

# Optimal monetary policy under menu costs

Daniele Caratelli	Basil Halperin
US Treasury, OFR	Stanford, DEL

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The views expressed are my own and do not necessarily reflect those of the OFR or the Department of Treasury.

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*Woodford 2003; Rubbo 2023*

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## **Criticism:**

- Theoretical critique: Not microfounded
- Empirical critique: State-dependent pricing is a better fit

► [examples](#)

*Nakamura et al 2018; Cavallo and Rigobon 2016; Alvarez et al 2018; Cavallo et al 2023*

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- **Stylized analytical model**
- **Quantitative model**

# Related literature

- Optimal monetary policy with sectors / relative prices, Calvo

*Aoki 2001, Woodford 2003, Benigno 2004, Wolman 2011, Rubbo 2023*

- Menu costs *assuming* inflation targeting, solve for optimal inflation target

*Wolman 2011, Nakov-Thomas 2014, Blanco 2021*

- Menu costs + trending productivities (no direct costs)

*Adam and Weber 2023*

- Optimal policy with menu costs w/out sectors

*Karadi, Nakov, Nuno, Pasten, and Thaler 2024*

- Non-normative menu cost literature

- \* Theoretical

*Golosov-Lucas 2007; Caballero-Engel 2007; Nakamura-Steinsson 2009;*

*Alvarez-Lippi-Paciello 2011; Midrigan 2011; Gertler-Leahy 2008; Auclert et al 2023*

- \* Empirical

*Nakamura et al 2018; Cavallo-Rigobon 2016; Alvarez et al 2018; Gautier-Le Bihan 2022*

# Roadmap

1. **Baseline model & optimal policy**
2. **Extensions**
3. **Comparison to Calvo model**
4. **Quantitative model**
5. **Conclusion and bigger picture**

# Model setup + household's problem

## General setup:

- Off-the shelf sectoral model with  $S$  sectors
- Each sector is a continuum of firms, bundled with CES technology
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$$\begin{aligned} \max_{C, N, M} \quad & \ln(C) - N + \ln\left(\frac{M}{P}\right) \\ \text{s.t.} \quad & PC + M = WN + D + M_{-1} - T \\ & C = \prod_{i=1}^S c_i^{1/S} \end{aligned}$$

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## Optimality conditions:

$$\begin{aligned} c_i &= \frac{1}{S} \frac{PC}{p_i} \\ PC &= M \\ W &= M \end{aligned}$$



## Intermediate firms: price setting with menu costs

**Technology:** firm  $j \in [0, 1]$  in sector  $i$

$$y_i(j) = A_i \cdot n_i(j)$$

**Demand:**  $y_i(j) = y_i \left( \frac{p_i(j)}{p_i} \right)^{-\eta}$

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$$\left( p_i y_i - \frac{W}{A_i} y_i (1 - \tau) \right) - W \psi \chi_i$$

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**Menu cost:**  $\psi$  extra units of labor

- $\chi_i$ : indicator for price change

$\implies$  **Direct cost of menu costs:** excess disutility of labor

$$N = \sum_i n_i + \psi \sum_i \chi_i$$

- Other specifications do not affect result ▶ more



## Menu costs induce an inaction region

Objective function of sector  $i$  firm:  $\left( p_i y_i - \frac{W}{A_i} y_i (1 - \tau) \right) - W \psi \chi_i$

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**Inaction region:** don't adjust iff  $p_i^* = \frac{W}{A_i}$  close to  $p_i^{\text{old}}$

# Optimal policy after a productivity shock

► Formal planner's problem

- Start at steady state: all sectors have  $A_i^{ss} = 1 \quad \forall i$ , so  $p_i^{ss} = W^{ss} \equiv 1$

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**Proposition 1:** there exists a threshold level of productivity  $\bar{A}$  s.t.:

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- If shock is small,  $A_1 < \bar{A}$ , then optimal policy ensures no sector adjusts:

$$p_i = p_i^{ss} \quad \forall i$$

# Large-enough shocks

$$\text{Recall: } p_i^* = MC_i = \frac{W}{A_i}$$





# Large-enough shocks

- Sector 1 productivity  $A_1 \uparrow$   
 $\implies$  relative price  $p_1/p_k$  should fall

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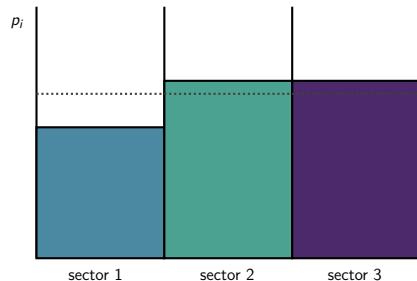
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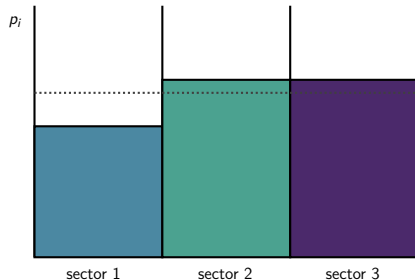
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**Inflation targeting**

