectors to adjust.

3. If neither of the two shocked sectors adjusts, then allocative efficiency is yet lower, but resources burned on menu costs are also lowered to zero.

Optimal policy trades off these considerations, and again faces the issue of adjustment externalities.

Proposition XXX in appendix XXX summarizes this formally. Again, a baseline case is that for shocks that are not "too small', nominal income targeting is always optimal (and may be so even for small shocks). Again, inflation targeting requires that *all* sectors adjust, thus wastefully causing *all* sectors to pay a menu cost, and is always strictly dominated by another policy.

[25] to doublecheck XXX

4.3.1 Knife-edge aggregate shocks

The one exception to the above claim that inflation targeting is always suboptimal is the knife-edge case when *all* sectors in the economy are affected by *precisely* the same productivity shock. In this case, the shock is simply an aggregate productivity shock – the usual kind of shock considered in much of the optimal policy literature – since it does not induce any desire to change *relative* prices.

In this case, stabilizing the price level *is*, in fact, superior to stabilizing nominal income: no prices need to change in order to ensure relative prices are correct, and therefore it is best to have none change to avoid wasteful menu costs. Under nominal income targeting, all firms would adjust and, for e.g. a positive aggregate shock, the aggregate price level would fall as real output rises.

While aggregate shocks are often a useful modeling device for obtaining clean analytical results, in reality it is unlikely that any shock in an economy has precisely homogeneous effects across all firms in the economy.

To the extent that there is *any* heterogeneity in the effect across sectors, it is always preferable to have the least-cost sectors adjust, and to have the highest cost sector leave its nominal price unchanged.

4.3.2 Time aggregation and shock synchronicity

What happens as the number of shocked sectors approaches the total number of sectors, S? As discussed in proposition 3, the welfare loss of inflation targeting over nominal income targeting is the result of inflation targeting unnecessarily forcing non-shocked firms to pay a menu cost to adjust. As the number of sectors *simultaneously* receiving a productivity shock increases, the number of sectors being forced to unnecessarily adjust falls – since it is optimal for an increasing number to adjust (for large enough shocks).

If *all* S sectors receive heterogeneous shocks simultaneously, then monetary policy can still be set to allow one sector to not adjust even as the other $S-1$ sectors adjust, thus economizing on one sector's worth of menu costs; whereas inflation targeting would still require all sectors to adjust, even this one. The quantitative welfare gains of nominal income targeting over inflation targeting would be reduced, under this interpretation. This is related to an issue of time aggregation: if time periods in the model are mapped to long-enough periods in reality, then all sectors will be measured as receiving a shock every period.

However, we think it is more natural to think of shocks as asynchronous. In reality, it seems more plausible to think of shocks as arriving in continuous time with something like a Poisson frequency: the probability of a shock is zero at any given instant, and there are gaps between shock arrival. Under this interpretation, truly only one sector is shocked at any given instant.

Of course, this discussion introduces time dynamics, which present additional complications not modeled in our baseline two-period model. We now turn to a quantitative model that can handle dynamic issues.

5 Quantitative model

Though the intuition is entirely captured by the static model, we show that even in a quantitative dynamic model calibrated to the US economy nominal income targeting dominates inflation targeting.

5.1 Model description and solution method

The closest model to ours in the literature is Nakamura and Steinsson (2010) who consider a multisector menu cost model. Unlike their model, our firms are subject to sector-specific shocks which will make the computation a bit more cumbersome. In this dynamic version of the model only the problem faced by households and intermediate firms change so we discuss these in detail below.

Household. The household chooses consumption, C_t, labor N_t, money balances, M_t, and bonds, B_t to maximize the present discounted value of utility. The problem faced by the household then is

$$\max_{\{C_t,N_t,B_t,M_t\}_{t=0}^{\infty}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\gamma}}{1-\gamma}-\omega\frac{N_t^{1+\varphi}}{1+\varphi}+\ln\left(\frac{M_t}{P_t}\right)\right]$$

$$\text{s.t.} \qquad P_tC_t+B_t+M_t\leq R_tB_{t-1}+W_tN_t+M_{t-1}+\sum_{i=1}^{S}\int_0^1\Pi_{t,i}(j)dj-T_t$$

To consume and save (the left hand side of the budget constraint) the household uses previous savings, money holdings, labor earnings, the profits from the firms, net of the lump sum tax imposed by the government (the right hand side of the budget constraint).

Intermediate Firms. The firm maximizes the present discounted value of real profits, $\frac{\Pi_{t,i}(j)}{P_t}$. In any given period the firm chooses whether to update its price, $\chi_{t,i}(j)=1$ or not.

If the firm decides to change its price, $p_{t,i}(j)$, it must pay a menu cost equal to a share ψ of total profits. The problem faced by each intermediate firm then is

$$
\begin{aligned}
V\left(\frac{p_{t-1,i}(j)}{P_t}, \{p_{t,i}\}_i, \{A_{t,i}\}_i, a_{t,i}(j)\right) = \max_{p_{t,i}(j), \chi_{t,i}(j)} \Bigg\{ & \frac{\Pi_{t,i}(j)}{P_t} \\
& + \mathbb{E}\left[\frac{1}{R_t} V\left(\frac{p_{t,i}(j)}{P_{t+1}}, \{p_{t+1,i}\}_i, \{A_{t+1,i}\}_i, a_{t+1,i}(j)\right)\right] \Bigg\} \\
\text{s.t.} \quad \Pi_{i,t}(j) = & \left[p_{t,i}(j) y_{t,i}(j) - W_t n_{t,i}(j)(1-\tau)\right](1 - \chi_{t,i}(j)\psi) \\
y_{t,i}(j) = & y_{t,i}\left(\frac{p_{t,i}}{p_{t,i}(j)}\right)^{\eta} \\
y_{t,i}(j) = & A_{t,i} a_{t,i}(j) n_{t,i}(j)^{\alpha}
\end{aligned}
$$

The relevant state variables necessary for firm to make its decisions are real price it enters the period with, $\frac{p_{t-1,i}(j)}{P_t}$, the aggregate prices, $\{p_{t,i}\}_{i=1}^{S}$ and P_t derived from these, the aggregate states, $\{A_{t,i}\}$, and the idiosyncratic productivity shock it is subject to in period t. We further follow Nakamura and Steinsson (2010) in assuming that, to a first order around the steady state, $\hat{Y} = \hat{C}$ and $\hat{N} = \hat{Y}$. These assumption allow us to simplify the firm's problem and not add the aggregate wage, W_t and the aggregate labor supply, N_t as state variables.

Model Solution. Because of the sector-level productivity shocks we must resort to a quantitative solution method that allows for aggregate shocks. As Nakamura and Steinsson (2010) we use a version of Krusell and Smith (1998). Firms form expectations of the sector-level prices using a VAR(1), that is they predict the price of sector s at time t according to

$$
\log\left(p_{t,i}\right) \quad = \quad \phi_0\left(\{a_{i,t}\}_i\right) + \sum_{i=1}^{S} \phi_i\left(\{a_{i,t}\}_i\right) \log\left(p_{t,s-1}\right) \tag{21}
$$

where ϕ_0 and ϕ_i depend on the realization of the states just as in the original Krusell and Smith (1998).

5.2 Calibration

We closely follow Nakamura and Steinsson (2010) in calibrating the model.

5.3 Welfare Comparison: Nominal Wage vs. Inflation Targeting

While idiosyncratic shocks had no role to play in the static version of the model, they do show up in this quantitative version. Replicating the experiment in the static model including idiosyncratic shocks points to the potential importance of these for the quantitative results. We run the same experiment where sector $s = 1$ firms are subject to both sector-level and idiosyncratic productivity shocks while sector $s = \{2, \ldots, S\}$ firms are only subject to idiosyncratic productivity shocks. We then simulate the economy for 100 periods and keep track of how many times on average sector 1 versus sector k firms changed prices. This is shown in Figure (1) for the periods in which sector 1 is hit by a negative productivity shock (red bars), a positive shock (green bars), and no shock at all (blue bars). While in the benchmark calibration sectoral and idiosyncratic productivity shocks have the same standard deviation (solid bars), in the counterfactual calibration the standard deviation of idiosyncratic shocks is double that of sectoral shocks. If our

