ECON 165, Review Section # 9

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Plan for Today

- Risk Aversion Recap
- UIP Deviation and Risk
- Practice
- Course Evaluation: Canvas, EvaluationKIT, or Axess

Risk Aversion Recap

- **Risk Neutral** agents
  - $*$ Indifferent towards risk
  - $*$ \$10 for sure $=$ \$5 with prob. $\frac{1}{2}$ and \$15 with probability $\frac{1}{2}$.
- **Risk Averse** agents
  - $*$ Do not like risk
  - $*$ \$10 for sure $>>$ \$5 with prob. $\frac{1}{2}$ and \$15 with probability $\frac{1}{2}$.
- E.g.

$$\mathbb{E}[u\,(\text{risky bet})] = \frac{1}{2}\left[\ln(5) + \ln(15)\right] = 2.16 < 2.30 = \ln(10) = u\,(\text{risless bet})$$

A UIP Model with Risky Investors

- Investors are risk averse with utility and their lifetime utility is

$$u\left(D_1\right) + \beta \mathbb{E} u\left(D_2(s)\right)$$

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$$u\left(D_1\right)+\beta \mathbb{E} u\left(D_2(s)\right)$$

- Investors have the option to invest in a domestic bond B_1 and a foreign one B_1^*. The bonds are risk-free and so they bear interest r and r^* respectively.

- Set up the investor's problem:
  - ∗ What is the objective function?
  - ∗ What do the investors y choose?
  - ∗ What are the budget constraints?

Risky UIP Setup

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$$\begin{aligned} \max_{B_1, B_1^*} \quad & u(D_1) + \beta \mathbb{E}\left[u(D_2(s))\right] \\ \text{s.t.} \quad & D_1 = Y_1 - B_1 - S_1 B_1^* \\ & D_2(s) = Y_2(s) + (1+r) B_1 + S_2(s)(1+r^*) B_1^* \end{aligned}$$

- FOCs?

Risky UIP Setup

$$
\begin{aligned}
\max_{B_1, B_1^*} \quad & u\left(D_1\right)+\beta \mathbb{E}\left[u\left(D_2(s)\right)\right] \\
\text{s.t.} \quad & D_1 = Y_1 - B_1 - S_1 B_1^* \\
& D_2(s) = Y_2(s) + (1+r) B_1 + S_2(s)\left(1+r^*\right) B_1^*
\end{aligned}
$$

- FOCs?

B_1 :

$$u'\left(D_1\right)=\beta(1+r) \mathbb{E}\left[u'\left(D_2(s)\right)\right]$$

B_2 :

$$S_1 u'\left(D_1\right)=\beta\left(1+r^*\right) \mathbb{E}\left[S_2(s) u'\left(D_2(s)\right)\right]$$

Deriving UIP Condition with Risk, pg. 1

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$$
\begin{aligned}
u'(D_1) &= \beta(1+r)\mathbb{E}\left[u'(D_2(s))\right] \\
S_1 u'(D_1) &= \beta(1+r^*)\mathbb{E}\left[S_2(s)u'(D_2(s))\right] \\
\implies 1 &= \beta(1+r)\mathbb{E}\left[\frac{u'(D_2(s))}{u'(D_1)}\right] := (1+\boldsymbol{r})\mathbb{E}\left[\boldsymbol{M(s)}\right] \\
\text{and } S_1 &= \beta(1+r^*)\mathbb{E}\left[S_2(s)\frac{u'(D_2(s))}{u'(D_1)}\right] := (1+\boldsymbol{r^*})\mathbb{E}\left[\boldsymbol{S_2(s)M(s)}\right] \\
\text{recall } & \quad Cov(X,Y) = \mathbb{E}[XY] - \mathbb{E}[X]\,\mathbb{E}[Y] \\
\implies \mathbb{E}\left[S_2(s)M(s)\right] &= Cov\left(S_2(s), M(s)\right) + \mathbb{E}\left[S_2(s)\right]\mathbb{E}\left[M(s)\right]
\end{aligned}
$$