$$W^f - S\psi$$

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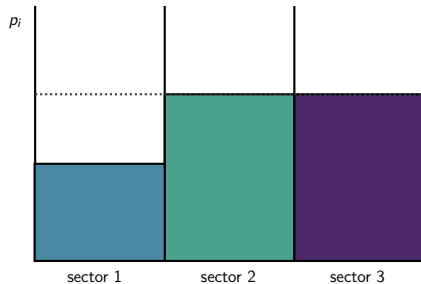
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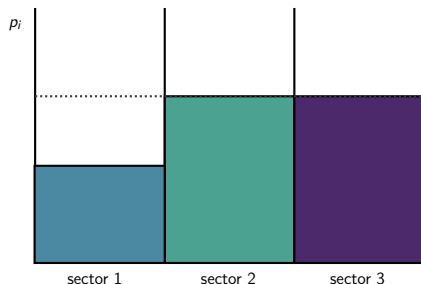
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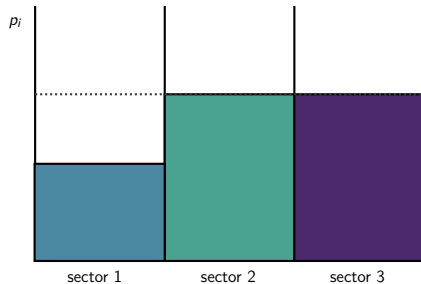
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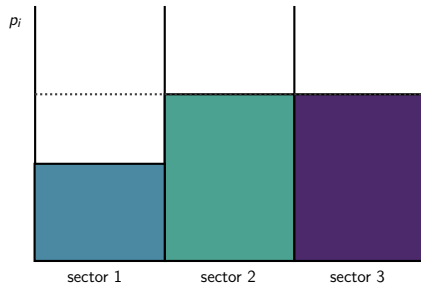
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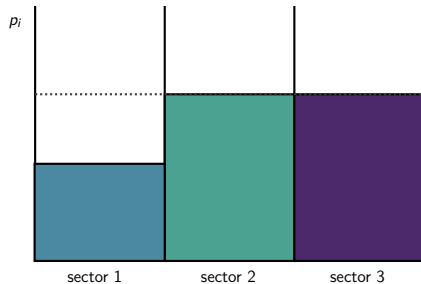
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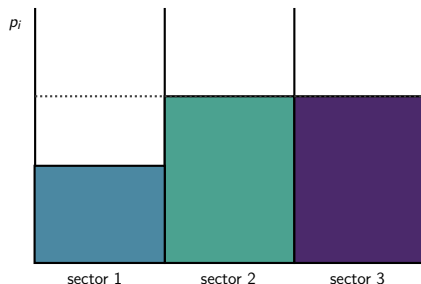
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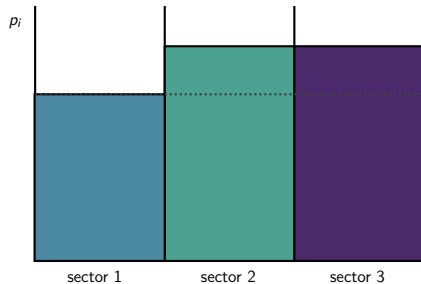
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**Only sectors  $k$  adjusts**  
 $W^f - (S-1)\psi$

## Small shocks: state dependent optimal policy

► math

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**Lemma 2:**  $\exists \bar{A}$  such that

$$\mathbb{W}_{\text{only 1 adjusts}} > \mathbb{W}_{\text{none adjust}}$$

iff  $A_1 > \bar{A}$ . Furthermore,  $\bar{A}$  is increasing in  $\psi$ .

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## 1. Calibrated models.

- (1) Measure *frequency of price adjustment*
- (2) Build structural model
- (3)  $\implies$  *calibrate* menu costs to fit

Nakamura and Steinsson (2010):

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Blanco et al (2022):

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**2. Direct measurement.** For *physical* adjustment costs,

Levy et al (1997, QJE): 5 grocery chains

- 0.7% revenue

Dutta et al (1999, JMCB): drugstores

- 0.6% revenue

Zbaracki et al (2003, Restat): mfg

- 1.2% revenue

# Extensions

- Generalized functional forms
- Multiple shocks / production networks
- Heterogenous costs
- Sticky wages

► more

# Generalization: stabilize nominal MC of unshocked firms

Generalized model:

- Any (HOD1) aggregator:  
 $C = F(c_1, \dots, c_S)$
- DRS production technology:  
 $y_i(j) = A_i n_i(j)^\alpha, \alpha \in (0, 1]$
- Any preferences quasilinear in labor:  
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**Nominal MC:**

$$MC_i(j) = \left[ \alpha \frac{W}{A_i^\alpha} (y_i p_i^\eta)^{\alpha-1} \right]^\theta$$
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**Extended Proposition 1:**

Stabilize **nominal marginal costs of unshocked firms**  $\implies Y \uparrow, P \downarrow$

# Production networks

## Baseline model:

- Production technology:

$$y_i = A_i n_i$$

## Roundabout production network:

- Production technology:

$$y_i = A_i n_i^\beta l_i^{1-\beta}$$
$$l_i = \prod_{k=1}^S l_i(k)^{1/S}$$

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 $\equiv W^\beta P^{1-\beta}$

# Why not inflation targeting?

- *Why* then is optimal policy in multisector Calvo **inflation targeting**? *Aoki, Rubbo*

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*Aoki, Rubbo*

- **Menu costs are *nonconvex*:**

$$\psi \cdot \mathbb{I}\{p_i \neq p_i^{ss}\}$$

# Why not inflation targeting?

- *Why* then is optimal policy in multisector Calvo inflation targeting?