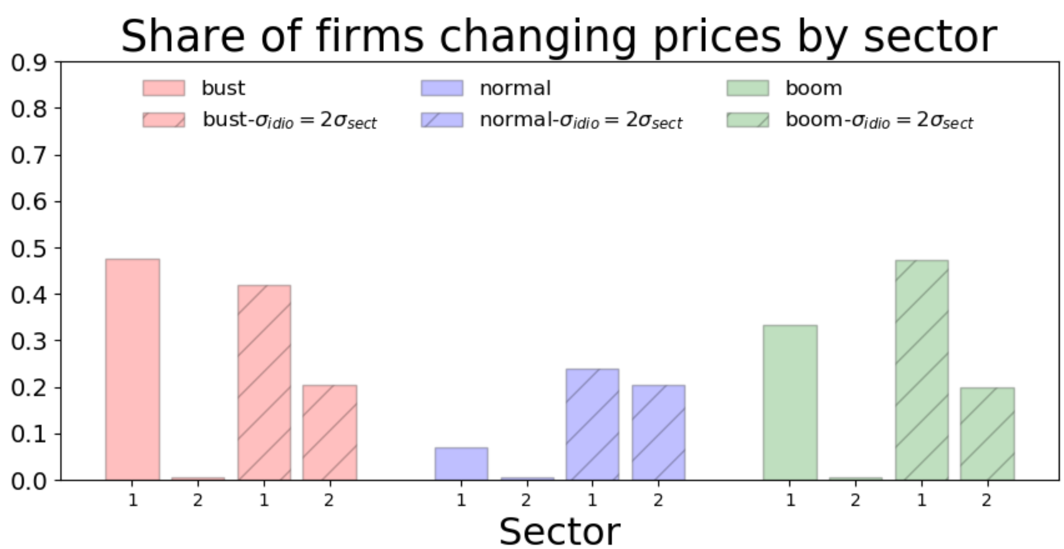

Figure 1: Share of firms changing prices by sector over a 100 periods simulation. Solid bars are for the calibration in which idiosyncratic and sectoral shocks have the same standard deviation, hatched bars are for the calibration in which sectoral shocks have twice the standard deviation of idiosyncratic shocks.

mechanism is to have bite then idiosyncratic shocks must not be much more volatile than sectoral ones. The literature suggests exactly this: sector-level productivity is about 30% more volatile than idiosyncratic productivity as Yeh (2017) shows[26].

Ultimately, however, the question is whether price level or nominal wage targeting is superior. To do this we simulate the above model, where all sectors and all shocks are subject to both sectoral and idiosyncratic shocks, for 250 periods under the two different

[26] See Table IV (Relative contribution of firm-specific and macro-sectoral components).

monetary policy regimes and evaluate welfare. Welfare under the nominal wage targeting regime is approximately 1% higher, indicating this policy dominates.

6 Conclusion

This paper concludes that central banks should implement a nominal wage target rather than inflation targeting, the standard answer in the literature. We show this analytically in a multisector economy where firms pay menu costs to adjust prices. Because of sectoral shocks, in this economy as in the real world, relative prices should change, otherwise firms subject to shocks experience distorted relative prices relative to their marginal costs. Because of menu costs firms can choose when to update their prices and thus firms within sectors coordinate their price changes upon the realization of sector-level shocks. Nominal wage targeting allows for only shocked firms to reset their prices, achieving efficient relative prices while minimizing the menu costs expended. Inflation target, on the other hand, would only be able to achieve efficient relative prices by forcing *all* firms to reset their prices, maximizing the menu costs expended.

This paper complements much of the literature on optimal monetary policy. While other research has already shown the optimality of nominal wage (or similarly nominal income) targeting in economies with sticky wages, with financial frictions, or informational frictions, this is the first paper to show that nominal wage targeting is also optimal in the canonical New-Keynesian sticky-price model.

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A Optimal Policy Proofs.

Claim A.1 (Price change rule)**.** There exist sS bounds $(\overline{\lambda} p_{[-1]}, \underline{\lambda} p_{[-1]})$ such that:

1. If $\frac{W}{A_i} \in (\overline{\lambda} p_{[-1]}, \underline{\lambda} p_{[-1]})$, then firms in sector i choose not to adjust
2. Otherwise, firms in sector i do adjust

That is, $\underline{\lambda}, \overline{\lambda}$ are defined by setting (22) with equality and solving for $\frac{W}{A_i} / p_{[-1]}$. Furthermore:

Proof. Recall that the general price changing decision is

$$\frac{1}{\eta}[1-\psi] > \left[\frac{W/A_i}{p_{[-1]}}\right]^{\eta} \left[\left(\frac{W/A_i}{p_{[-1]}}\right)^{-1} - \frac{\eta-1}{\eta}\right] \tag{22}$$

Consider the case in which $\eta > 2$, and $\psi \in [0,1)$. Rewriting (22) with $x \equiv \frac{W}{A_i}/p_{[-1]}$, rearranging gives, and looking for its zeros gives

$$F(x) = -\frac{\eta-1}{\eta} x^{\eta} + x^{\eta-1} - \frac{1-\psi}{\eta} \tag{23}$$

According to Descartes' rule of signs[27], there being two sign changes in the coefficients, there are at most two positive roots. Evaluated at $x = 0$, $F(\cdot)$ gives that $F(0) = -\frac{1-\psi}{\eta} < 0$. For $x \to \infty$ we have

$$\lim_{x\to\infty} F(x) = \lim_{x\to\infty} -\frac{\eta-1}{\eta} x^{\eta} + x^{\eta-1} - \frac{1-\psi}{\eta} = \infty$$

Evaluated at $x = 1$, $F(1) = \frac{\psi}{\eta} - 1 > 0$ since ψ is strictly less than 1 and $\eta > 2$. The intermediate value theorem then implies that between 0 and 1 there exist at least one positive real root and similarly between 1 and $+\infty$ there exists at least one positive real root. In other words this means there are at least two positive real roots. But, as we said earlier, there are at most two positive real roots. We conclude there are exactly two real roots, $\underline{\lambda}$ and $\overline{\lambda}$ such that $\underline{\lambda} < 1 < \overline{\lambda}$.

Equation (23) implies that whenever $x \in (\underline{\lambda}, \overline{\lambda})$ then the firm does not reset its price. In other words, if $x = \frac{W/A_i}{p_{[-1]}} \in (\underline{\lambda}, \overline{\lambda}) \iff \frac{W}{A_i} \in \left(\underline{\lambda} p_{[-1]}, \overline{\lambda} p_{[-1]}\right)$ the firm decides not to change price. Otherwise, the firm changes its price. □

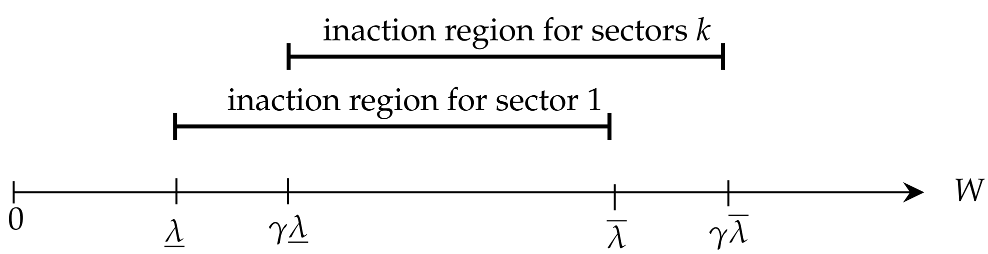
[27] Descartes' rule of signs applies to natural exponents but an easy way around this is to (i) approximate the exponents, η and $\eta - 1$, by rationals with a large denominator n, (ii) replace x by x^n, that changes no roots and Descartes' rule of signs applies.

Note that given $p^* = 1$, Claim A.1 implies that after the shock to sector 1, sectors $s > 1$ have the inaction region $W \in \left(\underline{\lambda}, \overline{\lambda}\right)$ while sector $s = 1$, which is subject to a positive productivity shock $\gamma > 1$, has $W \in \left(\gamma\underline{\lambda}, \gamma\overline{\lambda}\right)$. Next we investigate when these two regions overlap.

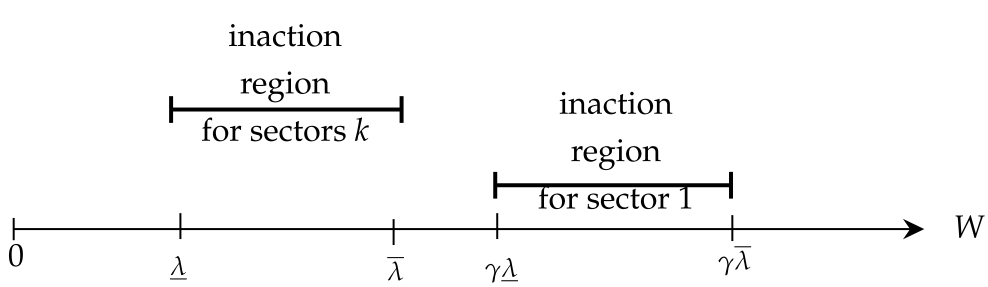
Claim A.2 (Inaction region threshold γ_0)**.** There exists a threshold level of productivity γ_0 such that:

1. If $\gamma > \gamma_0$, then the inaction regions do not overlap
2. Otherwise, they do

Proof. If $\gamma < \gamma_0$, then the inaction regions overlap:



Otherwise they do not:



Thus γ_0 is such that:

$$\gamma_0 = \overline{\lambda}/\underline{\lambda} \tag{24}$$

□

Claim A.3 (Equilibrium Regimes)**.** There are four equilibrium regimes with the allocations listed below.

