Summer Intern Lectures

Time Series Analysis Team

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1 Endogenous Grid Method

These notes are an adaptation of Josep Pijoan-Mas' notes with a few steps explained more in detail as well as the addition of some Julia code.

1.1 Simple consumption/savings model

Consider the dynamic programming problem

$$\begin{gathered} v(a,y) = \max_{a',n} \left\{ u(c,n) + \beta E_y \left[v(a',y') \right] \right\} \\ \text{s.t. } a' + c \leq aR + nyw, \\ a' \geq -b,\ c \geq 0. \end{gathered}$$

where the income process is a Markov chain, i.e. y' only depends on y and some stochastic factor. Substituting the (binding) budget constraint, gives us the FOCs are:

a')

$$-u_1(c,n) + \beta E_y \left[v_1(a',n') \right] = 0$$

n)

$$u_1(c,n)yw + u_2(c,n) = 0$$

and the Envelope condition is:

a)

$$v_1(a,y) = u_1(c,n)R.$$

Including the Envelope condition in the first FOC and assuming we can isolate n from the second FOC gives us:

$$u_1(c,n) = \beta R E_y \left[u_1(c',n') \right] \tag{1}$$

$$n = n(c,y) \tag{2}$$

1.2 Euler Equation Iteration

Here follow the steps for the Euler Equation iteration.

1. Guess a policy function for consumption $\boldsymbol{c = g_0^c(a, y)}$. This will be updated and will determine our convergence.

2. Substitute this rule on the right-hand side of (1) and get:

$$u_1(c) = \beta R E_y \left[u_1(g_0^c(a', y'), n') \right],$$

Assuming $u_1(\cdot)$ is invertible, invert the equation just derived in order to get consumption:

$$c = u_1^{-1} \left(\beta R E_y \left[u_1(g_0^c(a', y'), n') \right] \right) =: \tilde{g}_0^c(a', y)$$

Notice that $\tilde{g}_0^c(\cdot)$ depends on a' because, in the above expression those are known at at 0 and it depends on y because the expectation is taken over income and next period income only depends on current income.
This formulation tells us that consumption today is a function of assets tomorrow and income today. So, knowing y and a', we can pin down c.

3. If the asset grid is $A = \{a_1, \ldots a_m\}$ and the income process is discrete, i.e. $Y = \{y_1, \ldots, y_n\}$ and Markovian (as we are assuming) with transition matrix Γ, we can write

$$c = \tilde{g}_0^c(a_i, y_j) = u_1^{-1} \left(\beta R \sum_l \Gamma_{j,l} u_1(g_0^c(a_i, y_l), n') \right)$$

Notice further that A is the asset grid for tomorrow's assets.

Next, using the (binding) budget constraint we can deduce current assets given tomorrow's assets and current income, namely

$$\begin{aligned} a_i + c &= a_{i,j} R + n y_j w \\ \Rightarrow a_{i,j}^* &= \frac{a_i + c - n y_i w}{R} \end{aligned}$$

If hours worked were exogenous we would be done, however, we can, making some separability assumptions, say as before that $n = n(c, y_j) = n(g_c^0(a_i, y_j), y_j)$ and so

$$a_{i,j}^* = \frac{a_i + c - n y_i w}{R} \tag{3}$$

2 Julia example

Here is the example...