*Aoki, Rubbo*

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- **With *convex* menu costs:**

e.g. Rotemberg,  $\psi \cdot (p_i - p_i^{ss})^2$

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$$\Delta \equiv \sum_{i=1}^S \int_0^1 \left[ \frac{p_i(j)}{p_i} \right]^{-\eta} dj$$

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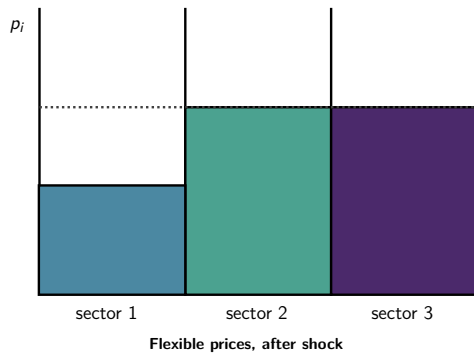
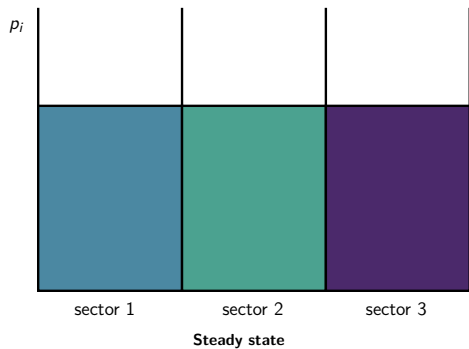
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**Convex costs  $\implies$  smooth price changes across sectors**

## Calvo diagram: shocking sector-1 productivity





# Calvo diagram: shocking sector-1 productivity

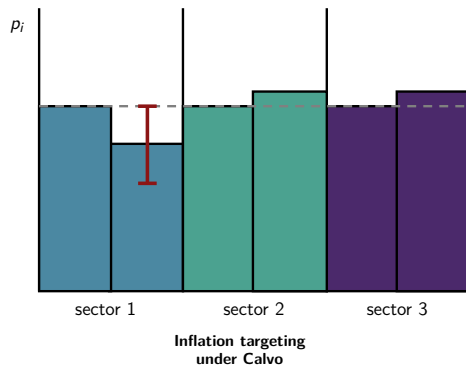
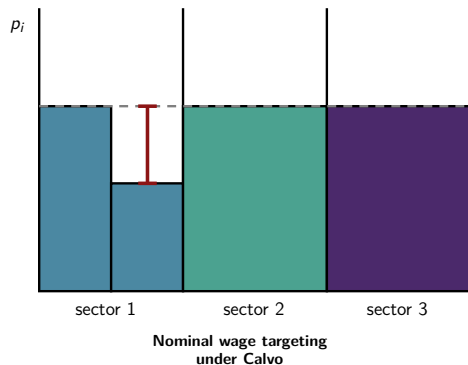
► math



*Lots of price dispersion: only one sector*

# Calvo diagram: shocking sector-1 productivity

► math

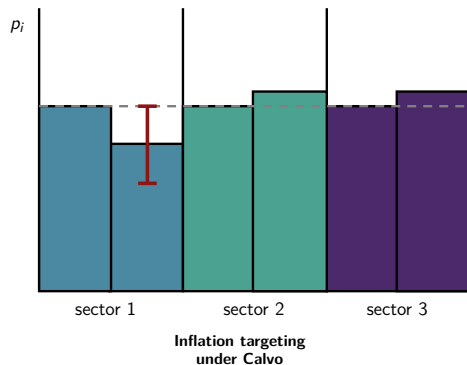
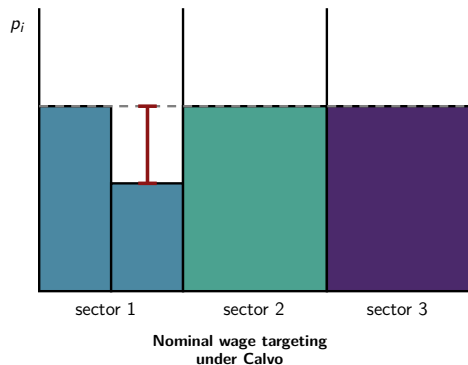


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# Calvo diagram: shocking sector-1 productivity

► math



*Lots of price dispersion: only one sector*

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**Convex costs  $\implies$  smooth price changes across sectors**

## Quantitative model: setup

Does nominal wage target dominate inflation target in **quantitative model**?