Proof. Fix γ, the only policy choice the central bank makes is M. By symmetry across sectors in the initial steady state and symmetry within sectors, there are only four possible equilibrium regimes:

A no sector changes price

B all sectors changes price

C only sector $s = 1$ changes price

D only sectors $s \in \{2, \dots S\}$ change price

To compute the equilibrium allocations follow these steps:

1. Prices in each sector (and associated indicators $\chi_i(j) = \chi_i$ for whether or not a sector changes price) are derived from the profit maximization problem. Specifically, if a firm resets it sets the price to its nominal marginal cost:

$$p_1(j) = p_1 \in \left\{ \frac{M}{\gamma}, 1 \right\}$$
$$p_{s>1}(j) = p_{s>1} \in \{M, 1\}$$

2. The aggregate price level is implied by the price aggregator:

$$P = S\Pi_i p_i^{1/S} = S \left[p_1^{\frac{1}{S}} p_{s>1}^{\frac{S-1}{S}} \right]$$

3. This implies consumption from money demand and output from the aggregate resource constraint:

$$C = Y = \frac{M}{P} = \frac{M}{S} p_1^{\frac{-1}{S}} p_{s>1}^{\frac{1-S}{S}}$$

4. This implies firm-level and sector-level output from sectoral demand, and associated labor supplies from production functions

$$\begin{aligned}
y_i &= \frac{1}{S}\frac{PY}{p_i} = \frac{1}{S}\frac{1}{p_i}M & \implies y_1 &= \frac{1}{S}\frac{1}{p_1}M \\
& & y_{s>1} &= \frac{1}{S}\frac{1}{p_{s>1}}M \\
n_i &= \frac{1}{A_i} y_i & \implies n_1 &= \frac{1}{\gamma}\frac{1}{S}\frac{1}{p_1}M \\
& & n_{s>1} &= \frac{1}{S}\frac{1}{p_{s>1}}M \\
N &= \sum_i n_i & &= \frac{M}{S}\left[\frac{1}{\gamma}\left(\frac{1}{p_1}\right) + (S-1)\left(\frac{1}{p_{s>1}}\right) \right]
\end{aligned}$$

5. Giving welfare:

$$
\begin{aligned}
\mathcal{W} &= \{2\ln C - N\} - \chi\psi \\
&= \left\{2\ln\left[\frac{M}{S}p_1^{\frac{-1}{S}}p_{s>1}^{\frac{1-S}{S}}\right] - \frac{M}{S}\left[\frac{1}{\gamma}\left(\frac{1}{p_1}\right) + (S-1)\left(\frac{1}{p_{s>1}}\right)\right]\right\} - \chi\psi
\end{aligned}
$$

The equilibrium welfare functions $\mathcal{W}(M)$ in each regime are

A When no firm changes price $p_1^A = p_{s>1}^A = 1$, implying implies $P^A = S$. Substituting accordingly gives $C^A = Y^A = \frac{M}{S}$, $y_1^A = \frac{M}{S}$, $y_{s>1}^A = \frac{M}{S}$, $n_1^A = \frac{M}{\gamma S}$, $n_{s>1}^A = \frac{M}{S}$, and $N^A = \frac{M}{S}\left[\frac{1}{\gamma} + (S-1)\right]$. This implies further that welfare is

$$
\mathcal{W}^A(M) \quad = \quad 2\ln\left(\frac{M}{S}\right) - \frac{M}{S}\left[\frac{1}{\gamma} + (S-1)\right] \tag{25}
$$

B When all firms change price $p_1^B = \frac{M}{\gamma}$ and $p_{s>1}^B = M$, implying implies $P^B = SM\gamma^{-\frac{1}{S}}$. Substituting accordingly gives $C^B = Y^B = \frac{\gamma^{\frac{1}{S}}}{S}$, $y_1^B = \frac{\gamma}{S}$, $y_{s>1}^B = \frac{1}{S}$, $n_1^B = \frac{1}{S}$, $n_{s>1}^B = \frac{1}{S}$, and $N^B = 1$. This implies further that welfare is

$$
\mathcal{W}^B(M) \quad = \quad 2\ln\left(\frac{\gamma^{\frac{1}{S}}}{S}\right) - 1 - \psi \tag{26}
$$

C When only sector $s = 1$ firms change their price $p_1^C = \frac{M}{\gamma}$ and $p_{s>1}^C = 1$, implying implies $P^C = S\left(\frac{M}{\gamma}\right)^{\frac{1}{S}}$. Substituting accordingly gives $C^C = Y^C = \frac{M^{\frac{S-1}{S}}}{S}\gamma^{\frac{1}{S}}$, $y_1^C = \frac{\gamma}{S}$, $y_{s>1}^C = \frac{M}{S}$, $n_1^C = \frac{1}{S}$, $n_{s>1}^C = \frac{M}{S}$, and $N^C = \frac{1}{S} + \frac{S-1}{S}M$. This implies further that welfare is

$$
\mathcal{W}^C(M) \quad = \quad 2\ln\left(\frac{M^{\frac{S-1}{S}}\gamma^{\frac{1}{S}}}{S}\right) - \left[\frac{1}{S} + \frac{S-1}{S}M\right] - \frac{\psi}{S} \tag{27}
$$

D When only sectors $s > 1$ firms change their price $p_1^D = 1$ and $p_{s>1}^D = M$, implying that $P^D = S(M)^{\frac{S}{S-1}}$. Substituting accordingly gives $C^D = Y^D = \frac{M^{\frac{1}{S}}}{S}$, $y_1^D = \frac{M}{S}$, $y_{s>1}^D = \frac{1}{S}$, $n_1^D = \frac{M}{\gamma S}$, $n_{s>1} = \frac{1}{S}$, and $N = \frac{1}{S}\left[\frac{M}{\gamma} + (S-1)\right]$. This implies that welfare is

$$
\mathcal{W}^D \quad = \quad 2\ln\left[\frac{M^{\frac{1}{S}}}{S}\right] - \frac{1}{S}\left[\frac{M}{\gamma} + (S-1)\right] - \frac{S-1}{S}\psi \tag{28}
$$

$\square$

Claim A.4 (Optimal Policy within Regimes)**.** Here we list optimal welfare in each regime under the binding and non-binding constraints.

Proof. In each case we consider the optimal M and corresponding $\mathcal{W}$ both when the constraints bind and when they don't.

A Taking a derivative of $\mathcal{W}^A$ with respect to M gives

$$\max_M \ln\left(\frac{M}{S}\right) - \frac{M}{S}\left(S - 1 + \frac{1}{\gamma}\right)$$
$$\text{s.t.}\gamma\underline{\lambda} \le M \le \overline{\lambda}$$

implying for the interior optimum

$$M^A_{\text{int}} = \frac{S}{S - 1\frac{1}{\gamma}} \tag{29}$$

$$\mathcal{W}^A_{\text{int}} = -\ln\left(S - 1 + \frac{1}{\gamma}\right) - 1 \tag{30}$$

Next we check the boundaries.

Regime A: upper bound. The upper bound binds whenever

$$\frac{S}{S - 1 + \frac{1}{\gamma}} > \overline{\lambda}$$

Note that the denominator is positive since we assume $\gamma > 1$ and $S > 1$. Thus we can rearrange

$$S > \overline{\lambda}S - \overline{\lambda} + \frac{\overline{\lambda}}{\gamma}$$
$$\Rightarrow S(1 - \overline{\lambda}) + \overline{\lambda} > \frac{\overline{\lambda}}{\gamma}$$

There are now two cases:

(a) $\boldsymbol{S\left(1 - \overline{\lambda}\right) + \overline{\lambda} \le 0}$. But this can never happen since $\gamma > 1, \overline{\lambda} > 1$. If this is ever the case the upper bound of the constraint does not bind.