Deriving UIP Condition with Risk, pg. 1

$$\begin{aligned}
\implies 1 &= (1+r)\mathbb{E}\left[M(s)\right] = (1+r^*)\,\frac{\mathbb{E}\left[S_2(s)M(s)\right]}{S_1} \\
(1+r) &= (1+r^*)\,\frac{\mathbb{E}\left[S_2(s)M(s)\right]}{S_1\mathbb{E}\left[M(s)\right]} \\
&\quad \text{substituting using the } Cov(\cdot) \text{ formula} \\
(1+r) &= (1+r^*)\,\frac{\mathbb{E}\left[S_2(s)\right]\mathbb{E}\left[M(s)\right] + Cov\left(S_2(s), M(s)\right)}{S_1\mathbb{E}\left[M(s)\right]} \\
(1+r) &= (1+r^*)\,\frac{\mathbb{E}\left[S_2(s)\right]}{S_1} + (1+r^*)\,\frac{Cov\left(S_2(s), M(s)\right)}{S_1\mathbb{E}\left[M(s)\right]} \\
(1+r) &= (1+r^*)\,\frac{\mathbb{E}\left[S_2(s)\right]}{S_1} + \frac{(1+r^*)}{S_1}Cov\left(\frac{S_2(s)}{S_1}, \frac{M(s)}{\mathbb{E}\left[M(s)\right]}\right)
\end{aligned}$$

Deriving UIP Condition with Risk, pg. 2

$$\underbrace{(1+r) = (1+r^*)\frac{\mathbb{E}\left[S_2(s)\right]}{S_1}}_{\text{UIP}} + Cov\left((1+r^*)\frac{S_2(s)}{S_1}, \frac{M(s)}{\mathbb{E}\left[M(s)\right]}\right)$$

Deriving UIP Condition with Risk, pg. 2

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Deriving UIP Condition with Risk, pg. 2

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- $(1+r^*)\frac{S_2(s)}{S_1}$: return to home lender investing abroad
- $\beta\frac{u'(D_2(s))}{u'(D_1)}$: value of \$1 tomorrow relative to today
  - if $D_2(s) < D_1 \Rightarrow$ this is large and vice versa
- if foreign return is low when $D_2(s)$ is low then $Cov(\cdot) < 0$ and $(1+r^*)\frac{\mathbb{E}[S_2(s)]}{S_1} > (1+r)$. Investor needs a higher return because the carry will tend to deliver lower interest when the she is most in need.

Practice # 1

Consider a risk-neutral investor that has the option to invest in a domestic bond with interest r and a foreign bond with interest r^*. The nominal exchange rate today is $\frac{1}{S_1}$ and tomorrow, because of uncertainty, it is $\frac{1}{S_2(s)}$ is state s. The cost of converting a unit of currency to another is c. What is the “new” UIP condition?

Practice # 1

Solution:

$$\max_{B_1, B_1^*} \quad D_1 + \beta \mathbb{E}\left[D_2(s)\right]$$

$$\text{s.t.} \quad D_1 = Y_1 - B_1 - \frac{S_1 B_1^*}{1-c}$$

$$\text{and} \quad D_2(s) = Y_2(s) + (1+r)B_1 + S_2(2)(1-c)(1+r^*)B_1^*$$

FOC B_1 :

$$\lambda_1 = \beta\lambda_2(1+r)$$

FOC B_1^* :

$$\lambda_1 \frac{S_1}{1-c} = \lambda_2 \beta \mathbb{E}\left[S_2(2)\right](1+r^*)(1-c)$$

$$(1+r) = (1+r^*)\frac{\mathbb{E}\left[S_2(2)\right]}{S_1}(1-c)^2$$

Practice # 2

Agents in countries A and B consume one unit a tradable good T at price 1\$ and one unit of a non-tradable good NT. In country A the income is \$100 while in country B the income is 50€. If moving countries and finding a job is costless, can you determine what the price of the NT good is in B if its price in country A is \$2 and the nominal exchange rate is 3\$ per 1€?

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Solution:

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Solution:

$$P^A = \frac{1}{2} \cdot 1\$ + \frac{1}{2} \cdot 2\$ = 1.5\$ \quad \text{and} \quad P^B = \frac{1}{2} \cdot \frac{1€}{3\$} \cdot 1\$ + \frac{1}{2} \cdot x€ = ?€$$

$$\begin{aligned} 100\$ &= e^{\text{PPP}} \cdot \frac{3\$}{€} \cdot 50€ \Rightarrow e^{\text{PPP}} = \frac{2}{3} \\ e^{\text{PPP}} = \frac{\frac{1}{S} P^B}{P^A} &= \frac{2}{3} \Rightarrow P^B = \frac{2}{3} \frac{3€}{1\$} \$1.5 = 3€ \\ \Rightarrow x &= 2\left(3€ - \frac{1}{6}€\right) = 5.6€ \end{aligned}$$

Speed Round

- If a country pegs its currency to the USD (e.g. Hong Kong), even with risk-averse investors the standard UIP condition hold.
- Inflation differences across countries are mostly due to differences in tradable inflation.
- Fixed (not per unit) costs cannot account for deviations in UIP.