**Household:** **dynamic** with more **general functional forms**

$$\begin{aligned} & \max_{\{C_t, N_t, B_t, M_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\gamma}}{1-\gamma} - \omega \frac{N_t^{1+\varphi}}{1+\varphi} + \ln \left( \frac{M_t}{P_t} \right) \right] \\ \text{s.t.} \quad & P_t C_t + B_t + M_t \leq R_t B_{t-1} + W_t N_t + M_{t-1} + D_t - T_t \end{aligned}$$

### Firms

- \* final and sectoral good producers: same as in static model

## Quantitative model: intermediate firms

**Intermediate firms:** **idiosyncratic** shocks, **Calvo+** price setting, and **DRS**

$$\begin{aligned} \max_{p_{it}(j), \chi_{it}(j)} \quad & \sum_{t=0}^{\infty} \mathbb{E} \left[ \frac{1}{R^t P_t} \{ p_{it}(j) y_{it}(j) - W_t n_{it}(j) (1 - \tau) - \chi_{it}(j) \psi W_t \} \right] \\ \text{s.t.} \quad & y_{it}(j) = A_{it} a_{it}(j) n_{it}(j)^{\alpha} \\ & \psi_{it}(j) = \begin{cases} \psi & \text{w/ prob. } 1 - \nu \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

productivity distribution is mixture between AR(1) and uniform (**fat tail**)

$$\log(a_{it}(j)) = \begin{cases} \rho_{\text{idio}} \log(a_{it-1}(j)) + \varepsilon_{it}^{\text{idio}}(j) & \text{with prob. } 1 - \varsigma \\ \mathcal{U}[-\log(\underline{a}), \log(\bar{a})] & \text{with prob. } \varsigma \end{cases}$$

# Calibration

Two sets of parameters to calibrate:

(1) standard or drawn from literature and

	Parameter (monthly frequency)	Value	Target
$\beta$	Discount factor	0.99835	2% annual interest rate
$\omega$	Disutility of labor	1	standard
$\varphi$	Inverse Frisch elasticity	0	Golosov and Lucas (2007)
$\gamma$	Inverse EIS	2	standard
$S$	Number of sectors	6	Nakamura and Steinsson (2010)
$\eta$	Elasticity of subst. between sectors	5	standard value
$\alpha$	Returns to scale	0.6	standard value
$\tau$	Labor subsidy	0.2	$1/\eta$

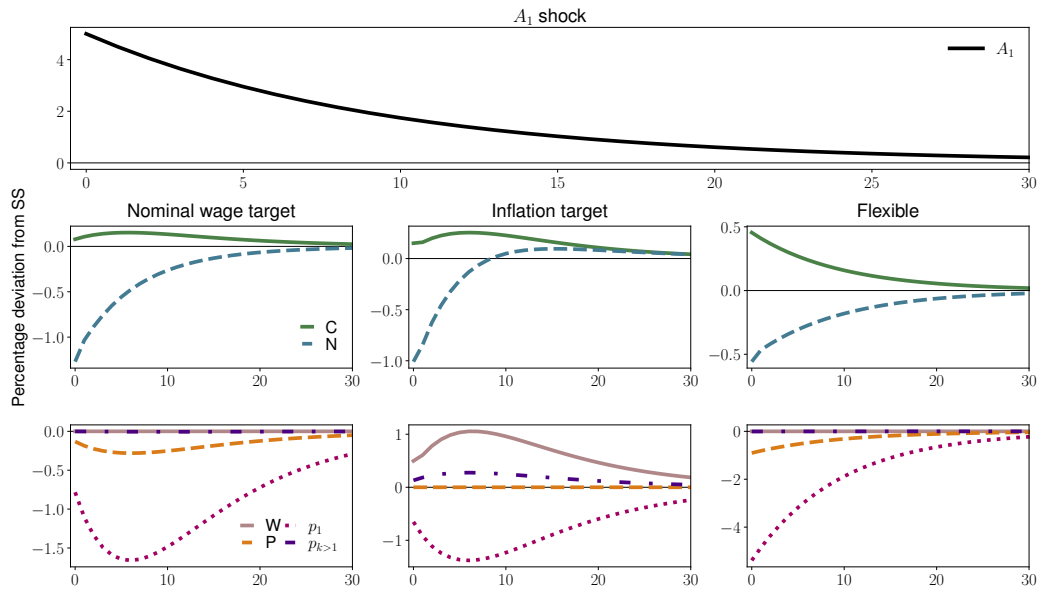
# Calibration

Two sets of parameters to calibrate:

(1) standard or drawn from literature and (2) calibrated by **SMM** targeting

	Parameter (monthly frequency)	Value	Target
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$\alpha$	Returns to scale	0.6	standard value
$\tau$	Labor subsidy	0.2	$1/\eta$
$\sigma_{\text{idio}}$	Standard deviation of idio. shocks	0.044	menu cost expenditure / revenue 1%(1.1%)
$\rho_{\text{idio}}$	Persistence of idio. shocks	0.995	share of price changers 8.7% (8.3%)
$\psi$	Menu cost	0.1	median absolute price change 8.5% (8.7%)
$\nu$	Calvo parameter	0.075	Q1 absolute price change 4.5% (4.2%)
$\zeta$	Fat tail parameter	0.0016	Q3 absolute price change 20.4% (14.8%)
			kurtosis of price changes 3.609 (2.755)

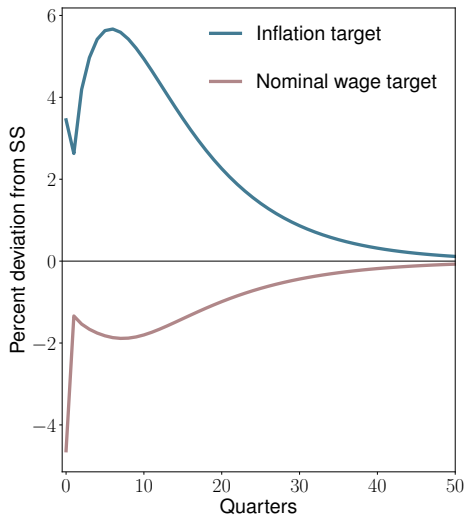
# Exercise: perfect foresight sectoral shock



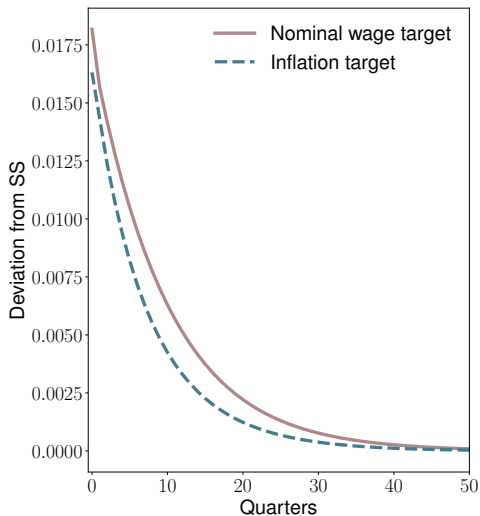


# Policy comparison: menu costs and welfare

## Real menu cost expenditure

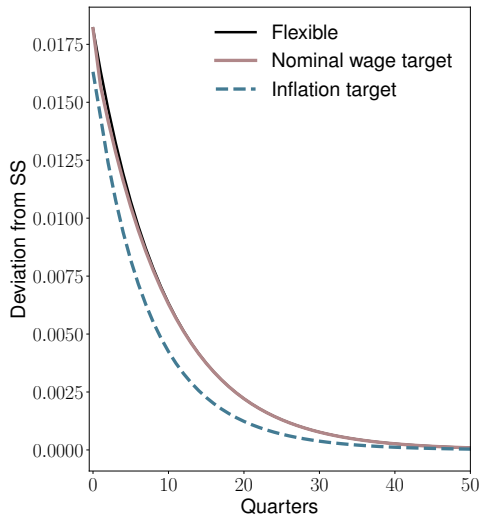


## Welfare



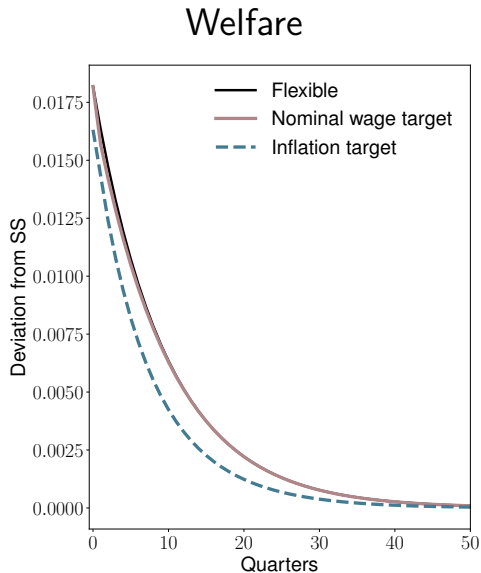
# Policy comparison: consumption equivalence

## Welfare



# Policy comparison: consumption equivalence

- Consider **welfare** under  $W$  targeting



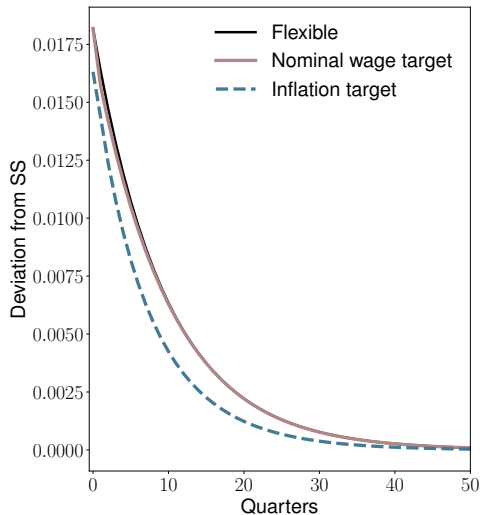
# Policy comparison: consumption equivalence