(b) $\boldsymbol{S\left(1-\overline{\lambda}\right)+\overline{\lambda}>0}$. Rearranging gives

$$\gamma > \frac{\overline{\lambda}}{S\left(1-\overline{\lambda}\right)+\overline{\lambda}}$$

Thus, the upper bound binds if:

$$\begin{aligned} \gamma &> \overline{\gamma}_A & S\left(1-\overline{\lambda}\right)+\overline{\lambda}>0 \\ \overline{\gamma}_A &\equiv \frac{\overline{\lambda}}{S\left(1-\overline{\lambda}\right)+\overline{\lambda}} \end{aligned} \tag{31}$$

In this case, when $\gamma > \overline{\gamma}_A$ and $S\left(1-\overline{\lambda}\right)+\overline{\lambda}>0$, then

$$M_{ub}^{A} = \overline{\lambda} \tag{32}$$

$$\mathcal{W}_{ub}^{A} = \ln\left[\frac{1}{S}\overline{\lambda}\right] - \frac{1}{S}\overline{\lambda}\left[S-1+\frac{1}{\gamma}\right] \tag{33}$$

Recall however, that for regime A to even be feasible, it is necessary for $\gamma < \gamma_0 = \frac{\overline{\lambda}}{\underline{\lambda}}$. Next we check wether this upper bound $\overline{\gamma}_A < \gamma_0$, if not, given for the constraint to bind $\gamma > \overline{\gamma}_A$, it cannot be that the constraint binds.

$$\begin{aligned} \overline{\gamma}_A &< \gamma_0 \\ \iff \frac{\overline{\lambda}}{S(1-\overline{\lambda})+\overline{\lambda}} &< \frac{\overline{\lambda}}{\underline{\lambda}} \\ \iff \frac{1}{S(1-\overline{\lambda})+\overline{\lambda}} &< \frac{1}{\underline{\lambda}} \\ \iff \underline{\lambda} &< S(1-\overline{\lambda})+\overline{\lambda} \\ \iff S\left(\overline{\lambda}-1\right) &< \overline{\lambda}-\underline{\lambda} \end{aligned}$$

We show that this inequality never holds (and hence the upper bound is never binding) by evaluating this expression for $\eta = 2$, $\eta \to \infty$ and considering how the inequality behaves for intermediate η's.

$\eta = 2$ In this case we have that $\overline{\lambda} = 1+\sqrt{\psi}$ and $\underline{\lambda} = 1-\sqrt{\psi}$. Thus the inequality of interest becomes

$$S\sqrt{\psi} < 2\sqrt{\psi}$$

which does not hold for $S \geq 2$. In particular, the difference $S\left(\overline{\lambda}-1\right)-\overline{\lambda}+\underline{\lambda} >$

0

$\eta \to \infty$ Sending $\eta \to \infty$ in equation (22) delivers

$$\left(\frac{W/A_i}{p_{[-1]}}\right)^{\eta}\left(\frac{1}{\frac{W/A_i}{p_{[-1]}}}-1\right)=0$$

and so there is a unique $\lambda=\overline{\lambda}=\underline{\lambda}=1$. This means that $S\left(\overline{\lambda}-1\right)-\overline{\lambda}+\underline{\lambda}=0$

$\eta \in (2,\infty)$ Next we show that $\frac{\partial}{\partial\eta}\left[S\left(\overline{\lambda}-1\right)-\overline{\lambda}+\underline{\lambda}\right]=(S-1)\frac{\partial\overline{\lambda}}{\partial\eta}+\frac{\partial\underline{\lambda}}{\partial\eta}<0$, that is it is monotonically decreasing in η and could never go below 0 since it is identically 0 for $\eta\to\infty$. To do so, apply the implicit function theorem on equation (22). Define

$$F(\lambda,\eta)=\lambda^{\eta}\left[\frac{1}{\lambda}-\frac{\eta-1}{\eta}\right]-\frac{1}{\eta}(1-\psi)$$

and so

$$\frac{\partial F}{\partial\lambda}=-(\lambda-1)\lambda^{\eta-2}(\eta-1)$$

and

$$\begin{aligned}\frac{\partial F}{\partial\eta}&=\frac{1-\psi}{\eta^2}+\left(\frac{\eta-1}{\eta^2}-\frac{1}{\eta}\right)\lambda^{\eta}+\left(\frac{1}{\lambda}-\frac{\eta-1}{\eta}\right)\lambda^{\eta}\log(\lambda)\\&=\frac{1-\psi}{\eta^2}-\frac{1}{\eta^2}\lambda^{\eta}+\left(\frac{1}{\lambda}-\frac{\eta-1}{\eta}\right)\lambda^{\eta}\log(\lambda)\end{aligned}$$

The implicit function theorem implies

$$\frac{\partial\lambda}{\partial\eta}=-\frac{\frac{\partial F}{\partial\eta}}{\frac{\partial F}{\partial\lambda}}=\lambda^{(2-\eta)}\cdot\frac{\frac{1-\psi}{\eta^2}-\frac{1}{\eta^2}\lambda^{\eta}+\left(\frac{1}{\lambda}-\frac{\eta-1}{\eta}\right)\lambda^{\eta}\log(\lambda)}{(\lambda-1)\cdot(\eta-1)}\tag{34}$$

All that is now needed is for $(S-1)\frac{\partial\overline{\lambda}}{\partial\eta}+\frac{\partial\underline{\lambda}}{\partial\eta}<0$. To show this follow the steps below:

i. First show that $\overline{\lambda}>1$ and $\underline{\lambda}<1$, $\frac{\partial\overline{\lambda}}{\partial\eta}<0$ and $\frac{\partial\underline{\lambda}}{\partial\eta}>0$.
ii. Second, show that $|\overline{\lambda}-1|\leq|1-\underline{\lambda}|$, (see `https://www.desmos.com/calculator/cbikavhjxq`: the red line is further away from 1 to the right of 1)
iii. Third, note that $|\frac{\partial\overline{\lambda}}{\partial\eta}|>|\frac{\partial(2-\underline{\lambda})}{\partial\eta}|>|\frac{\partial\underline{\lambda}}{\partial\eta}|$, (see `https://www.desmos.com/calculator/kzwzjepvuq`: the green line is above 0)
iv. This means that for any η,ψ, we have that $\frac{\partial\overline{\lambda}}{\partial\eta}+\frac{\partial\underline{\lambda}}{\partial\eta}<0$ and so, since $S\geq 2$, this implies $(S-1)\frac{\partial\overline{\lambda}}{\partial\eta}+\frac{\partial\underline{\lambda}}{\partial\eta}<0$.

We thus have that at $S\left(\overline{\lambda}-1\right)-\overline{\lambda}+\underline{\lambda}\geq 0$ for all value $\eta\geq 2$ and thus the $\overline{\gamma}_A>\gamma_0$, in turn meaning the upper bound of the constraint can never hold.

With this we conclude that the upper bound is never binding.

Regime A: lower bound. For the lower bound, noting $\underline{\lambda}$ is independent of γ, this binds if $\gamma>\underline{\gamma}_A$, where $\underline{\gamma}_A$ solves $M^A_{int}(\gamma)=\gamma\underline{\lambda}$, i.e.:

$$\underline{\gamma}_A\equiv\frac{\frac{1}{\underline{\lambda}}S-1}{S-1} \tag{35}$$

In this case when $\gamma>\underline{\gamma}_A$

$$M^A_{lb}=\gamma\underline{\lambda} \tag{36}$$

$$\mathcal{W}^A_{lb}=\ln\left[\frac{1}{S}\gamma\underline{\lambda}\right]-\frac{1}{S}\gamma\underline{\lambda}\left[S-1+\frac{1}{\gamma}\right] \tag{37}$$

B In regime B everyone changes price so welfare is independent of monetary policy. Modulo the menu costs, this case is identical to the flexible price scenario. In particular

$$\begin{aligned}\mathcal{W}^B&=\left\{\ln\left[\frac{1}{S}\gamma^{1/S}\right]-1\right\}(1-\psi)\\&=\mathcal{W}^f-\psi\end{aligned} \tag{38}$$

C The central bank's problem is:

$$\begin{aligned}&\max_M\left\{\ln\left[\frac{1}{S}\gamma^{\frac{1}{S}}\cdot M^{\frac{S-1}{S}}\right]-\left[\frac{S-1}{S}M+\frac{1}{S}\right]\right\}-\frac{1}{S}\psi\\&\text{s.t. }\ \underline{\lambda}<M<\min(\gamma\underline{\lambda},\overline{\lambda})\end{aligned}$$

The interior optimum is:

$$M^C_{int}=1 \tag{39}$$

$$\mathcal{W}^C_{int}=\mathcal{W}^f-\frac{1}{S}\psi \tag{40}$$

The interior optimum in Regime C implements nominal wage targeting: $W(M^C_{int})=W^{ss}$. NGDP is also constant.

Regime C: lower bound. The lower bound $\underline{\lambda} < 1$ and so never binds.

Regime C: upper bound. Additionally, if $\gamma > \gamma_0$ so that the upper bound is $\gamma\overline{\lambda} > 1$, then the upper bound does not bind. However, if $\gamma < \gamma_0$, then the upper bound may bind: if $\gamma < \gamma_C$, where

$$\gamma_C \equiv 1/\underline{\lambda} \tag{41}$$

In which case

$$M^C_{ub} = \gamma\underline{\lambda} \tag{42}$$

$$\mathcal{W}^C_{ub} = \ln\left[\frac{1}{S}\gamma\underline{\lambda}^{\frac{S-1}{S}}\right] - \left[\frac{S-1}{S}\gamma\underline{\lambda} + \frac{1}{S}\right] - \frac{1}{S}\psi \tag{43}$$

D The central bank's problem is:

$$\max_M \left\{\ln\left[\frac{1}{S}M^{\frac{1}{S}}\right] - \left[\frac{S-1}{S} + \frac{1}{S}\frac{M}{\gamma}\right]\right\} - \frac{S-1}{S}\psi$$
$$\text{s.t.} \quad \max(\gamma\underline{\lambda}, \overline{\lambda}) < M < \gamma\overline{\lambda}$$

The interior optimum is:

$$M^D_{int} = \gamma \tag{44}$$

$$\mathcal{W}^D_{int} = \mathcal{W}^f - \frac{S-1}{S}\psi \tag{45}$$

Regime D: upper bound. The upper bound $\gamma\overline{\lambda} > \gamma$ never binds, being $\overline{\lambda} > 1$.

Regime D: lower bound. If $\gamma > \gamma_0$ so that the lower bound is $\gamma\underline{\lambda} < \gamma$, then the lower bound does not bind. However, if $\gamma < \gamma_0$, then the lower bound may bind: if $\gamma < \gamma_D$, where

$$\gamma_D \equiv \overline{\lambda}$$

In which case

$$M^D_{lb} = \overline{\lambda} \tag{46}$$

$$\mathcal{W}_{lb}^{D} = \ln\left[\frac{\overline{\lambda}^{1/S}}{S}\right] - \left[\frac{S-1}{S} + \frac{1}{S}\frac{\overline{\lambda}}{\gamma}\right] - \frac{S-1}{S}\psi \tag{47}$$

□

Lemma 5 (C dominates D and B)**.** C-interior strictly dominates B and D (for $S > 2$). In other words: if $\gamma > \gamma_C$, then B and D should never be implemented.

Proof. This is immediate from inspection of equations (38, 40, and 45). Intuitively, they all lead to undistorted relative prices, the $\mathcal{W}^f$ term, but with different menu cost expenditures, namely ψ in regime B, $\frac{\psi}{S}$ in regime C, and $\frac{S-1}{S}\psi$ in regime D. □

Lemma 6 (C vs. A)**.** Fixing other parameters, there is a threshold level for the productivity shock γ^*, below which A-interior dominates C-interior and above which C-interior dominates A-interior. Furthermore, this γ^* is strictly increasing in ψ: the larger the menu cost, the higher the productivity shock that is required for it to be optimal to have prices change[28].

Proof. Define the welfare difference between A-interior and C-interior as a function of productivity by $f(\gamma)$:

$$\begin{aligned} f(\gamma) &= \mathbb{W}^A\left(M_{int}^A\right) - \mathbb{W}^C\left(M_{int}^C\right) \\ &= \left\{-\ln\left[S-1+\frac{1}{\gamma}\right]-1\right\} - \left\{\ln\left[\frac{1}{S}\gamma^{1/S}\right]-1-\frac{1}{S}\psi\right\} \\ &= -\ln\left[S-1+\frac{1}{\gamma}\right] - \ln\left[\frac{1}{S}\gamma^{1/S}\right] + \frac{1}{S}\psi \end{aligned} \tag{48}$$

Observe:

1. If no productivity shock, $\gamma = 1$, then $f(\cdot)$ is positive, as long as menu cost is nonzero: regime A economizes on menu costs and dominates regime C

$$f(1) = \frac{1}{S}\psi$$

2. As $\gamma \to \infty$, then $f(\gamma) \to -\infty$, i.e. regime C dominates: the welfare cost of distorted prices becomes very large

$$\lim_{\gamma\to\infty} f(\gamma) = -\ln(S-1) - \lim_{\gamma\to\infty}\left\{\ln\left[\frac{1}{S}\gamma^{1/S}\right]\right\} + \frac{1}{S}\psi \to -\infty$$

[28]This is also invertible: fixing γ, there is a threshold value $\psi^*(\gamma)$ above which A-interior dominates C-interior and vice versa; and $\psi^*(\gamma)$ is increasing in γ.

3. f is continuous in γ (on the domain $\gamma > 1$)

4. Note the welfare difference $f(\cdot)$ is strictly monotonically decreasing in the productivity shock γ, i.e. $f'(\gamma) < 0$:

$$f'(\gamma) = \frac{1}{\gamma^2}\frac{1}{S-1+\frac{1}{\gamma}} - \frac{1}{S}\frac{1}{\gamma}$$
$$= \frac{1}{\gamma}\left[\frac{1}{1+\gamma(S-1)} - \frac{1}{S}\right]$$
$$< 0$$

5. Given that $f(\gamma)$ is continuous in γ, by the intermediate value theorem, there exists some $\gamma^* \in (1, \infty)$ such that $f(\gamma^*) = 0$ and welfare under the two regimes is the same.

6. To see that γ^* is increasing in ψ, observe that (for $S > 2$), increasing ψ shifts the entirety of $f(\cdot)$ up:

$$\frac{\partial f}{\partial \psi} = \frac{1}{S} > 0$$

Thus we have proved all the claims in the lemma. □

Lemma 7 (Properties of $\gamma^*, \gamma_0, \gamma_C, \underline{\gamma}_A, \gamma_D$)**.** The ordering delimiting γ_0, γ_A, and γ_C (and the respective regimes), is given by the following information:

γ^* : When $\psi = 0$, the threshold $\gamma^* = 1$. When $\psi = 1$, the threshold γ^* is *finite*.

γ_0 : $\gamma_0(\psi = 0) = 1/1 = 1$, $\gamma_0(\psi = 1) = \frac{\eta}{\eta-1}/0 = \infty$

γ_C : $\gamma_C(\psi = 0) = 1/1 = 1$, $\gamma_C(\psi = 1) = 1/0 = \infty$

$\underline{\gamma}_A$: $\underline{\gamma}_A(\psi = 0) = \frac{S-1}{S-1} = 1$, $\underline{\gamma}_A(\psi = 1) = \frac{\frac{1}{0}S-1}{S-1} = \infty$, $\underline{\gamma}_A > \gamma_C$, $\underline{\gamma}_A < \gamma_0$

Proof. γ^* :

(a) When $\psi = 0$, then $f = -\ln[S - 1 + 1/\gamma] - \ln[(1/S)\gamma^{1/S}] < 0$. We know from proposition 2 that this is decreasing in γ . And note that $f(\psi = 0, \gamma = 1) = 0$, implying that $\gamma^* = 1$ if $\psi = 0$. So f evaluated at $\psi = 0$ is negative for $\gamma > 1$, that is $f(\psi = 0, \gamma > 1) < 0$.

(b) To see that $\gamma^* < \infty$ when $\psi = 1$,

$$\equiv f(\psi = 1) = -\ln\left[S - 1 + \frac{1}{\gamma}\right] - \ln\left[\frac{1}{S}\gamma^{1/S}\right] + \frac{1}{S}$$

The fact that there is a finite γ that solves this is clear looking at the above expression:

i. Let $f_1(\gamma) = -\ln[S - 1 + 1/\gamma]$ and $f_2(\gamma) = -\frac{1}{S}\ln(\gamma) + \ln(S) + \frac{1}{S}$

ii. f_1 is increasing in γ while f_2 is decreasing

iii. $f_1(1) = -\ln(S) < f_2(1) = \ln(S) + 1/S$

iv. $f_1(\infty) = -\ln(S-1) > f_2(\infty) \to -\infty$

v. Thus f_1 and f_2 intersect

γ_0 : Follows immediately from the analogous properties on $\underline{\lambda}(\psi), \overline{\lambda}(\psi)$

γ_C : Follows immediately from the analogous properties on $\underline{\lambda}(\psi)$ and definitions of γ_0, γ_C

γ_A : Properties on ψ follow immediately from properties on $\underline{\lambda}(\psi)$. To prove that $\underline{\gamma}_A > \gamma_C$, note

$$\begin{aligned}
\underline{\gamma}_A = \frac{\frac{1}{\underline{\lambda}}S - 1}{S-1} &> \frac{1}{\underline{\lambda}} = \gamma_C \\
\frac{1}{\underline{\lambda}}S - 1 &> \frac{1}{\underline{\lambda}}(S-1) \\
-1 &> -1/\underline{\lambda}
\end{aligned}$$

which holds since $\underline{\lambda} \in (0,1)$.
To prove that $\underline{\gamma}_A < \gamma_0$ also holds, we again use the variation in η to prove it. For $\eta = 2$ we have $\underline{\lambda} = 1 - \sqrt{\psi}$ while $\overline{\lambda} = 1 + \sqrt{\psi}$. Then the expression becomes $\frac{\frac{S}{1-\sqrt{\psi}} - 1}{S-1} < \frac{1+\sqrt{\psi}}{1-\sqrt{\psi}}$ which boils down to $S > 2$ which is an initial assumption made so this is indeed true. For $\eta \to \infty$ we have that $\gamma_0 = 1$ and $\underline{\gamma}_A = 1$ and so $\underline{\gamma}_A = \gamma_0$. Next we show that for $\eta \in (2, \infty)$ that $\gamma_0 - \underline{\gamma}_A$ is monotonically decreasing. This expression becomes

$$\gamma_0 - \underline{\gamma}_A = \frac{1}{S-1}\frac{1}{\underline{\lambda}}\left((S-1)\overline{\lambda} - S + \underline{\lambda}\right)$$

But, recall, we already showed that $\frac{\partial \overline{\lambda}}{\partial \eta} + \frac{\partial \underline{\lambda}}{\partial \eta} < 0$ (see proof to Claim 0.4). Furthermore, we showed that $\frac{\partial \underline{\lambda}}{\partial \eta} > 0$. Thus the second factor, $\frac{1}{\underline{\lambda}}$ is decreasing while the term $(S-1)\overline{\lambda} + \underline{\lambda}$ is also decreasing, making the entire expression decreasing (monotonically) in η. Since at $\eta \to \infty$ this expression equals 0 and at $\eta = 2$ it is positive, it must be so along the entire interval $(2, \infty)$.