- Consider welfare under  $W$  targeting
- Extra  $C$  needed to match welfare under flexible prices

$$\begin{aligned} & \sum_t \beta^t U((1 + \lambda) C_t, N_t) \\ &= \sum_t \beta^t U(C_t^{\text{flex}}, N_t^{\text{flex}}) \end{aligned}$$

- $\lambda^W = 0.002\%$        $\lambda^P = 0.025\%$

## Welfare



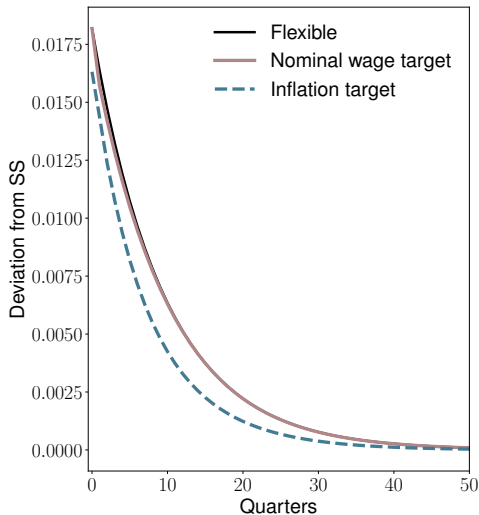
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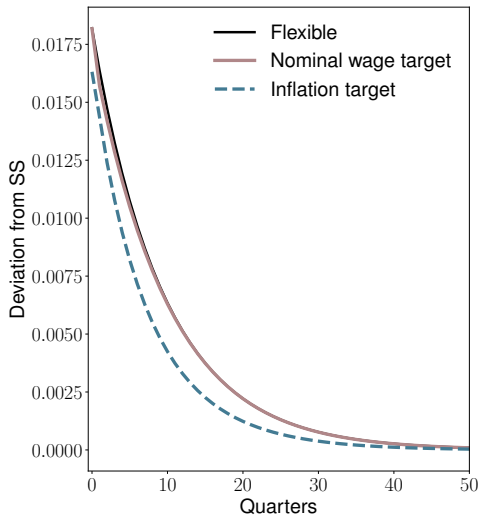
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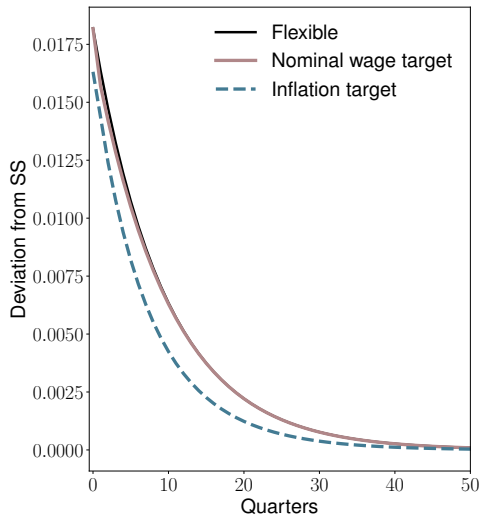
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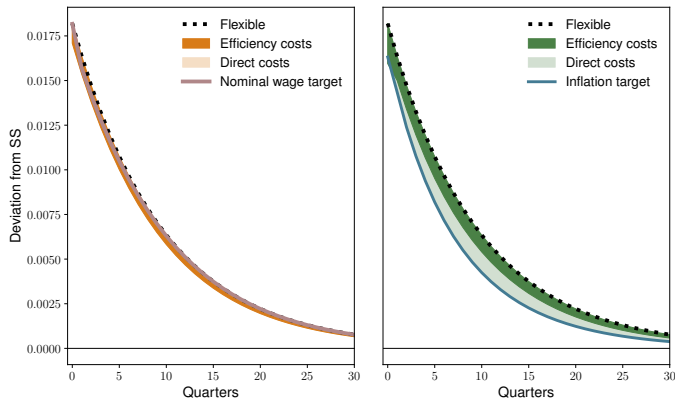
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## Welfare



# Decomposing welfare

## Welfare response to $A_1$ shock



1. **Direct costs:**  $\psi\chi_t$ , disutility of labor from menu costs
2. **Efficiency costs:** welfare loss from incorrect relative prices
3. Improvement from both channels



# Welfare over the business cycle

- Shock sector productivities according to

$$\log(A_t) = \rho_A \log(A_{t-1}) + \varepsilon_A$$

- $\varepsilon_A = 0.95$      $\sigma_A \sim \mathcal{N}(0, 0.0161)$

*Garin, Pries, and Sims (2018)*

- Consumption equivalence results:
  - \*  $\lambda^P = 0.01\%$  versus  $\lambda^W = 0.003\%$

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⇒ Nominal wage targeting dominates inflation targeting in quantitative model

# Conclusion

**Inflation should be countercyclical** after sectoral shocks

Rationale:

- Inflation targeting **forces firms to adjust unnecessarily**, which is costly
- Nominal wage targeting does not and still achieves “correct” relative prices

**This aligns with the implications of other recent work:**

- Calvo sticky wages
- Incomplete markets/financial frictions: Sheedy (2014), Werning (2014)
- Information frictions: Angeletos and La'O (2020)
- Sticky prices [**new**]: **Caratelli and Halperin (2024)**

Thank you!