□

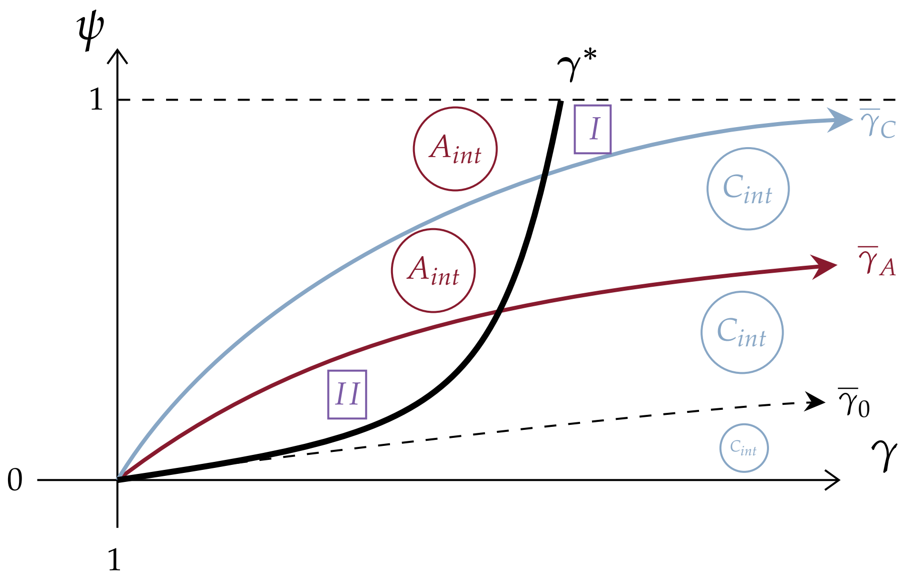


Figure 2: Segmentation of space.

These results allow us to depict the segmentation of the (γ, ψ) space as in Figure 2. As argued above, there are two regions however in which the central bank is constrained and cannot implement the interior solution, we call these regions I and II. Next we make a simple and reasonable parameter restriction that allows us to ignore region I.

Proposition 8 (ruling out regime I)**.** There exists $\overline{\psi} > 0$ such that as long $\psi < \overline{\psi}$ then the interior optimum of regime C is always feasible *when* regime C dominates regime A. $\overline{\psi} \approx 0.47$ and is exactly the solution to

$$-\ln\left[S - 1 + \underline{\lambda}(\psi)\right] = -\ln(S) - \frac{1}{S}\ln\left(\underline{\lambda}(\psi)\right) - \frac{\psi}{S} \tag{49}$$

Proof. Regime C dominates regime A when $f(\gamma) \leq 0$; and the regime C interior optimum is feasible when $\gamma > \gamma_C = 1/\underline{\lambda}$. Then, $\overline{\psi}$ will be defined as the menu cost that leaves the planner indifferent between regimes C and A when productivity is at its lowest possible value consistent with feasibility of the interior optimum of regime C. That is, $\overline{\psi}$ is implicitly defined by $f(\gamma_C = 1/\underline{\lambda}) = 0$, where $f(\cdot) = 0$ requires:

$$-\ln\left[S - 1 + 1/\gamma\right] - 1 = \ln\left[\frac{1}{S}\gamma^{1/S}\right] - 1 - \frac{\psi}{S}$$

Consider first the case for $\eta = 2$ which will implicitly define $\overline{\psi}$ solving the following system

$$\begin{cases} -\ln\left[S - 1 + 1/\gamma\right] - 1 & = \ln\left[\frac{1}{S}\gamma^{1/S}\right] - 1 - \frac{\psi}{S} \\ \gamma = \frac{1}{1-\sqrt{\psi}} \end{cases}$$

Start by simplifying the first of these

$$\begin{aligned}
\ln\left(\frac{1}{S}\gamma^{\frac{1}{S}}\right) + \ln\left(S - 1 + \frac{1}{\gamma}\right) &= \frac{\psi}{S} \\
&\text{plugging in for } \gamma = \frac{1}{1-\sqrt{\psi}} \\
\ln\left(\frac{1}{S}\left(\frac{1}{1-\sqrt{\psi}}\right)^{\frac{1}{S}}\right) + \ln\left(S - \sqrt{\psi}\right) &= \frac{\psi}{S} \\
&\text{using the properties of logs} \\
\frac{1}{S}\ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \ln\left(\frac{1}{S}\left(S - \sqrt{\psi}\right)\right) &= \frac{\psi}{S} \\
&\text{multiplying by } \frac{S}{S} \\
\frac{1}{S}\ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \frac{1}{S}\ln\left(\frac{1}{S}\left(S - \sqrt{\psi}\right)\right)^{S} &= \frac{\psi}{S} \\
\implies \ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \ln\left(\frac{1}{S}\left(S - \sqrt{\psi}\right)\right)^{S} &= \psi \\
&\text{adjusting the second addend}
\end{aligned}$$

$$\implies \ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \ln\left(1-\frac{\sqrt{\psi}}{S}\right)^S \; = \; \psi$$

Next, consider this expression when $S \to \infty$.

$$\begin{aligned}
\lim_{S\to\infty} \quad & \ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \ln\left(1-\frac{\sqrt{\psi}}{S}\right)^S = \psi \\
& \ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \ln\left(\lim_{S\to\infty} 1-\frac{\psi}{S}\right)^S = \psi \\
& \ln\left(\frac{1}{1-\sqrt{\psi}}\right) - \sqrt{\psi} = \psi
\end{aligned}$$

whose solution we define as $\overline{\psi} \approx 0.47$. Next we show that, around this second root $\overline{\psi}$, ψ such that this holds is decreasing in S using the Implicit Function theorem applied to $F(\psi, S) = \ln\left(\frac{1}{1-\sqrt{\psi}}\right) + \ln\left(1-\frac{\sqrt{\psi}}{S}\right)^S - \psi$ or, better yet, for ease of derivation, denoting $x = \sqrt{\psi}$, applied to

$$F(x,S) = \ln\left(\frac{1}{1-x}\right) + \ln\left(1-\frac{x}{S}\right)^S - x^2$$

Its derivatives are

$$\begin{cases} \frac{\partial F}{\partial S} & = \ln\left(1-\frac{x}{S}\right) + \frac{x}{S-x} \\ \frac{\partial F}{\partial x} & = \frac{1}{1-x} - \frac{S}{S-x} - 2x \end{cases}$$

Note that $\frac{\partial F}{\partial S} \geq 0$ always holds. The second, $\frac{\partial F}{\partial x} \geq 0$ always holds if $x \geq \overline{\psi}^2$[29]. This means that $\frac{\partial x}{\partial S} = -\frac{\frac{\partial F}{\partial S}}{\frac{\partial F}{\partial x}} < 0$, implying that the root ψ of $F(\psi, S)$ is decreasing in S. This means that the smallest ψ such that $F(\psi, S) > 0$ is $\overline{\psi} \approx 0.47$, being this defined for $S \to \infty$. Restricting $\psi < \overline{\psi}$, a most reasonable restriction given empirical estimates, there is thus no Region I to worry about for $\eta = 2$. Next we generalize this result to $\eta > 2$ by showing that the threshold ψ such that

$$\ln\left(\frac{\gamma^{\frac{1}{S}}}{S}\right) + \ln\left(S-1+\frac{1}{\gamma}\right) = \frac{\psi}{S}$$

[29] One could iteratively show that $\frac{\partial x}{\partial S} < 0$ for the non-zero root by evaluating the derivative $\frac{\partial F}{\partial x}$ as S decreases

is greater than $\overline{\psi}$ if $\eta > 2$. Again, denote by $F(\psi, \eta)$ the residual function, that is

$$F(\psi, \eta) = \ln\left(\frac{\gamma^{\frac{1}{S}}(\psi, \eta)}{S}\right) + \ln\left(S - 1 + \frac{1}{\gamma(\psi, \eta)}\right) - \frac{\psi}{S}$$

We need to confirm that $\frac{\partial\psi}{\partial\eta} > 0$, that is the root ψ increases as η increases. First compute the individual partial derivatives

$$\begin{aligned}
\frac{\partial F(\psi,\eta)}{\partial\eta} &= \frac{1}{S}\frac{1}{\gamma(\psi,\eta)}\frac{\partial\gamma(\psi,\eta)}{\partial\eta} - \frac{1}{S-1+\frac{1}{\gamma(\psi,\eta)}}\left(\frac{\frac{\partial\gamma(\psi,\eta)}{\partial\eta}}{(\gamma(\psi,\eta))^2}\right) \\
&= \frac{1}{\gamma(\psi,\eta)}\frac{\partial\gamma(\psi,\eta)}{\partial\eta}\left[\frac{1}{S} - \frac{1}{\gamma(\psi,\eta)(S-1)+1}\right]
\end{aligned}$$

and

$$\frac{\partial F(\psi,\eta)}{\partial\psi} = \frac{1}{\gamma(\psi,\eta)}\frac{\partial\gamma(\psi,\eta)}{\partial\psi}\left[\frac{1}{S} - \frac{1}{\gamma(\psi,\eta)(S-1)+1}\right] - \frac{1}{S}$$

and so, by the implicit function theorem

$$\frac{\partial\psi}{\partial\eta} = -\frac{\frac{\partial F(\psi,\eta)}{\eta}}{\frac{\partial F(\psi,\eta)}{\partial\psi}} = -\frac{\frac{1}{\gamma(\psi,\eta)}\frac{\partial\gamma(\psi,\eta)}{\partial\eta}\left[\frac{1}{S} - \frac{1}{\gamma(\psi,\eta)(S-1)+1}\right]}{\frac{1}{\gamma(\psi,\eta)}\frac{\partial\gamma(\psi,\eta)}{\partial\psi}\left[\frac{1}{S} - \frac{1}{\gamma(\psi,\eta)(S-1)+1}\right] - \frac{1}{S}} \tag{50}$$

Taking one step at a time, consider the case of $S \to \infty$, then this expression becomes

$$\left.\frac{\partial\psi}{\partial\eta}\right|_{S\to\infty} = -\frac{\frac{\partial\gamma(\psi,\eta)}{\partial\eta}}{\frac{\partial\gamma(\psi,\eta)}{\partial\psi}} \tag{51}$$

Because here $\gamma = \frac{1}{\lambda}$ and we know from earlier that $\frac{\partial\lambda}{\partial\eta} > 0$, we have that the numerator $\frac{\partial\gamma(\psi,\eta)}{\partial\eta} < 0$. To say something about the denominator we must look at $\frac{\partial\lambda}{\partial\psi}$. Recall from 22 this is implicitly defined by the function

$$F(\lambda, \psi) = -\frac{\eta-1}{\eta}\lambda^{\eta} - \lambda^{\eta-1} - \frac{1}{\eta}(1-\psi)$$

and, using the implicit function theorem implies

$$\frac{\partial \lambda}{\partial \psi} = -\frac{\frac{\partial F(\lambda,\psi)}{\partial \psi}}{\frac{\partial F(\lambda,\psi)}{\partial \lambda}}$$

We have already computed

$$\frac{\partial F}{\partial \lambda} = -(\lambda - 1)\lambda^{\eta-2}(\eta - 1)$$

while it is easy to see that

$$\frac{\partial F}{\partial \psi} = \frac{1}{\eta}$$

implying that

$$\frac{\partial \lambda}{\partial \psi} = \frac{\frac{1}{\eta}}{(\lambda - 1)\lambda^{\eta-2}(\eta - 1)} < 0$$

given that $\eta \geq 2$ but $\underline{\lambda}$ is always less than 1. Since $\gamma = \frac{1}{\underline{\lambda}}$, this implies $\frac{\partial \gamma}{\partial \psi} > 0$ and in turn

$$\frac{\partial \psi}{\partial \eta} > 0$$

and so, at least as $S \to \infty$, the threshold ψ equating the value in regimes A and C is increasing in η, meaning that $\overline{\psi} \approx 0.47$ is still a valid lower bound to exclude regime I.

What about for $S < \infty$? Recall, in this case the relevant expression is

$$\frac{\partial \psi}{\partial \eta} \quad = \quad -\frac{\frac{\partial F(\psi,\eta)}{\eta}}{\frac{\partial F(\psi,\eta)}{\partial \psi}} = -\frac{\frac{1}{\gamma(\psi,\eta)}\frac{\partial \gamma(\psi,\eta)}{\partial \eta}\left[\frac{1}{S} - \frac{1}{\gamma(\psi,\eta)(S-1)+1}\right]}{\frac{1}{\gamma(\psi,\eta)}\frac{\partial \gamma(\psi,\eta)}{\partial \psi}\left[\frac{1}{S} - \frac{1}{\gamma(\psi,\eta)(S-1)+1}\right] - \frac{1}{S}}$$

While we have computed $\frac{\partial \gamma}{\partial \eta}$ and $\frac{\partial \gamma}{\partial \psi}$, we have to determine the signs of the other components. Clearly $\frac{1}{\gamma} > 0$ (and in fact we also know that it will be less than 1 since $\gamma > 1$). Also note, because $\gamma > 1$, that $\gamma(S-1) + 1 > S$ and so $\frac{1}{\gamma(\psi,\eta)(S-1)+1} > 0$. This is sufficient to say that the whole numerator of this expression is still overall negative. For the denominator, apply the chain rule first to determine $\frac{\partial \gamma}{\partial \psi}$, given that $\gamma(\psi) = \frac{1}{\lambda(\psi)}$.

$$\frac{\partial \gamma}{\partial \psi} \quad = \quad -\left(\frac{1}{\lambda(\psi)^2}\right)^2 \frac{\partial \lambda}{\partial \psi}$$

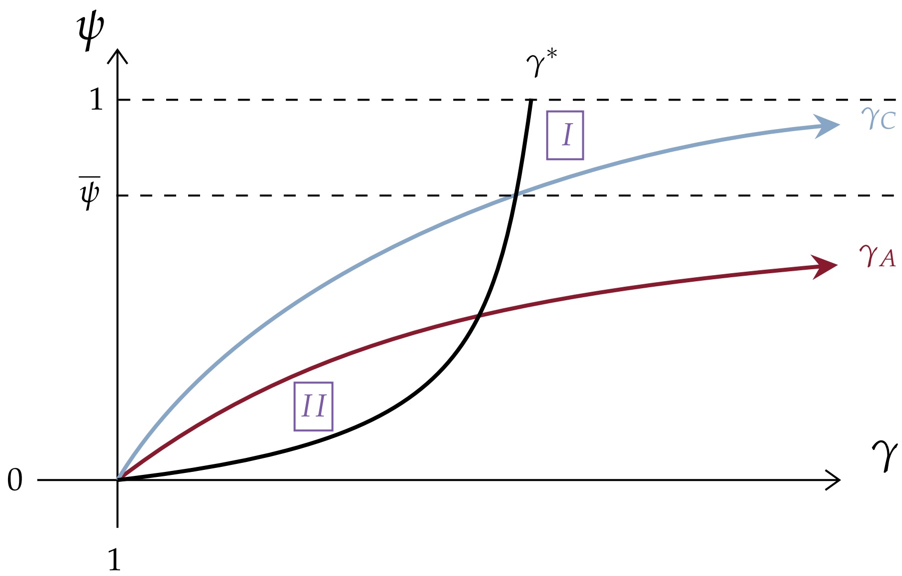


Figure 3: Space segmentation with $\overline{\psi}$

$$
\begin{aligned}
&= -\frac{1}{\eta}\frac{1}{\lambda^2}\frac{1}{(\lambda-1)\lambda^{\eta-2}(\eta-1)} \\
&= -\frac{1}{\eta}\frac{1}{(\lambda-1)\lambda^{\eta}(\eta-1)}
\end{aligned}
$$

and so the denominator in the expression for $\frac{\partial\psi}{\partial\eta}$ becomes

$$
-\frac{1}{\eta}\frac{1}{(\lambda-1)\lambda^{\eta}(\eta-1)}\frac{1}{\gamma}\left[\frac{1}{S}-\frac{1}{\gamma(S-1)+1}\right]-\frac{1}{S}
$$

which, it can be verified, is always positive for $\gamma > 1$, $S > 3$, $\lambda < 1$, and $\eta > 2$.

This means that also the denominator is still positive which in turn means $\frac{\partial\psi}{\eta} > 0$ regardless of S. This allows us to conclude that $\overline{\psi} \approx 0.47$ is a sufficient lower bound to exclude region I. The new segmentation of the space is shown in Figure (3).

□

Lemma 9 (Region II Threshold)**.** Suppose (γ, ψ) are in region 2 and that furthermore $W^C(\gamma, \psi) > W^{\partial A}(\gamma, \psi)$ then, $\forall \gamma' > \gamma$ such that (γ', ψ) is in region 2, we have that $W^C(\gamma', \psi) > W^{\partial A}(\gamma', \psi)$.