# Formally: Social planner's problem

► back

$$\max_{X \in \{A, B, C, D\}} \mathbb{U}^X$$

$$\mathbb{U}^A = \left\{ \begin{array}{ll} \max_M & \ln[M] - M[S - 1 + 1/\gamma] \\ \text{s.t.} & \min(\gamma\lambda_1, \lambda_2) \leq M \leq \max(\gamma\lambda_1, \lambda_2) \end{array} \right\}$$

$$\mathbb{U}^B = \left\{ \ln \left[ \frac{1}{S} \gamma^{1/S} \right] - 1 - \psi \right\}$$

$$\mathbb{U}^C = \left\{ \begin{array}{ll} \max_M & \ln \left[ \left( \frac{\gamma}{S} \right)^{\frac{1}{S}} \cdot M^{\frac{S-1}{S}} \right] - \left[ (S-1)M + \frac{1}{S} \right] - \frac{1}{S}\psi \\ \text{s.t.} & \lambda_1 < M < \min(\gamma\lambda_1, \lambda_2) \end{array} \right\}$$

$$\mathbb{U}^D = \left\{ \begin{array}{ll} \max_M & \ln \left[ S^{\frac{1-S}{S}} M^{\frac{1}{S}} \right] - \left[ \frac{S-1}{S} + \frac{M}{\gamma} \right] - \frac{S-1}{S}\psi \\ \text{s.t.} & \max(\gamma\lambda_1, \lambda_2) < M < \gamma\lambda_2 \end{array} \right\}$$

$$\text{where } \lambda_1 = \frac{1}{S} (1 - \sqrt{\psi}), \quad \lambda_2 = \frac{1}{S} (1 + \sqrt{\psi})$$

# Adjustment externalities

► back

Example: Social planner's *constrained* problem for “neither adjust”

$$\max_M U(C(M), N(M)) \quad (1)$$

$$\text{s.t. } D_1^{\text{adjust}} < D_1^{\text{no adjust}} \quad (2)$$

$$D_k^{\text{adjust}} < D_k^{\text{no adjust}} \quad (3)$$

$$\implies M_{\text{unconstrained}}^*$$

Social planner's *unconstrained* problem: maximize (1), without constraints

$$\implies M_{\text{constrained}}^*$$

**Adjustment externality:**  $M_{\text{unconstrained}}^* \neq M_{\text{constrained}}^*$

# Alternative menu cost formulations

► back

**Labor costs:** Welfare mechanism is *higher labor*

$$\begin{aligned} & \text{profits}_i - W\psi \cdot \chi_i \\ \implies N &= \sum n_i + \psi \sum \chi_i \end{aligned}$$

**Real resource cost:** Welfare mechanism is *lower consumption*

$$\begin{aligned} & \text{profits}_i \cdot (1 - \psi \cdot \chi_i) \\ \implies C &= Y \left( 1 - \psi \sum_i \chi_i \right) \end{aligned}$$

**Direct utility cost:** Welfare mechanism is *direct*

$$\text{utility} - \psi \cdot \sum \chi_i$$

# Heterogeneity: “least-cost avoider principle”

► back

**Proposition 5:** Suppose sector  $i$  has mass  $S_i$  and menu cost  $\psi_i$ . Suppose further

$$S_1\psi_1 < \sum_{k>1} S_k\psi_k.$$

Then optimal policy is exactly as in proposition 1, modulo changes in  $\bar{A}$ .

- *Proof:* Follows exactly as in proof of proposition 1.



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► back

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- *Proof:* Follows exactly as in proof of proposition 1.

**Interpretation 1:** monetary “least-cost avoider principle”

**Interpretation 2:** “stabilizing the stickiest price”

# Multiple shocks: general case

► back

**Proposition 7:** Consider an arbitrary set of productivity shocks to the baseline model,  $\{A_1, \dots, A_S\}$ .

- Conditional on sectors  $\Omega \subseteq \{1, \dots, S\}$  adjusting, optimal policy is given by setting  $M = M_\Omega^* \equiv \frac{S-\omega}{\sum_{i \notin \Omega} \frac{1}{A_i}}$ , where  $\omega \equiv |\Omega|$ .
- The optimal set of sectors that should adjust,  $\Omega^*$ , is given by comparing welfare under the various possibilities for  $\Omega$ , using  $W_\Omega^*$  defined in the paper.
- Nominal wage targeting is exactly optimal if the set of sectors which should not adjust are unshocked:  $A_i = 1 \quad \forall i \notin \Omega^*$ .