Proof. First recall that:

$$\begin{aligned} W^{\partial A}(\gamma,\psi) &= \ln\left(\frac{1}{S}\gamma\left(1-\sqrt{\psi}\right)\right)-\frac{1}{S}\gamma\left(1-\sqrt{\psi}\right)\left(S-1+\frac{1}{\gamma}\right) \\ W^{C}(\gamma,\psi) &= \ln\left(\frac{1}{S}\gamma^{\frac{1}{S}}\right)-1-\frac{\psi}{S} \end{aligned}$$

Then consider $W^{C}(\gamma,\psi)-W^{\partial A}(\gamma,\psi)$ for a fixed ψ, which I denote $h(\gamma)$

$$\begin{aligned} h(\gamma) &= W^{C}(\gamma,\psi)-W^{\partial A}(\gamma,\psi) \\ &= \ln\left(\frac{1}{S}\gamma^{\frac{1}{S}}\right)-1-\frac{\psi}{S}-\ln\left(\frac{1}{S}\gamma\left(1-\sqrt{\psi}\right)\right)+\frac{1}{S}\gamma\left(1-\sqrt{\psi}\right)\left(S-1+\frac{1}{\gamma}\right) \\ &\text{taking the derivative wrt } \gamma \\ h'(\gamma) &= \frac{1}{S}\frac{1}{\gamma}-\frac{1}{\gamma}+\frac{1}{S}\left(1-\sqrt{\psi}\right)(S-1) \\ h'(\gamma) &= \frac{S-1}{S}\left[\left(1-\sqrt{\psi}\right)-\frac{1}{\gamma}\right] \end{aligned}$$

Note that $h'(\gamma)>0$ if and only if $1-\frac{1}{\gamma}>\sqrt{\psi}$.

Recall that in order for (γ,ψ) to be in region II, $\psi<\psi^{A}=\left[\frac{(\gamma-1)(S-1)}{1+\gamma(S-1)}\right]^{2}$. Notice further that:

$$\begin{aligned} 1-\frac{1}{\gamma}-\sqrt{\psi}^{A} &= 1-\frac{1}{\gamma}-\frac{(\gamma-1)(S-1)}{1+\gamma(S-1)} \\ &= \frac{\gamma-1}{\gamma}-\frac{(\gamma-1)(S-1)}{1+\gamma(S-1)} \\ &= (\gamma-1)\left[\frac{1}{\gamma}-\frac{S-1}{1+\gamma(S-1)}\right] \\ &= (\gamma-1)\frac{1}{\gamma\left[1+\gamma(S-1)\right]}>0 \end{aligned}$$

Because $\psi^{A}>\psi$ it must be that

$$1-\frac{1}{\gamma}>\sqrt{\psi^{A}}>\sqrt{\psi}$$

and so $h'(\gamma)>0$. This means that, indeed, even in region II, we have that for $\gamma(\psi)>\gamma^{*}(\psi)$ then regime C is optimal and otherwise regime A is. $\square$

Theorem 10 (Optimal Monetary Policy)**.** WRITE BETTER If γ is the productivity shock that hits sector 1 and the menu cost is $\psi<\overline{\psi}$, then there exist a threshold $\gamma^{*}(\psi)$ such that:

- if $\gamma \geq \gamma^*(\psi)$ the optimal monetary policy is nominal wage targeting,

- if $\gamma < \gamma^*(\psi)$ then the optimal monetary is for prices to remain fixed. Note that, because $\gamma < \gamma^*(\psi)$, there is only a small change in productivity and optimal policy is approximately nominal wage targeting.

B Optimal policy when $\psi > \overline{\psi}$

The optimal policy characterized in Theorem 2 relies on assumption 2 that menu costs are below a maximum level, $\psi < \overline{\psi}$. What happens if this condition does not hold?

We first emphasize that this possibility is empirically irrelevant. The *only* parameter affecting $\overline{\psi}$ is the number of sectors S, with $\overline{\psi}$ decreasing in S. Under assumption 1, as $S \to \infty$, then $\overline{\psi}$ asymptotes to approximately 0.468 at its smallest. This is roughly an order of magnitude higher than any plausible calibration of model parameters would suggest (as discussed in further detail in section 5) and compared to what common sense would suggest.

If, however, it were the case that $\psi > \overline{\psi}$, then what would optimal policy be? Refer to figure **??**. If $\psi > \overline{\psi}$ but $\gamma < \gamma^*$, then the standard regime **A** rigid price equilibrium is still optimal; and if $\gamma > \gamma_C$, then the standard regime **C** equilibrium is still optimal with nominal wage stabilization. However, the region of parameter space denoted region **I** remains to be characterized.

In region **I**, we show in appendix XXX that there are further thresholds $\overline{\gamma}_{C1}$ and $\overline{\gamma}_{C2}$ with $\gamma^* \leq \overline{\gamma}_{C1} \leq \overline{\gamma}_{C2} \leq \gamma_C$. For $\gamma \in (\gamma^*, \overline{\gamma}_{C1}$, then the standard rigid price regime **A** equilibrium is optimal. For $\gamma \in (\overline{\gamma}_{C1}, \overline{\gamma}_{C2})$, then implementing the flex-price allocation via regime **D** is optimal. For $\gamma \in (\overline{\gamma}_{C2}, \gamma_C)$, then it is optimal to implement regime **C**, but because $M = 1$ is not incentive-compatible the central bank must cut the money supply to $M = \hat{M}_C < 1$ to induce firms in sector 1 to cut their price. Because firms in sector k remain stuck with their old price, the flex-price allocation is not achieved, but the distortion is less than had no sector adjusted price.

C Calvo World with Sectors

As in our model the first sector has a positive productivity shock γ while all others sectors maintain the same productivity. Here we follow Rubbo (her appendix).

C.1 Symmetric World

The inflation in sector i is:

$$\pi_1 = \delta\left(\frac{\log\gamma}{S} - \log\gamma\right) = \frac{\delta(1-S)}{S}\log\gamma$$

$$\pi_{s>1} = \delta\left(\frac{\log\gamma}{S} - 0\right) = \frac{\delta}{S}\log\gamma$$

And so

$$\begin{aligned} \pi &= \sum_{s=1}^{S}\frac{1}{S}\pi_s = \frac{\delta}{S}\log\gamma\left(\frac{1-S}{S} + \frac{1}{S} + \dots\frac{1}{S}\right) \\ &= \sum_{s=1}^{S}\frac{1}{S}\pi_s = \frac{\delta}{S}\log\gamma\left(\frac{1-S}{S} + \frac{S-1}{S}\right) \\ &= 0 \end{aligned}$$

And so in the symmetric Calvo scenario with a horizontal economy, the optimal policy is inflation targeting. But to confirm how this differs from ours, we need to look at how the nominal wage moves.

Because there is no inflation, nominal and real wages are the same. Thus the change in the nominal wage is nothing but

$$dw = \frac{1}{S}\log\gamma \tag{52}$$

This is a very different result from ours in which nominal wages are fixed so that the non-shocked sectors see no change in marginal cost. In the Cavlo model, on the other hand, nominal marginal costs change as follows in sector 1 and in sector $s > 1$:

$$\begin{aligned} dMC &= \frac{1-S}{S}\log\gamma \\ dMC &= \frac{1}{S}\log\gamma \end{aligned}$$

and so all sectors see changes in their nominal marginal costs and hence have, if allowed, the temptation to reset their prices.

C.2 Aoki World

Finally, we consider Aoki's model (nested within this framework) for S-many sectors. In the first version the shocked sector has flexible pricing, in the second version the shocked sector will have sticky pricing.

$\delta_1 = 1$

With $\delta_1 = 1$ and $\delta_{s>1} = \delta$, in Rubbo's notation, $\overline{\delta}_w = \frac{1+(S-1)\delta}{S}$ and $\overline{\delta}_A = 1$. The optimal inflation in each sector is

$$\pi_1 = -\log\gamma \tag{53}$$

$$\pi_{s>1} = 0 \tag{54}$$

The change in the nominal wage is 0, just as in our case. Thus, our intuition completely carries through in the Aoki world but for different reasons: we want to stabilize the nominal wages of the non-shocked sectors (in our case the largest mass of sectors), while in Aoki he wants to stabilize the prices of the sticky sector (and hence their marginal cost and, they not being hit by productivity shocks in our scenario, the aggregate wage).

$\delta_{s>1} = 1$

With $\delta_1 = \delta$ and $\delta_{s>1} = 1$, in Rubbo's notation, $\overline{\delta}_w = \frac{\delta+(S-1)}{S}$ and $\overline{\delta}_A = \delta$. The optimal inflation in each sector is

$$\pi_1 = 0 \tag{55}$$

$$\pi_{s>1} = \log\gamma \tag{56}$$

The change in the nominal wage is $\log\gamma$ which is not equal to 0 as in our result. This is because now the flexible price sector (the one which changes its price) is not subject to a productivity shock. So, if it wants to increase its price relative to sector 1, it must see a change in the nominal marginal cost (equal here to the nominal wage) that is just as big.