# Price adjustment frequency tracks inflation

► back

**Calvo/TDP models:** frequency of price adjustment is exogenous to inflation

**Menu cost models:** frequency of price adjustment  $\uparrow$  if inflation  $\uparrow$

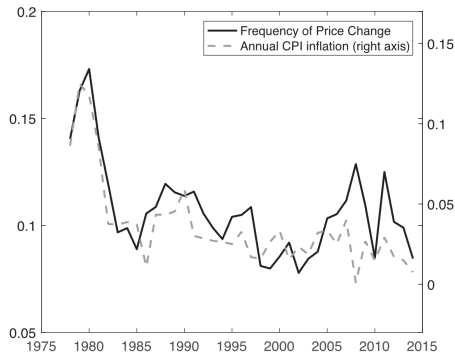


FIGURE XIV

Frequency of Price Change in U.S. Data

**Figure:** Nakamura et al (2018)

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► back

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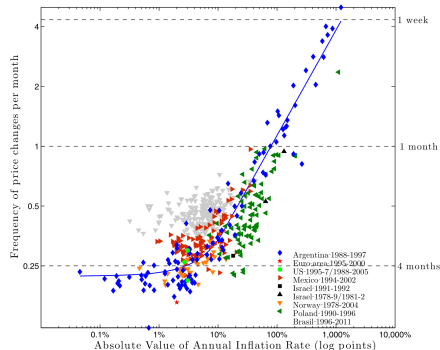


FIGURE VI

The Frequency of Price Changes ( $\lambda$ ) and Expected Inflation: International Evidence

Figure: Alvarez et al (2018)

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► back

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(a) Frequency of Adjustment

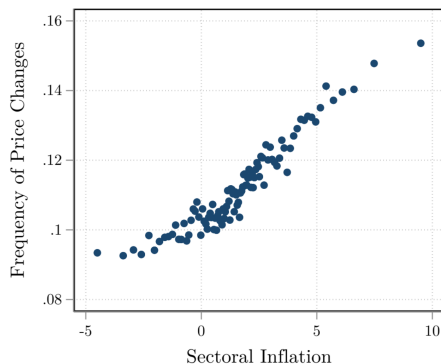


Figure: Blanco et al (2022)

# Price adjustment frequency tracks inflation

► back

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Figure 1: Frequency of price changes

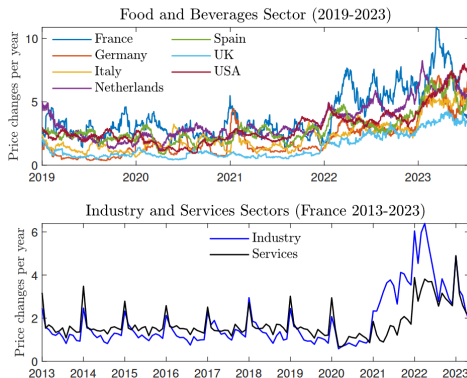
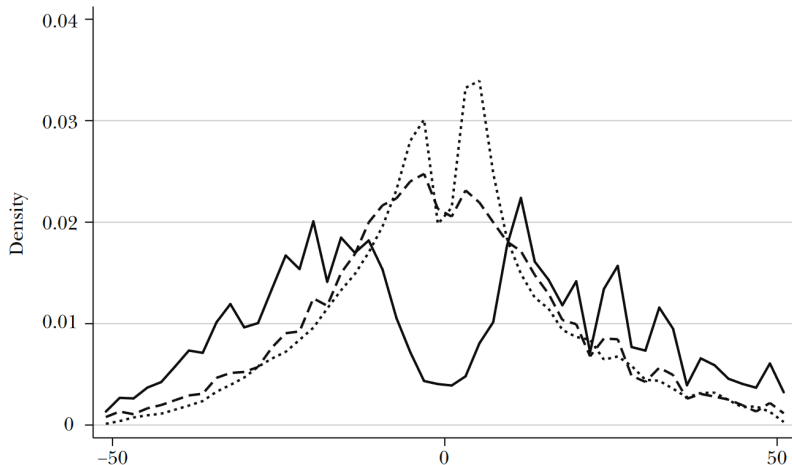


Figure: Cavallo et al (2023)

# Evidence of inaction regions

*Figure 8*

**The Distribution of the Size of Price Changes in the United States**



# The welfare loss of inflation targeting

► back

**“Inflation targeting”:**  $P = P^{ss}$  (while having correct relative prices)

**Proposition 2:** Suppose  $A_1 > \bar{A}$ .

Then:

- Inflation targeting requires all sectors adjust their prices
- Welfare loss from inflation targeting  $\propto$  size of menu costs

$$\mathbb{W}^* - \mathbb{W}^{IT} = (S - 1)\psi$$



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What are menu costs?

- **Physical adjustment costs.**  
Baseline interpretation.

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What are menu costs?

- **Physical adjustment costs.** Baseline interpretation.
- **Information costs.** Fixed costs of information acquisition / processing.
  - \* Results unchanged
- **Behavioral costs.** Consumer *distaste* for price changes.
  - \* Results unchanged